ttH and tH Searches at CMS

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Top Quark Physics at the Precision Frontier

Fermilab LPC

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Photo: Amy Scroggins, Fermilab Today

Why?

- What place does a discussion on ttH / tH searches have at a workshop on precision top quark physics?
  - Challenging topology to isolate – searches suffer from huge backgrounds and/or difficult to mitigate systematic uncertainties
  - Some systematics are not just difficult to mitigate but difficult to accurately understand their impact
  - CMS and ATLAS have only so far established the ttH process at the 3-4 std. dev. level, and it's been a long road to get to that point
    - *It's very likely there!*
  - Significant uncertainty remains in trying to pin down the ttH content in the data
    - *We can't yet precisely say how much though …*
- Looking forward however, the inclusion of top-Higgs analyses in a precision top quark physics program is *essential*

A brief interlude into Higgs physics...

Studies of Higgs Couplings

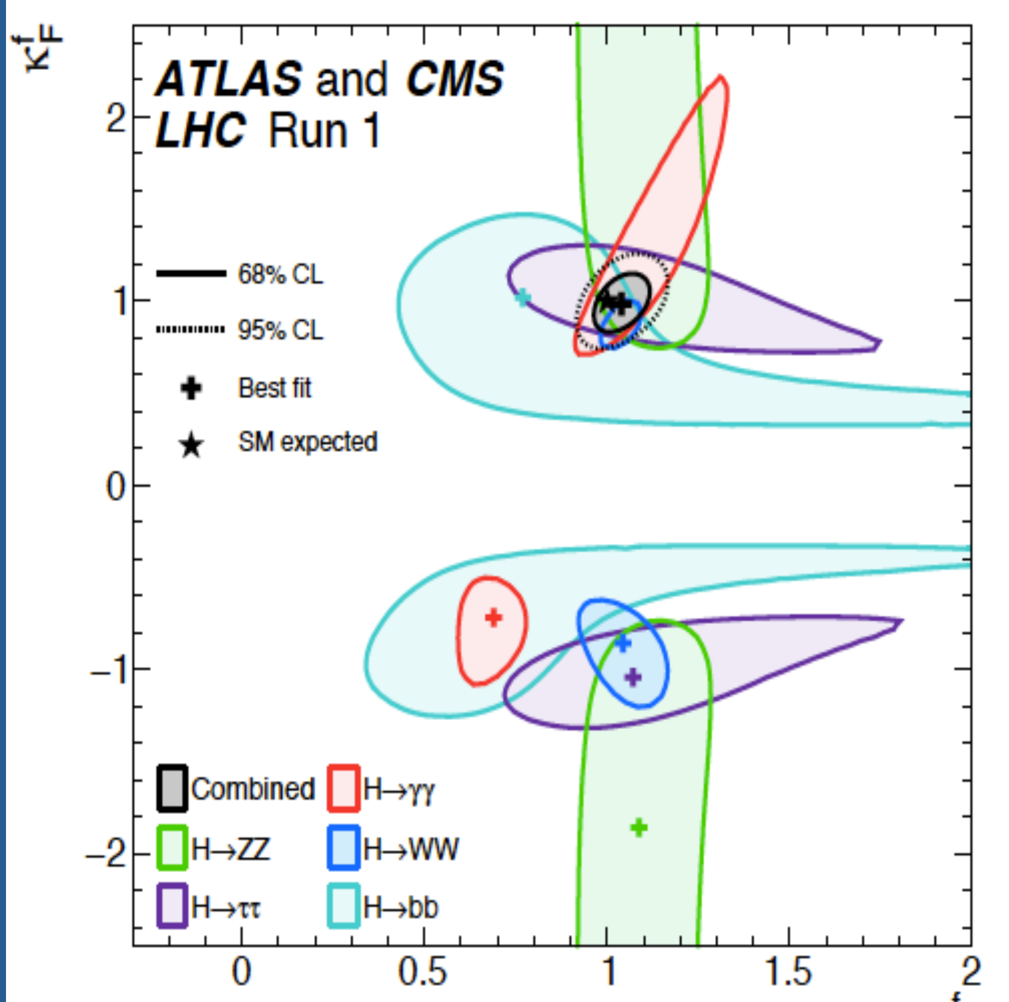
- Higgs couplings to particles of the SM a primary target of the characterization campaign
- Look for deviations from SM predictions for couplings to vector bosons and fermions

$$g_{HVV} = 2\frac{m_V^2}{v} \qquad g_{Hff} = \frac{m_f}{v}$$

$$\downarrow$$

$$g_{HVV} = \kappa_V\left(2\frac{m_V^2}{v}\right) \qquad g_{Hff} = \kappa_f\left(\frac{m_f}{v}\right)$$

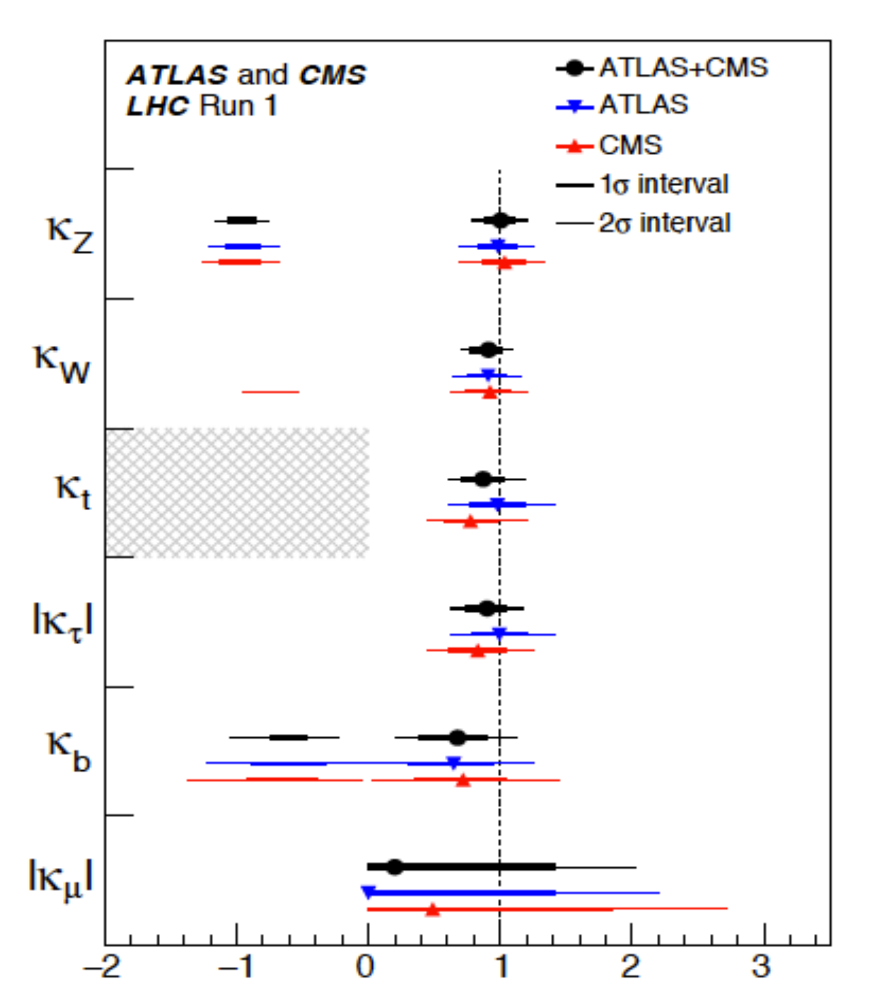
- Here assume same κ_V for each vector boson, and same κ_F for all fermions
- $(\kappa_V, \kappa_F) = (1.0, 1.0)$ is the SM



JHEP08(2016)045

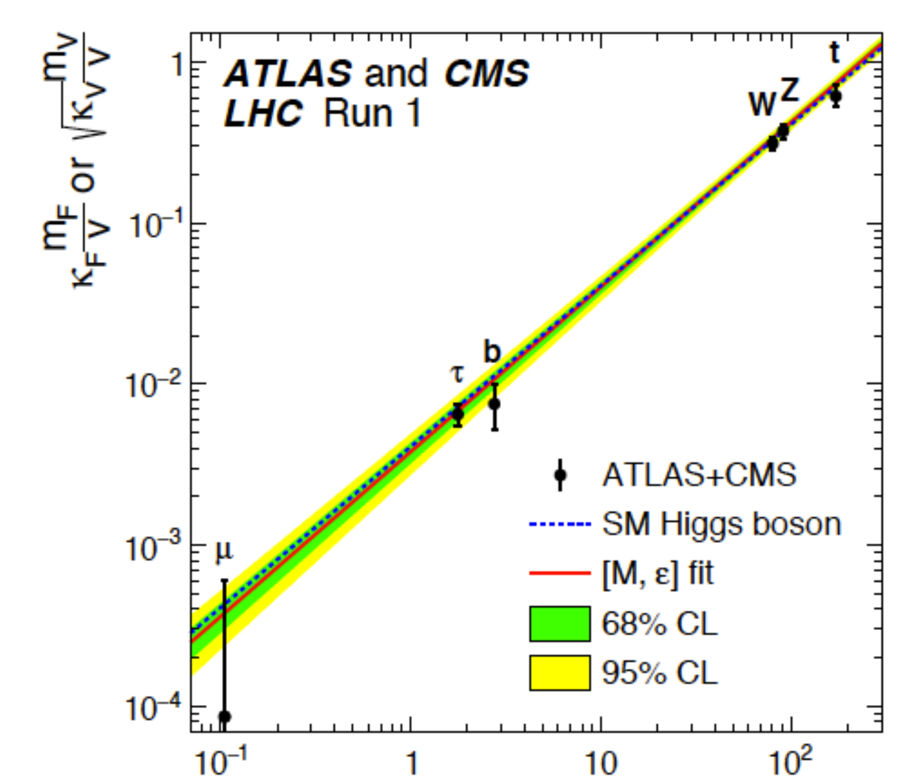
Fermionic and bosonic coupling modifiers look very SM-like

Studies of Higgs Couplings



JHEP08(2016)045

- Assume SM only particles participating in loop-mediated processes and BR(BSM)=0
- Examine prominent unique couplings that are accessible



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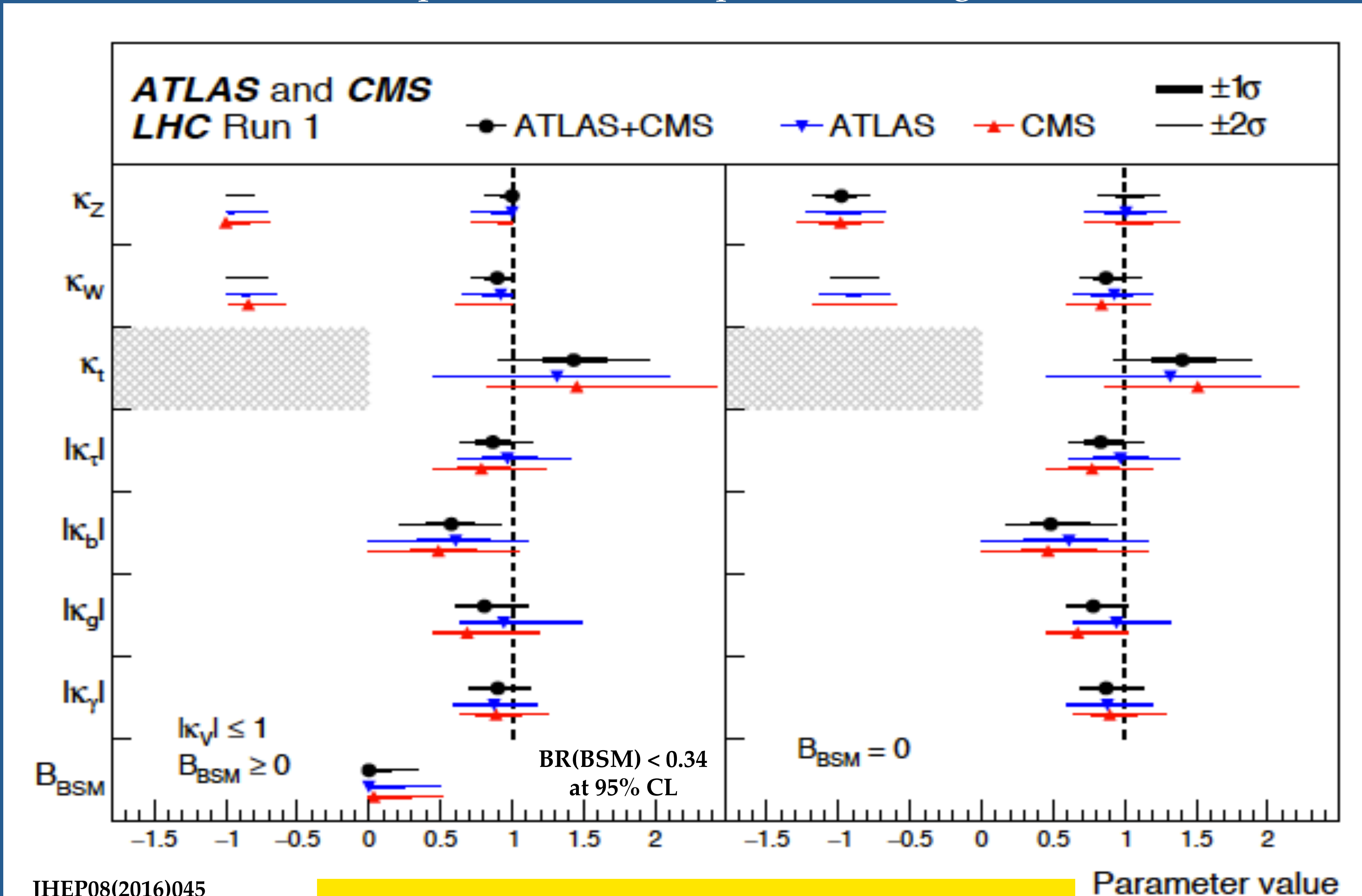
Particle-specific coupling modifiers look very SM-like assuming no influential BSM content

Studies of Higgs Couplings

But if one allows for the presence of BSM particles, things look a little less settled.



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Allowance for BSM contributions leaves a good deal of phase space remaining open.

Studies of Higgs Couplings

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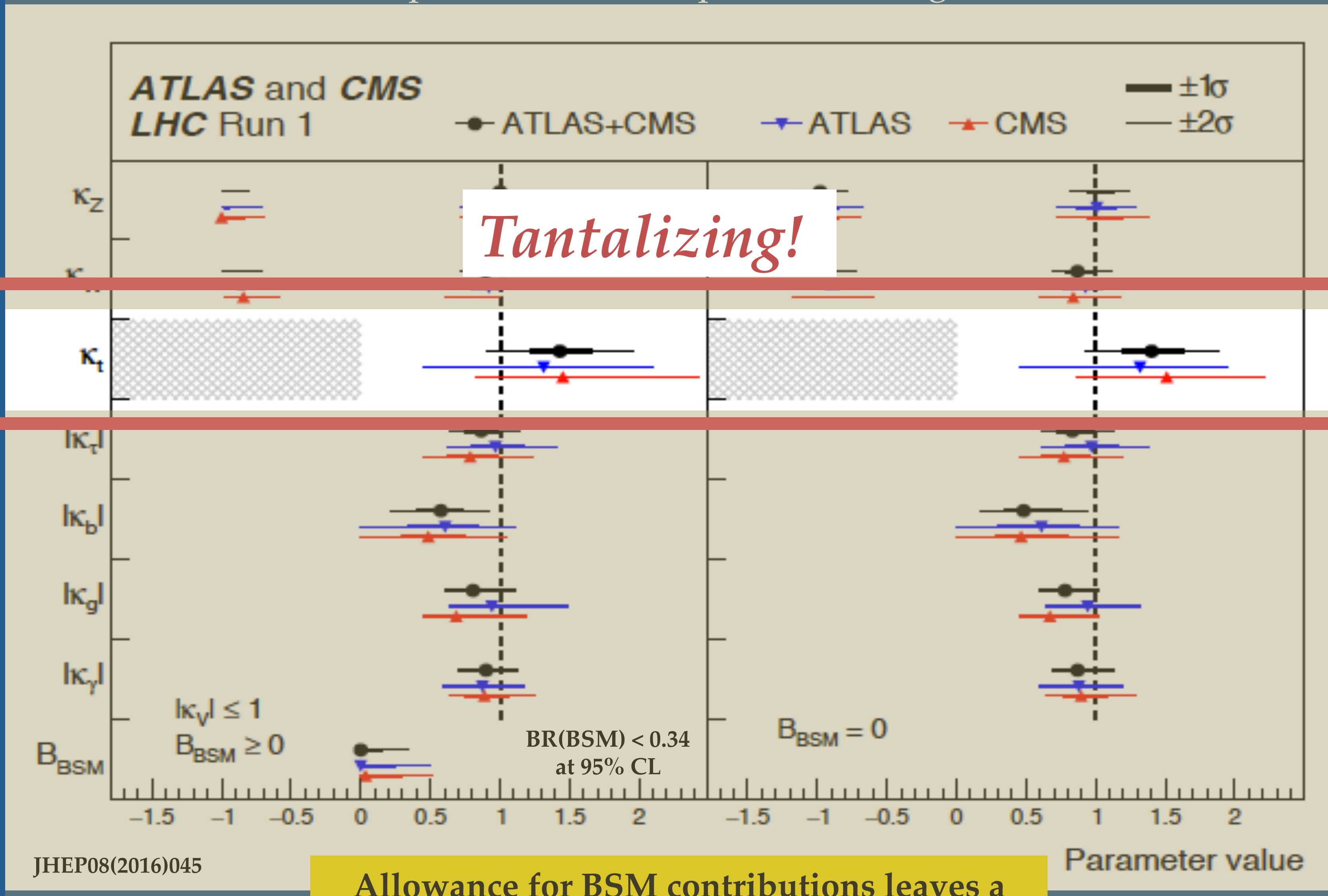
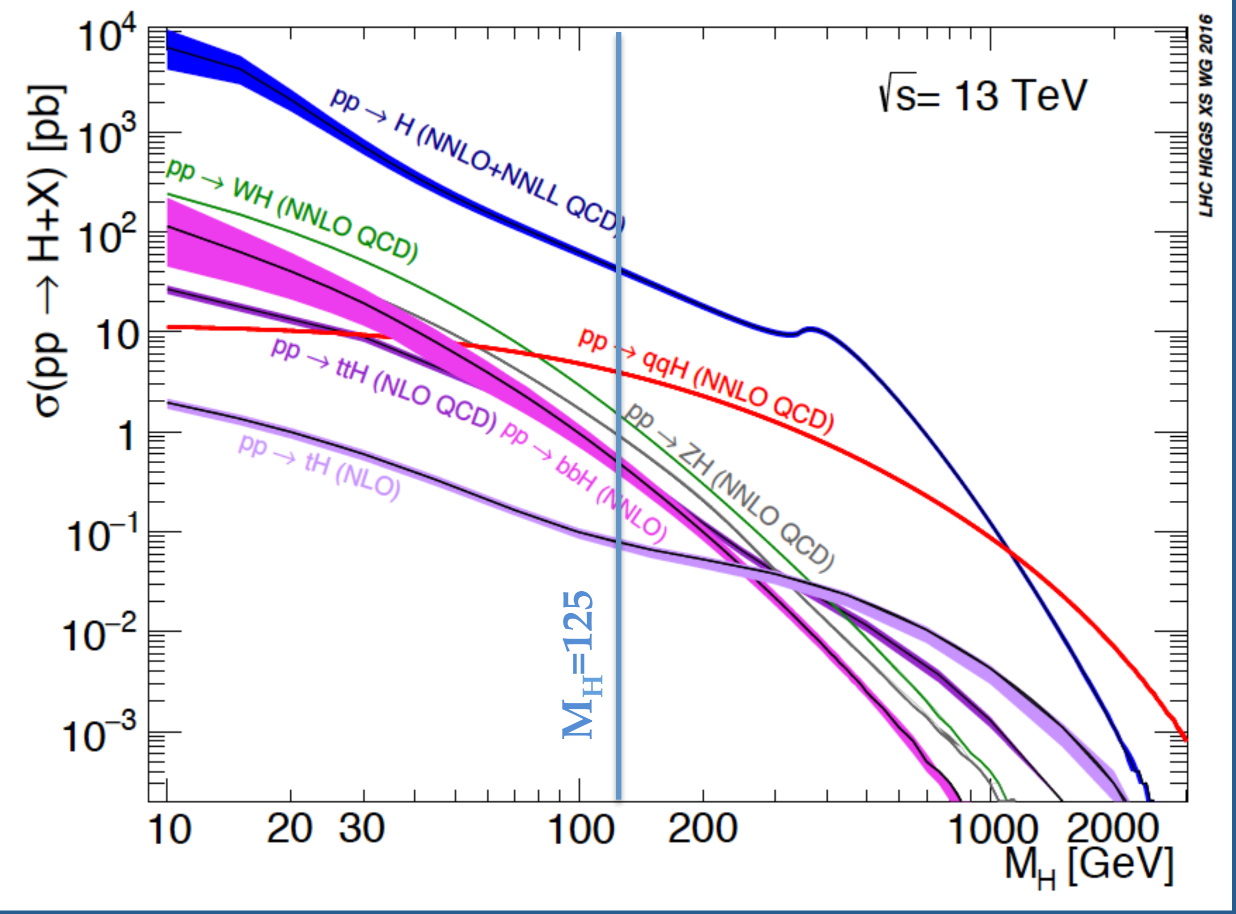


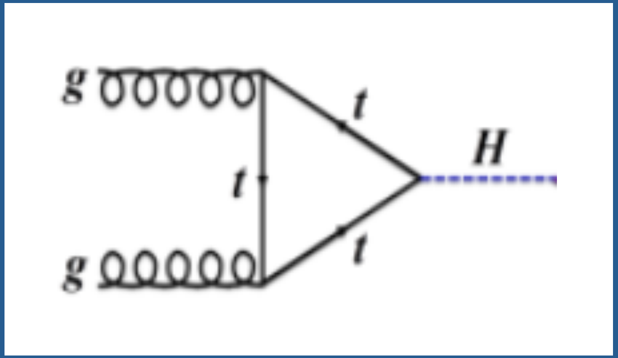
Tantalizing!

Allowance for BSM contributions leaves a good deal of phase space remaining open.

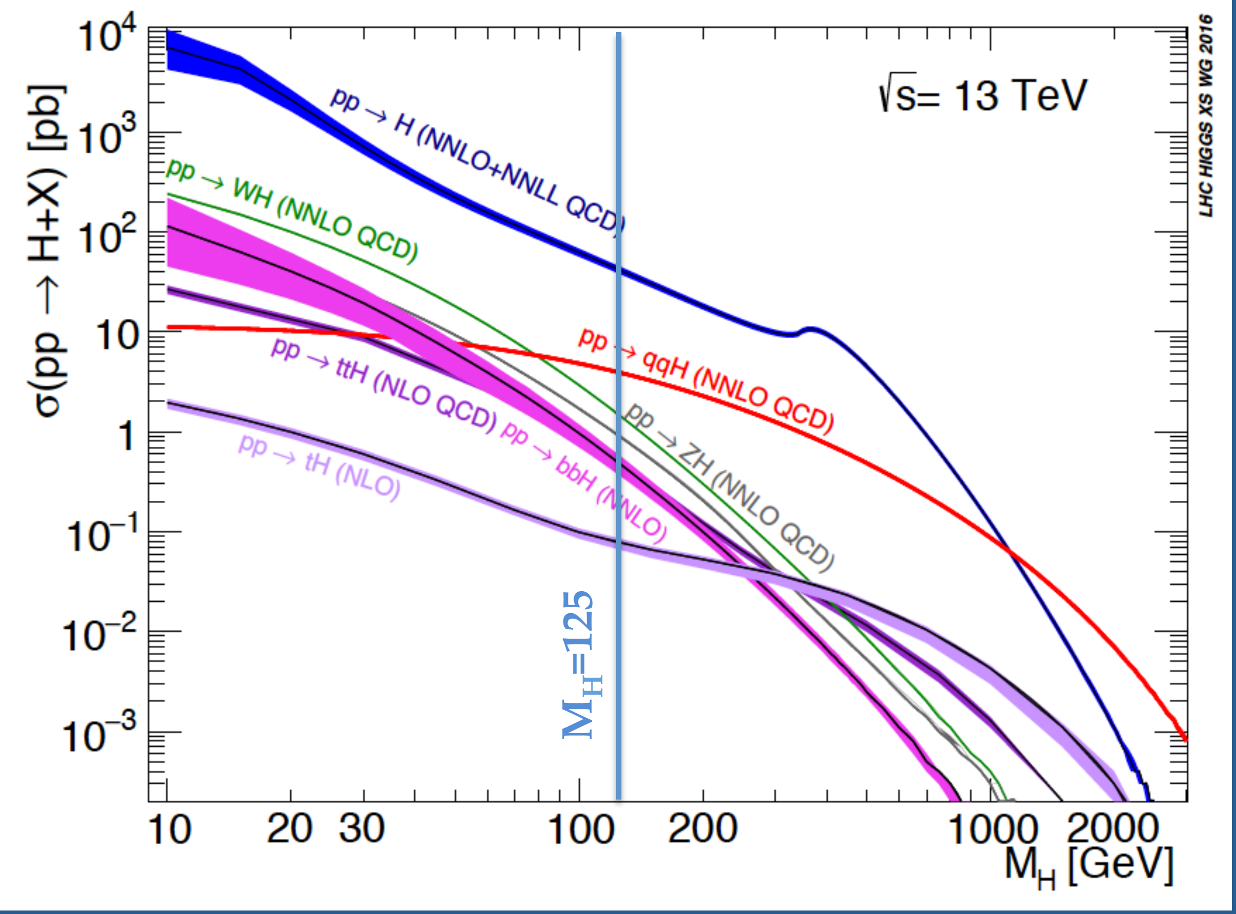
Loops and Higgs and Top



- Workhorse analyses already probe the top-Higgs coupling, though there are issues…
- Consider gluon fusion:



Loops and Higgs and Top



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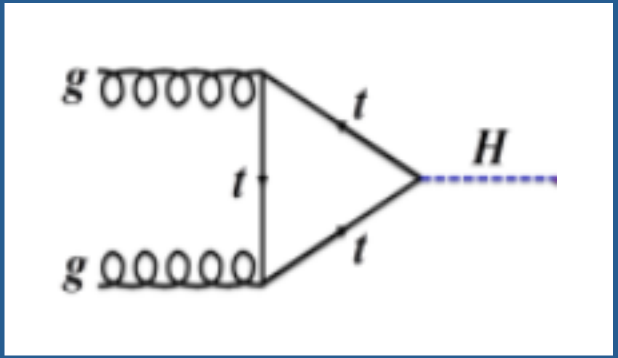
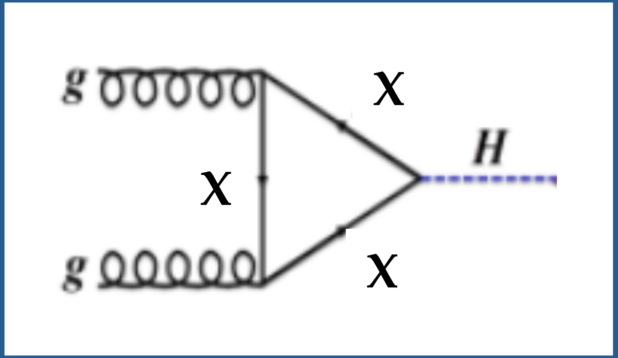
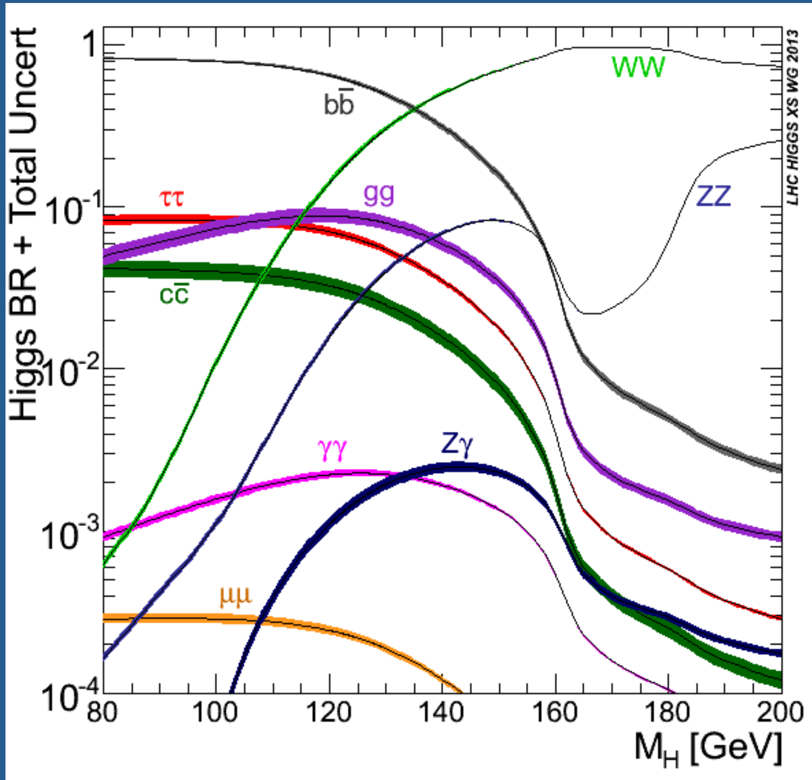


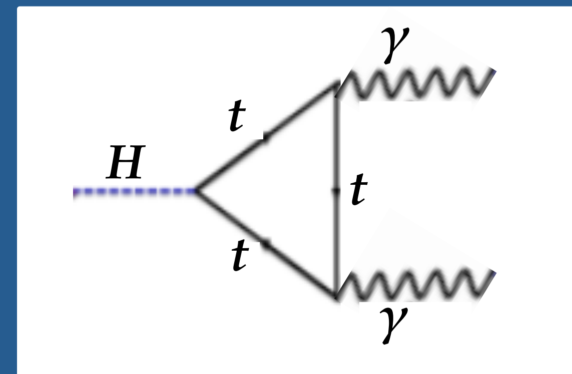
- But what about this?



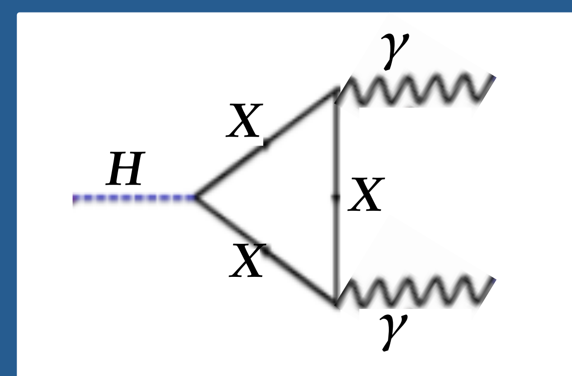
Loops and Higgs and Top



- Similar problems on the decay side:



…but what about…



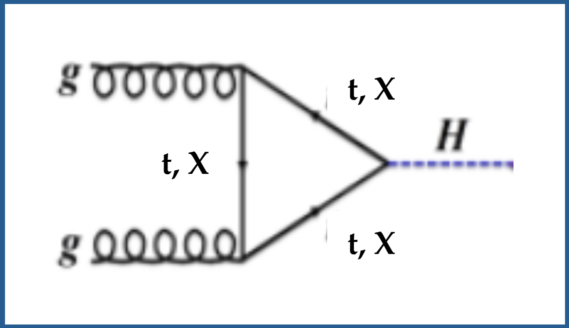
The results of the multi-channel fits indicate that the couplings that "are SM-like".

To really know what is going on, **we need a direct probe** of the top-Higgs coupling…

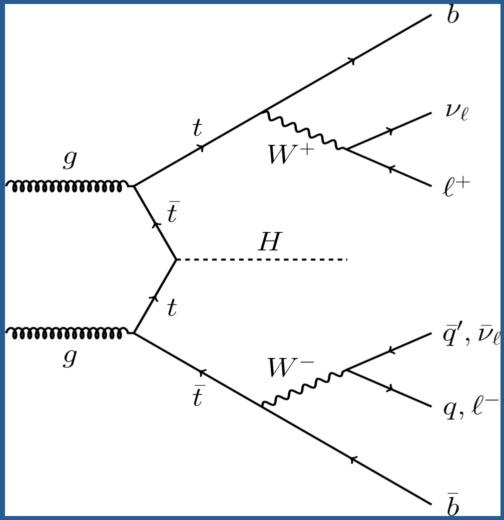
Motivation: The top-Higgs Coupling

- Within the SM, the Higgs coupling to the top quark, Y_t, is predicted to be **by far the largest** of all the fermionic couplings
  - m_{top} implies relatively large coupling, $Y_t \sim 1$
    - ~x30 larger than Y_b
    - ~x100 larger than Y_τ
  - Strongest coupling among all known SM particles
- **Fermionic couplings:**
  - LHC analyses have so far only been able to probe the Yukawa couplings Y_b and Y_τ directly
- Also Y_t will be the easiest (and perhaps only) up-type fermion coupling to probe
- Best channel: **ttH production**
  - tHX production also accessible – sensitivity to sign of Y_t

Loop-induced processes, for example:



ttH production:



<u>Imperative</u>:
Absolutely need to measure Y_t directly to know the true nature of the couplings of the new boson.

Summary of CMS $t\bar{t}H$ Analyses

<table>
<tr><th colspan="2"></th><th colspan="2">H → bb</th><th colspan="2">H → ττ</th><th rowspan="2">H → WW,ZZ</th><th rowspan="2">H → γγ</th></tr>
<tr><th colspan="2"></th><th></th><th></th><th>$\tau_{had}\,\tau_{had}$</th><th>$\tau_{had} + \tau_{lep}$</th></tr>
<tr><td colspan="2" rowspan="2">7 TeV</td><td rowspan="2">CMS-HIG-12-035
(NN)</td><td rowspan="3">JHEP 1305 (2013) 145
(NN)</td><td></td><td></td><td></td><td rowspan="2">various</td></tr>
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(ME)</td><td></td><td colspan="2" rowspan="2">CMS-HIG-13-020
(SS-2lep, 3lep, 4 lep)</td><td rowspan="2">various</td></tr>
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(BDT)</td></tr>
<tr><td colspan="6">JHEP 09(2014)087</td></tr>
<tr><td rowspan="2">13 TeV</td><td>2015</td><td colspan="2">CMS-HIG-16-004</td><td></td><td colspan="2">CMS-HIG-15-008</td><td></td></tr>
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</table>

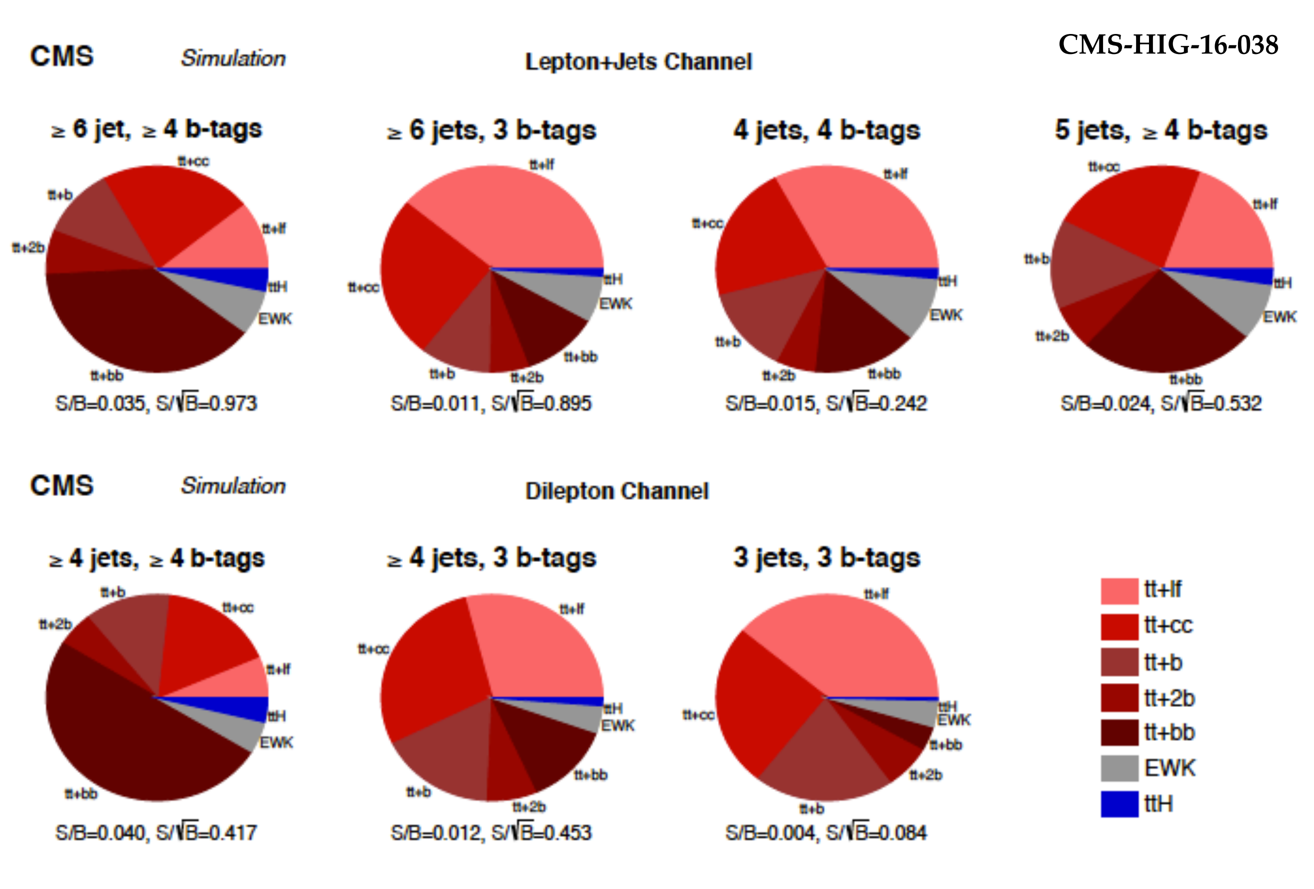
Summary of CMS $t\bar{t}H$ Analyses

	$H \to bb$		$H \to \tau\tau$	Multilepton		$H \to \gamma\gamma$
			$\tau_{had}\,\tau_{had}$	$\tau_{had} + \tau_{lep}$		
7 TeV	CMS-HIG-12-035 (NN)	**JHEP 1305 (2013) 145 (NN)**				various
8 TeV	**EPJC 75 (2015) 251 (ME)**			CMS-HIG-13-020 (SS-2lep, 3lep, 4 lep)		various
	CMS-HIG-13-019 (BDT)					
	JHEP 09(2014)087					
13 TeV 2015	CMS-HIG-16-004			CMS-HIG-15-008		
2016	**CMS-HIG-16-038**		**CMS-HIG-17-003**		**CMS-HIG-17-004**	**CMS-HIG-16-020**

ttH, H → bb

Overview: H→bb

- H→bb is a prime target of ttH analyses:
  - Largest Higgs BR for M_H=125
- CMS considers two topologies:
  - **Single-lepton channel:** one high p_T isolated e/μ, ≥4 jets, ≥2 b tags
  - **Dilepton channel:** two opposite-sign leptons, ≥2 jets, ≥2 b tags
- Split selected events into categories based on jet, b-tags

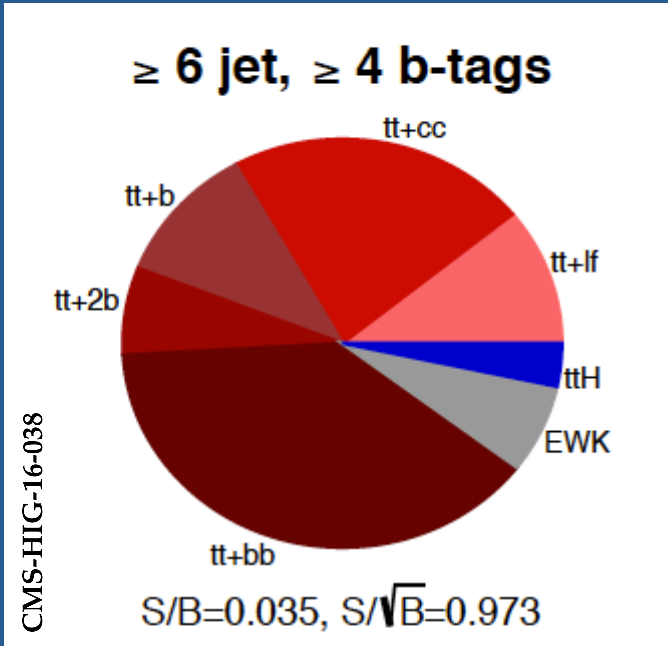


A discriminant is devised in each category for signal extraction and a simultaneous fit is performed across all categories.

Low-signal categories serve to help constrain backgrounds.

Details of signal extraction in backup.

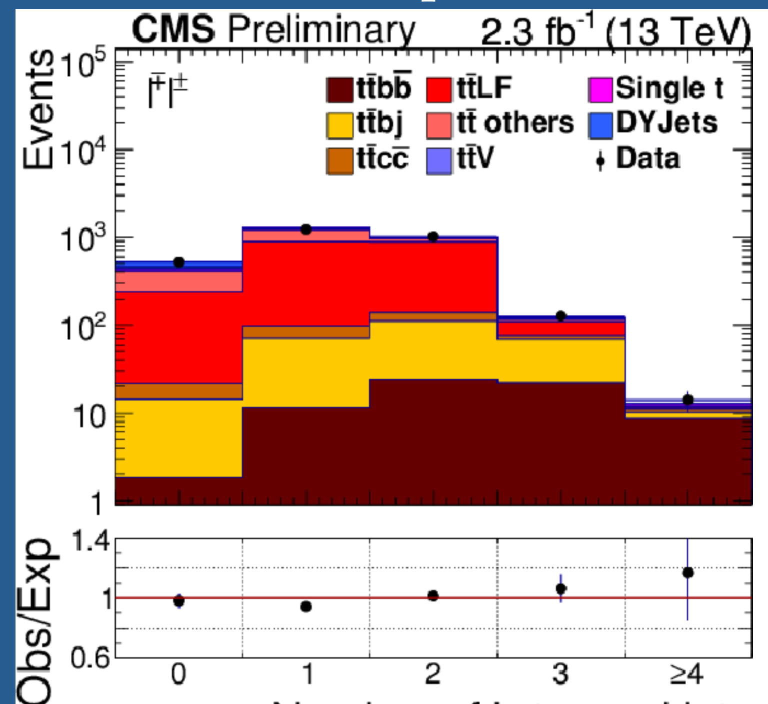
Big Issue: Understanding the tt+HF Background



- tt+bb production poses irreducible background:
  - Poorly known theoretically
  - Measurements of ttbb CRUCIAL

- Modeling of tt+jets process:
  - Powheg+Pythia8, normalized to NNLO prediction
  - Separate templates for tt + b, tt + bb, tt + 2b, tt + cc, tt + LF
  - 50% rate uncertainty per tt + HF process, uncorrelated in final fit
    - *Among the leading uncertainties*
  - Add. sources include parton shower, hadronisation, PDF, ISR/FSR

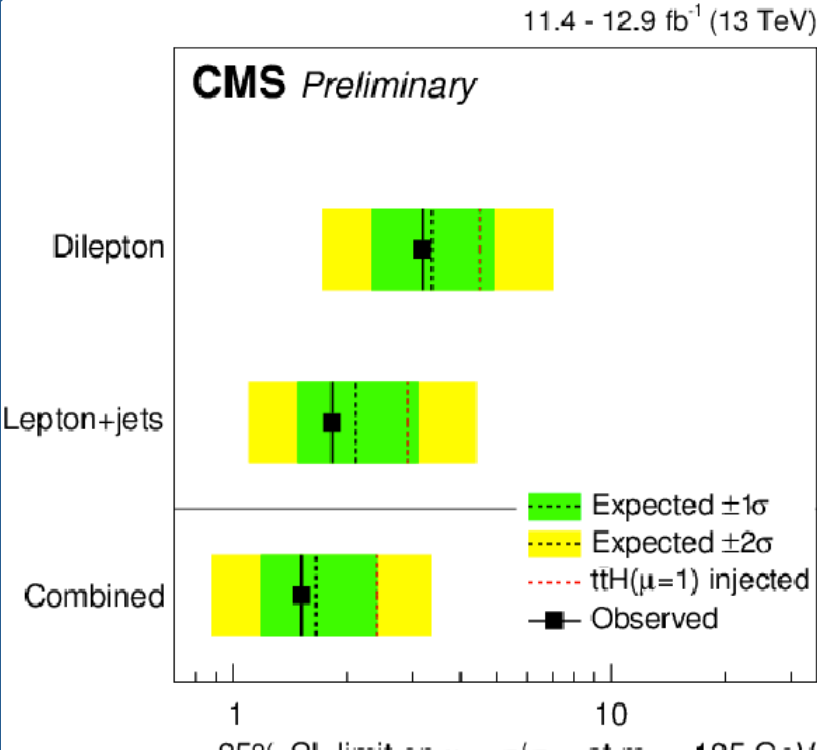
Measurement of ttbb production at CMS



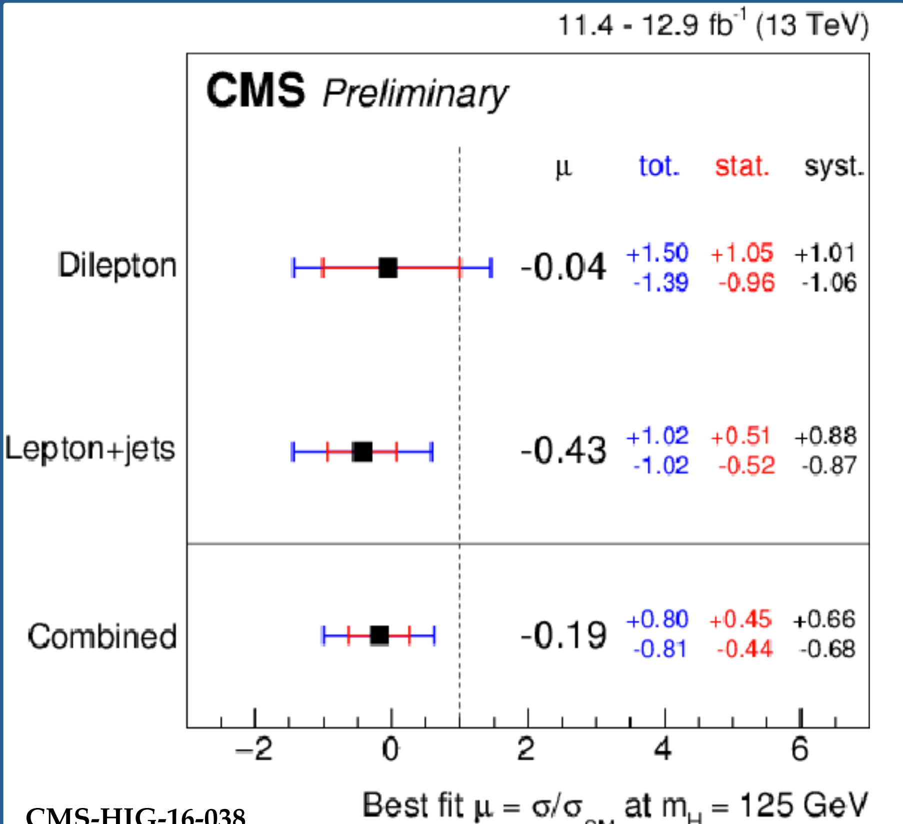
CMS-TOP-16-010

ttH,H→bb: Results

- Not yet achieving sensitivity to SM-level of ttH,Hbb production
- Upper limit on $\mu = \sigma / \sigma_{SM}$:
  - $\mu < 1.5$ (1.7) obs (exp) at 95% CL



CMS-HIG-16-038

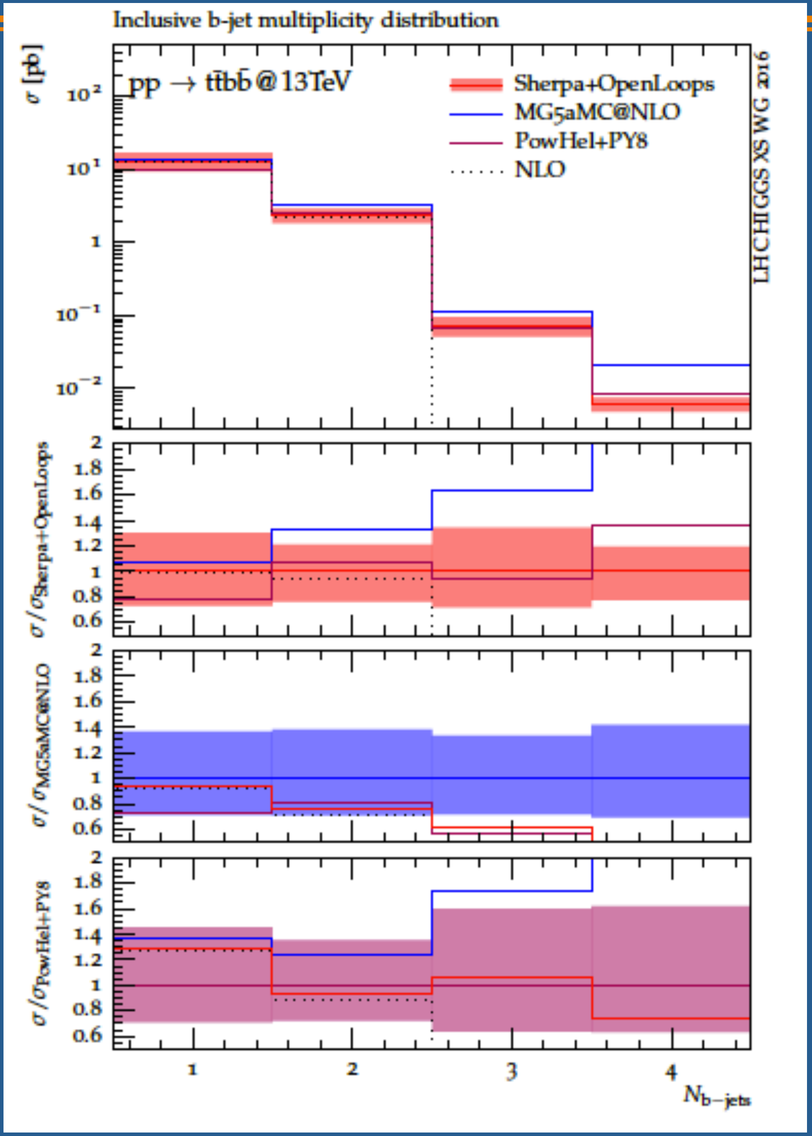


CMS-HIG-16-038

- Best fit: $\mu = -0.19\ ^{+0.45}_{-0.44}(\text{stat})\ ^{+0.66}_{-0.68}(\text{syst})$
- Systematics limited…how can we improve?

The Key: Understanding tt+HF

- Canonical ad-hoc 50% rate uncertainty on all tt+HF processes
  - tt+charm even less known than tt+b-jets
  - Huge impact on analyses
- The tt+b-jets process is poorly understood
  - Only recently do we have NLO calculations for the xsec
  - And even more recently NLO ME+PS events for use in analyses
- But NLO ≠ better, necessarily, if the predictions are poor
- Focus currently:
  - Compare various NLO ME+PS events for tt+bb
    - Consistency under well-defined conditions?
    - New scale treatment in MG5_aMC@NLO 2.5.4?
  - How do these state-of-the-art tools compare to CMS data
    - Need control regions independent from ttH signal-extraction campaign



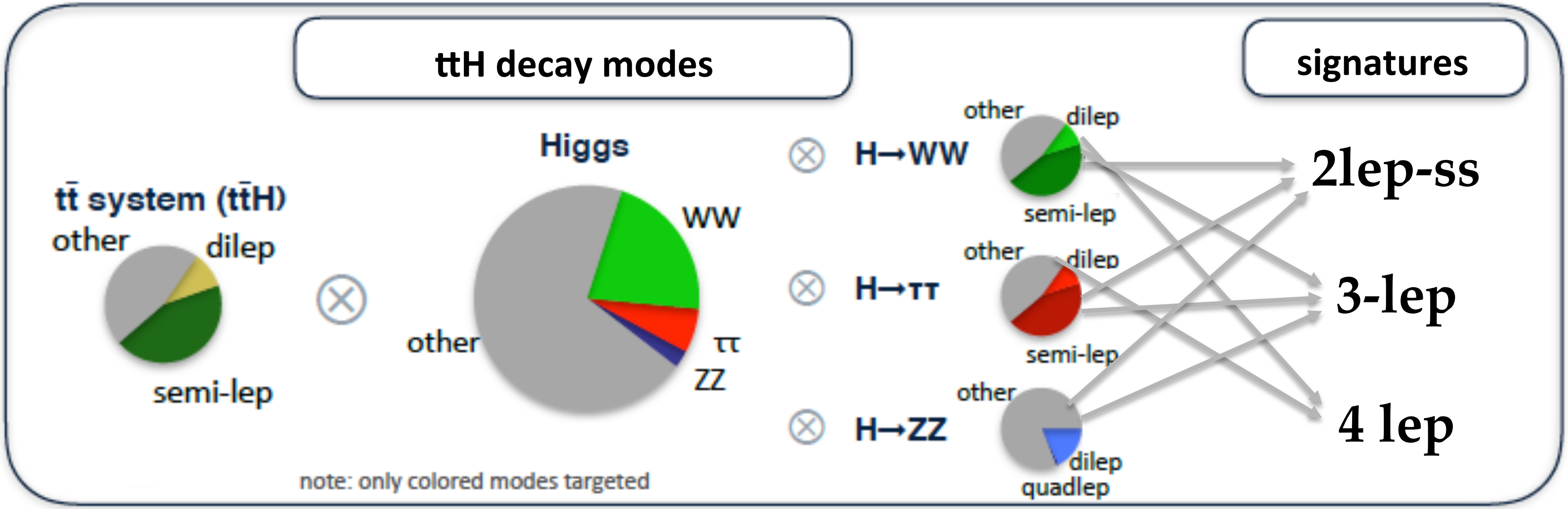
See Section I.6.8 of **Yellow Report 4** of the Higgs Cross Section Working Group

$t\bar{t}H$, H → multileptons (WW, ZZ, ττ)

ttH, H → multileptons

- ttH, H→leptons:
  - Targeted Higgs decays and BR H→WW* (~20%) ,ττ (6%), ZZ (3%)
  - leptons originate from Higgs and top system
- Targeted experimental signatures include multiple leptons
  - 2 same-sign leptons (2lss)
  - 3 leptons
  - 4 leptons

Event selection and signal extraction details in the backup



Low-rate but relatively low-background signatures.

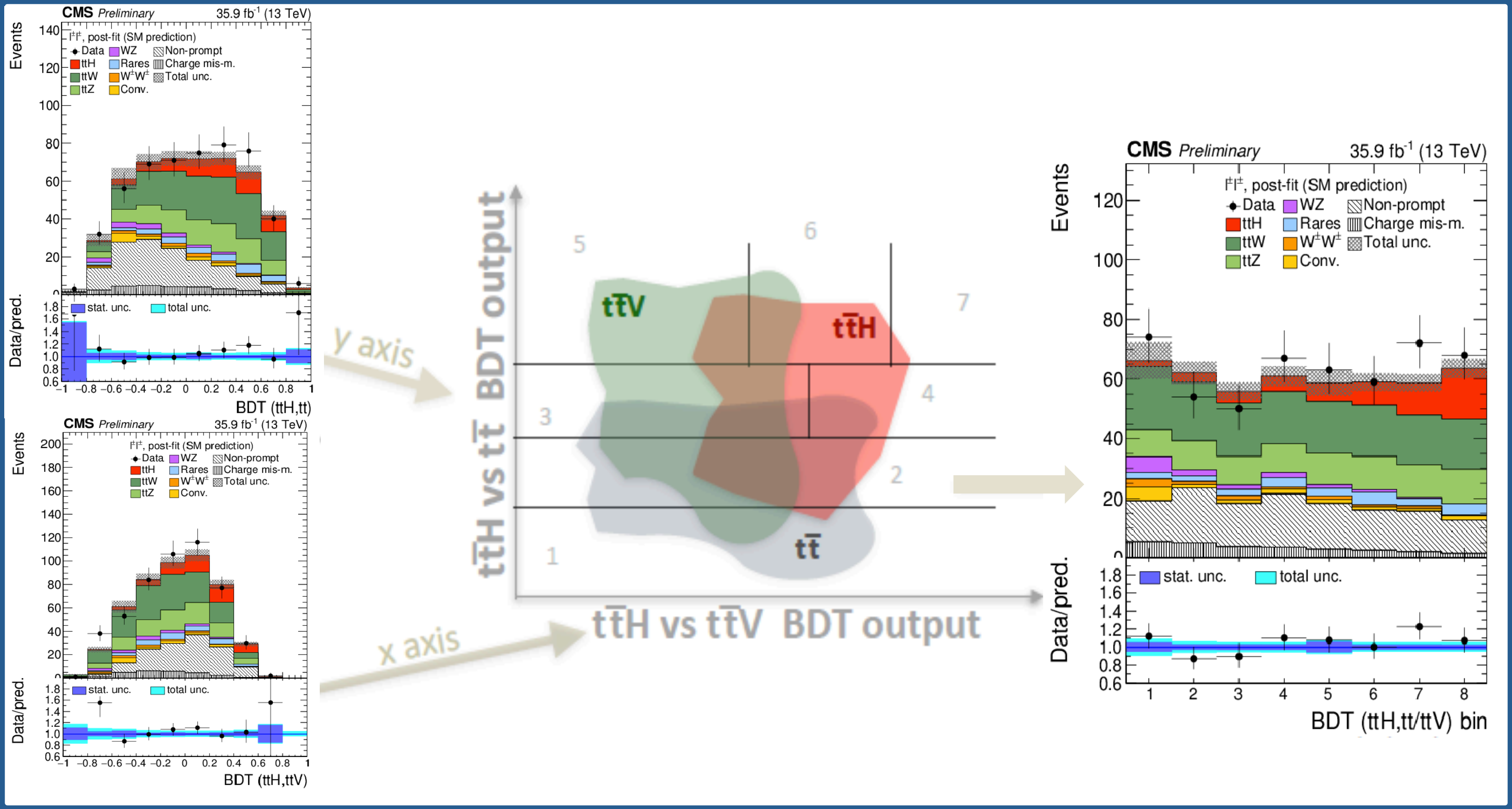
ttH, H → multileptons: Signal Extraction

- Further categorization occurs to exploit different signal purities available via b-tagging requirements and the charge asymmetry present in ttW
- Signal extraction:
  - 2lss, 3lep: Two BDT classifiers one each targeting tt+jets and ttV (V=W,Z)
  - 4lep: simple counting experiment

BDT inputs

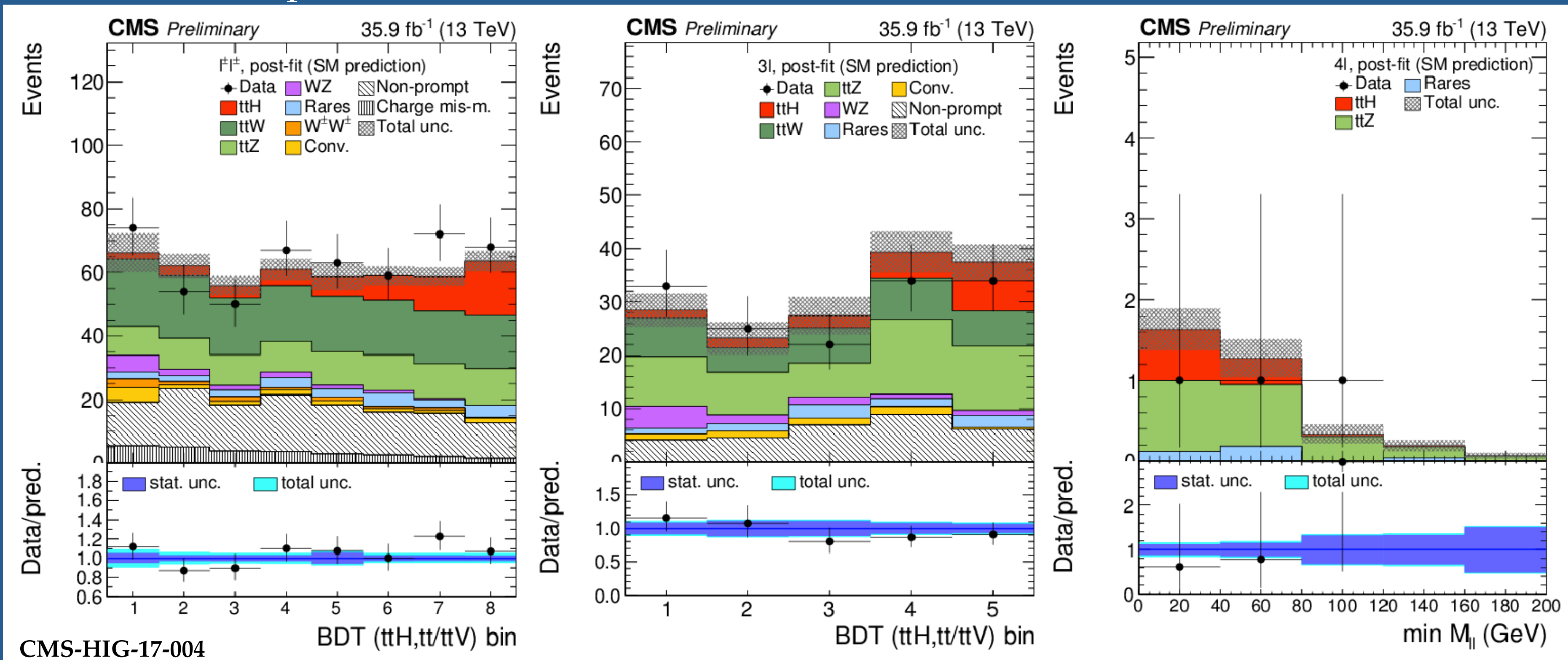
max eta
N_{jets}
min DelR(l,j)
M_{trans}
BDT_{hadtop}
$BDT_{higgsdecay}$
Max P_T
Min P_T

MEM
(3lep only)



ttH, H → multileptons: Final Fit

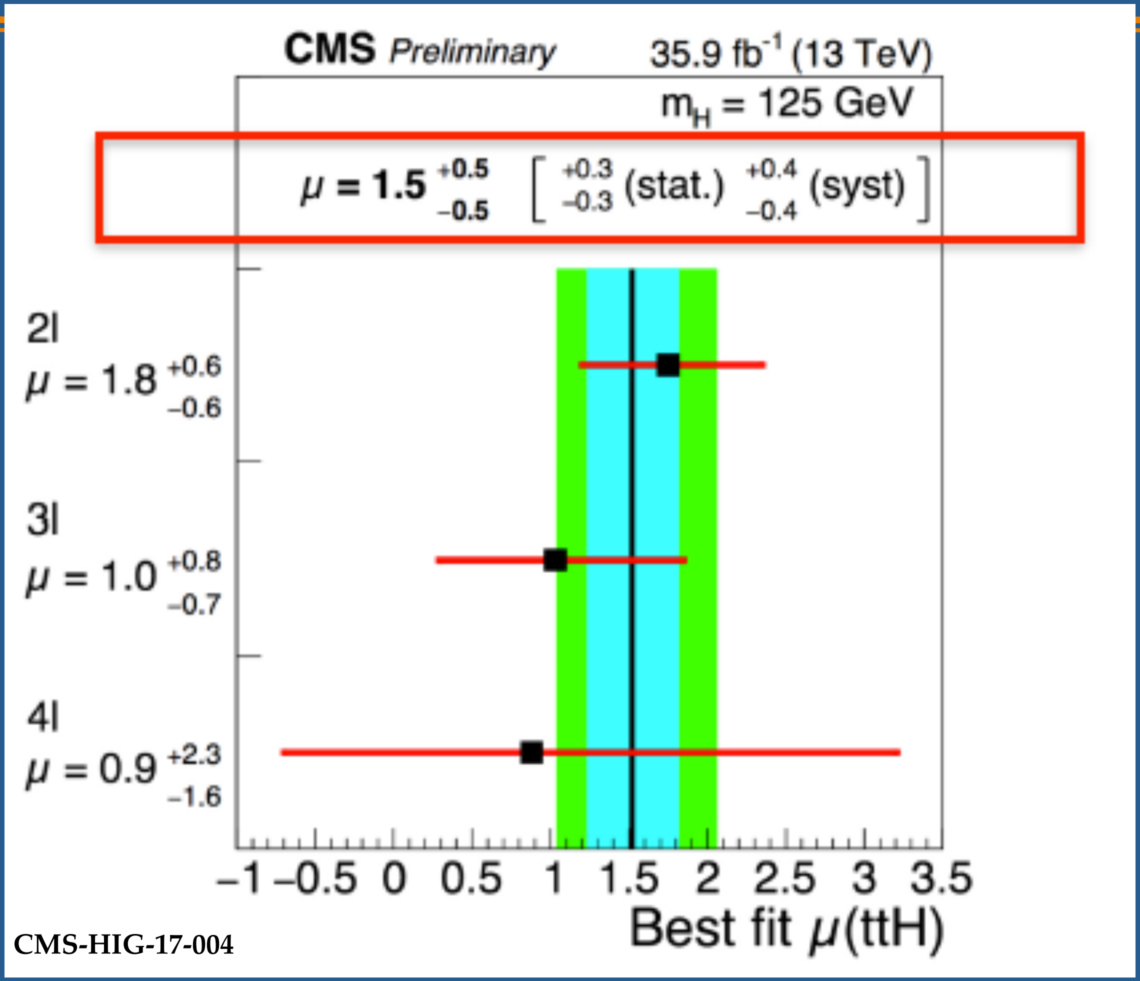
2lep-ss 3lep 4lep



- Final fit simultaneous across all categories, post-fit distributions shown here
- Starting to see sensitivity to SM-level of ttH production – no scaling factors on the red histograms here!

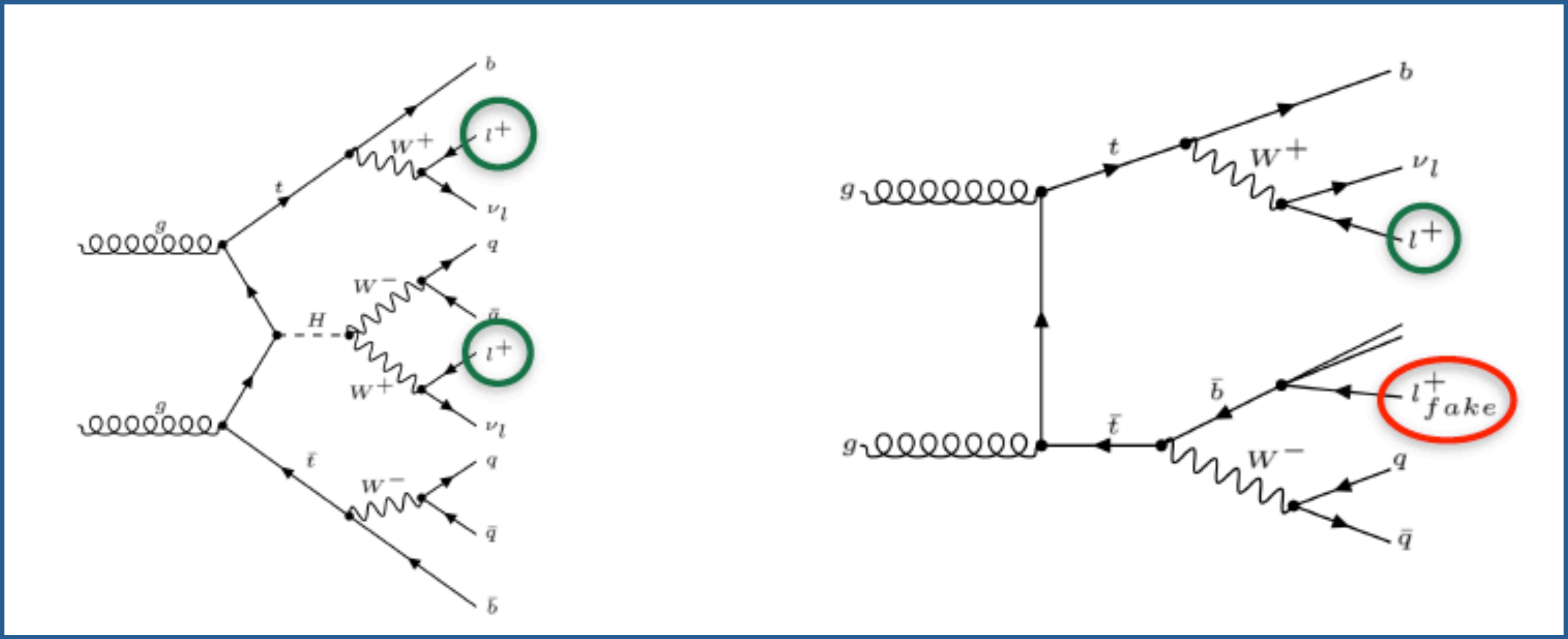
- Fit prefers *a slight bit more signal* in the 2lep-ss channels than expected in the SM

ttH, H → multileptons: Results



- Best fit: $\mu = 1.5 \,^{+0.3}_{-0.3}\text{(stat)} \,^{+0.4}_{-0.4}\text{(syst)}$
- **Significance of observation is 3.3σ**, whereas the expectation, assuming SM-level of ttH production was 2.4σ
- Systematics limited…how can we improve?

Leading systematic uncertainties



- Opportunities:
  - Improve our understanding of authentic leptons but from non-prompt sources
  - Theoretical cross sections on ttW and ttW:
    - NLO currently good to ~±15%, driven by missing higher order terms
    - NNLO tricky computationally

ttH, H $\rightarrow$ $\tau\tau$

$t\bar{t}H$, H → ττ

- Historically, $t\bar{t}H$,H→ττ analyses were folded in to other endeavors:
  - In 8 TeV, signatures containing two hadronic tau decays were included in a “hadronic decays” analysis, along with H→bb
  - The 2lep-ss + hadronic-tau category was considered in the context of an earlier iteration of the $t\bar{t}H$, H→mulitleptons search in 13 TeV.

- We do a fine job at taus here at CMS, no need for these signatures to play a subordinate role in other analyses

- First time in top-Higgs campaign at CMS: Dedicated $t\bar{t}H$,H→ττ analysis focusing on the 2016 13 TeV data sample

ttH, H → ττ: Event Selection

Common selection

- ≥ 1 tight lepton
- ≥ 1 tight tau (medium WP)
- b-tag requirement: ≥ 1 b-medium OR ≥ 2 b-loose
- $m(\ell\ell) > 12$ GeV

$1\ell + 2\tau_{had}$

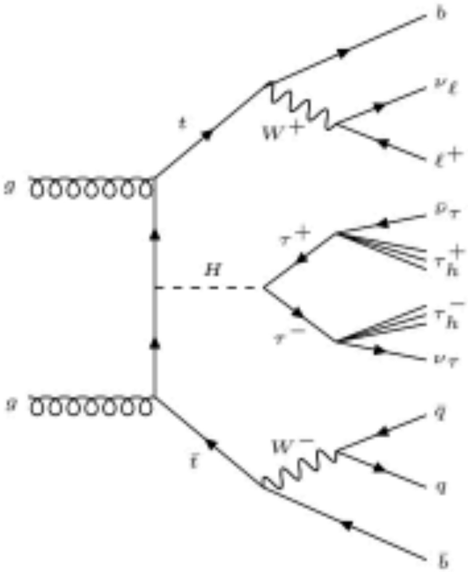
- ==1 tight lepton, pT > 25(30) μ(e)
- ≥ 2 tight τ_{had} w/ lead pT > 30
- OS requirement on 2 leading τ_{had}
- ≥ 3 jets

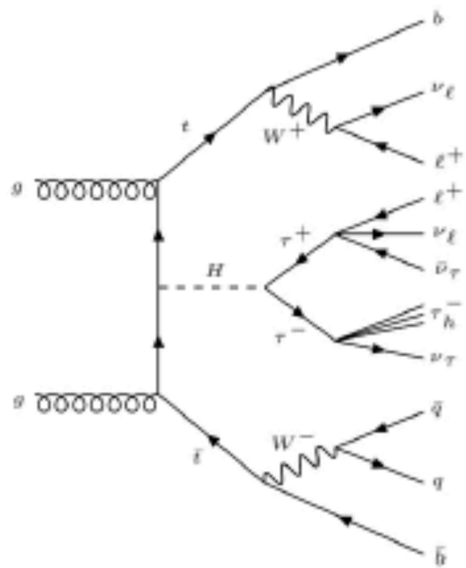
$2\ell SS + 1\tau_{had}$

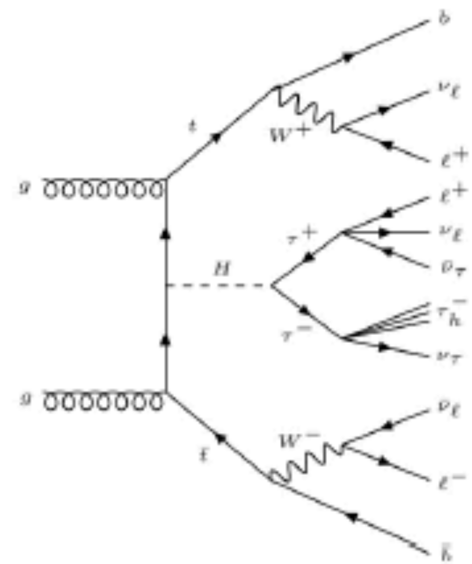
- ==2tight leps, pT > 25,10(15) μ(e)
- ≥ 1 medium τ_{had}
- SS requirement, tight-charge on leps
- ≥ 3 jets
- Z veto + MET LD in ee events

$3\ell + 1\tau_{had}$

- ≥3 tight leps, pT > 25,10,10
- ≥ 1 medium τ_{had}
- lep charges + τ_{had} charge = 0
- ≥ 2 jets
- Z veto + MET LD in SFOS events



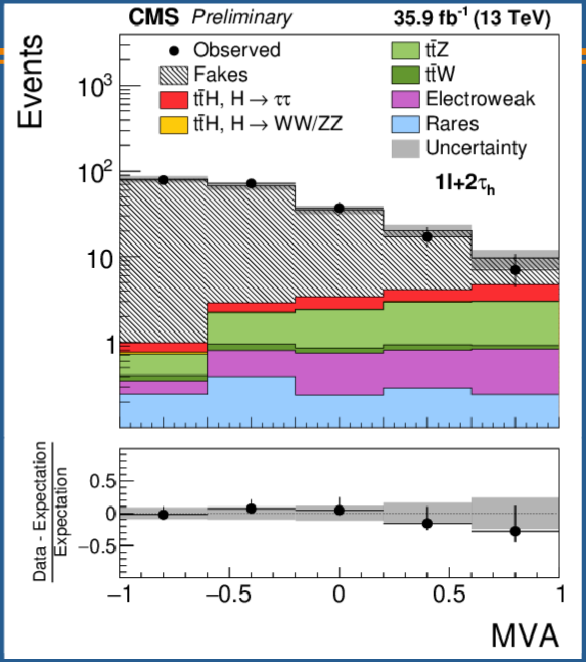


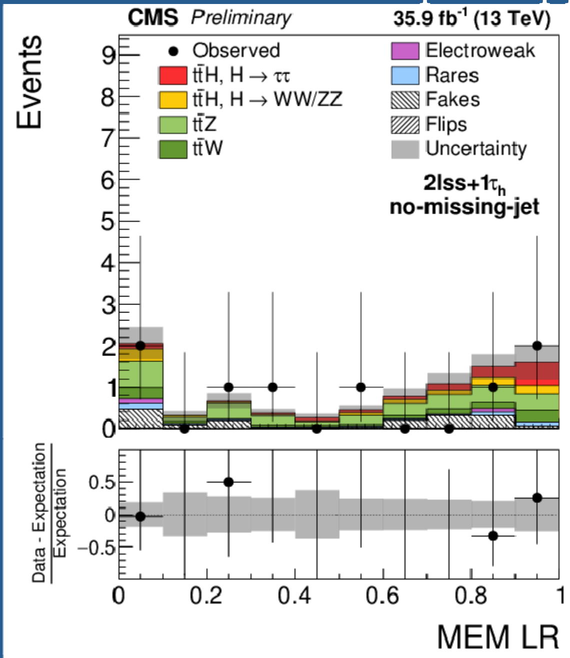


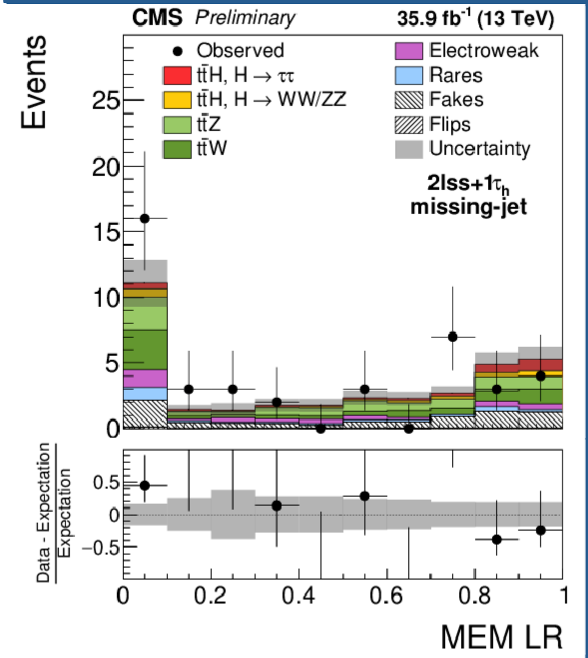
Designed to be orthogonal to ttH,multileptons selection to ease future combination.

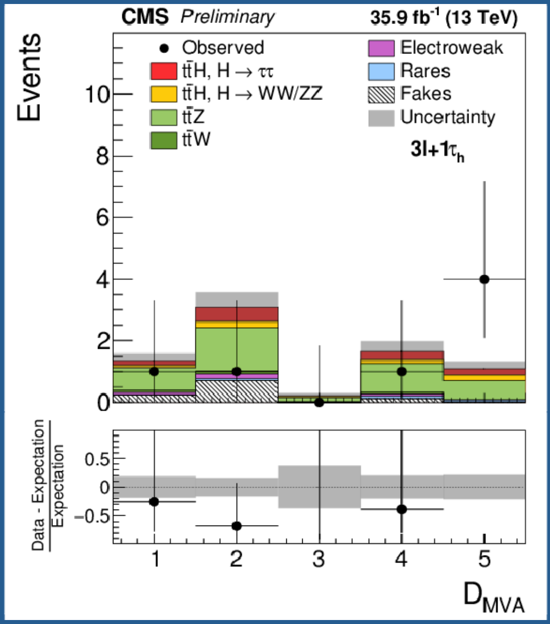
ttH, H → ττ: Signal Extraction

- Extraction methods:
  - **1lep + 2τ_h**: BDT, ttH v. tt+jets
  - **2lep-ss + 1τ_h**: MEM-based discriminant
    - Also split further by whether there is evidence of a hadronically decaying W. aka, “missing jets” v. “no-missing-jets”
  - **3lep + 1τ_h**: Two BDTs, one for tt+jets and one for ttV
    - Significant amount of commonality with ttH,multilepton approach

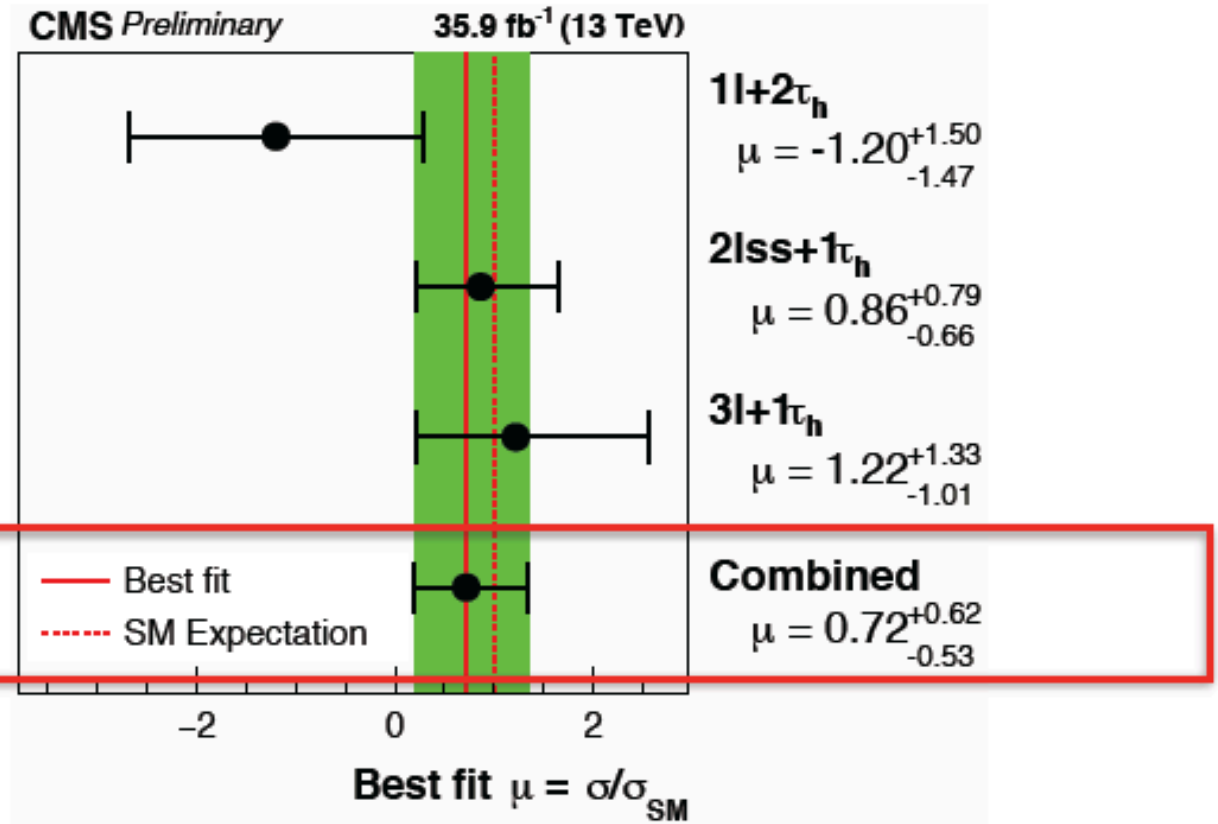








ttH, H → ττ: Results



CMS-HIG-17-003

- Best fit: $\mu = 0.72\,^{+0.62}_{-0.53}$ (stat $\oslash$ syst)
- Significance of observation is 1.4σ, whereas the expectation, assuming SM-level of ttH production was 1.8σ

ttH, H → $\gamma\gamma$

ttH, H → γγ

CMS-HIG-16-040

- Very rare process – yet very pure signature
- Important:
  - Completely reconstructible final state
  - No combinatoric background
  - Hence, only ttH search channel in which one can reconstruct a clear mass peak!
- Event selection:
  - 2 photons (requirements on BDTγID and EM deposits), $|\eta| < 2.5$
  - (sub)leading γ $p_T/m_{\gamma\gamma} > 0.5$ (0.25)
  - $100 < m_{\gamma\gamma} < 180$ GeV
  - Categorize events according to ttbar system decay:
    - Leptonic:
      - ≥1 p_T>20 e or μ far from γ and M_Z, ≥2 p_T>25 jets, ≥1 b-tag
    - Hadronic:
      - special BDT event classifier
      - ==0 e or μ, ≥3 pT>25 jets, ≥1 b-tag

So considering a window of $M_{\gamma\gamma}$ = 125 ± 1.5 GeV, there will be ~4.5 background events in the ttH Leptonic category.

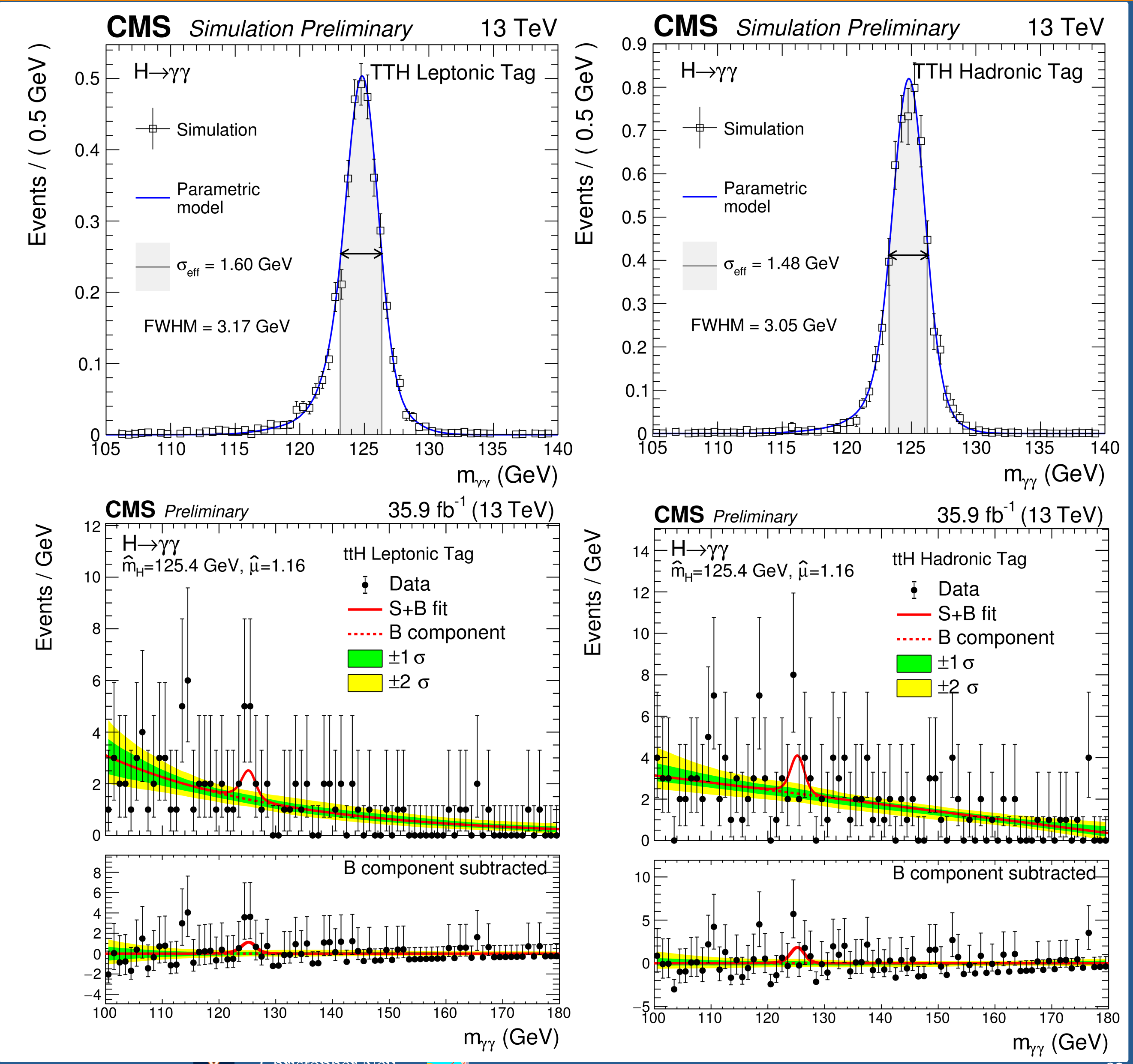
S/B ~ 0.85

Event Categories	SM 125 GeV Higgs boson expected signal											Bkg
	Total	ggH	VBF	ttH	bbH	tHq	tHW	WH lep	ZH lep	WH had	ZH had	(GeV^{-1})
ttH Hadronic	5.85	10.99 %	0.70 %	77.54 %	2.02 %	4.13 %	2.02 %	0.09 %	0.05 %	0.63 %	1.82 %	2.40
ttH Leptonic	3.81	1.90 %	0.05 %	87.48 %	0.08 %	4.73 %	3.04 %	1.53 %	1.15 %	0.02 %	0.02 %	1.50

ttH, H → γγ

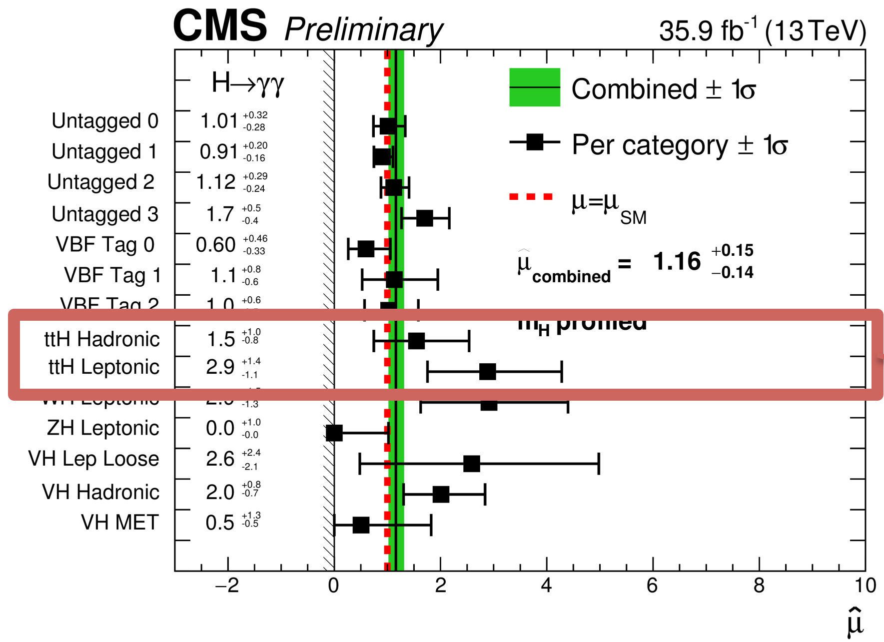
CMS-HIG-16-040

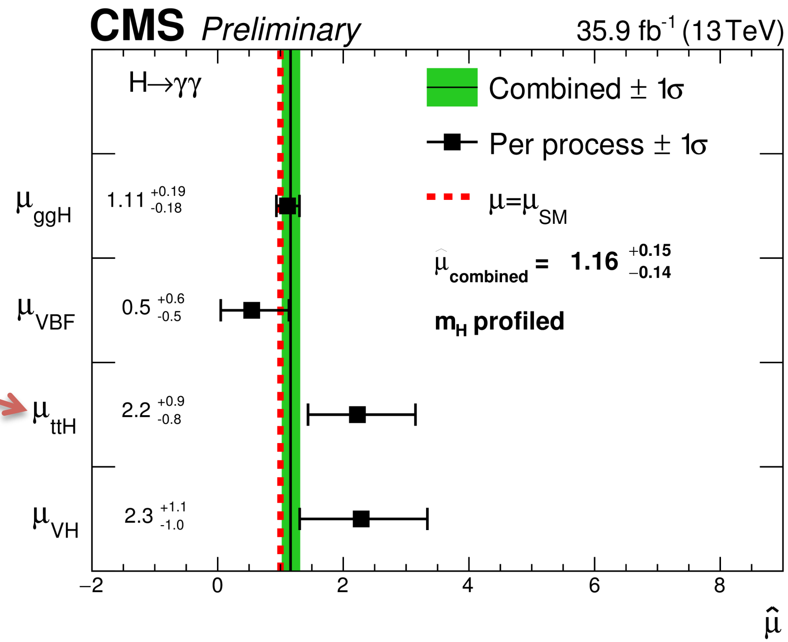
- Backgrounds so low – allows for very simple signal extraction:
  - Determine signal shape in $m_{\gamma\gamma}$ exploiting superior resolution of CMS crystal ECAL
  - Assume a falling exponential in $m_{\gamma\gamma}$ for the uncorrelated diphoton background
  - See what amount of signal is favored in the data for a specific M_H hypothesis



ttH, H → γγ

CMS-HIG-16-040

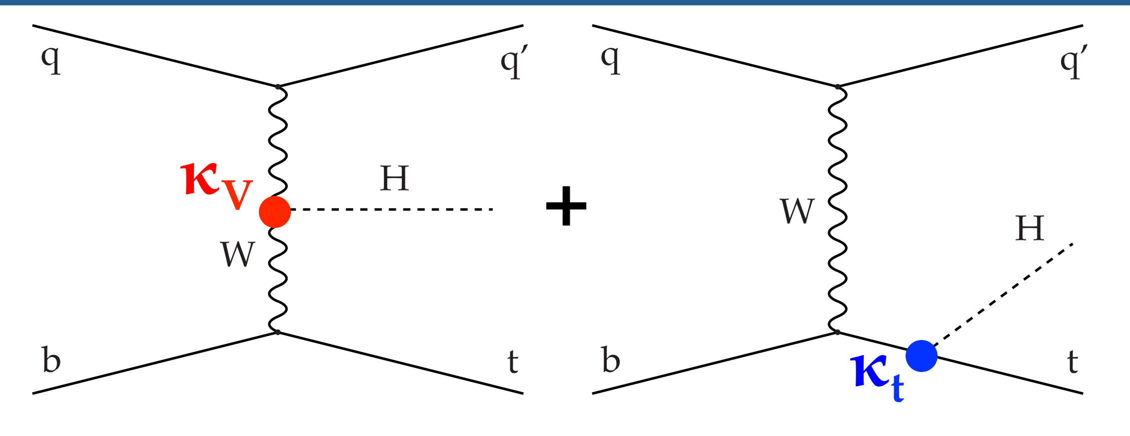




- Results from two ttH categories combined:
  - $\mu_{ttH} = 2.2^{+0.9}_{-0.8}$, assuming $M_H = 125.4$ GeV
  - Uncertainty driven by statistics
- Largely an afterthought…but will be a workhorse
  - Many recent changes in analysis of full 2016 data sample targeted for improving ttH sensitivity
  - *Good things come to those who wait…and build a solid analysis in the meantime*

Single top + Higgs Searches

tHq Analyses: Different Approach to Y_t



$\kappa_V = g_{HVV} / g_{HVV(SM)}$

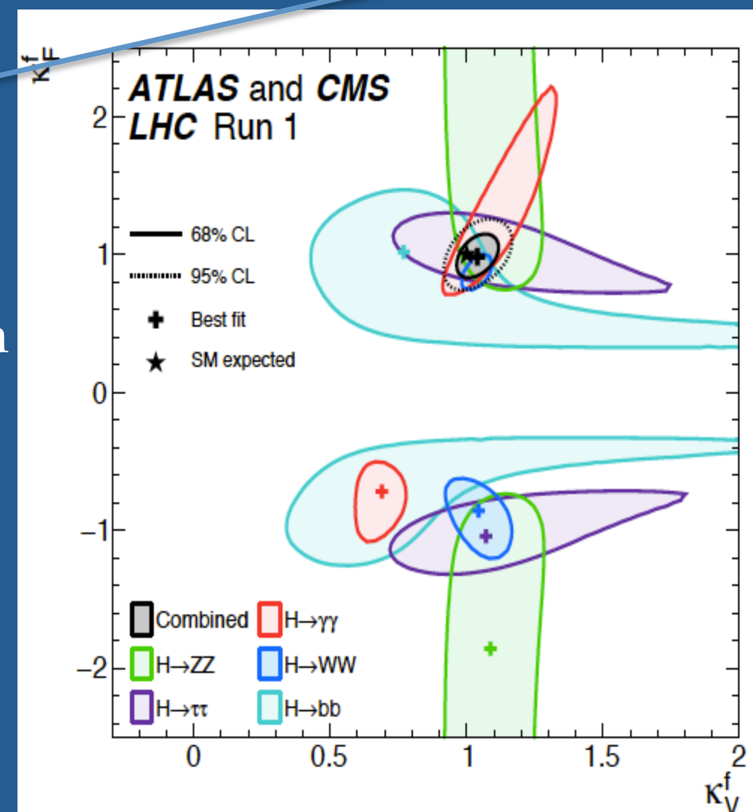
$Y_t = \kappa_t \, Y_{t(SM)}$

$\sigma(tHq) \approx a\,\kappa_t^2 + b\,\kappa_V^2 + \underbrace{c\,\kappa_t\,\kappa_V}$

- Hence this process is dependent on the *sign* of the top-Higgs coupling

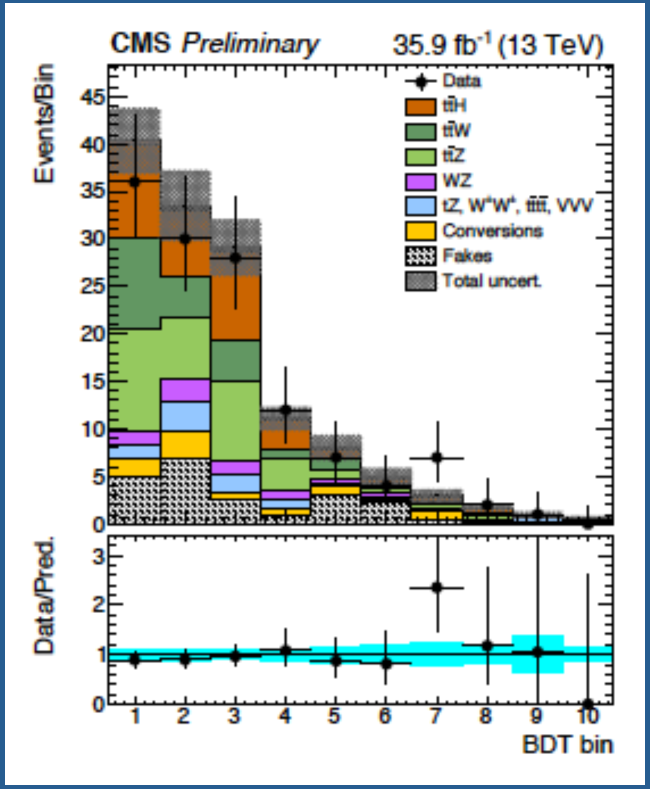
- Interference effects suppress tHq production in the SM, but if Y_t is negative there is considerable enhancement:
  - For example, at 13 TeV,
    - $\sigma_{SM}(tHq) = 71$ fb
    - $\sigma(tHq, Y_t = -1) = 739$ fb (x10 effect)

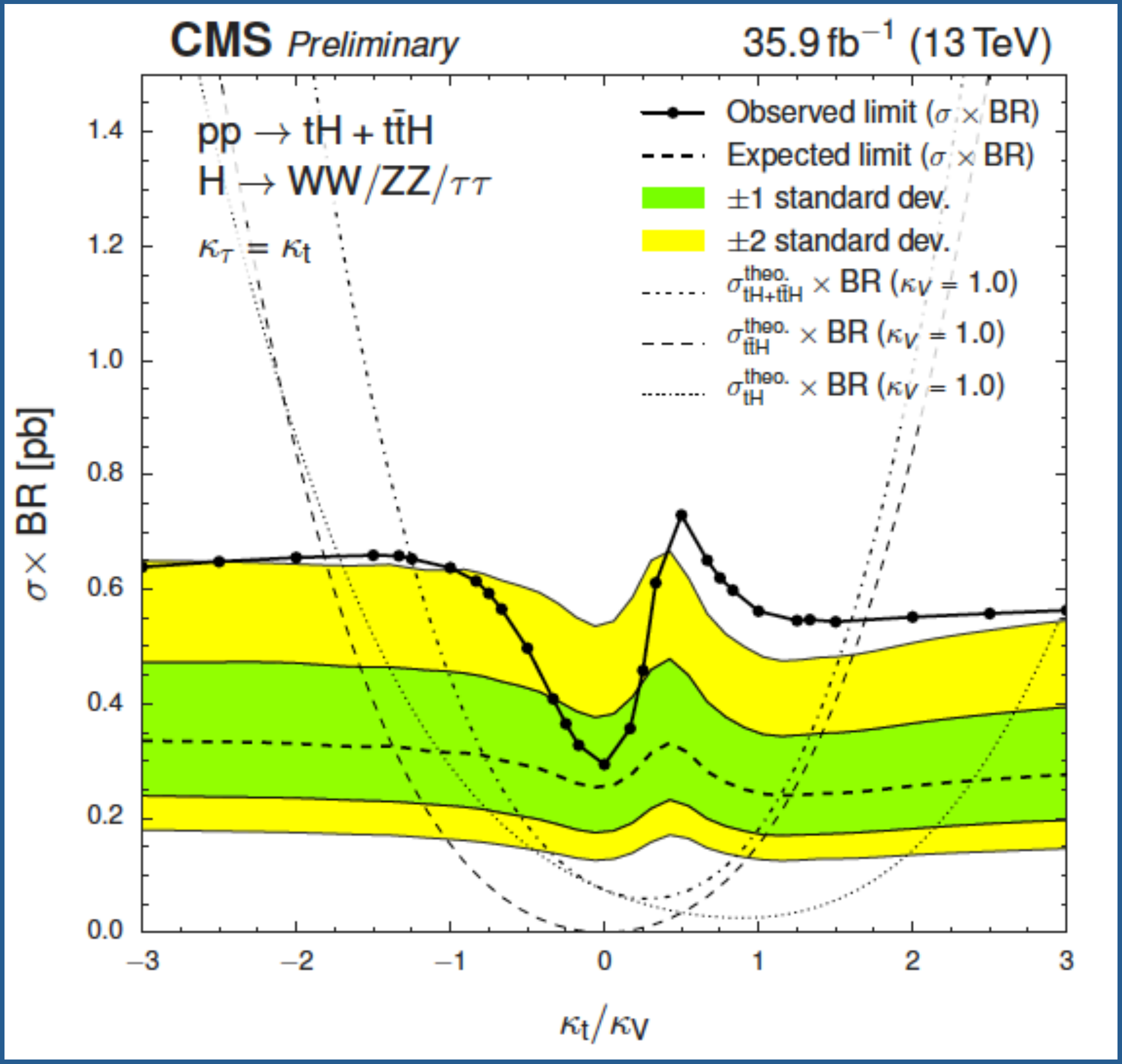
- ttH is far less sensitive to this negative coupling



tHq in Multilepton Signatures at 13 TeV

CMS-HIG-17-005





- Similar event categorization as in ttH,multilepton search
- For $\kappa_t / \kappa_V = -1$, observed (expected) upper limit on $\sigma \times BR = 0.64$ (0.32) pb

Summary

- Higgs physics has now moved from the search and discovery phase into a precision measurement era
- Characteristics of this Higgs boson need to be measured with high precision. The measurement campaign has so far revealed no significant deviations from the predictions of the SM

- A few crucial ones remain to be measured – the most foremost being the coupling between the top quark and the Higgs boson

2017 Status, 13 TeV analyses	**Signal Strength** $\mu = \sigma/\sigma_{SM}$
H→bb	$-0.19\ ^{+0.45}_{-0.44}(\text{stat})\ ^{+0.66}_{-0.68}(\text{syst})$
multileptons	$1.5\ ^{+0.3}_{-0.3}(\text{stat})\ ^{+0.4}_{-0.4}(\text{syst})$
H→ ττ	$0.72\ ^{+0.62}_{-0.53}$ (stat ⦸ syst)
H→γγ	$2.2^{+0.9}_{-0.8}$ (stat ⦸ syst)

- First direct measurement of the top-Higgs coupling is among the primary goals of the LHC physics program.

What's Next

- Near term:
  - Establish ttH in all accessible decay channels
  - We have some work to do to make this happen:
    - Improve understanding of tt+HF process and uncertainties
    - Improve theoretical understanding of ttV
    - Improve upon already-mature treatment of non-prompt leptons

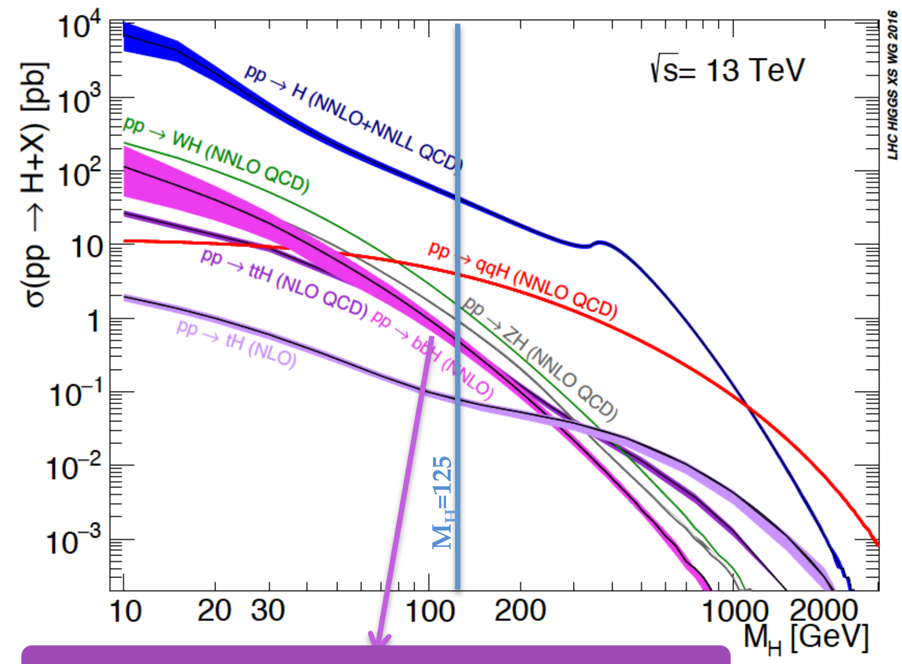
- Longer term
  - SM-driven backgrounds to ttH, H→γγ, ie ttγγ* at NLO
  - Refine background models
  - Increase purity
  - Differential cross sections

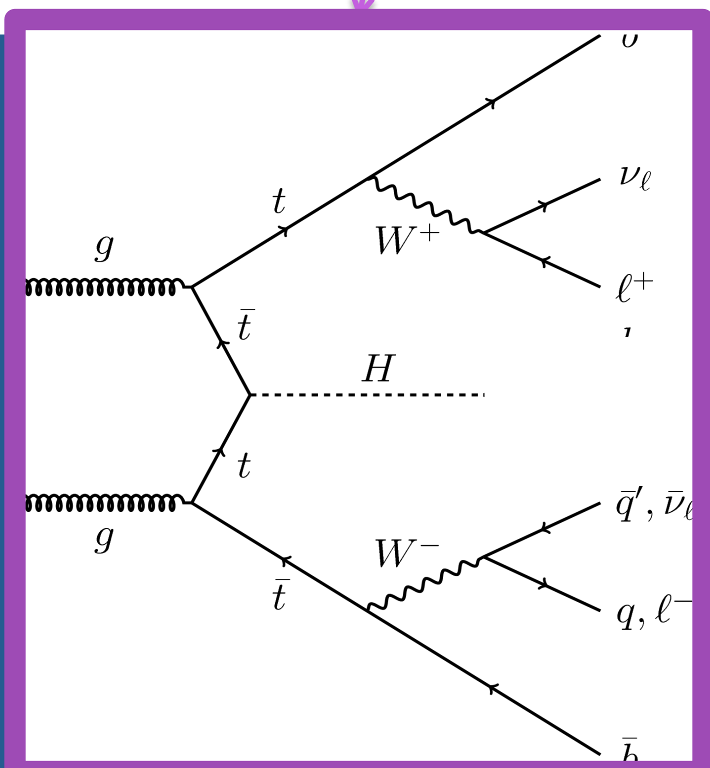
Things like EFTs / top partners / exotic 4th gen / 2HDM / etc look like SM top-Higgs…until you look closely, in the tails.

We will enter that regime in the future – best to lay the groundwork now.

Backup

ttH: Experimental Challenges



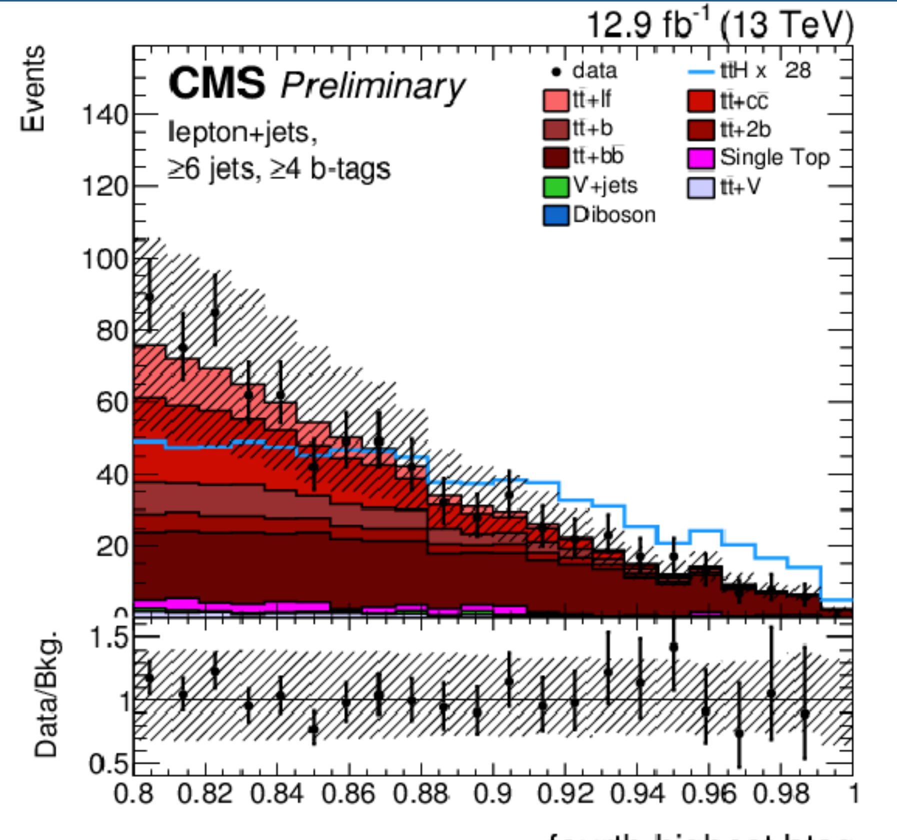


- Higgs production in association with a top-quark pair
  - Comparatively small production cross section wrt other Higgs production channels
  - Spectacular signature – rich final state
- ttH production cross section dwarfed by main background, tt+jets:

root(s) [TeV]	7	8	13
σ (ttH (125)) [fb]	90	130	510
σ (tt+jets) [fb]	177000	253000	830000
Ratio	5.0E-4	5.1E-4	6.1E-4

ttH,H→bb: Signal Extraction

- Single variable not very sensitive
  - Jet energy resolution
  - Combinatorics in jet assignment
- Different choices and combinations of more advanced classifiers
  - Boosted-Decision-Trees (BDTs) and Neural Networks (NNs)
    - Combination of various input variables, trained per category
    - Separation against all tt +X processes
    - Used in signal vs. background separation and in event reconstruction
  - Matrix-Element-Method (MEM) classifiers
    - Likelihood of event kinematics under signal or background hypothesis
    - Particularly powerful against difficult tt + bb background



CMS-HIG-16-038

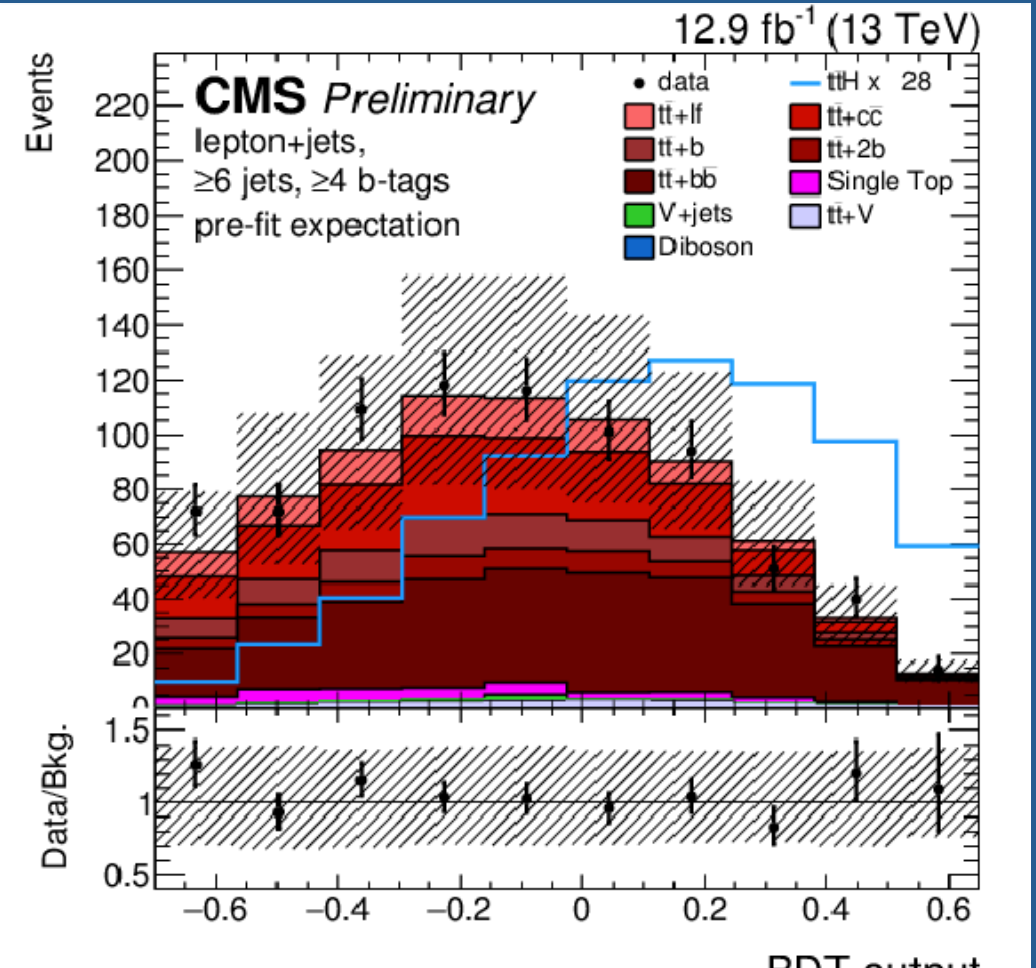
BDT inputs in SL ≥6j, ≥4t category:
- Best Higgs candidate mass
- $M(t_i,t_j)$ closest to 125
- $M_{l,v,jets}$
- 4^{th} and 5^{th} highest b-tag disc score
- Sum P_T of all jets, lepton, MET

ttH,H→bb: Signal Extraction

- Great deal of power added to the final analysis by combining these different multivariate approaches.

- For instance, the MEM discriminant can be used as an input to a BDT along with other kinematic variables

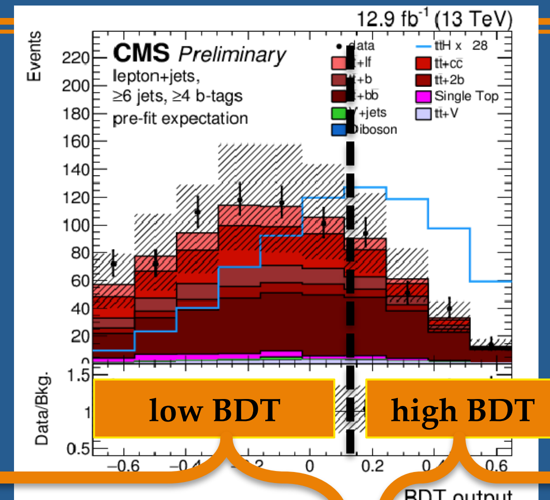
- Or as is done here, further categorize according to BDT score and then use the MEM in each subcategory…



CMS-HIG-16-038

ttH,H→bb: Signal Extraction

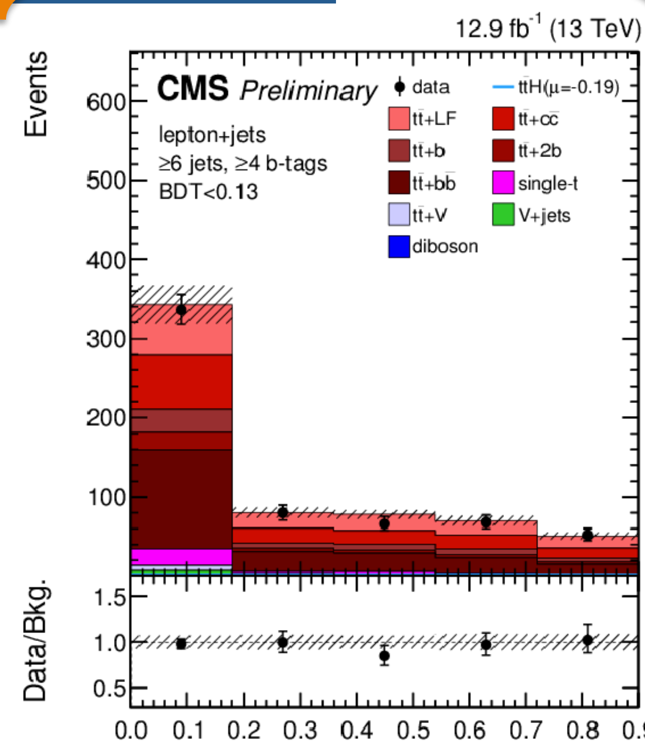
	4 jets	5 jets	≥6 jets
3 tags			low/high BDT
≥4 tags	low/high BDT	low/high BDT	low/high BDT



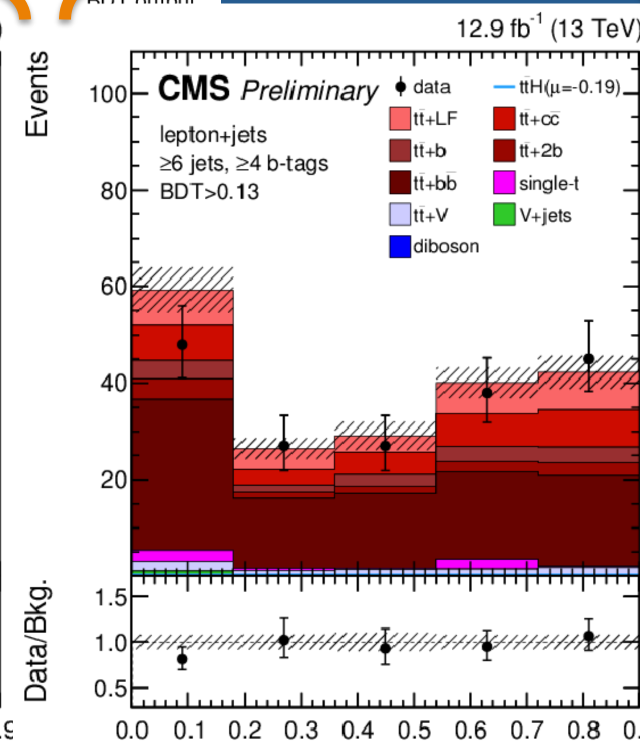
Similar approach is used in the DL categories *except* 3j,3t (BDT fit only)

Approach:

1. Divide events by jet,tag multipliticyt
2. Subdivide by low/high BDT score
3. Fit MEM discriminant in each subdividded region

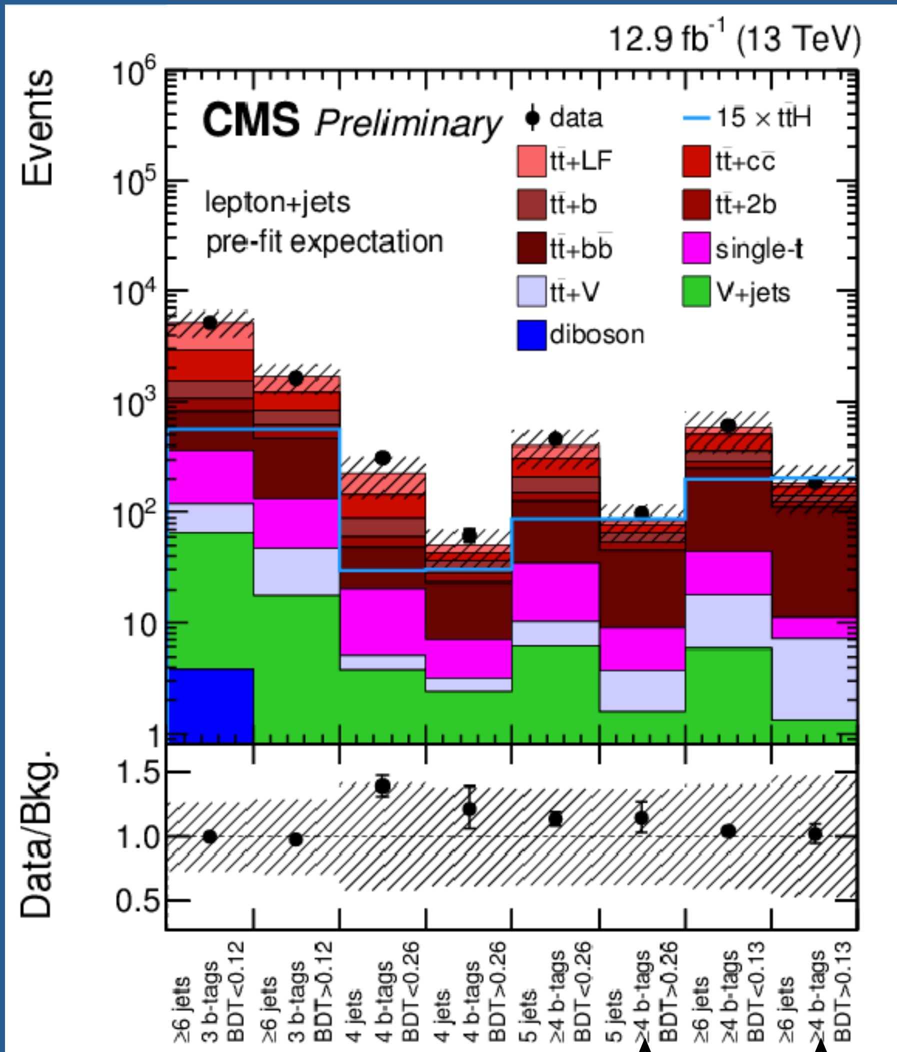


CMS-HIG-16-038

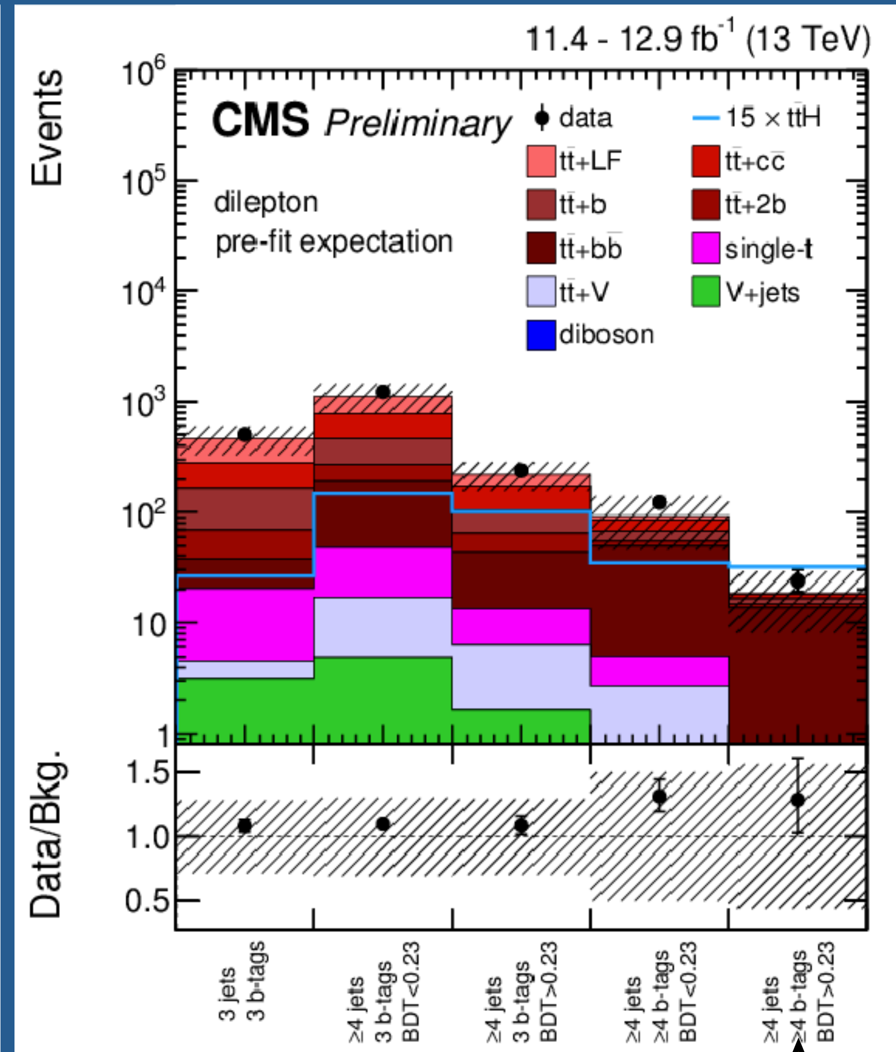


CMS-HIG-16-038

ttH,H→bb: Results



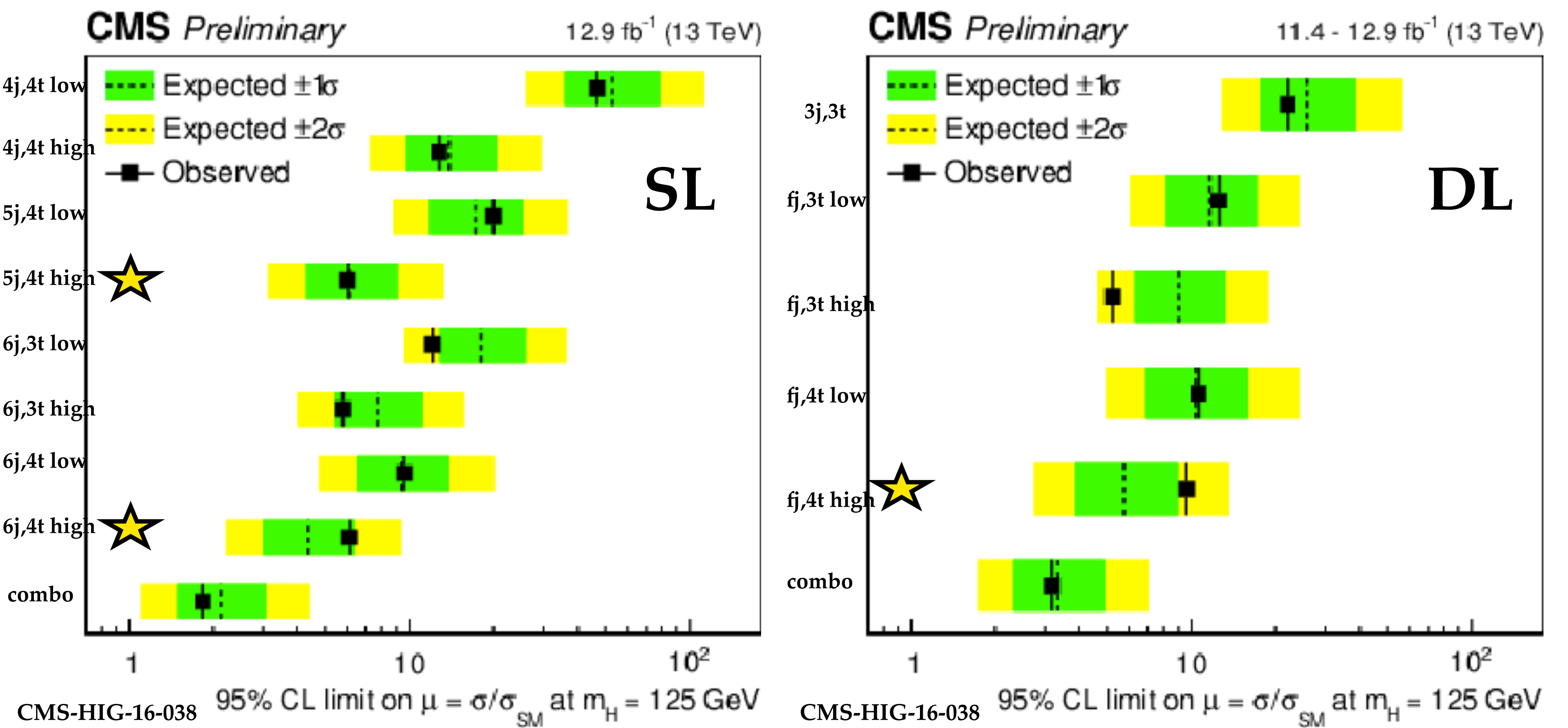
CMS-HIG-16-038



CMS-HIG-16-038

★ = highest signal/background, hence highest sensitivity categories

ttH,H→bb: Results

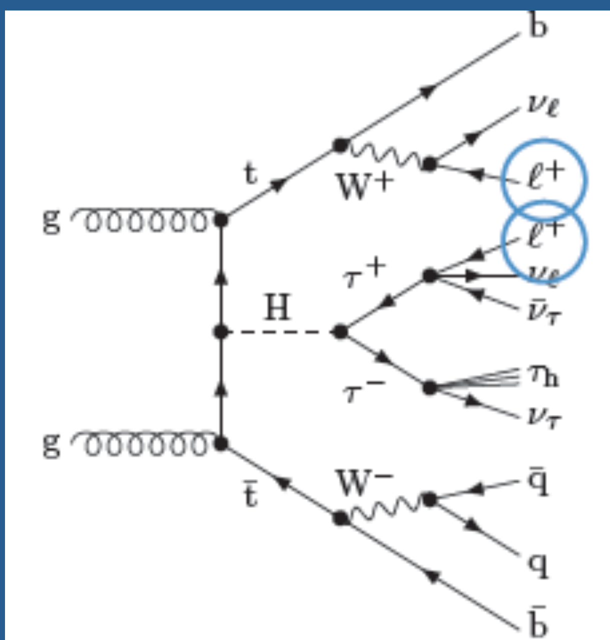


- Highest signal purity / sensitivity bins show upward fluctuation
- Compensated by overall deficits elsewhere

Search for ttH Production, H→Multileptons

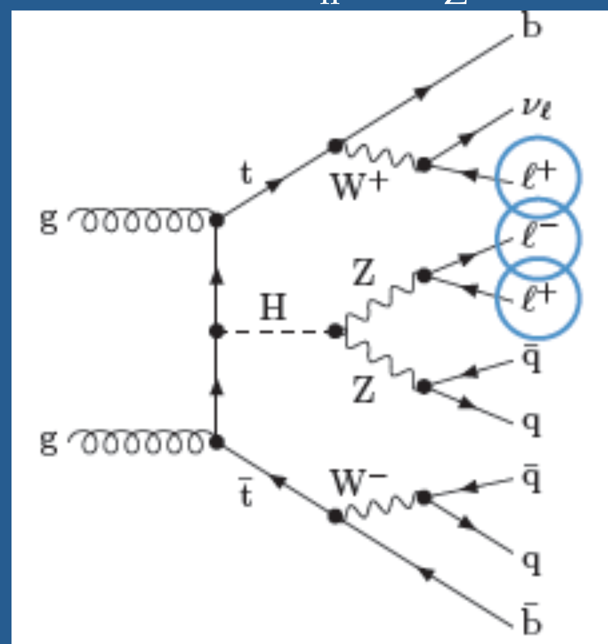
Same-sign dilepton

2 e/μ with $p_T > 20$
≥4 jets with $p_T > 25$
≥1 b-tagged jet



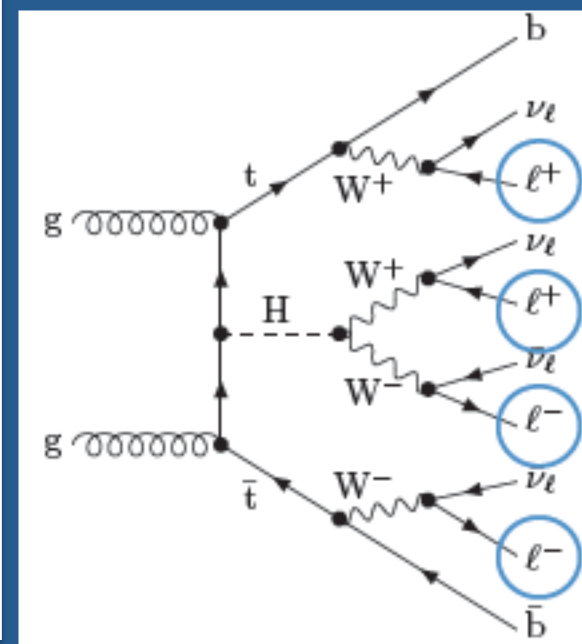
Three lepton

1 e/μ with $p_T > 20$
1 e/μ with $p_T > 10$
1 e(μ) with $p_T > 7(5)$
≥2 jets with $p_T > 25$
≥1 b-tagged jet
Veto $M_{ll} \sim M_Z$



Four lepton

1 e/μ with $p_T > 20$
1 e/μ with $p_T > 10$
2 e(μ) with $p_T > 7(5)$
≥2 jets with $p_T > 25$
≥1 b-tagged jet
Veto $M_{ll} \sim M_Z$



Low-rate but relatively low-background signatures.

$t\bar{t}H$, H → multileptons: Per-Category Yields

CMS-HIG-17-004

	$\mu\mu$	$e\mu$	ee
$t\bar{t}W$	51.0 ± 0.6 (stat.) ± 6.9 (syst.)	72.8 ± 0.7 (stat.) ± 10.2 (syst.)	20.5 ± 0.4 (stat.) ± 3.1 (syst.)
$t\bar{t}Z/\gamma^*$	17.7 ± 0.8 (stat.) ± 2.9 (syst.)	47.3 ± 1.6 (stat.) ± 9.0 (syst.)	17.5 ± 1.0 (stat.) ± 3.6 (syst.)
WZ	4.2 ± 0.6 (stat.) ± 4.1 (syst.)	7.0 ± 0.8 (stat.) ± 6.8 (syst.)	1.8 ± 0.4 (stat.) ± 1.7 (syst.)
Rare SM bkg.	4.2 ± 1.5 (stat.) ± 3.0 (syst.)	13.3 ± 1.9 (stat.) ± 9.3 (syst.)	4.8 ± 1.1 (stat.) ± 3.6 (syst.)
WWss	3.5 ± 0.6 (stat.) ± 2.5 (syst.)	4.1 ± 0.6 (stat.) ± 3.2 (syst.)	1.4 ± 0.3 (stat.) ± 1.2 (syst.)
Conversions		7.8 ± 2.5 (stat.) ± 2.3 (syst.)	3.6 ± 3.5 (stat.) ± 1.7 (syst.)
Charge mis-meas.		16.4 ± 0.2 (stat.) ± 9.1 (syst.)	10.5 ± 0.2 (stat.) ± 5.9 (syst.)
Non-prompt leptons	38.7 ± 1.6 (stat.) ± 20.5 (syst.)	61.8 ± 2.0 (stat.) ± 13.0 (syst.)	17.7 ± 1.1 (stat.) ± 5.4 (syst.)
All backgrounds	120.3 ± 2.5 (stat.) ± 11.7 (syst.)	231.2 ± 4.3 (stat.) ± 13.3 (syst.)	77.9 ± 4.0 (stat.) ± 9.0 (syst.)
$t\bar{t}H$ signal	20.1 ± 0.5 (stat.) ± 2.1 (syst.)	27.9 ± 0.5 (stat.) ± 3.0 (syst.)	8.0 ± 0.3 (stat.) ± 1.1 (syst.)
Data	150	268	89

	3L	4L
$t\bar{t}W$	32.8 ± 1.0 (stat.) ± 4.9 (syst.)	
$t\bar{t}Z/\gamma^*$	49.8 ± 3.9 (stat.) ± 11.1 (syst.)	2.15 ± 0.24 (stat.) ± 0.44 (syst.)
WZ	9.1 ± 0.9 (stat.) ± 4.0 (syst.)	
Rare SM bkg.	8.8 ± 4.3 (stat.) ± 5.9 (syst.)	0.27 ± 0.16 (stat.) ± 0.19 (syst.)
WWss		
Conversions	5.3 ± 1.2 (stat.) ± 4.0 (syst.)	
Charge mis-meas.		
Non-prompt leptons	30.8 ± 1.5 (stat.) ± 10.9 (syst.)	
All backgrounds	137.3 ± 6.2 (stat.) ± 12.4 (syst.)	2.42 ± 0.28 (stat.) ± 0.56 (syst.)
$t\bar{t}H$ signal	19.5 ± 1.0 (stat.) ± 3.0 (syst.)	1.00 ± 0.09 (stat.) ± 0.11 (syst.)
Data	148	3

tHq 8TeV Analyses

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tHq, H → bb

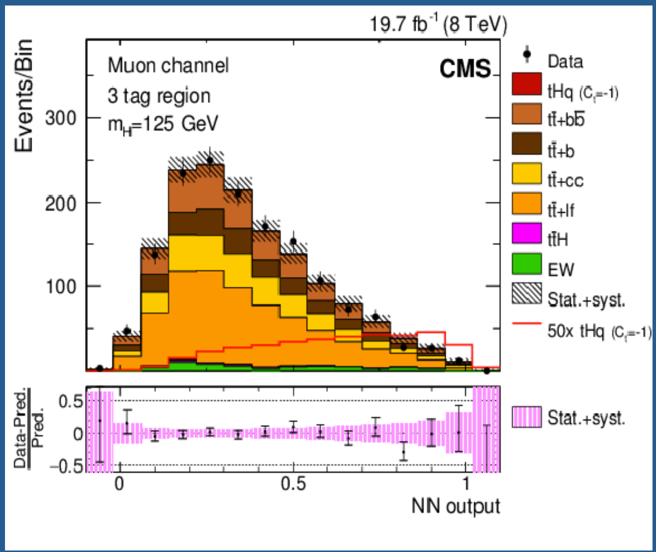
Leptonic W decay
3-,4-tag categories
MVA for tHq v. ttbar

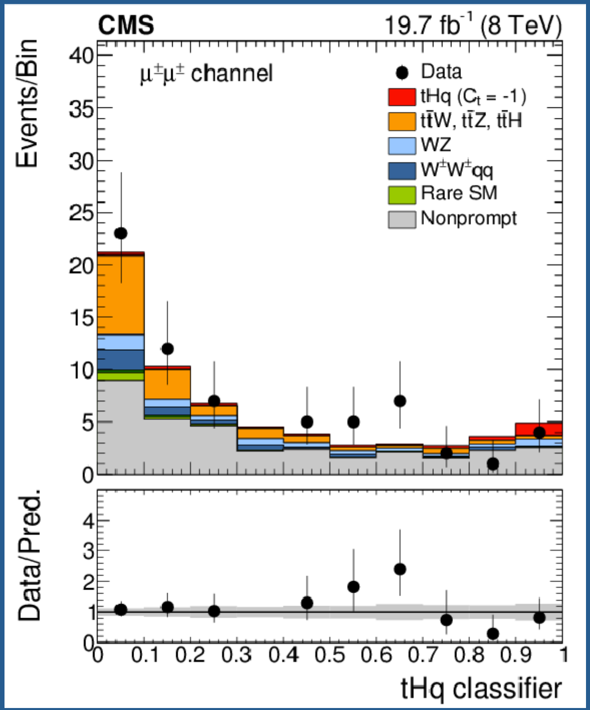
tHq, H → WW, ττ

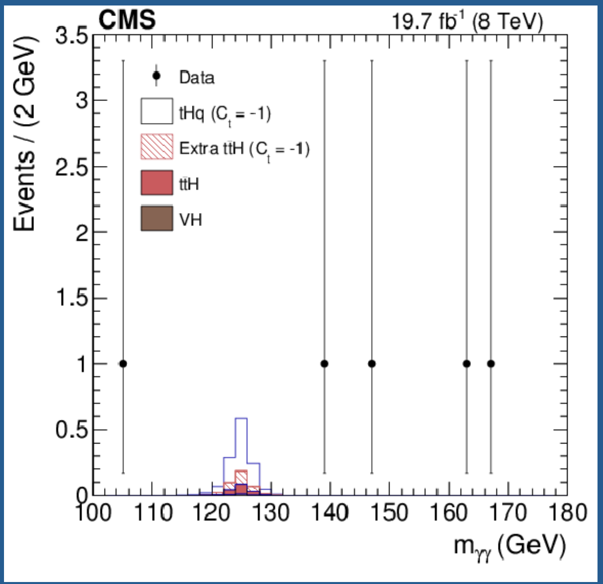
Same-sign 2lep and 3lep
MVA for tHq v. bkgd

tHq, H → γγ

Enhancement on
production and decay side.
No events survive in data.







Expected (observed) upper limits:

$\sigma/\sigma(Y_t = -1) < 5.4\ (7.6)$

$\sigma/\sigma(Y_t = -1) < 5.0\ (6.7)$

$\sigma/\sigma(Y_t = -1) < 4.1\ (4.1)$

Combined upper limit $\sigma/\sigma(Y_t = -1) < 2.0\ (2.8)$