A New Definition of the Quark Mass

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Top Quark Physics at the Precision Frontier
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Outline

- Original motivation
- The *minimal renormalon-subtracted* (MRS) mass
- Results for all quark masses *except* top
- Speculation about applications to the top-quark mass

Heavy-light Meson Masses in HQET

- From HQET (or other approaches to the $1/m_h$ expansion):

$$M_{H_J} = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- For ~20 years, I've wanted to vary m_h and use this formula to determine $\bar{\Lambda}$, μ_π^2, and $\mu_G^2(m_b)$ from lattice QCD [arXiv:hep-ph/0006345].

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1 for B, $-\frac{1}{3}$ for B^*

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$$M_{H_J} = m_h + \boxed{\text{long-distance QCD} \; \therefore \; \text{nonperturbative}}$$

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What’s a Quark Mass?

- You can’t put a quark on a scale and weigh it.
- Need definition, preferably regularization-independent, in QFT.
- Natural candidate is the “perturbative pole mass.” Alas, ambiguous:
    - physics—infrared gluons need to find a sink;
    - mathematics—obstruction to Borel summation of the perturbative series;
    - jargon—infrared renormalon;
    - numbers—$m_{\text{b,pole}}/\bar{m}_b = (1, 1.093, 1.143, 1.183, 1.224)$.

$$m_{t,\text{pole}}/\bar{m}_t = (1, 1.042, 1.053, 1.056, 1.058)$$

Short-Distance Definitions

- Usual work-around is to use a “short-distance” mass.
- The $\overline{\text{MS}}$ mass in dimensional regularization, $m_{h,\overline{\text{MS}}}(\mu)$; $\bar{m}_h \equiv m_{h,\overline{\text{MS}}}(\bar{m}_h)$:
  - spoils HQET power counting: $m_{\text{pole}} - \bar{m}_h \propto \alpha_s(\bar{m}_h)\bar{m}_h$.
- Other definitions subtract out infrared part:
  - “kinetic mass” (Uraltsev) via a Wilsonian renormalization;
  - “renormalon subtracted mass” (Pineda) subtracts out renormalon;
  - “MSR mass” (Hoang, Jain, Scimemi, Stewart) similarly (for top!!!);
  - all need another scale $1\ \text{GeV} < \nu_f < m_h$, or yet another $\alpha_s(\mu)$.

Pole Mass vs. $\overline{\text{MS}}$ Mass

- Consider the relation between the pole mass and the $\overline{\text{MS}}$ mass:

$$m_{\text{pole}} = \bar{m}\left(1 + \sum_{n=0}^{N} r_n \alpha_{\text{g}}^{n+1}(\bar{m}) + \mathrm{O}(\alpha_{\text{g}}^{N+2})\right)$$

where α_{g} is a scheme for α_{s} that simplifies the algebra:

$$\beta\left(\alpha_{\text{g}}(\mu)\right) = -\frac{\beta_0 \alpha_{\text{g}}^2(\mu)}{1 - (\beta_1/\beta_0)\alpha_{\text{g}}(\mu)}$$

$$\frac{1}{\alpha_{\text{g}}(\mu)} = \frac{1}{\alpha_{\overline{\text{MS}}}(\mu)} + b_1 + b_2 \alpha_{\overline{\text{MS}}}(\mu) + \cdots$$

α_{g} is regularization independent

- One finds $b_2 = \beta_2/\beta_0 - (\beta_1/\beta_0)^2$ $b_3 = \frac{1}{2}[\beta_3/\beta_0 - (\beta_1/\beta_0)^3]$, ...

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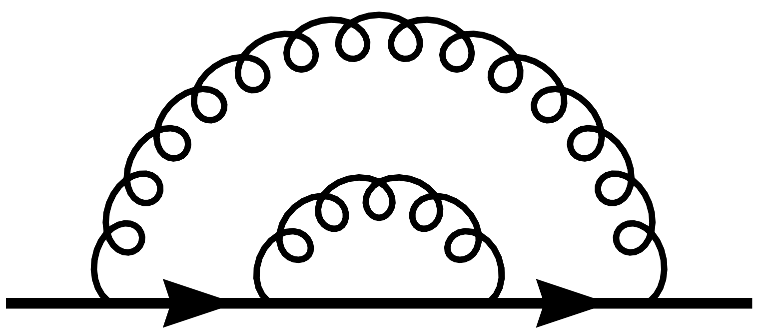
Infrared Properties

- The r_n are infrared finite and gauge-independent [hep-ph/9805215].
- The low loop-momentum parts of self energy diagrams cause the n^{th} coefficient to grows like $n!$



- Remarkably, the β function tells us almost everything about this growth:

$$r_n \sim R_n = R_0 (2\beta_0)^n \frac{\Gamma(n+1+b)}{\Gamma(1+b)}, \quad n \geq 0, \quad b = \frac{\beta_1}{2\beta_0^2} = \frac{231}{645} \text{ for } (n_f = 4)$$

only the overall normalization R_0 does not. Hence name “renormalon.”

Leading Infrared Renormalon

- Newly discovered formula [arXiv:1701.00347]:

$$R_0 = \sum_{k=0}^{\infty} r'_k \frac{\Gamma(1+b)}{\Gamma(2+k+b)} \frac{1+k}{(2\beta_0)^k}$$

$$r'_k = r_k - 2\left[\beta_0 k r_{k-1} + \beta_1 (k-1) r_{k-2} + \cdots + \beta_{k-1} r_0\right]$$

- We re-write the relation between the pole mass and the $\overline{\text{MS}}$ mass:

$$m_{\text{pole}} = \bar{m} + \bar{m} \sum_{n=0}^{\infty} [r_n - R_n] \alpha_{\text{g}}^{n+1}(\bar{m}) + \bar{m} \sum_{n=0}^{\infty} R_n \alpha_{\text{g}}^{n+1}(\bar{m})$$

and truncate the first sum as usual but carry out the second sum analytically.

Renormalon-a-Ding-Dong

- Use the technique of Borel resummation, one finds

$$\mu \sum_{n=0}^{\infty} R_n \alpha_{\rm g}(\mu) = \frac{R_0}{2\beta_0} \mu \int_0^{\infty} dz \, \frac{e^{-z/(2\beta_0 \alpha_{\rm g}(\mu))}}{(1-z)^{1+b}}$$
$$\equiv \mathscr{J}(\mu)$$

- The integrand has a branch point at $z = 1$. That's the ambiguity!

- Our suggestion:

  - Break the integral into an unambiguous part $z \in [0,1]$ and a totally ambiguous part $z \in [1,\infty]$.

Minimal Renormalon Subtraction

arXiv:1712.04983

- Splitting the integral (Brambilla, **Komijani**, ASK, Vairo):

$$\mathscr{J}(\mu) = \mathscr{J}_{\mathrm{MRS}}(\mu) + \delta m$$

$$\mathscr{J}_{\mathrm{MRS}}(\mu) = \frac{R_0}{2\beta_0}\mu \int_0^1 dz\, \frac{e^{-z/[2\beta_0\alpha_{\mathrm{g}}(\mu)]}}{(1-z)^{1+b}}$$

$$\delta m = \frac{R_0}{2\beta_0}\mu \int_1^\infty dz\, \frac{e^{-z/[2\beta_0\alpha_{\mathrm{g}}(\mu)]}}{(1-z)^{1+b}}$$

$$= -(-1)^b \frac{R_0}{2\beta_0}\Gamma(-b)\alpha_{\mathrm{g}}\mu\, \frac{e^{-1/[2\beta_0\alpha_{\mathrm{g}}(\mu)]}}{[2\beta_0\alpha_{\mathrm{g}}(\mu)]^b}$$

$$= -(-1)^b \frac{R_0}{2^{1+b}\beta_0}\Gamma(-b)\Lambda_{\overline{\mathrm{MS}}}$$

Minimal Renormalon Subtraction

arXiv:1712.04983

- Minimal renormalon-subtracted (MRS) mass:

$$\begin{aligned} m_{\mathrm{MRS}} &\equiv m_{\mathrm{pole}} - \delta m \\ &= \bar{m}\left(1 + \sum_{n=0}^{\infty} \left[r_n - R_n\right] \alpha_{\mathrm{g}}^{n+1}(\bar{m})\right) + \mathscr{J}_{\mathrm{MRS}}(\bar{m}) \end{aligned}$$

$$\mathscr{J}_{\mathrm{MRS}}(\bar{m}) = \frac{R_0}{2\beta_0} \bar{m} e^{-1/[2\beta_0 \alpha_{\mathrm{g}}(\bar{m})]} \Gamma(-b) \gamma^{\star}\left(-b, -[2\beta_0 \alpha_{\mathrm{g}}(\bar{m})]^{-1}\right)$$

- This function is easy enough to evaluate.

Remarks

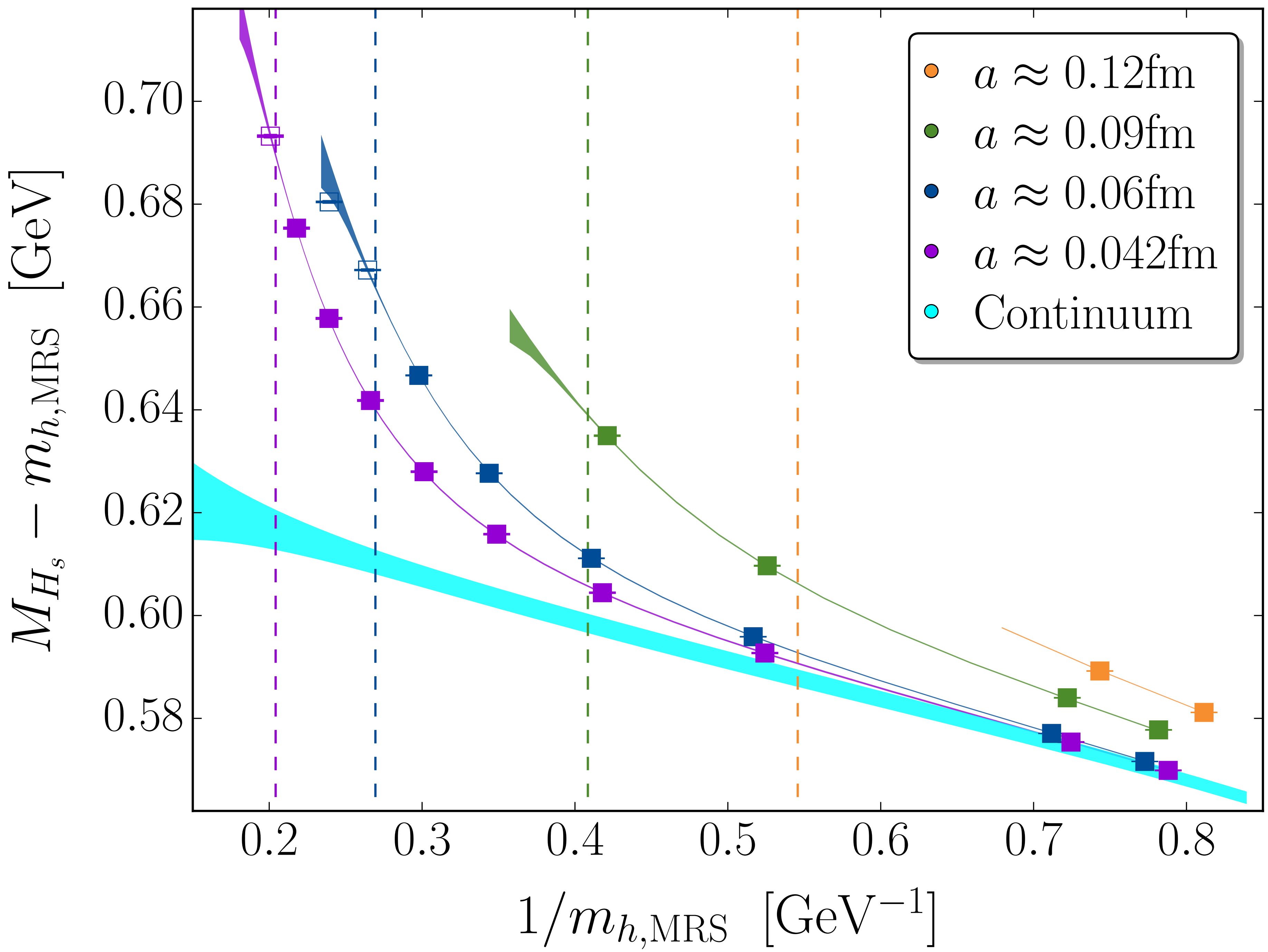
- MRS mass is a short-distance mass: subtract off long-range δm.
- No new scale: trim long-range field at $1/m_h$, not $1/\nu_f$.
- Numerically very stable: $m_{b,\mathrm{MRS}}/\bar{m}_b = (1.157, 1.133, 1.131, 1.132, 1.132)$.
  $m_{t,\mathrm{MRS}}/\bar{m}_t = (1.0687, 1.0576, 1.0573, 1.0574, 1.0574)$
- Has *same* asymptotic expansion in α_s as the pole mass.
- Makes HQET formula unambiguous (to order $1/m_h$):

$$M_{H_J} = m_{h,\mathrm{MRS}} + \bar{\Lambda}_{\mathrm{MRS}} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- Next step: fit this formula to lattice-QCD data!

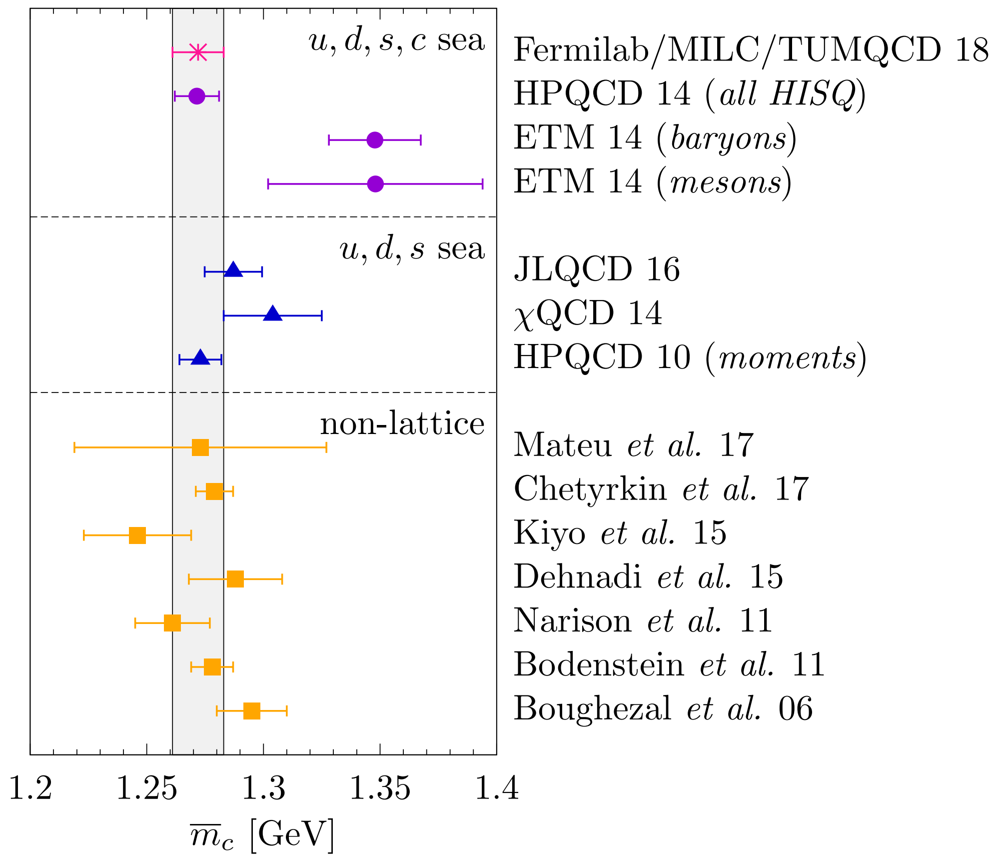
HQET Fit ⊕ Symanzik EFT ⊕ χPT

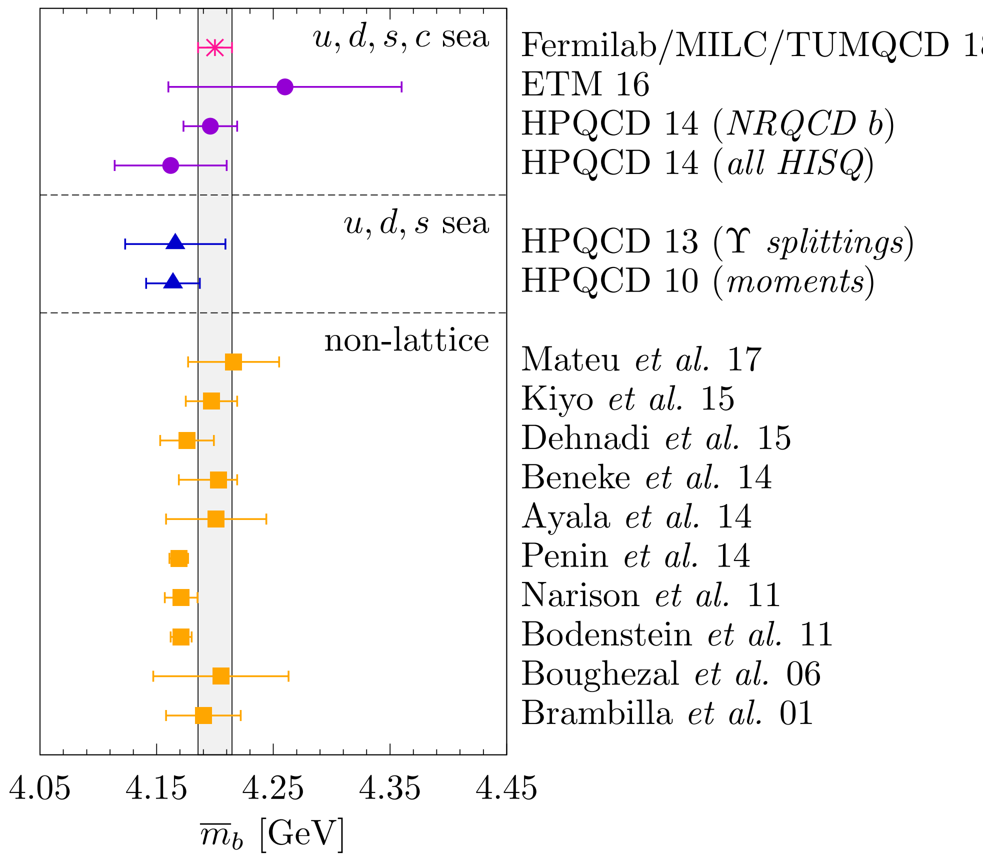
- Same correlators as decay constants.
- 20 ensembles
- 0.005–0.12% on meson M
- 5 (6) lattice spacings
- Snapshot at right ☛



Results & Comparisons

- Not quite finished, so these preliminary results are indicative:



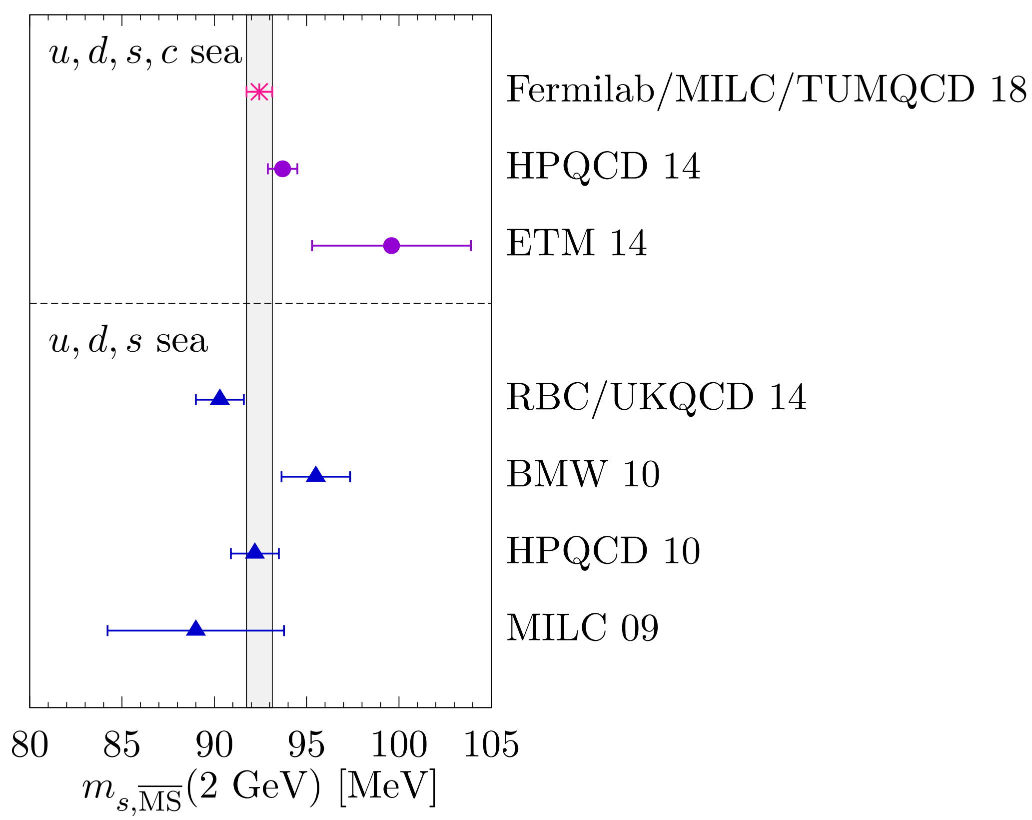


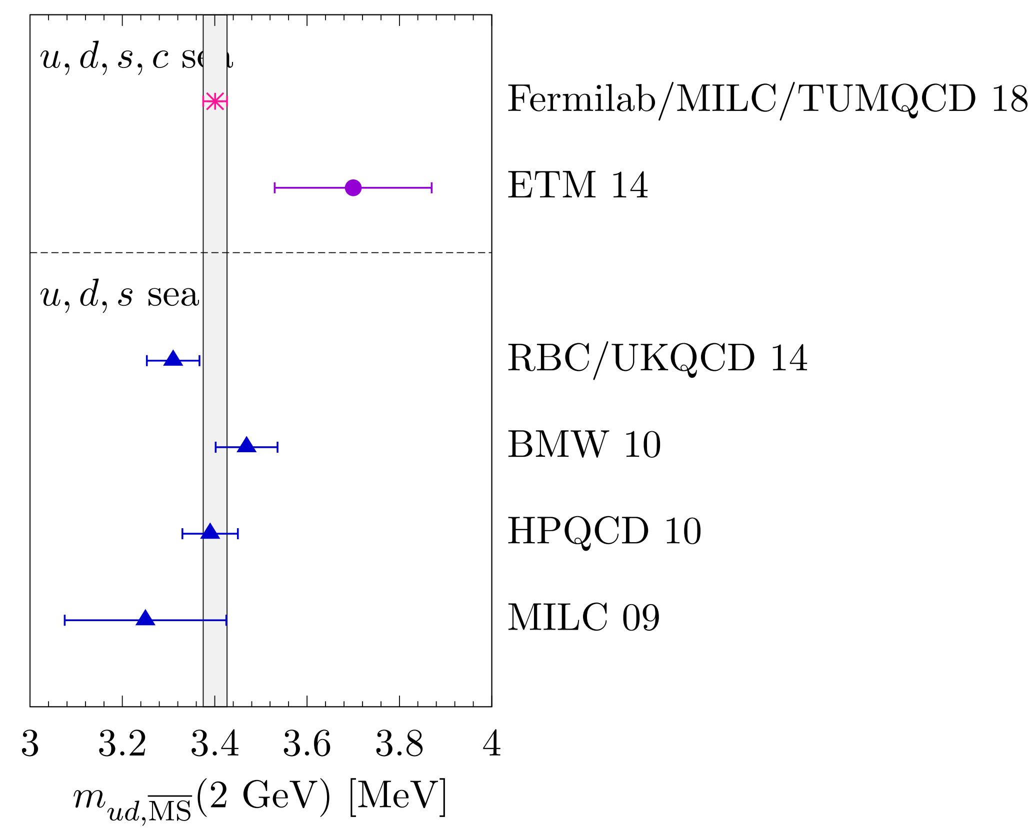
- To our knowledge, first results w/ order-α_s^5 running & order-α_s^4 matching.

- Precision: 0.3% for bottom to 0.5% for charm.

Results & Comparisons 2

- With mass ratios from light pseudoscalar meson:





- Most precise strange and “light” quark masses to date.

- Most precise quark masses for all quarks except top ($m_u > 50\sigma$).

Top Quark Physics

- Can the MRS mass be identified with the mass in Pythia?
  - It all the advantages without the disadvantage.
- Is there an observable that is analogous to the heavy-light meson mass?
  - The “hadron”—i.e., the color singlet—that the top quark is in is the “fat jet” containing all the decay products;
  - think about mass-sensitive properties of this object.
- What can be varied to separate the MRS mass from the rest of the jet?
  - The top-quark mass cannot be varied at will.

Thank you!

Note on Finite Width

- The finite width arises from an “absorptive” part in the self energy.
- No extra UV divergences here.
- The proofs of infrared finiteness and gauge independence go through if one finds the pole of the propagator in the complex plane.
- I still hear about people trying to take the real part, basing a mass on that, and putting the width back in by hand.
- Don’t do that. Don’t even socialize with those that do.