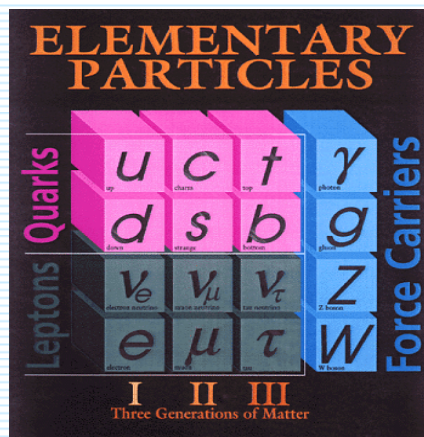


ISOLDE Workshop and Users meeting 2009
November 18-20, 2009, CERN

**Oscillations of lepton number, double electron
capture and atomic mass difference**

Fedor Šimkovic

**Comenius University, Bratislava
BLTP, JINR Dubna**



Standard Model

Lepton Universality

Particle	Symbol	Anti - p.	mass [MeV]	L_e	L_μ	L_τ	life - time [s]
electron	e^-	e^+	0.511	1	0	0	stable
el.neutrino	ν_e	$\bar{\nu}_e$	$< 2.2 \cdot 10^{-6}$	1	0	0	stable
muon	μ^-	μ^+	105.6	0	1	0	$2.2 \cdot 10^{-6}$
muon neutr.	ν_μ	$\bar{\nu}_\mu$	< 0.19	0	1	0	stable
tau	τ^-	τ^+	1777.	0	0	1	$2.9 \cdot 10^{-13}$
tau neutrino	ν_τ	$\bar{\nu}_\tau$	< 18.2	0	0	1	stable

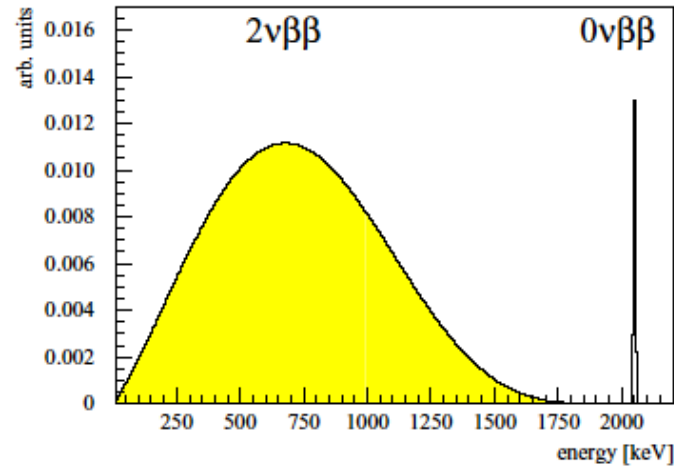
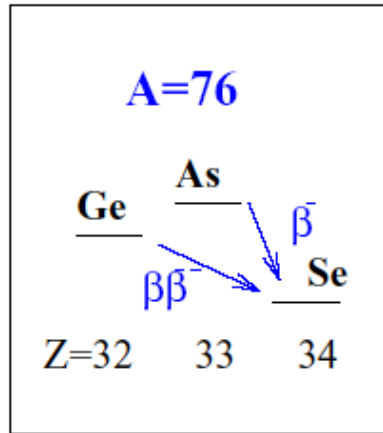
Lepton Family Number Violation

NEW PHYSICS massive neutrinos, SUSY...

Total Lepton Number Violation

$\nu_{e,\mu\tau} \leftrightarrow \nu_{e,\mu\tau}, \quad \bar{\nu}_{e,\mu\tau} \leftrightarrow \bar{\nu}_{e,\mu\tau}$	<i>observed</i>	$\nu_{e,\mu\tau} \leftrightarrow \bar{\nu}_{e,\mu\tau}$	<i>not observed</i>
$\mu^+ \rightarrow e^+ + \gamma$	$R \leq 1.2 \times 10^{-11}$	$K^+ \rightarrow \pi^- + e^+ + \mu^+$	$R \leq 5 \times 10^{-10}$
$\mu^+ \rightarrow e^+ + e^- + e^+$	$R \leq 1.0 \times 10^{-12}$	$\tau^- \rightarrow \pi^- + \pi^+ + e^+$	$R \leq 1.9 \times 10^{-6}$
$K^+ \rightarrow \pi^+ + e^- + \mu^+$	$R \leq 4.7 \times 10^{-12}$	$W^- + W^- \rightarrow e^- + e^-$	
$\tau^- \rightarrow e^- + \mu^+ + \mu^-$	$R \leq 1.8 \times 10^{-6}$	$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$	$T^{0\nu} \geq 1.9 \times 10^{-25}$
$Z^0 \rightarrow e^\pm + \mu^\mp$	$R \leq 1.7 \times 10^{-6}$	$\mu_b^- + (A, Z) \rightarrow (A, Z-2) + e^+$	$R \leq 3.6 \times 10^{-11}$
$\mu_b^- + (A, Z) \rightarrow (A, Z) + e^-$	$R \leq 1.2 \times 10^{-11}$	$e^- + e^- \rightarrow \pi^- + \pi^-$?

$\beta\beta$ -decay



$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$

Observed for 10 isotopes: ^{48}Ca , ^{76}Ge , ^{82}Se , ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{128}Te , ^{130}Te , ^{150}Nd , ^{238}U , $T_{1/2} \approx 10^{18}-10^{24}$ years

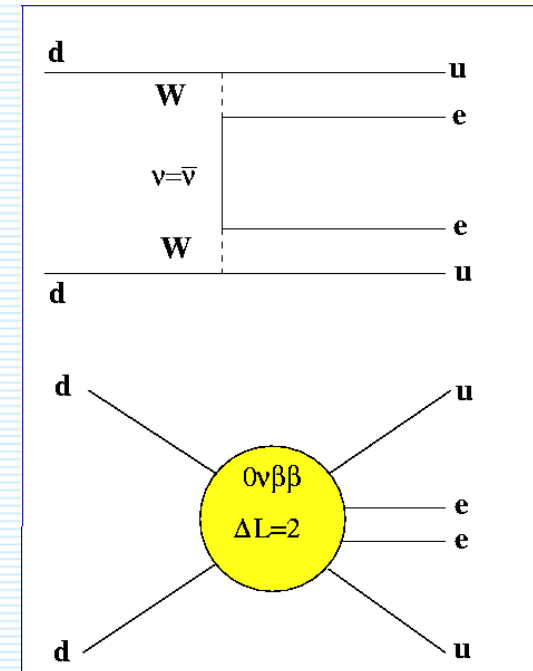
1967: ^{130}Te , Kirsten et al, Takaoka et al, (geochemical)

1987: ^{82}Se , Moe et al. (direct observation)

2008: ^{100}Mo , NEMO 3 coll. $\sim 300\,000$ events

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$

SM forbidden ,not observed yet: $T_{1/2} (^{76}\text{Ge}) > 10^{25}$ years



The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.


$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z) |M'^{0\nu}|^2 |\langle m_{\beta\beta} \rangle|^2,$$

$$m_{\beta\beta} = \sum_{i=1}^3 U_{ei}^2 m_i$$

Absolute ν mass scale

Normal or inverted Hierarchy of ν masses

CP-violating phases

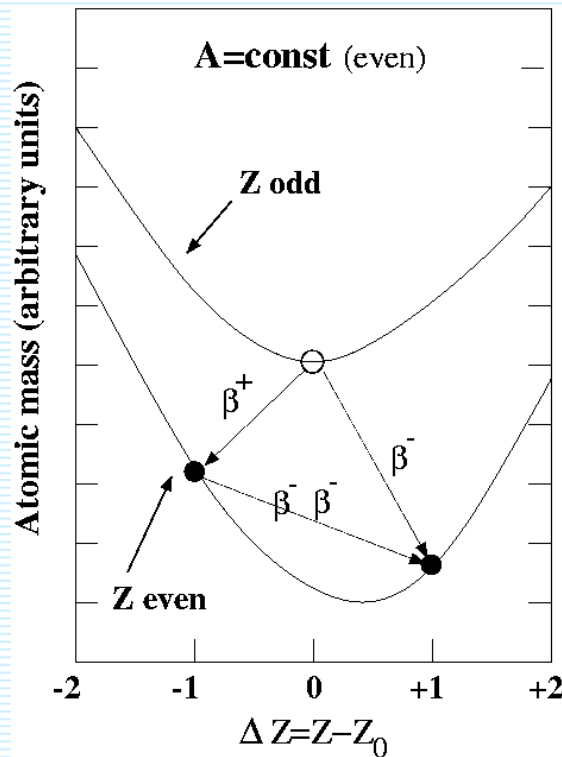


$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei

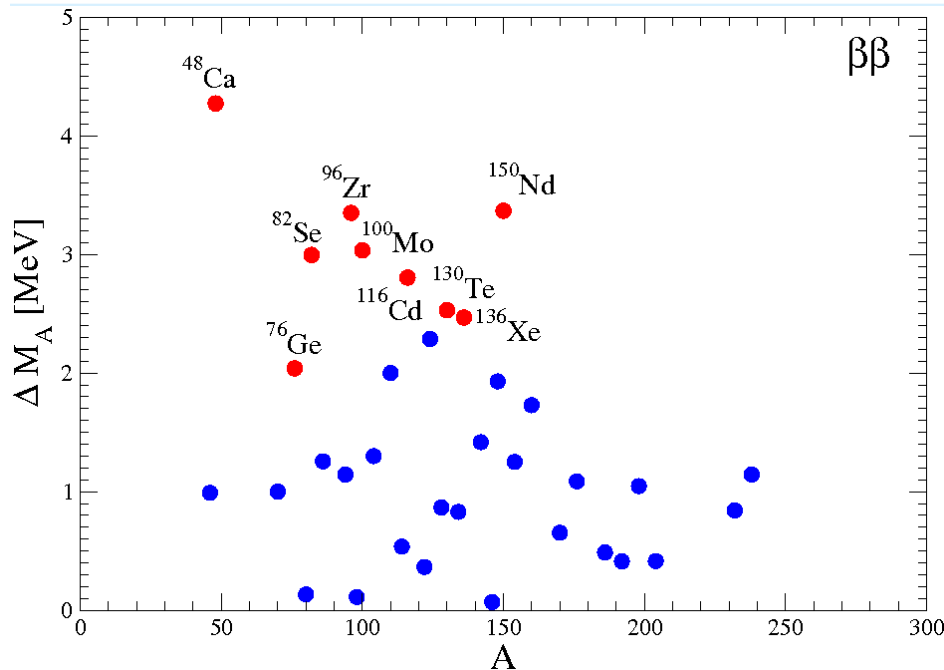


$\beta\beta$ emitters with $Q_{\beta\beta} > 2$ MeV

Transition	$Q_{\beta\beta}$ (keV)	Abundance (%) ($^{232}\text{Th} = 100$)
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	2013	12
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2040	8
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	2288	6
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2479	9
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2533	34
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2802	7
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2995	9
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3034	10
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3350	3
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3667	6
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4271	0.2

Double Beta Decay Nuclei of experimental interest

Emission of two electrons
 $(A,Z) \rightarrow (A,Z+2) + e + e$

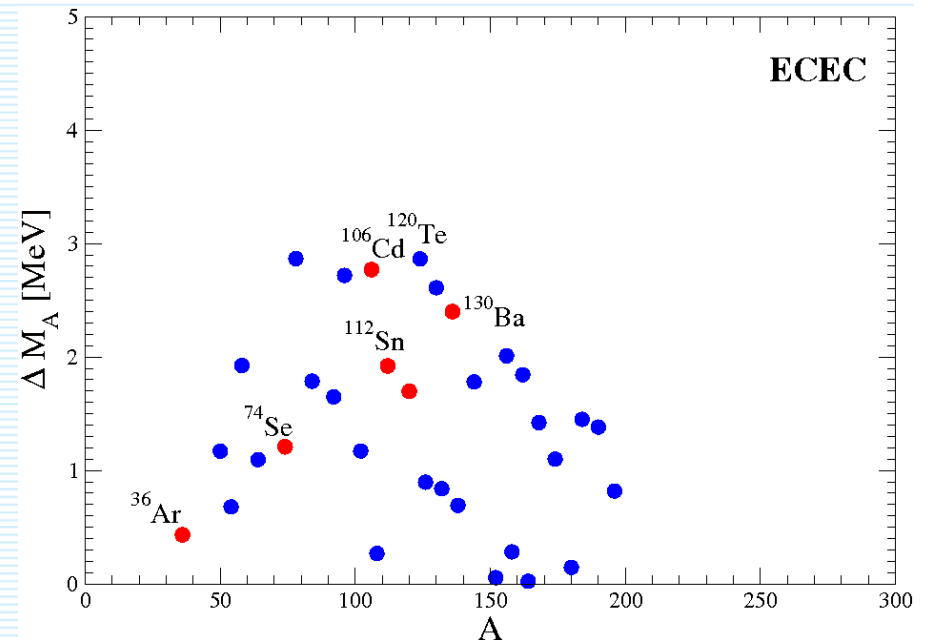


Preferable nuclear systems
 with large ΔM_A (E^5)

11/19/2009

Fedor Simkovic

Double electron capture
 $e_b + e_b + (A,Z) \rightarrow (A,Z)^*$



Nuclear systems with small
 ΔM_A might be also
 important (resonant
 enhancement)
 Signal from γ - and X-rays

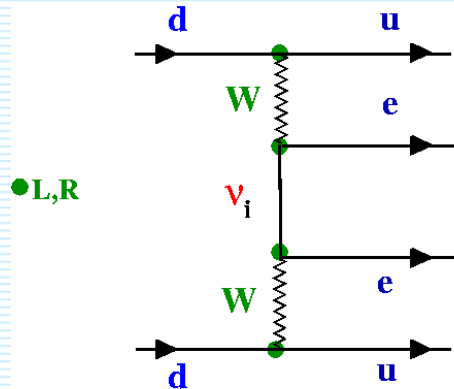
The $0\nu\beta\beta$ -decay mechanisms

Two basic categories are
long-range (exchange of light Majorana ν)
and
short-range (exchange of heavy ν , squarks, gluinos ...)
contributions to the $0\nu\beta\beta$ -decay

Light neutrino Mass mechanism

$$\mathcal{H}_W^\beta = \frac{G_F}{\sqrt{2}} \bar{e} \gamma_\alpha (1 + \gamma_5) \nu_e j_\alpha + h.c.$$

$$\nu_{eL} = \sum_k U_{lk}^L \chi_{kL}$$



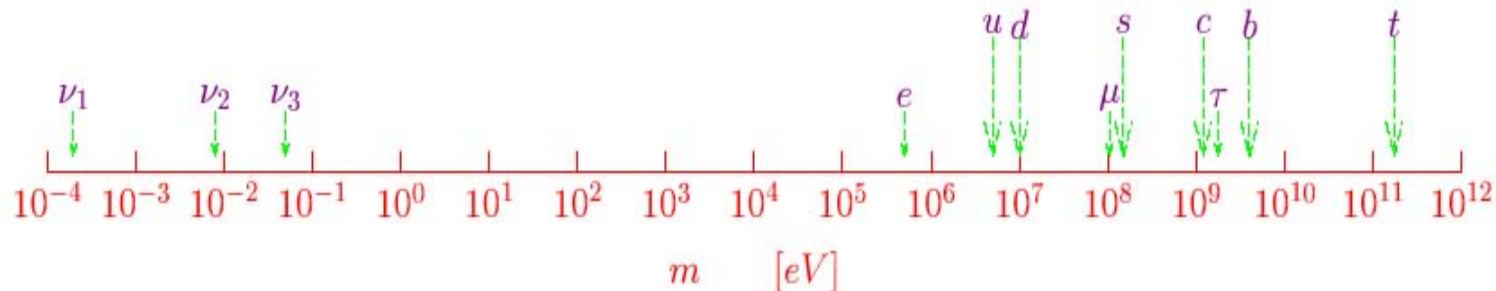
$$\begin{aligned} \langle \nu_{eL}(x_1) \nu_{eL}^T(x_2) \rangle &= - \sum_k (U_{ek}^L)^2 \xi_k \frac{1 + \gamma_5}{2} S_k(x_1 - x_2) \frac{1 + \gamma_5}{2} C \\ &= \frac{i}{(2\pi)^4} \sum_k (U_{ek}^L)^2 \xi_k m_k \int \frac{e^{iq(x_1 - x_2)} dq}{q^2 + m_k^2} \frac{1 + \gamma_5}{2} C \end{aligned}$$

Effective mass of
Majorana neutrinos

$$m_{\beta\beta} = \sum_k (U_{ek}^L)^2 \xi_k m_k$$

$$\begin{pmatrix} \bar{\nu}_L & \overline{(\nu_R)^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}$$

$$\begin{aligned} m_1 &= m_D^2 / M_R \ll m_D & m_2 &\approx M_R \\ \nu_1 &= \nu_L - m_D / M_R (\nu_R)^c & \nu_2 &= \nu_R + m_D / M_R (\nu_L)^c \end{aligned}$$



Squark mixing RPV SUSY

Neutrino vertex

$$\mathcal{L}^{LH} = \frac{G_F}{\sqrt{2}} \sum_i U_{ei} (\bar{e} \gamma_\alpha (1 - \gamma_5) \nu) (\bar{u} \gamma^\alpha (1 - \gamma_5) d) + h.c. \quad (V - A)$$

R-parity violating SUSY vertex

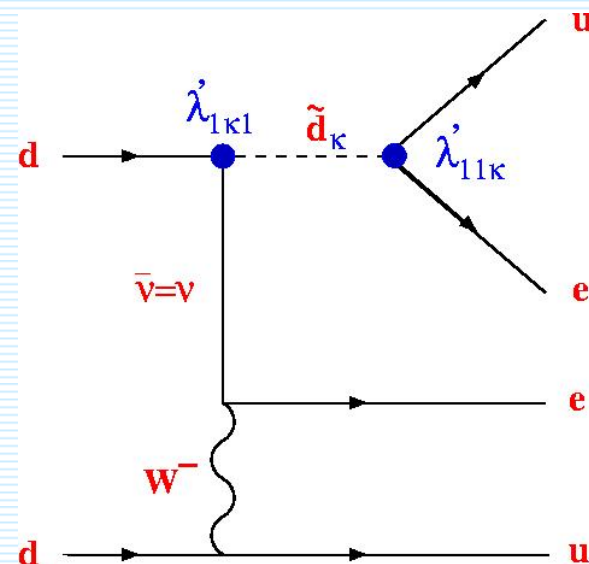
Hirsch, Klapdor-Kleingrothaus, Kovalenko
PLB 372 (1996) 181

$$\begin{aligned} \mathcal{L}_{SUSY}^{eff} = & \frac{G_F}{\sqrt{2}} \left(\frac{1}{4} \eta_{(q)LR} \sum_i U_{ei}^* (\bar{\nu} (1 + \gamma_5) e) (\bar{u} (1 + \gamma_5) d) \right. \\ & \left. + \frac{1}{8} \eta_{(q)LR} \sum_i U_{ei}^* (\bar{\nu} \sigma_{\alpha\beta} (1 + \gamma_5) e) (\bar{u} \sigma^{\alpha\beta} (1 + \gamma_5) d) + h.c. \right) \end{aligned} \quad \begin{matrix} (S, P) \\ (Tensor) \end{matrix}$$

Paes, Hirsch, Klapdor-Kleingrothaus,
PLB 459 (1999) 450

LN-violating parameter

$$\eta_{(q)LR} = \sum_k \frac{\lambda'_{11k} \lambda'_{1k1}}{8\sqrt{2}G_F} \sin 2\theta_{(k)}^d \left(\frac{1}{m_{\tilde{d}_1(k)}^2} - \frac{1}{m_{\tilde{d}_2(k)}^2} \right)$$



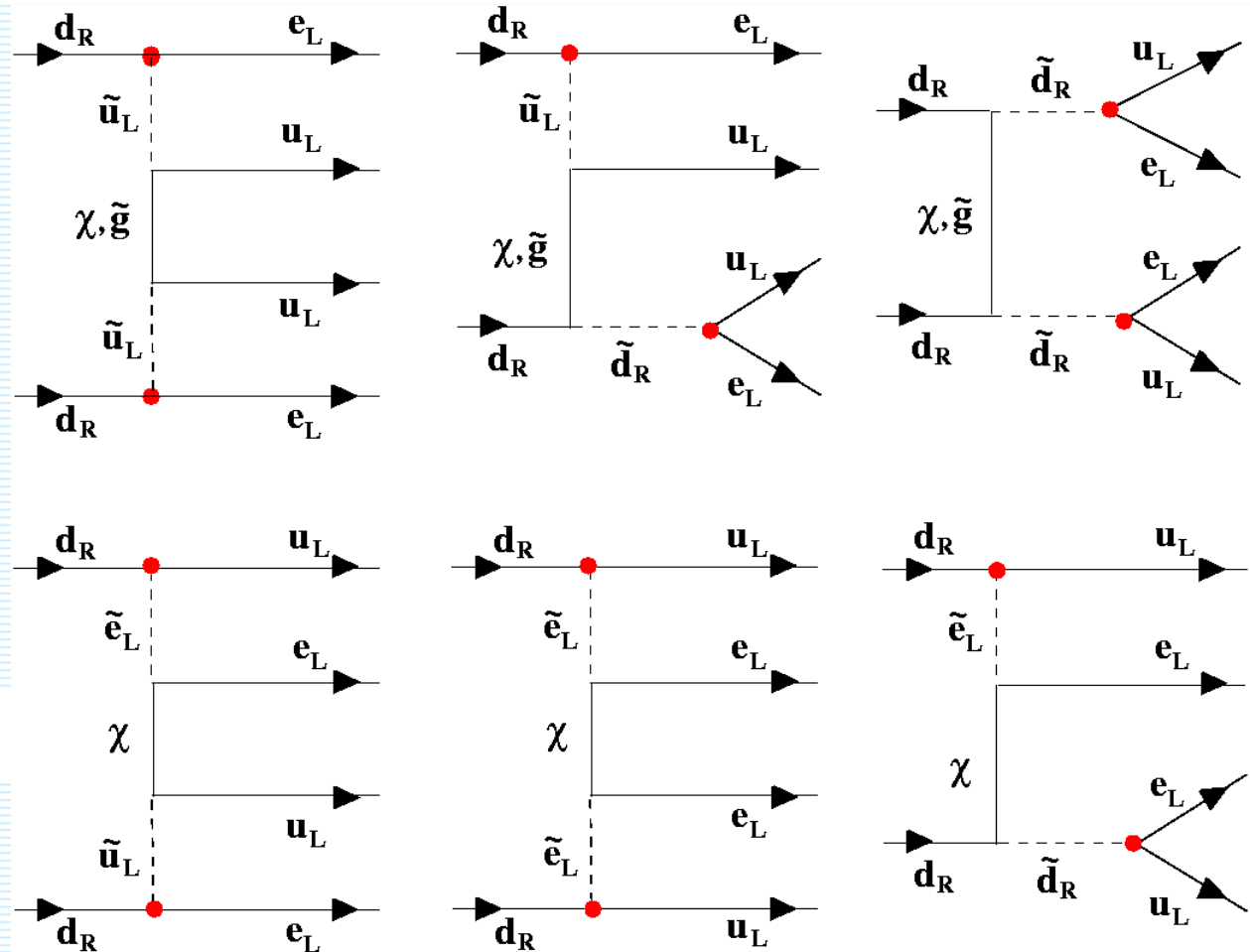
gluino/neutralino exchange R-parity breaking SUSY mechanism of the $0\nu\beta\beta$ -decay

quark-level diagrams

$$d+d \rightarrow u + u + e^- + e^-$$

exchange of
squarks,
neutralinos
and
gluinos

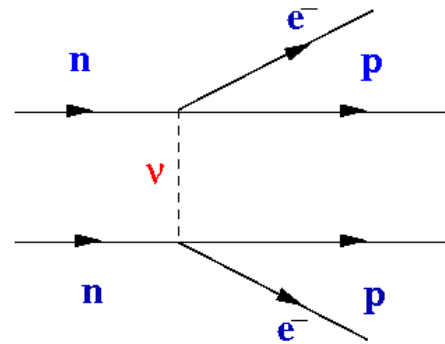
$(\lambda'_{111})^2$ mechanism



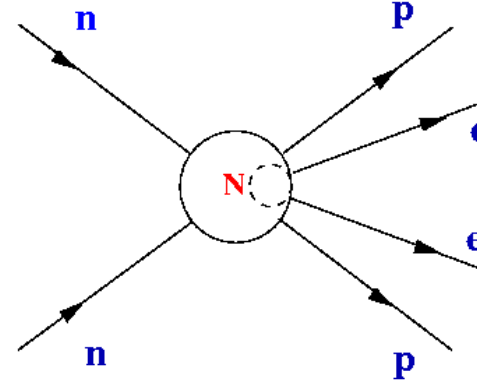
• R-parity violation

nucleon level

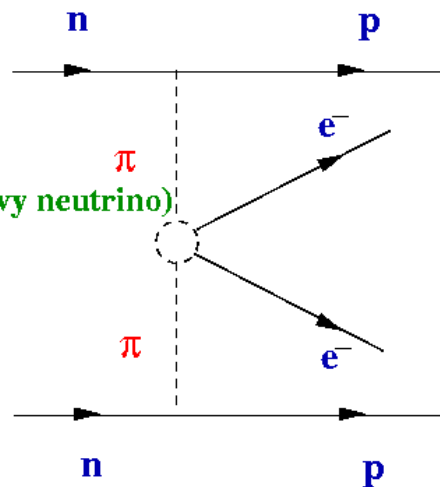
Light neutrino exchange



Heavy neutrino exchange



two-pion exchange (heavy neutrino)



The $0\nu\beta\beta$ -decay NMEs

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+ , 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$ -decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge the quality of the result.

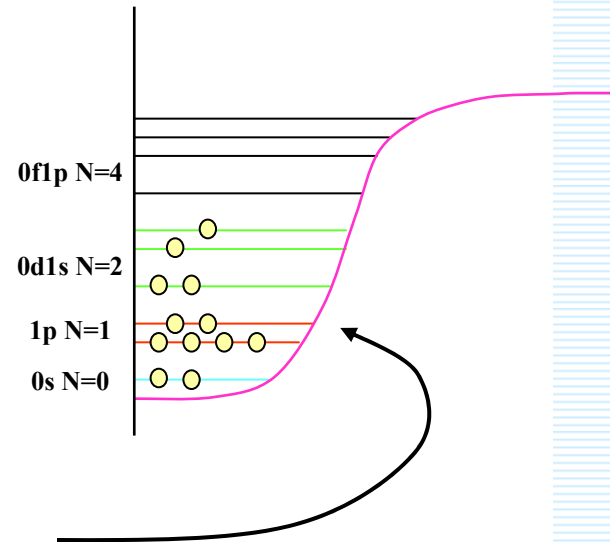
Many-body Hamiltonian

- Start with the many-body Hamiltonian

$$H = \sum_i \frac{\vec{p}_i^2}{2m} + \sum_{i < j} V_{NN}(\vec{r}_i - \vec{r}_j)$$

- Introduce a mean-field U to yield basis

$$H = \sum_i \left(\frac{\vec{p}_i^2}{2m} + U(r_i) \right) + \underbrace{\sum_{i < j} V_{NN}(\vec{r}_i - \vec{r}_j) - \sum_i U(r_i)}_{\text{Residual interaction}}$$



The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- The **mean field** determines the shell structure
- In effect, nuclear-structure calculations rely on **perturbation theory**

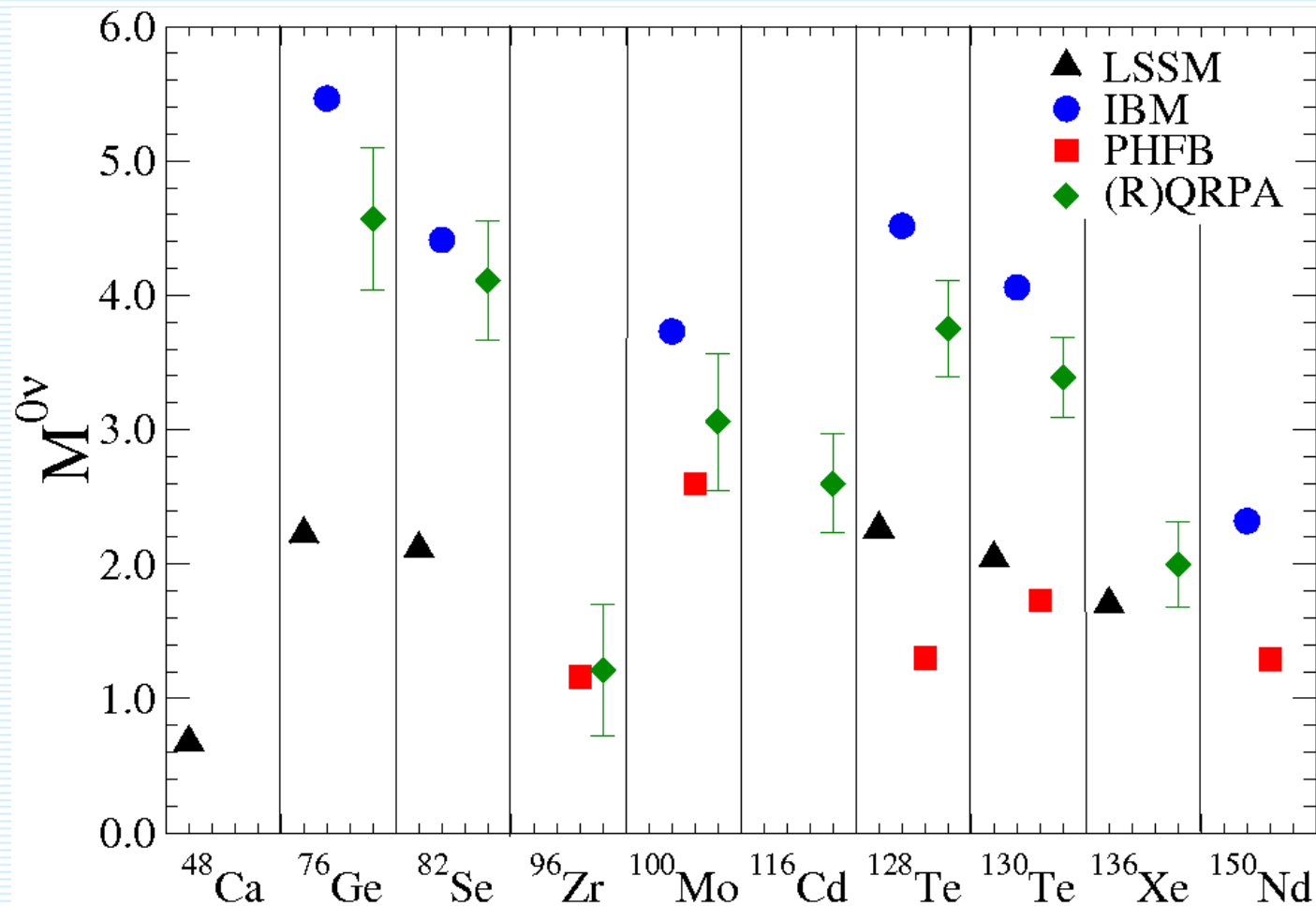
Two complementary procedures are commonly used:

- **Nuclear shell model (NSM)**
- **Quasiparticle Random Phase Approximation (QRPA)**

*In **NSM** a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$ -decay calculations*

*In **QRPA** a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more $0nbb$ -decay calculations*

The $0\nu\beta\beta$ -decay NMEs (Status:2009)



Nobody is perfect:

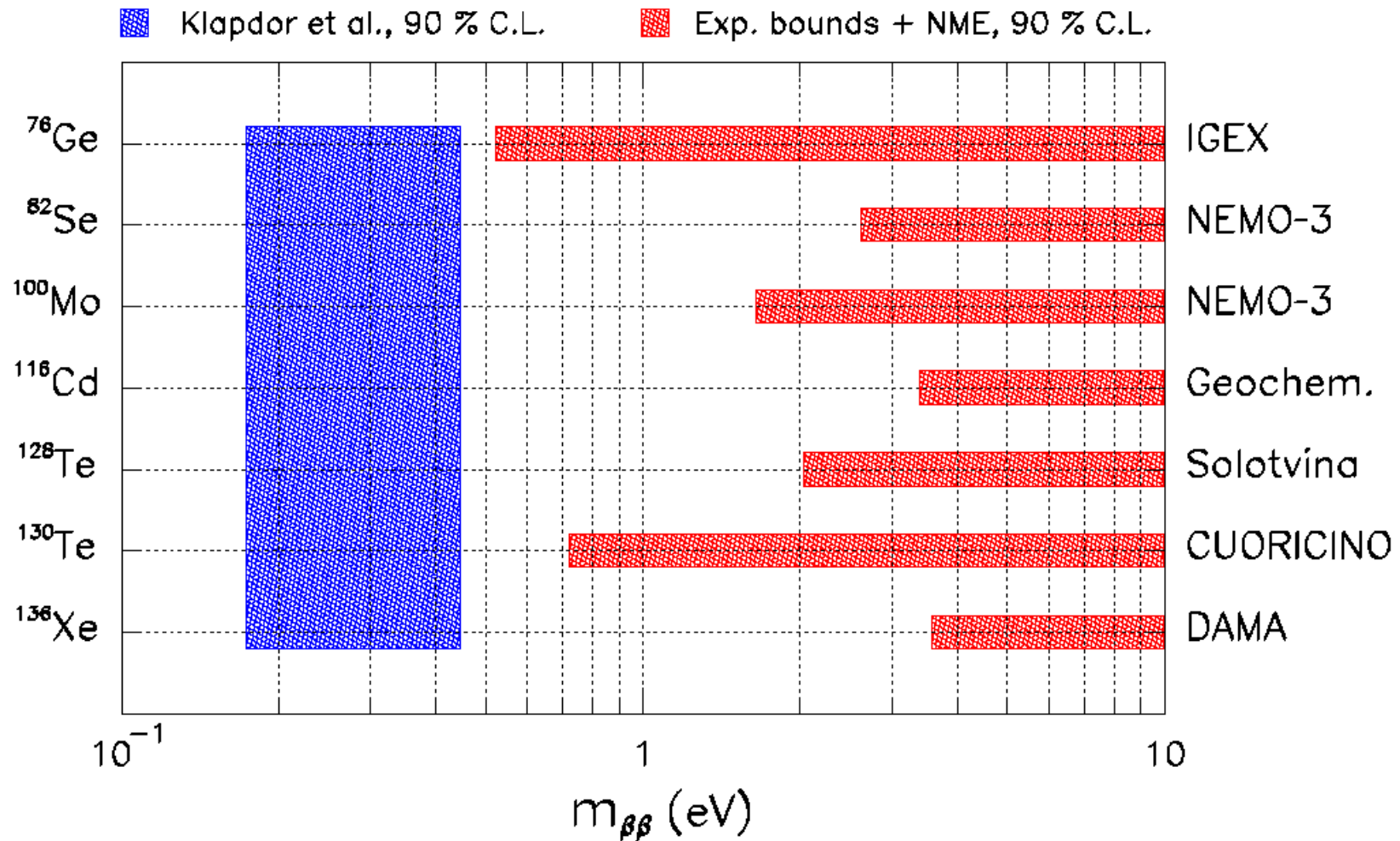
LSSM (small m.s., negative parity states)

PHFB (GT force neglected)

IBM (Hamiltonian truncated)

(R)QRPA (g.s. correlations not accurate enough)

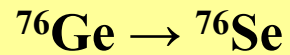
A claim of evidence and other experiments (current status)



Constraining the $0\nu\beta\beta$ -decay NMEs

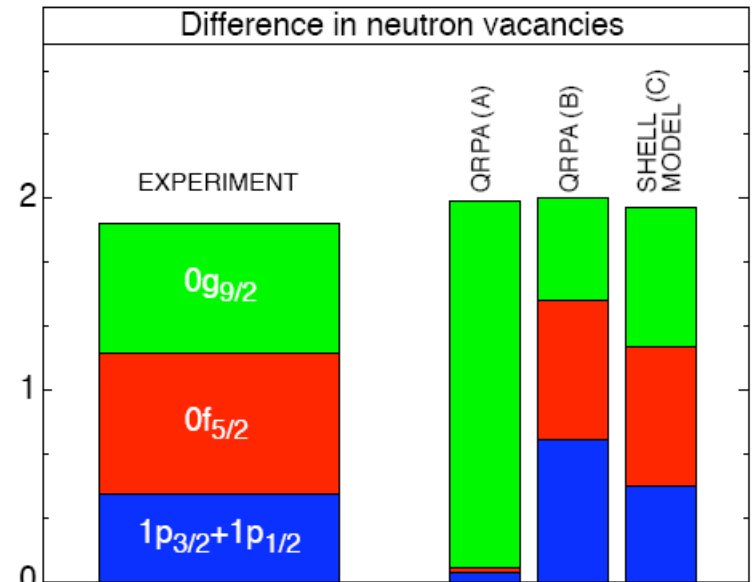
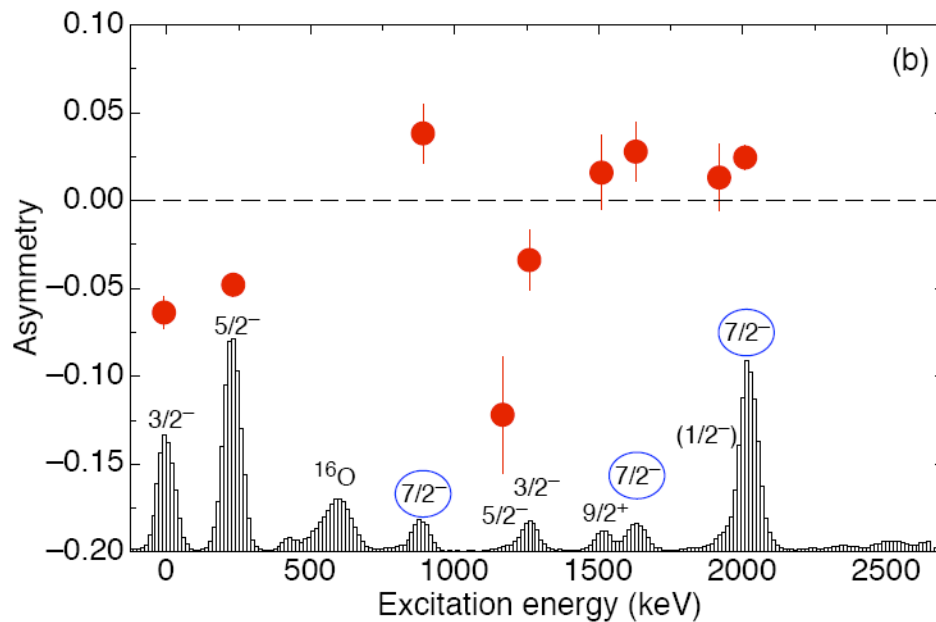
*Nucleons that change from neutrons to protons
are valence neutrons*

Proton,
neutron
removing
transfer reaction

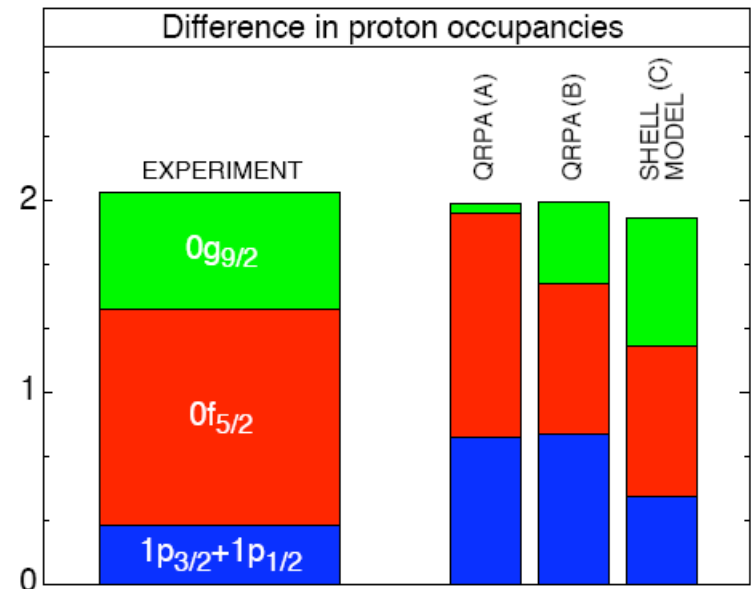


John Schiffer,
P. Grabmayr *et al*

$$n_j^{exp} = \langle 0_{init}^+ | \sum_m c_{j,m}^+ c_{j,m} | 0_{init}^+ \rangle$$



Kay *et al.*, PRC **79**, 021301 (2009)



QRPA(A) \equiv BCS (WS)

QRPA(B) \equiv BCS (AWS) Suhonen, Civitarese, *ovic*
PLB **668**, 277 (2008)

How can we take into account theoretically the constraint represented by the experimentally determined occupancies?

**The experiment fixes
for the final nucleus**

$$n_j^{\text{exp}} = \langle 0^+_{\text{init}} | \sum c_{j,m}^+ c_{j,m} | 0^+_{\text{init}} \rangle \text{ and the same}$$

particle creation and
annihilation operators

In BCS $n_j^{\text{BCS}} = v_j^2 \times (2j+1)$ depends only on v_j which in turn depends on the mean field eigenenergies

In QRPA the ground state includes correlations and thus

$$n_j^{\text{QRPA}} = (2j+1) \times [v_j^2 + (u_j^2 - v_j^2) \xi_j]$$

$$\xi_j = (2j+1)^{-1/2} \langle 0^+_{\text{qrpa}} | [a_j^+ a_j]^0 | 0^+_{\text{qrpa}} \rangle \text{ depends on the quasiparticle content of the correlated ground state}$$

quasiparticle creation and
annihilation operators

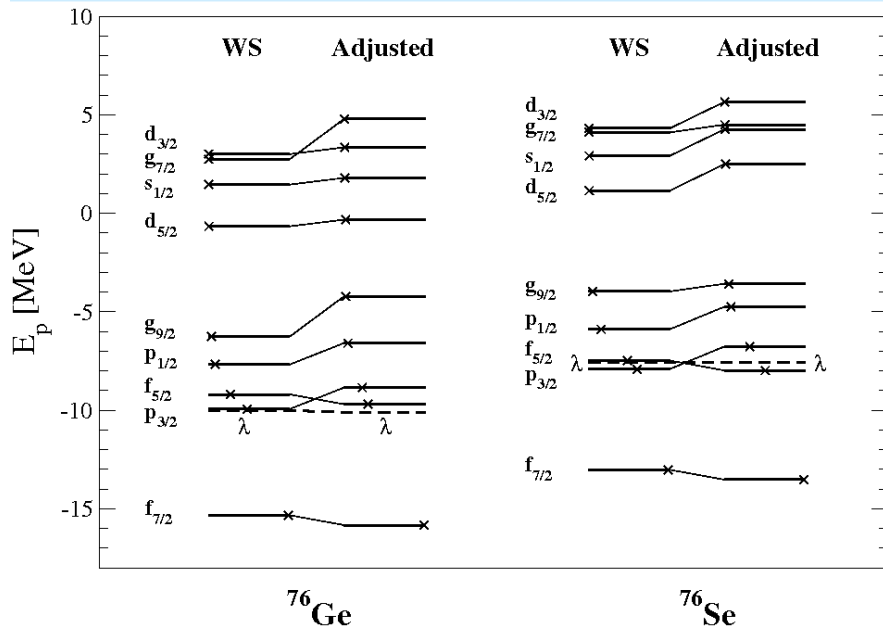
Initial and adjusted mean field levels

While n_j^{exp} and n_j^{BCS} are constrained by $\sum n_j = N$ (or Z) the n_j^{QRPA} are not constrained by that requirement. The particle number is not conserved, even on average. Thus the QRPA must be modified to remedy this \Rightarrow **Selfconsistent Renormalized QRPA**

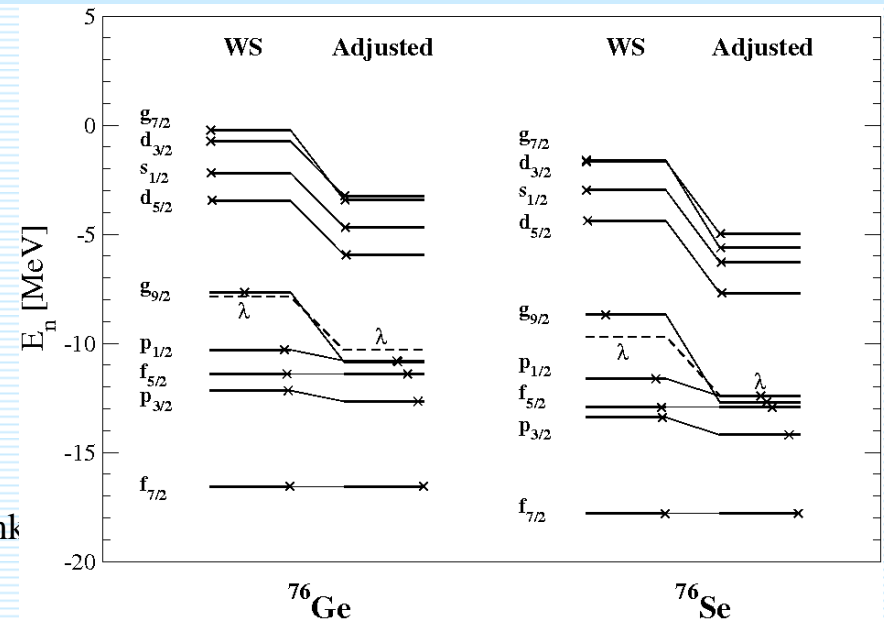
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	prev.	new
Jastrow s.r.c.	4.24(0.44)	3.49(0.23)
UCOM s.r.c.	5.19(0.54)	4.60(0.39)

	^{76}Ge				^{76}Se			
neut.	BCS	Q	S	exp	BCS	Q	S	exp
p	5.65	5.27	4.64	4.9 ± 0.2	5.57	5.05	4.12	4.4 ± 0.2
$f_{5/2}$	5.54	5.12	4.34	4.6 ± 0.4	5.53	5.00	3.63	3.8 ± 0.4
$f_{7/2}$	7.91	7.67	7.62	-	7.90	7.54	7.37	-
$s_{1/2}$	0.01	0.05	0.07	-	0.01	0.04	0.08	-
$d_{3/2}$	0.03	0.14	0.15	-	0.02	0.14	0.16	-
$d_{5/2}$	0.09	0.30	0.36	-	0.07	0.27	0.39	-
$g_{7/2}$	0.14	0.53	0.48	-	0.12	0.56	0.58	-
$g_{9/2}$	4.63	4.78	6.35	6.5 ± 0.3	2.78	3.55	5.66	5.8 ± 0.3
prot.	BCS	Q	S	exp	BCS	Q	S	exp
p	2.23	2.34	1.75	1.77 ± 0.15	2.77	2.76	2.28	2.08 ± 0.15
$f_{5/2}$	1.61	2.27	2.08	2.04 ± 0.25	2.95	2.97	3.03	3.16 ± 0.25
$f_{7/2}$	7.83	7.19	7.13	-	7.76	7.12	7.06	-
$s_{1/2}$	0.00	0.02	0.03	-	0.00	0.03	0.04	-
$d_{3/2}$	0.01	0.07	0.07	-	0.01	0.09	0.09	-
$d_{5/2}$	0.01	0.12	0.15	-	0.02	0.17	0.18	-
$g_{7/2}$	0.02	0.19	0.16	-	0.03	0.31	0.27	-
$g_{9/2}$	0.29	0.85	0.62	0.23 ± 0.25	0.46	1.15	1.04	0.84 ± 0.25

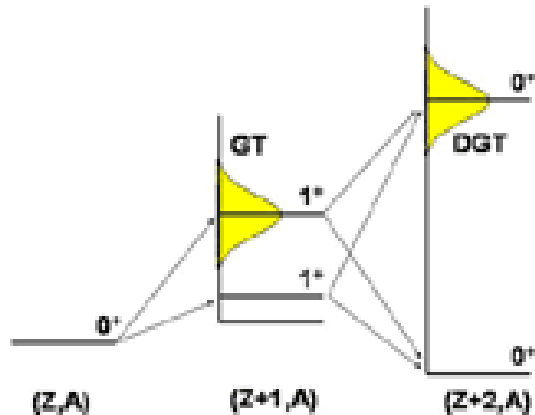
F.Š., A. Faessler, P. Vogel, PRC 79, 015502 (2009)



dor Simk



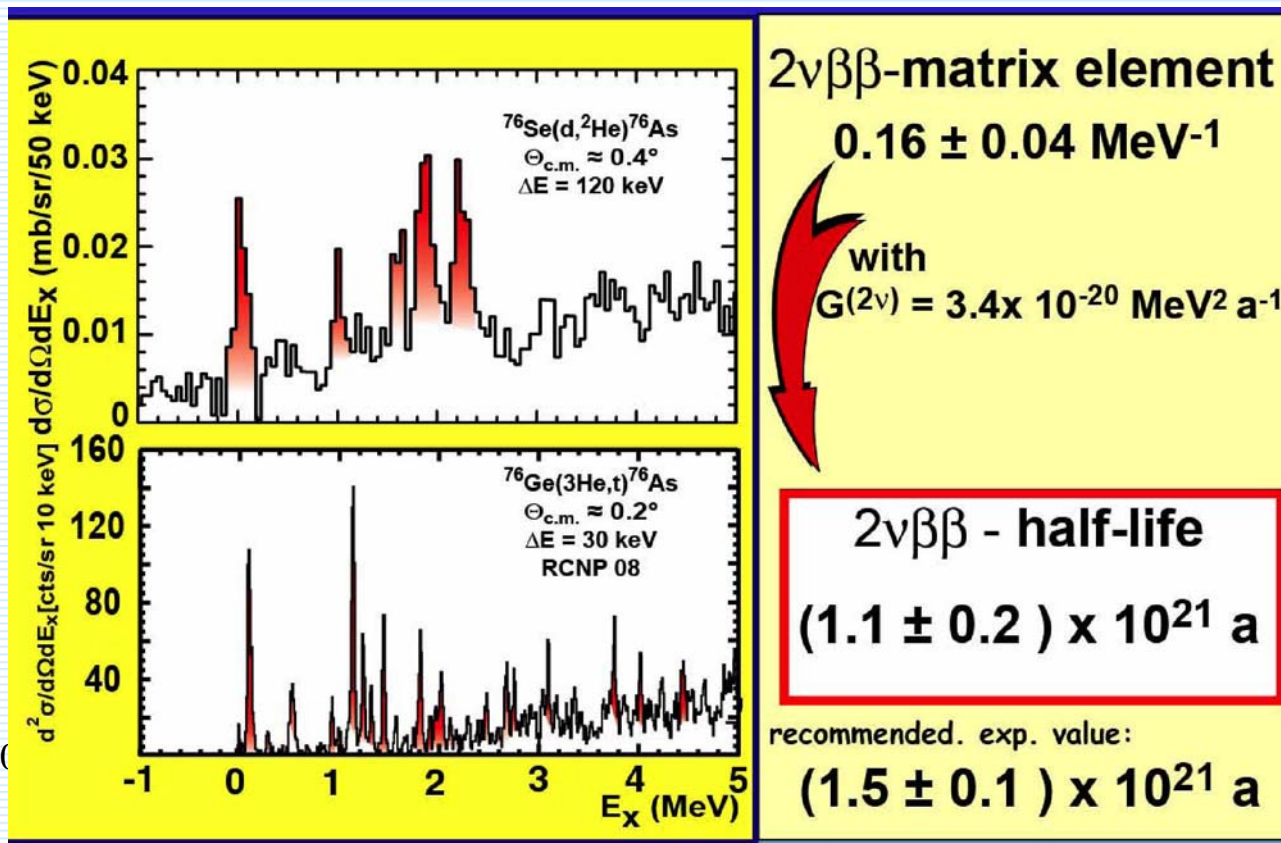
Constraining the $0\nu\beta\beta$ -decay NME



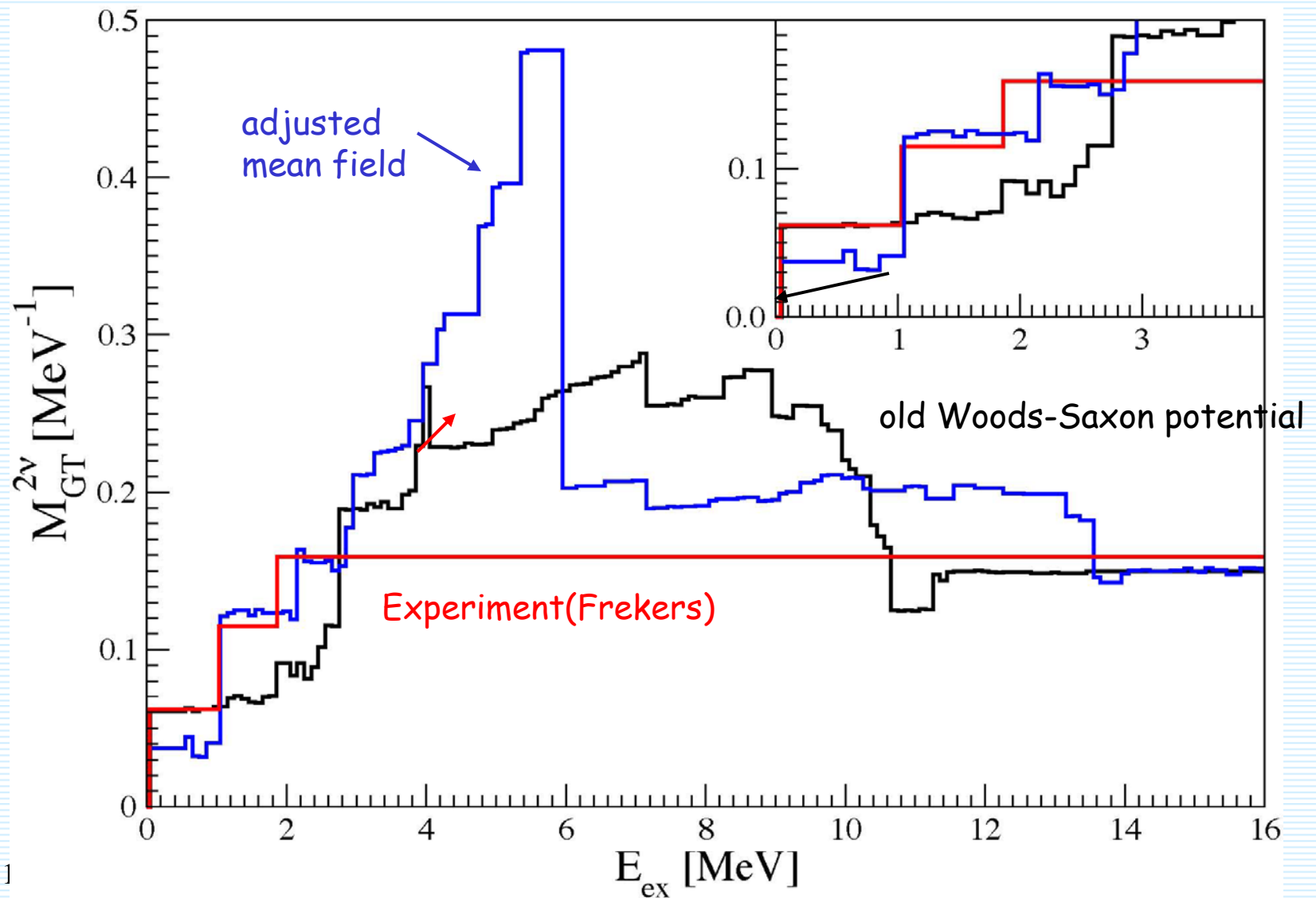
charge-exchange
reactions

(t, ^3He)
(d, ^2He)

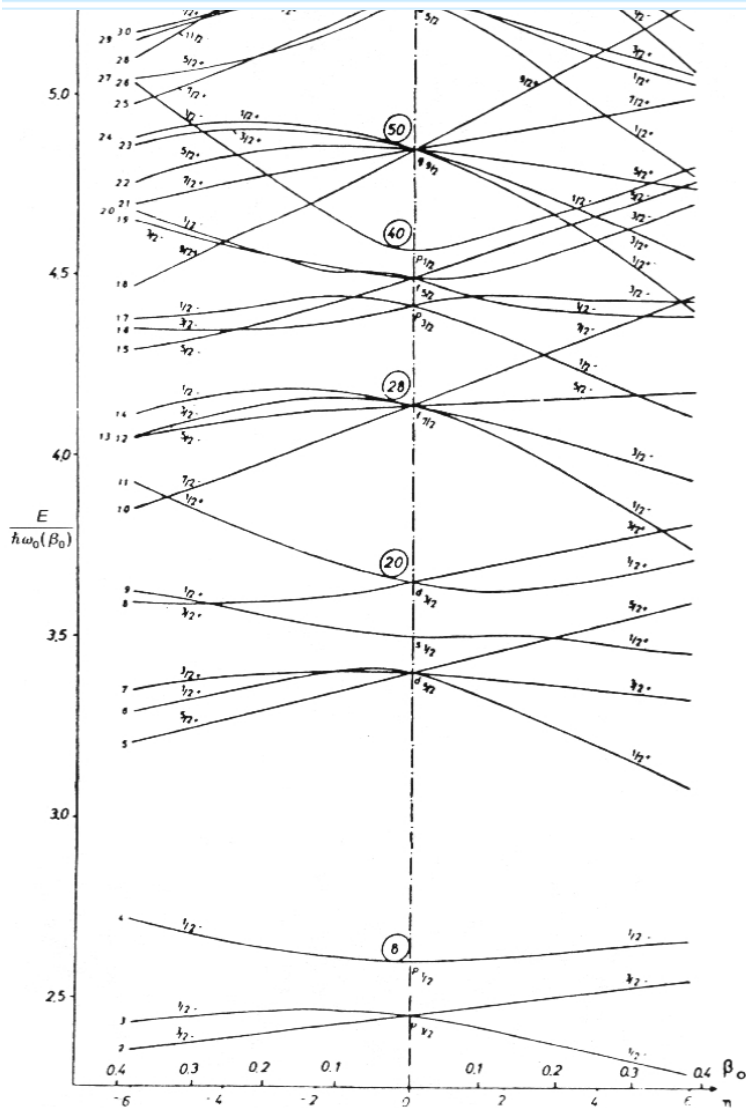
From D. Frekers, RIKEN 2008 lecture
The cross sections give $B(\text{GT})$ for β^+ and β^- ,
product of the amplitudes $(B(\text{GT})^{1/2})$ gives
the numerator of the $M^{2\nu}$ matrix element.



Staircase plot (running sum) of the contributions to the $2\nu\beta\beta$ decay ($^{76}\text{Ge} \rightarrow ^{76}\text{Se}$)



Shell structure of the mean field changed



Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

Exp. I (nuclear reorientation method)

Exp.II (based on measured E2 trans.)

Theor. I (Rel. mean field theory)

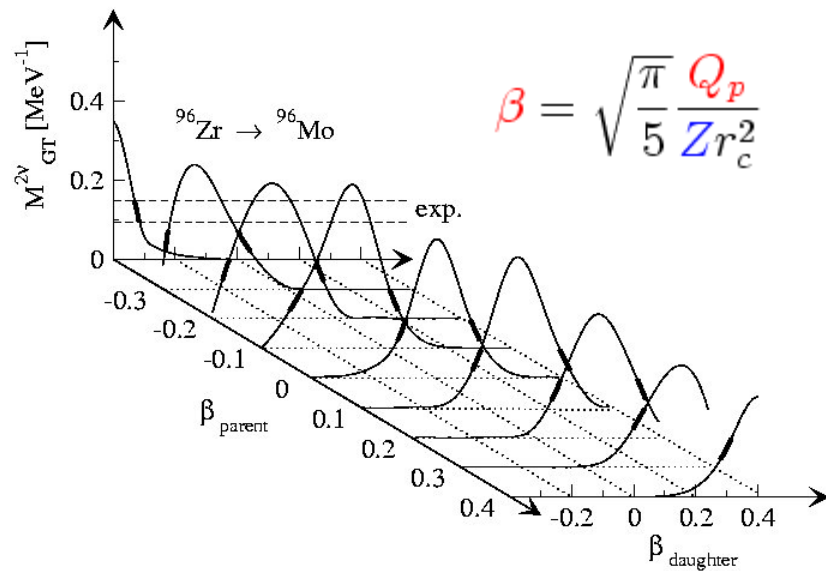
Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)

Till now, in the QRPA-like calculations of the $0\nu\beta\beta$ -decay NME spherical symmetry was assumed

The effect of deformation on NME has to be considered

Nucl.	Exp. I	Exp. II	Theor. I	Theor. II
⁴⁸ Ca	0.00	0.101	0.00	0.00
⁴⁸ Ti	+0.17	0.269	-0.01	0.00
⁷⁶ Ge	+0.09	0.26	0.16	0.14
⁷⁶ Se	+0.16	0.31	-0.24	-0.24
⁸² Se	+0.10	0.19	0.13	0.15
⁸² Kr		0.20	0.12	0.07
⁹⁶ Zr		0.081	0.22	0.22
⁹⁶ Mo	+0.07	0.17	0.17	0.08
¹⁰⁰ Mo	+0.14	0.23	0.25	0.24
¹⁰⁰ Ru	+0.14	0.22	0.19	0.16
¹¹⁶ Cd	+0.11	0.19	-0.26	-0.24
¹¹⁶ Sn	+0.04	0.11	0.00	0.00
¹²⁸ Te	+0.01	0.14	-0.00	0.00
¹²⁸ Xe		0.18	0.16	0.14
¹³⁰ Te	+0.03	0.12	0.03	0.00
¹³⁰ Xe		0.17	0.13	-0.11
¹³⁶ Xe		0.09	0.00	0.00
¹³⁶ Ba		0.12	0.00	0.00
¹⁵⁰ Nd	+0.37	0.28	0.22	0.24
¹⁵⁰ Sm	+0.23	0.19	0.18	0.21

New Suppression Mechanism of the DBD NME



The suppression of the NME depends on
relative deformation of initial and final nuclei

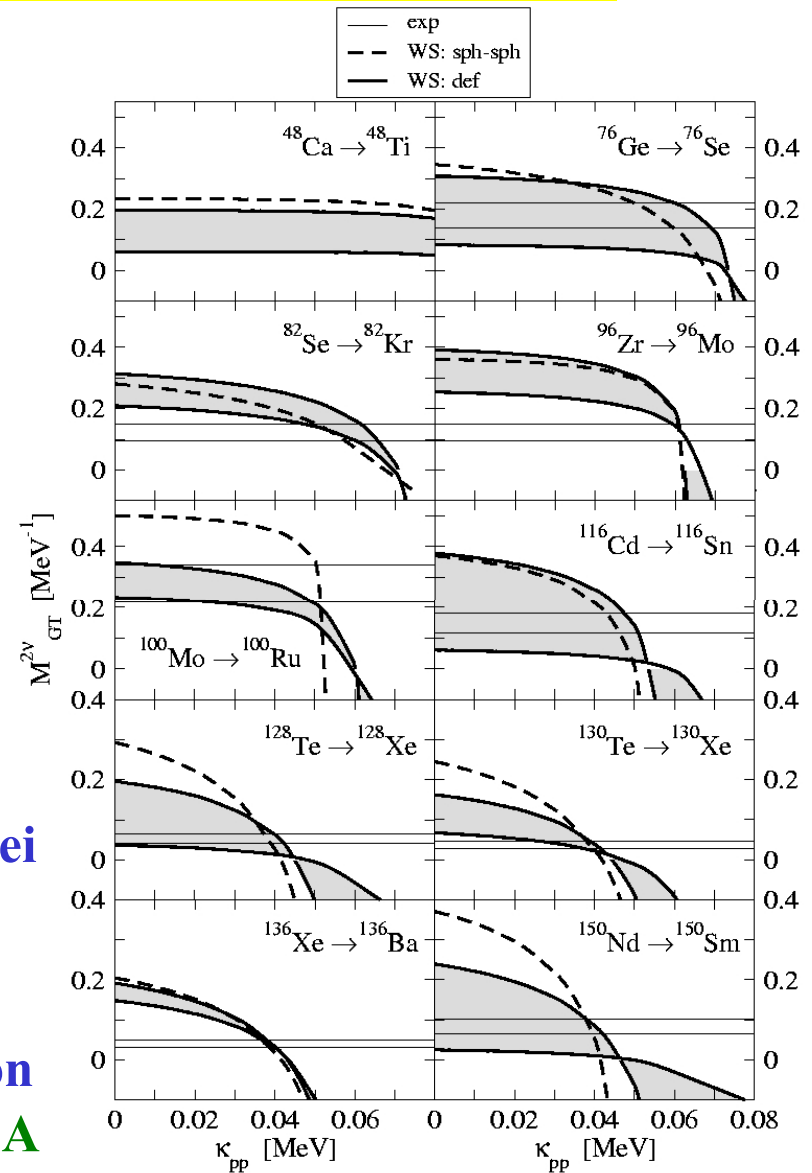
F.Š., Pacearescu, Faessler.

NPA 733 (2004) 321

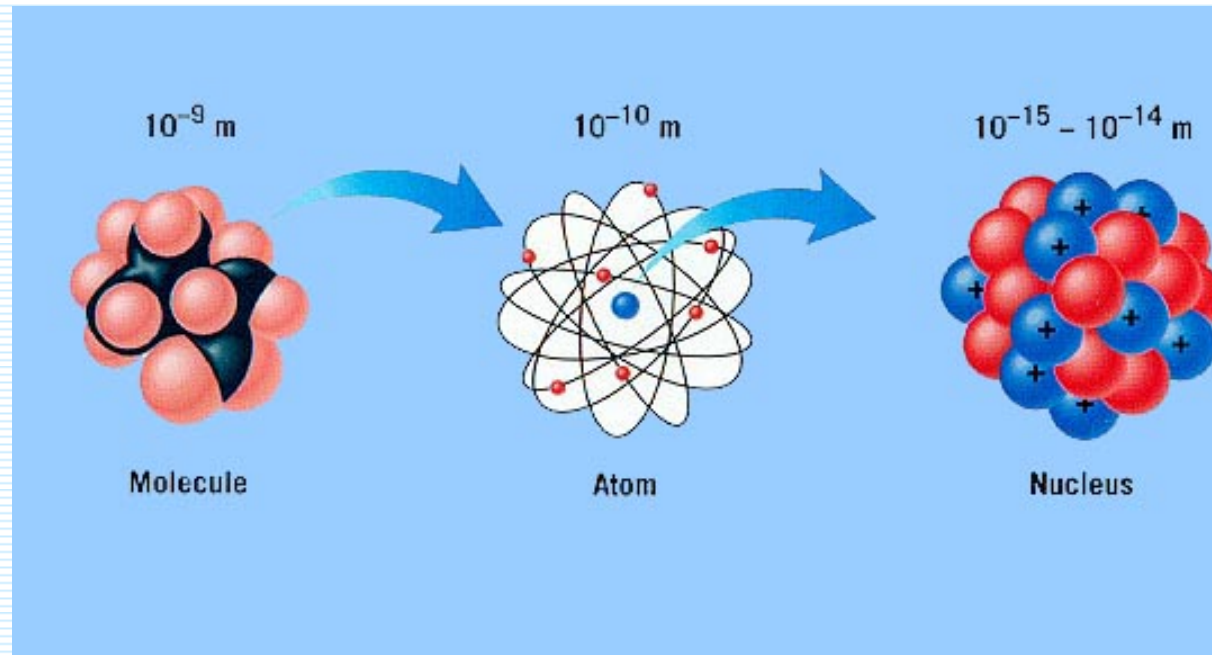
Systematic study of the deformation effect on
the $2\nu\beta\beta$ -decay NME within deformed QRPA

Alvarez, Sarriguren, Moya, Pacearescu, Faessler, F.Š.,

Phys. Rev. C 70 (2004) 321



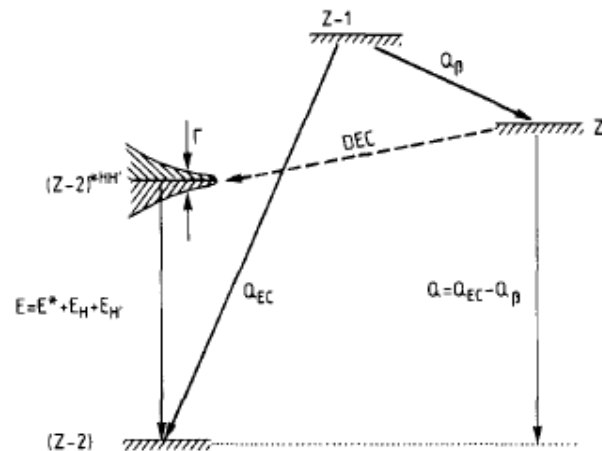
Neutrinoless Double Electron Capture



Neutrinoless double electron capture (resonance transitions) $(A,Z) \rightarrow (A,Z-2)^{*HH'}$

J. Bernabeu, A. DeRujula, C. Jarlskog,
Nucl. Phys. B 223, 15 (1983)

DEC transitions, abundance, daughter nuclear excitation, atomic vacancies
and figure of merit of some isotopes [10]



Atom mixing amplitude ΔM

$$E \simeq E^* + E_H + E_{H'},$$

$$\Gamma \simeq \Gamma^* + \Gamma_H + \Gamma_{H'}.$$

Decay rate

$$\frac{1}{\tau} \simeq \frac{(\Delta M)^2}{(Q - E)^2 + \frac{1}{4}\Gamma^2} \Gamma,$$

2νECEC-background
depends strongly
on Q-value

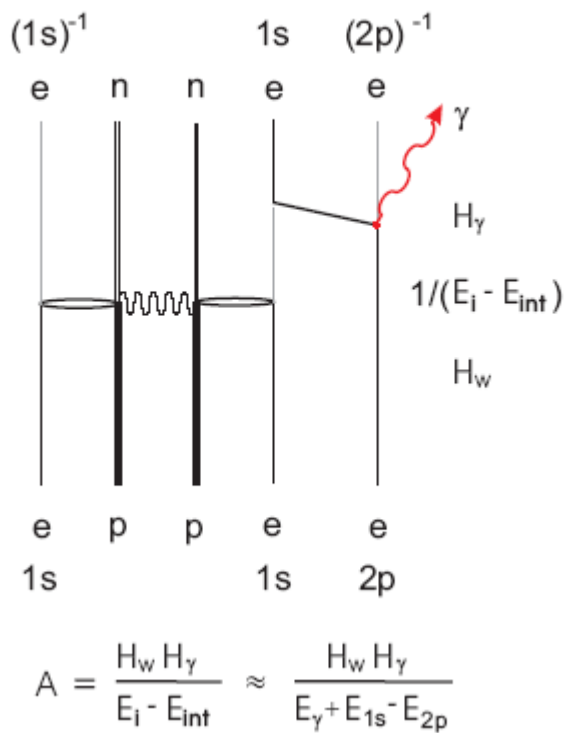
Transition $Z \rightarrow Z - 2$	Z-natural abundance in %	Nuclear excitation E^* (in MeV), J^P	Atomic vacancies H, H'	Figure of merit $Q - E$ (in keV)
$^{74}_{34}\text{Se} \rightarrow ^{74}_{32}\text{Ge}$	0.87	1.204 (2^+)	2S(P), 2S(P)	2 ± 3
$^{78}_{36}\text{Kr} \rightarrow ^{78}_{34}\text{Se}$	0.36	2.839 (2^+) 2.864 (?)	1S, 1S	$^{19}_{-6} \pm 10$
$^{102}_{46}\text{Pd} \rightarrow ^{102}_{44}\text{Ru}$	1	1.103 (2^+) 1.107 (4^+)	1S, 1S	$^{29}_{25} \pm 9$
$^{106}_{48}\text{Cd} \rightarrow ^{106}_{46}\text{Pd}$	1.25	2.741 (?)	1S, 1S	-8 ± 10
$^{112}_{50}\text{Sn} \rightarrow ^{112}_{48}\text{Cd}$	1.01	1.871 (0^+)	1S, 1S	-3 ± 10
$^{130}_{56}\text{Ba} \rightarrow ^{130}_{54}\text{Xe}$	0.11	2.502 (?) 2.544 (?)	1S, 1S 1S, 2S(P)	$^{8}_{-6} \pm 13$
$^{152}_{64}\text{Gd} \rightarrow ^{152}_{62}\text{Sm}$	0.20	0 (0^+)	1S, 2S	4 ± 4
$^{162}_{68}\text{Er} \rightarrow ^{162}_{66}\text{Dy}$	0.14	1.783 (2^+)	1S, 2S	1 ± 6
$^{164}_{68}\text{Er} \rightarrow ^{164}_{66}\text{Dy}$	1.56	0 (0^+)	2S, 2S	9 ± 5
$^{168}_{70}\text{Yb} \rightarrow ^{168}_{68}\text{Er}$	0.14	1.355 (1^-) 1.393 (?)	1S, 2S 2S, 2S	$^{-1}_{8} \pm 4$
$^{180}_{74}\text{W} \rightarrow ^{180}_{72}\text{Hf}$	0.13	0 (0^+) 0.093 (2^+)	1S, 1S 1S, 3S	$^{26}_{-4} \pm 17$
$^{196}_{80}\text{Hg} \rightarrow ^{196}_{78}\text{Pt}$	0.15	0.689 (2^+)	1S, 2S	26 ± 9

Modes of the $0\nu\text{ECEC}$ -decay:

$$e_b + e_b + (A, Z) \rightarrow (A, Z-2) + \gamma + 2\gamma + e^+e^- + M$$

$$e_b + e_b + (A, Z) \rightarrow (A, Z-2) + \gamma$$

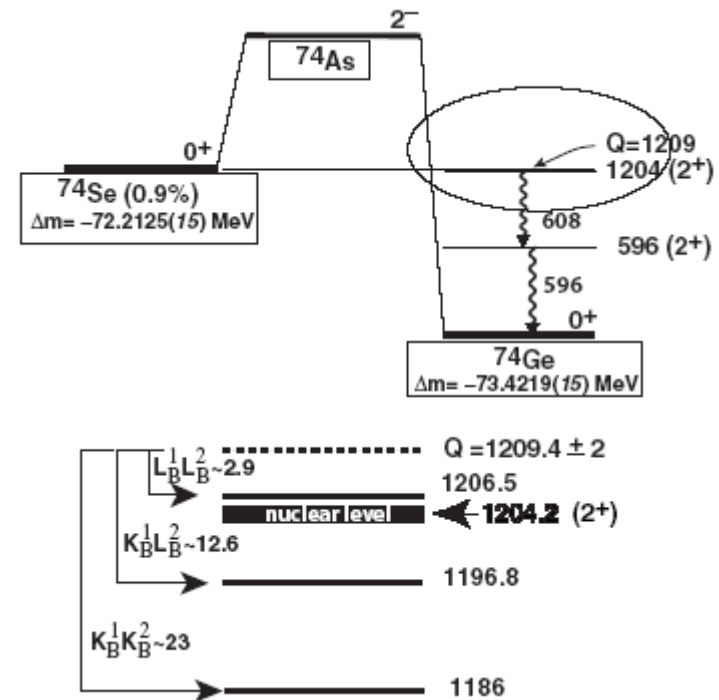
THE RESONANT SITUATION



Neutrinoless double electron capture (perturbation theory approach)

Theoretically, not well understood yet:

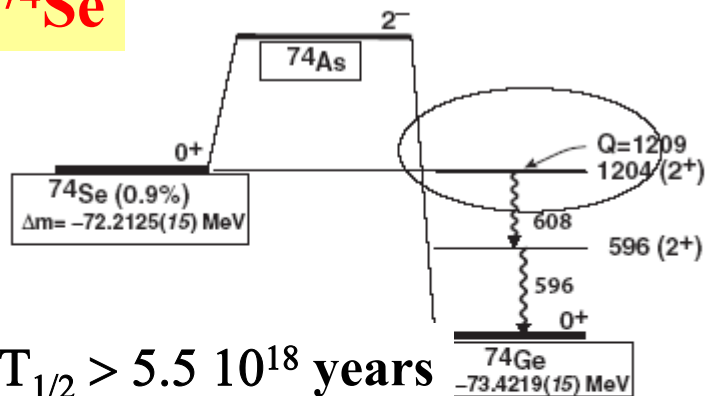
- which mechanism is important?
- which transition is important?



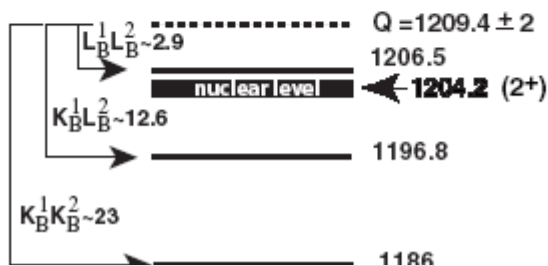
$$\Gamma^{0\nu\gamma} = \frac{\Gamma^r(2p \rightarrow 1s)}{[E_\gamma - Q_{\text{res}}]^2 + [\Gamma^r/2]^2} |R_{0\nu}^{\text{cc}}|^2$$

$$\text{Fed } Q_{\text{res}} = E_{s_{1/2}} - E_{p_{1/2}}$$

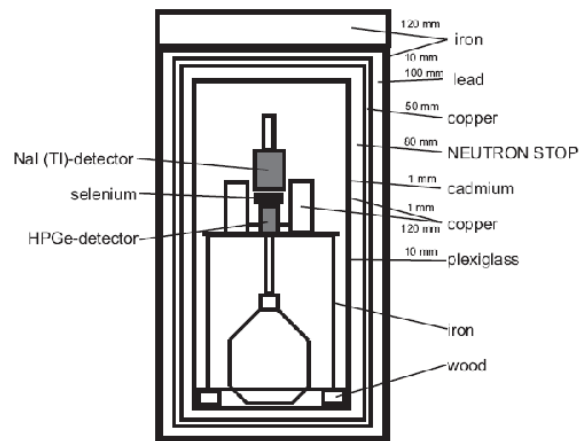
^{74}Se



$T_{1/2} > 5.5 \cdot 10^{18}$ years



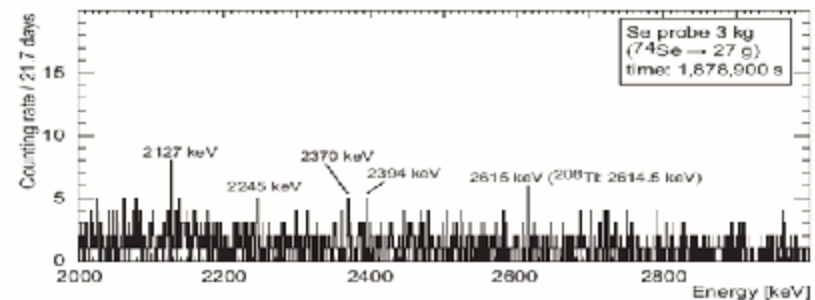
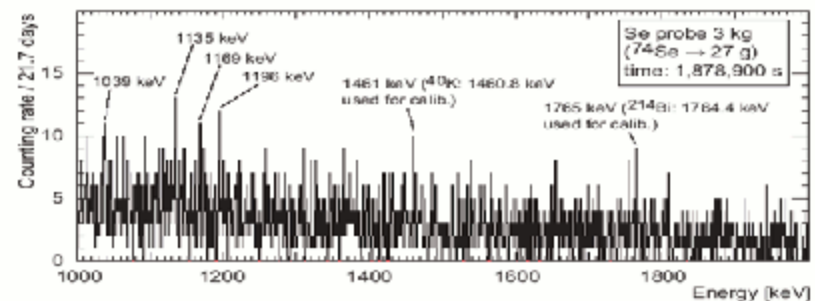
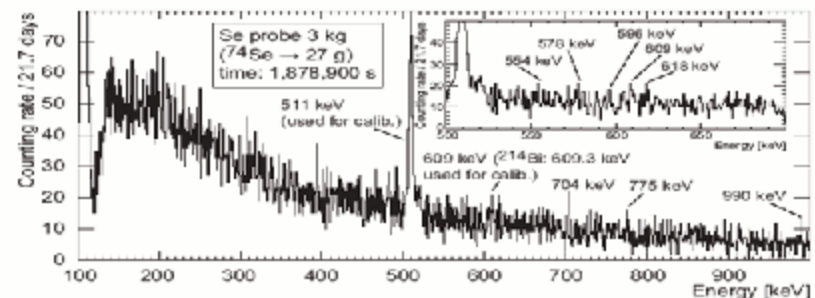
**A.S. Barabash et al.,
NPA 785 (2007) 371**



11/19/200

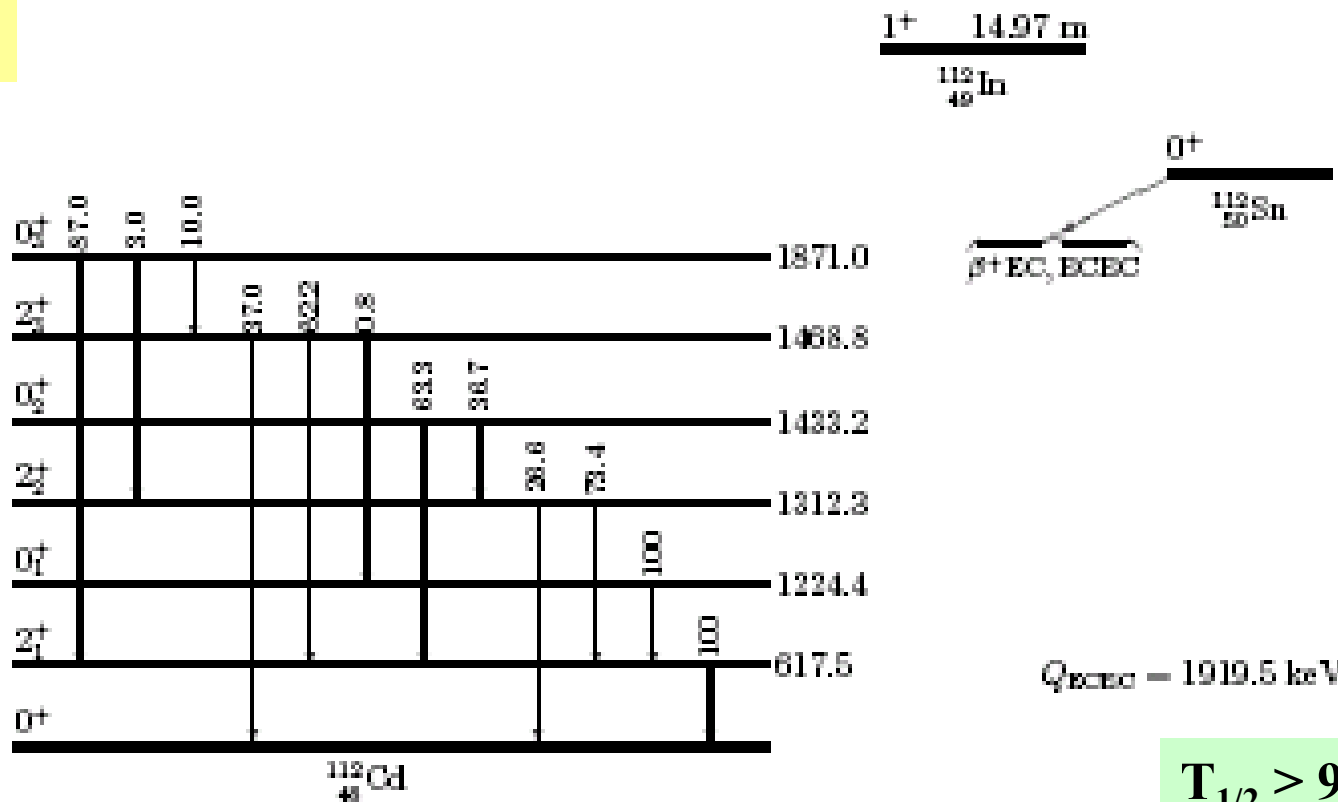
Experimental activities (^{74}Se)

**Muenster and Bratislava
groups exp. in Bratislava
Frekers et al., to be submitted**



Experimental activities (^{112}Sn)

^{112}Sn



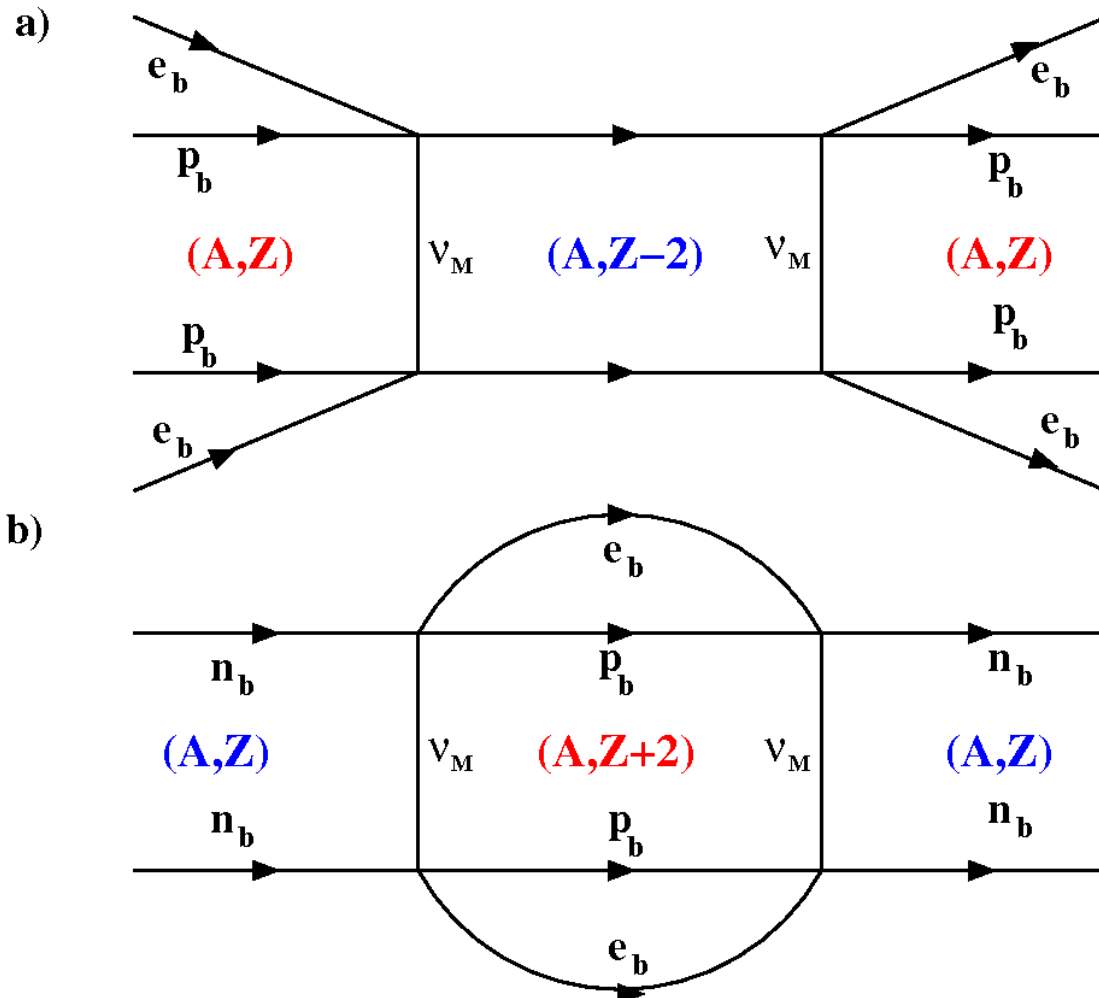
$T_{1/2} > 9.2 \cdot 10^{19}$ years

In comparison with the $0\nu\beta\beta$ -decay disfavoured due:

- process in the 3-rd (4th) order in electroweak theory
- bound electron wave functions

favoured: **resonant enhancement ?**

A.S. Barabash et al.,
NPA 807 (2008) 269



**Mixing of neutral atoms
and total
lepton number oscillation**

$$n + n \leftrightarrow p + p + e_b^- + e_b^-$$

$$(A, Z) \Longleftrightarrow (A, Z + 2)^{**}$$

$$(A, Z) \Longleftrightarrow (A, Z - 2)^{**}$$

LVN Potential

$$V^{LVN} \simeq m_{\beta\beta} G_F^2 < \frac{1}{4\pi r_a} > \Psi_1(0)\Psi_2(0)$$

$$V^{LVN} \sim 10^{-24} \text{ eV}$$

$$m_\nu = 0.5 \text{ eV}, \quad Z = 30, \quad n_i = 1, \quad l_i = 0$$

Oscillations of stable atoms ($\Gamma=0$)

$$| \langle f | e^{-iH_{eff}t} | i \rangle |^2 = \frac{4V^2}{(M_i - M_f)^2} \sin^2 [t (M_i - M_f)/2]$$

$$[t (M_i - M_f)] \leq 1 \quad | \langle f | e^{-iH_{eff}t} | i \rangle |^2 = V^2 t^2$$

$$[t (M_i - M_f)] \geq 1 \quad | \langle f | e^{-iH_{eff}t} | i \rangle |^2 \approx \frac{V^2}{(M_i - M_f)^2}$$

$$\begin{array}{l} {}^{164}_{68}Er \rightarrow {}^{164}_{66}Dy \\ (M_i - M_f) = 24.1 \text{ keV} \end{array} \quad | \langle f | e^{-iH_{eff}t} | i \rangle |^2 \leq 3 \cdot 10^{-55}$$

Double electron capture ($\Gamma \neq 0$) (resonant enhancement of atom)

$$\begin{aligned} \Gamma &= 4 \times 10^{-7} Z^4 \text{ eV} \\ &= 0.3 \text{ eV} \quad (Z = 30) \end{aligned}$$

$$\begin{aligned} R_{max} &= \frac{1 \text{ ton}}{M_i} \times \frac{4V^2}{\Gamma} \\ &\sim 10^4 \text{ yr}^{-1} \end{aligned}$$

Mass difference $\gg \Gamma$

$$\Gamma_1 = \frac{4V^2}{4(M_i - M_f)^2 + \Gamma^2} \Gamma$$

$$R \sim R_{max} \frac{\Gamma^2}{(M_i - M_f)^2} \sim 10^{-3} \text{ yr}^{-1}$$

Mass difference $\sim \text{keV}$

Different types of Oscillations (Effective Hamiltonian)

$$H_{eff}^{K_0 \bar{K}_0} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

**Oscillations of $\nu_I - \nu_I$,
(lepton flavor)**

**Oscillation of K_0 -anti $\{K_0\}$
(strangeness)**

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

**Oscillation of n -anti $\{n\}$
(baryon number)**

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

**Oscillation of atoms
(total lepton number)**

Eigenvalues

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1,$$

$$\lambda_- = M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_1$$

Fedor

Full width of unstable atom/nucleus

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2},$$

$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}.$$

Double electron capture



Relativistic electron w.f. ($j=1/2, l=0, l'=1$)

$$\Psi_{jm}^{(\alpha)}(\vec{x}) = \begin{pmatrix} f_\alpha(r) \Omega_{jlm} \\ (-1)^{\frac{1+l+l'}{2}} g_\alpha(r) \Omega_{jl'm} \end{pmatrix} \quad l = j \pm 1/2, \quad l' = 2j - l$$

Potential

$$V^{1s_{1/2}1s_{1/2}}(0_3^+) = \frac{1}{4\pi} m_e (G_\beta^2 m_e^4) \frac{m_{\beta\beta}}{m_e} \frac{1}{R m_e} \frac{(\bar{f}_{1s_{1/2}})^2}{4\pi m_e^3} g_A^2 M^{0\nu}(0_3^+).$$

Width

$$\Gamma^{ECEC} = \frac{|V^{1s_{1/2}1s_{1/2}}(0_3^+)|^2}{(M_i - M_f)^2 + \frac{\Gamma_X^2}{4}} \Gamma_X$$

Matrix element

Exc. state	E_{ex} (MeV)	$M^{0\nu}$
$0_{\text{g.s.}}^+$	0	2.69
0_1^+ (1 ph.)	1.224	3.02
0_2^+ (2 ph.)	1.433	0.90
0_3^+ (1 ph.)	1.224	2.78

Double electron capture of ^{112}Sn (perspectives of search)

F. Šimkovic, M. Krivoruchenko, A. Faessler, to be submitted

$M_i - M_f$	$T_{1/2}^{\text{EDEC}}$ ($m_{\beta\beta} = 50 \text{ meV}$)
1 keV	$2.44 \cdot 10^{31} \text{ years}$
100 eV	$2.45 \cdot 10^{29} \text{ years}$
10 eV	$2.91 \cdot 10^{27} \text{ years}$
0 eV	$4.67 \cdot 10^{26} \text{ years}$

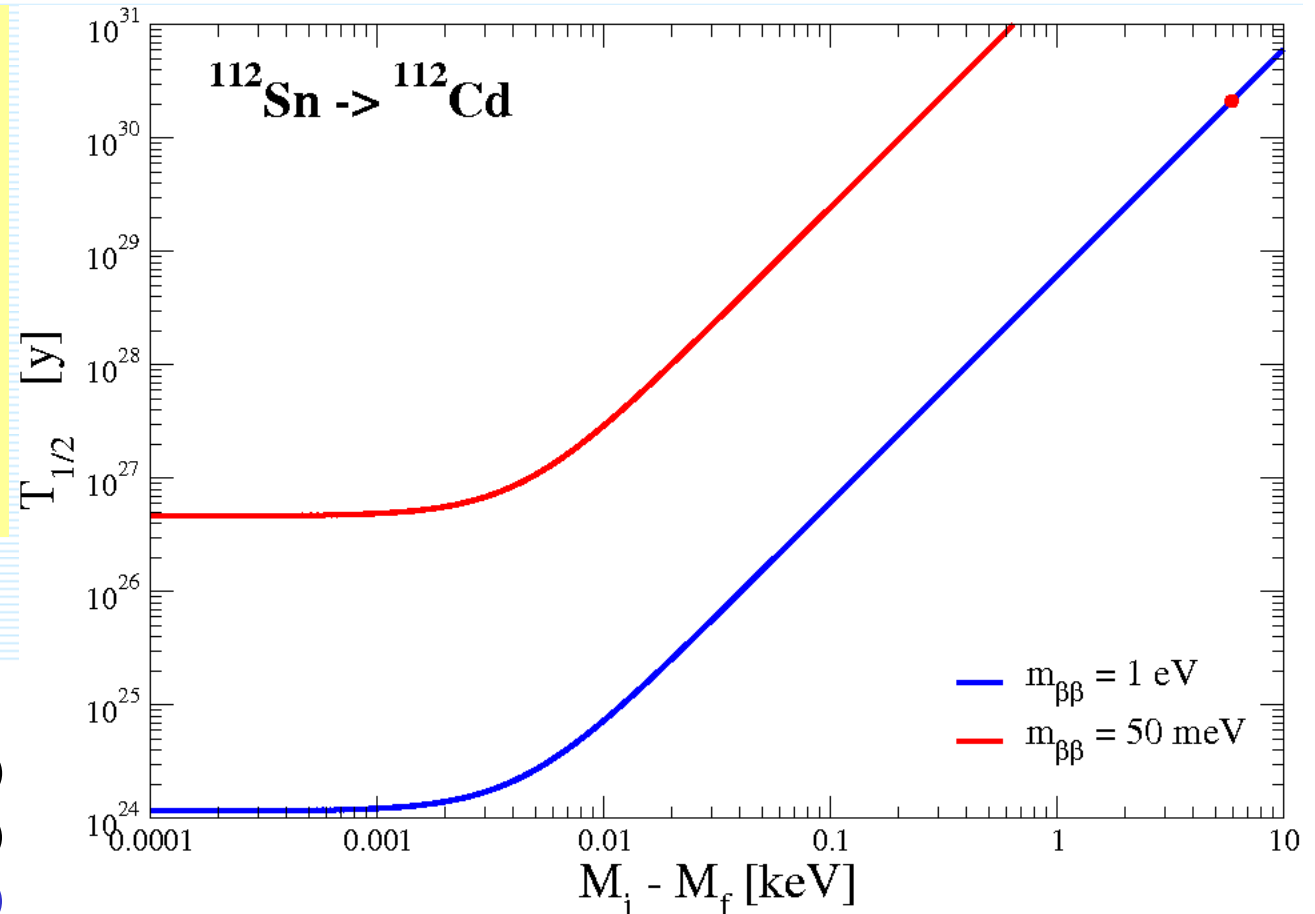
No $2\nu\text{EDEC}$ background!

$$T_{2\nu\text{EDEC}} = 1.7 \cdot 10^{22} \text{ y } (0^+_{\text{g.s.}})$$

$$7.4 \cdot 10^{24} \text{ y } (0^+_1)$$

$$5.4 \cdot 10^{34} \text{ y } (0^+_3)$$

Domin, Kovalenko, F. Š., Semenov,
NPA 753, 337 (2005)



$$T_{1/2}^{0\nu} (^{76}\text{Ge}) = (2.95 - 5.74) \cdot 10^{26} \text{ years for } m_{\beta\beta} = 50 \text{ meV}$$

$J^\pi=0^+$ **Calculated double electron capture half-lives ($m_{\beta\beta} = 1$ eV)**

Transition	$M_{A,Z-2}^* - M_{A,Z-2}$	$M_{A,Z-2}^{**} - M_{A,Z}$	Holes	$T_{1/2}^{\min}$	$T_{1/2}$
$^{112}_{50}\text{Sn} \rightarrow ^{112}_{48}\text{Cd}^*$	1871 ± 0.2	$-5.9 \pm 4.2 \pm 2.7$	$1s_{1/2} \ 1s_{1/2}$	2×10^{24}	8×10^{30}
$^{152}_{64}\text{Gd} \rightarrow ^{152}_{62}\text{Sm}$	0	$-0.3 \pm 2.5 \pm 2.5$	$1s_{1/2} \ 2s_{1/2}$	5×10^{24}	9×10^{29}
	0	$5.9 \pm 2.5 \pm 2.5$	$1s_{1/2} \ 3s_{1/2}$	4×10^{25}	8×10^{29}
	0	$7.4 \pm 2.5 \pm 2.5$	$1s_{1/2} \ 4s_{1/2}$	8×10^{26}	10^{33}
$^{148}_{64}\text{Gd} \rightarrow ^{148}_{62}\text{Sm}^*$	3045 ± 2	$5.7 \pm 2.5 \pm 2.5$	$2s_{1/2} \ 2s_{1/2}$	8×10^{25}	3×10^{32}
	3045 ± 2	$11.8 \pm 2.5 \pm 2.5$	$2s_{1/2} \ 3s_{1/2}$	3×10^{26}	8×10^{33}
	3045 ± 2	$13.3 \pm 2.5 \pm 2.5$	$2s_{1/2} \ 4s_{1/2}$	4×10^{27}	2×10^{35}
	3045 ± 2	$6.6 \pm 2.5 \pm 2.5$	$2p_{1/2} \ 2p_{1/2}$	2×10^{29}	2×10^{36}
$^{156}_{66}\text{Dy} \rightarrow ^{156}_{64}\text{Gd}^*$	1988.5 ± 0.2	$7.0 \pm 6.6 \pm 2.5$	$2s_{1/2} \ 2s_{1/2}$	2×10^{27}	8×10^{31}
	1988.5 ± 0.2	$7.9 \pm 6.6 \pm 2.5$	$2p_{1/2} \ 2p_{1/2}$	8×10^{29}	4×10^{35}

Transition	J^P	$M_{A,Z-2}^* - M_{A,Z-2}$	$M_{A,Z-2}^{**} - M_{A,Z}$	Holes	$\tilde{T}_{1/2}^{\min}$	$\tilde{T}_{1/2}$
$^{162}_{68}\text{Er} \rightarrow ^{162}_{66}\text{Dy}^*$	1^+	1745.716 ± 0.007	$-10.1 \pm 3.5 \pm 2.5$	$1s_{1/2} \ 1s_{1/2}$	8×10^{23}	2×10^{29}
$^{156}_{66}\text{Dy} \rightarrow ^{156}_{64}\text{Gd}^*$	1^+	1965.950 ± 0.004	$-12.5 \pm 6.6 \pm 2.5$	$1s_{1/2} \ 2s_{1/2}$	10^{25}	3×10^{30}
	1^+	1965.950 ± 0.004	$-5.8 \pm 6.6 \pm 2.5$	$1s_{1/2} \ 3s_{1/2}$	2×10^{26}	2×10^{31}
	1^-	1946.375 ± 0.006	$8.4 \pm 6.6 \pm 2.5$	$1s_{1/2} \ 2s_{1/2}$	8×10^{26}	4×10^{31}
$^{74}_{34}\text{Se} \rightarrow ^{74}_{32}\text{Ge}^*$	2^+	1204.204 ± 0.007	$3.0 \pm 1.7 \pm 1.6$	$2p_{1/2} \ 2p_{3/2}$	10^{36}	10^{45}

Lepton number and parity
oscillations

$0^+ \rightarrow 2^+$ strongly suppressed,
 $p_{3/2}$ -electron needed (squared R/a_B-factor)

Q-value measurements
Klaus Blaum “LAUNCH09 (Nov. 09)”

$\beta\beta$		
Decay	Q-value	Precision
$^{76}\text{Ge} - ^{76}\text{Se}$	2039.006(50) G. Douysset et al., PRL 86, 4259 (2001)	6E-10
$^{130}\text{Te} - ^{130}\text{Xe}$	2527.518(13) M. Redshaw et al., PRL 102, 212502 (2009)	1E-10
$^{136}\text{Xe} - ^{136}\text{Ba}$	2457.83(37) M. Redshaw et al., PRL 98, 053003 (2007)	3E-09
ECEC		
$^{112}\text{Sm} - ^{112}\text{Cd}$	1919.82(16) S. Rahaman et al., PRL 103, 042501 (2009)	1E-09
$^{120}\text{Te} - ^{120}\text{Sm}$	1714.81(1.25) N. Scielzo et al., PRC 80, 025501 (2009)	1E-08

$0\nu\beta\beta$ -decay and $0\nu\text{ECEC}$ study

There is a need for supporting experiments

Nuclear matrix elements:

- *Mean field* *p and n removing transfer reactions*
- *β^- and β^+ strengths* *Charge-changing experiments*
- *deformation* *Exp. to remeasure deformation needed*
- *$2\nu\beta\beta$ -decay* *Double beta decay experiments*

Q-value measurement:

$0\nu\beta\beta$: $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$, $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$, $^{100}\text{Mo} \rightarrow ^{100}\text{Mo}$, $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$

$0\nu\text{ECEC}$: $^{162}\text{Er} \rightarrow ^{162}\text{Dy}$, $^{156}\text{Dy} \rightarrow ^{156}\text{Gd}$, $^{202}\text{Pb} \rightarrow ^{202}\text{Hg}$,
($^{74}\text{Se} \rightarrow ^{74}\text{Ge}$, $^{130}\text{Ba} \rightarrow ^{130}\text{Xe}$, $^{78}\text{Kr} \rightarrow ^{78}\text{Se}$...)

What is the nature of neutrinos?



ν \Rightarrow
theory



Only the $0\nu\beta\beta$ -decay can answer this fundamental question

^{76}Ge

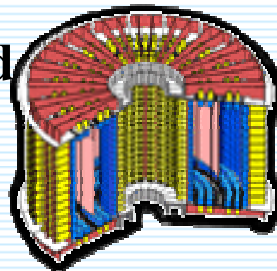


^{130}Te

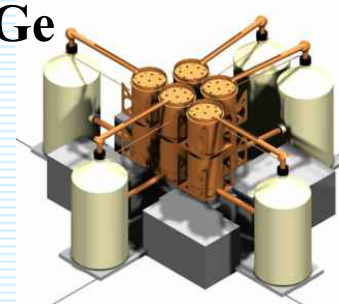


^{82}Se

^{150}Nd

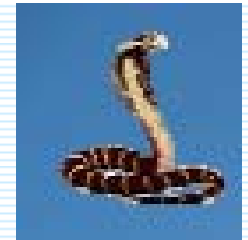


^{76}Ge



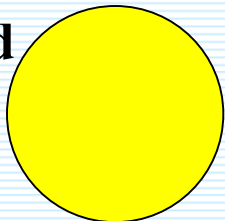
^{106}Cd

^{116}Cd



^{100}Mo

^{150}Nd



^{136}Xe



+ (?)

Double electron
capture

(Muenster, Dresden,
Jyvaskula, Bratislava...col.)