Spectral properties and scattering in chaotic billiards and nuclei

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CERN 2009

- Classical billiards and quantum billiards
- Random Matrix Theory (Wigner 1951 – Dyson 1962)
- Spectral properties of billiards and mesoscopic systems
- Wave functions and doorway state phenomena
- Chaotic quantum scattering with and without T-invariance

Supported by the DFG within the SFB 634

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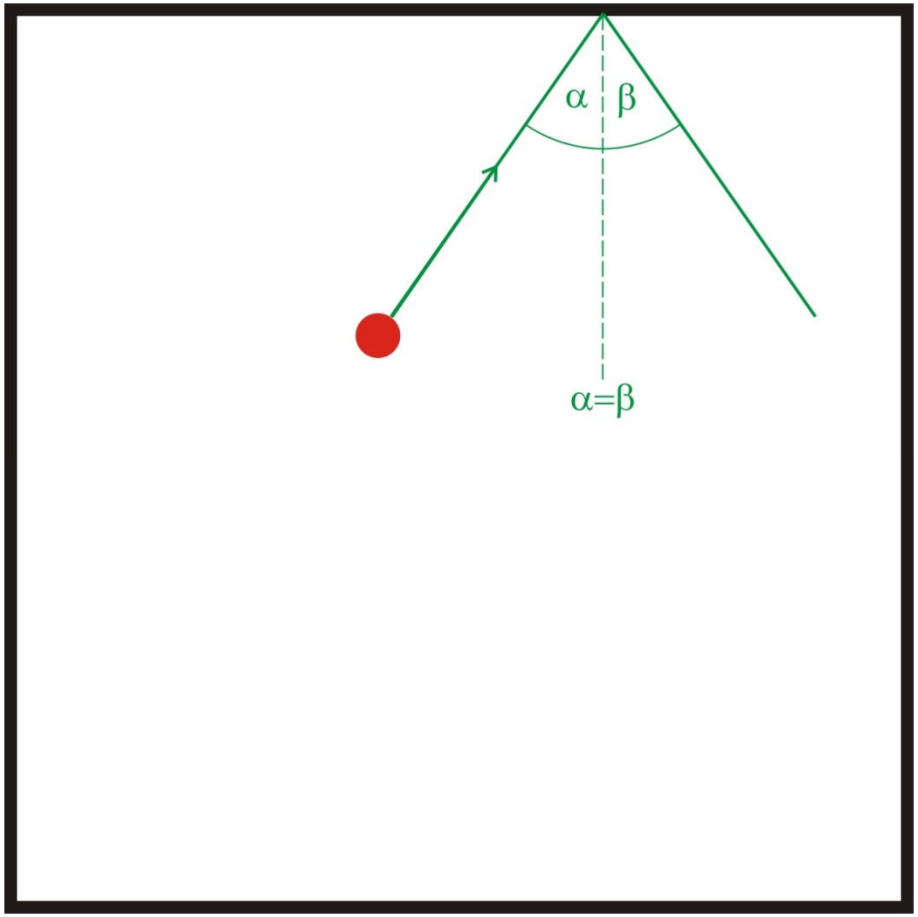
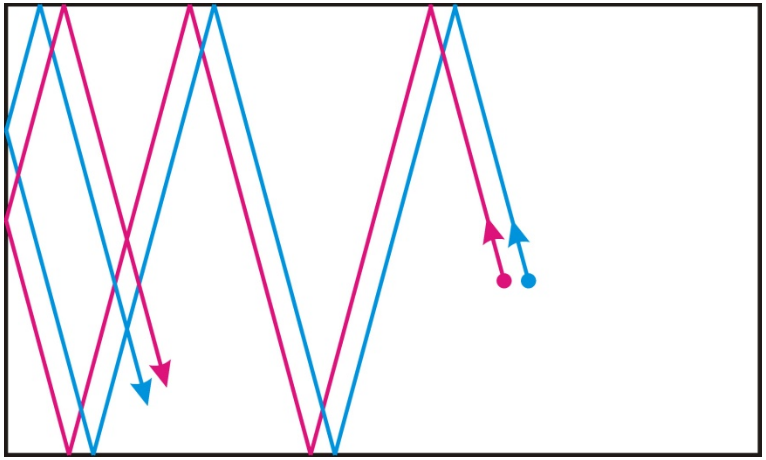
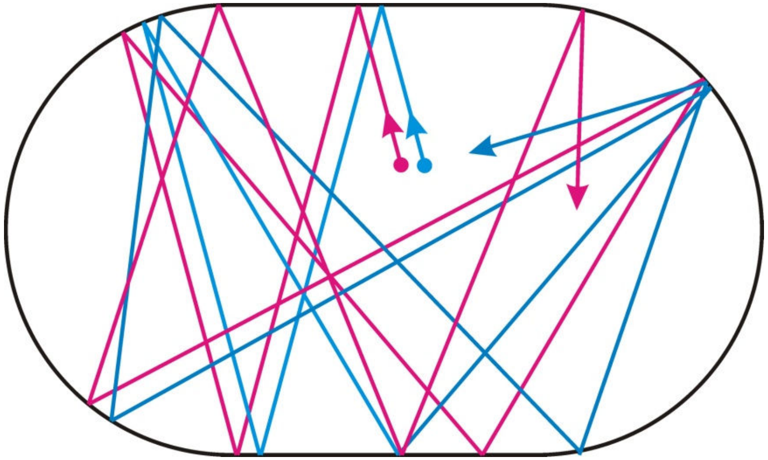
Classical Billiard



Regular and Chaotic Dynamics

Regular	Bunimovich stadium (chaotic)





- Energy and p_x^2 are conserved
- Equations of motion are integrable
- Predictable for infinite long times

- Only energy is conserved
- Equations of motion are not integrable
- Predictable for a finite time only

Small Changes → Large Actions

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- Sensitivity of the solutions of a deterministic problem with respect to small changes in the initial conditions is called Deterministic Chaos.

- Beyond a fixed, for the system characteristic time becomes every prediction impossible. The system behaves in such a way as if not determined by physical laws but randomness.

Our Main Interest

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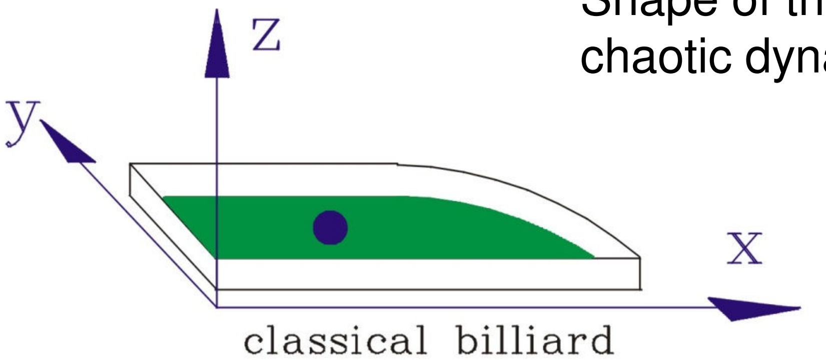
- How are these properties of classical systems transformed into corresponding quantum-mechanical systems?

  $\rightarrow$ Quantum chaos?

- What might we learn from generic features of billiards and mesoscopic systems (hadrons, nuclei, atoms, molecules, metal clusters, quantum dots)?

The Quantum Billiard and its Simulation

Shape of the billiard implies chaotic dynamics



Schrödinger $\leftrightarrow$ Helmholtz

quantum billiard

$$\left(\Delta + k^2\right)\Psi = 0$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

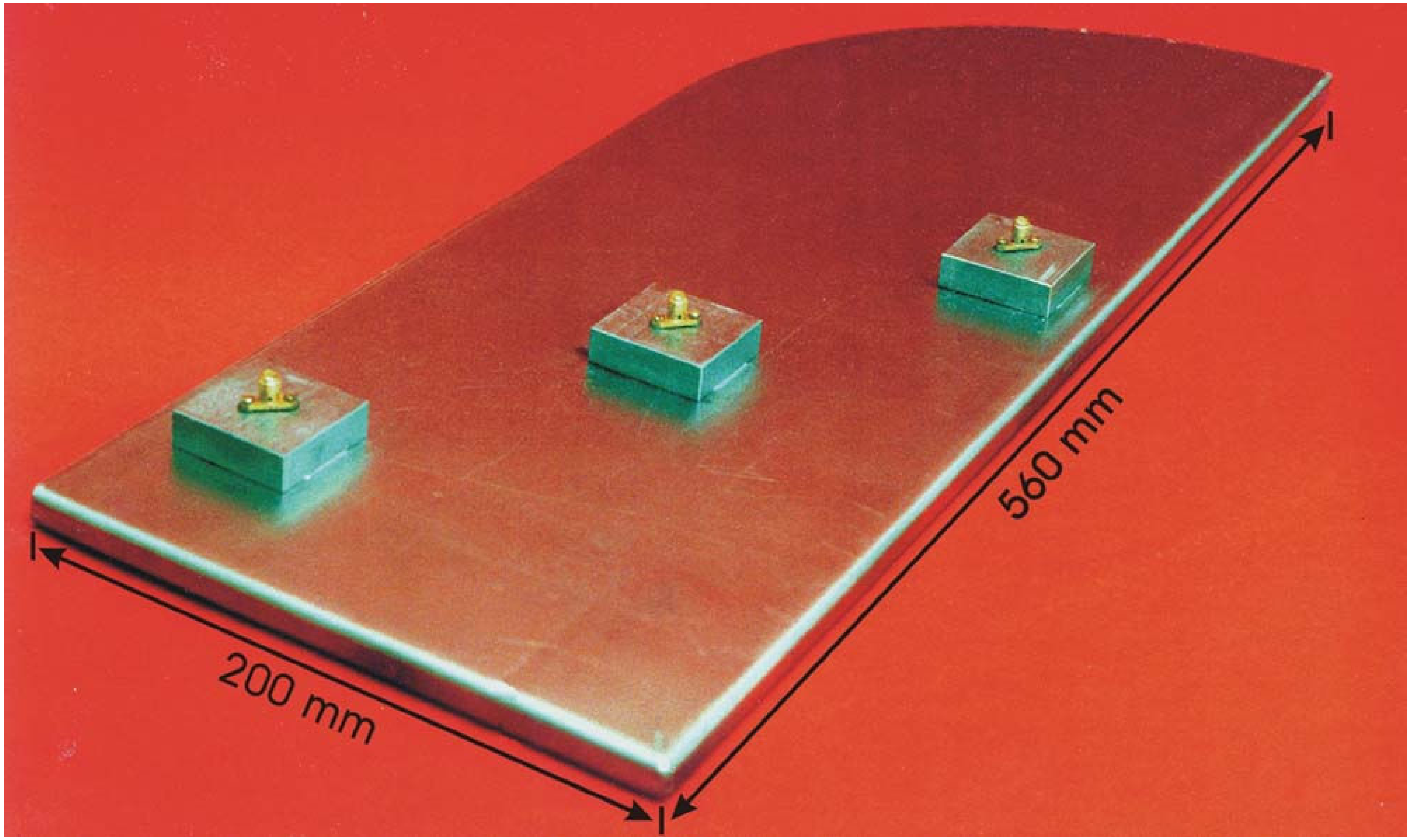
2D microwave cavity: $h_z < \lambda_{min}/2$

$$\left(\Delta + k^2\right)E_z = 0$$

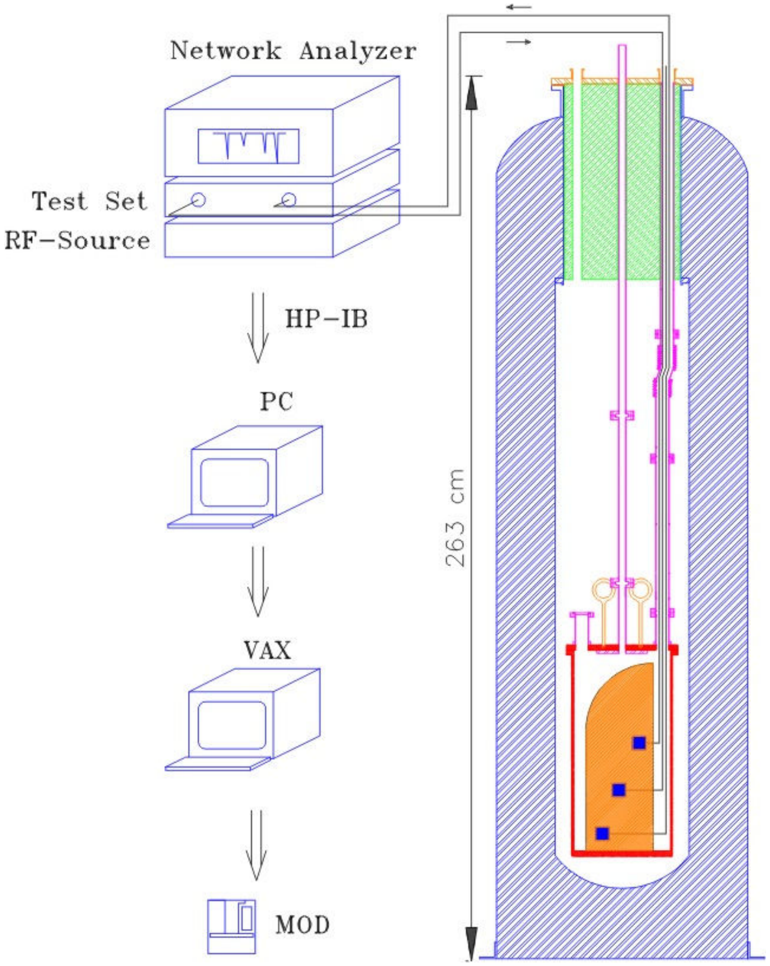
$$k = \frac{2\pi f}{c}$$

Helmholtz equation and Schrödinger equation are equivalent in 2D. The motion of the quantum particle in its potential can be simulated by electromagnetic waves inside a two-dimensional microwave resonator.

Superconducting Niobium Microwave Resonator



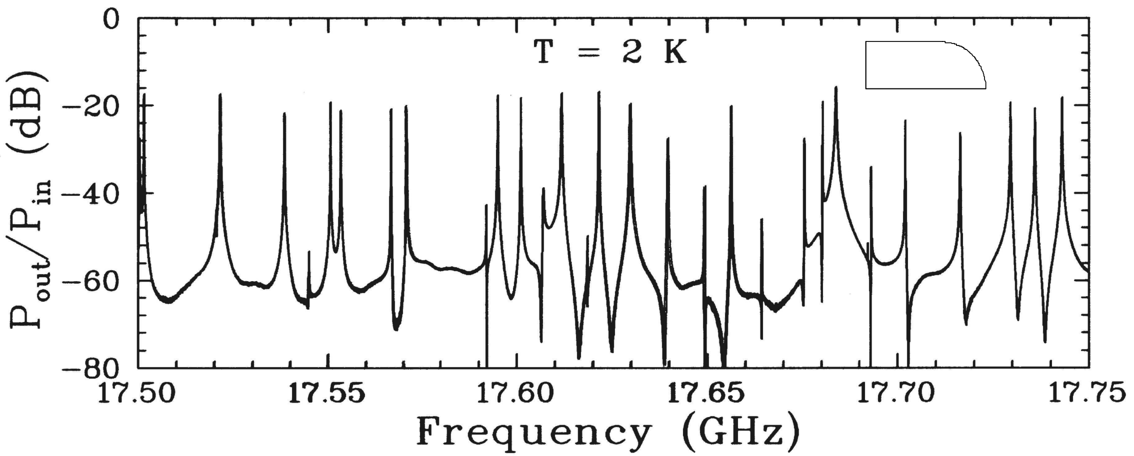
Experimental Setup



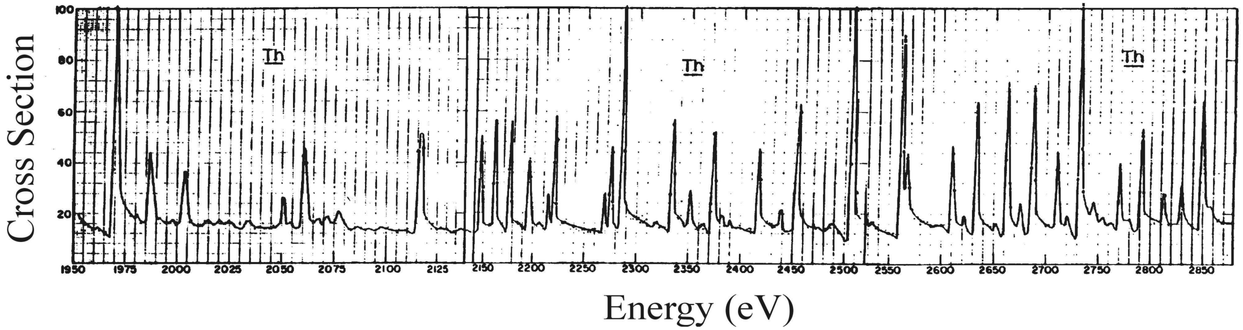
- Superconducting cavities
- LHe (T = 4.2 K)
- f = 45 MHz ... 50 GHz
- $10^3 \ldots 10^4$ eigenfrequencies
- $Q = f/\Delta f \approx 10^6$

Stadium Billiard $\leftrightarrow$ n + ^{232}Th

Transmission spectrum for the stadium billiard



Spectrum of neutron resonances in ^{232}Th + n



Niels Bohr's Model of the Compound Nucleus

February 29, 1936 NATURE 351

News and Views

Neutron Capture and Nuclear Constitution

The new views of nuclear structure and the processes involved in neutron capture, presented by Prof. Niels Bohr in an address which appears elsewhere in this issue, were expounded by him in a lecture to the Chemical and Physical Society of University College, London, on February 11 and were illustrated by two pictures here reproduced. The first of these is intended to convey an idea of events arising out of a collision between a neutron and the nucleus. Imagine a shallow basin with a number of billiard balls in it as shown in the accompanying figure. If the basin were empty, then upon striking a ball from the outside, it would go down one slope and pass out on the opposite side with its original velocity. But with other balls in the basin, there would not be a free passage of this kind. The struck ball would divide its energy first with one of the balls in the basin, these two would similarly

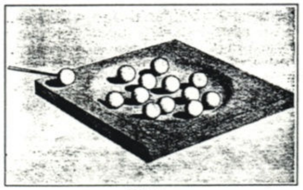


Fig. 1.

share their energies with others, and so on until the original kinetic energy was divided among all the balls. If the basin and the balls are regarded as perfectly smooth and elastic, the collisions would continue until the kinetic energy happens again to be concentrated upon a ball close to the edge. This ball would then escape from the basin and the remainder of the balls would be left with insufficient total energy for any of them to climb the slope. The picture illustrates, therefore, "that the excess energy of the incident neutron will be rapidly divided among all the nuclear particles with the result that for some time afterwards no single particle will possess sufficient kinetic energy to leave the nucleus".

Nuclear Energy Levels

The second figure illustrates the character of the distribution of energy levels for a nucleus of not too small atomic weight. The lowest lines represent the levels with an excitation of the same order of magnitude as ordinary excited γ-ray states. According to the views developed in Prof. Bohr's address, the levels will for increasing excitation rapidly become closer to one another and will, for an excitation of about 15 million electron volts, corresponding to a collision between a nucleus and a high-speed neutron, be continuously distributed, whereas in the region of small excess energy of about 10 million volts excitation they will still be sharply separated. This is illustrated by the two lenses of high magnification placed over the level-diagram in the two above-mentioned regions. The dotted line in the middle of the field of the lower magnifying glass represents zero excess energy, and the fact that one of the levels

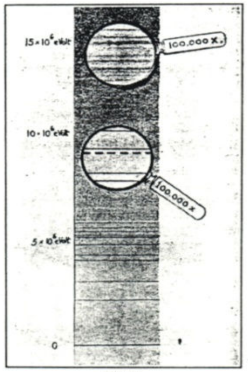


Fig. 2.

is very close to this line (about ½ volt distant) corresponds to the possibility of selective capture for very slow neutrons. The average distance between the neighbouring levels will in this energy region be about ten volts as estimated from the statistics for the occurrence of selective capture. The diagram shows no upper limit to the levels, and these actually extend to very high energy values. If it were possible to experiment with neutrons or protons of energies above a hundred million volts, several charged or uncharged particles would eventually leave the nucleus as a result of the encounter; and, adds Prof. Bohr, "with particles of energies of about a thousand million volts, we must even be prepared for the collision to lead to an explosion of the whole nucleus".

Random Matrices $\leftrightarrow$ Level Schemes

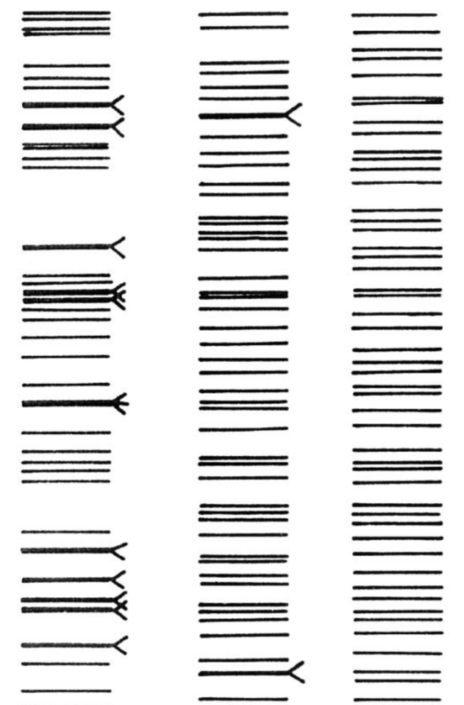
Random Matrix

$$H = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

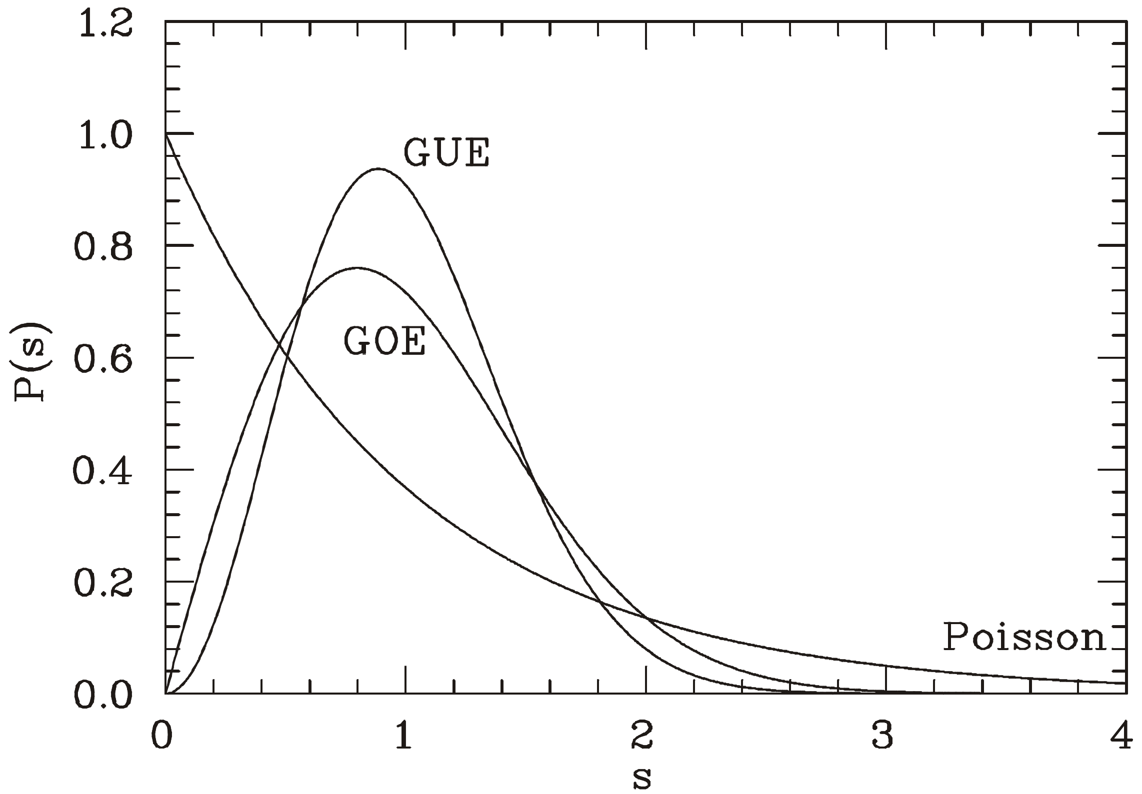
Eigenvalues

$$H\phi_n = E_n \phi_n$$

Level Schemes



Nearest Neighbor Spacings Distribution

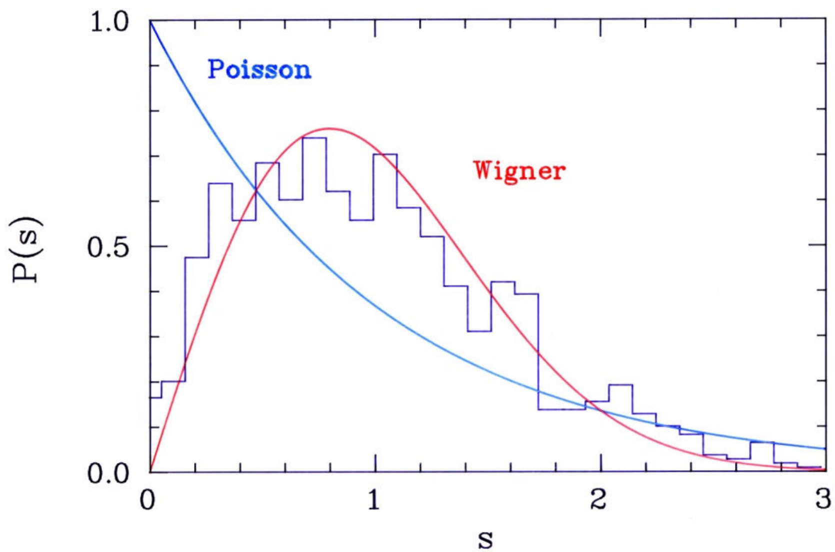


GOE and GUE ↔ "Level Repulsion"

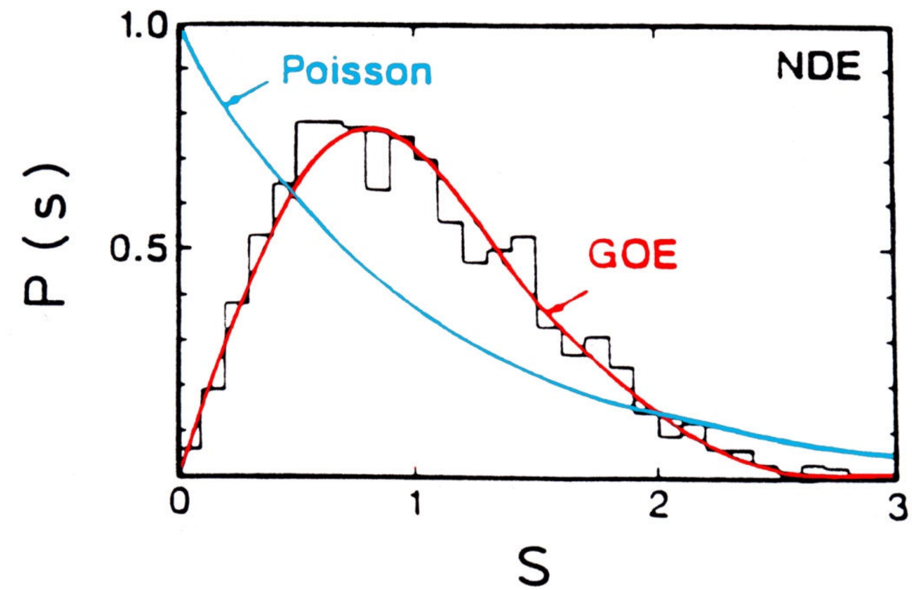
Poissonian Random Numbers ↔ "Level Clustering"

Nearest Neighbor Spacings Distribution

stadium billiard



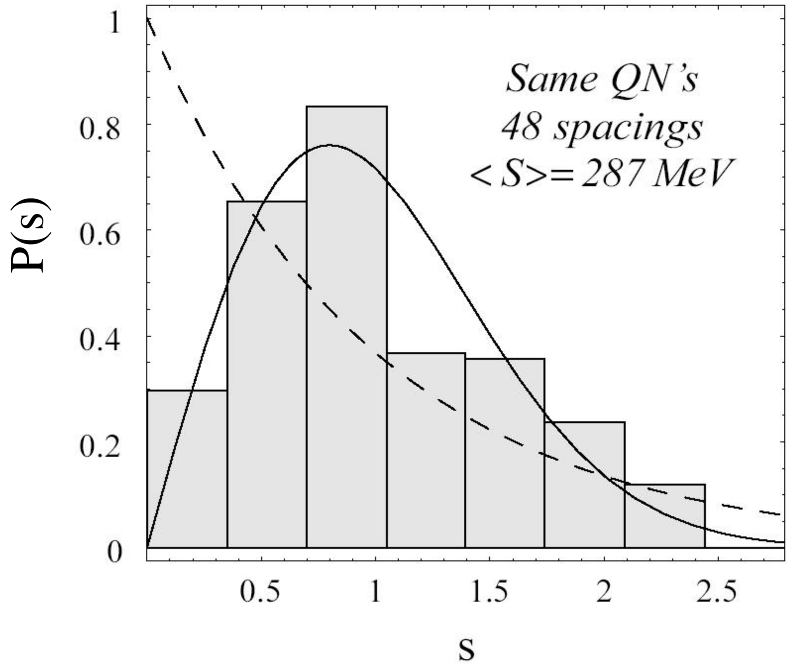
nuclear data ensemble



- Universal (generic) behavior of the two systems

Universality in Mesoscopic Systems: Quantum Chaos in Hadrons

- Combined data from measured baryon and meson mass spectra up to 2.5 GeV (from PDG)
- Spectra can be organized into multiplets characterized by a set of definite quantum numbers: isospin, spin, parity, baryon number, ...

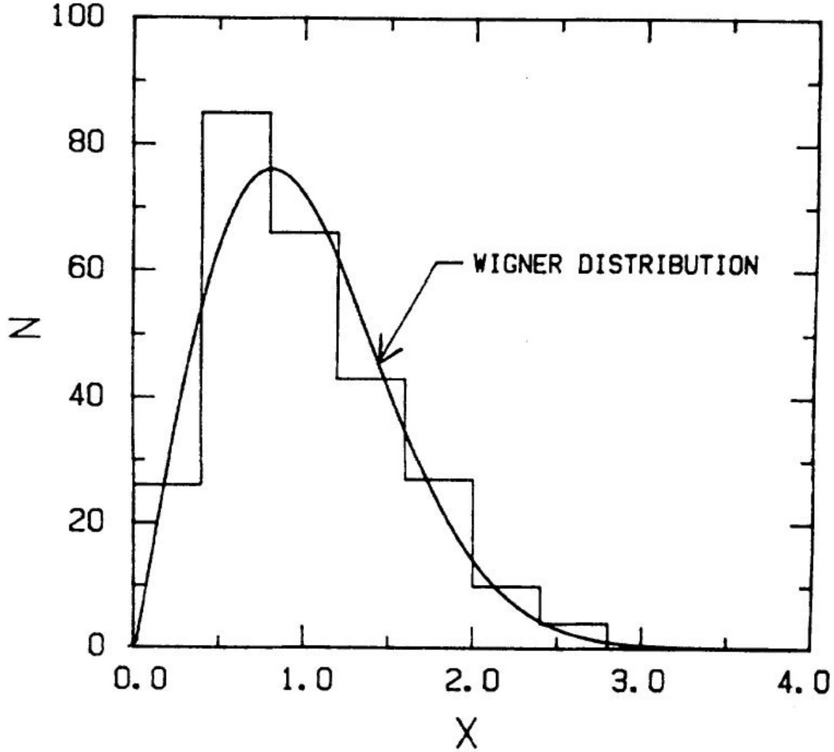


- Scale: 10^{-16} m

Pascalutsa (2003)

Universality in Mesoscopic Systems: Quantum Chaos in Atoms

- 8 sets of atomic spectra of highly excited neutral and ionized rare earth atoms combined into a data ensemble
- States of same total angular momentum and parity

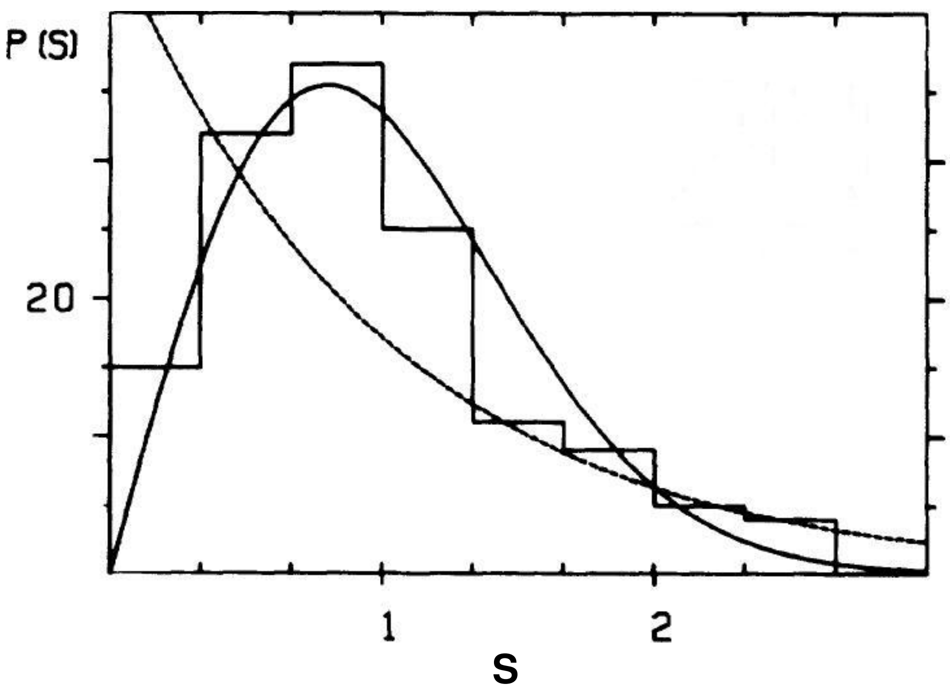


- Scale: 10^{-10} m

Camarda + Georgopulos (1983)

Universality in Mesoscopic Systems: Quantum Chaos in Molecules

- Vibronic levels of NO_2
- States of same quantum numbers



- Scale: 10^{-9} m

Zimmermann et al. (1988)

How is the Behavior of the Classical Systems Transferred to the Quantum System?

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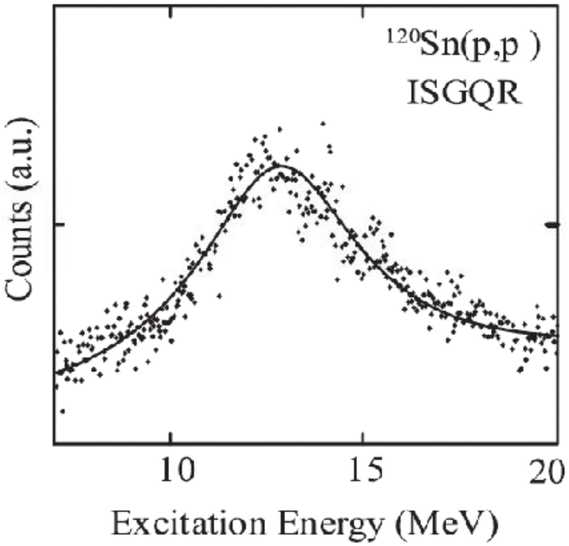
- There is a one-to-one correspondence between billiards and mesoscopic systems.

  → Bohigas' conjecture:

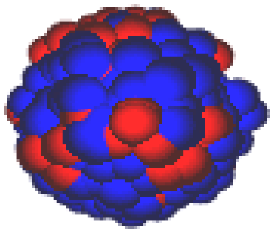
  ”The spectral properties of a generic chaotic system coincide with those of random matrices from the GOE“.

- Next: scattering in billiards and nuclei

Electric Giant Quadrupole Resonance in Nuclei-Strength Function Phenomena



Quadrupole

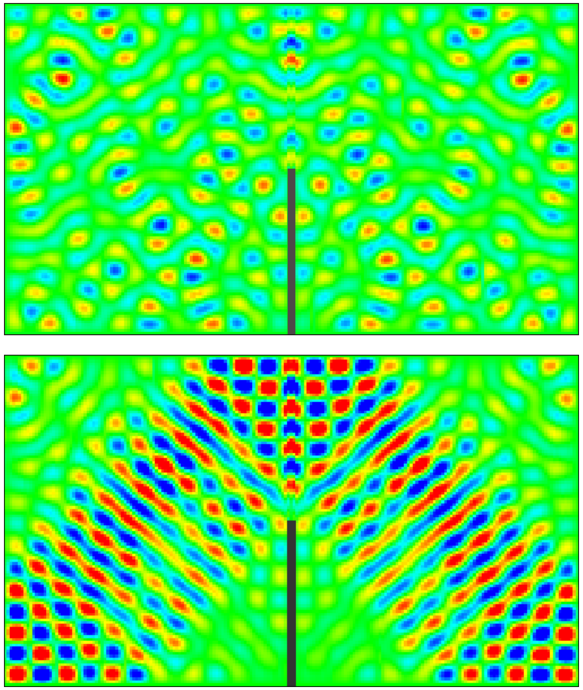


$\Delta L = 2$
$\Delta T = 0$
$\Delta S = 0$

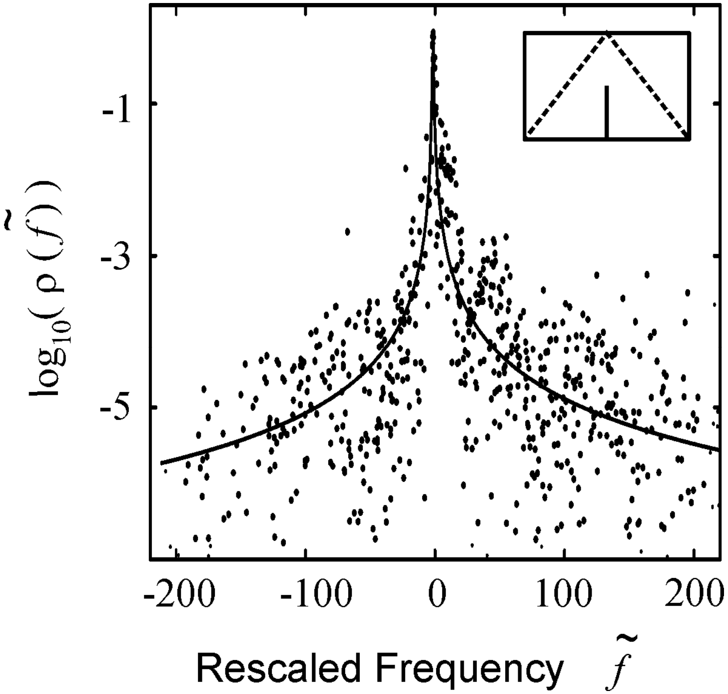
- Cross section contains huge number of individual states (fragmentation) which are not fully resolved
- GQR is the doorway state into the more complicated states at high excitation energy → density around GQR is BW:

$$\rho_{GQR} = \frac{1}{\pi} \frac{\Gamma/2}{(E - E_{GQR})^2 + \Gamma^2/4}$$

Superscars in the Barrier Billiard as a Strength Function Phenomenon

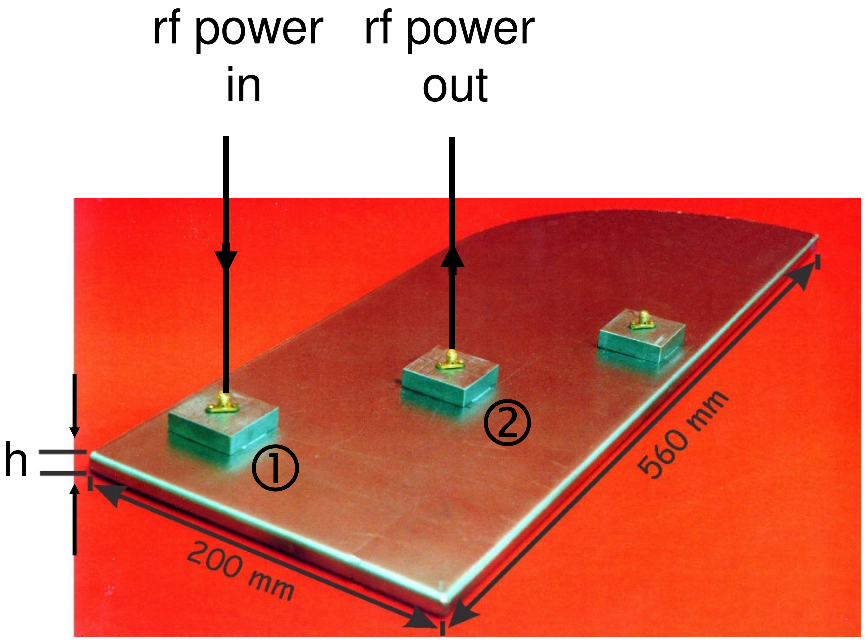


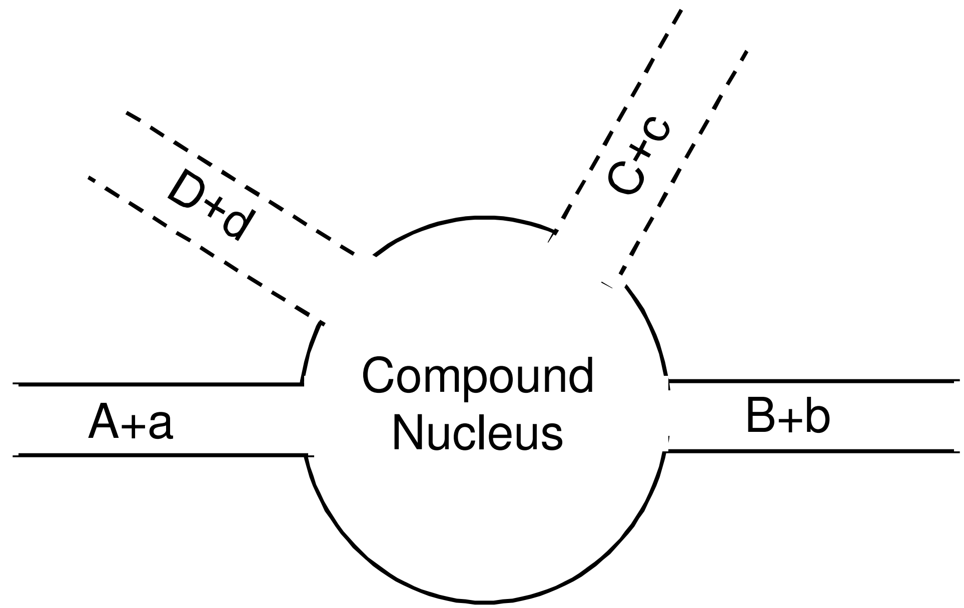
- Chaotic wave function
- N = 613
- Superscar wave function
- N = 587



- Doorway state and spreading width
- Breit-Wigner shape

Microwave Resonator as a Model for the Compound Nucleus





- Microwave power is **emitted** into the resonator by antenna ① and the output signal is **received** by antenna ② → **Open scattering system**
- The antennas act as **single scattering channels**
- Absorption into the walls is modelled by **additive channels**

Scattering Matrix Description

- Scattering matrix for both scattering processes

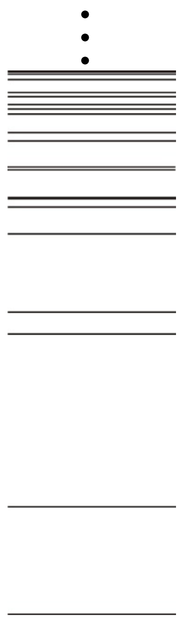
$$\hat{S}(f) = I - 2\pi i \hat{W}^T \left(f I - \hat{H} + i\pi \hat{W}\hat{W}^T \right)^{-1} \hat{W}$$

Compound-nucleus reactions		**Microwave billiard**
nuclear Hamiltonian	$\leftarrow \hat{H} \rightarrow$	resonator Hamiltonian
coupling of quasi-bound states to channel states	$\leftarrow \hat{W} \rightarrow$	coupling of resonator states to antenna states and to the walls

- Experiment: complex S-matrix elements
- **RMT description**: replace $\hat{H}$ by a GOE / GUE matrix for T-inv / T-noninv systems

Excitation Spectra

atomic nucleus

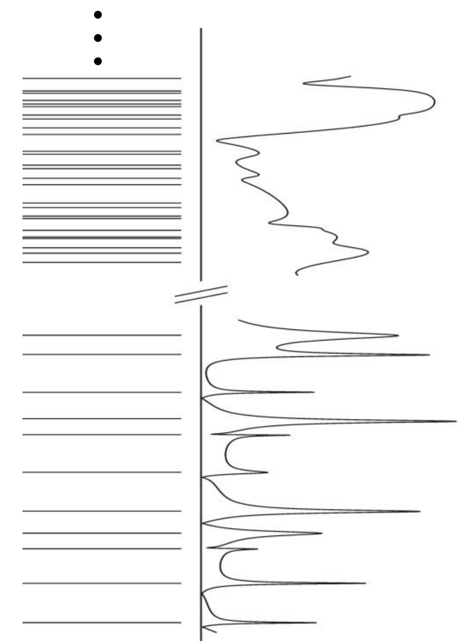


$\rho \sim \exp(E^{1/2})$

overlapping resonances
for Γ/D>1
Ericson fluctuations

isolated resonances
for Γ/D<<1

microwave cavity



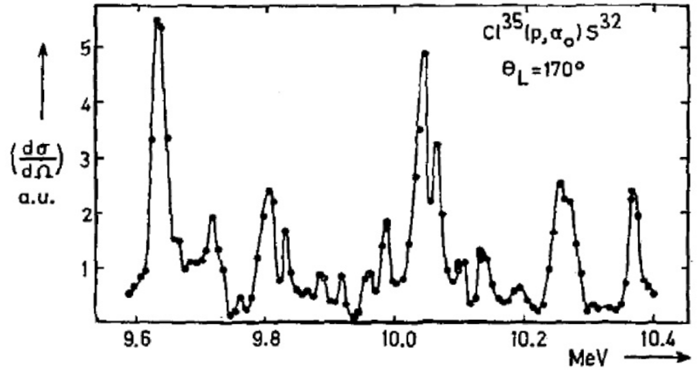
$\rho \sim f$

- Universal description of spectra and fluctuations: Verbaarschot, Weidenmüller + Zirnbauer (1984)

Ericson‘s Prediction

- Ericson fluctuations (1960):

$$\left|C(\varepsilon)\right|^2 \propto \frac{\Gamma_{coh}^2}{\Gamma_{coh}^2 + \varepsilon^2}$$



- Autocorrelation function is Lorentzian
- Measured 1964 for overlapping compound nuclear resonances

P. v. Brentano et al., Phys. Lett. 9, **48** (1964)

- Now observed in many different systems: molecules, quantum dots, laser cavities,…
- Applicable for Γ/D >> 1 and for many open channels only

Exact RMT Result for GOE Systems

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- Verbaarschot, Weidenmüller and Zirnbauer (VWZ) 1984 for arbitrary Γ/D
- VWZ-integral: $C = C(T_i,\ D\ ;\ \varepsilon)$

$$C_{ab}(\epsilon) = \frac{1}{8}\int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda\, \mu(\lambda,\lambda_1,\lambda_2) \times \exp\left(-i\pi\epsilon(\lambda_1+\lambda_2+2\lambda)/D\right) \times J_{ab}(\lambda,\lambda_1,\lambda_2) \times \prod_e \frac{(1-T_e\lambda)}{((1+T_e\lambda_1)(1+T_e\lambda_2))^{1/2}}$$

$$\mu(\lambda,\lambda_1,\lambda_2) = \frac{\lambda(1-\lambda)|\lambda_1-\lambda_2|}{(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2(\lambda_1\lambda_2(1+\lambda_1)(1+\lambda_2))^{1/2}}$$

$$J_{ab}(\lambda,\lambda_1,\lambda_2) = \delta_{ab}\,T_a^2\,(1-T_a) \times \left(\frac{\lambda_1}{1+T_a\lambda_1} + \frac{\lambda_2}{1+T_a\lambda_2} + \frac{2\lambda}{1-T_a\lambda}\right) + (1+\delta_{ab})\,T_a\,T_b + \left(\frac{\lambda_1(1+\lambda_1)}{(1+T_a\lambda_1)(1+T_b\lambda_1)} + \frac{\lambda_2(1+\lambda_2)}{(1+T_a\lambda_2)(1+T_b\lambda_2)} + \frac{2\lambda(1-\lambda)}{(1-T_a\lambda)(1-T_b\lambda)}\right)$$

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First test of VWZ in the intermed

statistical significance only achie

- Note: nuclear cross section fluctuation experiments yield only $|S|^2$

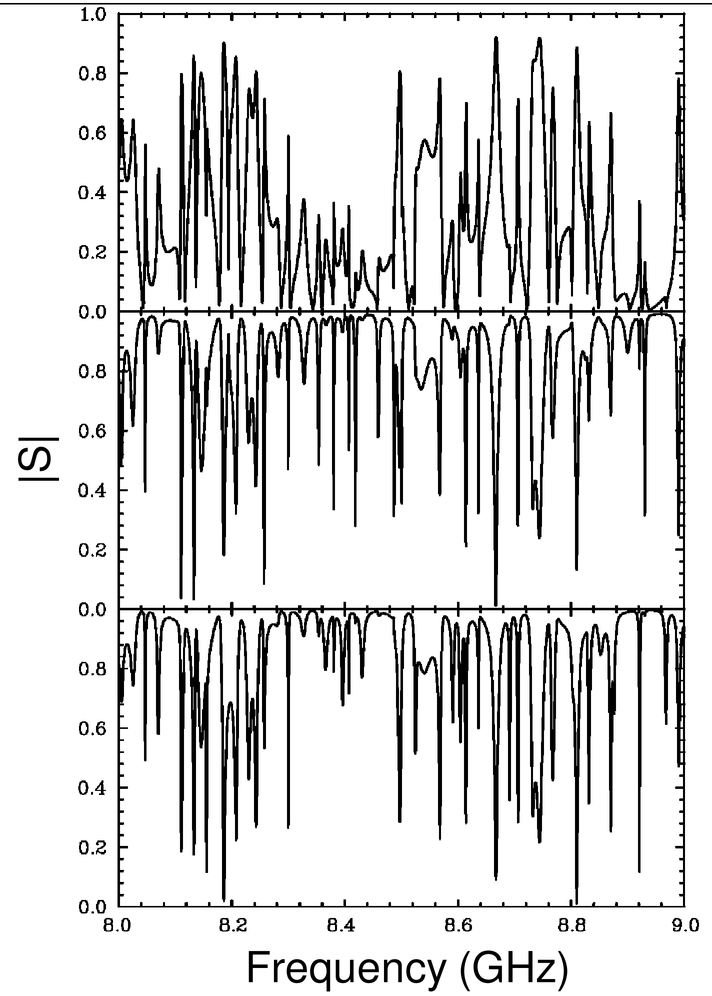
Spectra of S-Matrix Elements

Example: 8-9 GHz, $\Gamma/D \cong 0.2$

S_{12} →

S_{11} →

S_{22} →



Road to Analysis of the Measured Fluctuations

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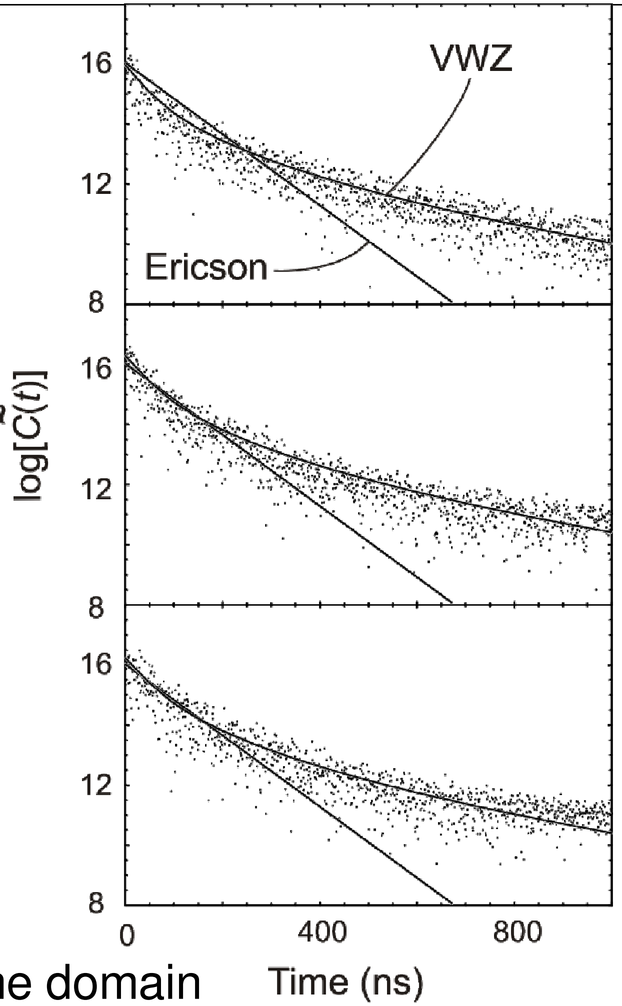
- Problem: adjacent points in C(ε) are correlated

$$C(\varepsilon) = \left\langle S(f)S^*(f+\varepsilon)\right\rangle - \left\langle S\ (f)\right\rangle\left\langle S^*(f+\varepsilon)\right\rangle$$

- Solution: FT of C(ε) → uncorrelated Fourier coefficients $\tilde{C}(t)$
  Ericson (1965)

- Development: non Gaussian fit and test procedure

Fourier Transform vs. Autocorrelation Function



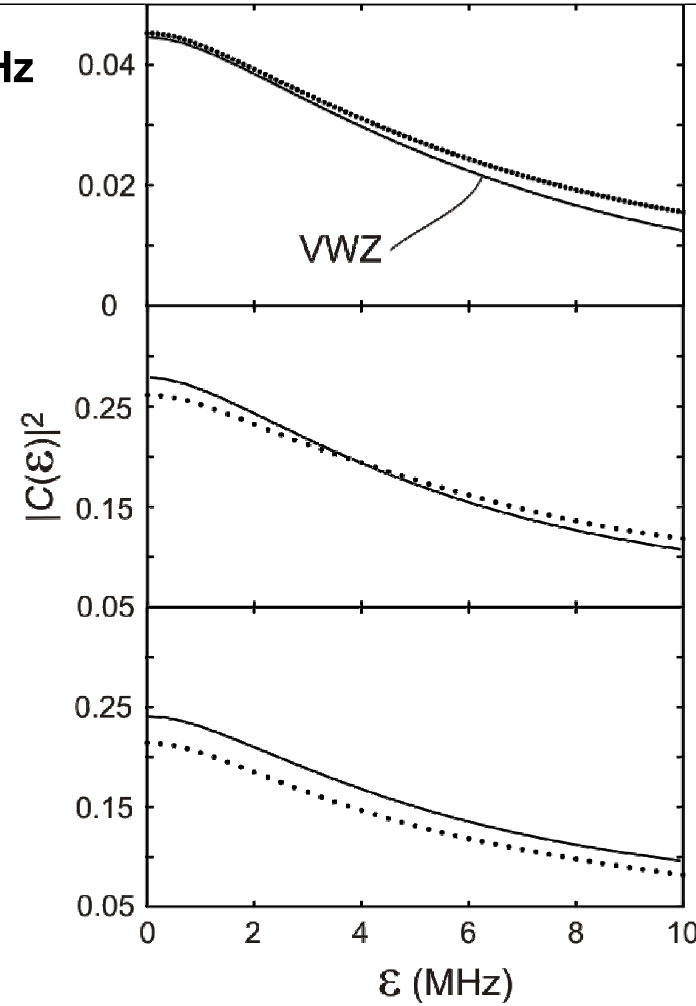
Time domain

Example 8-9 GHz

← S_{12}

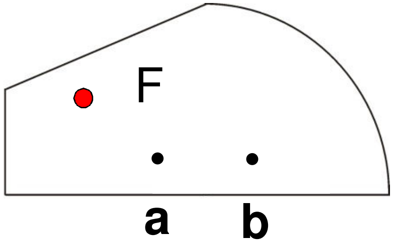
← S_{11}

← S_{22}

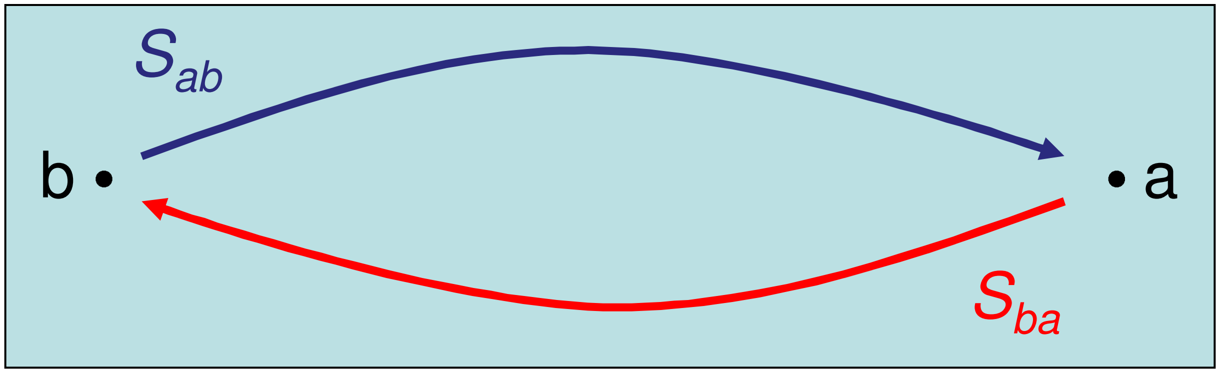


Induced Time-Reversal Symmetry Breaking (TRSB) in Billiards

- T-symmetry breaking caused by a magnetized ferrite

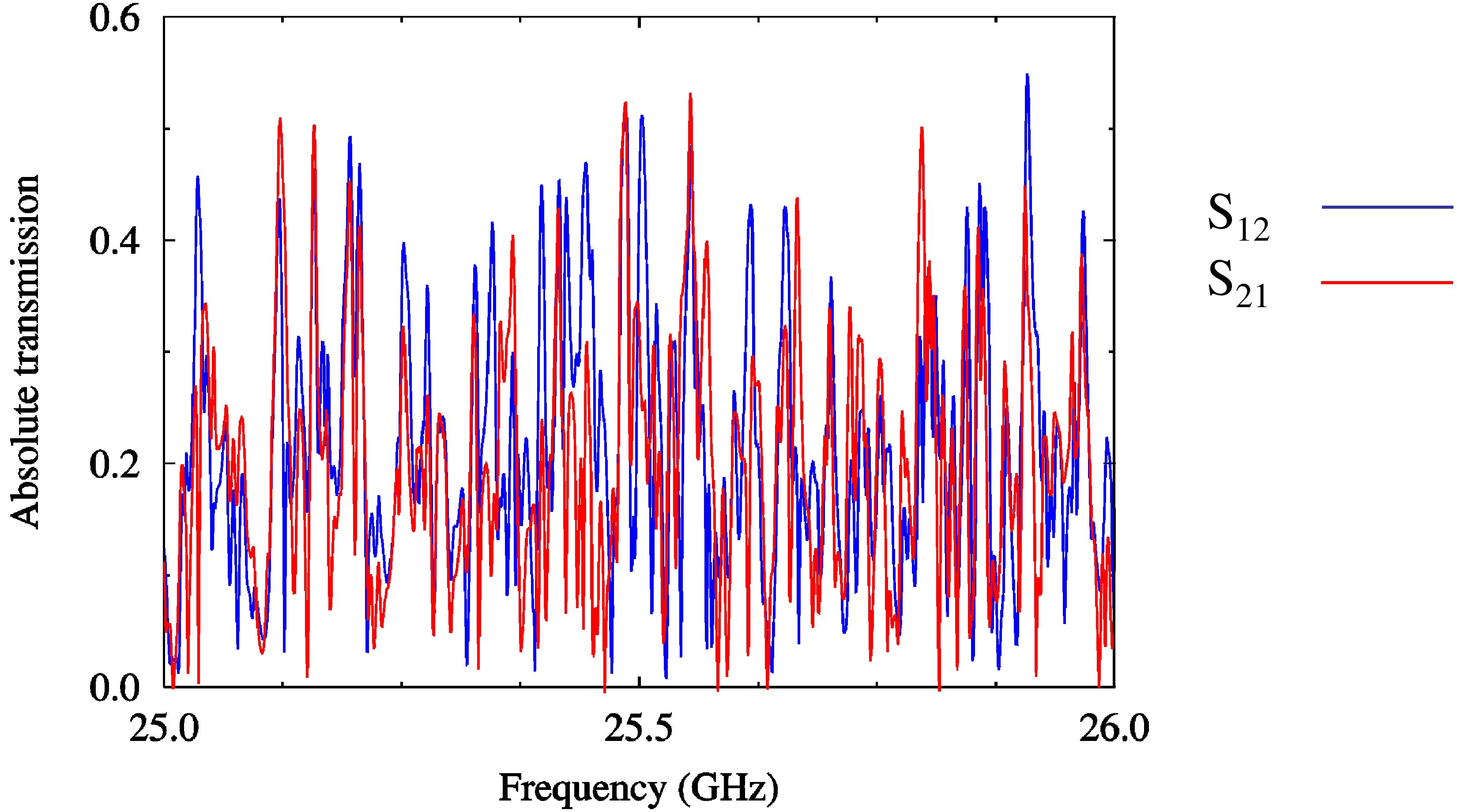


- Ferrite features Ferromagnetic Resonance (FMR)
- Coupling of microwaves to the FMR depends on the direction a $\leftrightarrows$ b



- Principle of detailed balance: $|S_{ab}|^2 = |S_{ba}|^2$
- Principle of reciprocity: $S_{ab} = S_{ba}$

Violation of Reciprocity



- Clear violation of reciprocity in the regime of Γ/D ≈ 1

Analysis of Fluctuations with Crosscorrelation Function

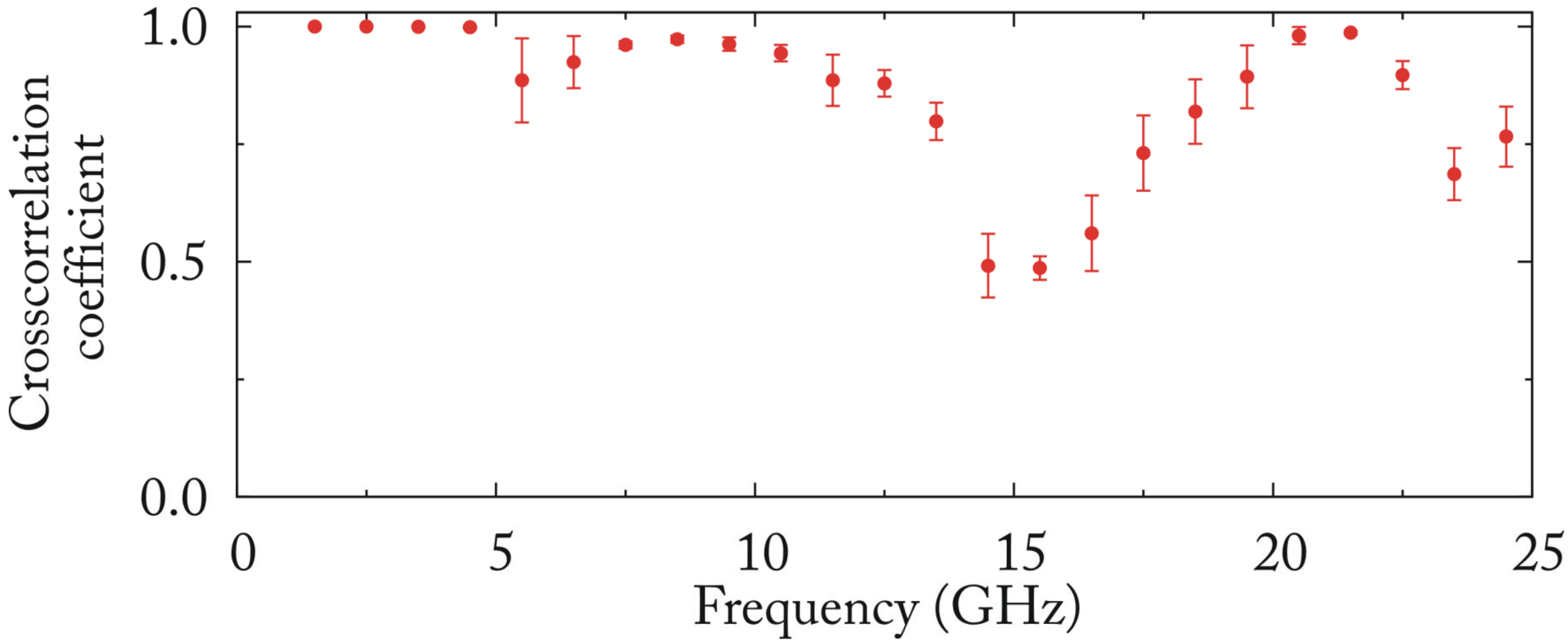
Crosscorrelation function:

$$C(S_{12}, S_{21}^{*}, \varepsilon) = \left\langle S_{12}(f)\, S_{21}^{*}(f+\varepsilon)\right\rangle - \left\langle S_{12}(f)\right\rangle \left\langle S_{21}^{*}(f)\right\rangle$$

- Determination of T-breaking strength from the data

- Special interest in first coefficient (ε = 0)

Experimental Crosscorrelation coefficients

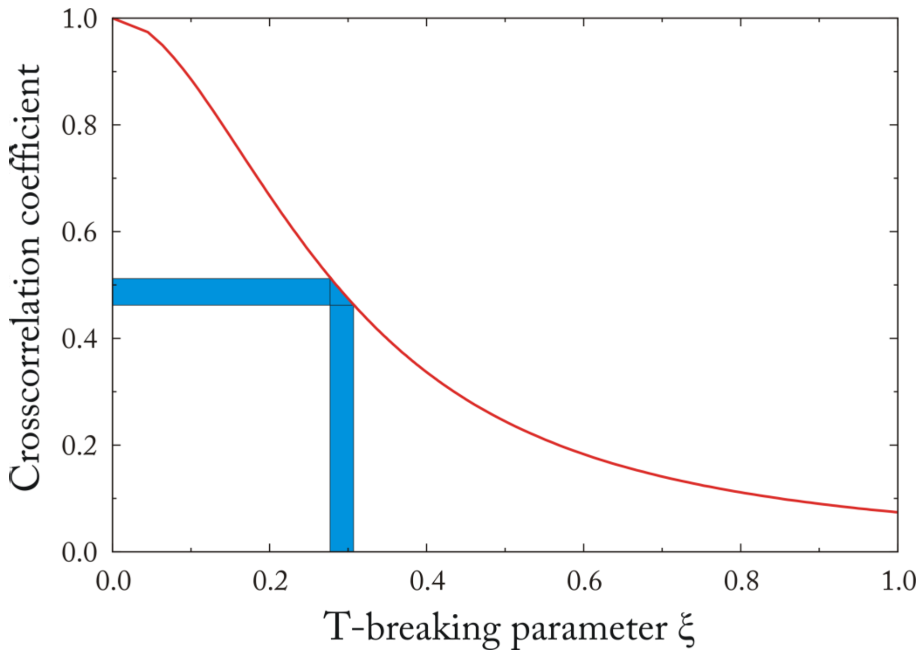


- $C(S_{12}, S_{21}^{*}) = \begin{cases} 1 & \text{for GOE} \\ 0 & \text{for GUE} \end{cases}$

- Data: TRSB is incomplete → mixed GOE/GUE system

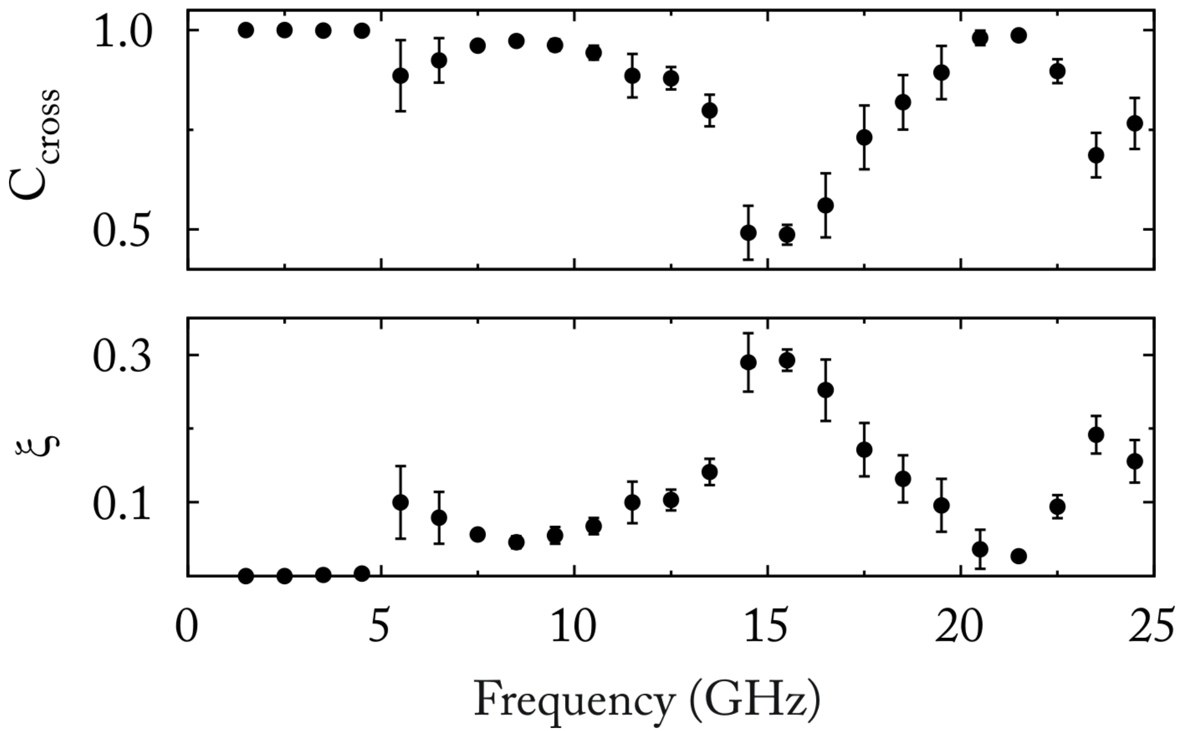
Exact RMT Result for Partial T Breaking

- RMT analysis based on Pluhař, Weidenmüller, Zuk, Lewenkopf and Wegner, 1995



- RMT $\rightarrow \quad H = H^s + i\left(\pi\xi/\sqrt{N}\right)H^a, \quad \xi = \begin{cases} 0 & \text{for GOE} \\ 1 & \text{for GUE} \end{cases}$

Determination of T-Breaking Strength



- B. Dietz *et al.*, Phys. Rev. Lett. **103**, 064101 (2009).

„Wo das Chaos auf die Ordnung trifft, gewinnt meist das Chaos, weil es besser organisiert ist.“

”Where chaos meets order, wins mostly chaos, since it is better organized.“

Friedrich Nietzsche (1844-1900)