Nonlinear Zeeman effect in boronlike highly charged ions

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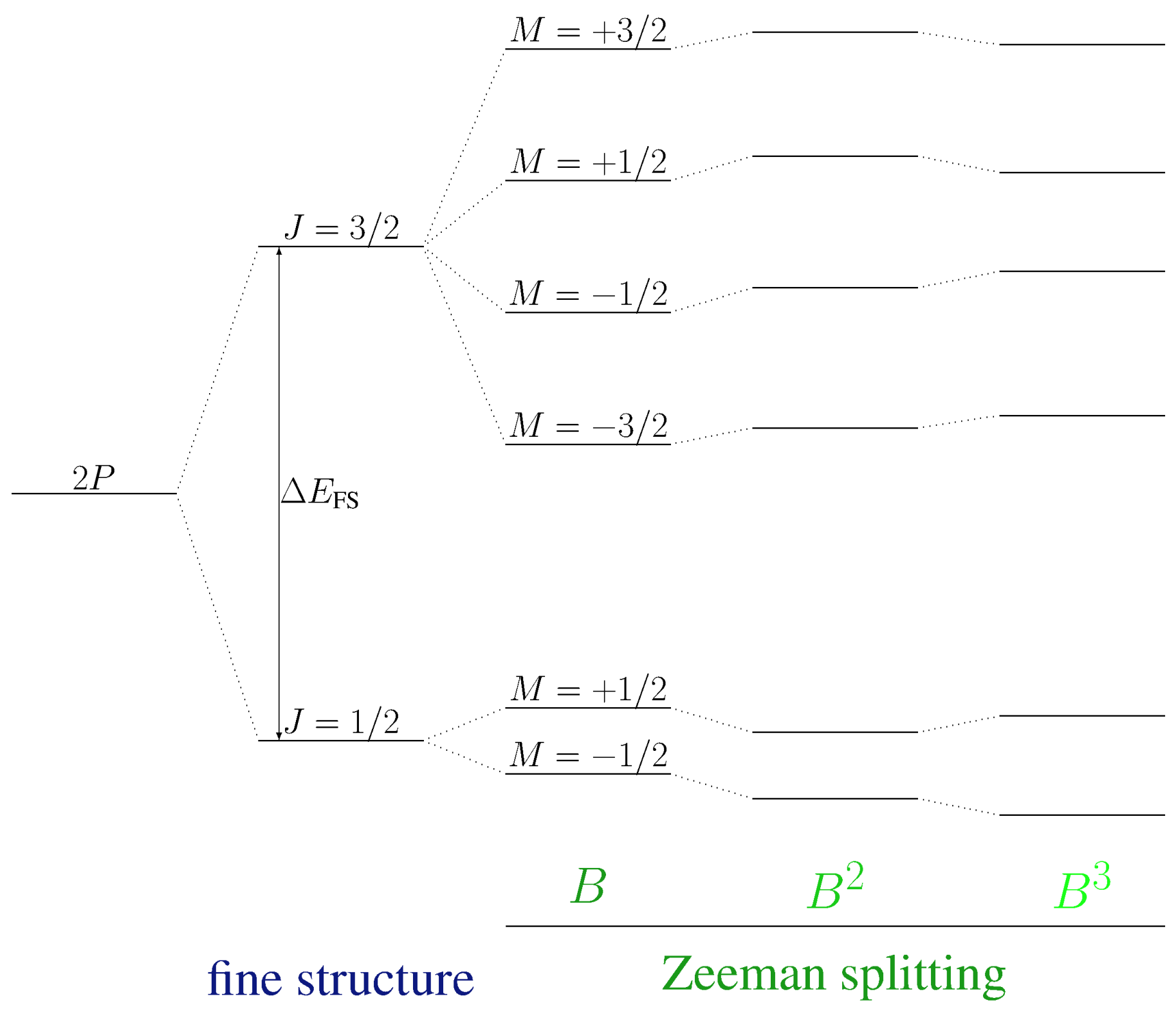
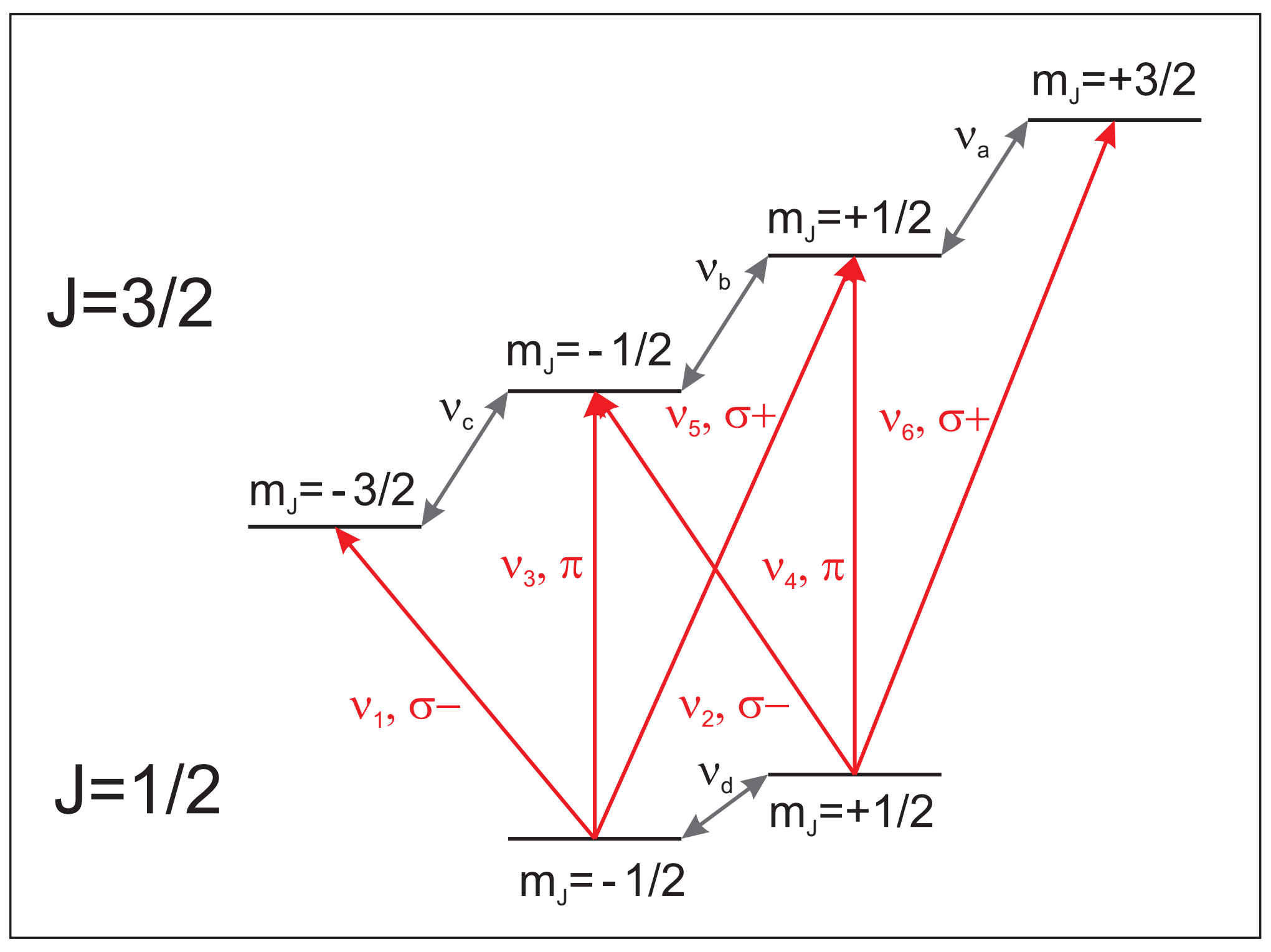
Motivation

- High-precision determination of α from the g factor of heavy boronlike ions [1]
- Test of purely relativistic one- and many-electron QED effects in magnetic field

Zeeman splitting in boronlike ion



Experiment ARTEMIS @ GSI



$$\lambda_1 \approx \ldots \approx \lambda_6 \approx 442 nm$$
$$\lambda_d \approx 2\lambda_a \approx 2\lambda_b \approx 2\lambda_c \approx 4.61 mm$$

Zeeman effect in ^{40}Ar^{13+}

$$\Delta E(B) = \Delta E^{(1)}(B) + \Delta E^{(2)}(B) + \Delta E^{(3)}(B) + \ldots$$
$$\Delta E^{(1)} = g_J \mu_0 B M_J$$

g-factor values from [4]:

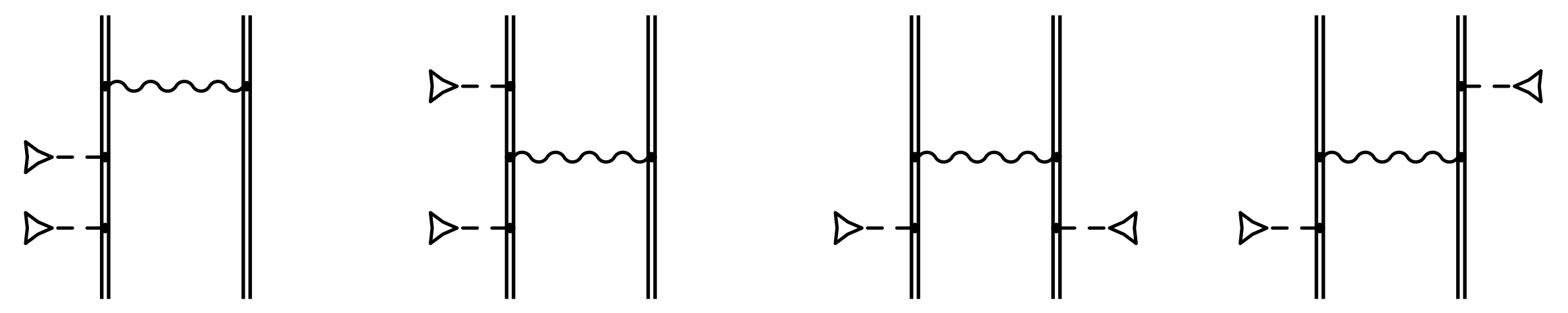
$$g_{1/2} = 0.663647(1) \qquad g_{3/2} = 1.332291(3)$$

$$\Delta E^{(2)} = g_J^{(2)}(M_J)(\mu_0 B)^2/(mc^2)$$
$$\frac{\Delta E^{(2)}(B)}{\Delta E^{(1)}(B)} \sim \frac{\Delta E^{(1)}(B)}{\Delta E_{\rm FS}} \sim 10^{-4}$$

$$\Delta E^{(3)} = g_J^{(3)}(M_J)(\mu_0 B)^3/(mc^2)^2$$
$$\frac{\Delta E^{(3)}(B)}{\Delta E^{(1)}(B)} \sim \left(\frac{\Delta E^{(1)}(B)}{\Delta E_{\rm FS}}\right)^2 \sim 10^{-8}$$

Quadratic effect

One-photon-exchange diagrams



One-photon-exchange correction

		$2P_{1/2}\ \ M_J = 1/2$	
Z		CH	LDF
	$g_0^{(2)}$	-2.0285×10^6	-2.1108×10^6
8	$\Delta g_{\rm int,1}^{(2)}$	-0.1564×10^6	-0.0836×10^6
	$g_{\rm int}^{(2)}$	-2.1849×10^6	-2.1945×10^6
	$g_0^{(2)}$	-3.6782×10^4	-3.7135×10^4
18	$\Delta g_{\rm int,1}^{(2)}$	-0.1470×10^4	-0.1146×10^4
	$g_{\rm int}^{(2)}$	-3.8253×10^4	-3.8281×10^4
	$g_0^{(2)}$	-2.3238×10	-2.3176×10
82	$\Delta g_{\rm int,1}^{(2)}$	0.0087×10	0.0020×10
	$g_{\rm int}^{(2)}$	-2.3150×10	-2.3155×10
		$2P_{3/2}\ \ M_J = 1/2$	
Z		CH	LDF
	$g_0^{(2)}$	2.0405×10^6	2.1231×10^6
8	$\Delta g_{\rm int,1}^{(2)}$	0.1592×10^6	0.0860×10^6
	$g_{\rm int}^{(2)}$	2.1998×10^6	2.2092×10^6
	$g_0^{(2)}$	3.8189×10^4	3.8556×10^4
18	$\Delta g_{\rm int,1}^{(2)}$	0.1611×10^4	0.1270×10^4
	$g_{\rm int}^{(2)}$	3.9800×10^4	3.9827×10^4
	$g_0^{(2)}$	6.5589×10	6.5561×10
82	$\Delta g_{\rm int,1}^{(2)}$	0.0970×10	0.1001×10
	$g_{\rm int}^{(2)}$	6.6560×10	6.6562×10

Calculation method

$$\Delta\tilde{\varepsilon}_a = \sum_{\tilde b}\left(\langle \tilde a\tilde b|I(0)|\tilde a\tilde b\rangle - \langle \tilde a\tilde b|I(\tilde\omega_{ab})|\tilde b\tilde a\rangle\right) - \langle \tilde a|V_{\rm scr}|\tilde a\rangle,$$

where $\tilde a$ and $\tilde b$ are wave functions with magnetic field included, $\tilde\omega_{ab} = \tilde\varepsilon_a - \tilde\varepsilon_b$

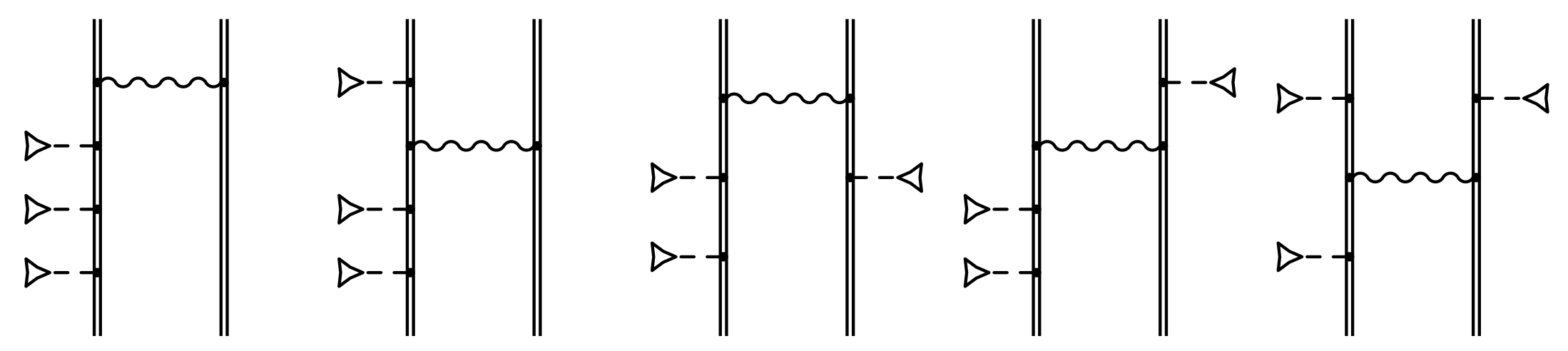
$$\Delta g_{\rm int,1}^{(k)} \equiv \Delta\varepsilon_a^{(k)} = \frac{1}{k!}\frac{d^k\Delta\tilde\varepsilon_a}{d(\mu_0 B)^k}\bigg|_{B=0}$$

Zeeman shifts for B-like argon at 7 Tesla

J, M_J	$\Delta E_A^{(1)}/h$ [GHz]	$\Delta E_A^{(2)}/h$ [kHz]	$\Delta E_A^{(3)}/h$ [Hz]
$3/2, +3/2$	195. 793	74.	0.
$3/2, +1/2$	65. 264	3189.	−153.
$3/2, -1/2$	−65. 264	3189.	153.
$3/2, -3/2$	−195. 793	74.	0.
$1/2, +1/2$	32. 510	−3069.	153.
$1/2, -1/2$	−32. 510	−3069.	−153.

Qubic effect

One-photon-exchange diagrams



One-photon-exchange correction

		$2P_{1/2}\ \ M_J = 1/2$	
Z		CH	LDF
	$g_0^{(3)}$	6.231×10^{12}	6.746×10^{12}
8	$\Delta g_{\rm int,1}^{(3)}$	0.964×10^{12}	0.538×10^{12}
	$g_{\rm int}^{(3)}$	7.195×10^{12}	7.284×10^{12}
	$g_0^{(3)}$	2.156×10^9	2.198×10^9
18	$\Delta g_{\rm int,1}^{(3)}$	0.175×10^9	0.138×10^9
	$g_{\rm int}^{(3)}$	2.331×10^9	2.336×10^9
	$g_0^{(3)}$	4.5726×10^3	4.5650×10^3
82	$\Delta g_{\rm int,1}^{(3)}$	0.0946×10^3	0.1028×10^3
	$g_{\rm int}^{(3)}$	4.6672×10^3	4.6679×10^3
		$2P_{3/2}\ \ M_J = 1/2$	
Z		CH	LDF
	$g_0^{(3)}$	-6.231×10^{12}	-6.746×10^{12}
8	$\Delta g_{\rm int,1}^{(3)}$	-0.964×10^{12}	-0.538×10^{12}
	$g_{\rm int}^{(3)}$	-7.195×10^{12}	-7.284×10^{12}
	$g_0^{(3)}$	-2.156×10^9	-2.198×10^9
18	$\Delta g_{\rm int,1}^{(3)}$	-0.175×10^9	-0.138×10^9
	$g_{\rm int}^{(3)}$	-2.331×10^9	-2.336×10^9
	$g_0^{(3)}$	-4.6062×10^3	-4.5986×10^3
82	$\Delta g_{\rm int,1}^{(3)}$	-0.0941×10^3	-0.1024×10^3
	$g_{\rm int}^{(3)}$	-4.7004×10^3	-4.7011×10^3

Cubic effect as g-factor correction ($B = 1$ Tesla)

$$\delta g = (\mu_0 B)^2 g^{(3)}(M_J)/M_J$$

Z	$2P_{1/2}$	$2P_{3/2}$
8	1.869×10^{-7}	-1.869×10^{-7}
12	2.731×10^{-9}	-2.731×10^{-9}
18	5.995×10^{-11}	-5.995×10^{-11}
24	4.581×10^{-12}	-4.581×10^{-12}
54	4.381×10^{-15}	-4.385×10^{-15}
82	1.198×10^{-16}	-1.206×10^{-16}

References

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