A fundamental constant as basis for the SI unit kilogram

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Only few weeks remaining, a 125 years old small cylinder of platinum-iridium, the International Prototype of the Kilogram, IPK, still defines the SI unit of mass. In November 2018 CGPM, the general conference on weights and measures, will meet to redefine four of the seven SI units on the base of fundamental constants. For the kilogram the Avogadro experiment provides a link of the kilogram to the atomic mass constant m_u by counting atoms in a given amount of mass – here a kilogram of a ²⁸Si single crystal. But how to "count" 10²⁵ atoms as the age of the universe is only 10¹⁷ seconds? The solution is a crystal – a very good crystal of best purity, highest quality and perfection of crystalline order. With this it is possible to calculate the number of atoms if one only knows the distance of the atoms in the crystal and the macroscopic dimension of an artifact of this crystal. For the determination of Avogadro's constant, a sphere was chosen. Silicon crystals, which occur face-centered cubic, are available in high perfection by special institutes for crystal growth and the form of a sphere was selected as the obvious form of a cube failed because of the stability of its edges.

For the measurement of the Avogadro constant [1] four quantities are to be determined:

 $N_A = \frac{M_{Si} \cdot V_{sphere}}{n \cdot v_{uc} \cdot m_{sphere}}$, with n = 8 the number of atoms per cubic unit cell

Herein the quotient of macroscopic volume V_{sphere} and the volume of the unit cell v_{uc} of silicon atoms gives the number of atoms of that sphere. Mass of the sphere m_{sphere} and molar mass M_{Si} considering the mass of the entity so that the number of atoms per mole is derived.

The measurements are divided into crystal measurements which determine parameters typical for the whole silicon crystal, here the molar mass and the volume of the unit cell, and the properties which are related to the artifact produced from the crystal, here mass and volume of a test sphere.

For the measurement of the molar mass a new technique, the isotope dilution mass spectrometry IDMS, demanded highly isotopically enriched material to allow relative uncertainties at the 10^{-9} level. 99.99% ²⁸Si material was received from Russian institutes, unfortunately for a price of about 1 Mio ϵ /1kg-sphere. The lattice parameter is determined by COXI, combined optical and X-ray interferometry. Probes of the crystal are arranged in a Laue interferometer and the movement is measured with an optical interferometer [2].



From a Si crystal two spheres are produced. They are manufactured with outstanding perfection regarding roundness, roughness and subsurface damage of the crystal. The spheres must be measured for mass and volume. For the mass the sphere is compared to the national prototype of kilogram, a Pt-Ir cylinder, considering different volume, surface and material. For the volume of the sphere an optical interferometer is used. It consists of two high performance objectives with spherical reference faces which spacing is determined. With the sphere inserted the resulting gaps between sphere and the respective objective are measured.

This interferometer resolves deviations from roundness in the sub-nm range and yields full topographies of the silicon spheres. A relative volume uncertainty of $7 \cdot 10^{-9}$ could be achieved.

- [1] G Bartl et al., Metrologia 54 (2017) 693-715
- [2] E Massa et al., Metrologia 48 (2011) S37-S43