Atomic photoexcitation by twisted light

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Crucial paper for this talk: New J. Phys. 20 (2018) 023032

for experts

- There are three ways (at least) to discuss twisted photons:
  - Bessel beams
  - Bessel-Gauss beams
  - Laguerre-Gauss beams
- The last is shortchanged in this talk, because of time.

for everyone: Topics

- History (brief)
- Twisted photon basics
- Theory results in atomic photoexcitation
  (Experimental results, 2nd floor, Physics building, JGU, Mainz)
- End

Some history

- Waves diffract (mostly)
- Plane waves don’t: they are too bland
- 1987: Durnin points out existence of structured waves that also don’t diffract.
- Structured means there are hot spots and cold spots in the wave front, and nondiffracting means the hot spots don’t spread out.

History: Durnin

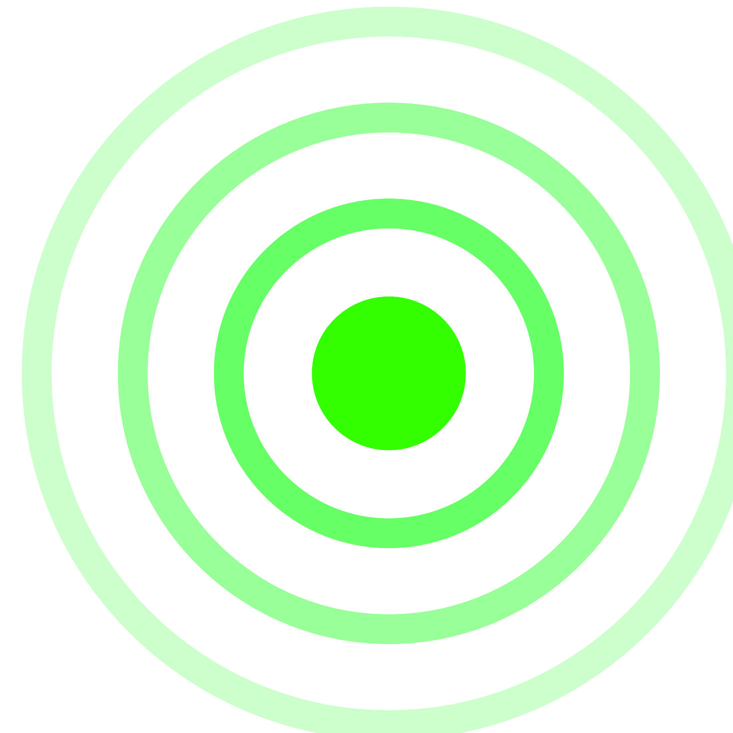
- Waves satisfy the Helmholtz equation

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\psi(t, \overrightarrow{x}) = \left(\nabla^2 + k^2\right)\psi(t, x, y, z) = 0$$

- Monochromatic: $e^{-i\omega t}$
- Traveling in z-direction, non-diffractive, must have $e^{i\beta z}$
- Solution $\psi = e^{i(\beta z - \omega t)} J_0(\alpha\rho)$
- where $\alpha \to k_\perp, \quad \beta \to k_z, \quad k_\perp^2 + k_z^2 = k^2 \ ; \quad \rho = \sqrt{x^2 + y^2}$

Bessel wave

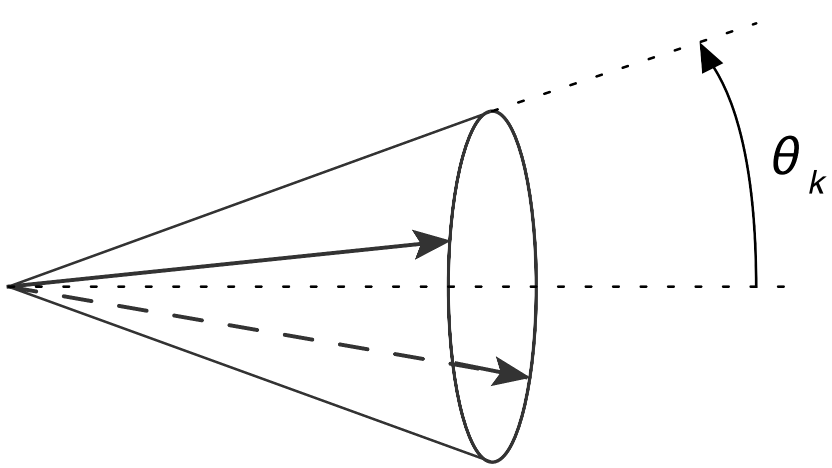
- Wave front coming at you:



- Bullseye pattern, vortex center (vortex line)
- Hot spot in center (for J_0)

Wavenumber space

- In QM, momentum space
- Recall $\psi = e^{i(\beta z - \omega t)} J_0(\alpha \rho)$
- Fourier transform, $\tilde{\psi}(t, k_z, k_\perp, \phi_k) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t} \delta(k_z - \beta) \delta(k_\perp - \alpha)$
- Every component in wave number space has same k_z, same $k_\perp$, but all possible azimuthal angles ϕ_k



- Component momenta form a cone, opening angle or "pitch angle" θ_k

γ angular momentum

- Initiating paper: Allen *et al.*, 1992
- Can discuss what they did classically or QM
- Classically, for plane wave, RH polarization,

$$angular\ momentum = +\frac{energy}{angular\ frequency}$$

- QM, for single photon,

$$angular\ momentum = +\hbar$$

- Now (QM version),

$$angular\ momentum = +(any\ integer) \times \hbar$$

- How?

Twisted photons

- New solution to Helmholtz equation,

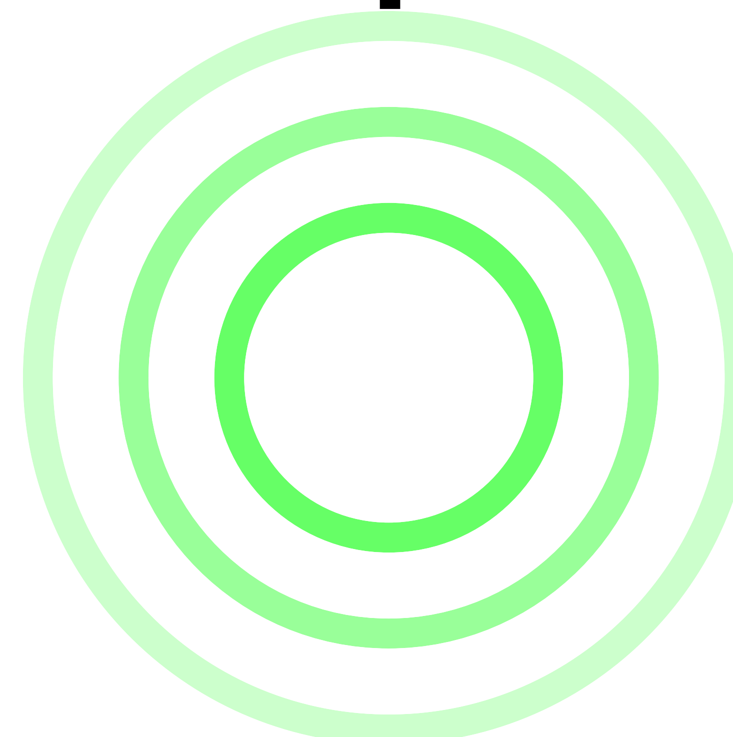
$$\psi(t,z,\rho,\phi_\rho) = e^{i(\beta z-\omega t)}\, e^{im_\gamma\phi_\rho}\, J_{m_\gamma}(\alpha\rho)$$

- or

$$\tilde{\psi}(t,k_z,k_\perp,\phi_k) = \frac{(2\pi)^2}{k_\perp}\, e^{-i\omega t}\, \delta(k_z-\beta)\,\delta(k_\perp-\alpha)\, i^{-m_\gamma}\, e^{im_\gamma\phi_k}$$

- Same, except that phase changing around edge of cone
- If $L_z = -\,i\hbar\frac{\partial}{\partial\phi_\rho}$ have $L_z = m_\gamma\hbar$
- Above for scalar photons (beloved of some theorists), results to be shown are for (real world) vector photons.
- Vectors photons have additional QN, helicity of plane wave component states = $\Lambda = \pm 1$.

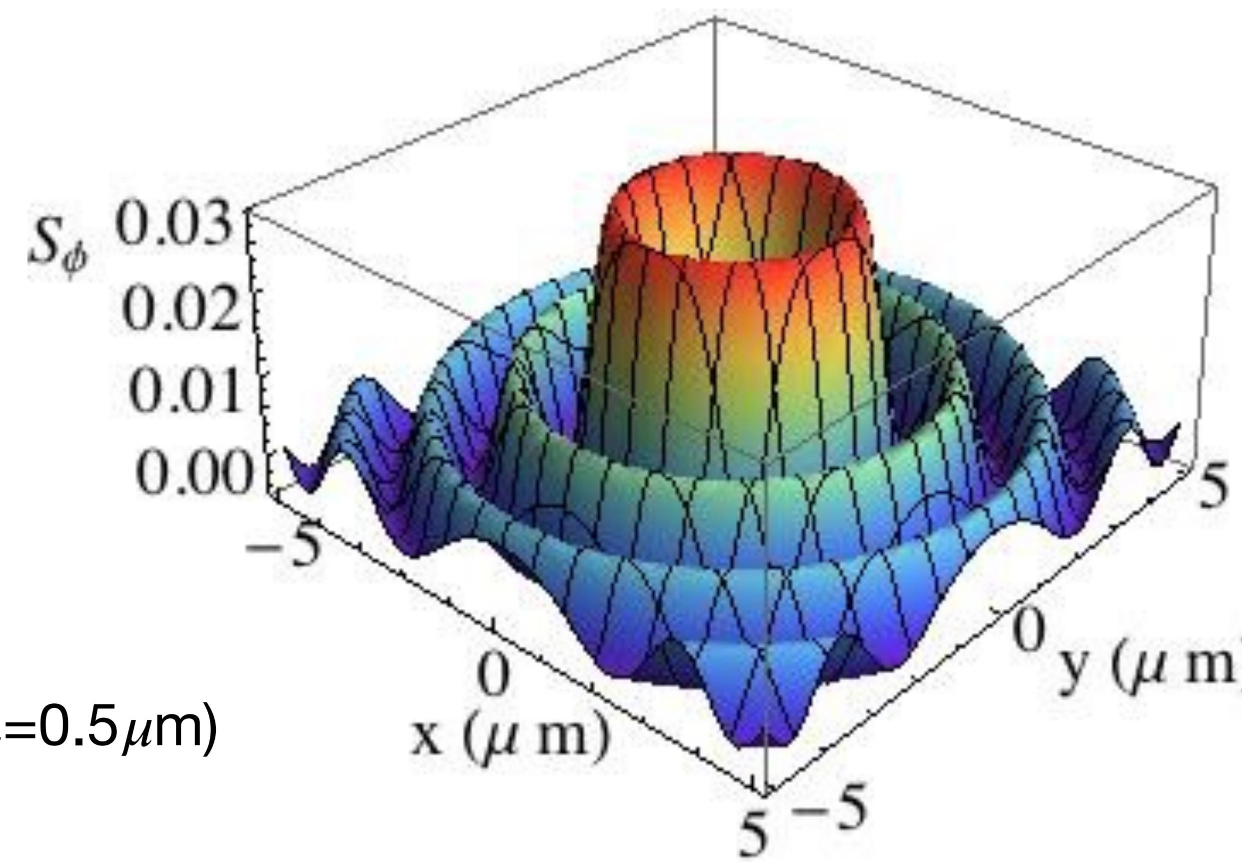
Show Poynting vector plots



- Wave front for $J_{m_\gamma}, \;\; m_\gamma \neq 0$, has hole in center

- Plane wave has Poynting vector only in z-direction

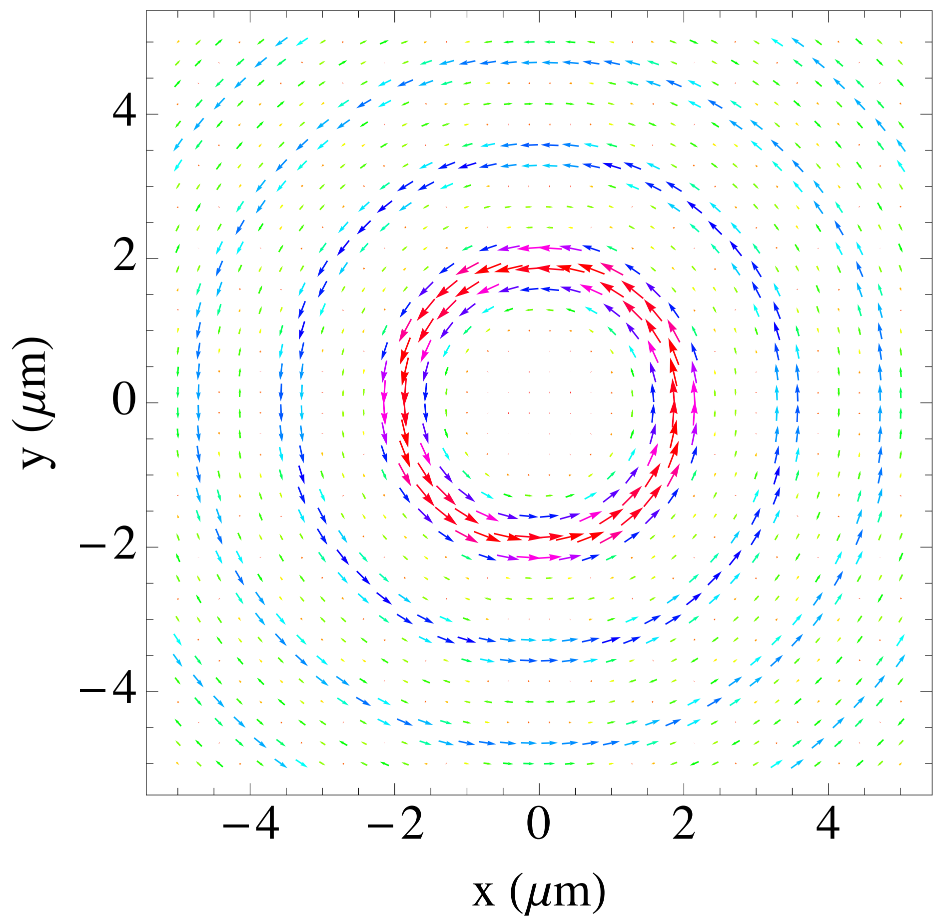
- Twisted photon also has azimuthal component of Poynting vector, S_ϕ



(figure for m_γ=4, θ_k=0.2, λ=0.5μm)

One more plot

- Previous figure gave magnitude of Sϕ; here indicate direction:



also for m_γ=4, θ_k=0.2, λ=0.5μm

- Swirling gives photon orbital angular momentum in z-dir.

- Spin of photon projects to $\Lambda \cos(\theta_k)$ in z-dir.

- Total projected angular momentum of state is m_γ

Selection rules

- Photoexcitation, initial state $\{j_i, m_i\}$ goes to final state $\{j_f, m_f\}$
- Plane wave photon, $m_f - m_i = \Lambda$ (always),
  $| j_f - j_i | = 1$ (usually)
  ($| j_f - j_i | = 1$ are E1 transitions).
- Twisted photon and direct hit (vortex line passing through atom's center), gives

  $$m_f - m_i = m_\gamma \quad \text{(may be >> 1)}$$
- Extraordinary! Off axis can get more ordinary.
- Selectively excite higher angular momentum higher atomic states (or higher nuclear/nucleon resonances).

Atomic possibilities

- See that on the nose strikes give quantum number changes not possible with plane wave photons
- Predict photoexcitation rates when atom and vortex line are offset by measured amount

Results

- (Next slides)
- Target is $^{40}Ca^+$ ions
- Ground state has S-state valence electron
- All transitions are S -> D, ≈ 729 nm wavelength
- Specifically, to $D_{5/2}$, with applied magnetic field Zeeman separating the different final J_z
- Figures to be shown here all have initial S_z = -1/2
- There is data on later figures.
(Also published data in Nature Comm., Dec. 2016)

Practical item: Bessel-Gauss

- Bessel beam falls only slowly in transverse direction ($\propto 1/\rho^{1/2}$). Require unlimited energy. Can't be made.

- Real beams can be Bessel-Gauss, e.g., for scalar case

$$\psi(t, z = 0\,, \rho, \phi_\rho) = A\, e^{-i\omega t}\, e^{i m_\gamma \phi_\rho}\, J_{m_\gamma}(\alpha\rho)\, e^{-\rho^2/w_0^2}$$

- For w_0 >> wavelength, diffraction spread slow and can be ignored under actual experimental conditions.

- Three parameters (ω given):

  - A — overall amplitude

  - θ_k — pitch angle ($\alpha = k_\perp = k \sin(\theta_k)$)

  - w_0 — width of beam

Off axis transitions

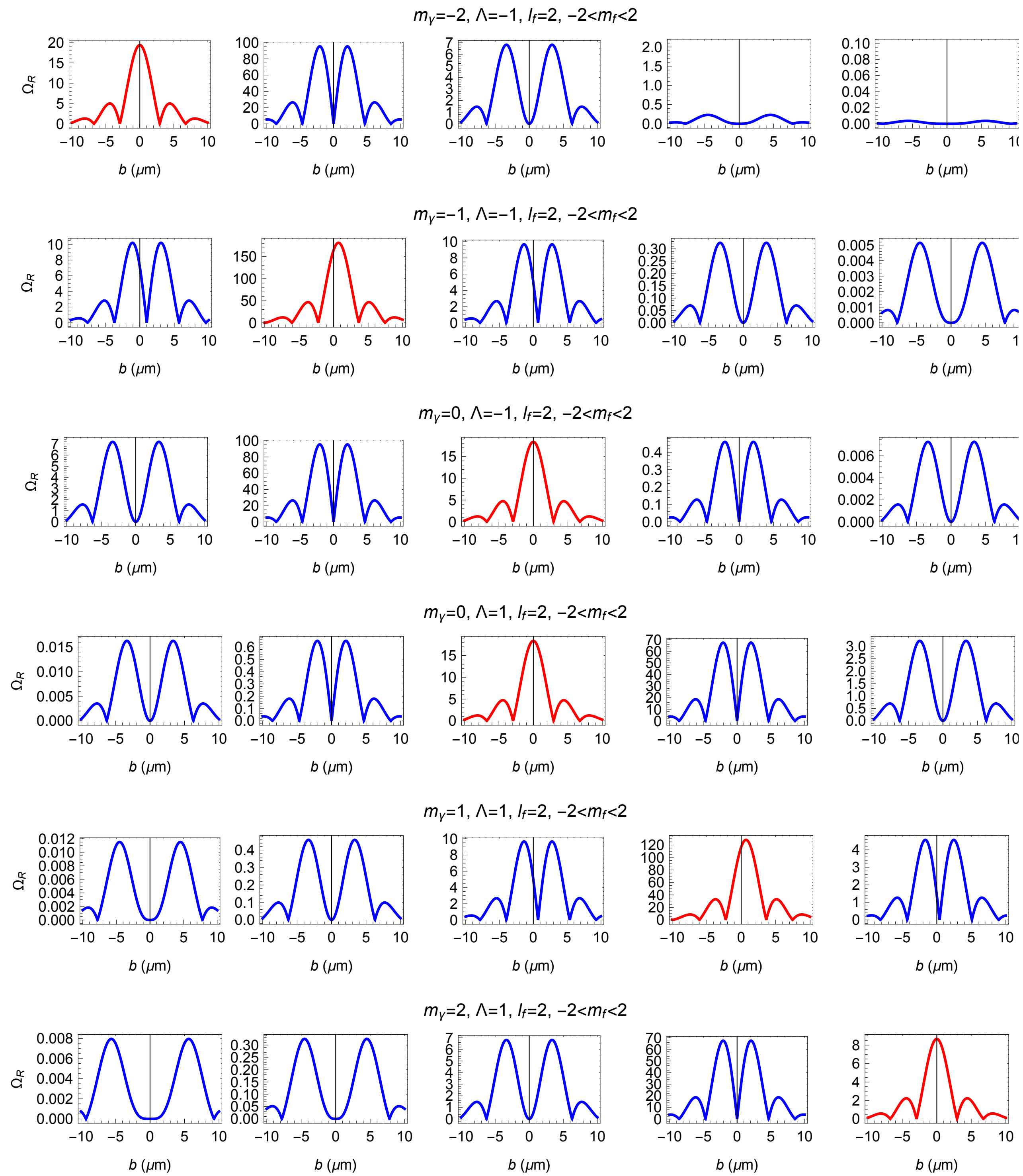
- Qualitative: Electric field swirls as seen already for Poynting vector.
- Atom smaller than structure scale of wave front
- Atom on vortex line sees circular swirling. Electron will absorb all photon's angular momentum, if transition occurs.
- Farther out, the atom being small sees a roughly spatially constant E-field. Transitions will be largely electric transitions like that produced by plane wave.

Theory results

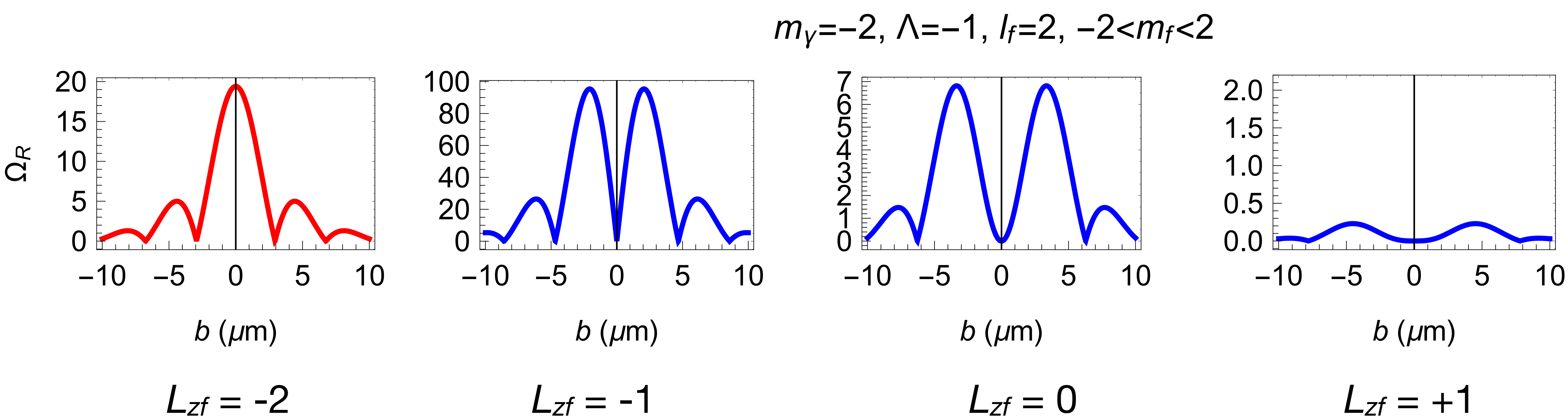
Bessel−Gauss beam calculations for target electron in S−state with $s_z = -1/2$

Pitch angle = 0.095 radians, Gaussian (1/e) width = 6.8 μm

With 0.% by amplitude of opposite photon helicity added in



Part of top row



- Code: red is most interesting curve for twisted photon
- L_{zf} = Λ is only case possible for untwisted photons
- L_{zf} = 1 result rather low.

… one more thing

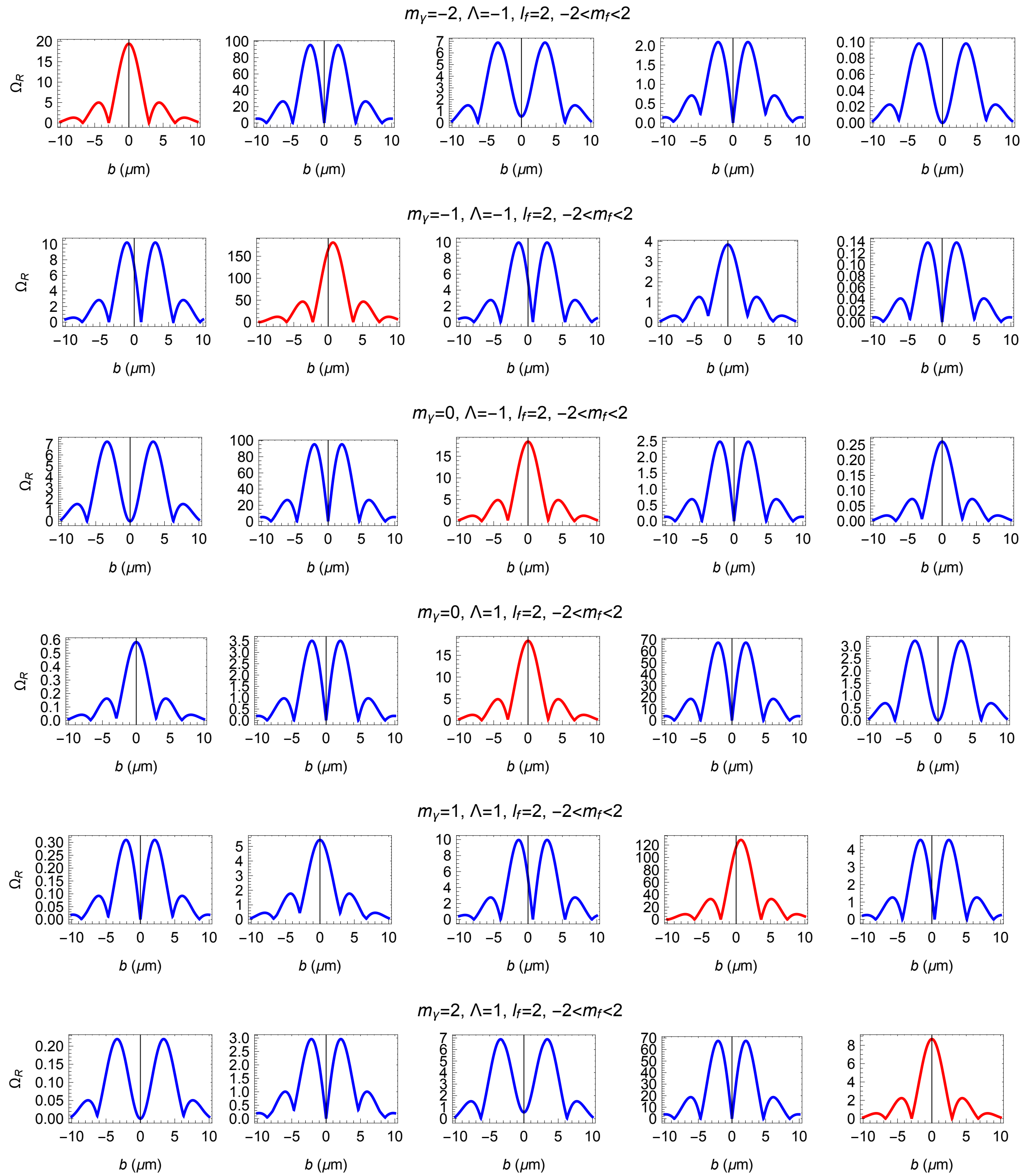
- There may be leakage of opposite helicity photons into beam. See effect of 3% by amplitude (9×10^{-4} by intensity) of opposite helicity photon.

Theory results again

Bessel−Gauss beam calculations for target electron in S−state with $s_z = -1/2$

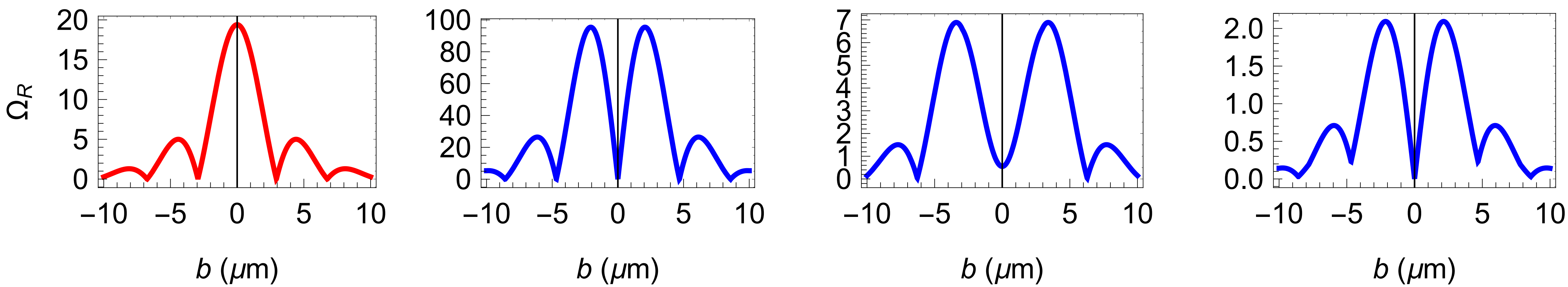
Pitch angle = 0.095 radians, Gaussian (1/e) width = 6.8 μm

With 3.% by amplitude of opposite photon helicity added in



Similar part of top row

m_γ=−2, Λ=−1, l_f=2, −2<m_f<2



L_{zf} = -2 L_{zf} = -1 L_{zf} = 0 L_{zf} = +1

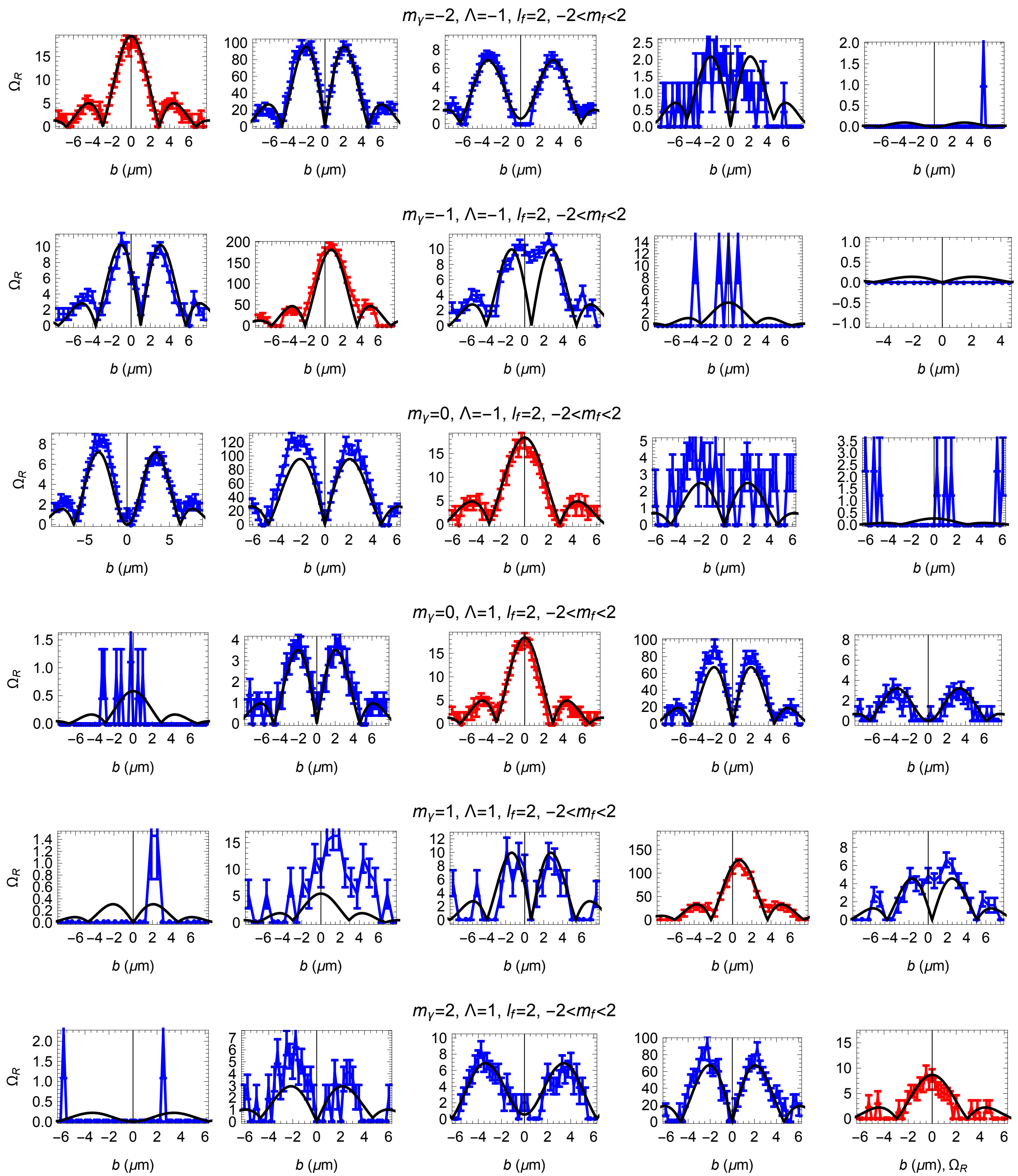
- L_{zf} = 1 much changed. Change in rest not noticeable.

Results with data

Bessel−Gauss beam calculations for target electron in S−state with $s_z = -1/2$

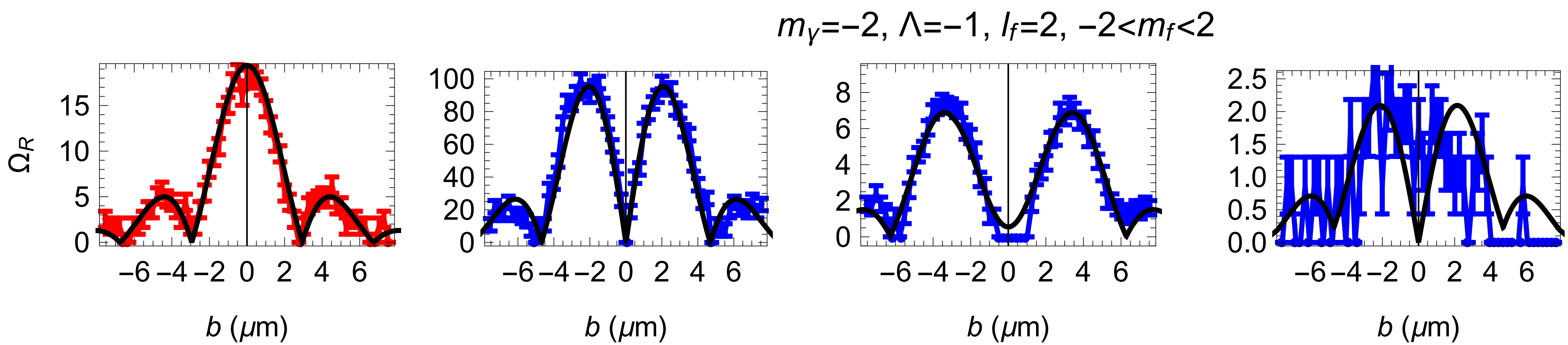
Pitch angle = 0.095 radians, Gaussian (1/e) width = 6.8 μm

With 3.% by amplitude of opposite photon helicity added in



more focused

With 3.% by amplitude of opposite photon helicity added in

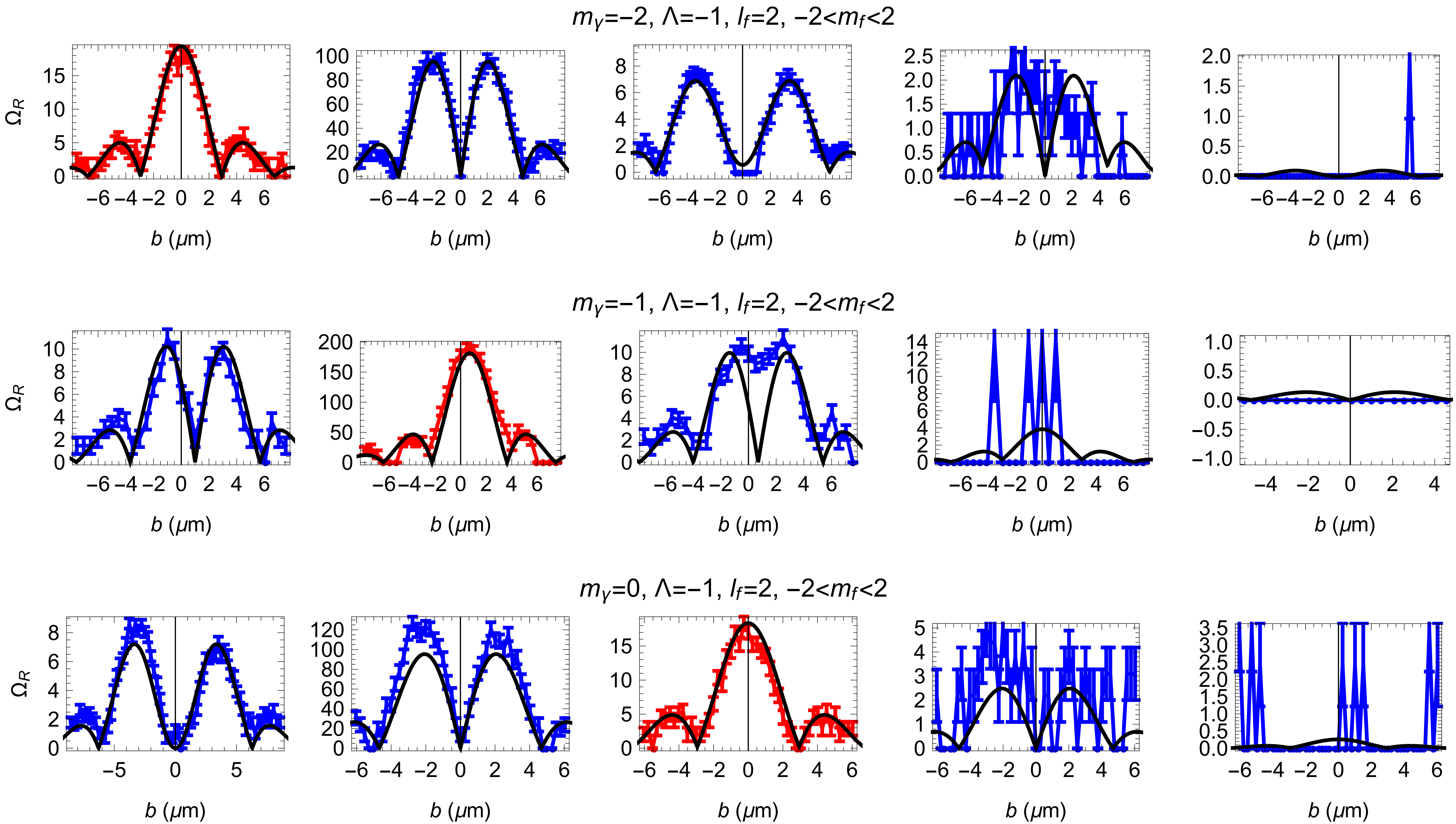


L_{zf} = -2 L_{zf} = -1 L_{zf} = 0 L_{zf} = +1

- First graph used to set A, θ_k, w_0
- L_{zf} = 1 used to set admixture of opposite helicity γ
- Other 26 graphs follow

the upper half

With 3.% by amplitude of opposite photon helicity added in



Penultimate comments

- Twisted photons exist and give extraordinary transitions
  - Significant rates for transitions with large angular momentum transfer, in situations where plane wave gives zero.
    First shown on 2nd floor, Staudingerweg 7
- Detailed theory works
  - Few parameters fit to one data set
  - Obtain accurate predictions for remaining data
  - allows determining beam characteristics

Comments for nuclear apps.

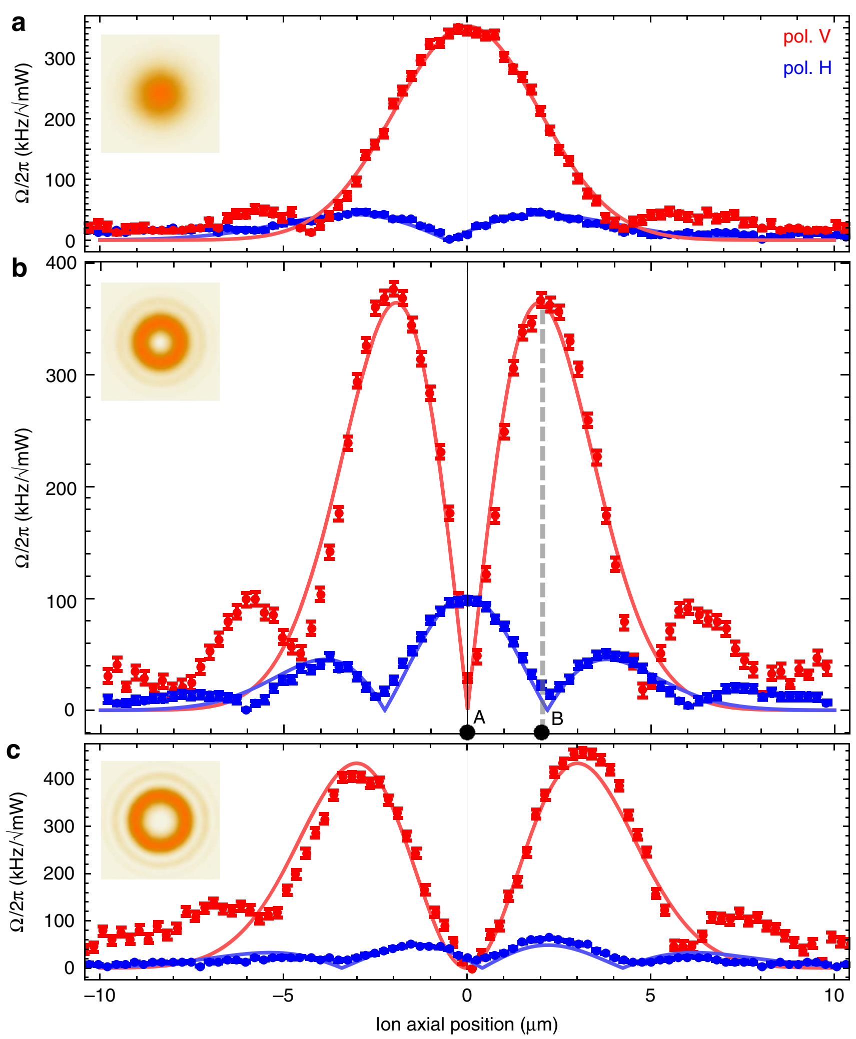
- Note on scales in atomic case
    - atom ca. 10^{-1} nm
    - location of single atom to ca. 10 nm
    - wavelength ca. 10^{3} nm
    - hole in wavefront several wavelengths wide
- Hope for nuclear/nucleon analogs?
    - nucleon ca. 1 fm (or 0.84087(39) fm)
      nucleus ca. 10 fm
    - location placement ?
    - 1 GeV photon -> wavelength ca. 1 fm
    - width of hole is large

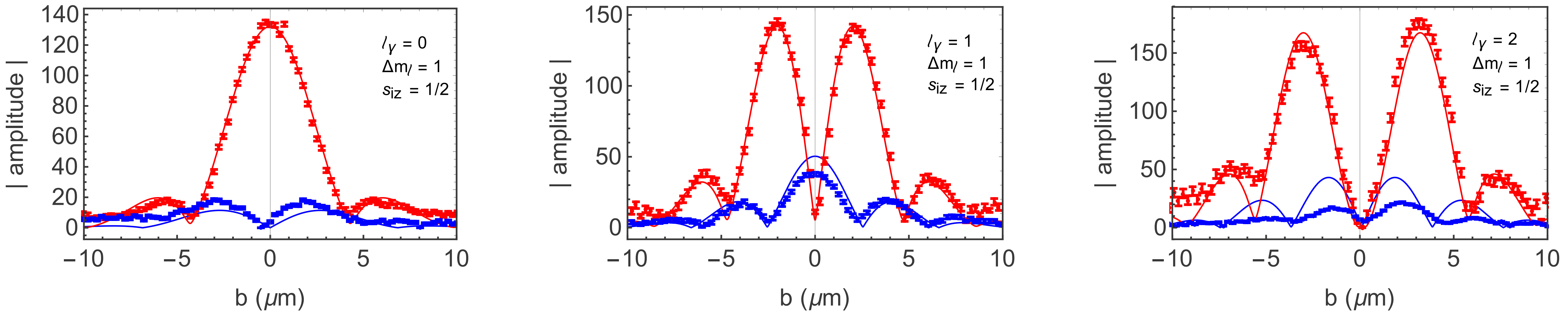
Thanks for listening

- Omitted topics
  - atomic recoil
  - radiation pressure and Poynting vector
  - circular dichroism on spherical targets
  - twisted light on N -> Delta(1232) transition
  - not to mention Laguerre-Gauss beams, and long distance propagation of Bessel-Gauss.

Beyond the end

Nat. Comm. 2016, Fig 3





Twisted vector photons

- Can obtain or visualize L_z in other ways
- First, noting we so far have scalar photons (beloved of some theorists), will switch to vector photons.
- Easiest is to note matrix elements of fields with standardly normalized states, scalar and vector:

$$\langle 0 \,|\, \psi(0) \,|\, \overrightarrow{k} \rangle = 1$$

$$\langle 0 \,|\, A_\mu(0) \,|\, \overrightarrow{k}, \Lambda \rangle = \varepsilon_\mu(k, \Lambda)$$

- These are for plane waves
- Λ is helicity of the plane wave photon state (= ±1)

Vector photons

- Momentum space is expansion in plane waves. Becomes,

$$A_\mu^{(m_\gamma)}(t, k_z, k_\perp, \phi_k, \Lambda) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t}\, \delta(k_z - \beta)\, \delta(k_\perp - \alpha)\, i^{-m_\gamma}\, e^{i m_\gamma \phi_k}\, \varepsilon_\mu(\overrightarrow{k}, \Lambda)$$

- Coordinate space expression medium long, will show on demand.

- Can work out electric and magnetic fields, and Poynting vector

Twisted γ in coordinate space

$$\mathscr{A}^{\mu}_{k_\perp k_z m_\gamma \Lambda}(x) = -i\Lambda\, e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \Big\{ e^{-i\Lambda\phi_\rho} \cos^2\frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho)\, \eta^\mu_\Lambda + \frac{i}{\sqrt{2}} \sin\theta_k J_{m_\gamma}(k_\perp\rho)\, \eta^\mu_0 - e^{i\Lambda\phi_\rho} \sin^2\frac{\theta_k}{2} J_{m_\gamma+\Lambda}(k_\perp\rho)\, \eta^\mu_{-\Lambda} \Big\}$$

where

$$\eta_{\pm 1} = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) = \frac{1}{\sqrt{2}} e^{\pm i\phi_\rho}\left(\mp\hat{\rho} - i\hat{\phi}\right), \quad \eta_0 = (0,0,0,1) = \hat{z}$$

fields:

$$E_\rho = -\omega\, e^{i(k_z z - \omega t + m_\gamma\phi_\rho)} \left[\cos^2\frac{\theta_k}{2} J_{m_\gamma-\Lambda}(k_\perp\rho) + \sin^2\frac{\theta_k}{2} J_{m_\gamma+\Lambda}(k_\perp\rho)\right],$$

$$E_\phi = -i\Lambda\omega\, e^{i(k_z z - \omega t + m_\gamma\phi_\rho)} \left[\cos^2\frac{\theta_k}{2} J_{m_\gamma-\Lambda}(k_\perp\rho) - \sin^2\frac{\theta_k}{2} J_{m_\gamma+\Lambda}(k_\perp\rho)\right],$$

$$E_z = i\Lambda\omega\, e^{i(k_z z - \omega t + m_\gamma\phi_\rho)} \sin\theta_k\, J_{m_\gamma}(k_\perp\rho)\,.$$

and

$$\vec{B} = -i\Lambda\vec{E}$$

Poynting vector (either Λ)

$$S_\rho = 0$$

$$S_\phi = \omega^2 \sin\theta_k \, J_{m_\gamma}(k_\perp \rho)\left(\cos^2\frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) + \sin^2\frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho)\right)$$

$$S_z = \omega^2 \left(\cos^4\frac{\theta_k}{2} J^2_{m_\gamma - \Lambda}(k_\perp \rho) - \sin^4\frac{\theta_k}{2} J^2_{m_\gamma + \Lambda}(k_\perp \rho)\right)$$

For electron accelerators

- Make high energy photons by backscattering optical photons off energetic electrons

- Jentschura-Serbo (2011) showed this backscattering maintains the twistedness.

- Achievable energy, if electron energy is $E_e = \gamma m_e$, and initial energy is ω_1,

$$\omega_2 \approx \frac{4\gamma^2\omega_1}{1 + 4\gamma\omega_1/m_e}$$

- For 0.5μm light (2.48 eV) in,

  get 1.11 GeV photons out for 6 GeV electrons, or
  get 3.75 GeV photons out for 12 GeV electrons.

electron accelerators

- Lots of energy to excite baryon resonances, with hoped for angular momentum selectivity
- There is study group at JLab (Joe Grimes et al.)
- Claim that twisted photon beam of 10^{34} $cm^{-2}sec^{-1}$ luminosity is possible
- Beam steering may be crucial

Angular momentum

- 1992, Allen *et al*. show solutions for photons with arbitrary angular momentum in direction of motion
- 4649 citations on Google Scholar as of 13 May 2018 (or 2500 on ADS and even 128 on SPIRES)
- Did for Laguerre-Gauss, works also for Bessel beams