Atomic photoexcitation by twisted light

Carl E. Carlson
William and Mary & JGU, Mainz (Visitor)

PSAS_2018
International Conference on Precision Physics of Simple Atomic Systems
Vienna, Austria, 14-18 May 2018

Collaborators: Andrei Afanasev, Asmita Mukherjee, Maria Solyanik, Christian Schmiegelow, Jonas Schulz, Ferdinand Schmidt-Kaler,
for experts

- There are three ways (at least) to discuss twisted photons:
  - Bessel beams
  - Bessel-Gauss beams
  - Laguerre-Gauss beams

- The last is shortchanged in this talk, because of time.
for everyone: Topics

- History (brief)
- Twisted photon basics
- Theory results in atomic photoexcitation (Experimental results, 2nd floor, Physics building, JGU, Mainz)
- End
Some history

• Waves diffract (mostly)

• Plane waves don’t: they are too bland

• 1987: Durnin points out existence of structured waves that also don’t diffract.

• Structured means there are hot spots and cold spots in the wave front, and nondiffracting means the hot spots don’t spread out.
Waves satisfy the Helmholtz equation

\[
\left( \nabla^2 + \frac{\omega^2}{c^2} \right) \psi(t, \vec{x}) = (\nabla^2 + k^2) \psi(t, x, y, z) = 0
\]

Monochromatic: \( e^{-i\omega t} \)

Traveling in \( z \)-direction, non-diffractive, must have \( e^{i\beta z} \)

Solution

\[
\psi = e^{i(\beta z - \omega t)} J_0(\alpha \rho)
\]

where \( \alpha \rightarrow k_\perp \), \( \beta \rightarrow k_z \), \( k_\perp^2 + k_z^2 = k^2 \); \( \rho = \sqrt{x^2 + y^2} \)
Bessel wave

- Wave front coming at you:
- Bullseye pattern, vortex center (vortex line)
- Hot spot in center (for $J_0$)
Wavenumber space

- In QM, momentum space

- Recall \( \psi = e^{i(\beta z - \omega t)} J_0(\alpha \rho) \)

- Fourier transform, \( \tilde{\psi}(t, k_z, k_\perp, \phi_k) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t} \delta(k_z - \beta) \delta(k_\perp - \alpha) \)

- Every component in wave number space has same \( k_z \), same \( k_\perp \), but all possible azimuthal angles \( \phi_k \)

- Component momenta form a cone, opening angle or "pitch angle" \( \theta_k \)
\( \gamma \) angular momentum

- Initiating paper: Allen et al., 1992
- Can discuss what they did classically or QM
- Classically, for plane wave, RH polarization,
  \[
  \text{angular momentum} = + \frac{\text{energy}}{\text{angular frequency}}
  \]
- QM, for single photon,
  \[
  \text{angular momentum} = + \hbar
  \]
- Now (QM version),
  \[
  \text{angular momentum} = + (\text{any integer}) \times \hbar
  \]
- How?
Twisted photons

• New solution to Helmholtz equation,

\[
\psi(t, z, \rho, \phi_\rho) = e^{i(\beta z - \omega t)} e^{im_\gamma \phi_\rho} J_{m_\gamma}(\alpha \rho)
\]

• or

\[
\tilde{\psi}(t, k_z, k_\perp, \phi_k) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t} \delta(k_z - \beta) \delta(k_\perp - \alpha) i^{-m_\gamma} e^{im_\gamma \phi_k}
\]

• Same, except that phase changing around edge of cone

• If \( L_z = -i\hbar \frac{\partial}{\partial \phi_\rho} \) have \( L_z = m_\gamma \hbar \)

• Above for scalar photons (beloved of some theorists), results to be shown are for (real world) vector photons.

• Vectors photons have additional QN, helicity of plane wave component states \( \Lambda = \pm 1 \).
Show Poynting vector plots

- Wave front for $J_{m_r}$, $m_r \neq 0$, has hole in center

- Plane wave has Poynting vector only in z-direction

- Twisted photon also has azimuthal component of Poynting vector, $S_\phi$

(figure for $m_r=4$, $\theta_k=0.2$, $\lambda=0.5\mu m$)
One more plot

- Previous figure gave magnitude of $S\phi$; here indicate direction:

- Swirling gives photon orbital angular momentum in z-dir.

- Spin of photon projects to $\Lambda \cos(\theta_k)$ in z-dir.

- Total projected angular momentum of state is $m_\gamma$

also for $m_\gamma=4$, $\theta_k=0.2$, $\lambda=0.5\mu$m
Selection rules

- Photoexcitation, initial state \( \{j_i, m_i\} \) goes to final state \( \{j_f, m_f\} \)

- Plane wave photon, \( m_f - m_i = \Lambda \) (always),
  \[ |j_f - j_i| = 1 \] (usually)
  \( (|j_f - j_i| = 1 \text{ are E1 transitions}). \)

- Twisted photon and direct hit (vortex line passing through atom’s center), gives
  \[ m_f - m_i = m_r \] (may be >> 1)

- Extraordinary! Off axis can get more ordinary.

- Selectively excite higher angular momentum higher atomic states (or higher nuclear/nucleon resonances).
Atomic possibilities

- See that on the nose strikes give quantum number changes not possible with plane wave photons
- Predict photoexcitation rates when atom and vortex line are offset by measured amount
Results

• (Next slides)

• Target is $^{40}$Ca$^+$ ions

• Ground state has S-state valence electron

• All transitions are S -> D, $\approx$ 729 nm wavelength

• Specifically, to D$_{5/2}$, with applied magnetic field Zeeman separating the different final J$_z$

• Figures to be shown here all have initial S$_z$ = -1/2

• There is data on later figures. (Also published data in Nature Comm., Dec. 2016)
Practical item: Bessel-Gauss

• Bessel beam falls only slowly in transverse direction ($\propto 1/\rho^{1/2}$). Require unlimited energy. Can’t be made.

• Real beams can be Bessel-Gauss, e.g., for scalar case

$$\psi(t, z = 0, \rho, \phi_{\rho}) = A e^{-i\omega t} e^{im_\gamma \phi_{\rho}} J_{m_\gamma}(\alpha \rho) e^{-\rho^2/w_0^2}$$

• For $w_0 >>$ wavelength, diffraction spread slow and can be ignored under actual experimental conditions.

• Three parameters ($\omega$ given):

  • $A$ — overall amplitude

  • $\theta_k$ — pitch angle ($\alpha = k_\perp = k \sin(\theta_k)$)

  • $w_0$ — width of beam
Off axis transitions

- Qualitative: Electric field swirls as seen already for Poynting vector.

- Atom smaller than structure scale of wave front

- Atom on vortex line sees circular swirling. Electron will absorb all photon’s angular momentum, if transition occurs.

- Farther out, the atom being small sees a roughly spatially constant E-field. Transitions will be largely electric transitions like that produced by plane wave.
Theory results

Bessel–Gauss beam calculations for target electron in S–state with $s_z = -1/2$

Pitch angle = 0.095 radians, Gaussian (1/e) width = 6.8 μm

With 0.0% by amplitude of opposite photon helicity added in

$m_{\gamma}=-2, \Lambda=-1, l=2, -2<m_{\gamma}<2$

$m_{\gamma}=-1, \Lambda=-1, l=2, -2<m_{\gamma}<2$

$m_{\gamma}=0, \Lambda=-1, l=2, -2<m_{\gamma}<2$

$m_{\gamma}=0, \Lambda=1, l=2, -2<m_{\gamma}<2$

$m_{\gamma}=1, \Lambda=1, l=2, -2<m_{\gamma}<2$

$m_{\gamma}=2, \Lambda=1, l=2, -2<m_{\gamma}<2$
Part of top row

$m_y=2, \Lambda=-1, l_f=2, -2<m_f<2$

- Code: red is most interesting curve for twisted photon
- $L_{zf} = \Lambda$ is only case possible for untwisted photons
- $L_{zf} = 1$ result rather low.
... one more thing

- There may be leakage of opposite helicity photons into beam. See effect of 3% by amplitude ($9 \times 10^{-4}$ by intensity) of opposite helicity photon.
Theory results again

Bessel–Gauss beam calculations for target electron in $S$–state with $x_c = -1/2$

Pitch angle $= 0.095$ radians, Gaussian (1/e) width $= 6.8 \mu m$

With 3\% by amplitude of opposite photon helicity added in
Similar part of top row

\[ \Omega_R \]

\( m_y = -2, \Lambda = -1, l_f = 2, -2 < m_f < 2 \)

\( L_{zf} = -2 \)
\( L_{zf} = -1 \)
\( L_{zf} = 0 \)
\( L_{zf} = +1 \)

- \( L_{zf} = 1 \) much changed. Change in rest not noticeable.
Results with data

Bessel–Gauss beam calculations for target electron in S–state with $s_z = -1/2$

Pitch angle = 0.095 radians, Gaussian (1/c) width = 6.8 μm

With 3% by amplitude of opposite photon helicity added in
more focused

With 3.9 % by amplitude of opposite photon helicity added in

\[ m_y = -2, \Lambda = -1, l_f = 2, -2 < m_f < 2 \]

- \( L_{zf} = -2 \)
- \( L_{zf} = -1 \)
- \( L_{zf} = 0 \)
- \( L_{zf} = +1 \)

- First graph used to set \( A, \theta_k, \omega_0 \)

- \( L_{zf} = 1 \) used to set admixture of opposite helicity \( \gamma \)

- Other 26 graphs follow
the upper half

With 3% by amplitude of opposite photon helicity added in

\[ m_\gamma = -2, \Lambda = -1, l_r = 2, -2 < m_r < 2 \]

\[ m_\gamma = -1, \Lambda = -1, l_r = 2, -2 < m_r < 2 \]

\[ m_\gamma = 0, \Lambda = -1, l_r = 2, -2 < m_r < 2 \]
Penultimate comments

• Twisted photons exist and give extraordinary transitions
  • Significant rates for transitions with large angular momentum transfer, in situations where plane wave gives zero. First shown on 2nd floor, Staudingerweg 7

• Detailed theory works
  • Few parameters fit to one data set
  • Obtain accurate predictions for remaining data
  • allows determining beam characteristics
Comments for nuclear apps.

- Note on scales in atomic case
  - atom ca. $10^{-1}$ nm
  - location of single atom to ca. 10 nm
  - wavelength ca. $10^3$ nm
  - hole in wavefront several wavelengths wide

- Hope for nuclear/nucleon analogs?
  - nucleon ca. 1 fm (or 0.84087(39) fm)
    nucleus ca. 10 fm
  - location placement?
  - 1 GeV photon -> wavelength ca. 1 fm
  - width of hole is large
Thanks for listening

• Omitted topics

  • atomic recoil

  • radiation pressure and Poynting vector

  • circular dichroism on spherical targets

  • twisted light on N -> Delta(1232) transition

  • not to mention Laguerre-Gauss beams, and long distance propagation of Bessel-Gauss.
Beyond the end
Measurements of the ac-Stark shift are carried out at positions marked with the ion. Note that in insets show beam intensity images taken with a CCD camera placed before the focusing lenses, revealing the same non-ideal LG outer ring structure as found as Supplementary Data.

Beam profiles

Excitation profiles in units of power-normalized Rabi frequency as a function of the ion position across three different beams:

\[ \Omega \propto \sqrt{\frac{\Delta m}{C_6}} \]

\[ \Delta m = \frac{1}{2} \]

\[ \text{Ion axial position (µm)} \]

\[ |\text{amplitude}| \]

Nat. Comm. 2016, Fig 3
Twisted vector photons

• Can obtain or visualize $L_z$ in other ways

• First, noting we so far have scalar photons (beloved of some theorists), will switch to vector photons.

• Easiest is to note matrix elements of fields with standardly normalized states, scalar and vector:

$$\langle 0 | \psi(0) | \vec{k} \rangle = 1$$

$$\langle 0 | A_\mu(0) | \vec{k}, \Lambda \rangle = \epsilon_\mu(k, \Lambda)$$

• These are for plane waves

• $\Lambda$ is helicity of the plane wave photon state ($= \pm 1$)
Vector photons

- Momentum space is expansion in plane waves. Becomes,

\[
A^{(m)}_{\mu}(t, k_z, k_\perp, \phi_k, \Lambda) = \frac{(2\pi)^2}{k_\perp} e^{-i\omega t} \delta(k_z - \beta) \delta(k_\perp - \alpha) i^{-m} e^{im \phi_k} \epsilon_{\mu}(\vec{k}, \Lambda)
\]

- Coordinate space expression medium long, will show on demand.

- Can work out electric and magnetic fields, and Poynting vector
Twisted $\gamma$ in coordinate space

\[ \mathcal{A}^{\mu}_{k_\perp k_z m_\gamma \Lambda}(x) = -i\Lambda e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \left\{ e^{-i\phi_\rho} \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) \eta^\mu_{\Lambda} \right. \\
\left. + \frac{i}{\sqrt{2}} \sin \theta_k J_{m_\gamma}(k_\perp \rho) \eta_0^\mu - e^{i\phi_\rho} \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \eta_{-\Lambda}^\mu \right\} \]

where

\[ \eta_{\pm 1} = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0) = \frac{1}{\sqrt{2}} e^{\pm i\phi_\rho} \left( \mp \hat{\rho} - i \hat{\phi} \right), \quad \eta_0 = (0,0,0,1) = \hat{z} \]

fields:

\[ E_\rho = -\omega e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \left[ \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) + \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \right] , \]

\[ E_\phi = -i\Lambda \omega e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \left[ \cos^2 \frac{\theta_k}{2} J_{m_\gamma - \Lambda}(k_\perp \rho) - \sin^2 \frac{\theta_k}{2} J_{m_\gamma + \Lambda}(k_\perp \rho) \right] , \]

\[ E_z = i\Lambda \omega e^{i(k_z z - \omega t + m_\gamma \phi_\rho)} \sin \theta_k J_{m_\gamma}(k_\perp \rho) . \]

and

\[ \vec{B} = -i\Lambda \vec{E} \]
Poynting vector (either $\Lambda$)

\[ S_\rho = 0 \]

\[ S_\phi = \omega^2 \sin \theta_k J_{m_{\gamma}}(k_{\perp \rho}) \left( \cos^2 \frac{\theta_k}{2} J_{m_{\gamma}-\Lambda}(k_{\perp \rho}) + \sin^2 \frac{\theta_k}{2} J_{m_{\gamma}+\Lambda}(k_{\perp \rho}) \right) \]

\[ S_z = \omega^2 \left( \cos^4 \frac{\theta_k}{2} J^2_{m_{\gamma}-\Lambda}(k_{\perp \rho}) - \sin^4 \frac{\theta_k}{2} J^2_{m_{\gamma}+\Lambda}(k_{\perp \rho}) \right) \]
For electron accelerators

• Make high energy photons by backscattering optical photons off energetic electrons

• Jentschura-Serbo (2011) showed this backscattering maintains the twistedness.

• Achievable energy, if electron energy is $E_e = \gamma m_e$, and initial energy is $\omega_1$,

$$\omega_2 \approx \frac{4\gamma^2 \omega_1}{1 + 4\gamma \omega_1/m_e}$$

• For 0.5 $\mu$m light (2.48 eV) in,
  get 1.11 GeV photons out for 6 GeV electrons, or
  get 3.75 GeV photons out for 12 GeV electrons.
electron accelerators

- Lots of energy to excite baryon resonances, with hoped for angular momentum selectivity

- There is study group at JLab (Joe Grimes et al.)

- Claim that twisted photon beam of $10^{34}$ cm$^{-2}$sec$^{-1}$ luminosity is possible

- Beam steering may be crucial
Angular momentum

- 1992, Allen et al. show solutions for photons with arbitrary angular momentum in direction of motion

- 4649 citations on Google Scholar as of 13 May 2018 (or 2500 on ADS and even 128 on SPIRES)

- Did for Laguerre-Gauss, works also for Bessel beams