Radiative finite-size corections to the Lamb shift in muonic atoms

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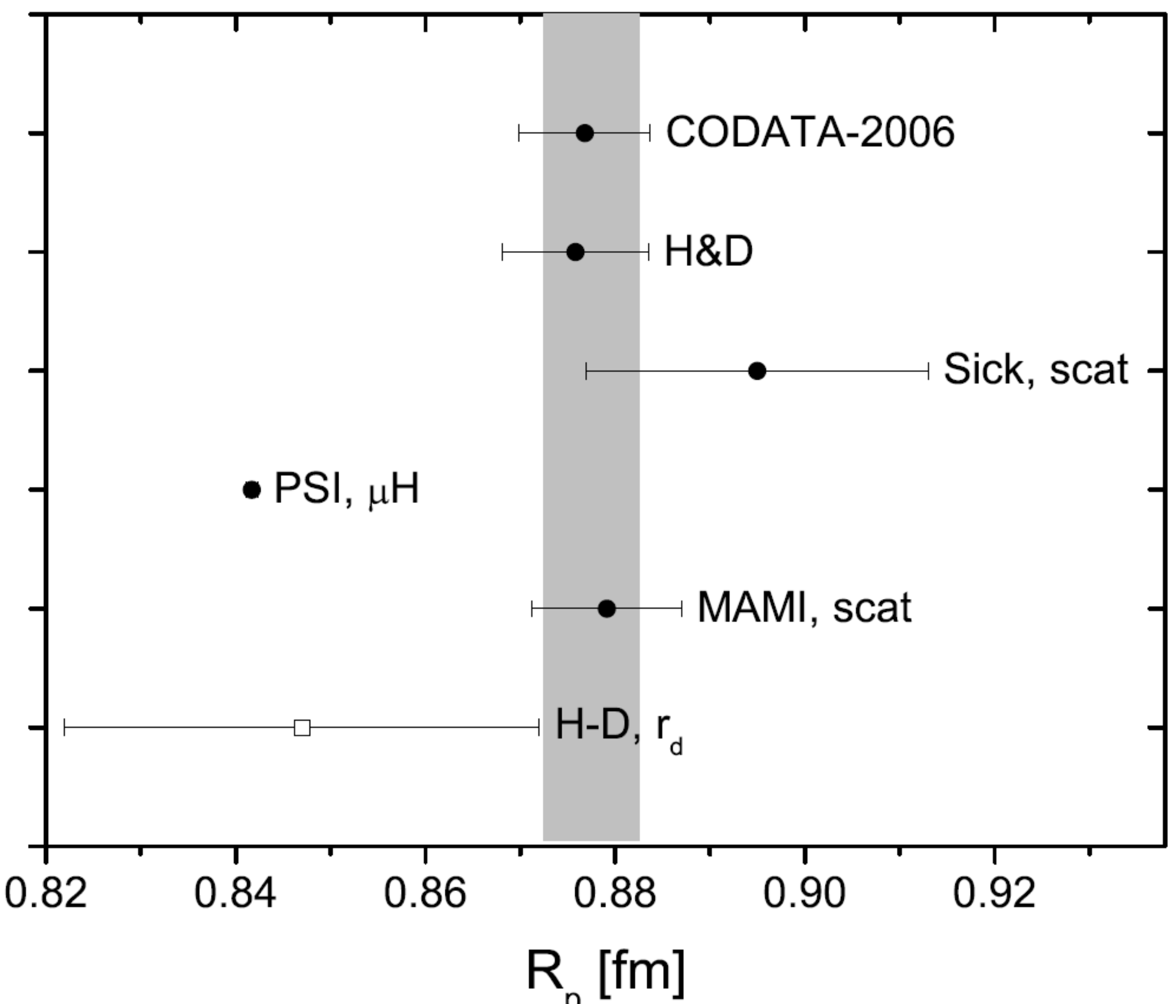
Different methods to determine the proton charge radius (2010)



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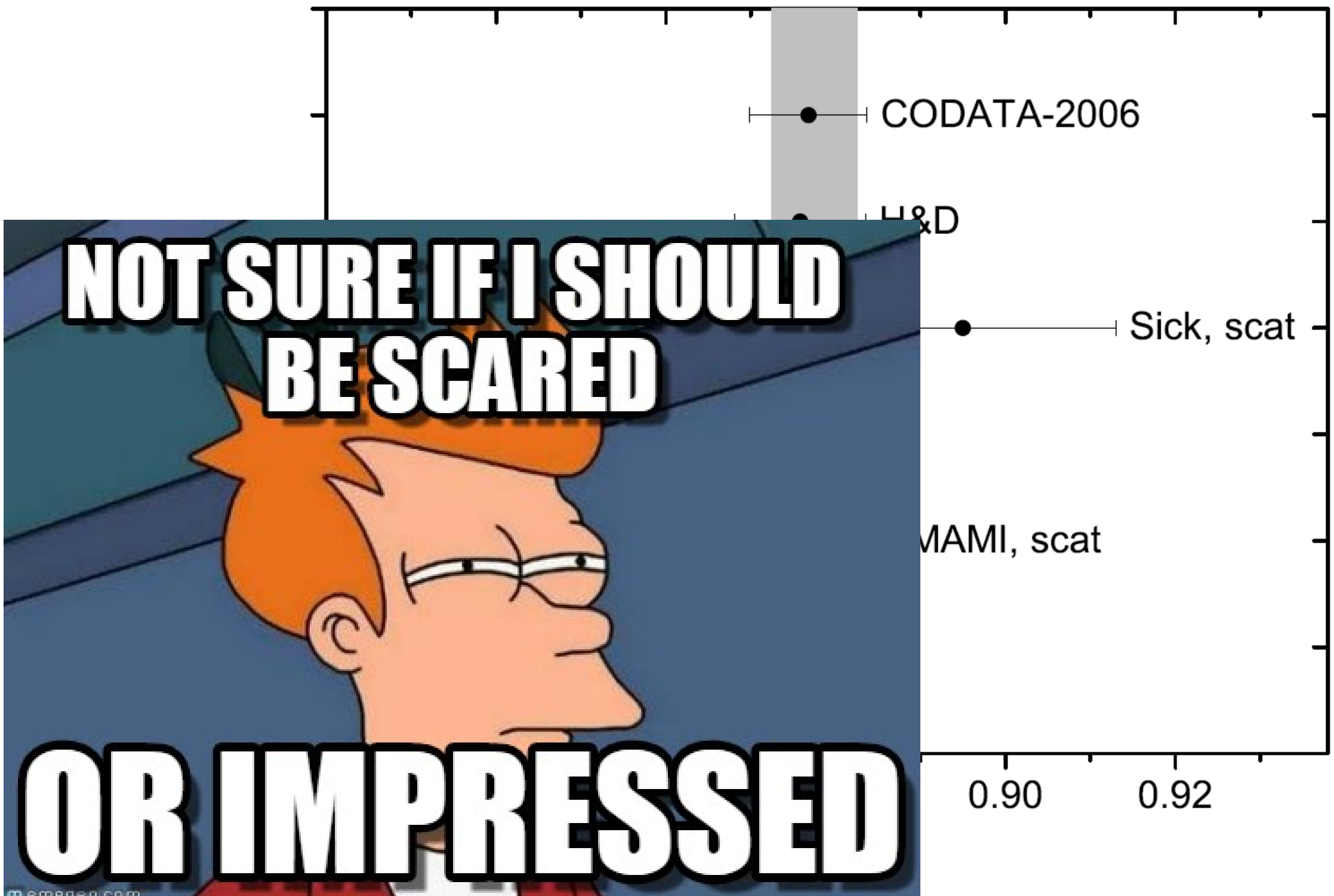
Different methods to determine the proton charge radius



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The Lamb shift in muonic hydrogen: experiment

The size of the proton

Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

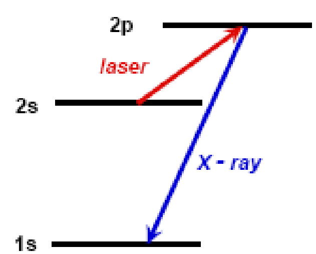

Fig. 16. Level scheme of the PSI experiment on the Lamb shift in a muonic hydrogen [88] (not to scale). The hyperfine structure is not shown.

The Lamb shift in muonic hydrogen: experiment

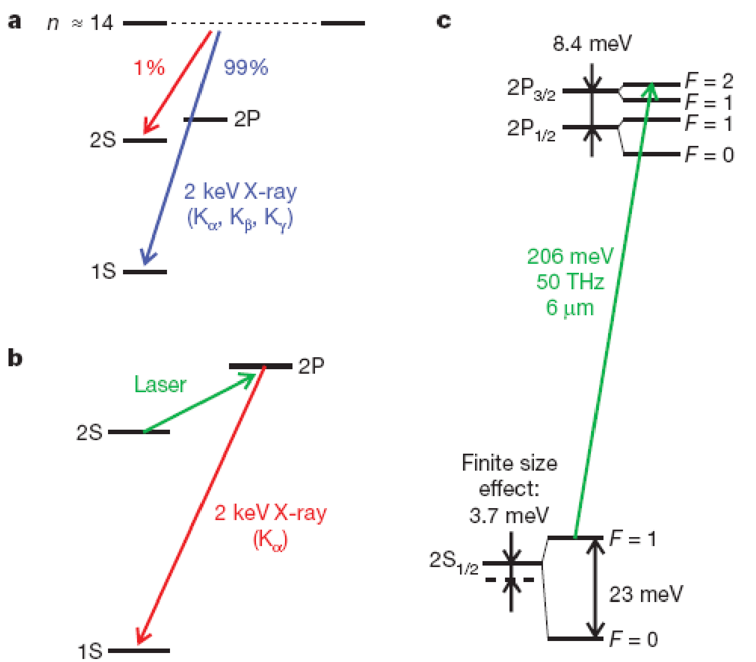

Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. **a**, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6$ μm.

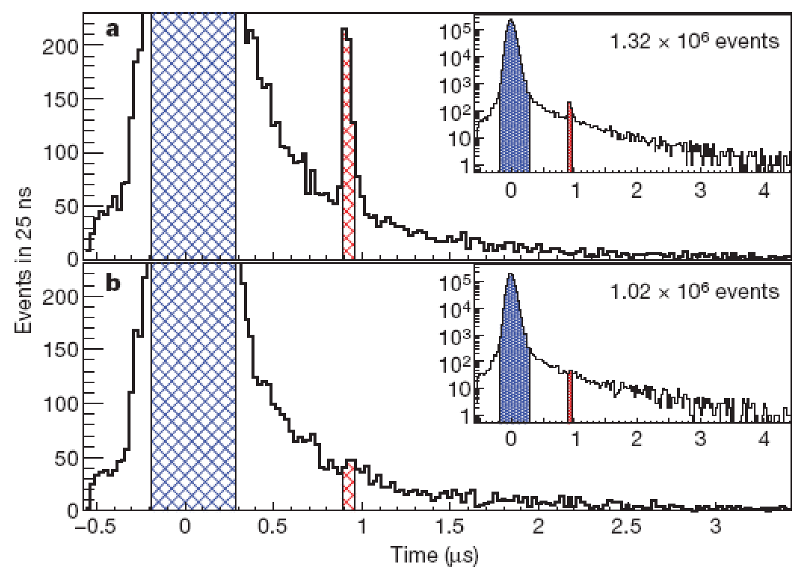

Figure 4 | Summed X-ray time spectra. Spectra were recorded on resonance (**a**) and off resonance (**b**). The laser light illuminates the muonic atoms in the laser time window $t \in [0.887, 0.962]$ μs indicated in red. The 'prompt' X-rays are marked in blue (see text and Fig. 1). Inset, plots showing complete data; total number of events are shown.

The Lamb shift in muonic hydrogen: experiment

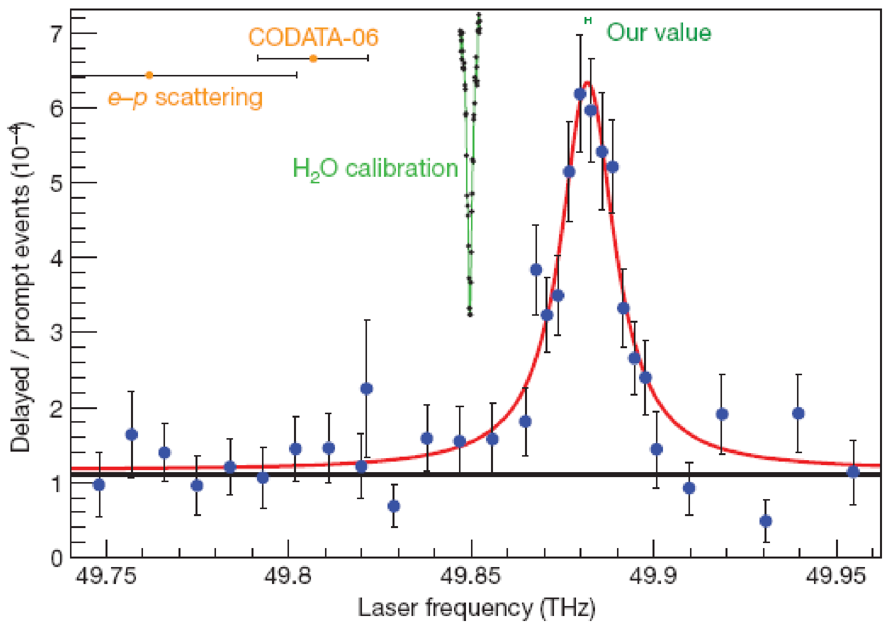

Figure 5 | Resonance. Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a χ^2/d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA[3] or electron scattering[1,2] are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the $\pm$1 s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

Theory of H and μH:

- Rigorous
- Ab initio
- Complicated
- Very accurate
- Partly not cross checked
- Needs no higher-order proton structure

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The *th* uncertainty is much below the level of the discrepancy

Spectroscopy of H and μH:

- Many transitions in different labs.
- One lab dominates.
- Correlated.
- Metrology involved.
- The discrepancy is much below the line width.
- Sensitive to various systematic effects.

- One experiment
- A correlated measurement on μD
- No real metrology
- Discrepancy is of few line widths.
- Not sensitive to many perturbations.

H vs μH:

- μH: **much** more sensitive to the R_p term:
  - less accuracy in theory and experiment is required;
  - easier for estimation of systematic effects etc.
- H experiment: easy to see a signal, hard to interpret.
- μH experiment: hard to see a signal, easy to interpret.

Proton radius puzzle

Big puzzle:

- What value is better?
- Plausible answer:

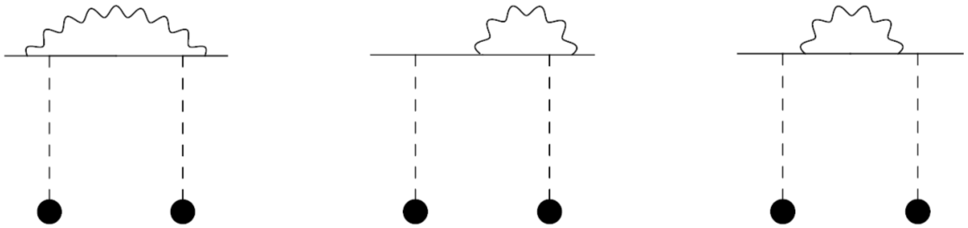
from μ H

Small puzzle:

- How to get an acurate value of R_p?
- The higher-order finite-size terms are needed.
- They involve scattering data or models.

Example: radiative finite-size corrections

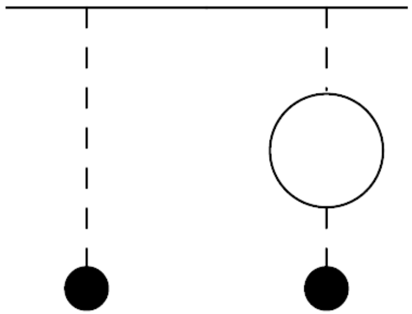
$$\Delta E_{\rm fns:lead}(nl) = \frac{2}{3}\frac{(Z\alpha)^4 m_r}{n^3}(m_r R_N)^2\,\delta_{l0}$$



$$\Delta E_{\rm fns:SE}(ns) \;=\; \left(4\ln 2 - \frac{23}{4}\right)\alpha(Z\alpha)\Delta E_{\rm fns:lead}\,.$$

A.I. Milstein, O.P. Sushkov, and I.S. Terekhov, Phys. Rev. Lett. **89**, 283003 (2002); Phys. Rev. A **67**, 062103 (2003).

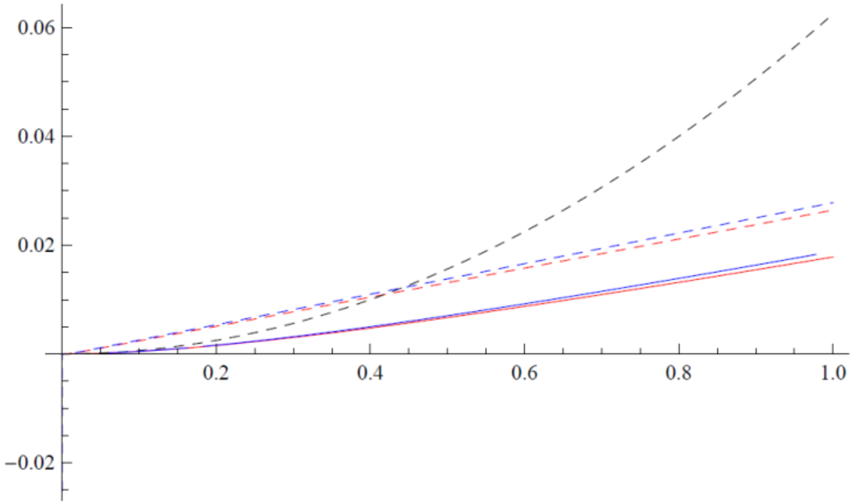
Example: radiative finite-size corrections

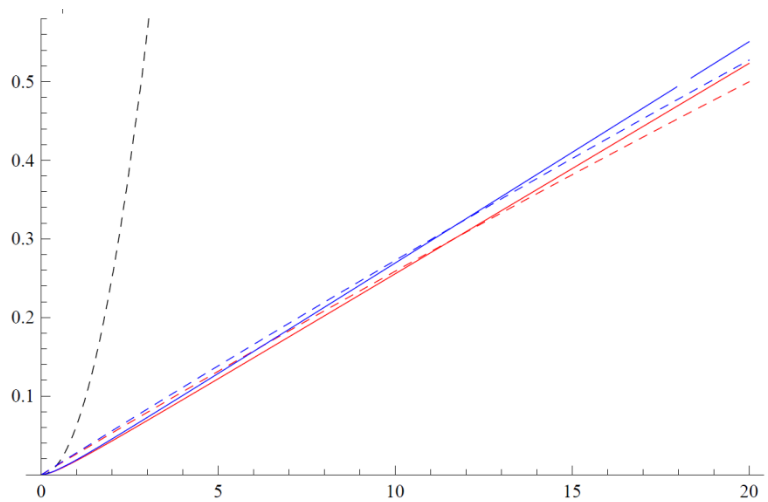


$$\Delta E_{\text{fns:VP}}(ns) \;=\; \frac{3}{4}\,\alpha(Z\alpha)\Delta E_{\text{fns:lead}}$$

K. Pachucki, Phys. Rev. A **48**, 120 (1993).
M.I. Eides and H. Grotch, Phys. Rev. A **56**, R2507 (1997).

Medium Z muonic atoms: μVP

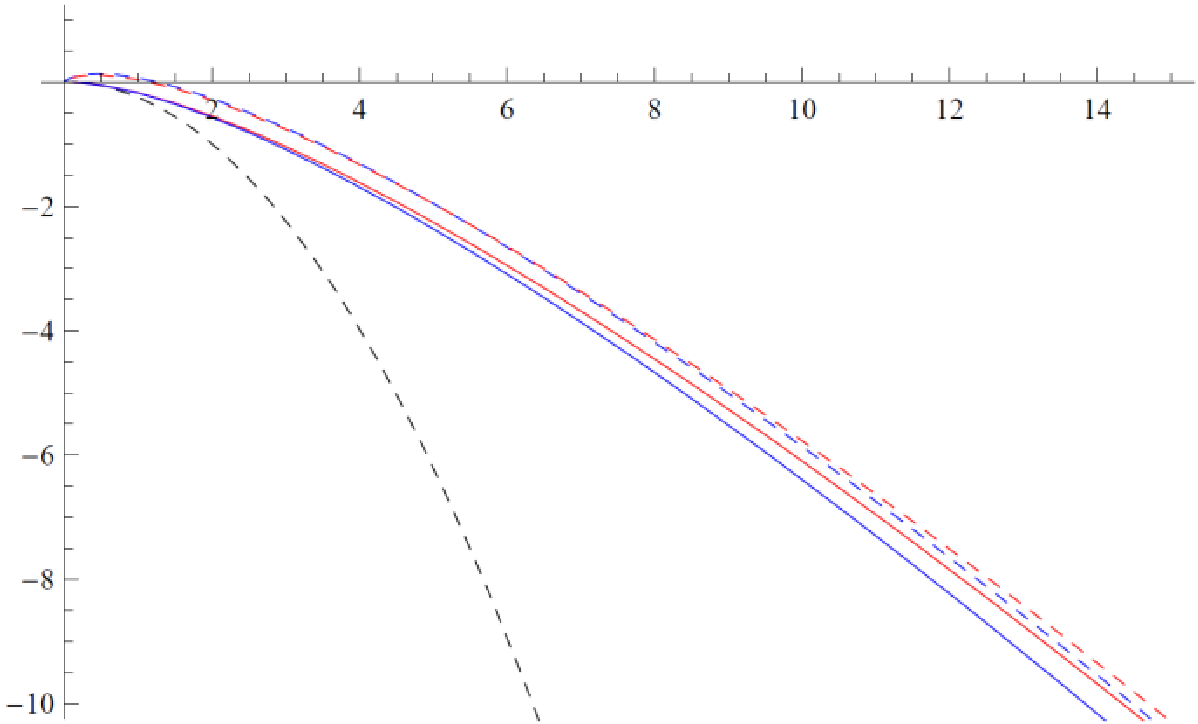




Limit of electronic atoms: $mR_N << 1$

Muonic H: $mR_N = 0.4$

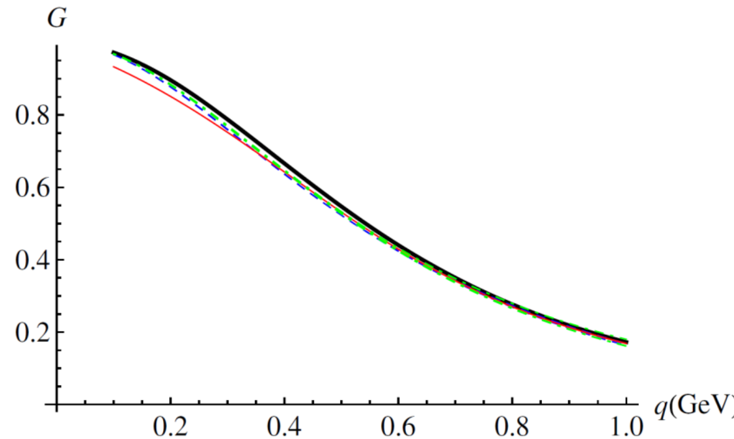
Medium Z muonic atoms: SE

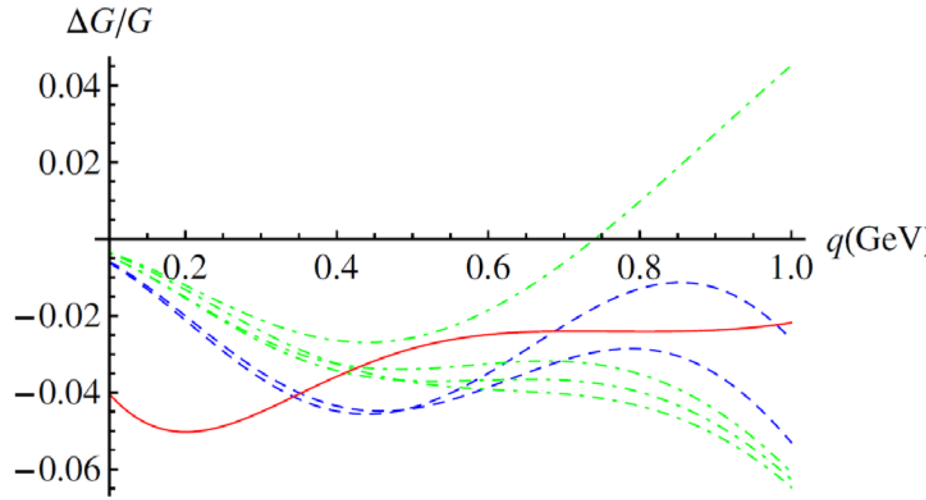


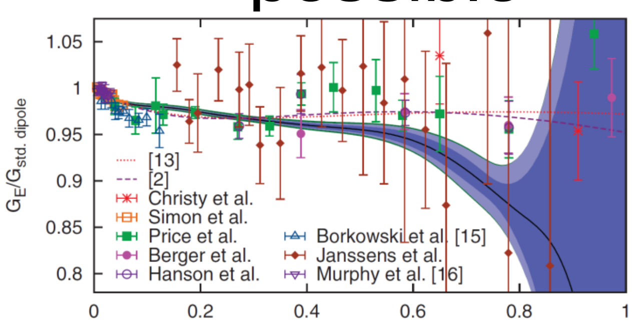
Muonic hydrogen: method

We use fits:

- momentum space
- over all area of q
- realistic low q
- realistic high q
- as different as possible







Muonic hydrogen: results

fit	R_E [fm]	ΔE_{vp} $[\alpha(Z\alpha)^5 m_r^3/m^2]$	ΔE_{se} $[\alpha(Z\alpha)^5 m_r^3/m^2]$
(44)	0.81	0.00542	−0.0382
(50)	0.90	0.00622	−0.0460
(49)	0.90	0.00618	−0.0455
(45)	0.86	0.00588	−0.0426
(46)	0.88	0.00601	−0.0439
(47)	0.87	0.00592	−0.0429
(48)	0.88	0.00600	−0.0438

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Editors' Suggestion

Lamb shift and fine structure at $n = 2$ in a hydrogenlike muonic atom with the nuclear spin $I = 0$

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Radiative nonrecoil nuclear finite size corrections of order $\alpha(Z\alpha)^5$ to the Lamb shift in light muonic atoms

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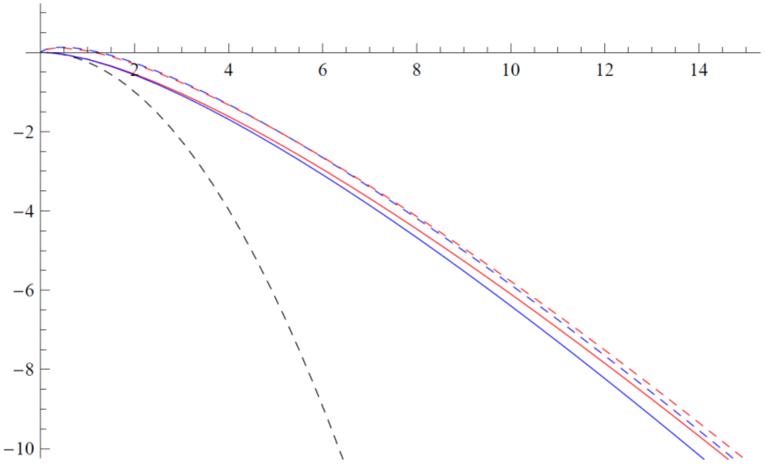
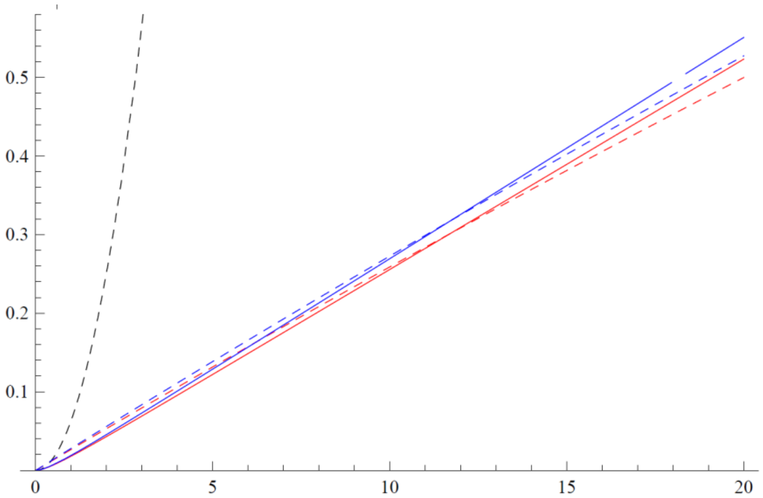
Asymptotics at $mR_N >> 1$

Self energy



Muonic vacuum polarization



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