Stable tetraquarks; bound-electron g
Outline:
- two quarks and two anti-quarks: like Ps-molecule
- effective anti-quark --> resulting new type of hadron
- PLB 778 (2018) 233 with Bo Leng and M. B. Voloshin

Stability of tetrons
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- g-factor of a bound electron: new level of precision
Color interaction between two heavy quarks

Hadrons have no net color:

Meson
- Antigreen
- Green

Baryon
- Red
- Blue
- Green

Antibaryon
- Antigreen
- Antiblue
- Antired
Field point of view: electrostatics

Electric fields:

\[ E = \frac{q r}{4\pi \varepsilon_0 r^3} \quad q \text{ at the origin} \]

\[ E' = \frac{q'(r - r')}{4\pi \varepsilon_0 |r - r'|^3} \quad q' \text{ at } r' \]

Energy density:

\[ \frac{\varepsilon_0}{2} (E + E')^2 \]

Interaction energy:

\[ U_{\text{int}} = \frac{\varepsilon_0}{2} \int 2E' \cdot Ed^3r \]

\[ = \frac{\varepsilon_0 q'q}{(4\pi \varepsilon_0)^2} \int \frac{(r - r') \cdot r}{|r - r'|^3} r^3 d^3r \]

\[ = \frac{q'q}{16\pi^2 \varepsilon_0} \int \frac{r - r' \cos \theta}{|r - r'|^3} r^3 d^3r = \frac{q'q}{4\pi \varepsilon_0 r'} \]
Similar cancellation happens in QCD

One-gluon-exchange approximation: the same spatial dependence.

However, the color charge opens more options.

Example: quark-antiquark states
Quark-antiquark: color singlet or octet

One of these three states, singlet, is the least colorful: most attractive

(the most complete cancellation of fields)

\[ 3 \otimes \bar{3} = 1 \oplus 8 \]
Quark-quark: anti-triplet or sextet

Emerging anti-triplet: the least colorful combination; attractive.

However, the cancellation is not as complete as in quark-antiquark pairs, so the attraction is not as strong. An interesting competition between heavy and lighter degrees of freedom!

\[3 \otimes 3 = \overline{3} \oplus 6\]
Attractive potential between heavy quarks

Singlet: quark-antiquark

\[- \frac{N_c^2 - 1}{2N_c} \frac{\alpha_s}{r} \quad N_c = 3 \rightarrow \quad - \frac{4 \alpha_s}{3 r} \sim N_c \quad \text{for} \quad N_c \rightarrow \infty\]

Anti-triplet: quark-quark

\[- \frac{N_c + 1}{2N_c} \frac{\alpha_s}{r} \quad N_c = 3 \rightarrow \quad - \frac{2 \alpha_s}{3 r}\]

If the QQ are sufficiently heavy, they form a compact bound state that, from far away, looks like an antiquark: can bind two antiquarks like in an antibaryon (resembling helium).

Bohr radius of the QQ state: \[r_{QQ} \sim \frac{1}{\alpha_s M}\]
Interaction of the QQ with antiquarks

First consider a solvable model with heavy antiquarks: single-gluon exchanges $\rightarrow$ Coulomb-like potential.

$$V_{q\bar{Q}} = -\frac{N_c \alpha_s}{2} \left( \frac{1}{r_q} + \frac{1}{r_{q'}} \right) \text{ again } \sim N_c$$

(total color cancellation)

Characteristic distance of the antiquarks from the QQ-nucleus:

$$R_q \sim \frac{1}{N_c \alpha_s m}$$

No auto-dissociation provided that

$$R_q \gg r_{Q\bar{Q}}$$

Auto-dissociation when

$$\frac{N_c^3 m^2}{M^2} \sim 1$$
Results of variational calculation

Critical mass ratio $f$ as a function of the number of colors: confirms breakup when

$$\frac{N_c^3 m^2}{M^2} \sim 1$$

Based on Mariusz Puchalski's code.

Conclusion: tetrons only possible when $Q=b$ and $q=u,d$, maybe $s$. 

Different approach: Karliner+Rosner PRL 119, 202001 (2017) 1707.07666
Eichten+Quigg PRL 119, 202002 (2017) 1707.09575
Bound-electron $g$-factor
Bound-electron $g$-2: binding and loops

$$g = 2 - \frac{2 (Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$

$$+ \frac{\alpha}{\pi} \left[ 1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 (a_{41} \ln Z\alpha + a_{40}) + \ldots \right]$$

$$+ \left( \frac{\alpha}{\pi} \right)^2 \left[ -0.65.. \left( 1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \ldots \right]$$

Breit 1928

Pachucki, Jentschura, Yerokhin (2004)

two-loop corrections

Pachucki, AC
Jentschura, Yerokhin (2005)
Next stage:

\[ \frac{\alpha}{\pi} (Z\alpha)^5 \quad \text{and} \quad \left( \frac{\alpha}{\pi} \right)^2 (Z\alpha)^5 \]
One- and two-loop binding corrections at \((Z\alpha)^5\)

One-loop: Pachucki+Puchalski, PRA96 (2017) 032503

\[ \Delta g \sim \alpha (Z\alpha)^5 \]

Conceptual breakthrough

\[ \Delta g \sim \alpha^2 (Z\alpha)^5 \]

Sources of \(\alpha^2 (Z\alpha)^5\) effects:
- additional short-distance potential generated by Lamb-shift diagrams
- energy-dependence of the Lamb shift (and B-field change of energy)
- modification of the electron response to the B-field
- modification of the B-field by the Coulomb field

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**Two-Loop Binding Corrections to the Electron Gyromagnetic Factor**

Andrzej Czarnecki,¹ Matthew Dowling,¹ Jan Piclum,¹,² and Robert Szafron¹,³

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**Physical Review Letters 120, 043203 (2018)**
Crucial tool: Laporta algorithm

Integration by parts

Identities among loop integrals with various powers of propagators

System of difference equations

Solution in terms of a basis of master integrals

Milestone papers:
Tkachev PLB 100 (1981) 65
Chetyrkin and Tkachev NPB 192 (1981) 159
Laporta IJMP 15 (2000) 5087
Smirnov JHEP 0810 (2008) 107
Short-distance potential at $O(\alpha^2 (Z\alpha)^2)$

+ 18 other diagrams with two-loop self-energy

First computed for Lamb shift in hydrogen:
  Pachucki, PRL 72, 3154 (1994);

Improved precision by reduction to master integrals:
  Dowling, Mondejar, Piclum, AC, PRA 81, 022509 (2010).
Modified electron response to the B-field

A group of about a hundred 3-loop diagrams: automatically generated from Lamb. Reduced to the same master integrals.

Gauge dependent: need $g_3$ to cancel the $\xi$-dependence.

Propagators depend on the energy of the electron

The energy is shifted in the magnetic field,

$$\Delta E = -\mu \cdot B = \frac{e}{2m} g s \cdot B$$

$$g_3 = g \frac{d \text{Lamb}}{dE} = (2 + \frac{\alpha}{\pi}) \frac{d \text{Lamb}}{dE}$$
Summary

Tetrons: a new type of hadrons; stable with respect to strong-interactions, very narrow states (decay only weakly).

Studied with variational methods developed in atomic theory.

Bound-electron $g$-factor: effects $\alpha^2 (Z \alpha)^5$ are now under control.

\[
g^{(2)} = \left( \frac{\alpha}{\pi} \right)^2 \left[ b_{00} \left(1 + \frac{\alpha^2}{6} \right) + \alpha^4_Z \left( b_{40} + b_{41} L \right) + \alpha^5_Z b_{50} + \ldots \right]
\]

\[
b_{50} = b_{50}^{VP} + \Delta b_{50}
\]

\[
\Delta b_{50} = 4.7304(9)
\]
Extra slides
Result: short-distance potential

\[ \Delta V = c \alpha^2 (Z\alpha)^2 \delta^3(r) \]

Lamb shift

\[ \Delta E = -\frac{7.72381(4)}{\pi} \alpha^2 (Z\alpha)^5 m \]

Correction to \( g \)

\[ \Delta g = \frac{4\Delta E}{m} \]

Karshenboim, PLA 266 (2000) 380
Coulomb field modifies the magnetic field

“Magnetic loop”: a vacuum-polarization effect,

\[ g_{\text{MLPH}}^{\text{MLPH}} = \left( -\frac{7543}{16200} - \frac{303587}{10125\pi} + \frac{92368}{2025\pi} \ln 2 \right) \alpha^2 \alpha_Z^5 \]

\[ g_{\text{MLVP}}^{\text{MLVP}} = \left( \frac{628}{8505\pi} - \frac{1}{54} \right) \alpha^2 \alpha_Z^5 \]

Note: technically simpler --> analytical result. This part is numerically small.
TABLE I. Bound electron $g$ factor for helium, carbon, and silicon ions. The error related to missing higher order contributions is estimated by $(\alpha/\pi)^2 \alpha_s^2 \ln^3 \alpha_s^{-2}$.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$^4\text{He}^+$</th>
<th>$^{12}\text{C}^5+$</th>
<th>$^{28}\text{Si}^{13+}$</th>
<th>Source</th>
</tr>
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<tbody>
<tr>
<td>Dirac/Breit value</td>
<td>1.999 857 988 825 37(6)</td>
<td>1.998 721 354 392 1(6)</td>
<td>1.993 023 571 557(3)</td>
<td>Ref. [20]</td>
</tr>
<tr>
<td>+ other known corrections</td>
<td>2.002 177 406 711 41(55)</td>
<td>2.001 041 590 168 6(12)</td>
<td>1.995 348 957 825(39)</td>
<td>Refs. [16,22]a</td>
</tr>
<tr>
<td>$g^{SE}$</td>
<td>0.000 000 000 000 02</td>
<td>0.000 000 000 005 0</td>
<td>0.000 000 000 348</td>
<td>This work</td>
</tr>
<tr>
<td>$g^{LBL}$</td>
<td>-0.000 000 000 000 01</td>
<td>-0.000 000 000 001 5</td>
<td>-0.000 000 000 102</td>
<td>This work</td>
</tr>
<tr>
<td>$g^{ML}$</td>
<td>0.000 000 000 000 00</td>
<td>0.000 000 000 006</td>
<td>0.000 000 000 038</td>
<td>This work</td>
</tr>
<tr>
<td>$(\alpha/\pi)^2 \alpha_s^2 \ln^3 \alpha_s^{-2}$</td>
<td>0.000 000 000 000 00(3)</td>
<td>0.000 000 000 000 00(93)</td>
<td>0.000 000 000 000(583)</td>
<td>This work</td>
</tr>
<tr>
<td>Total</td>
<td>2.002 177 406 711 42(55)</td>
<td>2.001 041 590 172 7(94)</td>
<td>1.995 348 958 109(584)</td>
<td>This work</td>
</tr>
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