Testing fundamental interactions on light atoms

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1. Introduction

HYDROGENIC systems such as H, He^+ or μH are being considered for low-energy tests of the Standard model and for determination of fundamental constants. Comparison of accurate experimental values for transition frequencies with theoretical predictions gives us information about the extent to which energy levels can by predicted by the Standard Model. Any discrepancy could be signal of new physics or incorrect values of physical constants. One can reverse the problem and use the comparison of experimental values with theoretical predictions to extract values of fundamental constants. Example of this is comparison of the Lamb shift in muonic hydrogen with electronic one. This provides us with significantly different values of proton charge radius

$$\sqrt{\langle r_p^2\rangle}_{eH} = 0.8770(45)\,\text{fm}$$
$$\sqrt{\langle r_p^2\rangle}_{\mu H} = 0.8409(4)\,\text{fm}$$

Present electronic and muonic hydrogen theory allows accurate determination of the proton charge radius from measured transition frequencies, and the comparison between electronic and muonic results stands as a low-energy test of the Standard Model. Purpose of our calculations is to bring the high accuracy achieved for hydrogenic levels also to few-electron atomic and molecular systems and in particular for helium and helium-like ions.

2. Method

Our calculation is based on the Nonrelativistic QED (NRQED) expansion. The basic assumption of the NRQED is that E can be expanded in a power series of the fine-structure constant α,

$$E\left(\alpha,\frac{m}{M}\right) = \alpha^2\,E^{(2)}\left(\frac{m}{M}\right) + \alpha^4\,E^{(4)}\left(\frac{m}{M}\right) + \alpha^5\,E^{(5)}\left(\frac{m}{M}\right) + \alpha^6\,E^{(6)}\left(\frac{m}{M}\right) + \alpha^7\,E^{(7)}\left(\frac{m}{M}\right) + \dots\,, \qquad (1)$$

where m/M is the electron-to-nucleus mass ratio and the expansion coefficients $E^{(n)}$ may contain finite powers of $\ln\alpha$. The coefficients $E^{(i)}(m/M)$ are further expanded in powers of m/M,

$$E^{(i)}\left(\frac{m}{M}\right) = E^{(i,0)} + \frac{m}{M}\,E^{(i,1)} + \left(\frac{m}{M}\right)^2 E^{(i,2)} + \dots\,. \qquad (2)$$

The expansion coefficients in Eqs. (1) and (2) can be expressed as expectation values of some effective Hamiltonians with the nonrelativistic wave function.

The first term of the NRQED expansion of the bound-state energy, $E^{(2,0)} \equiv E$, is the nonrelativistic eigenvalue of the Schrödinger-Coulomb Hamiltonian in the infinite nuclear mass limit, which for helium-like atoms reads

$$H_0 \equiv H = \frac{p_1^2}{2} + \frac{p_2^2}{2} - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r}\,, \qquad (3)$$

where $r \equiv r_{12}$. The finite nuclear mass corrections to $E^{(2,0)}$ can be obtained perturbatively. The next term of the expansion, $E^{(4)}$, is the leading relativistic correction induced by the Breit Hamiltonian $H^{(4)}$ and the corresponding recoil addition $\delta_M H^{(4)}$. In the next-order contribution, $E^{(5)}$, the QED corrections start to appear. Finally, the next expansion term $E^{(6)}$ is the higher-order QED correction, whose general form is

$$E^{(6)} = \langle H^{(6)}\rangle + \left\langle H^{(4)}\,\frac{1}{(E-H)'}\,H^{(4)}\right\rangle. \qquad (4)$$

The recoil correction $\delta_M H^{(6)}$ was calculated by us in Refs. [1, 2].

3. Transition frequencies and isotope shift in helium

We calculated [1, 2] the binding energies to order $\alpha^6\,m^2/M$, including also the higher-order recoil corrections $E^{(2,3)}$ and $E^{(4,2)}$ [3]. The uncertainty of the total energies is exclusively defined by the $\alpha^7\,m$ contribution, whose complete form is unknown at present. The results for various transitions in ^{4}He are presented in the Table 1.

Table 1: *Comparison of the theoretical predictions for various transitions in ^{4}He with the experimental results, in MHz.*

	Theory	Experiment
1^1S	5 945 204 173 (36)	5 945 204 212 (6)
2^1S	960 332 038.0(1.9)	960 332 041.01(15)
1^1S–2^1S	4 984 872 135 (36)	4 984 872 315 (48)
2^3S–3^3D_1	786 823 848.4 (1.3)	786 823 850.002 (56)
2^1S–2^1P_1	145 622 891.5(2.3)	145 622 892.886 (183)
2^1P_1–3^1D_2	448 791 397.4(0.4)	448 791 399.113 (268)
2^3P_0–3^3D_1	510 059 754.0 (0.7)	510 059 755.352 (28)
2^3P–2^3S	276 736 495.4 (2.0)	276 736 495.649 (2)
2^3S–2^1P_1	338 133 594.9 (1.4)	338 133 594.4 (5)
2^1S–2^3S	192 510 703.4 (0.8)	192 510 702.148 72(20)

The isotope shift is defined, for the spinless nuclei, as the difference of the transition frequencies of different isotopes of the same element. In order to separate out the effects of the nuclear spin in ^{3}He, the isotope shift of the $2S$ and $2P$ levels is defined as the shift of the centroid energies, which are the average over all fine and hyperfine energy sublevels,

$$E(2^{2S+1}L) = \frac{\sum_{J,F}(2F+1)\,E(2^{2S+1}L_{J,F})}{(2I+1)\,(2S+1)\,(2L+1)}\,, \qquad (5)$$

where $^{2S+1}L_{J,F}$ denotes the state with electron angular momentum L, spin S, and total momentum J, whereas F is the total momentum of the atom. Comparing theoretical calculations with experimental data we can obtain ^{3}He–^{4}He nuclear charge radii difference δr^2 from the isotope shift of the 2^3S–2^3P and 2^3S–2^1S transitions. Results are presented in Table 2.

Table 2: *Determinations of the nuclear charge difference of ^{3}He and ^{4}He, $\delta r^2 \equiv r^2(^3\text{He}) - r^2(^4\text{He})$ from different measurements. $\delta E = E_{exp} - E_{theo}$ is the part of the isotope shift induced by the finite nuclear size, represented as $\delta E = C\,\delta r^2$, with C being the coefficient calculated from theory.*

Rengelink *et al.* [5]	
$\delta E\,(2^1S - 2^3S)$	-223.5(1.5) kHz
C	-214.66 (2) kHz/fm^2 [4]
δr^2	1.041 (7) fm^2 [2, 5]
Cancio Pastor *et al.* [6, 7]	
$\delta E\,(2^3P - 2^3S)$	-1 295.4 (3.3) kHz
C	-1212.2 (1) kHz/fm^2 [4]
δr^2	1.069 (3) fm^2 [1]
Shiner *et al.* [8]	
$\delta E\,(2^3P - 2^3S)$	-1286.7 (3.5) kHz
C	-1212.2 (1) kHz/fm^2 [4]
δr^2	1.061 (3) fm^2 [1]

4. Higher-order nuclear structure corrections

The nuclear structure corrections are usually divided into the elastic and the inelastic parts. The elastic part (also referred to as the finite nuclear size correction) is induced by a static distribution of the nuclear charge and can be obtained by solving the Dirac equation. The inelastic nuclear correction is much more complicated; it encompass the nuclear dipole polarizability and higher-order contributions. To deal with the nuclear corrections, one performs an expansion of the binding energy in powers of the fine structure constant α and examines the expansion terms one after another.

The leading nuclear effect is of order α^4 and of a pure elastic origin,

$$E^{(4)}_{\text{fns}} = \frac{2\pi}{3}\,Z\,\alpha\,\langle r^2\rangle\,\phi^2(0)\,. \qquad (6)$$

The first-order $O(\alpha)$ nuclear-structure correction has both elastic and inelastic parts and was extensively studied both for the electronic and the muonic atoms.

The next-order $O(\alpha^2)$ nuclear structure correction comes from the three-photon exchange between the bound lepton and the nucleus. Only the elastic part of this correction has been addressed in the literature so far. Here we show that the inelastic $O(\alpha^2)$ contribution is significant and partially cancels its elastic counterpart [10]. Results of our calculations are presented in Table 3.

Table 3: *Numerical results for the three-photon exchange nuclear structure corrections. Numerical values include the leading recoil effect by the multiplicative reduced-mass prefactor $(\mu/m)^3$. Elastic contributions are obtained with the exponential parametrization of the nuclear charge distribution. The first two results are in Hz, while the remaining are in meV.*

Transition	Elastic	Elastic + inelastic
$E^{(6)}(2S-1S, e\text{H})$	-584	$-928\,(344)$
$E^{(6)}(2S-1S, e\text{D}-e\text{H})$	$-2\,846$	$-2\,029\,(41)$
$E^{(6)}(2P_{1/2}-2S, \mu\text{H})$	$-0.001\,27$	$-0.001\,27\,(27)$
$E^{(6)}(2P_{1/2}-2S, \mu\text{D})$	$-0.006\,56$	$0.002\,19\,(88)(27)$
$E^{(6)}(2P_{1/2}-2S, \mu^3\text{He}^+)$	$-0.384\,7$	
$E^{(6)}(2P_{1/2}-2S, \mu^4\text{He}^+)$	$-0.304\,8$	

Our results for the three-photon exchange nuclear structure corrections affect determinations of the hydrogen-deuterium nuclear charge radii differences derived from the spectroscopic observations of the isotope shifts in electronic and muonic hydrogen and deuterium.

For the electronic H-D isotope shift of the $1S$–$2S$ transition, our result shifts the total theoretical prediction by 0.8 kHz Our review of the present status of theory of the H-D isotope shift leads us to the updated result for the nuclear charge radius difference determined from the measurement of the H-D isotope shift of the $1S$–$2S$ transition [9],

$$\delta r^2[\text{electronic}] \equiv r_d^2 - r_p^2 = 3.820\,70\,(31)\,\text{fm}^2\,, \qquad (7)$$

For muonic hydrogen and deuterium, our result for the inelastic three-photon exchange nuclear structure contribution to the $2P_{1/2}$–$2S$ transition energy of $0.008\,75\,(88)$ meV shifts the deuteron-proton charge radius difference determined in Ref. [11] by 0.0014 fm^2, with the result

$$\delta r^2[\text{muonic}] \equiv r_d^2 - r_p^2 = 3.8126\,(34)\,\text{fm}^2\,. \qquad (8)$$

The results derived from the electronic and muonic atoms disagree by about $2\,\sigma$, which confirms the discrepancy previously observed in Ref. [11]

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