A theory for TMCI in the presence of Harmonic Cavities + RW

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LBNL

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U.S. DEPARTMENT OF ENERGY | Office of Science

ALS-U ADVANCED LIGHT SOURCE

Outline

- The ALS-U Project status, main parameters
  - The importance of RW
- TMCI simulations with Harmonic Cavities (HCs) + RW that motivated this study
- Refresher on TMCI mode-analysis theory (no HCs)
- How to extend mode-analysis theory to TMCI w/ HCs
  - & how to handle numerical difficulties.
- Theory, simulations benchmark
- Conclusions & outlook

ALSu: a DOE Project in the conceptual-design stage

schedule still uncertain

Present ALS:

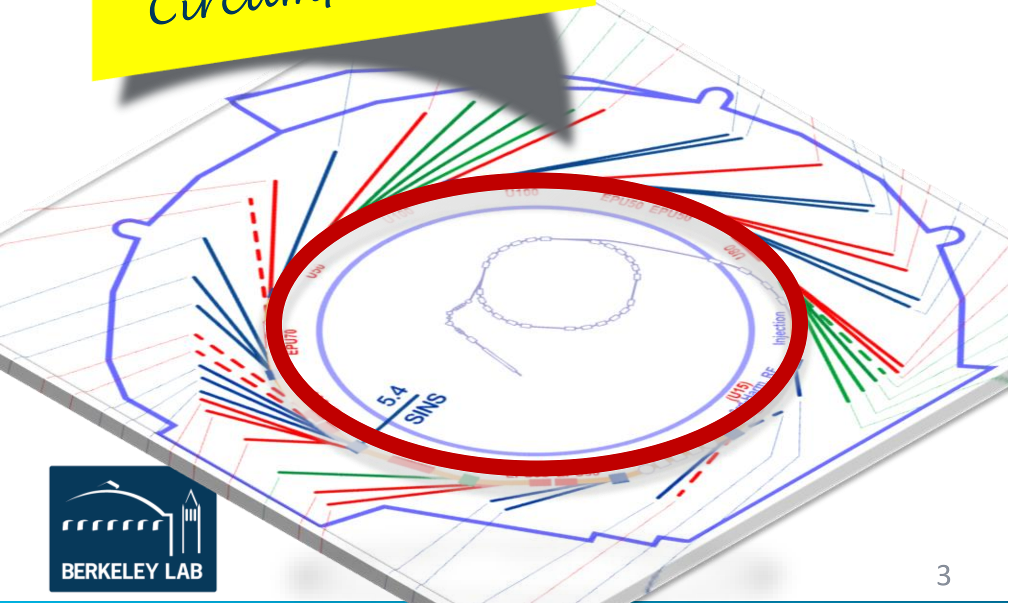
Energy: 1.9 GeV
Current: 500mA
Emittance: 2nm

Beam Size @IDs
$\sigma_x/\sigma_y \sim 250/9\ \mu m$

TBA lattice
Circumf. = 196.8m

- **ALSu target beam/lattice specs**
  - $\varepsilon_x \simeq \varepsilon_y \lesssim 75\ pm$ (full coupling)
  - $\beta_x \sim \beta_y \lesssim 3$ m in straight sections
    - $\sigma_x \sim \sigma_y \sim 10\ \mu m$ @IDs
  - 2 GeV
  - 500 mA

- **Features**
  - 9BA lattice
  - Maintain 12 period layout
  - 4 new IDs/beamlines
  - SuperBends
  - Swap-out on-axis injection (2nm beam, Accumulator)



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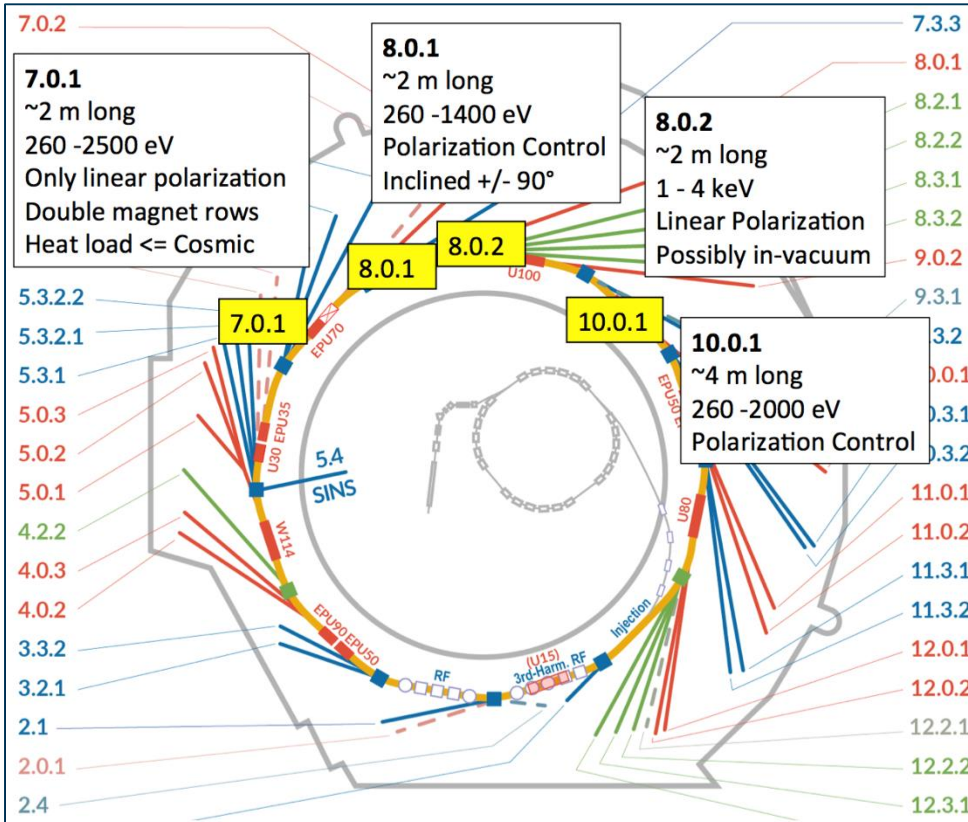
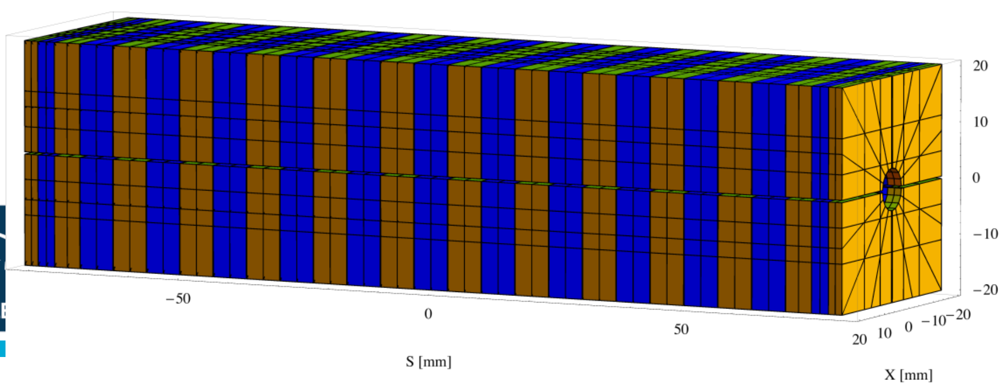
Project includes 4 new beamlines and supporting IDs (tentative)

- **Narrow ID gap/ vacuum chamber aperture**
- **Delta-type IDs (r=2mm chamber)?**
  - Exploiting small round beam
- **Cu + NEG $(1\mu m?)$ coating**
- **Most of existing ALS IDs to be inherited. Vacuum chambers?**





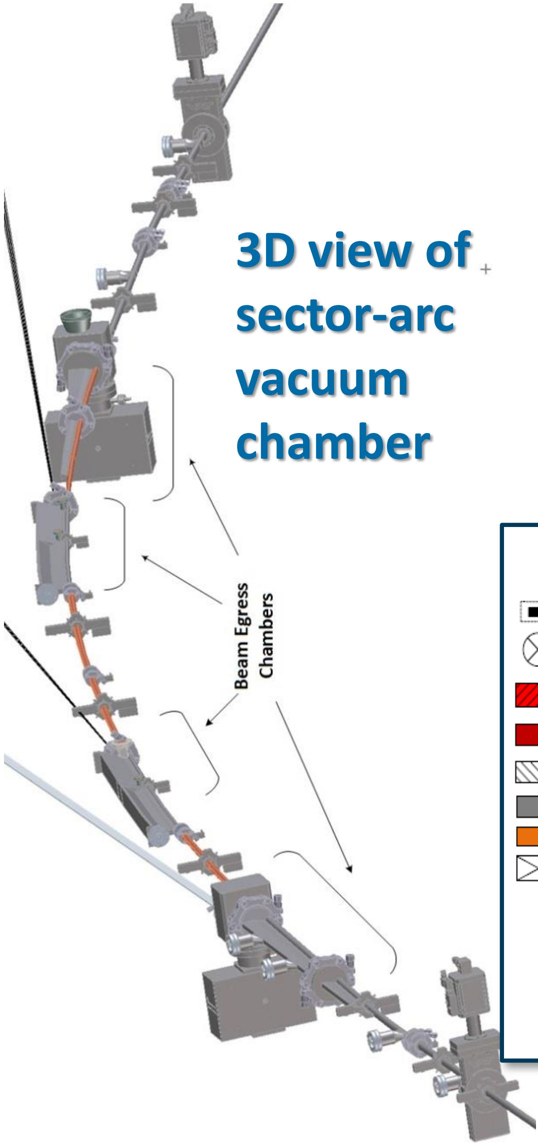
Concept of Delta ID

$\lambda_u = 26.7\text{mm}$

Magnetic bore diameter =7.5mm

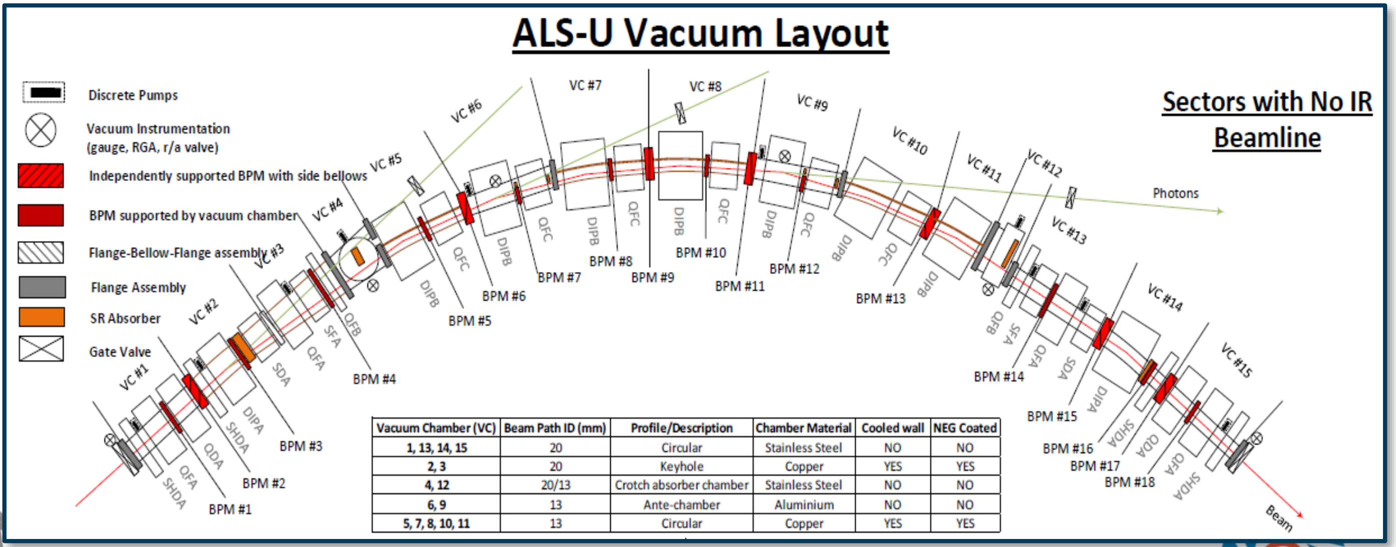
Stay-clear beam diameter=4mm

Most of the sector-arc (round) vacuum chambers to have r=6.5mm radius



3D view of sector-arc vacuum chamber

- **Swap-out of low-emittance beam allows for narrow-aperture chamber**
  - High-gradient magnets using conventional technology
- **Preliminary concept of vacuum chamber in the sector-arcs**
  - Combination of r=6.5mm and r=10mm round chambers
  - Narrower chambers to be NEG coated



Vacuum Chamber (VC)	Beam Path ID (mm)	Profile/Description	Chamber Material	Cooled wall	NEG Coated
1, 13, 14, 15	20	Circular	Stainless Steel	NO	NO
2, 3	20	Keyhole	Copper	YES	YES
4, 12	20/13	Crotch absorber chamber	Stainless Steel	NO	NO
6, 9	13	Ante-chamber	Aluminium	NO	NO
5, 7, 8, 10, 11	13	Circular	Copper	YES	YES

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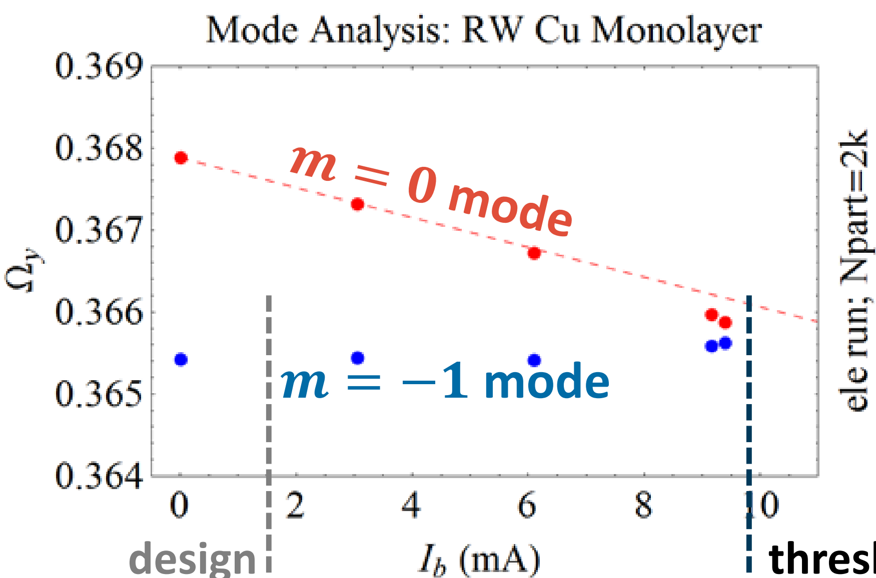
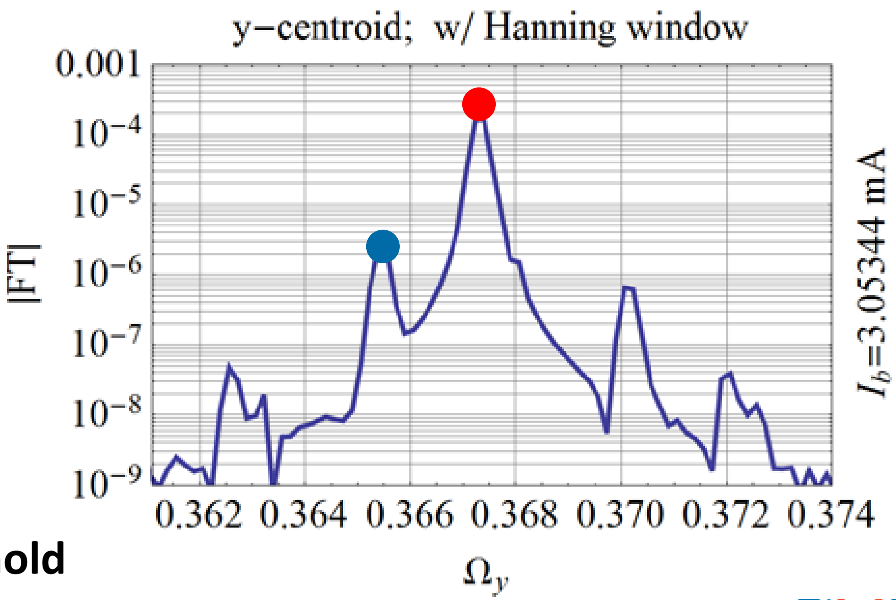
Expect large RW contribution to the impedance budget

- Multi-bunch, single-bunch
- Longitudinal and especially transverse
  - $1/r^3$ scaling of RW transverse impedance
- NEG coating

TMCI in the ALSu*

RW-Z Cu-monolayer; no Harmonic Cavities; zero chroms (elegant simulations)





**Outdated Z-machine model*

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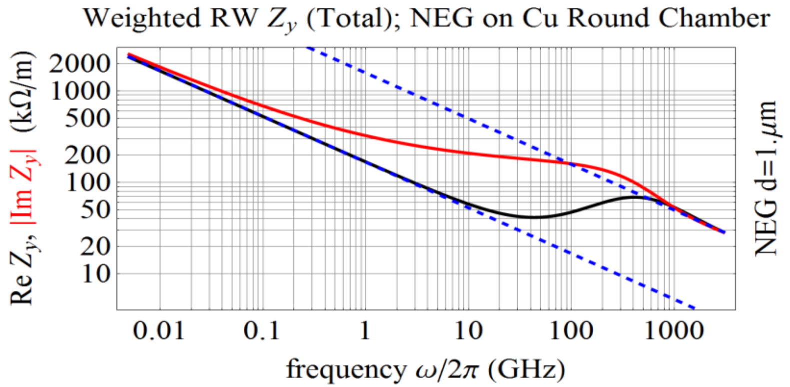
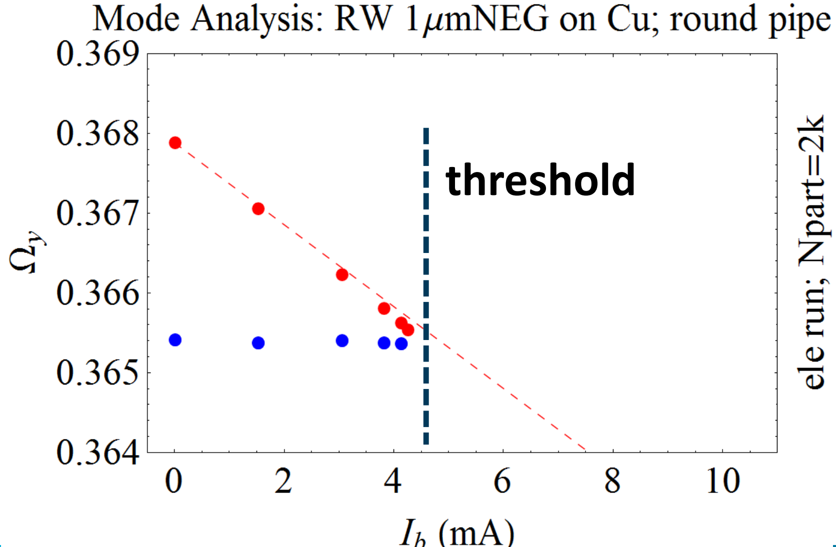
Simulations with Harmonic Cavities showed unexpected result: is Landau not doing his job ???

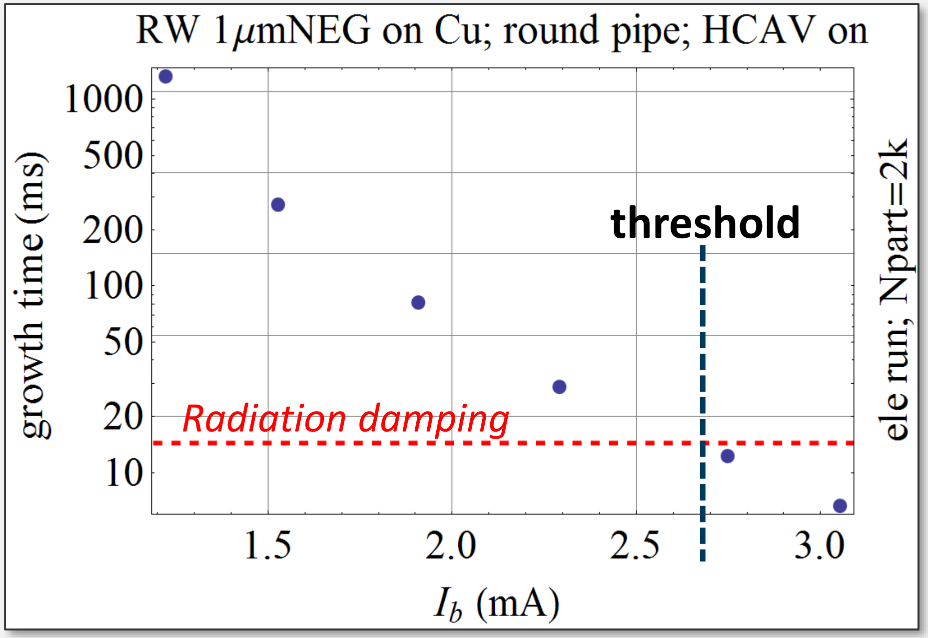
Including NEG lowers threshold by factor ~2 (still no HCs)





- "Ideal" setting for HCs (maximum flattening of RF voltage, profile)
- Motion always unstable or very small instability threshold?

Growth rate with HCs



Surprisingly, not much literature on theory of transverse instabilities with HCs. & Somewhat contradictory claims

- **Cullinan et al.:**
  - PRAB 2016
  - Multi-bunch + chromaticities. Careful macro-particle simulations + mode analysis for linear RF voltage (no HCs)
  - TMCI regime excluded
  - **Conclusion: HCs help**

- **S. Krinksy:**
  - Unpublished 2005 (early NSLS-II studies) + NSLS-II conference papers with collaborators
  - Macroparticle simulations using home-made Matlab code
  - Single-bunch instability. Zero chromaticities. No radiation damping.
  - **Conclusion: in the presence of RW, HCs lower instability threshold**
    - For broad-band resonator Z , HCs may not have much effect, depending on parameters

- **Y. Chin et al.:**
  - Part. Accel. 1985 (!)
  - Mode analysis of Sacherer's integral equation
  - Effect of HCs on single-particle dynamics as a small perturbation + 'hand waving' extrapolation to case of cubic RF voltage
  - **Conclusion: HCs cause instability at any current (broad-band resonator model of impedance)**

TMCI theory (no Harmonic Cavities) on a cheat-sheet.
(localized RW Impedance)

Unpertubed (long.) dynamics: harmonic oscillator

Amplitude(action)-angle variables
$z = r(J_z)\cos\varphi_z$

$r = \left(\frac{2J_z \alpha c}{\omega_{s0}}\right)^{1/2}$

Beam equilibrium as a Gaussian
$\exp\left(-\frac{z^2}{2\sigma_{z0}^2} - \frac{\delta^2}{2{\sigma_{\delta 0}}^2}\right)$

Linearized Vlasov equation

Mode expansion of perturbation to equilibrium

$$g_1(J_z, \varphi_z; \Omega) = \sum_{m=-\infty}^{\infty} R_m(J_z; \Omega) e^{im\varphi_z}$$

Sacherer's equation for radial component R_m of m-mode

$$(\Delta\hat{\Omega} - m)R_m(\rho) + i\hat{I}_0 e^{-\rho^2/2} \sum_{m'=-\infty}^{\infty} \int_0^{\infty} R_{m'}(\rho')\mathcal{G}_{m,m'}(\rho,\rho')\rho' d\rho' = 0,$$

$\rho = r/\sigma_{z0}$

Kernel (DC σ_c model of Impendance)

$$\mathcal{G}_{m,m'}(\rho,\rho') = i^{(m-m')} \int_{-\infty}^{\infty} d\kappa \frac{\mathrm{sign}(\kappa) - i}{\sqrt{|\kappa|}} J_m(\kappa\rho) J_{m'}(\kappa\rho').$$

Note : Bessel functions come from integrals over z-trajectories $J_m \sim \int_0^{2\pi} d\varphi_z e^{-i(m\varphi_z + r\kappa\cos\varphi_z)}$

Dimensionless current- parameter

Kernel expressed in terms of Hypergeometric functions

$$\int_0^{\infty} \frac{d\kappa}{\sqrt{\kappa}} J_\mu(\kappa\rho_>) J_\nu(\kappa\rho_<) = \frac{\Gamma(a)}{\Gamma(1-b)\Gamma(1+\nu)} \frac{1}{\sqrt{2\rho_>}} \frac{\rho_<^\nu}{\rho_>^\nu} \, {}_2F_1\left(b, a, 1+\nu, \frac{\rho_<^2}{\rho_>^2}\right)$$

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How to solve Sacherer's integral equation

- Approximate finite-dimension representation of integral equation
  - Expand beam density perturbation in terms of orthogonal polynomials (conventional approach)
  - Radial function $R_m(J)$ on grid (done here)

Problem reduced to determination of eigenvalues of matrix M

$$\det\left(\mathbf{1}\Delta\hat{\Omega} - M\right) = 0.$$

$$M_{m,m',n,n'} = m\delta_{m,m'}\delta_{n,n'} - i\hat{I}e^{-\rho_n^2/2}\mathcal{G}_{m,m'}(\rho_n,\rho_{n'})\rho_{n'}\Delta\rho.$$

Extrapolate to case with HCs?

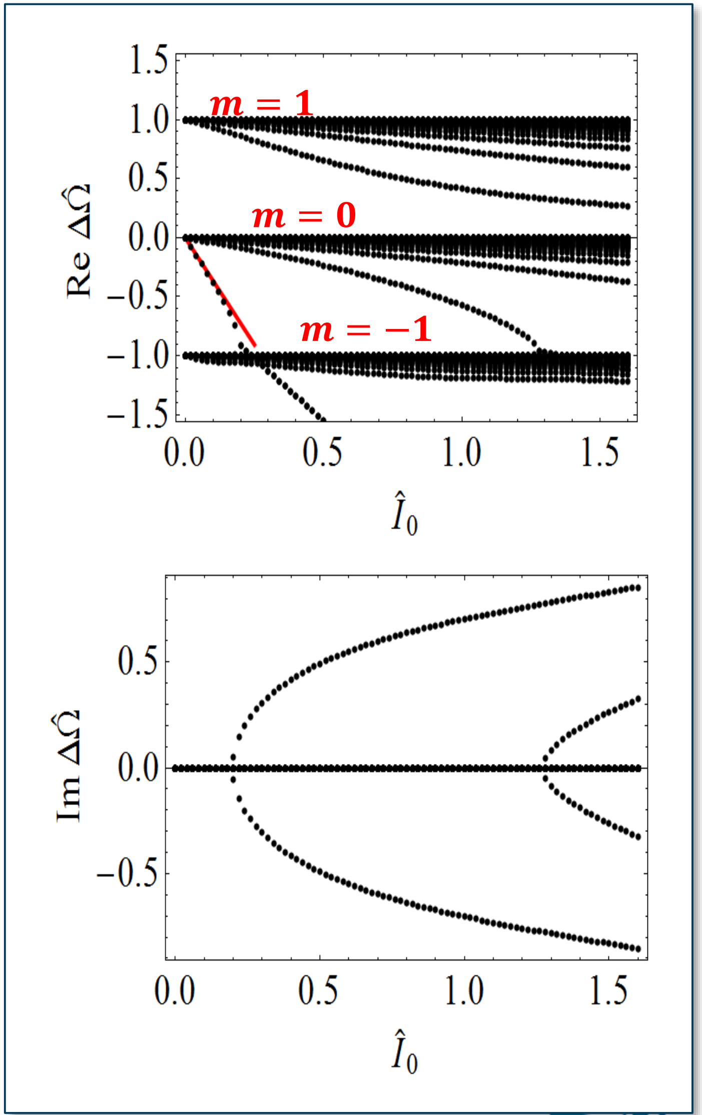
$$\hat{I}_0 = \frac{N r_c c}{(2\pi)^{3/2}\gamma\nu_{s0}b^3\sqrt{c\sigma_c\sigma_{z0}}}\frac{\beta_y L_u}{2\pi} \simeq 0.2$$

ν_s gets smaller ☹

σ_z gets larger ☺

Who wins?

Text-book picture: Instability emerges from crossing of m=0 and m=-1 modes



Toward a mode-analysis theory that includes HCs

Unpertubed (long.) dynamics: unharmonic oscillator

Main RF cavity *Harmonic RF cavity*

$$V_{\rm rf}(z) = V_1 \sin(k_1 z + \phi_1) + V_n \sin(k_n z + \phi_n)$$

$$V_{\rm rf}(z) \simeq z^3[(n^2-1)/6]k_1^3 V_1 \cos\phi_1 .$$

Cubic approx.

$$\mathcal{H} = \alpha c\frac{\delta^2}{2} + \alpha c q\frac{z^4}{4}$$

Hamiltonian w/ quartic potential for motion in RF bucket

$$q = \frac{n^2-1}{6}\frac{eV_1k_1^3}{\alpha c E_0 T_0}\cos\phi_1 \simeq \frac{4}{3}\frac{\omega_{s0}^2 k_1^2}{(\alpha c)^2}$$

3^{rd}-HCs

Negligible radiation loss

Toward a mode-analysis theory that includes HCs

Unpertubed (long.) dynamics: <u>unharmonic</u> oscillator

Amplitude(action)-angle variables $z \simeq r(J_z)\cos\varphi_z$

Exact solution:
Jacobi elliptic function

$$z = r\mathrm{cn}(2\hat{K}\varphi_z/\pi; 1/2)$$

$$z \simeq r(J_z)\cos\varphi_z$$

- Error for dropping higher harmonics of φ_z is <6%
- Formally the same as in the linear (no HC) case
- Bessel functions still appear in the kernel of integral equation: $J_m \sim \int_0^{2\pi} d\varphi_z e^{-i(m\varphi_z + r\kappa\cos\varphi_z)}$
- No conceptual difficulty in doing an exact calculation in terms of generalized 'Bessel' functions (just more numerical work) $\hat{J}_m \sim \int_0^{2\pi} d\varphi_z e^{-i[m\varphi_z + r\kappa\,\mathrm{cn}(\frac{2\hat{K}\varphi_z}{\pi};\frac{1}{2})]}$

$$\omega_s(r) = \frac{2\pi}{T_s} = \frac{\pi}{2\hat{K}}\sqrt{q\alpha c r}$$

Synchrotron tune is linear with amplitude

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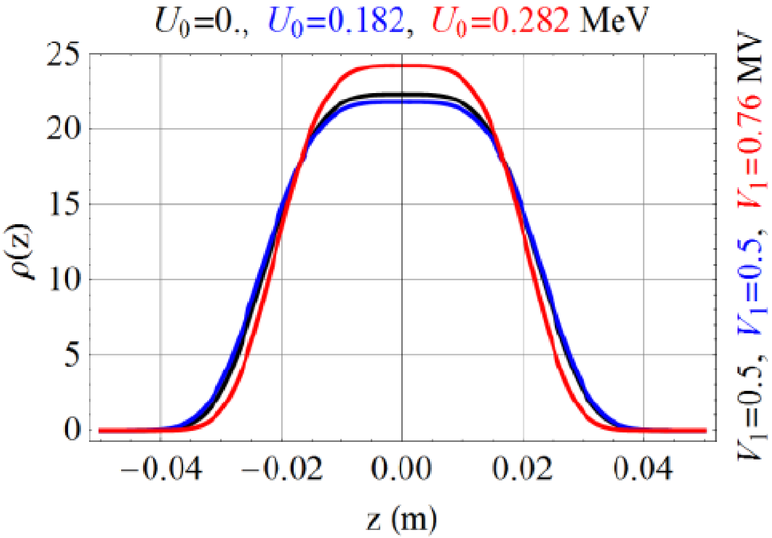
Toward a mode-analysis theory that includes HCs

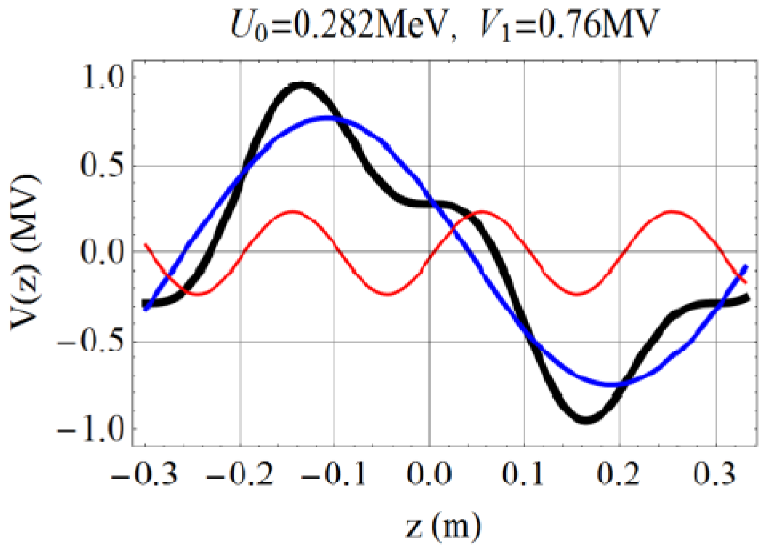
Unpertubed (long.) dynamics: unharmonic oscillator

Amplitude(action)-angle variables
$z \simeq r(J_z)\cos\varphi_z$

Beam equilibrium

$$g_0(r) = \frac{2^{3/4}}{\Gamma(1/4)^2\sigma_z\sigma_\delta}\exp\left(-h_1\frac{r^4}{\sigma_z^4}\right)$$





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Linearized Vlasov equation

Sacherer's equation for radial R_m component of m-mode

$$(\Delta\hat{\Omega} - m\rho)R_m(\rho) + i\hat{I}e^{-h_1\rho^4}\sum_{m'=-\infty}^{\infty}\int_0^{\infty} R_{m'}(\rho')\mathcal{G}_{m,m'}(\rho,\rho')\rho'^2 d\rho' = 0.$$

Main difference is singularity of the integral equation (factor multiplying R_m vanishes for some ρ)

Same kernel as in linear case

Modified equilibrium ($h_1 \simeq 0.114$ is a number)

Similar current parameter

$$\hat{I} = \frac{N r_c c}{\pi^{5/2}\gamma\langle\nu_s\rangle b^3\sqrt{c\sigma_\delta\sigma_z}}\frac{\beta_y L_u}{2\pi}$$

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A digression into *plasma physics, longitudinal microwave instability*: why is the singularity problematic?

Equation for 1D plasma-waves, microwave instability, has a similar singular nature:

$$(p-\Omega)f(p) + i\hat{I}p\,\frac{e^{-p^2/2}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(p\,')dp\,' = 0$$

Eigen-functions can be highly singular (Dirac distributions). "Van-Kampen modes":

$$f(p) = -i\hat{I}\mathcal{P}\,\frac{1}{\sqrt{2\pi}}\,\frac{p\,e^{-p^2/2}}{p-\Omega} + \lambda(\Omega)\delta(p-\Omega)$$

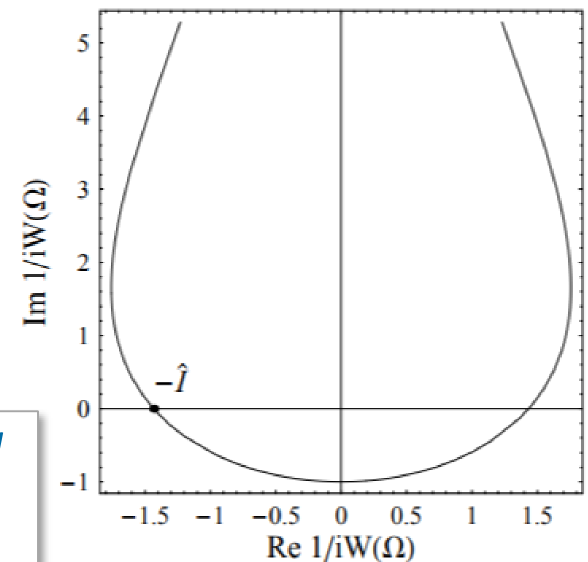
Discretize equation ≡ represent a δ-function by ordinary functions.
Orthogonal polynomials? *Bad idea…*
Eigenfunctions on a grid? *Better, but still have convergence problems*

Preferred approach: divide by $(p-\Omega)$ and integrate to derive the dispersion equation:

$$1 = -i\hat{I}\,\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{p\,e^{-p^2/2}}{p-\Omega}dp.$$

This is the equation one would want to discretize to do things numerically

The "Onion": stability analysis for the μwave-instability dispersion equation



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Preferred numerical approach: regularize Sacherer's integral equation before solving it

- Sweeping the singularity under the integral rug by making a change of the unknown radial function:

New unknown — Old unknown

$$S_m(\rho) = (\Delta\hat{\Omega} - m\rho) R_m(\rho) e^{h_1 \rho^4}$$

Regularized Sacherer's equation

$$S_m(\rho) + i\hat{I} \sum_{m'=-\infty}^{\infty} \int_0^{\infty} \frac{S_{m'}(\rho') e^{-h_1 \rho'^4}}{\Delta\hat{\Omega} - m'\rho'} \mathcal{G}_{m,m'}(\rho,\rho') \rho'^2 d\rho' = 0.$$

- *Finite-dim. approx. now expected to converge*
- *Secular equation is now trascendental vs. polynomial: but Newton method worked well here*
- *(Note: dispersion equation is defined for* $Im\,\Delta\,\widehat{\Omega} > 0$*)*

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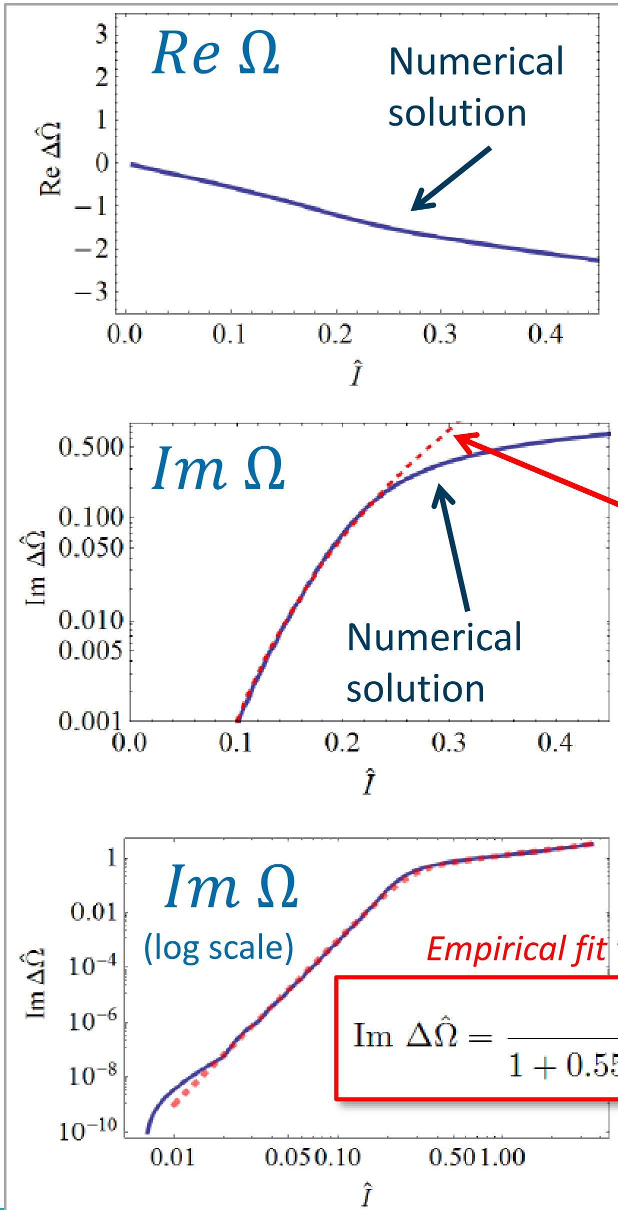
Numerical search of most unstable mode

Mode frequency vs current



Empirical fit to numerical solution

$$\mathrm{Im}\ \Delta\hat{\Omega} = \frac{(2^{5/3}\hat{I})^6}{1 + 0.55 \times (4\hat{I})^5[1 + \tanh(\hat{I}/2)]}.$$

- A root of the dispersion Eq. with $Im\ \Omega > 0$ exists for any value of the current.
- Motion is always unstable (no radiation here)
- Consistent with growth rate $\propto \hat{I}^6$ for small $\hat{I}$ (this is the current range of more practical interest)

$$\mathrm{Im}\ \Delta\hat{\Omega} = (2^{5/3}\hat{I})^6$$

Conjecture that this may be the exact asymptotic form in the low $\hat{I}$ limit

40 radial points; 3 azimuthal modes

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Restating result in practical form

- Enter radiation: threshold appears when instability growth time equals damping time τ_y
- Case where $\mathrm{Im}\ \Delta\hat{\Omega} = (2^{5/3}\hat{I})^6$

Natural bunch length

Threshold w/ HCs

Threshold w/o HCs

$$N_c \simeq 1.15 \times N_{c0} \left(\frac{T_0}{\tau_y \nu_{s0}}\right)^{1/6} \left(\frac{\sigma_{z0}}{\sigma_z}\right)^{1/3}$$

~0.52 for ALSu

Lengthened bunch length

~0.62 for ALSu

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**The fine print: formula valid for small radiation losses; 3rd harmonic cavities. Ideal HC setting*

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Macroparticle simulations (elegant) confirm the $\propto \hat{I}^6$ power law for growth rate

Theory vs. simulation

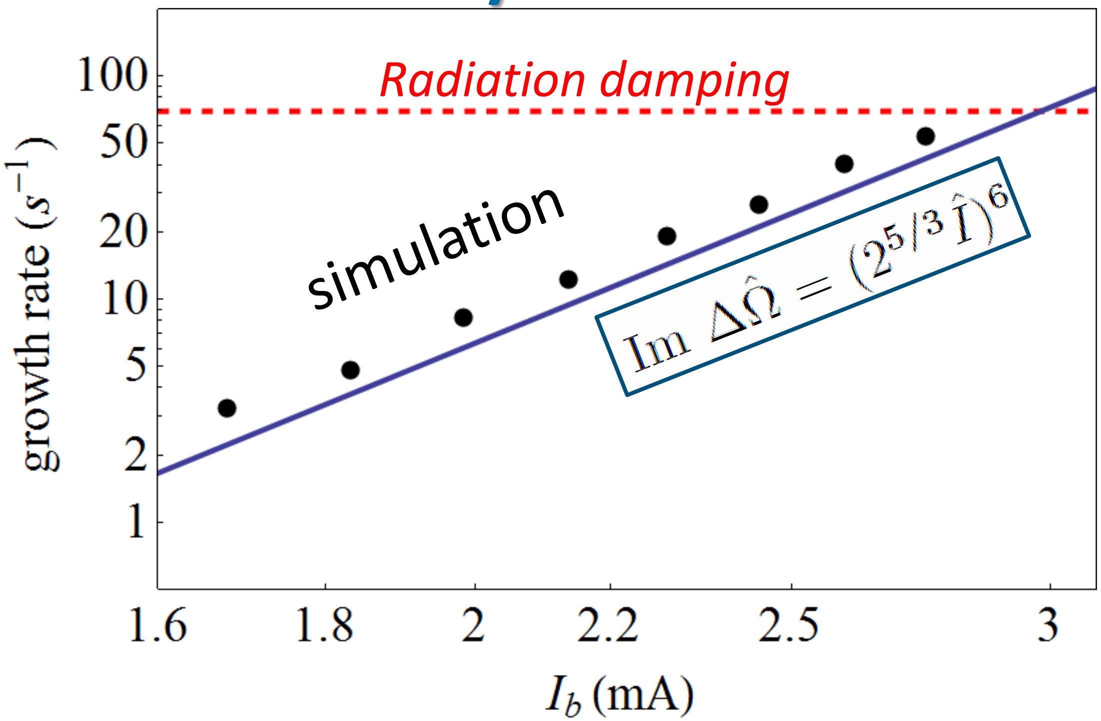

TABLE I: Beam/machine parameters loosely based on ALS-U

Ring circumference	C	196.5 m
Beam energy	E_0	2 GeV
Design bunch current	I_b	1.76 mA
Vertical tune	ν_y	20.368
Momentum compaction	α	2.79×10^{-4}
Natural energy spread	σ_δ	0.835×10^{-3}
Energy loss per turn	U_0	182 keV
Vertical damping time	τ_y	14.4 ms
Main rf cavity voltage	V_1	0.76 MV
Main rf cavity frequency		500 MHz
Harmonic rf cavity frequency		1.5 GHz
Rms bunch length (no HCs)	σ_{z0}	3.2 mm
Linear synchr. tune (no HCs)	ν_{s0}	2.3×10^{-3}
Rms bunch length with HCs	σ_z	13 mm
Avg. synchr. tune with HCs	$\langle \nu_s \rangle$	0.44×10^{-3}
Total ID length	L_u	40 m
ID vacuum chamber radius	b	3 mm
Avg. beta function along IDs	β_y	3 m

- Toy model for ALS-U RW transverse impedance with ten $10m$ long IDs chambers ($b = 3mm$ radius; Cu)
- No radiation damping in simulation

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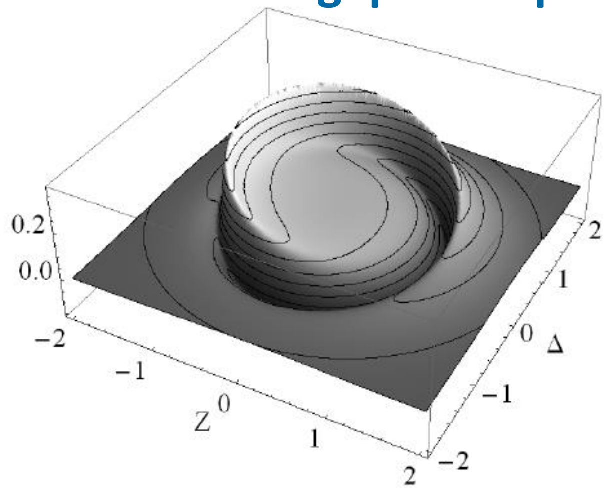
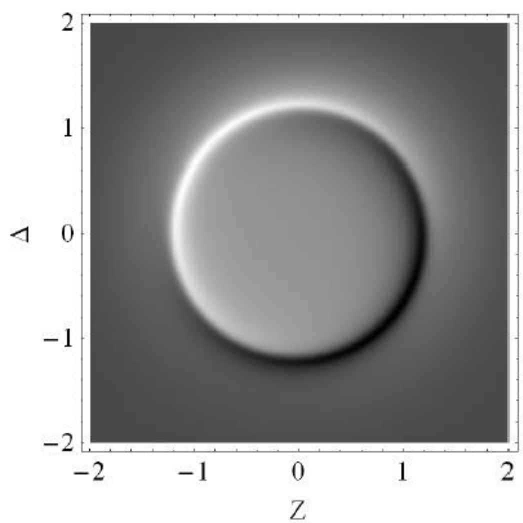
Taking a peek at the unstable mode

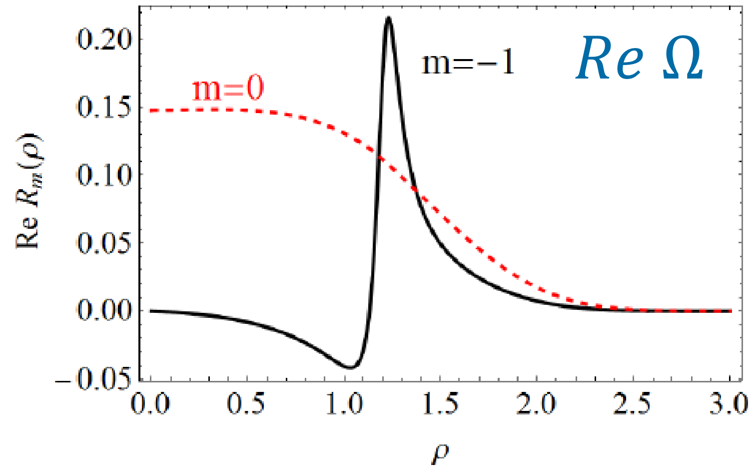
- Unstable mode is a mixture of mostly m=0 and m=-1

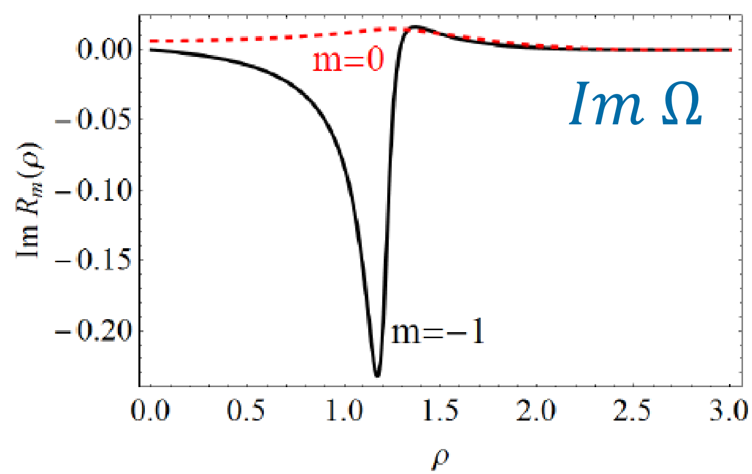
Density plot of unstable perturbation (normalized long. phase space)





$m = -1$ component peaked at radius where coherent tuneshift equals synchrotron tune





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Conclusions & Outlook

- **HCs → Sacherer's singular integral equation**
  - Numerical difficulties of naïve discretization

- **Robust numerical method to solve the Sacherer's in the presence of singularity**

- **RW-dominated impedance (monolayer pipe, DC conductivity) → is always unstable (no radiation)**
  - Growth rate consistent with $\mathrm{Im}\,\Delta\,\widehat{\Omega} = \left(2^{\frac{5}{3}}\hat{I}\right)^{6}$ at low-currents.
  - Exact asymptotic solution? Rigorous proof?

- **With ALS-U like parameters HC could reduce the TMCI current-threshold to less than half (RW only, including radiation damping)**
  - Finite chromaticities come to the rescue (see back-up slide; Cullinan et al. work)

- **Expand theory to include**
  - Chromaticities
  - Exact account of unperturbed motion (use numerical canonical transformation)
  - Arbitrary settings of the HCs
  - Multi-bunches
  - Effect of feed-back systems?

Acknowledgements

R. Warnock (SLAC), and ALS AP group

Additional slides

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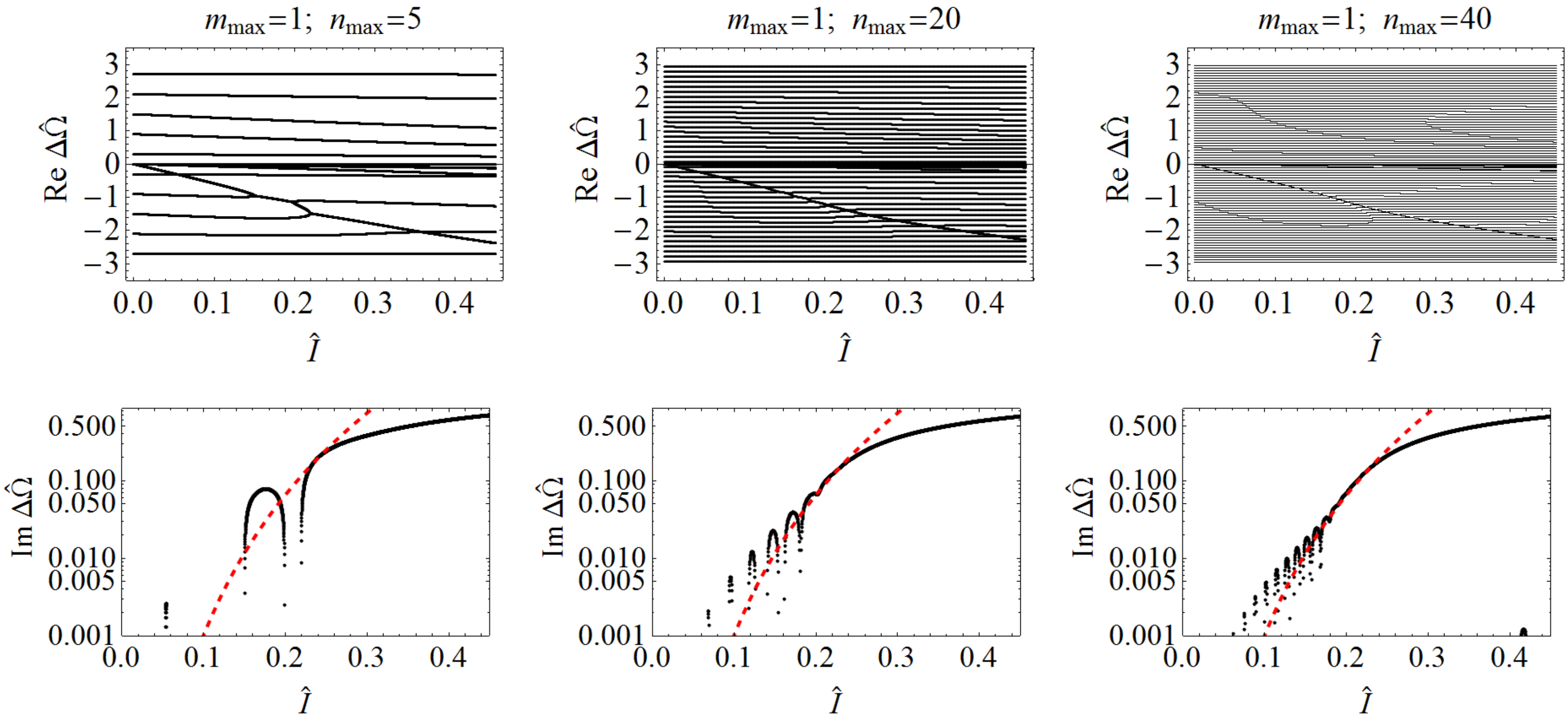
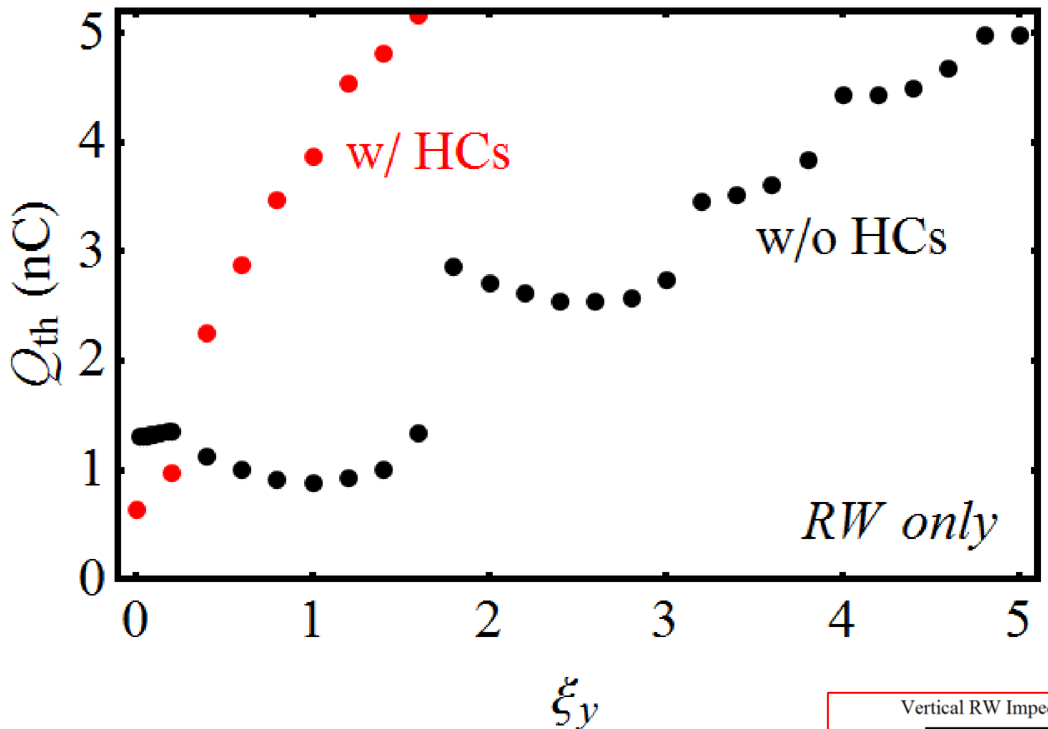
Solving the singular integral equation w/o regularization shows slow, (questionable?) convergence

Re and Im parts of eigenvalues for increasingly finer radial grid



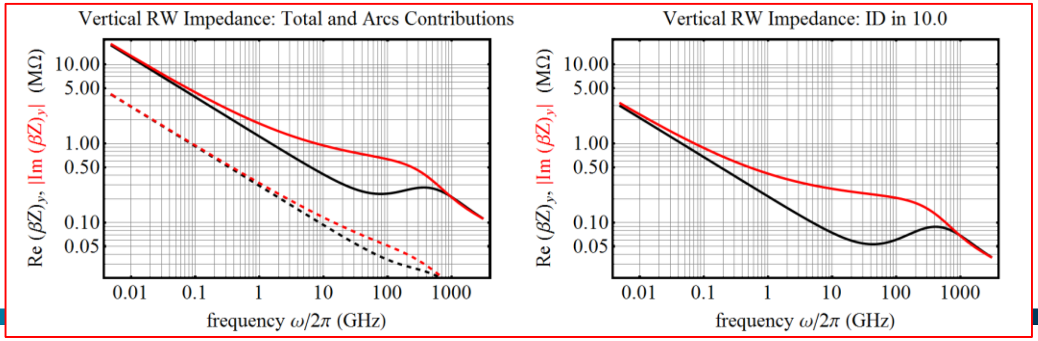
Finite chromaticities generally help more when HCs are present

Single-bunch instability threshold vs. ξ_y



- ALSu instability study with current RW Z model
  - Include account of NEG coating
- Design bunch charge Q = 1.15 nC

ALS-U RW Impedance model



Complete expression for current threshold

- More accurate than expression on slide 18 (arbitrary radiation energy loss)
- Still assumeing regime where $\mathrm{Im}\,\Delta\hat{\Omega} = (2^{5/3}\hat{I})^6$

$$N_c = N_{c0} \times \frac{\pi}{4 \times 2^{2/3}\hat{I}_{c0}} \left(\frac{1}{\tau_y h_2 \langle\omega_s\rangle}\right)^{1/6} \frac{\langle\nu_s\rangle}{\nu_{s0}} \left(\frac{\sigma_z}{\sigma_{z0}}\right)^{1/2}$$

$$\hat{I}_{c0} \simeq 0.197$$

$$h_2 = 2^{3/4}\pi^{3/2}/\Gamma(1/4)^2 \simeq 0.712$$