Status Tolerances studies for FCC-ee at 175 GeV

Sandra Aumon, CERN
On behalf of the FCC-ee Lattice Design team

Acknowledgments to Bernhard Holzer, Katsunobu Oide, Bastian Haerer, Tobias Tydecks

Future Circular Collider Study

- 100 km storage ring

- **FCC-hh** (=long-term goal):

→ High-energy hadron collider

→ Push the energy frontier to 100 TeV

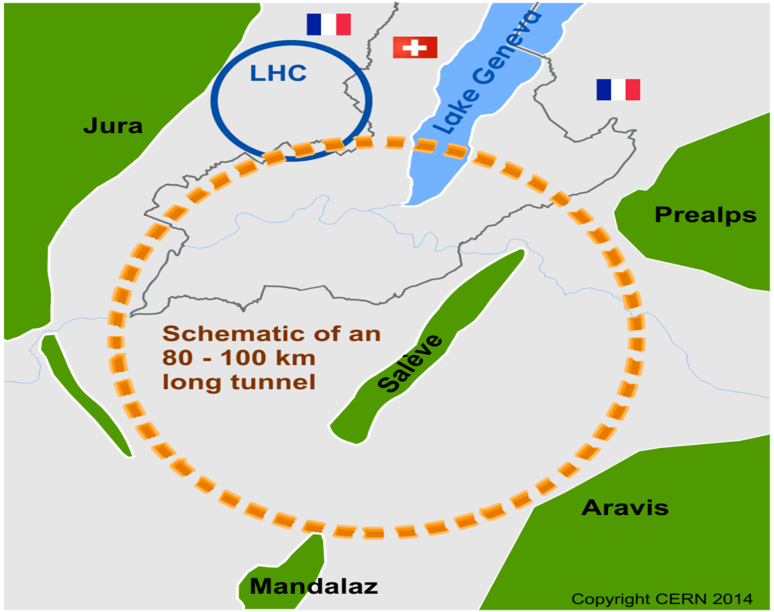
- **FCC-ee (TLEP):**

→ e^+/e^--collider as intermediate step

- **FCC-he**

→ Hadron-lepton collider option

→ Deep inelastic scattering



Physics goals of FCC-ee

Provide highest possible luminosity for a wide physics program ranging from the Z pole to the tt production threshold.

- Beam energy range from 45 GeV to 175 GeV

Main physics programs / energies (+ scan around central values):

- Z (45.5 GeV): Z pole, high precision of M_Z and $\Gamma_{Z,}$
- W (80 GeV): W pair production threshold,
- H (120 GeV): H production,
- T (175 GeV): tt threshold.

All energies quoted refer to BEAM energies

Challenges and constraints (1)

	Z	W	H	tt
Beam energy [GeV]	45.5	80	120	175
Beam current [mA]	1450	152	30	6.6
Bunches / beam	91500	5260	780	81
Bunch population [10^{11}]	0.33	0.6	0.8	1.7
Transverse emittance ε - Horizontal [nm] - Vertical [nm]	0.09 0.001	0.26 0.001	0.61 0.0012	1.3 0.0025
Momentum comp. [10^{-5}]	0.7	0.7	0.7	0.7
Betatron function at IP β* - Horizontal [mm] - Vertical [mm]	1000 2	1000 2	1000 2	1000 2
Energy loss / turn [GeV]	0.03	0.33	1.67	7.55
Total RF voltage [GV]	0.2	0.8	3	10

Small emittance ratio 0.2% requires

- Coupling correction
- Small vert. dispersion

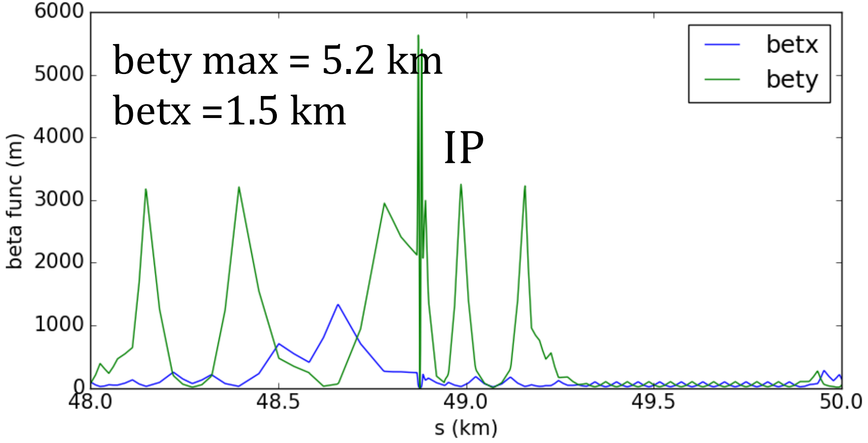
Small beta functions

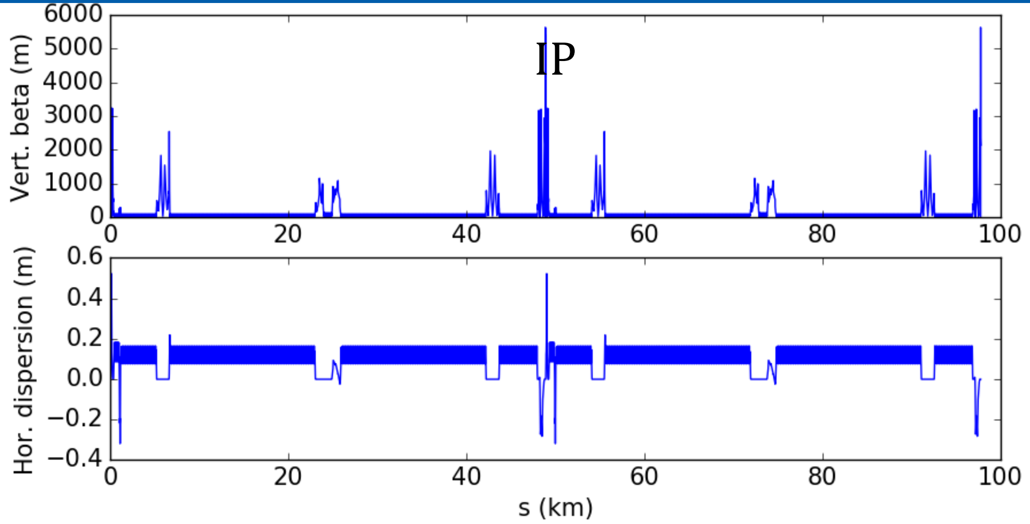
- make lattice sensitive towards FF misalignments
- require strong sextupoles (coupling)

FCC-ee Optic

Vertical beta function in the IR:

Hor. Dispersion along the ring:





beta function in the IR

Phase advance per cell: 90/90 deg.

Emittance tuning for electron machine

- <u>Horizontal emittance:</u>

$$\epsilon_x = \frac{C_g}{J_x}\gamma^2\theta^3 F \qquad F_{FODO} = \frac{1}{2\sin\psi}\frac{5+3\cos\psi}{1-\cos\psi}\frac{L}{l_B}$$

L: cell length
l_B: dipole length
ψ : phase advance/cell

- <u>Vertical emittance:</u>

$$\epsilon_y = \left(\frac{dp}{p}\right)^2 (\gamma D_y^2 + 2\alpha D_y D_y' + \beta D_y'^2)$$

- <u>Source of vertical emittance growth</u>

- vertical dispersion Dy
- betatron coupling
- opening angle → here negligible ~ $1/\gamma$

Source of vertical dispersion & coupling

- **Quadrupoles off-set**: dipolar kick *

$$B_x = k_1(y + \Delta y) = k_1 y + k_1 \Delta y$$

Constant term

Vertical dipole -> vertical dispersion

- **Sextupoles off-set** *

$$B_x = k_2 x(y + \Delta y) = k_2 xy + k_2 x \Delta y$$

$$B_y = k_2(x^2 - (y - \Delta y)^2) = k_2(x^2 - y^2) + k_2 y \Delta y - k_2 (\Delta y)^2$$

Skew quad (coupling) + vertical dipole

- **Quadrupole roll:** skew quad, coupling of the planes

Source emittance growth

* SY Lee “Accelerator Physics”

Challenges and constraints (4)

- Challenging and strict emittance budget (1.3nm, 2.5pm) and coupling ratio (0.2%)
- Very sensitive machine to alignment errors.
- Study strategy:
  - study each error separately
  - Establish appropriate correction scheme to get convergence on a maximum seed number (MADX fails easily to find closed orbit)
  - Merge step by step the different errors together, keeping the Final Focus Doublets for the end.
  - Unless mentioned, FF quadrupoles are left perfectly aligned.
  - Emittance calculations:

    → **Fully tapered machine: every magnet strength follows beam energy loss**

Correction methods

- Orbit correction with MICADO & SVD from MADX
  → Hor. corrector at each QF, Vert. corrector at each QD
  → BPM at each quadrupole

- Vertical dispersion and orbit:
  Orbit Dispersion Free Steering (DFS)

- Linear coupling:
  Linear Coupling resonant driving terms (RDT)
  → 1 skew at each sextupole + skews correctors at the IP

- Beta beating correction & Hor dispersion via Response Matrix:
  Rematching of the phase advance at the BPMs
  → 1 trim quadrupole at each sextupole

$$(\Delta\phi_{xy}, \Delta D_x, \Delta Q_x, \Delta Q_y) = \mathbf{R}\Delta k_1$$

Dispersion Free Steering: Principle

- Build numerically a matrix for vertical orbit (u) & dispersion (Du) response under a corrector kick (al)

$$\begin{pmatrix} (1-\alpha)\vec{u} \\ \alpha\vec{D}_u \end{pmatrix} + \begin{pmatrix} (1-\alpha)\mathbf{A} \\ \alpha\mathbf{B} \end{pmatrix}\vec{\theta} = 0$$

- Orbit response

$$A_{i,j} = \frac{\sqrt{\beta_i\beta_j}}{2\sin(\pi Q_y)}\cos(|\mu_i-\mu_j|-\pi Q_y)$$

- Dispersion response

$$\begin{aligned} B_{ij} \quad = \quad & \{\sum_l^{quad} \frac{K_l L_l \beta_l}{4\sin(\pi Q)^2}\cos(|\mu_i-\mu_l|-\pi Q)\cos(|\mu_l-\mu_j|-\pi Q) \\ & -\sum_m^{sext} \frac{K_{2,m} D_{x,m} L_m \beta_m}{4\sin(\pi Q)^2}\cos(|\mu_i-\mu_m|-\pi Q)\cos(|\mu_m-\mu_j|-\pi Q) \\ & -\frac{\cos(|\mu_i-\mu_j|-\pi Q)}{\sin(\pi Q)}\}\sqrt{\beta_l\beta_j} \end{aligned}$$

- SVD analysis to solve the system and find a solution

"Emittance optimization with dispersion free steering at LEP"
R. Assmann et al. Phys. Rev. ST Accel. Beams 3, 121001

Resonance driving terms (RDT)

Coupling RDT f_{1001}-f_{1010} are related to the coupling parameter via:

$$\Delta Q_{\min} = |C^-| = \left|\frac{4\Delta}{2\pi R}\oint ds f_{1001} e^{-i(\phi_x-\phi_y)+is\Delta/R}\right|,$$

References:
-Vertical emittance reduction and preservation in electron storage rings via resonance driving terms correction, A. Franchi et al, PRSTAB 14, 034002

f_{1001}-f_{1010} can be computed via analytical formulas, or via a matrix formalism with the coupling matrix:

$$f_{\substack{1001\\1010}} = \frac{\sum_w^W J_{w,1}\sqrt{\beta_x^w \beta_y^w}\, e^{i(\Delta\phi_{w,x} \mp \Delta\phi_{w,y})}}{4(1 - e^{2\pi i(Q_u \mp Q_v)})},$$

$$\Delta D_y = -(\Delta J_w) D_x \frac{\sqrt{\beta_y \beta_{y0}}}{2 sin(\pi Q)} cos(\pi Q - |\phi_{y0} - \phi_y|)$$

For FCC-ee, I build a RDT and vertical dispersion response matrix with skew quadrupole kick

$$\begin{pmatrix} \vec{f}_{1001} \\ \vec{f}_{1010} \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_c,$$

Jc are the skew strength

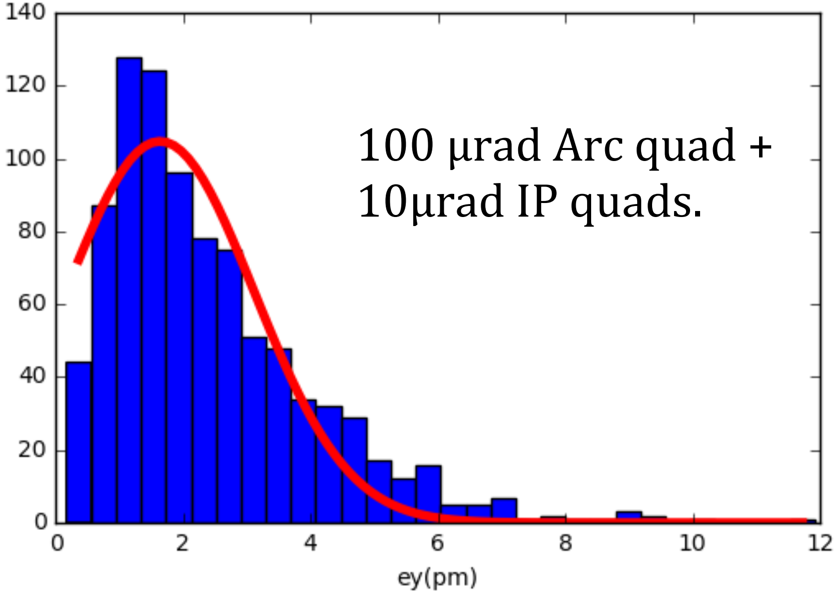
$$(\vec{D}_y) = -M\vec{J}$$

Roll angles in arc quadrupoles + IR quadrupoles

- 100 μrad RMS, gaussian distributed truncated 2.5 sigma in arc quads + 10μrad in IR quads

Without correction at 175 GeV:

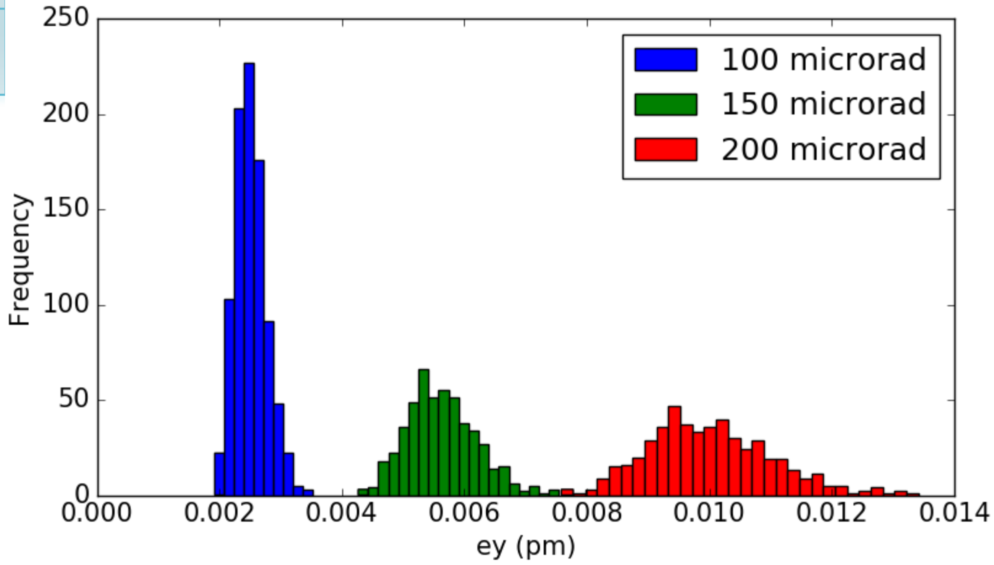
ey=2.47 pm (design vertical emittance value),
ey/ex~0.2 % coupling ratio
Dy=0.003 m



Roll angles in arc and IR quadrupoles

emit	100µrad	150µrad	200µrad
ε_y (pm)	0.002 +/-0.0002	0.005 +/-0.0005	0.01 +/-0.001
ε_x(nm)	1.34+/-5.0e-6	1.34+/-5.0e-6	1.34+/-5.0e-6
$\varepsilon_y/\varepsilon_x$	1.87e-6	4.2e-6	7.4e-6

skew coils at IP quads side to locally correct the coupling from the IR → can be optimised



100 µrad RMS, arc quads
+ 10µrad in IR quads
ey=2pm

100 µrad RMS, arc and IR quads
→ ey/ex~0.002 pm

Transverse displacement of arc quadrupoles

- Errors, **no strength in sextupoles**
- X-y orbits correction
- Dispersion Free Steering wo sextupole (y+Dy correction)
- Save x,x',y,y' at the beginning of the machine

- **Switch on sextupole** to +10% of their design current
  - coupling correction, tune matching
  - beta beat correction, Dx correction
  - coupling + Dy correction
  - increase by 10% the sextupole strength

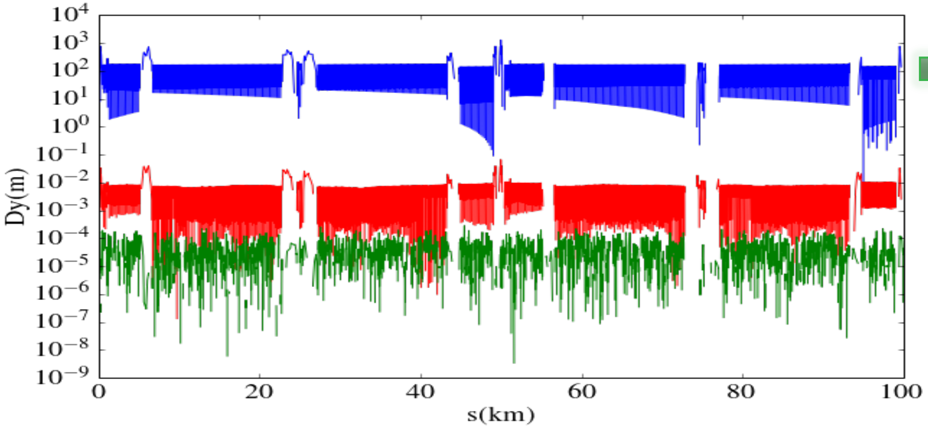
- Emittance computation

Manage by a python script.

up to 4h of computation time/ seeds

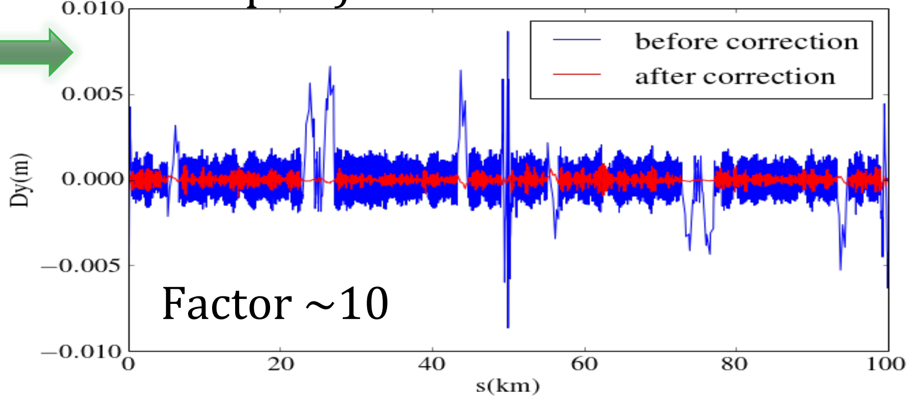
Transverse displacement of arc quadrupoles

Orbit correction +DFS (no sextupole)



Switch on sextupoles

Dispersion correction during the coupling correction (coupling due to sextupole)

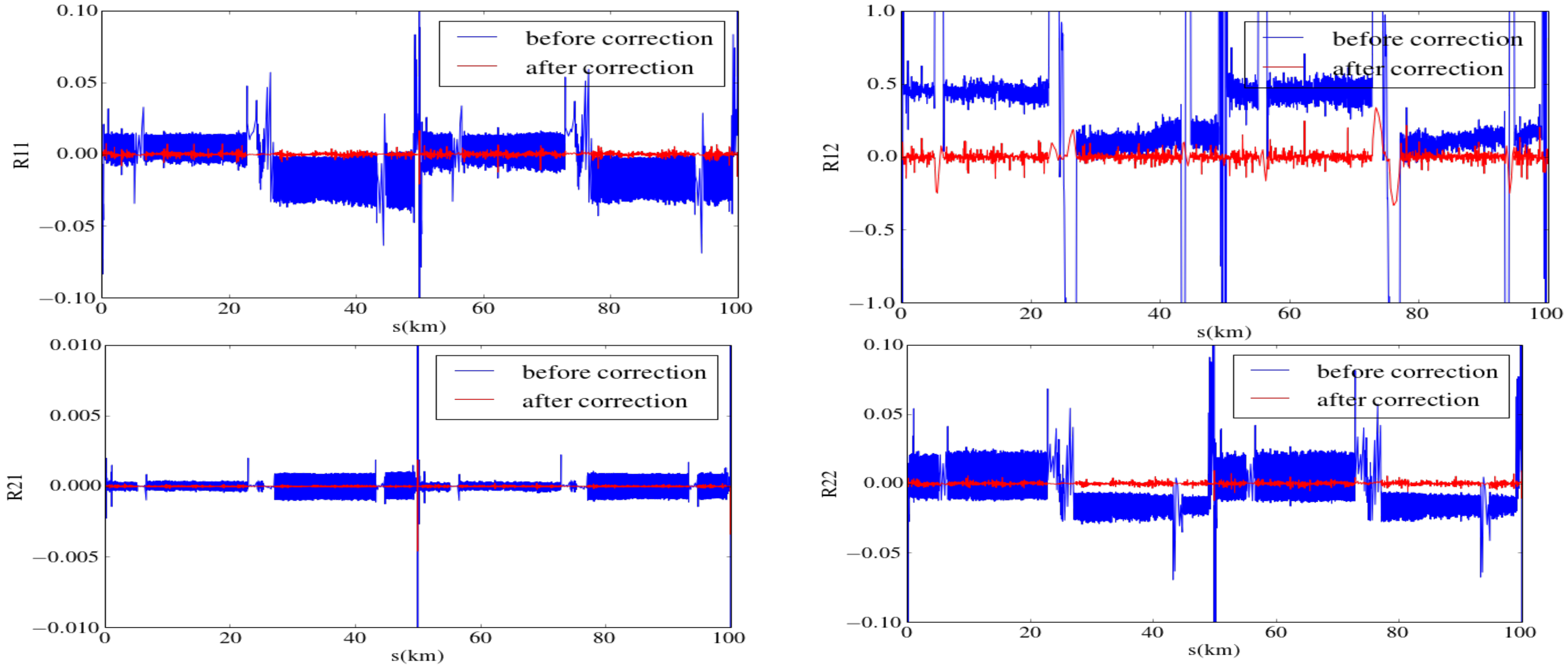


Init DY

After CO-correction Factor 2e4

DFS factor 50

Coupling matrix

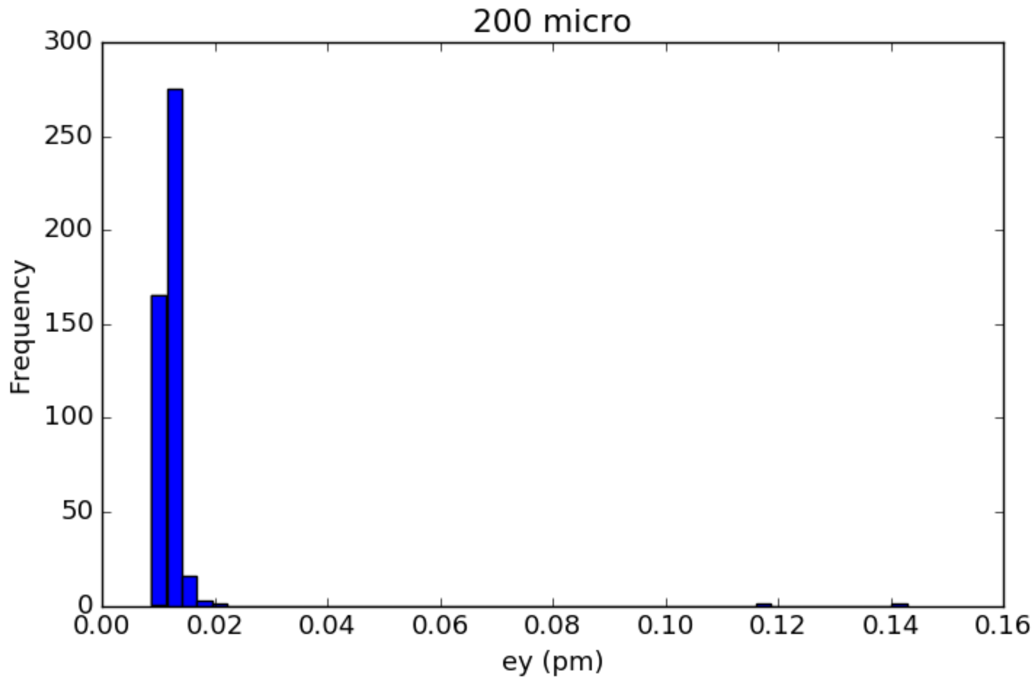


Transverse displacement of arc quadrupoles

Example of emittances with 200 μrad , IR perfectly aligned

90% seeds valid

emittance	$\Delta x = \Delta y = 200\mu m$
ε_y (pm)	0.012 +/-0.008
ε_x(nm)	1.51 +/-0.01
$\varepsilon_y/\varepsilon_x$ (%)	0.001



Sextupole transverse displacements

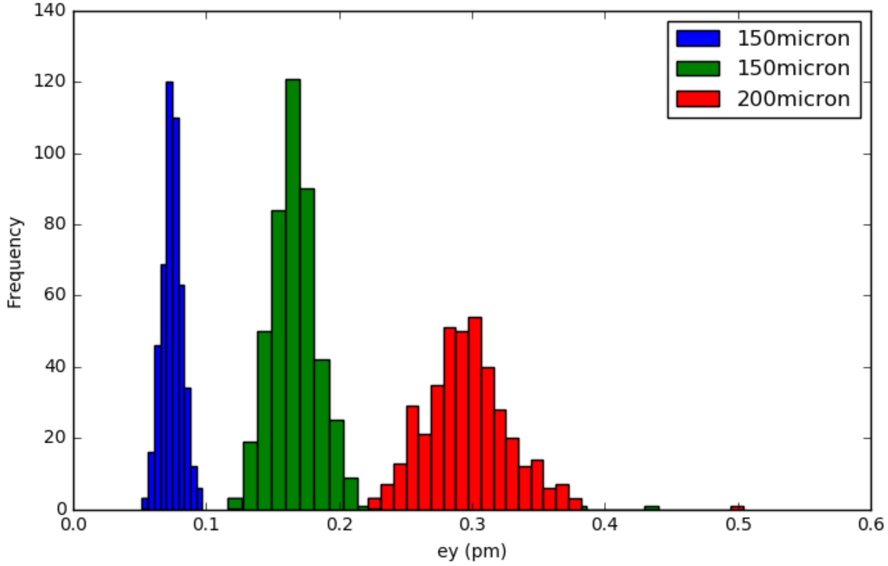
- Δy=10 μm RMS gaussian distributed truncated at 2.5 sigma
  No correction

$$\epsilon_y = 2.1\ pm, \epsilon_x = 1.26\ nm, \epsilon_y/\epsilon_x\ 0.0017$$

2.5 pm vertical emittance design value!

- After correction

Δx, Δy	εx (nm)	εy (pm)	εy/ex %
100	1.34+/-0.0001	0.074+/-0.008	0.005
150	1.34 +/- 0.0003	0.17+/-0.022	0.012
200	1.34 +/- 0.0001	0.3+/-0.03	0.022



Arc Quads & sextupoles misaligned

Strategy

- Errors **, no strength in sextupoles**
- X-y orbits correction
- Pure coupling correction
- Rematch the horizontal dispersion
- 1 step Dispersion Free Steering wo sextupole (Dy correction)
  +
  1 step coupling correction (kicker strength change the coupling configuration)
- Save x,x',y,y' at the beginning of the machine
- **Switch on sextupoles** to +10% of their design current
  - orbit corrections
  - coupling correction, tune matching
  - beta beat correction, Dx correction
  - coupling + Dy correction
  - increase by 10% the sextupole strength
- Emittance computation

7-8h up to one day of simulation/seed

Loop 20 times

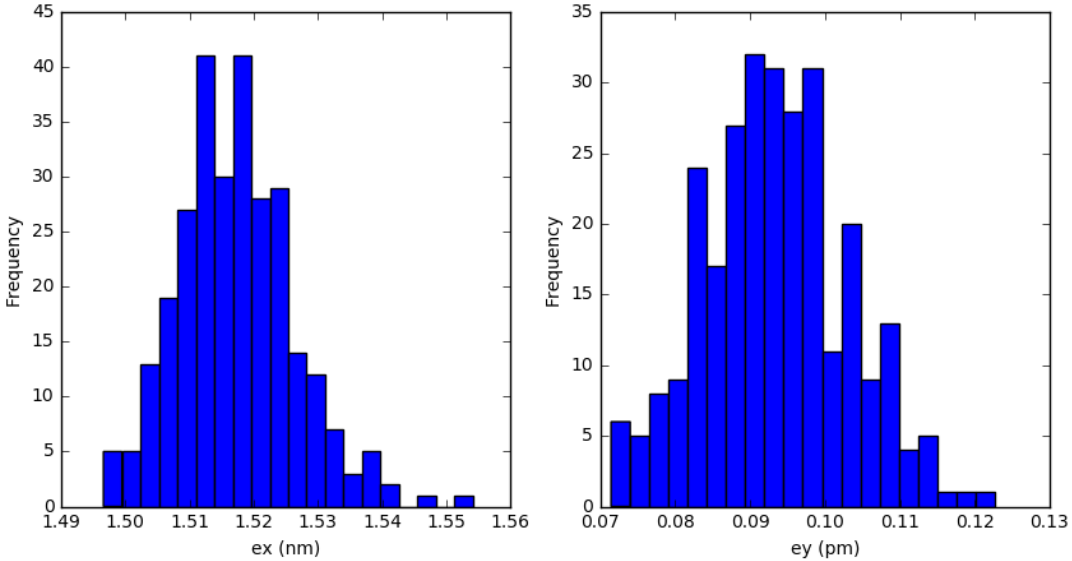
This avoid the tunes run of to resonance and maximize the number of seeds

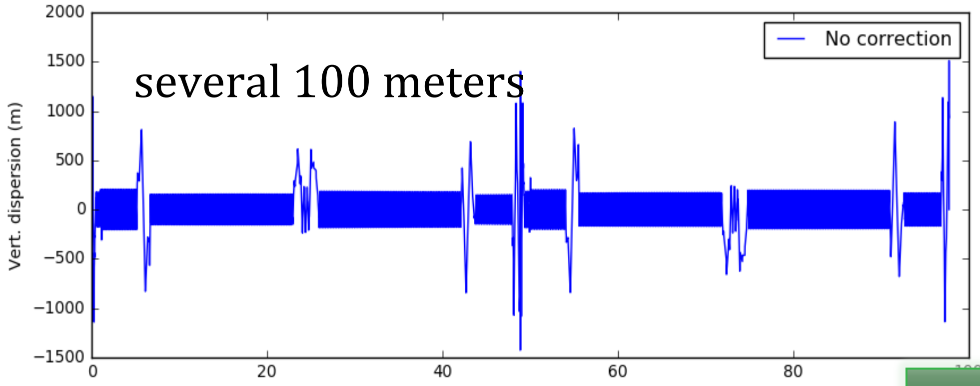
Arc Quads & sextupoles misaligned

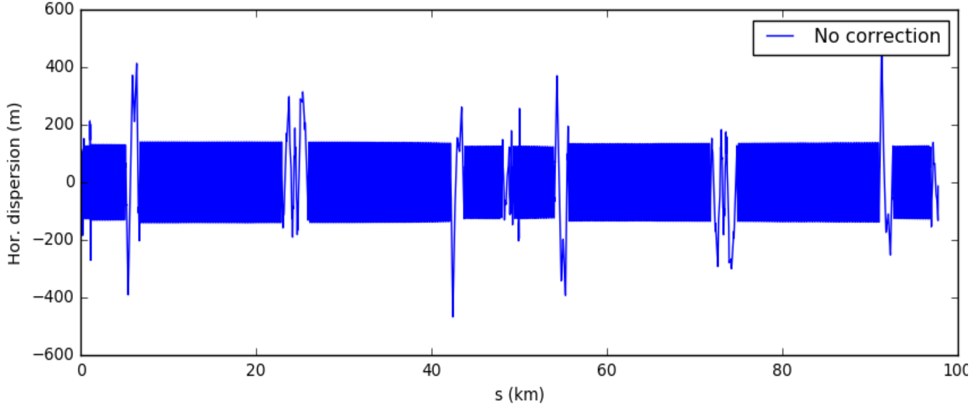
	Δx	Δy	Δθ
Arc quad	100	100	100
Sextupole	100	100	
IP quad	0	0	0

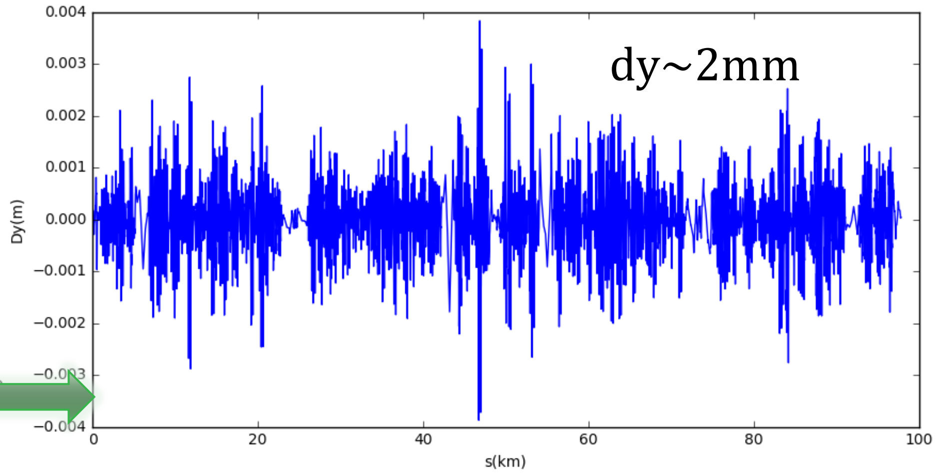
<u>Out of 500 seeds, 436 converged</u>

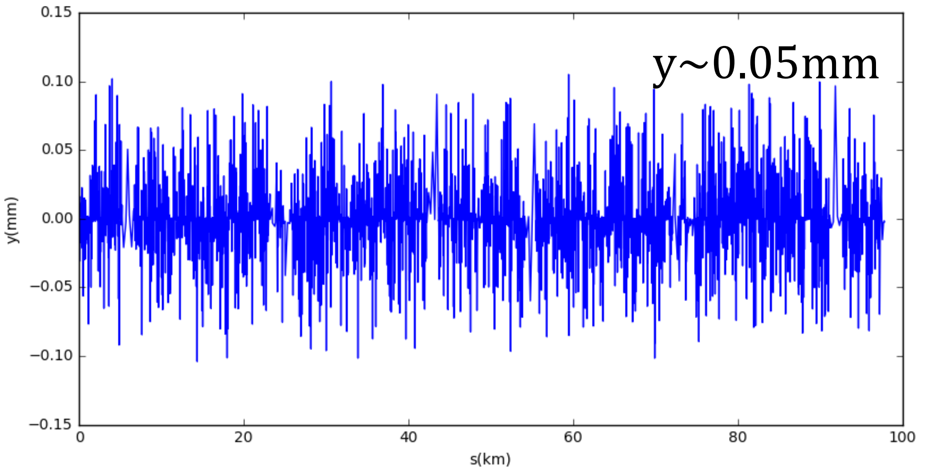
ε_y = 0.093 pm +/- 0.01
ε_x = 1.52 nm +/- 0.009
$\varepsilon_y/\varepsilon_x$ = 0.006% (limit 0.2%)




No correction
several 100 meters
Vert. dispersion (m)

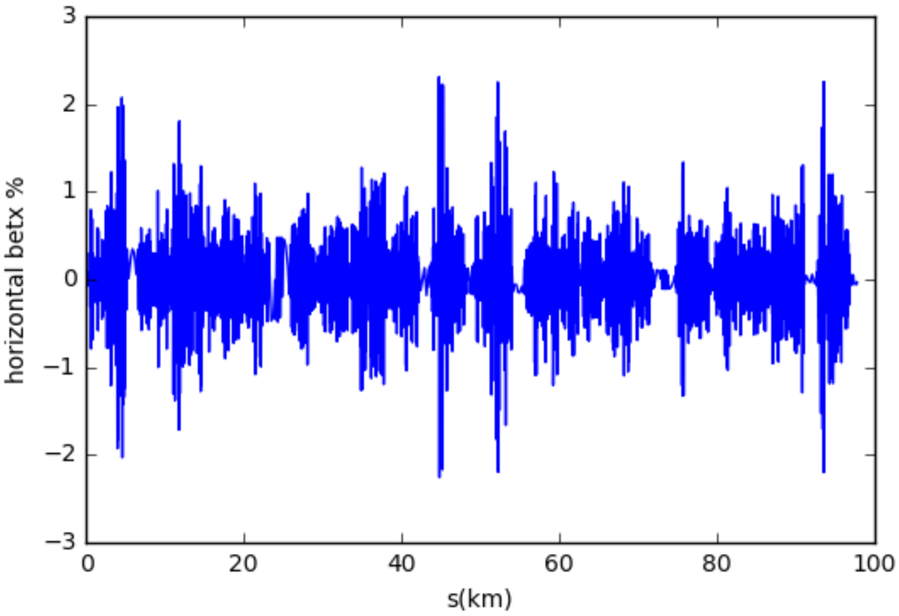

No correction
Hor. dispersion (m)
s (km)

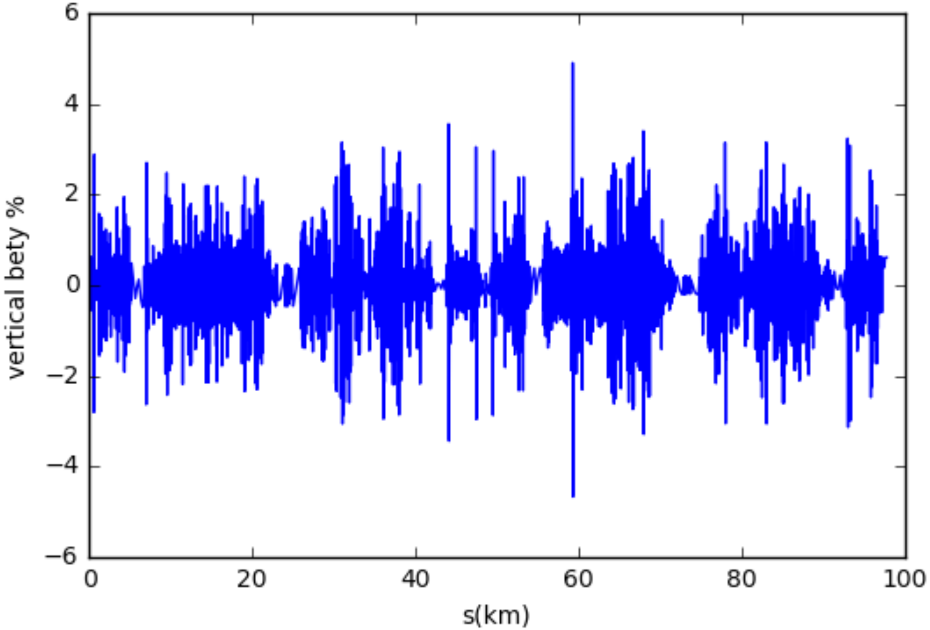

dy~2mm
Dy(m)
s(km)


y~0.05mm
y(mm)
s(km)

Arc Quads & sextupoles misaligned : beta beat

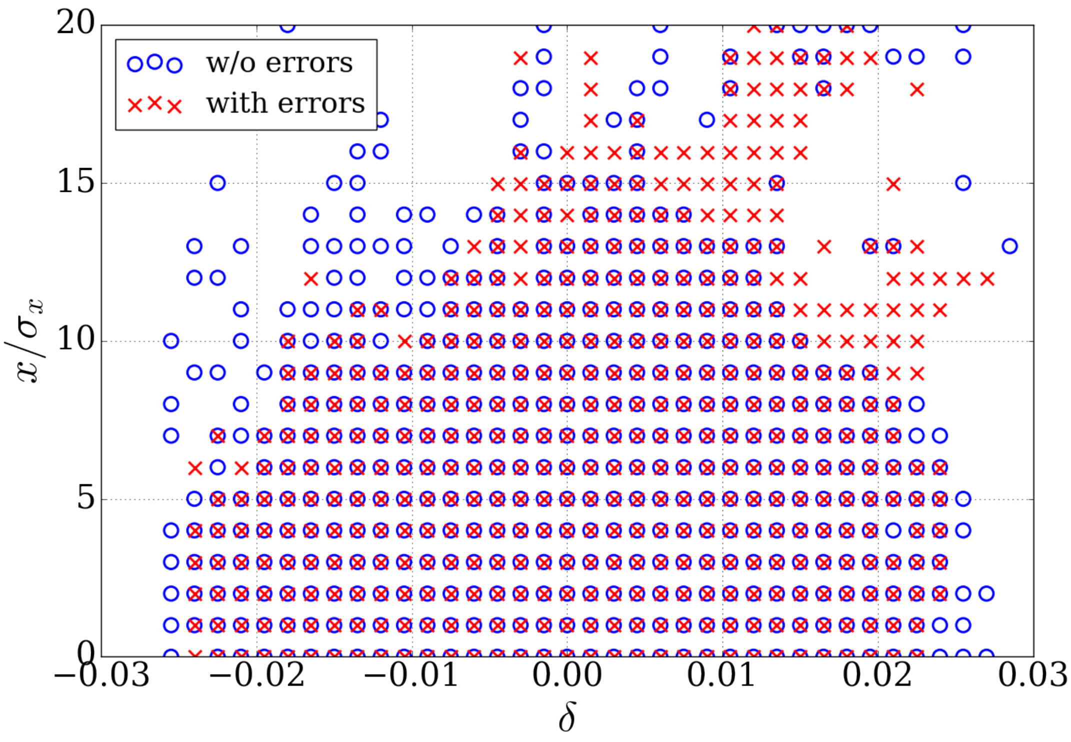
Beta beat





Momentum Acceptance with Errors

Tracking done by Tobias Tydecks



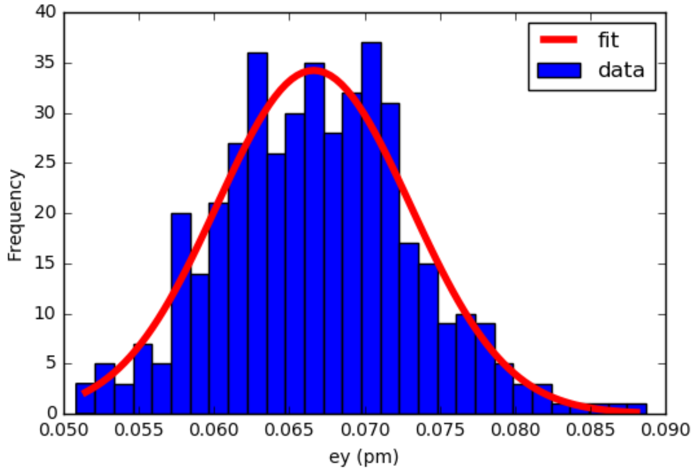
- Very preliminary results, still work on going.
- No misalignment in IR quadrupoles
- tracking for 100 turns (4 damping times) PTC
- One seed

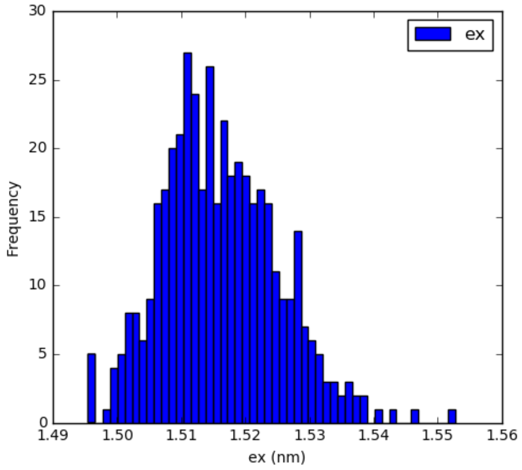
- No large Mom. Acceptance reduction

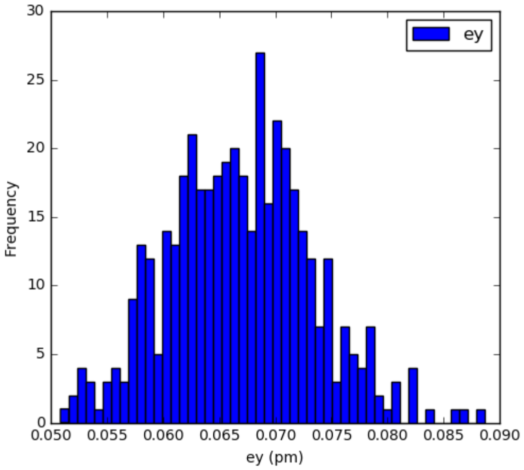
.. to treat the IR quadrupoles, it is better to start from a relax optics, 20mm beta star.

Misaligned Arc and IP elements 20mm beta*

	Δx	Δy	Δθ
Arc quad	100	100	100
Sextupole	100	100	
IP quad	100	100	100





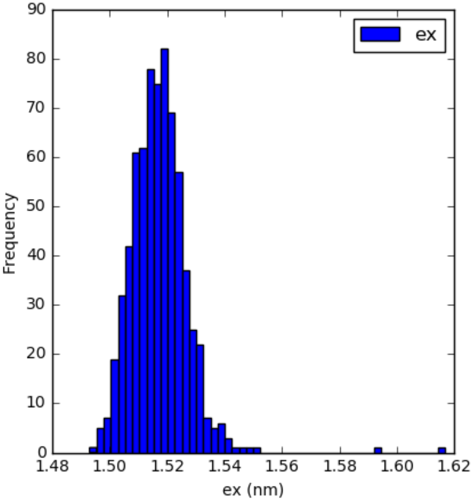


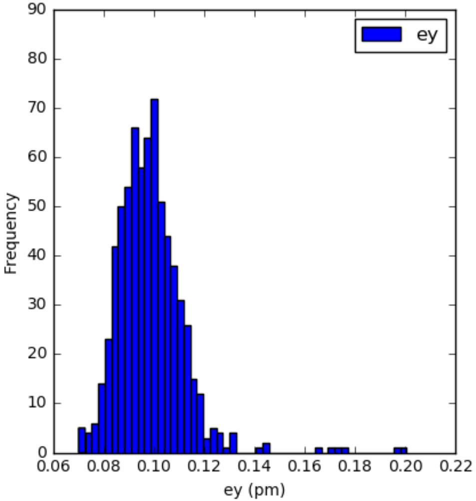
<u>Out of 500 seeds, 436 converged</u>

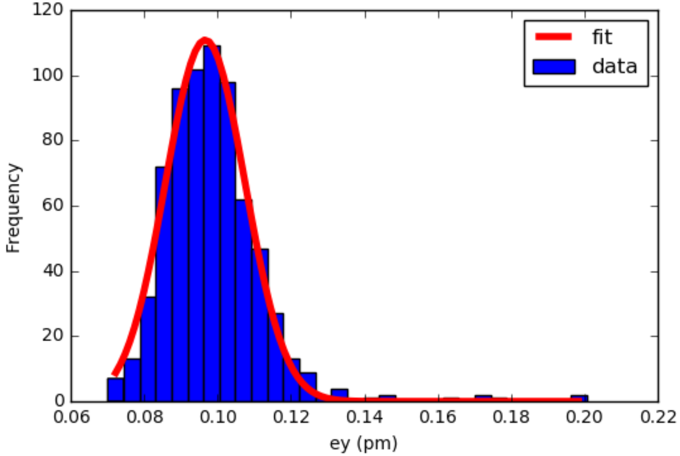
ε_y = 0.067 pm +/- 0.006
ε_x = 1.52 nm +/- 0.009
$\varepsilon_y/\varepsilon_x$ = 0.0044% (limit 0.2%)

Misaligned Arc and IP elements 2mm beta*

	Δx	Δy	Δθ
Arc quad	100	100	100
Sextupole	100	100	
IP quad	50	50	50





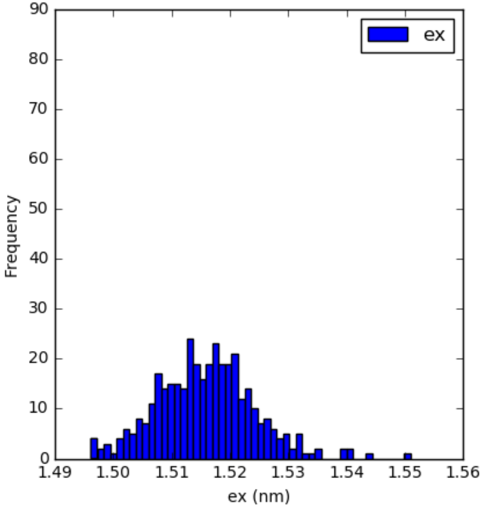


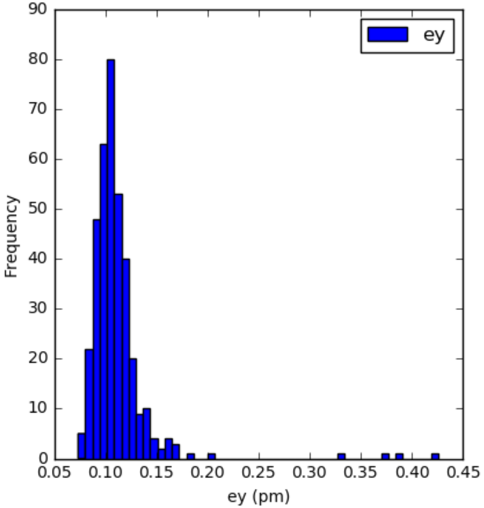
<u>Out of 1000 seeds, 700 converged</u>

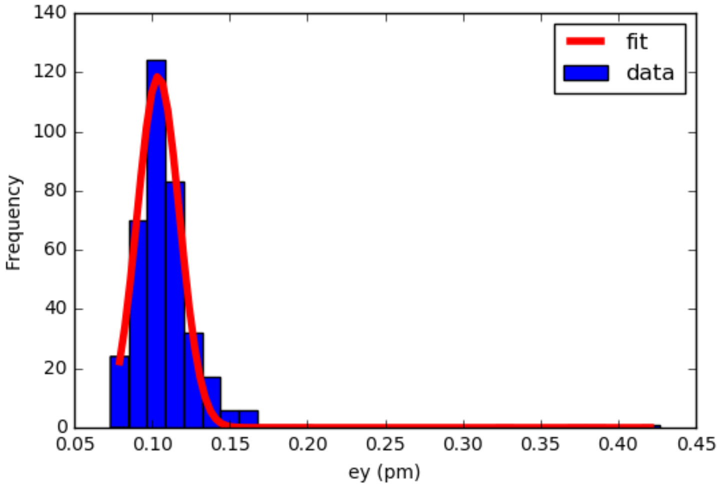
ε_y = 0.099 pm +/- 0.013
ε_x= 1.52 nm +/- 0.01
$\varepsilon_y/\varepsilon_x$= 0.0065% (limit 0.2%)

Misaligned Arc and IP elements 2mm beta*

	Δx	Δy	Δθ
Arc quad	100	100	100
Sextupole	100	100	
IP quad	100	100	100







Out of 1000 seeds, 369 converged

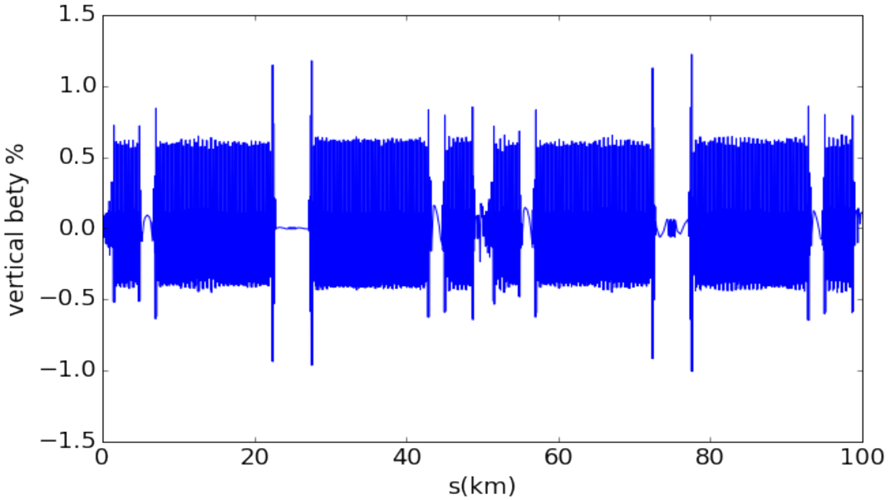
ε_y = 0.11 pm +/- 0.03
ε_x= 1.52 nm +/- 0.01
$\varepsilon_y/\varepsilon_x$= 0.0073% (limit 0.2%)

Conclusions - Outlooks

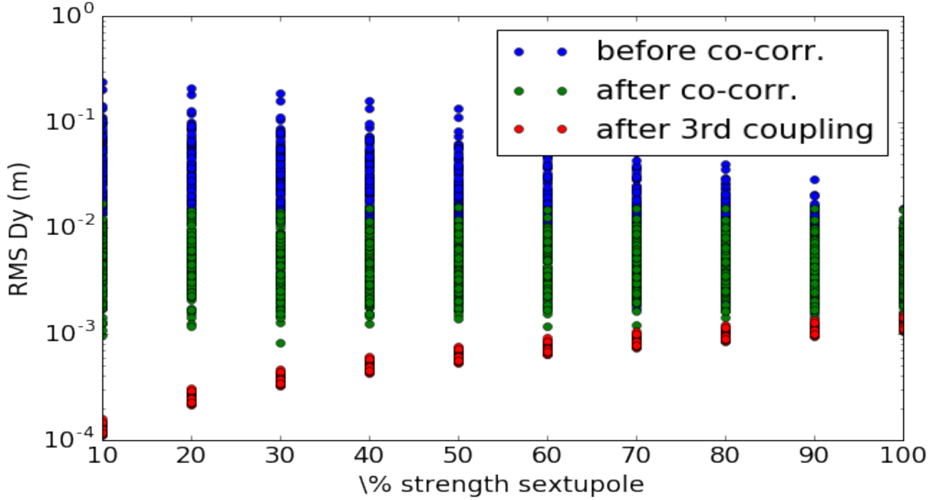
- FCC-ee is a 100km electron-positron collider
- With 100 µm – 100 µrad in Arc quads+Sextupoles and 50µm and 50 µrad IP quads → ey = 0.1 pm (limit 2pm)
  - Stray field solenoid not taken into account in the calculation and can take 30% of the total emittance budget.
  - Above those tolerances, convergence & numerical problems, not enough statistics.
  - Local correctors in IR mandatory.
- Optimization of the number of correctors. Can we preserve the luminosity with less correctors?
- next step: BPM errors and dipole roll still to be included.
- Field errors are still to be treated (chromaticity correction preservation, impact mom. accept. etc..)
- CDR for this year.

Sextupole transverse displacements, no roll

Vertical Δβ/β after beta beat correction (initial up to 50%)



Vertical dispersion during correction with sextupole misalignments



Horizontal Δβ/β after beta beat correction below 1%

Challenges and constraints (2)

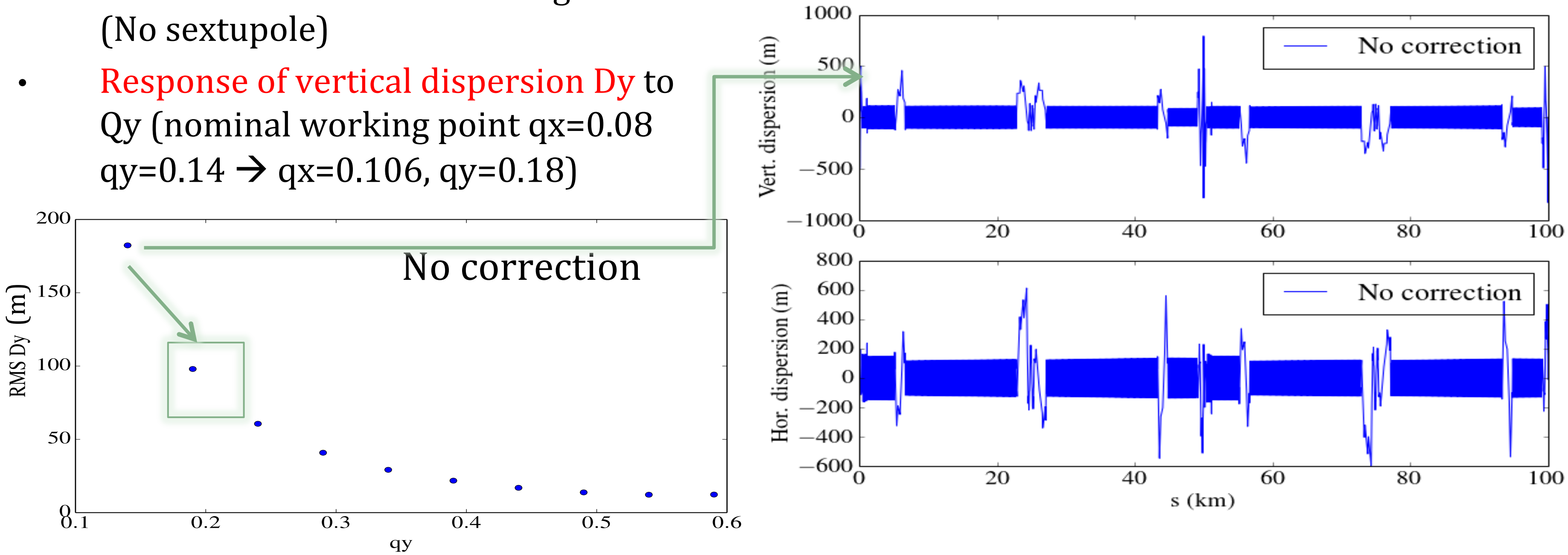
- Initial working point **Qx=0.08 & Qy=0.14** (for beam-beam effect)
  → 1/Sin(pi Q) amplification in orbit, dispersion, coupling response due to errors

$$
\begin{aligned}
B_{ij} \quad = \quad & \{\sum_{l}^{quad} \frac{K_l L_l \beta_l}{4\sin(\pi Q)^2} \cos(|\mu_i - \mu_l| - \pi Q)\cos(|\mu_l - \mu_j| - \pi Q) \\
& - \sum_{m}^{sext} \frac{K_{2,m} D_{x,m} L_m \beta_m}{4\sin(\pi Q)^2} \cos(|\mu_i - \mu_m| - \pi Q)\cos(|\mu_m - \mu_j| - \pi Q) \\
& - \frac{\cos(|\mu_i - \mu_j| - \pi Q)}{\sin(\pi Q)}\}\sqrt{\beta_l \beta_j}
\end{aligned}
$$

Vertical dispersion response to errors in quad/sextupole

Example of FCC-ee lattice sensitivity to errors

- Δy=Δx=100μm RMS displacement in arc quadrupoles, errors gaussian distributed truncated at 2.5sigma (No sextupole)
- Response of vertical dispersion Dy to Qy (nominal working point qx=0.08 qy=0.14 → qx=0.106, qy=0.18)
- Δy=Δx=100μm RMS displacement in arc quadrupoles (No sextupole, Qy=0.14)



Dynamical Aperture