Status Tolerances studies for FCC-ee at 175 GeV

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On behalf of the FCC-ee Lattice Design team

Acknowledgments to Bernhard Holzer, Katsunobu Oide, Bastian Haerer, Tobias Tydecks
Future Circular Collider Study

- 100 km storage ring

- **FCC-hh** (=long-term goal):
  → High-energy hadron collider
  → Push the energy frontier to 100 TeV

- **FCC-ee (TLEP)**:
  → $e^+/e^-$-collider as intermediate step

- **FCC-he**
  → Hadron-lepton collider option
  → Deep inelastic scattering
Physics goals of FCC-ee

Provide highest possible luminosity for a wide physics program ranging from the Z pole to the tt production threshold.

- Beam energy range from 45 GeV to 175 GeV

Main physics programs / energies (+ scan around central values):

- **Z (45.5 GeV):** Z pole, high precision of $M_Z$ and $\Gamma_Z$
- **W (80 GeV):** W pair production threshold,
- **H (120 GeV):** H production,
- **T (175 GeV):** tt threshold.

All energies quoted refer to BEAM energies
## Challenges and constraints (1)

<table>
<thead>
<tr>
<th></th>
<th>Z</th>
<th>W</th>
<th>H</th>
<th>tt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy [GeV]</td>
<td>45.5</td>
<td>80</td>
<td>120</td>
<td>175</td>
</tr>
<tr>
<td>Beam current [mA]</td>
<td>1450</td>
<td>152</td>
<td>30</td>
<td>6.6</td>
</tr>
<tr>
<td>Bunches / beam</td>
<td>91500</td>
<td>5260</td>
<td>780</td>
<td>81</td>
</tr>
<tr>
<td>Bunch population $[10^{11}]$</td>
<td>0.33</td>
<td>0.6</td>
<td>0.8</td>
<td>1.7</td>
</tr>
<tr>
<td>Transverse emittance $\epsilon$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Horizontal [nm]</td>
<td>0.09</td>
<td>0.26</td>
<td>0.61</td>
<td>1.3</td>
</tr>
<tr>
<td>- Vertical [nm]</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0012</td>
<td>0.0025</td>
</tr>
<tr>
<td>Momentum comp. $[10^{-5}]$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Betatron function at IP $\beta^*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Horizontal [mm]</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>- Vertical [mm]</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Energy loss / turn [GeV]</td>
<td>0.03</td>
<td>0.33</td>
<td>1.67</td>
<td>7.55</td>
</tr>
<tr>
<td>Total RF voltage [GV]</td>
<td>0.2</td>
<td>0.8</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

Small emittance ratio 0.2% requires
- Coupling correction
- Small vert. dispersion

Small beta functions
- make lattice sensitive towards FF misalignments
- require strong sextupoles (coupling)
Vertical beta function in the IR:

Hor. Dispersion along the ring:

\[ \text{bety max} = 5.2 \text{ km} \]
\[ \text{betx} = 1.5 \text{ km} \]

Phase advance per cell: 90/90 deg.
Emittance tuning for electron machine

• Horizontal emittance:
\[
\epsilon_x = \frac{C_g}{J_x} \gamma^2 \theta^3 F \\
F_{FODO} = \frac{1}{2 \sin \psi} \frac{5 + 3 \cos \psi}{1 - \cos \psi} \frac{L}{l_B} \quad \text{L: cell length} \\
l_B: dipole length \\
\psi: phase advance/cell
\]

• Vertical emittance:
\[
\epsilon_y = \left( \frac{dp}{p} \right)^2 (\gamma D_y^2 + 2\alpha D_y D_y' + \beta D_y'^2)
\]

• Source of vertical emittance growth

- vertical dispersion Dy
- betatron coupling
- opening angle \(\rightarrow\) here negligible \(\sim 1/\gamma\)
• **Quadrupoles off-set:** dipolar kick *
  \[ B_x = k_1(y + \Delta y) = k_1 y + k_1 \Delta y \]
  **Constant term**
  Vertical dipole -> vertical dispersion

• **Sextupoles off-set** *
  \[ B_x = k_2 x(y + \Delta y) = k_2 xy + k_2 x \Delta y \]
  \[ B_y = k_2(x^2 - (y - \Delta y)^2) = k_2(x^2 - y^2) + k_2 y \Delta y - k_2(\Delta y)^2 \]
  Skew quad (coupling) + vertical dipole

• **Quadrupole roll:** skew quad, coupling of the planes

* SY Lee “Accelerator Physics”
Challenges and constraints (4)

- **Challenging** and strict emittance budget (1.3nm, 2.5pm) and coupling ratio (0.2%)
- Very sensitive machine to alignment errors.
- Study strategy:
  - study each error separately
  - Establish appropriate correction scheme to get convergence on a maximum seed number (MADX fails easily to find closed orbit)
  - Merge step by step the different errors together, keeping the Final Focus Doublets for the end.
  - Unless mentioned, FF quadrupoles are left perfectly aligned.
  - Emittance calculations:

→ **Fully tapered machine: every magnet strength follows beam energy loss**
Correction methods

• Orbit correction with MICADO & SVD from MADX
  → Hor. corrector at each QF, Vert. corrector at each QD
  → BPM at each quadrupole

• Vertical dispersion and orbit:
  Orbit Dispersion Free Steering (DFS)

• Linear coupling:
  Linear Coupling resonant driving terms (RDT)
  → 1 skew at each sextupole + skews correctors at the IP

• Beta beating correction & Hor dispersion via Response Matrix:
  Rematching of the phase advance at the BPMs
  → 1 trim quadrupole at each sextupole

\[
(\Delta \phi_{xy}, \Delta D_x, \Delta Q_x, \Delta Q_y) = R \Delta \kappa_1
\]
Dispersion Free Steering: Principle

- Build numerically a matrix for vertical orbit (u) & dispersion (Du) response under a corrector kick (al)

\[
\begin{pmatrix}
(1 - \alpha)\hat{u} \\
\alpha \hat{D}_u
\end{pmatrix} + 
\begin{pmatrix}
(1 - \alpha)A \\
\alpha B
\end{pmatrix}\hat{\theta} = 0
\]

- Orbit response

\[A_{i,j} = \frac{\sqrt{\alpha_i\beta_j}}{2\sin(\pi Q_y)} \cos(|\mu_i - \mu_j| - \pi Q_y)\]

- Dispersion response

\[B_{ij} = \{ \sum_{l}^{\text{quad}} \frac{K_l L_l \beta_l}{4 \sin(\pi Q)^2} \cos(|\mu_i - \mu_l| - \pi Q) \cos(|\mu_l - \mu_j| - \pi Q) \\
- \sum_{m}^{\text{sext}} \frac{K_{2,m} D_{x,m} L_m \beta_m}{4 \sin(\pi Q)^2} \cos(|\mu_i - \mu_m| - \pi Q) \cos(|\mu_m - \mu_j| - \pi Q) \\
- \frac{\cos(|\mu_i - \mu_j| - \pi Q)}{\sin(\pi Q)} \} \sqrt{\alpha_i\beta_j}\]

- SVD analysis to solve the system and find a solution

“Emittance optimization with dispersion free steering at LEP”
Coupling RDT $f_{1001} - f_{1010}$ are related to the coupling parameter via:

$$\Delta Q_{\text{min}} = |C^-| = \left| \frac{4\Delta}{2\pi R} \int ds f_{1001} e^{-i(\phi_x - \phi_y) + is\Delta R} \right|,$$

$f_{1001} - f_{1010}$ can be computed via analytical formulas, or via a matrix formalism with the coupling matrix:

$$f_{1001}^{\text{1010}} = \frac{\sum_w W J_{w,1} \sqrt{\beta_x \beta_w} e^{i(\Delta \phi_{w,x} + \Delta \phi_{w,y})}}{4(1 - e^{2\pi i(Q_u + Q_v)})},$$

$$\Delta D_y = - (\Delta J_w) D_x \frac{\sqrt{\beta_y \beta_{y0}}}{2 \sin(\pi Q)} \cos(\pi Q - |\phi_{y0} - \phi_y|).$$

For FCC-ee, I build a RDT and vertical dispersion response matrix with skew quadrupole kick

$$\left( \begin{array}{c} \tilde{f}_{1001} \\ \tilde{f}_{1010} \end{array} \right)_{\text{meas}} = -M \tilde{J}_e,$$

$J_e$ are the skew strength

$$(\tilde{D}_y) = -M \tilde{J}$$
Roll angles in arc quadrupoles + IR quadrupoles

- 100 μrad RMS, gaussian distributed truncated 2.5 sigma in **arc quads** + 10μrad in **IR quads**
- **Without correction at 175 GeV:**

  ey=2.47 pm (design vertical emittance value),
  ey/ex~0.2 % coupling ratio
  Dy=0.003 m

100 μrad Arc quad + 10μrad IP quads.
### Roll angles in arc and IR quadrupoles

<table>
<thead>
<tr>
<th>emit</th>
<th>100μrad</th>
<th>150μrad</th>
<th>200μrad</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_y ) (pm)</td>
<td>0.002 +/- 0.0002</td>
<td>0.005 +/- 0.0005</td>
<td>0.01 +/- 0.001</td>
</tr>
<tr>
<td>( \varepsilon_x ) (nm)</td>
<td>1.34 +/- 5.0e-6</td>
<td>1.34 +/- 5.0e-6</td>
<td>1.34 +/- 5.0e-6</td>
</tr>
<tr>
<td>( \varepsilon_y / \varepsilon_x )</td>
<td>1.87e-6</td>
<td>4.2e-6</td>
<td>7.4e-6</td>
</tr>
</tbody>
</table>

100 μrad RMS, arc quads
+ 10μrad in IR quads
\( \varepsilon_y = 2 \) pm

100 μrad RMS, arc and IR quads
\( \rightarrow \varepsilon_y / \varepsilon_x \sim 0.002 \) pm

skew coils at IP quads side to locally correct the coupling from the IR can be optimised
Transverse displacement of arc quadrupoles

- Errors, **no strength in sextupoles**
- X-y orbits correction
- Dispersion Free Steering wo sextupole (y+Dy correction)
- Save x,x',y,y' at the beginning of the machine

- **Switch on sextupole** to +10% of their design current
  - coupling correction, tune matching
  - beta beat correction, Dx correction
  - coupling + Dy correction
  - increase by 10% the sextupole strength

- Emittance computation

Manage by a python script.

up to 4h of computation time/seeds
Orbit correction + DFS (no sextupole)  
Init DY  After CO-correction  DFS  Factor 2e4  factor 50

Switch on sextupoles

Dispersion correction during the coupling correction (coupling due to sextupole)

Factor ~10
Coupling matrix
Transverse displacement of arc quadrupoles

Example of emittances with 200 μrad, IR perfectly aligned

90% seeds valid

<table>
<thead>
<tr>
<th>emittance</th>
<th>Δx = Δy =200μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε_y (pm)</td>
<td>0.012 +/-0.008</td>
</tr>
<tr>
<td>ε_x (nm)</td>
<td>1.51 +/-0.01</td>
</tr>
<tr>
<td>ε_y/ε_x (%)</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Sextupole transverse displacements

- \(\Delta y = 10 \, \mu m \) RMS gaussian distributed truncated at 2.5 sigma
  
  No correction

  \( \epsilon_y = 2.1 \, \text{pm}, \epsilon_x = 1.26 \, \text{nm}, \epsilon_y / \epsilon_x \, 0.0017 \)

- After correction

\[
\begin{array}{|c|c|c|c|}
\hline
\Delta x, \Delta y & \epsilon_x (\text{nm}) & \epsilon_y (\text{pm}) & \epsilon_y / \epsilon_x \% \\
\hline
100 & 1.34 +/- 0.0001 & 0.074 +/- 0.008 & 0.005 \\
150 & 1.34 +/- 0.0003 & 0.17 +/- 0.022 & 0.012 \\
200 & 1.34 +/- 0.0001 & 0.3 +/- 0.03 & 0.022 \\
\hline
\end{array}
\]

2.5 pm vertical emittance design value!
Strategy

• Errors, no strength in sextupoles
• X-y orbits correction
• Pure coupling correction
• Rematch the horizontal dispersion

• 1 step Dispersion Free Steering wo sextupole (Dy correction)
  + 1 step coupling correction (kicker strength change the coupling configuration)

• Save x,x',y,y' at the beginning of the machine

• Switch on sextupoles to +10% of their design current
  - orbit corrections
  - coupling correction, tune matching
  - beta beat correction, Dx correction
  - coupling + Dy correction
  - increase by 10% the sextupole strength

• Emittance computation

7-8h up to one day of simulation/seed
Loop 20 times

This avoid the tunes run of to resonance and maximize the number of seeds
Arc Quads & sextupoles misaligned

<table>
<thead>
<tr>
<th></th>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
<th>$\Delta \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc quad</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Sextupole</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>IP quad</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Out of 500 seeds, 436 converged

$\varepsilon_y = 0.093$ pm $\pm 0.01$

$\varepsilon_x = 1.52$ nm $\pm 0.009$

$\varepsilon_y / \varepsilon_x = 0.006\%$ (limit 0.2%)
several 100 meters

\( dy \sim 2 \text{mm} \)

\( y \sim 0.05 \text{mm} \)
Arc Quads & sextupoles misaligned : beta beat

Beta beat

![Graph showing beta beat](image_url)
Momentum Acceptance with Errors

Tracking done by Tobias Tydecks

- Very preliminary results, still work on going.
- No misalignment in IR quadrupoles
- Tracking for 100 turns (4 damping times) PTC
- One seed
- No large Mom. Acceptance reduction
.. to treat the IR quadrupoles, it is better to start from a relax optics, 20mm beta star.
Misaligned Arc and IP elements 20mm beta*

<table>
<thead>
<tr>
<th></th>
<th>Δx</th>
<th>Δy</th>
<th>Δθ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc quad</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Sextupole</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>IP quad</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Out of 500 seeds, 436 converged

\[ \varepsilon_y = 0.067 \text{ pm} \pm 0.006 \]
\[ \varepsilon_x = 1.52 \text{ nm} \pm 0.009 \]
\[ \varepsilon_y / \varepsilon_x = 0.0044\% \text{ (limit 0.2\%)} \]
Misaligned Arc and IP elements 2mm beta*

<table>
<thead>
<tr>
<th></th>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
<th>$\Delta \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc quad</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Sextupole</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>IP quad</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Out of 1000 seeds, 700 converged

$\varepsilon_y = 0.099 \text{ pm} \pm 0.013$
$\varepsilon_x = 1.52 \text{ nm} \pm 0.01$
$\varepsilon_y/\varepsilon_x = 0.0065\% \text{ (limit 0.2\%)}$
**Misaligned Arc and IP elements 2mm beta***

<table>
<thead>
<tr>
<th></th>
<th>Δx</th>
<th>Δy</th>
<th>Δθ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc quad</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Sextupole</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>IP quad</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Out of 1000 seeds, 369 converged

ε_y = 0.11 pm +/- 0.03
ε_x = 1.52 nm +/- 0.01
ε_y/ε_x = 0.0073% (limit 0.2%)
Conclusions - Outlooks

- FCC-ee is a 100km electron-positron collider

- With 100 μm – 100 μrad in Arc quads+Sextupoles and 50μm and 50 μrad
  IP quads \(\rightarrow ey = 0.1 \text{ pm (limit 2pm)}\)
  - Stray field solenoid not taken into account in the calculation and can take 30% of the total
    emittance budget.
  - Above those tolerances, convergence & numerical problems, not enough statistics.
  - Local correctors in IR mandatory.

- Optimization of the number of correctors. Can we preserve the luminosity with less
  correctors?

- next step: BPM errors and dipole roll still to be included.

- Field errors are still to be treated (chromaticity correction preservation, impact mom.
  accept. etc..)

- CDR for this year.
Sextupole transverse displacements, no roll

Vertical $\Delta \beta/\beta$ after beta beat correction (initial up to 50%)

Horizontal $\Delta \beta/\beta$ after beta beat correction below 1%

Vertical dispersion during correction with sextupole misalignments
Challenges and constraints (2)

- Initial working point $Q_x=0.08$ & $Q_y=0.14$ (for beam-beam effect)
  $\rightarrow$ $1/\sin(\pi Q)$ amplification in orbit, dispersion, coupling response due to errors

$$B_{ij} = \left\{ \sum_{l}^{quad} \frac{K_{l}L_{l}\beta_{l}}{4\sin(\pi Q)^2} \cos(|\mu_{i} - \mu_{l}| - \pi Q) \cos(|\mu_{l} - \mu_{j}| - \pi Q) \right.$$ 
$$- \sum_{m}^{sext} \frac{K_{2,m}D_{x,m}L_{m}\beta_{m}}{4\sin(\pi Q)^2} \cos(|\mu_{i} - \mu_{m}| - \pi Q) \cos(|\mu_{m} - \mu_{j}| - \pi Q)$$ 
$$- \frac{\cos(|\mu_{i} - \mu_{j}| - \pi Q)}{\sin(\pi Q)} \right\} \sqrt{\beta_{i}\beta_{j}}$$

Vertical dispersion response to errors in quad/sextupole
• $\Delta y = \Delta x = 100\mu$m RMS displacement in arc quadrupoles, errors gaussian distributed truncated at 2.5sigma (No sextupole)

• Response of vertical dispersion $D_y$ to $Q_y$ (nominal working point $q_x = 0.08$ $q_y = 0.14 \rightarrow q_x = 0.106$, $q_y = 0.18$)

$\Delta y = \Delta x = 100\mu$m RMS displacement in arc quadrupoles (No sextupole, $Q_y = 0.14$)
Dynamical Aperture