# ANALYTICAL N-BPM METHOD

## IMPROVING ACCURACY AND ROBUSTNESS OF LINEAR OPTICS MEASUREMENTS

Andreas Wegscheider

PhD student at CERN and Hamburg University

Federal Ministry of Education and Research

DESY

Universität Hamburg
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MOTIVATION

BETA FUNCTION MEASUREMENT

ANALYTICAL N-BPM METHOD

SIMULATIONS

MEASUREMENTS

CONCLUSIONS

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# MOTIVATION

- Measurement and correction of focusing errors is of **great importance** in circular accelerators
  - **colliders** as well as **light sources**
- **N-BPM** method has been used in various machines
  - LHC
  - ALBA
  - ESRF
- Want to **improve** existing measurement techniques.

N-BPM method used Monte Carlo simulations to get systematic errors

- **time consuming**
- **failed for pushed optics**

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# $\beta$ FUNCTION MEASUREMENT

**beta from phase, 3-BPM method, N-BPM method**

# BETA FROM PHASE

The position of the beam at a position $s_i$ in the ring is

$$x(s_i) = A\cos(2\pi Q + \phi(s_i)) + x_{CO}$$

$Q$: machine tune, $\phi(s)$: betatron phase, $x_{CO}$: closed orbit

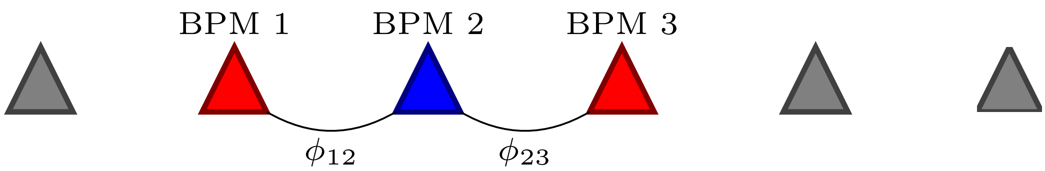
Measured with **Beam Position Monitors** (BPMs). The phase $\phi(s_i)$ is derived by a **harmonic analysis** of the BPM signal.

$\beta$ **from phase** using **three BPMs** via the following formula:

$$\beta(s_i) = \frac{\cot(\phi_{ij}) - \cot(\phi_{ik})}{\cot(\phi_{ij}^m) - \cot(\phi_{ik}^m)}$$

$\phi_{ij} = \phi_j - \phi_i$

*Assumption: relatively* ***small lattice imperfections*** *between the BPMs.*



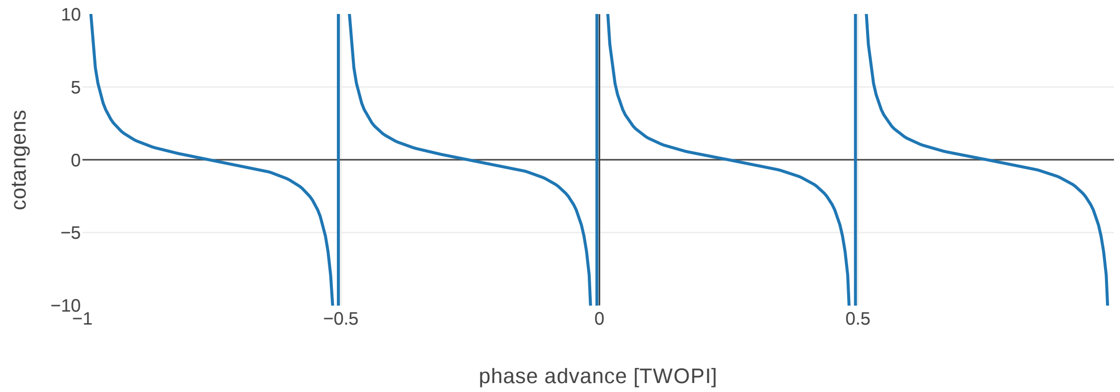

**blue**: probed BPM *(BPM i)* **red**: used BPM (*j*, *k*)

# BETA FROM PHASE

$$\beta(s_i) = \frac{\cot(\phi_{ij}) - \cot(\phi_{ik})}{\cot(\phi_{ij}^m) - \cot(\phi_{ik}^m)}$$

**sensitive** to the position of the BPMs with respect to each other.

The cotangens **enhances phase measurement errors** if $\phi_{ij} \approx n\pi \quad n \in \mathbb{N}$

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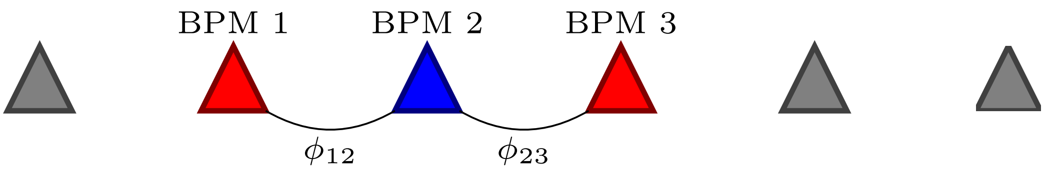
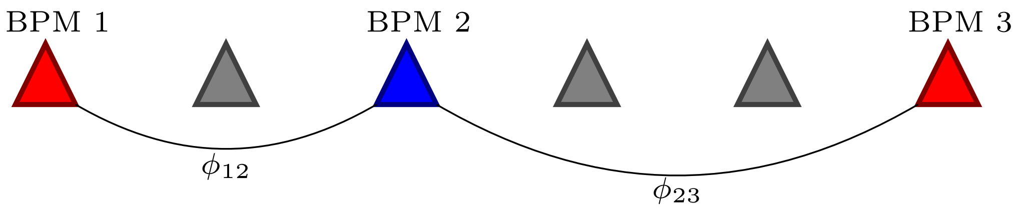
# SKIP BPMS / USE MULTIPLE COMBINATIONS



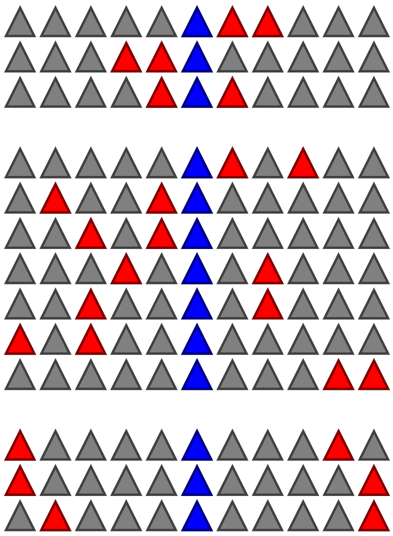




- **Skipping BPMs** to avoid unpreferable phase advances. But lattice imperfections deteriorate the quality of the result
- Take into account possible **sources of errors** between the BPMs
- Use **more than one combination** to increase the amount of information

**N-BPM method**

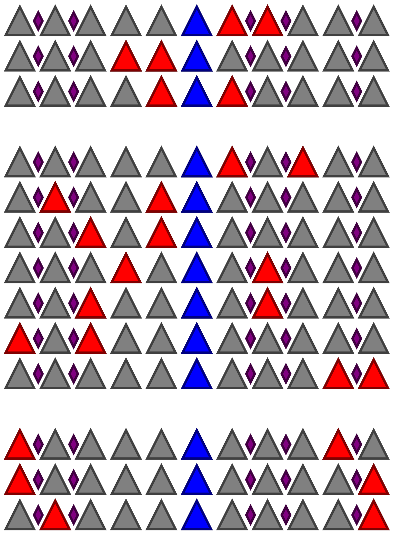
# ORIGINAL N-BPM METHOD

To get the $\beta$ function at the position of a BPM i: calculate **several combinations** $(i, j_l, k_l)$ and use the **mean value**.



# ORIGINAL N-BPM METHOD

But there are errors and far-away BPMs yield worse data.



# ORIGINAL N-BPM METHOD

To get the $\beta$ function at the position of a BPM i: calculate several combinations $(i, j_l, k_l)$

$$\beta_l(s_i) = \frac{\cot(\phi_{ij_l}) - \cot(\phi_{ik_l})}{\cot(\phi^m_{ij_l}) - \cot(\phi^m_{ik_l})}$$

The best estimation for the beta function is calculates by

$$\beta = \sum_l \beta_l(s_i) g_l$$

The weights $g_l$ are determined by a **least-squares** estimation

$$g_l = \frac{\sum_k V^{-1}_{ik}}{\sum_{i,j} V^{-1}_{ij}}$$

# ORIGINAL N-BPM METHOD

$$g_l = \frac{\sum_k V^{-1}_{ik}}{\sum_{i,j} V^{-1}_{ij}}$$

$\mathbf{V} = \mathbf{Cov}[\vec{\beta}]$ is the **covariance matrix** of $\vec{\beta} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}$

It can be calculated from the diagonal **error matrix** $\mathbf{M} = diag(\epsilon_1, \ldots, \epsilon_M)$ where the $\epsilon_\lambda$ are all the sources of error ($\Delta\phi, \Delta K_1, \Delta s, \ldots$):

$$\mathbf{V} = \mathbf{TMT}^{-1}$$

where $\mathbf{T}$ is the Jacobian

$$T_{l\lambda} = \left.\frac{\partial \beta_l}{\partial \epsilon_\lambda}\right|_{\delta\epsilon=0}$$

# ORIGINAL N-BPM METHOD

- **Only statistical errors** were calculated analytically. Systematic errors were calculated by Monte Carlo simulations
- Yields only approximated results and is **time consuming**. (LHC has many different optics)
- Pushed optics can cause a **crash** of the simulations.

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# ANALYTICAL N-BPM METHOD

**analytical calculation of the covariance matrix, removal of *bad* BPM combinations**

# CALCULATION OF THE CORRELATION MATRIX

The Jacobian $\mathbf{T}$ can be split into blocks

$$\mathbf{T} = \left( \mathbf{T}^{\phi} \ \mathbf{T}^{K} \ \mathbf{T}^{s} \right)$$

$\mathbf{T}^{\phi}$: phase uncertainties, $\mathbf{T}^{K}$: magnet field uncertainties, $\mathbf{T}^{s}$: longitudinal misalignments

The three parts of the matrix can be calculated:

$$\mathbf{T}^{\phi} = \left( \frac{\partial \beta_l}{\partial \phi_\lambda} \right)_{l\lambda} \quad \mathbf{T}^{K} = \left( \frac{\partial \beta_l}{\partial K_\mu} \right)_{l\mu} \quad \mathbf{T}^{s} = \left( \frac{\partial \beta_l}{\partial s_\nu} \right)_{l\nu}$$

The phase part is already known, for the others we need a relationship

$$\beta^{\mathrm{real}}(s_i) = \beta(\phi, \{\delta K_1\}_{\mathrm{acc}}, \{\delta s\}_{\mathrm{acc}}, \ldots) = \beta^m + \Delta\beta(\phi, \{\delta K_1\}_{\mathrm{acc}}, \{\delta s\}_{\mathrm{acc}}, \ldots)$$

$\{\delta K_1\}_{\mathrm{acc}}$: quadrupolar field errors in the accelerator

$\{\delta s\}_{\mathrm{acc}}$: longitudinal misalignments (BPMs and quads).

# EFFECT OF IMPERFECTIONS ON THE $\beta$ FUNCTION AND ITS MEASUREMENT

**new formula** to calculate $\beta$ function from

- **phase advances**
- **quadrupolar field errors**
- longitudinal **BPM misalignments**
- longitudinal **quadrupole misalignments**

$$\beta_l(s_i) \approx \frac{\cot\phi_{ij_l} - \cot\phi_{ik_l}}{\cot\phi^m_{ij_l} - \cot\phi^m_{ik_l} + g_{ij_l} - g_{ik_l}} \left[\beta^m(s_i) - 2\alpha^m(s_i)\delta s_i\right]$$

where the terms $g_{ij}$ collect the dependency on **lattice imperfections**.

$$g_{ij} = \mathrm{sgn}(i-j)\frac{\frac{1}{\beta^m(s_j)}\delta s_j - \frac{1}{\beta^m(s_i)}\delta s_i + \sum_{w\in I}\beta^m_w \delta K_{w,1}\sin^2\phi^m_{wj}}{\sin^2\phi^m_{ij}}$$

# CALCULATION OF THE CORRELATION MATRIX - CONCLUSION

$$T^K_{l\lambda} = \mp \frac{\beta^m(s_i)\beta^m(s_\lambda)}{\cot\phi^m_{ij_l} - \cot\phi^m_{ik_l}} \times \left( \frac{\sin^2\phi_{\lambda j_l}}{\sin^2\phi_{ij_l}} A_{ij_l}(\lambda) - \frac{\sin^2\phi_{\lambda k_l}}{\sin^2\phi_{ik_l}} A_{ik_l}(\lambda) \right)$$

and:

$$T^s_{l\lambda} = -2\alpha^m(s_i)\delta^\lambda_i \pm \frac{\frac{\mathrm{sgn}(i-j_l)}{\sin^2\phi^m_{ij_l}}\left(\frac{\beta^m(s_i)}{\beta^m(s_{j_l})}\delta^{j_l}_\lambda - \delta^i_\lambda\right) - \frac{\mathrm{sgn}(i-k_l)}{\sin^2\phi^m_{ik_l}}\left(\frac{\beta^m(s_i)}{\beta^m(s_{k_l})}\delta^{k_l}_\lambda - \delta^i_\lambda\right)}{\cot\phi^m_{ij_l} - \cot\phi^m_{ik_l}}$$

# REMOVAL OF BAD BPM COMBINATIONS

filter BPM combinations by **phase advances**

**bad**:

$\Delta\phi \in [n\pi - \delta, n\pi + \delta]$ for $n \in \mathbb{N}$, threshold $\delta$.

- enhances **phase errors**
- **numerically unstable**

If any of the four phase advances $\phi_{ij_l}, \phi_{ik_l}, \phi^m_{ij_l}, \phi^m_{ik_l}$ in

$$\beta_l(s_i) = \frac{\cot(\phi_{ij_l}) - \cot(\phi_{ik_l})}{\cot(\phi^m_{ij_l}) - \cot(\phi^m_{ik_l})}$$

is bad, the corresponding BPM combination is **disregarded**.

2016 and 2017 for the LHC: $\delta = 2\pi \times 10^{-2}$.

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**Evaluation of the new method**

# TEST SETUP

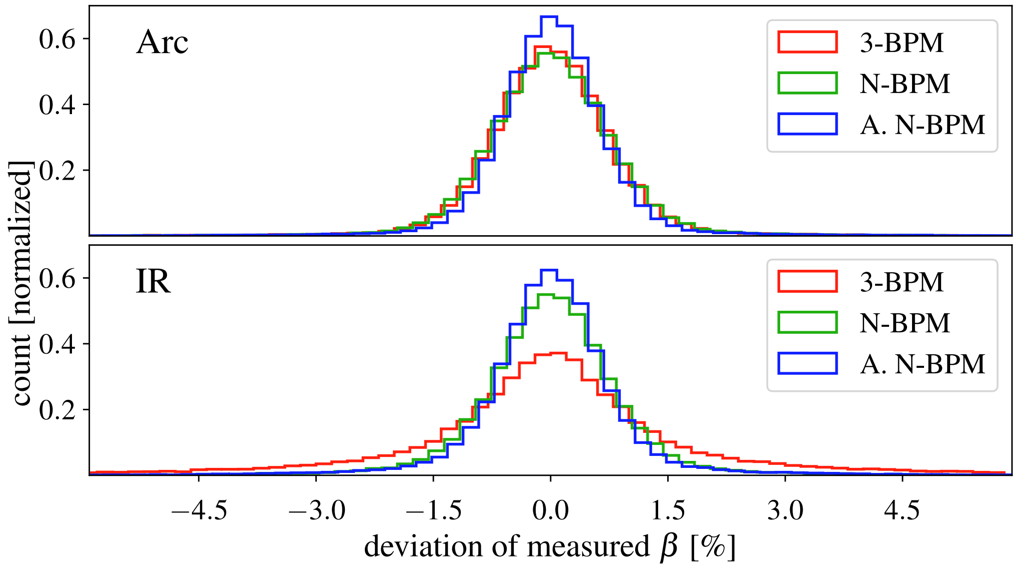
- generate many test lattices with **randomly distributed errors**
- simulate measurement by **tracking** a single particle via PTC
- use the LHC-OMC tools to **analyse** the data

Standard deviation of introduced errors:

| | $\frac{\sigma K}{K}\ 10^{-4}$ | $\sigma s$ [mm] | $\sigma x$ [mm] |
|---|---|---|---|
| MQ | 18 | 1.0 | - |
| MQM | 12 | 1.0 | - |
| MQY | 11 | 1.0 | - |
| MQX | 4 | 1.0 | - |
| MQW | 15 | 1.0 | - |
| MQT | 75 | 1.0 | - |
| MS | - | - | 0.3 |
| BPM | - | 1.0 | - |

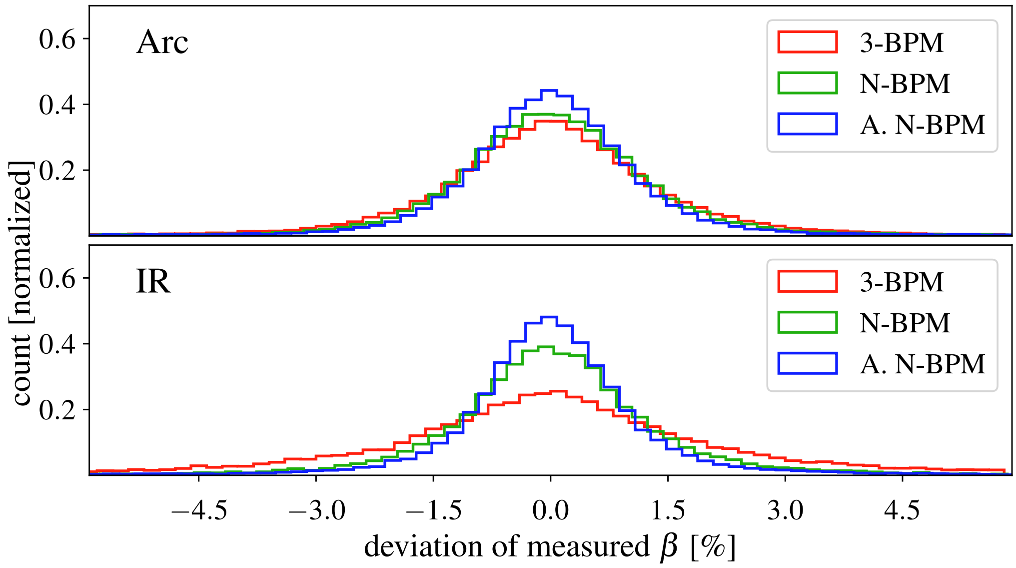
# NOMINAL LATTICE

**40 cm $\beta^*$ | collision tunes | collision energy**



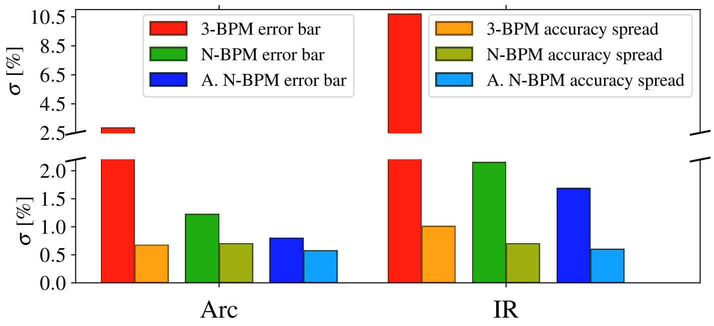
# HL-LHC LATTICE

**10 cm $\beta^*$ | collision tunes | collision energy | ATS optics**

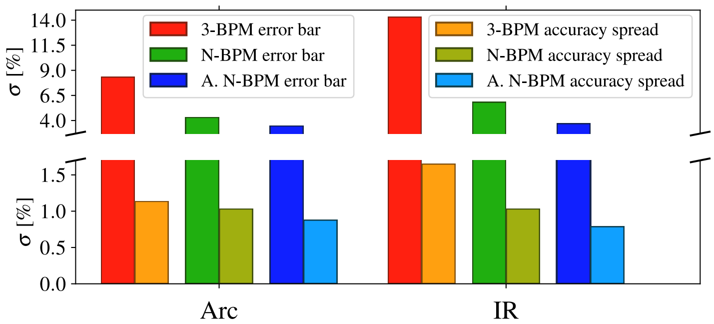


# ACCURACY AND PRECISION OF THE METHODS

**Nominal lattice**




**HL-LHC**



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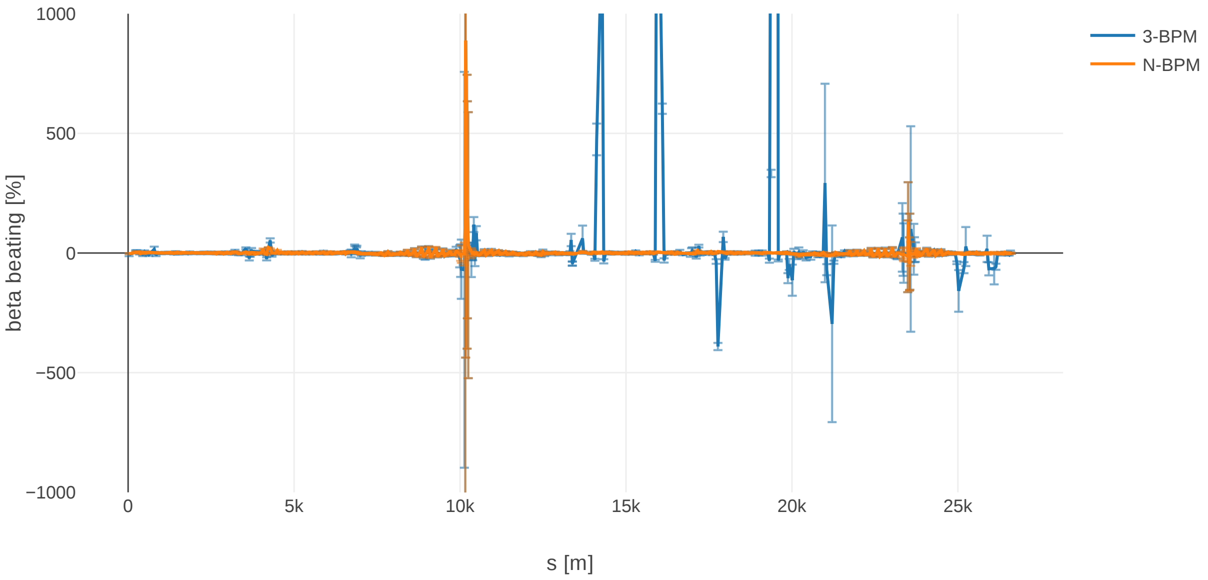
# MEASUREMENTS

**ATS machine development**, **10cm** $\beta^*$

# ATS MD IN OCTOBER 2016

## Beta beating plot at $\beta^* = 10cm$

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# CONCLUSIONS

**new method has been developed fully analytical** calculation of the covariance matrix

- faster
- more accurate
- avoids complication from failing simulations

**successfully used at CERN**:

- **LHC measurements** and **optics correction** (commisioning, machine development)
- first measurements at **PS**
- first measurements at **PS Booster**

**method independent of accelerator and accelerator type**

- Analytical N-BPM method can be used with **any kind** of accelerator
- linear optics analysis toolbox ready for integration of **new accelerators**

# THANK YOU VERY MUCH FOR YOUR ATTENTION