Constraining the SMEFT in the top sector at the LHC

Working document for the TOP LHC WG

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Contents

1 Introduction

The aim of this document is to collect the minimal information necessary for experimental collaborations to constrain dimension-six operators in the SMEFT from top-quark measurements at the LHC. It is organised in two main parts followed by several appendices.

In the first part, a minimal class of operators relevant for top-quark processes is presented together with possible extensions. The basic guiding principles are stated and the operators, organised in three main classes —four-quark, two-quark and two-quark-two-lepton— are explicitly written down following a well-defined naming and normalisation convention. In particular, the question of which non-trivial (flavour) symmetries can be employed to reduce the number of independent operators is addressed. The main production (and decay) channels involving a top quark at the LHC are then reviewed, and the relevant degrees of freedom (i.e. linear combination of operators) that can be probed in principle in each process is identified.

In the second part, recommendations on how to proceed in an actual analysis are provided. Limits set on operators in a series of theoretical studies are also presented in this part to provide a guidance for the experimental analyses.

2 Guiding principles

1. The so-called Warsaw basis of dimension-six operators [1] is adopted (see also Ref. [2, 3] for early specific discussions of top-quark related operators).
2. We wish to identify the linear combination of Warsaw-basis-operator coefficients which can in principle be probed in each process and therefore constitute independent degrees of freedom. Any observable would provide sensitivity to a combination of these.
3. As a starting point, we identify the linear combinations of Warsaw-basis operators that lead to combinations of physical fields (of given flavour and chirality) with distinct Lorentz and colour structures. This is necessary but not sufficient to achieve the above goal. Although physically distinct, the light-quark flavours and chiralities are for instance hardly distinguishable in practice. Further analysis, on an observable by observable basis, is thus needed.
4. To identify the leading effective-field-theory contributions to each top-quark-related process, the following criteria are relied on. The impact of relaxing some of them to obtain wider range of contributions will be studied and discussed. Several conventional *levels* will be defined.

   (a) Only the operators involving at least a top quark are considered. Other operators are assumed to be well constrained from non-top-quark-related processes. This may not be justified in some cases and this assumption may thus then be revisited.

   (b) We use minimal flavour violation, under the assumption of a unit CKM matrix and finite Yukawa couplings only for the top and bottom quarks, i.e. we impose a $U(2)_q \times U(2)_u \times U(2)_d$ flavour symmetry among the first two generations. This effectively reduces the number of four-fermion operator contributions considered. Less restrictive assumptions will be investigated and discussed.

   (c) As a starting point, we base ourselves on a tree-level description, working at zeroth order in the loop expansion. All tree-level contributions are then considered on an equal footing. The only hierarchy established between them is based on available experimental constraints. The dependence of a specific observable on a given effective-field-theory parameter can only be omitted if other measurements constrain that parameter much below the level of sensitivity of this observable. As a result, the inclusion of its dependence should affect neither the pre-existing constraints on that parameter, nor the resulting constraints on others in a combination of this new measurement with existing ones. This approach is very phenomenological and completely agnostic on specific theories. In practice, as further and further constraints are collected and combined a picture will progressively emerge of what specific measurement is particularly relevant to constrain a given direction in parameter space.

   The higher-order observable dependences in standard-model couplings could be considered in a second step. They could either induce corrections to existing tree-level contributions, or generate a dependence in a new effective-field-theory parameter. A discussion of next-to-leading order QCD effect on effective-field-theory predictions is planned.

3 Operator definitions

The definitions of the operators that will be relevant in our discussion here are reminded. Note the negative coupling order —or $\hbar$ dimension— of the Higgs vacuum expectation value is not compensated by any explicit standard-model coupling prefactor.

Four-quark operators:

$$O_{qq}^{1(ijkl)} = (\bar{q}_i \gamma^\mu q_j)(\bar{q}_k \gamma_\mu q_l), \tag{1}$$

$$O_{qq}^{3(ijkl)} = (\bar{q}_i\gamma^\mu\tau^I q_j)(\bar{q}_k\gamma_\mu\tau^I q_l), \tag{2}$$
$$O_{qu}^{1(ijkl)} = (\bar{q}_i\gamma^\mu q_j)(\bar{u}_k\gamma_\mu u_l), \tag{3}$$
$$O_{qu}^{8(ijkl)} = (\bar{q}_i\gamma^\mu T^A q_j)(\bar{u}_k\gamma_\mu T^A u_l), \tag{4}$$
$$O_{qd}^{1(ijkl)} = (\bar{q}_i\gamma^\mu q_j)(\bar{d}_k\gamma_\mu d_l), \tag{5}$$
$$O_{qd}^{8(ijkl)} = (\bar{q}_i\gamma^\mu T^A q_j)(\bar{d}_k\gamma_\mu T^A d_l), \tag{6}$$
$$O_{uu}^{(ijkl)} = (\bar{u}_i\gamma^\mu u_j)(\bar{u}_k\gamma_\mu u_l), \tag{7}$$
$$O_{ud}^{1(ijkl)} = (\bar{u}_i\gamma^\mu u_j)(\bar{d}_k\gamma_\mu d_l), \tag{8}$$
$$O_{ud}^{8(ijkl)} = (\bar{u}_i\gamma^\mu T^A u_j)(\bar{d}_k\gamma_\mu T^A d_l), \tag{9}$$
$$O_{quqd}^{1(ijkl)} = (\bar{q}_i u_j)\,\varepsilon\,(\bar{q}_k d_l), \tag{10}$$
$$O_{quqd}^{8(ijkl)} = (\bar{q}_i T^A u_j)\,\varepsilon\,(\bar{q}_k T^A d_l), \tag{11}$$

Two-quark operators:

$$O_{u\varphi}^{(ij)} = \bar{q}_i u_j \tilde{\varphi}\,(\varphi^\dagger\varphi), \tag{12}$$
$$O_{\varphi q}^{1(ij)} = (\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_i\gamma^\mu q_j), \tag{13}$$
$$O_{\varphi q}^{3(ij)} = (\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}_i\gamma^\mu\tau^I q_j), \tag{14}$$
$$O_{\varphi u}^{(ij)} = (\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_i\gamma^\mu u_j), \tag{15}$$
$$O_{\varphi ud}^{(ij)} = (\tilde{\varphi}^\dagger iD_\mu\varphi)(\bar{u}_i\gamma^\mu d_j), \tag{16}$$
$$O_{uW}^{(ij)} = (\bar{q}_i\sigma^{\mu\nu}\tau^I u_j)\,\tilde{\varphi}W^I_{\mu\nu}, \tag{17}$$
$$O_{dW}^{(33)} = (\bar{q}_i\sigma^{\mu\nu}\tau^I d_j)\,\varphi W^I_{\mu\nu}, \tag{18}$$
$$O_{uB}^{(ij)} = (\bar{q}_i\sigma^{\mu\nu}u_j)\quad\tilde{\varphi}B_{\mu\nu}, \tag{19}$$
$$O_{uG}^{(ij)} = (\bar{q}_i\sigma^{\mu\nu}T^A u_j)\,\tilde{\varphi}G^A_{\mu\nu}, \tag{20}$$

Two-quark-two-lepton operators:

$$O_{lq}^{1(ijkl)} = (\bar{l}_j\gamma^\mu l_j)(\bar{q}_k\gamma^\mu q_l), \tag{21}$$
$$O_{lq}^{3(ijkl)} = (\bar{l}_j\gamma^\mu\tau^I l_j)(\bar{q}_k\gamma^\mu\tau^I q_l), \tag{22}$$
$$O_{lu}^{(ijkl)} = (\bar{l}_j\gamma^\mu l_j)(\bar{u}_k\gamma^\mu u_l), \tag{23}$$
$$O_{eq}^{(ijkl)} = (\bar{e}_j\gamma^\mu e_j)(\bar{q}_k\gamma^\mu q_l), \tag{24}$$
$$O_{eu}^{(ijkl)} = (\bar{e}_j\gamma^\mu e_j)(\bar{u}_k\gamma^\mu u_l), \tag{25}$$
$$O_{lequ}^{1(ijkl)} = (\bar{l}_i e_j)\,\varepsilon\,(\bar{q}_k u_l), \tag{26}$$
$$O_{lequ}^{3(ijkl)} = (\bar{l}_i\sigma^{\mu\nu}e_j)\,\varepsilon\,(\bar{q}_k\sigma_{\mu\nu}u_l), \tag{27}$$
$$O_{ledq}^{(ijkl)} = (\bar{l}_i\sigma^{\mu\nu}e_j)(\bar{d}_k\sigma_{\mu\nu}q_l), \tag{28}$$

Baryon- and lepton-number-violating operators:

$$O_{duq}^{(ijkl)} = (\overline{d^c}_{i\alpha}u_{j\beta})(\overline{q^c}_{k\gamma}\varepsilon l_l)\,\epsilon^{\alpha\beta\gamma}, \tag{29}$$
$$O_{qqu}^{(ijkl)} = (\overline{q^c}_{i\alpha}\varepsilon q_{j\beta})(\overline{u^c}_{k\gamma}e_l)\,\epsilon^{\alpha\beta\gamma}, \tag{30}$$
$$O_{qqq}^{1(ijkl)} = (\overline{q^c}_{i\alpha}\varepsilon q_{j\beta})(\overline{q^c}_{k\gamma}\varepsilon l_l)\,\epsilon^{\alpha\beta\gamma}, \tag{31}$$
$$O_{qqq}^{3(ijkl)} = (\overline{q^c}_{i\alpha}\tau^I\varepsilon q_{j\beta})(\overline{q^c}_{k\gamma}\tau^I\varepsilon l_l)\,\epsilon^{\alpha\beta\gamma}, \tag{32}$$
$$O_{duu}^{(ijkl)} = (\overline{d^c}_{i\alpha}u_{j\beta})(\overline{u^c}_{k\gamma}e_l)\,\epsilon^{\alpha\beta\gamma}. \tag{33}$$

Non Hermitian operators are marked with a dot: $\dot{O}$. The corresponding effective Lagrangian then takes the form:

$$\mathcal{L} = \sum_a \frac{C_a}{\Lambda^2} O_a + \sum_a \left(\frac{C_a}{\Lambda^2} \dot{O}_a + \text{h.c.} \right) \tag{34}$$

where only the Hermitian conjugate of non Hermitian operators are added. Conventionally and unless otherwise specified, the arbitrary scale Λ will be set to 1 TeV. Equivalently, one could consider that numerical values quoted are in units of TeV^{-2} for the dimensionful $\tilde{C}_i \equiv C_i/\Lambda^2$. It is understood that the implicit sum over flavour indices only includes independent combinations, i.e., the symmetry in flavour space of the operator coefficients are taken into account.

4 Flavour assumptions

The minimal flavour violation expansion of quark bilinear coefficients is the following:

$$\bar{q}_i q_j : a_1 \mathbb{I} + a_2 Y_u Y_u^\dagger + a_3 Y_d Y_d^\dagger + \cdots \tag{35}$$

$$\bar{u}_i u_j : b_1 \mathbb{I} + b_2 Y_u^\dagger Y_u + \ldots \tag{36}$$

$$\bar{d}_i d_j : c_1 \mathbb{I} + c_2 Y_d^\dagger Y_d + \ldots \tag{37}$$

$$\bar{u}_i d_j : d_1 Y_u^\dagger Y_d + \ldots \tag{38}$$

$$\bar{q}_i u_j : e_1 Y_u + e_2 Y_u Y_u^\dagger Y_u + e_3 Y_d Y_d^\dagger Y_u + \ldots \tag{39}$$

$$\bar{q}_i d_j : f_1 Y_d + f_2 Y_d Y_d^\dagger Y_d + f_3 Y_u Y_u^\dagger Y_d + \ldots \tag{40}$$

where q, u, d are the left-handed quark doublet and right-handed quark singlet, and a_1, b_1, etc. are order-one coefficients. As a first approximation, we assume unit CKM matrix, and retain only the top and bottom Yukawas, so that $Y_u = \text{diag}(0, 0, y_t)$ and $Y_d = \text{diag}(0, 0, y_b)$. Denoting the left-handed quark doublet and right-handed quark singlets of the third generation as Q, t, and b,

$$\begin{aligned} \bar{q}_i q_i,\ \bar{u}_i u_i,\ \bar{d}_i d_i \quad & \text{are allowed in the first two generations,} \\ \bar{Q}Q,\ \bar{t}t,\ \bar{b}b,\ \bar{t}b,\ \bar{Q}t,\ \bar{Q}b \quad & \text{are allowed in the third generation.} \end{aligned}$$

The coefficient of the first-generation bilinears do not depend on the $i \in \{1, 2\}$ index which is thus implicitly summed over. Simply put, a $U(2)_q \times U(2)_u \times U(2)_d$ symmetry is assumed between the first two quark generations and no restriction is imposed on the third-generation bilinears. This assumption simplifies four-fermion operators but does therefore not affect third-generation two-fermion ones. Compared to flavour diagonality which would just force quarks and antiquarks to appear in same-flavour pairs,

- the right-handed charged current of the first generations ($\bar{u}d$, $\bar{d}u$) are forbidden,
- the chirality flipping bilinears of the first generations ($\bar{q}u$, $\bar{q}d$) are forbidden,
- the coefficients of the bilinears of the first and second generations are forced to be identical.

The above assumptions, i.e. MFV, diagonal CKM, and finite y_t, y_b are assumed throughout the rest of this note. The impact of less restrictive assumptions —like a $U(2)_{q+u+d}$ symmetry, or flavour diagonality— is planned to be examined.

In the lepton sector, it seems manageable to only assume flavour diagonality from the onset. This leaves independent parameters for each lepton-antilepton pair of a given generation.

5 Degrees of freedom and direct constraints

We determine in Appendix B, the linear combinations of Warsaw-basis operator coefficients that give rise, in the broken electroweak phase, to the interactions of physical fields with identical colour and Lorentz structures which are relevant for common top-quark decay and production processes at the LHC. These combinations will be the ones used as independent degrees of freedom and should span the whole effective-field-theory space, under the assumption specified earlier in this document, especially about the category of operators considered and about flavour.

As a reference, we collect here various limits set by theoretical studies on these degrees of freedom. We note here that there is no study considering all the operators listed above and therefore no marginalised constraints are available. Typically the one-operator fit (or the marginalised over a smaller operator subset) constraints are more stringent and therefore these should only serve as a guidance for potential sensitivity or fit studies. Direct limits are shown in Table 1, where available. Indirect limits from low-energy observables also exist, some of which can be very strong yet more model dependent and will be discussed in a dedicated Section.

6 A simple analysis strategy

The simplest strategy to constrain the standard-model effective field theory with measurements in the top sector could rely on fiducial observables defined at, and unfolded to, the particle level. It is meant to be simultaneously practical and useful on a long-term basis, being for instance able to benefit from future improvements in the accuracy of our EFT predictions. It should allow to derive robust global constraints and be applicable in a wide variety of situations —if not all them— but could however be far from optimal to constraints one operator at the time. Our bias is that global constraints have more physical value than individual ones (which does not mean that individual constraints contain no useful information). A tentative *recipe* could be the following:

1. Define observables at the particle level in a fiducial volume.
   - Several bins of a differential distribution would qualify as examples of observables.
   - One could also think about observables that are based on multivariate analysis techniques. The binned MVA output then yields the observables (or one bin in several MVA output, each maximizing sensitivity to different operator coefficients). The trained classifier(s) should then be provided, for instance as C++ code taking as input a particle-level event sample, or kinematic variables.
   - Statistically optimal observables [6–8] could also be very useful from both theoretical and experimental point of views. Their definitions rely on firm theoretical bases, encode our physical understanding instead of requiring a resource-intensive and opaque training. They moreover constitute a discrete set exactly sufficient to maximally exploit the available kinematic information on which the effect of higher-order corrections or systematic uncertainties can be transparently studied. Applications in experimental analyses include anomalous triple gauge coupling studies at LEP [9–12] and, recently, CP properties of the Higgs boson in di-tau final state [13].
2. Unfold the measurement of these observables, as well as the estimates for the various standard-model contributions, to the particle level.
   - For a suitably defined fiducial region, the particle and detector level should be sufficiently close to each other for the unfolding to be performed under the standard-model hypothesis only. Full simulation at various EFT parameter points would then be avoided altogether. Checks of this approximation can certainly be performed in case of doubts.
   - The *standard-model contributions* would include both the *signal* and *backgrounds* of a standard-model measurement. It is important to detail the background composition as these processes could have some EFT dependence which may be neglected at first but which could be desirable to account for at some later point in time.
3. Provide the statistical and systematics covariance matrices (in the Gaussian approximation) for the various standard-model contributions, across the various observables, or some higher-order expression for the likelihood.

Four-heavy		Indicative direct limits	
c^{1}_{QQ}	$\equiv 2C^{1(3333)}_{qq} - \frac{2}{3}C^{3(3333)}_{qq}$		
c^{8}_{QQ}	$\equiv 8C^{3(3333)}_{qq}$		
!c^{+}_{QQ}	$\equiv C^{1(3333)}_{qq} + C^{3(3333)}_{qq}$	[-2.92, 2.80] (E_{cut} =3 TeV) [4]	
c^{1}_{Qt}	$\equiv C^{1(3333)}_{qu}$	[-4.97,4.90] (E_{cut} =3 TeV) [4]	
c^{8}_{Qt}	$\equiv C^{8(3333)}_{qu}$	[-10.3,9.33] (E_{cut} =3 TeV) [4]	
c^{1}_{Qb}	$\equiv C^{1(3333)}_{qd}$		
c^{8}_{Qb}	$\equiv C^{8(3333)}_{qd}$		
c^{1}_{tt}	$\equiv C^{(3333)}_{uu}$	[-2.92,2.80] (E_{cut} =3 TeV) [4]	
c^{1}_{tb}	$\equiv C^{1(3333)}_{ud}$		
c^{8}_{tb}	$\equiv C^{8(3333)}_{ud}$		
Two-light-two-heavy			
$c^{3,1}_{Qq}$	$\equiv C^{3(ii33)}_{qq} + \frac{1}{6}(C^{1(i33i)}_{qq} - C^{3(i33i)}_{qq})$	[-0.66,1.24] [5]	[-3.11,3.10] [4]
$c^{3,8}_{Qq}$	$\equiv C^{1(i33i)}_{qq} - C^{3(i33i)}_{qq}$	[-6.06,6.73] [4]	
$c^{1,1}_{Qq}$	$\equiv C^{1(ii33)}_{qq} + \frac{1}{6}C^{1(i33i)}_{qq} + \frac{1}{2}C^{3(i33i)}_{qq}$	[-3.13,3.15] [4]	
$c^{1,8}_{Qq}$	$\equiv C^{1(i33i)}_{qq} + 3C^{3(i33i)}_{qq}$	[-6.92,4.93] [4]	
c^{1}_{Qu}	$\equiv C^{1(33ii)}_{qu}$	[-3.31,3.44] [4]	
c^{8}_{Qu}	$\equiv C^{8(33ii)}_{qu}$	[-8.13,4.05] [4]	
c^{1}_{Qd}	$\equiv C^{1(33ii)}_{qd}$	[-4.98,5.02] [4]	
c^{8}_{Qd}	$\equiv C^{8(33ii)}_{qd}$	[-11.7,9.39] [4]	
c^{1}_{tq}	$\equiv C^{1(ii33)}_{qu}$	[-2.84,2.84] [4]	
c^{8}_{tq}	$\equiv C^{8(ii33)}_{qu}$	[-6.80,3.49] [4]	
c^{1}_{tu}	$\equiv C^{(ii33)}_{uu} + \frac{1}{3}C^{(i33i)}_{uu}$	[-3.62,3.57] [4]	
c^{8}_{tu}	$\equiv 2C^{(i33i)}_{uu}$	[-8.05,4.75] [4]	
c^{1}_{td}	$\equiv C^{1(33ii)}_{ud}$	[-4.95,5.04] [4]	
c^{8}_{td}	$\equiv C^{8(33ii)}_{ud}$	[-11.8,9.31] [4]	
Two-heavy			
$c^{[I]}_{t\varphi}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(33)}_{u\varphi}\}$		
$c^{1}_{\varphi q}$	$\equiv C^{1(33)}_{\varphi q}$	[-3.1,3.1] [5]	
$c^{3}_{\varphi Q}$	$\equiv C^{3(33)}_{\varphi q}$	[-4.1,2.0] [5]	
$c_{\varphi t}$	$\equiv C^{(33)}_{\varphi u}$	[-9.7,8.3] [5]	
$c^{[I]}_{\varphi tb}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(33)}_{\varphi ud}\}$		
$c^{[I]}_{tW}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(33)}_{uW}\}$	c_{tW} [-4.0,3.5] [5]	
$c^{[I]}_{tB}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(33)}_{uB}\}$	c_{tB} [-6.9,4.6] [5]	
$c^{[I]}_{bW}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(33)}_{dW}\}$		
$c^{[I]}_{tG}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(33)}_{uG}\}$	c_{tG} [-1.32,1.24] [5]	
Two-heavy-two-lepton			
$c^{3(\ell)}_{Ql}$	$\equiv C^{3(\ell\ell 33)}_{lq}$		
$c^{-(\ell)}_{Ql}$	$\equiv C^{1(\ell\ell 33)}_{lq} - C^{3(\ell\ell 33)}_{lq}$		
$c^{(\ell)}_{Qe}$	$\equiv C^{(\ell\ell 33)}_{eq}$		
$c^{(\ell)}_{tl}$	$\equiv C^{(\ell\ell 33)}_{lu}$		
$c^{(\ell)}_{te}$	$\equiv C^{(\ell\ell 33)}_{eu}$		
$c^{S[I](\ell)}_{t}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{1(\ell\ell 33)}_{lequ}\}$		
$c^{T[I](\ell)}_{t}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{3(\ell\ell 33)}_{lequ}\}$		
$c^{S[I](\ell)}_{b}$	$\equiv {}^{[\mathrm{Im}]}_{\mathrm{Re}}\{C^{(\ell\ell 33)}_{ledq}\}$		

Table 1: Indicative direct limits on top operator coefficients for $\Lambda = 1$ TeV. For details on the fit procedure, information on the input data and set of operators over which the results are marginalised please consult the corresponding references.

N.B. The information above should be sufficient for anybody able to generate an EFT (or any NP) sample at the particle level to set constraints. To set global EFT constraints, one could further proceed as follows:

4. Compute, for each observable O^k, the linear S_i and quadratic S_{ij} EFT contributions, in addition to the standard-model contributions B_l already mentioned in item 2:

$$O^k = B_l^k + \frac{C_i}{\Lambda^2} S_i^k + \frac{C_i C_j}{\Lambda^4} S_{ij}^k. \tag{41}$$

   Higher-order terms can be generated when several operator insertions are possible at the amplitude level. In some cases (FCNCs for instance), the interference between standard-model amplitudes and EFT ones could be vanishingly small, leading to $S_i^k \simeq 0$.

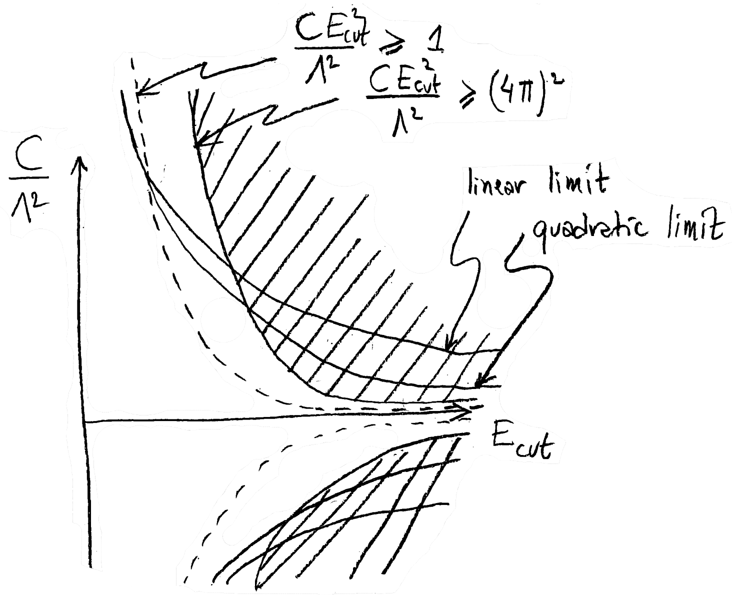
   - Quoting acceptances and efficiencies for each of those terms would be useful.
   - At leading order, with MG, if a different coupling order $\texttt{EFT}_\texttt{i}$ is given to each of the relevant N operator coefficient C_i, this can most easily be done by generating at most $N + N(N+1)/2$ samples with the $\texttt{EFT}_\texttt{i}\texttt{\^{}2==1}$ and $\texttt{EFT}_\texttt{i}\texttt{\^{}2==1 EFT}_\texttt{j}\texttt{\^{}2==1}$ specifications.
   - In principle, it is desirable to include all operators that contribute at some given loop order, say first at the tree level. Exceptions could be made for operators that are much better constrained than the others, at the time at which the limits are extracted. This comparison of the relative strength of different measurements could however evolve with time and the goodness of this approximation be re-evaluated.

5. Within some statistical framework, use the measured O^k, the estimated B_l^k with their statistical and systematic uncertainties, the S_i^k, S_{ij}^k to derive global constraints on the C_i operator coefficients.

   - It is instructive to also quote individual constraints, set by considering one operator at the time. A comparison between global and individual constraints gives some indication about degeneracies between parameters.
   - For easy combination with other measurements, provide the full covariance matrices in the Gaussian approximation (exact when the S_{ij}^k quadratic EFT contributions are dropped). Eigenvectors of V^{-1} with vanishing eigenvalues indicate which directions of the EFT parameter space remain unconstrained by the measured observables. Other eigenvectors and eigenvalues provide the constraints in other directions.
   - Repeat this procedure twice, with and without including the S_{ij}^k quadratic EFT contributions. The comparison between those two set of results would determine the validity of the linear EFT approximation. Remarkably, when it is accurate, the constraints obtained can easily be translated from one basis of dimension-six operators to the other and are therefore of greater generality.

6. Assessing the range of validity of the effective field theory requires information about E the characteristic scale of the process. In a production process at the LHC, the total centre-of-mass energy is often, theoretically, such a suitable quantity. Other proxies may also be considered like H_T, the scalar sum of all transverse energies. Their correlation with the physically relevant variables should then be studied, as done for instance in Ref. [14] (see Fig. 2 and related discussion).

   When it is practically feasible, displaying the variation of the limit as function of an upper cut on E at E_{cut} allows for the interpretation of EFT results in a large class of models [15]. One may consider doing this for the individual limits on each $\frac{C_i}{\Lambda^2}$ or (more usefully maybe) for the individual limits on the eigenvectors of the covariance matrix, obtained without S_{ij}^k quadratic contributions.



- One expect the limits to be progressively degraded as E_{cut} is pushed to lower and lower values. At high cut values, a plateau should be reached.
- Curves obtained for the limits derived with and without S^k_{ij} (one the same linear combination of $\frac{C_i}{\Lambda^2}$) could be compared.
- The EFT perturbativity requires that $\frac{C_i E^2_{\text{cut}}}{(4\pi)^2\Lambda^2}$ be smaller than a constant which would roughly be of order one if the normalization of C_i is *natural* to the observable considered (the exact condition would need to be determined on a case-by-case basis. This defines a hyperbola in the $(\frac{C_i}{\Lambda^2}, E_{\text{cut}})$ plane. All powers of C_i/Λ^2 can however become relevant as soon as $\frac{C_i E^2_{\text{cut}}}{\Lambda^2}$ approaches a constant of order one. If contributions from higher dimensional operators at the same order in $1/\Lambda$ had similar magnitude, they would also become relevant. The relative magnitude of these contributions requires a specific model or power counting to be determined.

  If it exists, the point at which the perturbativity hyperbola crosses limit curves establishes the minimal E_{cut} that has to be imposed for the limit computed perturbatively to make sense. For a sufficiently tight constraint, the limit curves would only cross the perturbativity hyperbola for a cut value beyond the maximal energy directly accessible in the process considered.

  If it exists, the point at which the $\frac{C_i E_{\text{cut}}}{\Lambda^2} \lesssim 1$ hyperbola crosses the limit curve provides a sense of where higher-order terms in the EFT expansion could become relevant. (One may expect the linear and quadratic limits to start diverging around that point.) When discussing the relevance of higher-order terms, more definite statements are difficult to make, as is always the case when trying to estimate the terms not computed in a truncated series. The first term could for instance have an accidental (or understood) suppression as it generally arises from the interference of a standard-model amplitude with a EFT one (see for instance Ref. [16]).

  When interpreting the EFT constraints in terms of specific models with some combination of couplings and a characteristic scale M (which is often identified with Λ) like the mass of a new particle, the validity of the EFT description roughly requires that $M > E_{\text{cut}}$. Stricter constraints may apply for instance if the particle of mass M has a large width.

A UFO Model documentation

In the current version, the following operators are included:

Two heavy fermion:

Operator	Name in param_card	Note
$c_{t\varphi}^{[I]}$	`ctp,ctpI`	"I"=imaginary part
$c_{\varphi Q}^{-}$	`cpQM`	
$c_{\varphi Q}^{3}$	`cpQ3`	
$c_{\varphi t}$	`cpt`	
$c_{\varphi b}$	`cpb`	Not in the table
$c_{\varphi tb}^{[I]}$	`cptb,cptbI`	
$c_{tW}^{[I]}$	`ctW,ctWI`	
$c_{tZ}^{[I]}$	`ctZ,ctZI`	
$c_{bW}^{[I]}$	`cbW,cbWI`	
$c_{tG}^{[I]}$	`ctG,ctGI`	

Two-heavy-two-lepton:

Operator	Name in param_card	Note
$c_{Ql}^{3(\ell)}$	`cQl3(l)`	Assuming flavor diagonality in the lepton sector
$c_{Ql}^{-(\ell)}$	`cQlM(l)`	(l)=1,2,3 = lepton flavor
$c_{Qe}^{(\ell)}$	`cQe(l)`	
$c_{tl}^{(\ell)}$	`ctl(l)`	
$c_{te}^{(\ell)}$	`cte(l)`	
$c_{t}^{S[I](\ell)}$	`ctlS[I](l)`	"I"=imaginary part
$c_{t}^{T[I](\ell)}$	`ctlT[I](l)`	
$c_{b}^{S[I](\ell)}$	`cblS[I](l)`	

Two-heavy-two-light:

Operator	Name in param_card	Note
$c_{Qq}^{3,1}$	`cQq13`	U(2) for u, d, q assumed
$c_{Qq}^{3,8}$	`cQq83`	i.e. flavor universal for light quarks
$c_{Qq}^{1,1}$	`cQq11`	
$c_{Qq}^{1,8}$	`cQq81`	
c_{Qu}^{1}	`cQu1`	
c_{Qu}^{8}	`cQu8`	
c_{Qd}^{1}	`cQd1`	
c_{Qd}^{8}	`cQd8`	
c_{tq}^{1}	`ctq1`	
c_{tq}^{8}	`ctq8`	
c_{tu}^{1}	`ctu1`	
c_{tu}^{8}	`ctu8`	
c_{td}^{1}	`ctd1`	
c_{td}^{8}	`ctd8`	

Four heavy fermion:

Operator	Name in param_card	Note
c^1_{QQ}	cQQ1	
c^8_{QQ}	cQQ8	
c^1_{Qt}	cQt1	
c^8_{Qt}	cQt8	
c^1_{Qb}	cQb1	
c^8_{Qb}	cQb8	
c^1_{tt}	ctt1	
c^1_{tb}	ctb1	
c^8_{tb}	ctb8	

A.1 Some general comments:

- Λ is conventionally fixed to 1 TeV. Equivalently, effective field theory input parameters can be thought about being the dimensionful $\tilde{c}_i \equiv c_i/\Lambda^2$ expressed in units of TeV^{-2}.
- Coupling order: each operator coefficient is assigned with coupling order "DIM6=1" only. This means the QED order may not be natural. For example, the ttg vertex from O_{tG} has QED order -1, that of the Higgs vev.
- CKM matrix is assumed to be unit.
- Masses of u, d, s, c, e, μ are zero.
- Goldstones are removed.
- Only the Hermitian conjugation of the non-hermitian operators are added.
- In $O_{t\varphi}$, the $\varphi^\dagger\varphi - v^2/2$ replaces the $\varphi^\dagger\varphi$. This is simply to avoid redefining the SM top quark mass and Yukawa.
- This is a LO model.

A.2 Syntax

In MadGraph5_aMC@NLO, import the model by

```
> import model dim6top_LO_UFO
```

To use five-flavour scheme:

```
> define p = p b b~
> define j = p
```

Processes can be generated by

```
> generate [process] DIM6=[n]
```

e.g.

```
> generate p p > t t~ DIM6=1
```

where [n] is the number of operator insertions allowed in one diagram. To generate only the leading diagrams in QCD, one could use:

```
> generate p p > [process] / a z h w+ DIM6=[n]
```

(for $t\bar{t}$, $t\bar{t}t\bar{t}$, $t\bar{t}Z$, $t\bar{t}\gamma$, $t\bar{t}H$ and $t\bar{t}W$).

Table 2: Linear and quadratic dependence on effective-field-theory parameters of some total cross section.

A.3 Benchmark results

We provide in Table 2 some numerical values for the linear and quadratic dependences of total cross sections on effective-field-theory parameters. They are meant to allow for the validation of each individual simulation setup. To be completed.

B Degrees of freedom for processes conserving flavour, baryon and lepton numbers

In this appendix, we examine the linear combinations of Warsaw-basis operator coefficients that give rise, in the broken electroweak phase, to the interactions of physical fields with identical colour and Lorentz structures which are relevant for common top-quark decay and production processes at the LHC. These combinations will be the ones used as independent degrees of freedom and should span the whole effective-field-theory space (even though each subsection only examines the dominant QCD contributions to a specific process), under the assumption specified earlier in this document, especially about the category of operators considered and about flavour.

B.1 Single top production & hadronic top decay

We consider first the Warsaw operator combinations which contribute to the $qb \to q't$ or $t \to bqq'$ processes.

B.1.1 Two-heavy-two-light contributions

In the Warsaw basis, under the above MFV assumption, two operators contribute to the single top process: $O_{qq}^{1(ijkl)}$ and $O_{qq}^{3(ijkl)}$. The operator coefficients have the following symmetries under permutations of their generation indices:

$$C^{(ijkl)} = C^{(klij)}, \qquad C^{(jilk)} = C^{(ijkl)*}, \qquad (\text{and thus } C^{(lkji)} = C^{(ijkl)*}),$$

which namely implies that $C^{(iijj)}$ and $C^{(ijji)}$ elements are real. From the $C^{(ijkl)}$ and $C^{(klij)}$ combinations, only one is thus retained in the sum over flavour indices in the effective-field-theory Lagrangian of Eq. (34). For each of these two operators, there are thus two independent assignments of third-generation indices to a quark-antiquark pairs:

$$C_{qq}^{1(ii33)} = C_{qq}^{1(ii33)*} = C_{qq}^{1(33ii)} = C_{qq}^{1(33ii)*}, \qquad C_{qq}^{1(i33i)} = C_{qq}^{1(i33i)*} = C_{qq}^{1(3ii3)} = C_{qq}^{1(3ii3)*}, \tag{42}$$

$$C_{qq}^{3(ii33)} = C_{qq}^{3(ii33)*} = C_{qq}^{3(33ii)} = C_{qq}^{3(33ii)*}, \qquad C_{qq}^{3(i33i)} = C_{qq}^{3(i33i)*} = C_{qq}^{3(3ii3)} = C_{qq}^{3(3ii3)*}. \tag{43}$$

To understand better the structure of their interferences with the standard-model amplitude it is useful to decompose them, using Fierz identities, onto quadrilinears featuring a heavy- and light-quark bilinear:

$$\begin{pmatrix} O_{qq}^{1(ii33)} \\ O_{qq}^{1(i33i)} \\ O_{qq}^{3(ii33)} \\ O_{qq}^{3(i33i)} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/6 & 1/6 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1/2 & -1/6 & 3 & -1 \end{pmatrix} \begin{pmatrix} (\bar{q}_i\gamma^\mu q_i)\,(\bar{Q}\gamma_\mu Q) \\ (\bar{q}_i\gamma^\mu \tau^I q_i)\,(\bar{Q}\gamma_\mu \tau^I Q) \\ (\bar{q}_i\gamma^\mu T^A q_i)\,(\bar{Q}\gamma_\mu T^A Q) \\ (\bar{q}_i\gamma^\mu \tau^I T^A q_i)\,(\bar{Q}\gamma_\mu \tau^I T^A Q) \end{pmatrix}. \tag{44}$$

Among those structures, only the $SU(2)$-triplet–colour-singlet one interferes with the standard-model amplitude, while the $SU(2)$-triplet–colour-octet one also gives an order-$1/\Lambda^4$ contribution to single top production. Therefore, the combinations of Warsaw operator coefficients that lead to contribution at the $1/\Lambda^2$ and $1/\Lambda^4$ order are respectively:

$$c_{Qq}^{3,1} \equiv C_{qq}^{3(ii33)} + \frac{1}{6}\left(C_{qq}^{1(i33i)} - C_{qq}^{3(i33i)}\right), \tag{45}$$

$$c_{Qq}^{3,8} \equiv C_{qq}^{1(i33i)} - C_{qq}^{3(i33i)}, \tag{46}$$

for both $i = 1$ and 2. Under our assumptions about flavour, those are the only four-quark degrees of freedom accessible in single top production.

B.1.2 Two-heavy contributions

The two-quark operator relevant for single top production are the following: $O_{\varphi q}^{3(ij)}$, $O_{\varphi ud}^{(ij)}$, $O_{uW}^{(ij)}$, $O_{dW}^{(ij)}$. The $C_{ij}^{3(\varphi q)*} = C_{ji}^{3(\varphi q)}$ coefficients are real. On the other hand, we distinguish the real and imaginary part of the three other operators coefficients:

$$c_{\varphi Q}^{3} \equiv C_{\varphi q}^{3(33)}, \tag{47}$$

$$c_{\varphi tb} \equiv \mathrm{Re}\{C_{\varphi ud}^{(33)}\}, \qquad c_{tW} \equiv \mathrm{Re}\{C_{uW}^{(33)}\}, \qquad c_{bW} \equiv \mathrm{Re}\{C_{dW}^{(33)}\}, \tag{48}$$

$$c_{\varphi tb}^{I} \equiv \mathrm{Im}\{C_{\varphi ud}^{(33)}\}, \qquad c_{tW}^{I} \equiv \mathrm{Im}\{C_{uW}^{(33)}\}, \qquad c_{bW}^{I} \equiv \mathrm{Im}\{C_{dW}^{(33)}\}. \tag{49}$$

They all contribute at the to order-$1/\Lambda^2$ interferences with standard-model amplitudes, that $c_{\varphi\varphi}$ and c_{bW} are however m_b suppressed.

B.2 Top pair production

We distinguish the contributions of four-quark operators with four heavy fields, with two heavy fields and two light ones, from that of two-quark operators featuring two heavy fields.

B.2.1 Four-heavy contributions

Four-quark operators containing two top and to bottom quarks provide a PDF-suppressed contribution to top pair production in the five-flavour scheme (or a NLO contribution in the four-flavour scheme). The relevant operators of the Warsaw basis are: $O_{qq}^{1(ijkl)}$, $O_{qq}^{3(ijkl)}$, $O_{ud}^{1(ijkl)}$, $O_{ud}^{8(ijkl)}$, $O_{qd}^{1(ijkl)}$, $O_{qd}^{1(ijkl)}$, $O_{qu}^{1(ijkl)}$, $O_{qu}^{1(ijkl)}$. All the corresponding $C^{(3333)} = C^{(3333)*}$ are real. Decomposing the quark doublet into its components and using Fierz identities, one obtains:

$$\begin{pmatrix} O_{qq}^{1(3333)} \\ O_{qq}^{3(3333)} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 1 & -2/3 & 8 & 1 \end{pmatrix} \begin{pmatrix} (\bar{t}\gamma^\mu P_L t)(\bar{t}\gamma_\mu P_L t) \\ (\bar{t}\gamma^\mu P_L t)(\bar{b}\gamma_\mu P_L b) \\ (\bar{t}\gamma^\mu T^A P_L t)(\bar{b}\gamma_\mu T^A P_L b) \\ (\bar{b}\gamma^\mu P_L b)(\bar{b}\gamma_\mu P_L b) \end{pmatrix} \tag{50}$$

Only the colour octet linear combinations interfere with the $b\bar{b} \to t\,\bar{t}$ QCD amplitude at order $1/\Lambda^2$:

$$c_{QQ}^{8} \equiv 8C_{qq}^{3(3333)}, \qquad c_{tb}^{8} \equiv C_{ud}^{8(3333)}, \qquad c_{Qb}^{8} \equiv C_{qd}^{8(3333)}, \qquad c_{Qt}^{8} \equiv C_{qu}^{8(3333)}, \tag{51}$$

while the colour singlet contributions are of order $1/\Lambda^4$:

$$c_{QQ}^{1} = 2C_{qq}^{1(3333)} - \frac{2}{3}C_{qq}^{3(3333)}, \qquad c_{tb}^{1} = C_{ud}^{1(3333)}, \qquad c_{Qb}^{1} = C_{qd}^{1(3333)}, \qquad c_{Qt}^{1} = C_{qu}^{1(3333)}. \tag{52}$$

B.2.2 Two-heavy-two-light contributions

Out of the seven contribution interfering with the QCD amplitude, only four linear combinations enter in the total unpolarized $p\,p \to t\,\bar{t}$ cross section [3]:

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} c_{Qq}^{3,8} \\ c_{Qq}^{1,8} \\ c_{Qu}^{8} \\ c_{Qd}^{8} \\ c_{tq}^{8} \\ c_{td}^{8} \\ c_{tu}^{8} \end{pmatrix} . \tag{53}$$

B.2.3 Four-quark $\bar{L}L\bar{L}L$ operators

The four-quark $\bar{L}L\bar{L}L$ operators that contribute to top pair production at hadron colliders are the same as the ones involved in single top production. Referring ourselves to Eq. (44), the colour octet combinations will interfere with the QCD $q\bar{q} \to t\,\bar{t}$ amplitude:

$$c_{Qq}^{1,8} \equiv C_{qq}^{1(i33i)} + 3C_{qq}^{3(i33i)}, \tag{54}$$

$$c_{Qq}^{3,8} \equiv C_{qq}^{1(i33i)} - C_{qq}^{3(i33i)}, \tag{55}$$

for both $i = 1$ and 2, while the colour singlet linear combinations contribute at the $1/\Lambda^4$ order:

$$c_{Qq}^{1,1} \equiv C_{qq}^{1(ii33)} + \frac{1}{6}C_{qq}^{1(i33i)} + \frac{1}{2}C_{qq}^{3(i33i)}, \tag{56}$$

$$c_{Qq}^{3,1} \equiv C_{qq}^{3(ii33)} + \frac{1}{6}\left(C_{qq}^{1(i33i)} - C_{qq}^{3(i33i)}\right), \tag{57}$$

for both $i = 1$ and 2. Note that the sum and difference of the $SU(2)$ singlet and triplet linear combinations of operators respectively contribute to the $u_i\bar{u}_i \to t\,\bar{t}$ and $d_i\bar{d}_i \to t\,\bar{t}$ subprocesses. Incidentally, decomposing further the quark doublets of Eq. (44) into their up and down components, one obtains:

$$\begin{pmatrix} O_{qq}^{1(ii33)} \\ O_{qq}^{1(i33i)} \\ O_{qq}^{3(ii33)} \\ O_{qq}^{3(i33i)} \end{pmatrix} = \begin{pmatrix} 1 & 1/3 & 1 & 1/3 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & -1 & 2/3 \\ 0 & 0 & 0 & 4 \\ 0 & 1/3 & 2 & -1/3 \\ 0 & 2 & 0 & -2 \\ 0 & 1/3 & 2 & -1/3 \\ 0 & 2 & 0 & -2 \\ 1 & 0 & -1 & 2/3 \\ 0 & 0 & 0 & 4 \\ 1 & 1/3 & 1 & 1/3 \\ 0 & 2 & 0 & 2 \end{pmatrix}^T \begin{pmatrix} (\bar{t}\gamma_\mu \quad P_L t)\,(\bar{u}_i\gamma^\mu \quad P_L u_i) \\ (\bar{t}\gamma_\mu\, T^A P_L t)\,(\bar{u}_i\gamma^\mu\, T^A P_L u_i) \\ (\bar{t}\gamma_\mu \quad P_L t)\,(\bar{d}_i\gamma^\mu \quad P_L d_i) \\ (\bar{t}\gamma_\mu\, T^A P_L t)\,(\bar{d}_i\gamma^\mu\, T^A P_L d_i) \\ (\bar{t}\gamma_\mu \quad P_L b)\,(\bar{d}_i\gamma^\mu \quad P_L u_i) \\ (\bar{t}\gamma_\mu\, T^A P_L b)\,(\bar{d}_i\gamma^\mu\, T^A P_L u_i) \\ (\bar{b}\gamma_\mu \quad P_L t)\,(\bar{u}_i\gamma^\mu \quad P_L d_i) \\ (\bar{b}\gamma_\mu\, T^A P_L t)\,(\bar{u}_i\gamma^\mu\, T^A P_L d_i) \\ (\bar{b}\gamma_\mu \quad P_L b)\,(\bar{u}_i\gamma^\mu \quad P_L u_i) \\ (\bar{b}\gamma_\mu\, T^A P_L b)\,(\bar{u}_i\gamma^\mu\, T^A P_L u_i) \\ (\bar{b}\gamma_\mu \quad P_L b)\,(\bar{d}_i\gamma^\mu \quad P_L d_i) \\ (\bar{b}\gamma_\mu\, T^A P_L b)\,(\bar{d}_i\gamma^\mu\, T^A P_L d_i) \end{pmatrix} . \tag{58}$$

B.2.4 Four-quark $\bar{R}R\bar{R}R$ operators

There are three four-quark operators of $\bar{R}R\bar{R}R$ type in the Warsaw basis: $O_{uu}^{(ijkl)}$, $O_{ud}^{1(ijkl)}$, $O_{ud}^{8(ijkl)}$ and the following independent assignments of third generation indices to $u\bar{u}$ pairs:

$$C_{uu}^{(ii33)} = C_{uu}^{(ii33)*} = C_{uu}^{(33ii)} = C_{uu}^{(33ii)*}, \quad \text{or} \quad C_{uu}^{(i33i)} = C_{uu}^{(i33i)*} = C_{uu}^{(3ii3)} = C_{uu}^{(3ii3)*}, \tag{59}$$

$$C_{ud}^{1(33ii)} = C_{ud}^{1(33ii)*}, \tag{60}$$

$$C_{ud}^{8(33ii)} = C_{ud}^{8(33ii)\,*}. \tag{61}$$

As in the $\bar{L}L\bar{L}L$ case, one can use Fierz identities to decompose the O_{uu}^{1} operators in terms of quadrilinears featuring either a colour octet or a colour singlet top bilinear:

$$\begin{pmatrix} O_{uu}^{(ii33)} \\ O_{uu}^{(i33i)} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1/3 & 2 \end{pmatrix} \begin{pmatrix} (\bar{u}_i\gamma^\mu u_i)\,(\bar{t}\gamma_\mu t) \\ (\bar{u}_i\gamma^\mu T^A u_i)\,(\bar{t}\gamma_\mu T^A t) \end{pmatrix}. \tag{62}$$

The colour octet combinations of Warsaw operator coefficients interfere with the $u\bar{u} \to t\,\bar{t}$ and $d\bar{d} \to t\,\bar{t}$ QCD amplitude at the $1/\Lambda^2$ order:

$$c_{tu}^{8} \equiv 2C_{uu}^{(i33i)}, \qquad c_{td}^{8} \equiv C_{ud}^{8(33ii)}, \tag{63}$$

while the colour singlet combinations only contribute at $1/\Lambda^4$:

$$c_{tu}^{1} \equiv C_{uu}^{(ii33)} + \frac{1}{3}C_{uu}^{(i33i)}, \qquad c_{td}^{1} \equiv C_{ud}^{1(33ii)}, \tag{64}$$

for both $i = 1$ and 2.

B.2.5 Four-quark $\bar{L}L\bar{R}R$ operators

There are four four-quark Warsaw-basis operators of $\bar{L}L\bar{R}R$ type: $O_{qu}^{1(ijkl)}$, $O_{qu}^{8(ijkl)}$, $O_{qd}^{1(ijkl)}$, $O_{qd}^{8(ijkl)}$ and six degrees of freedom relevant for top pair production:

$$C_{qu}^{1(ii33)} = C_{qu}^{1(ii33)\,*}, \qquad C_{qu}^{8(ii33)} = C_{qu}^{8(ii33)\,*}, \tag{65}$$

$$C_{qu}^{1(33ii)} = C_{qu}^{1(33ii)\,*}, \qquad C_{qu}^{8(33ii)} = C_{qu}^{8(33ii)\,*}, \tag{66}$$

$$C_{qd}^{1(33ii)} = C_{qd}^{1(33ii)\,*}, \qquad C_{qd}^{8(33ii)} = C_{qd}^{8(33ii)\,*}. \tag{67}$$

The colour octets interfere with the QCD $q\bar{q} \to t\,\bar{t}$ amplitude:

$$c_{tq}^{8} \equiv C_{qu}^{8(ii33)}, \qquad c_{Qu}^{8} \equiv C_{qu}^{8(33ii)}, \qquad c_{Qd}^{8} \equiv C_{qd}^{8(33ii)}, \tag{68}$$

while the colour singlets only contribute at the $1/\Lambda^4$ order:

$$c_{tq}^{1} \equiv C_{qu}^{1(ii33)}, \qquad c_{Qu}^{1} \equiv C_{qu}^{1(33ii)}, \qquad c_{Qd}^{1} \equiv C_{qd}^{1(33ii)}. \tag{69}$$

B.2.6 Two-heavy contributions

Only one two-quark operator contributes to top pair production and interferes at the $1/\Lambda^2$ order with the $gg \to t\,\bar{t}$ QCD amplitude: $O_{uG}^{(ij)}$ and we define for convenience $c_{tG} \equiv \mathrm{Re}\{C_{uG}^{(33)}\}$ and $c_{tG}^{I} \equiv \mathrm{Im}\{C_{uG}^{(33)}\}$.

B.3 Four top production

B.3.1 Four-heavy contributions

The following four-quark operators of the Warsaw basis can potentially contribute to four top production (and also enter top pair production at NLO, when a top quark loop is closed): $O_{qq}^{1(ijkl)}$, $O_{qq}^{3(ijkl)}$, $O_{uu}^{(ijkl)}$, $O_{qu}^{1(ijkl)}$, $O_{qu}^{8(ijkl)}$ with five real $C^{(3333)} = C^{(3333)\,*}$ coefficients. Given that only the $C_{qq}^{1(3333)} + C_{qq}^{1(3333)}$ linear combination gives rise to four-top contributions (see Eq. (50)), there are however only four independent degrees of freedom contributing to four top production:

$$c_{QQ}^{+} \equiv C_{qq}^{1(3333)} + C_{qq}^{3(3333)} = \frac{1}{2}c_{QQ}^{1} + \frac{1}{6}c_{QQ}^{8}, \qquad c_{tt}^{1} \equiv C_{uu}^{(3333)}, \tag{70}$$

$$c_{Qt}^{1} \equiv C_{qu}^{1(3333)}, \qquad c_{Qt}^{8} \equiv C_{qu}^{8(3333)}. \tag{71}$$

B.3.2 Two-heavy-two-light contributions

All four-quark operators yielding two-heavy-quark two-light-quark contributions to top pair production also contribute to four top production.

B.3.3 Two-heavy contributions

All two-quark operators of the Warsaw basis contributing to top pair production also contribute to four top production.

B.4 Top pair production in association with a Z boson or a photon

B.4.1 Four-heavy and two-heavy-light-quark contributions

All the operators giving four-heavy-quark and two-heavy-quark two-light-quark contributions to top pair production also enter in $pp \to t\bar{t}Z/\gamma$.

B.4.2 Two-heavy contributions

The following two-quark operators are relevant for $pp \to t\bar{t}Z$ and $pp \to t\,\bar{t}\gamma$: $O_{\varphi q}^{3(ij)}$, $O_{\varphi q}^{1(ij)}$, $O_{\varphi u}^{(ij)}$, $O_{uW}^{(ij)}$, $O_{uB}^{(ij)}$, $O_{uG}^{(ij)}$. Given the decomposition (in the unitary gauge)

$$\begin{pmatrix} O_{\varphi q}^{1(33)} \\ O_{\varphi q}^{3(33)} \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{+e}{2s_W c_W} & (\bar{t}\gamma^\mu P_L t) & Z_\mu & (v+h)^2 \\ \frac{-e}{2s_W c_W} & (\bar{b}\gamma^\mu P_L b) & Z_\mu & (v+h)^2 \\ \frac{g}{\sqrt{2}} & (\bar{t}\gamma^\mu P_L b) & W_\mu^+ & (v+h)^2 \\ \frac{g}{\sqrt{2}} & (\bar{b}\gamma^\mu P_L t) & W_\mu^- & (v+h)^2 \end{pmatrix}, \tag{72}$$

the linear combination of Warsaw-basis operators that can be probed in $pp \to t\,\bar{t}Z$ are:

$$\begin{aligned} c_{\varphi Q}^- &\equiv C_{\varphi q}^{1(33)} - C_{\varphi q}^{3(33)}, && (73)\\ c_{\varphi t} &\equiv C_{\varphi u}^{(33)}, && (74)\\ c_{tZ} &\equiv \mathrm{Re}\{\frac{c_W}{s_W} C_{uW}^{(33)} - \frac{s_W}{c_W} C_{uB}^{(33)}\} && (75)\\ &= \frac{c_W}{s_W} c_{tW} - \frac{s_W}{c_W} c_{tB}, && (76)\\ c_{tZ}^I &\equiv \mathrm{Im}\{\frac{c_W}{s_W} C_{uW}^{(33)} - \frac{s_W}{c_W} C_{uB}^{(33)}\} && (77)\\ &= \frac{c_W}{s_W} c_{tW}^I - \frac{s_W}{c_W} c_{tB}^I, && (78)\\ c_{tG} &\equiv \mathrm{Re}\{C_{uG}^{(33)}\}, && (79)\\ c_{tG}^I &\equiv \mathrm{Im}\{C_{uG}^{(33)}\}. && (80) \end{aligned}$$

where the sine and cosine of the weak mixing angle are abbreviated as $s_W \equiv \sin\theta_W$ and $c_W \equiv \cos\theta_W$. The linear combinations of Warsaw basis operators that enter in $pp \to t\,\bar{t}\gamma$ are:

$$\begin{aligned} c_{tA} &\equiv \mathrm{Re}\{s_W C_{uW}^{(33)} + c_W C_{uB}^{(33)}\} && (81)\\ &= c_{tW} + C_{tB}, && (82)\\ c_{tA}^I &\equiv \mathrm{Im}\{s_W C_{uW}^{(33)} + c_W C_{uB}^{(33)}\} && (83)\\ &= c_{tW}^I + C_{tB}^I, && (84)\\ c_{tG} &\equiv \mathrm{Re}\{C_{uG}^{(33)}\}, && (85)\\ c_{tG}^I &\equiv \mathrm{Im}\{C_{uG}^{(33)}\}. && (86) \end{aligned}$$

For both processes all combinations appear in interferences with the standard-model amplitude at the $1/\Lambda^2$ order. The combinations modifying the interactions of the bottom quarks: $c^{+}_{\varphi q} \equiv C^{1(33)}_{\varphi q} + C^{3(33)}_{\varphi q}$, $c_{\varphi b} \equiv C^{(33)}_{\varphi d}$, $c_{bZ} \equiv c_W C^{(33)}_{dW} - s_W C^{(33)}_{dB}$, $c_{bA} \equiv s_W C^{(33)}_{dW} + c_W C^{(33)}_{dB}$, and $c_{bG} \equiv C^{(33)}_{dG}$ also enter in the five-flavour scheme but are suppressed by the bottom PDF.

B.4.3 Two-heavy-two-lepton contributions

When the Z produced in associations with tops is decayed leptonically, operators giving rise to the interaction of two tops with two charged leptons also contribute to the full $pp \to t\,\bar{t}\,\ell^+\ell^-$ final state. They are $O^{1(ijkl)}_{lq}$, $O^{3(ijkl)}_{lq}$, $O^{(ijkl)}_{lu}$, $O^{(ijkl)}_{eq}$, and $O^{(ijkl)}_{eu}$. From the decomposition

$$\begin{pmatrix} O^{1(\ell\ell 33)}_{lq} \\ O^{3(\ell\ell 33)}_{lq} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} (\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)(\bar{t}\gamma_\mu P_L t) \\ (\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)(\bar{b}\gamma_\mu P_L b) \\ (\bar{\ell} \gamma^\mu P_L \ell)(\bar{t}\gamma_\mu P_L t) \\ (\bar{\nu}_\ell \gamma^\mu P_L \nu_\ell)(\bar{b}\gamma_\mu P_L b) \\ (\bar{\nu}_\ell \gamma^\mu P_L \ell)(\bar{b}\gamma_\mu P_L t) \\ (\bar{\ell} \gamma^\mu P_L \nu_\ell)(\bar{t}\gamma_\mu P_L b) \end{pmatrix}, \tag{87}$$

it is however seen that only the $C^{1(\ell\ell 33)}_{lq} - C^{1(\ell\ell 33)}_{lq}$ combination gives rise to an interaction of two tops with two charged leptons. The relevant linear combinations of Warsaw-basis operator coefficients are thus:

$$c^{-(\ell)}_{Ql} \equiv C^{1(\ell\ell 33)}_{lq} - C^{1(\ell\ell 33)}_{lq}, \qquad c^{(\ell)}_{tl} \equiv C^{(\ell\ell 33)}_{lu}, \qquad c^{(\ell)}_{Qe} \equiv C^{(\ell\ell 33)}_{eq}, \qquad c^{(\ell)}_{te} \equiv C^{(\ell\ell 33)}_{eu}, \tag{88}$$

where ℓ stands for the lepton flavour.

B.5 Top pair production in association with a Higgs boson

B.5.1 Four-heavy and two-heavy-two-light contributions

All operators contributing to top pair production contribute also to $pp \to t\bar{t}H$.

B.5.2 Two-heavy contributions

The two contributing operators are the chromomagnetic operator and the Yukawa operator: $O^{(ij)}_{uG}$, $O^{(ij)}_{u\varphi}$. The relevant linear combinations of Warsaw-basis operators are thus:

$$c_{tG} \equiv \mathrm{Re}\{C^{(33)}_{uG}\}, \qquad c^{I}_{tG} \equiv \mathrm{Im}\{C^{(33)}_{uG}\}, \qquad c_{t\varphi} \equiv \mathrm{Re}\{C^{(33)}_{u\varphi}\}, \qquad c^{I}_{t\varphi} \equiv \mathrm{Im}\{C^{(33)}_{u\varphi}\}. \tag{89}$$

We note here that the $O_{\varphi G} \equiv G^{A}_{\mu\nu} G^{\mu\nu A} (\varphi^\dagger \varphi)$ operator which does not contain top quarks also contributes to this process.

B.6 Top pair production in association with a W-boson

B.6.1 Four-heavy contributions

As in top pair production, four-heavy-quark operators only give PDF suppressed contribution in the five flavour scheme (and NLO contributions in the four-flavour scheme) to top production in association with a W boson. The relevant set is then identical to the one discussed for top pair production.

B.6.2 Two-heavy-two-light contributions

All the two-heavy-two-light linear combinations contributing to single top production also enter $pp \to t\bar{t}W^\pm$ production, but only at order $1/\Lambda^4$.

Among the ones that contribute to top pair production, only the ones featuring a left-handed pair of light quarks are relevant for $pp \to t\bar{t}W^\pm$ production. The colour octets contribute to order-$1/\Lambda^2$ interferences with the dominant standard-model amplitude:

$$c_{Qq}^{1,8} \equiv C_{qq}^{1(i33i)} + 3C_{qq}^{3(i33i)}, \tag{90}$$

$$c_{Qq}^{3,8} \equiv C_{qq}^{1(i33i)} - C_{qq}^{3(i33i)}, \tag{91}$$

$$c_{tq}^{8} \equiv C_{qu}^{8(ii33)}, \tag{92}$$

for both $i = 1$ and 2, while the colour-octet combinations contribute at order $1/\Lambda^4$:

$$c_{Qq}^{1,1} \equiv C_{qq}^{1(ii33)} + \frac{1}{6}C_{qq}^{1(i33i)} + \frac{1}{2}C_{qq}^{3(i33i)}, \tag{93}$$

$$c_{Qq}^{3,1} \equiv C_{qq}^{3(ii33)} + \frac{1}{6}\left(C_{qq}^{1(i33i)} - C_{qq}^{3(i33i)}\right), \tag{94}$$

$$c_{tq}^{1} \equiv C_{qu}^{1(ii33)}. \tag{95}$$

B.6.3 Two-heavy contributions

The c_{tG} and c_{tG}^{I} linear combinations interfere at order $1/\Lambda^2$ with the dominant standard-model amplitude.

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