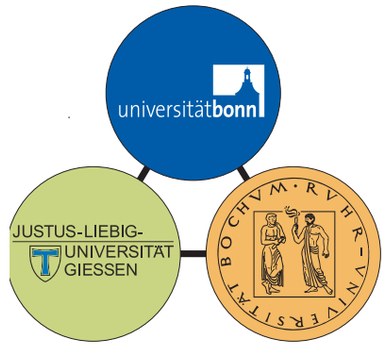
Baryons in the Dyson-Schwinger formalism

Christian S. Fischer

Justus Liebig Universität Gießen

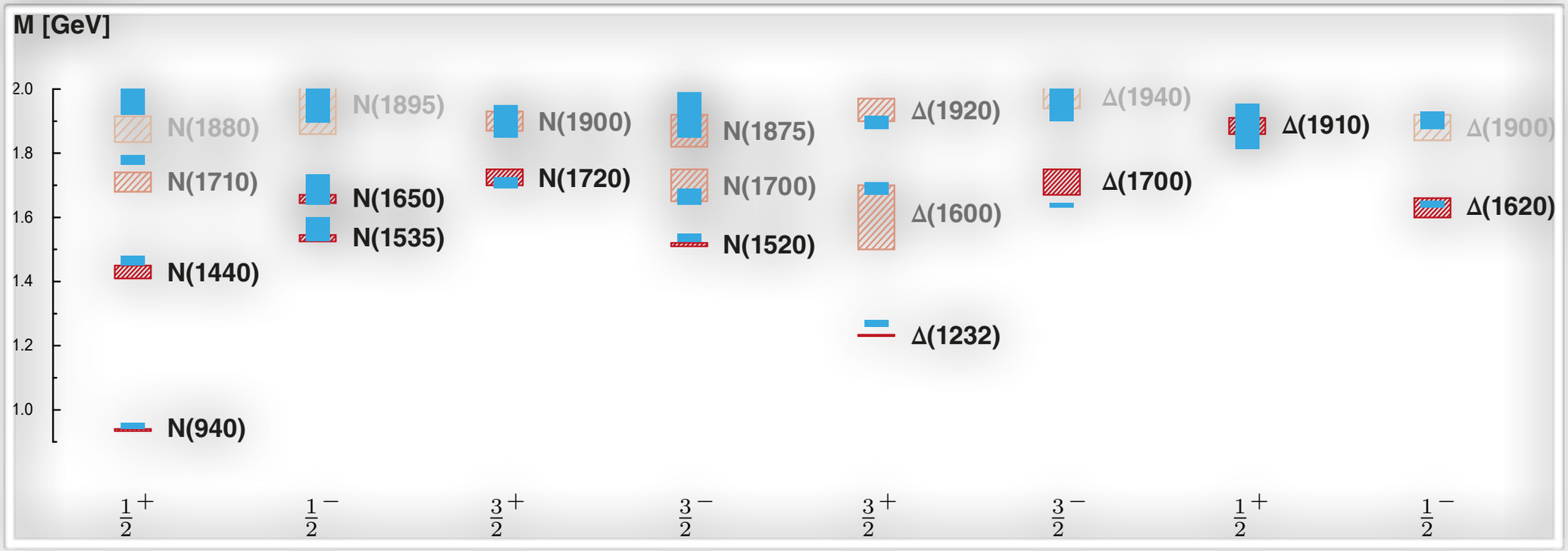
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]



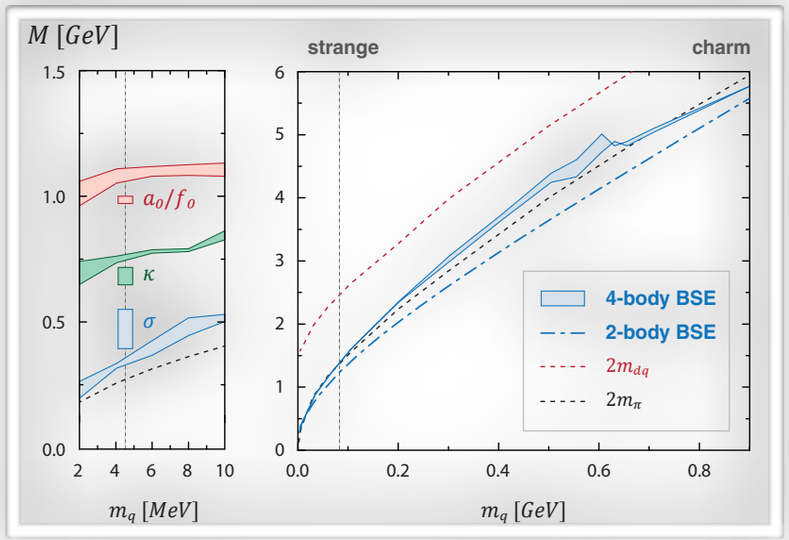
Overview - Take home messages

- Light and strange baryon spectrum:

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

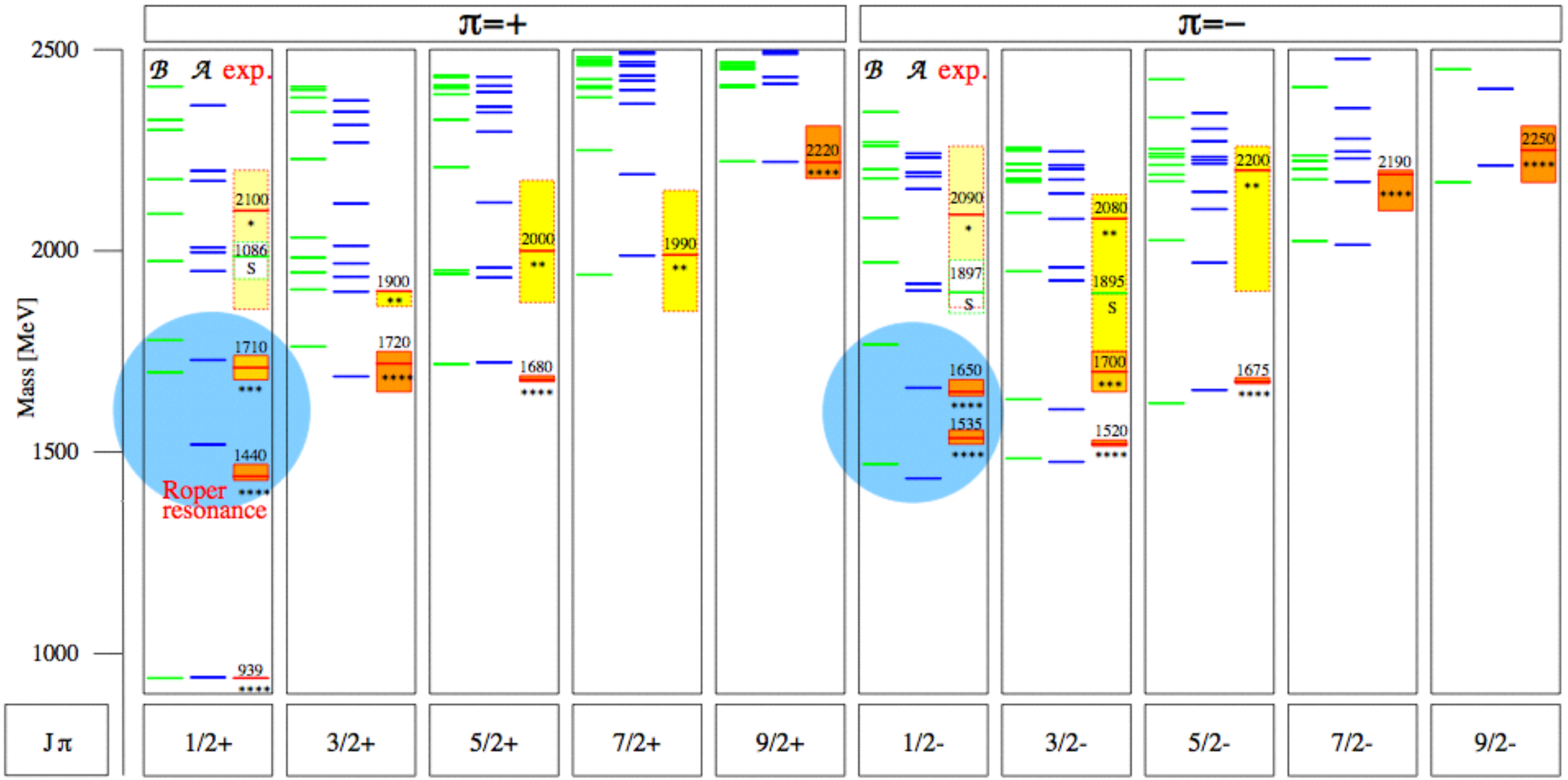


- Light tetraquarks:



Eichman, CF, Heupel, PLB 753 (2016) 282-287

Light baryon spectrum - quark model

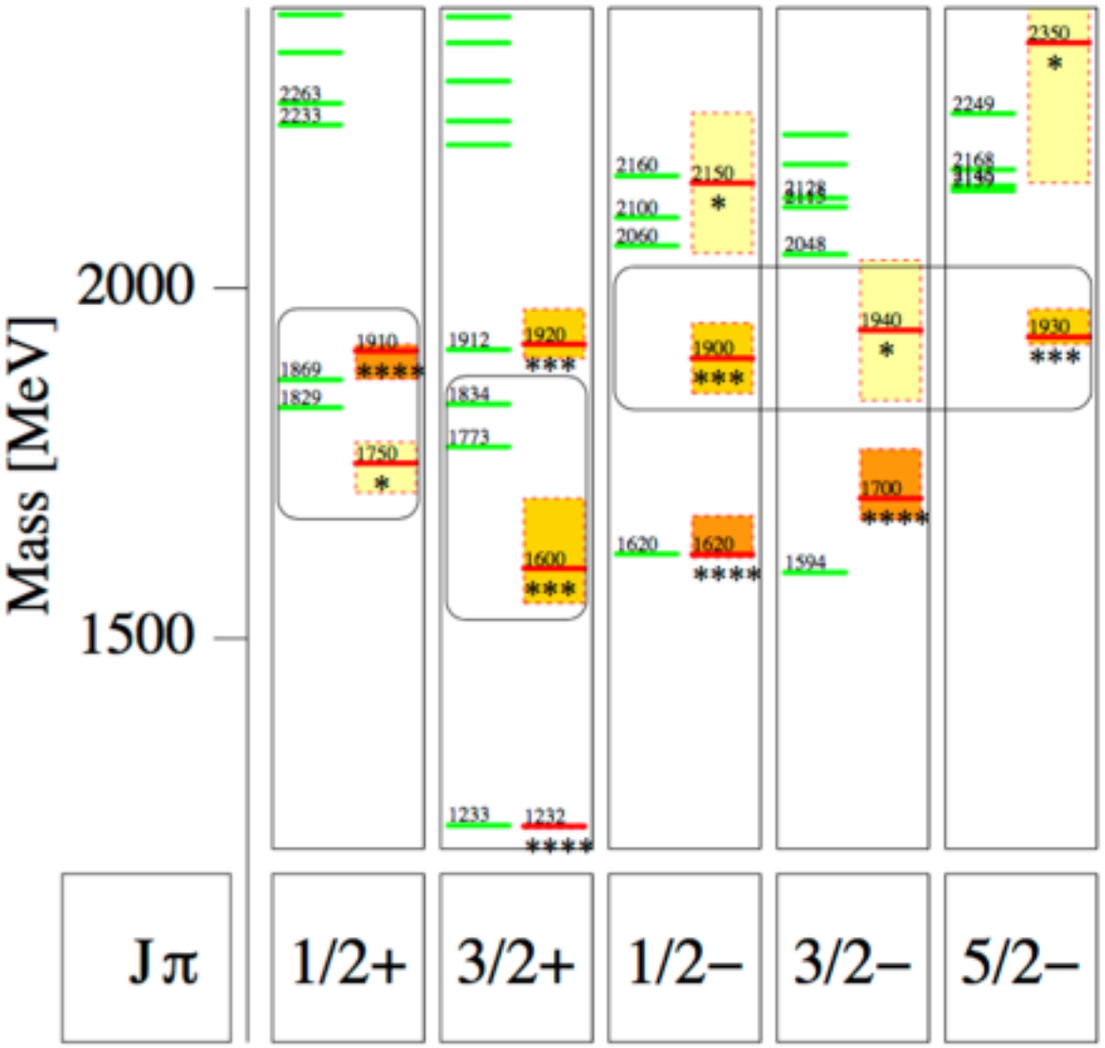


Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances': three-body vs. quark-diquark
- level ordering: $N_{\frac{1}{2}+}$ vs. $N_{\frac{1}{2}-}$

Flavored baryon spectrum - quark model

u/d - s - c - b: probe QCD at different scales

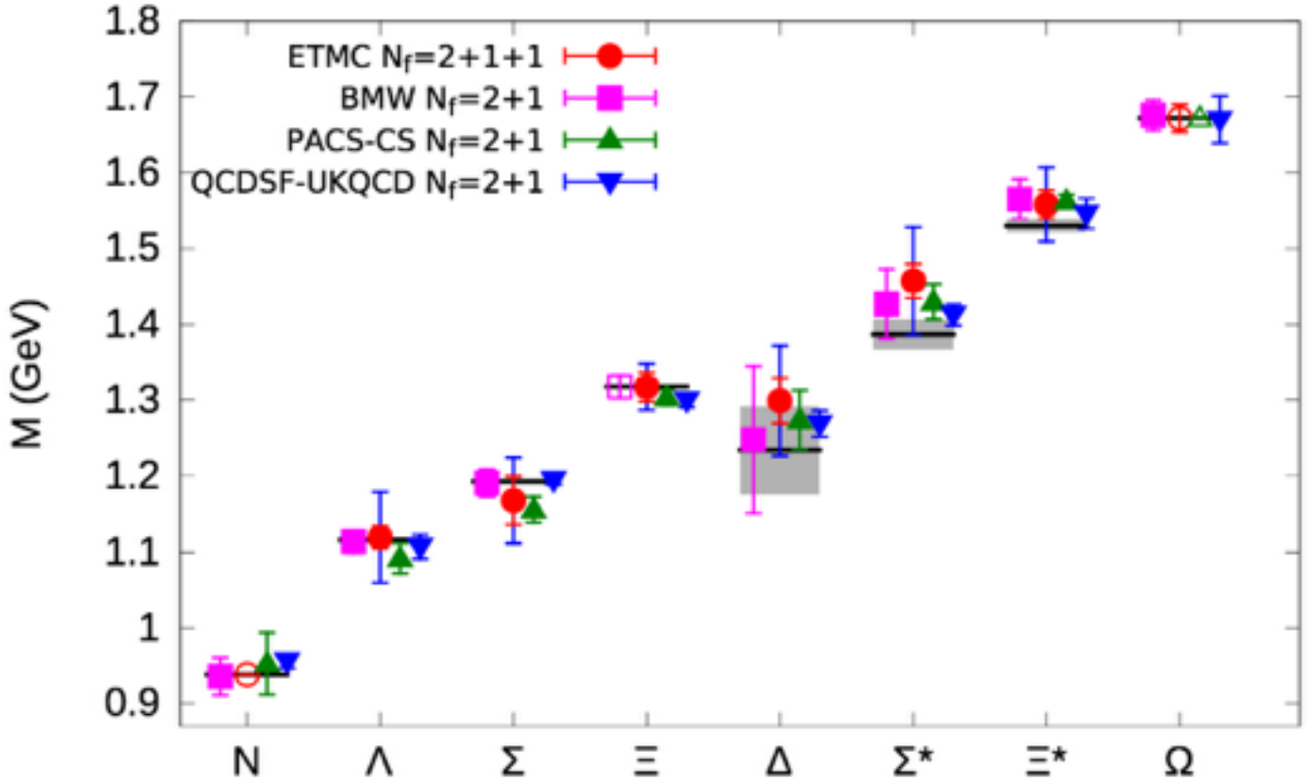


- need (effective) flavor dependent forces to explain spectrum
- models: parametrization
- should be determined from QCD

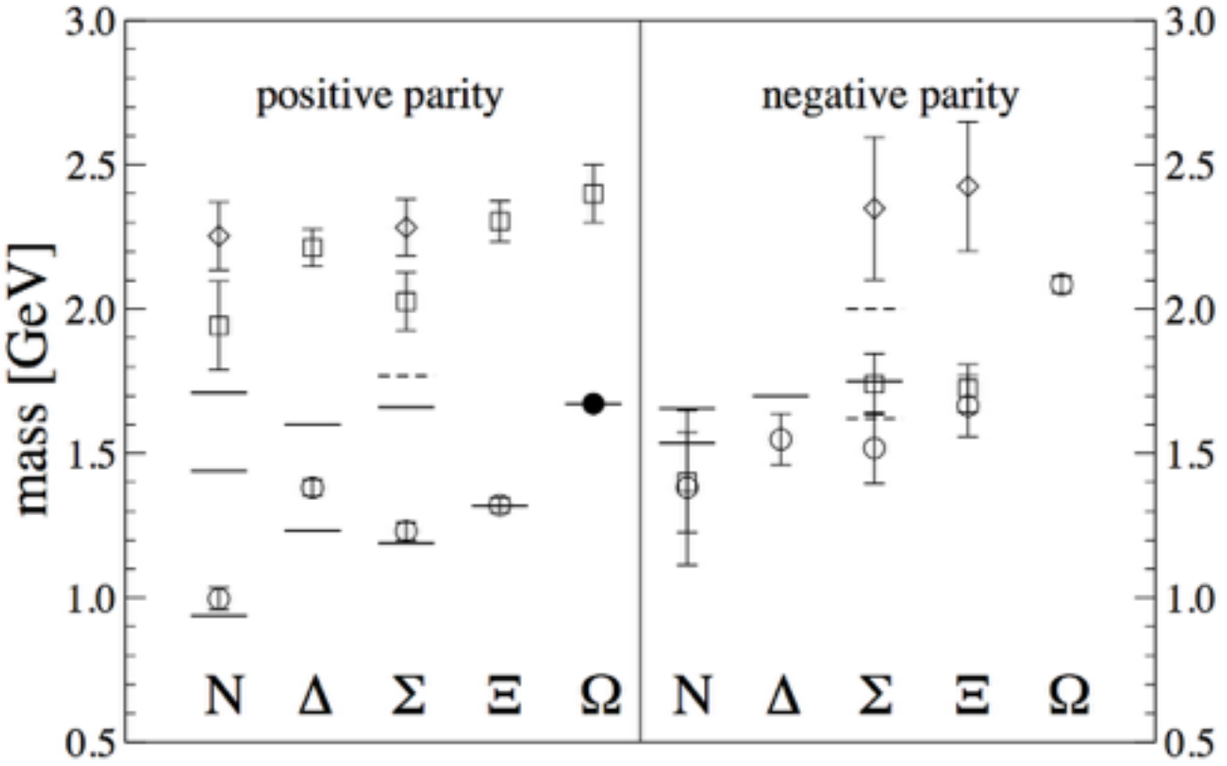
Ronniger, Metsch, EPJA 47 (2011) 162
see also Glozmann, Riska, Plessars et al.

Nonperturbative QCD: Lattice, Functional methods

Lattice QCD



Alexandrou et al. (ETMC), PRD 90 (2014) 7

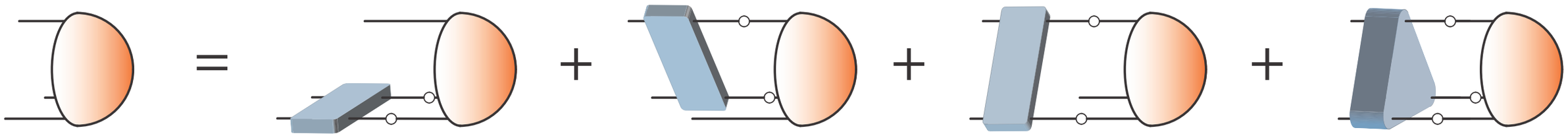


Engel et al. (BGR), PRD82 (2010) 034505

- baryon ground states well under control
- baryon excited states: very tough problem

Three-body vs. Diquark-quark approximation

Bethe-Salpeter equation for baryons:



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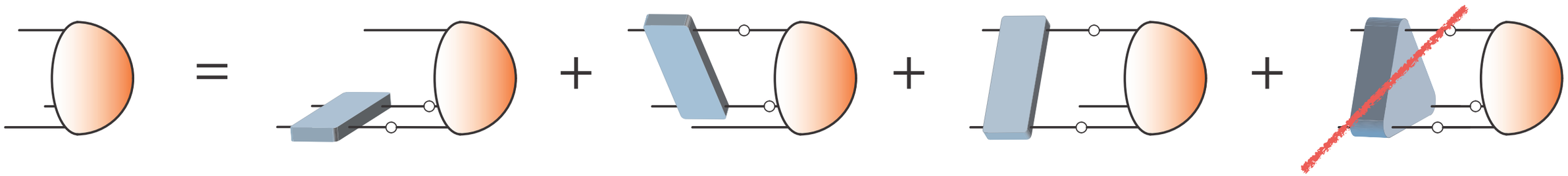

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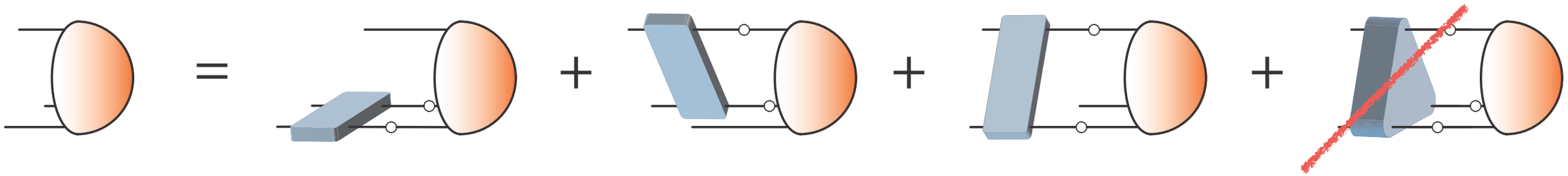
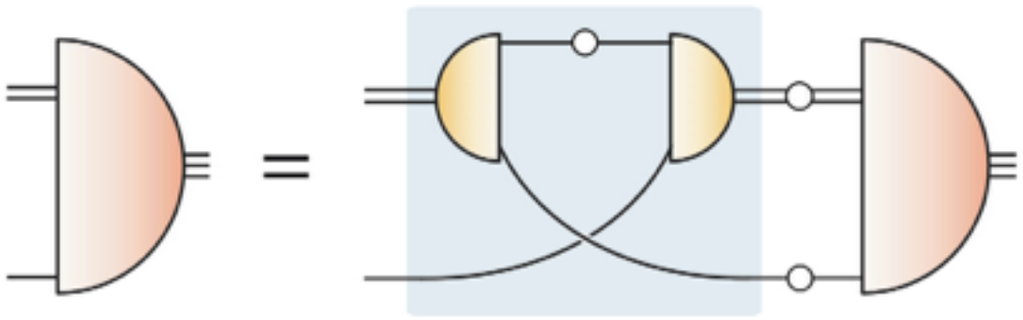
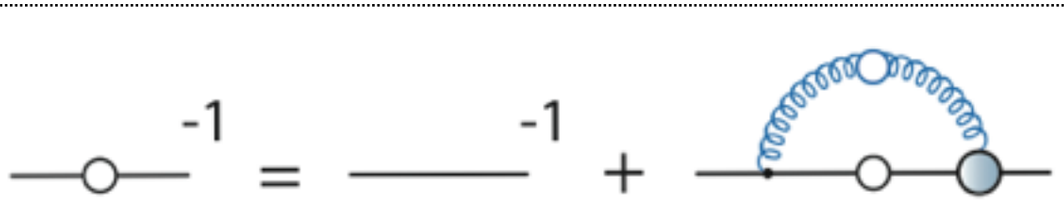
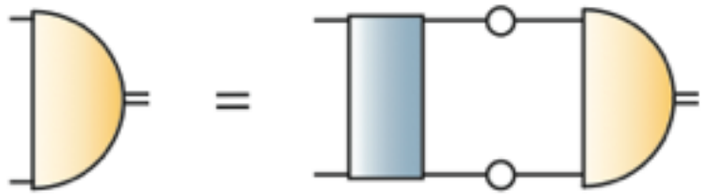


Diquark-quark approximation:







- Input: quark-gluon interaction
- Diquarks are NOT point like

Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

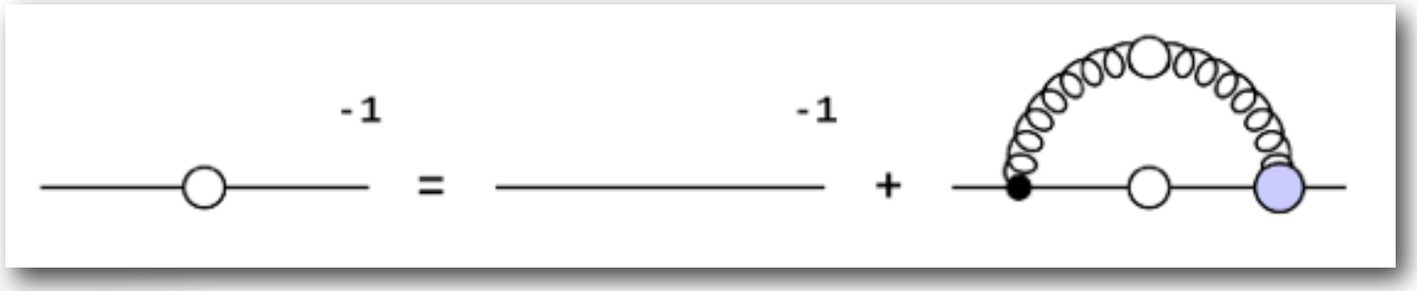
relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \left[F_1(P,p) \right. \qquad \text{s-wave}$$
$$+ F_2(P,p) i\not{P}$$
$$+ F_3(P,p) pP i\not{p} \qquad \text{p-wave}$$
$$\left. + F_4(P,p)[\not{p}, \not{P}]\right]$$

$$P : (-1)^{L+1}$$

- conventional states more complicated
  - baryon octet: 64 tensors with s,p,d wave
  - decuplet: 128 tensors with s,pd,f wave
- mesons: 'exotic' quantum numbers possible: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$

The DSE for the quark propagator

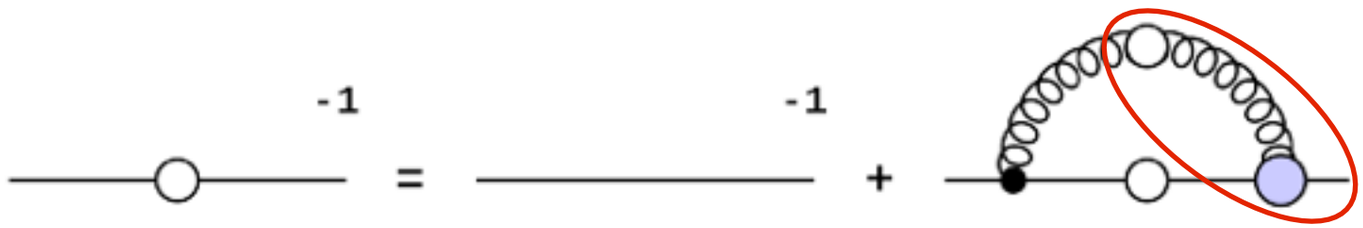


Approximations:

I) NJL/contact model:

II) Quark-diquark model: *ansatz for quark (and diquark wave function)*

III) Rainbow-ladder (RL):

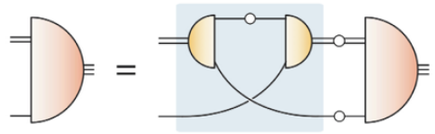


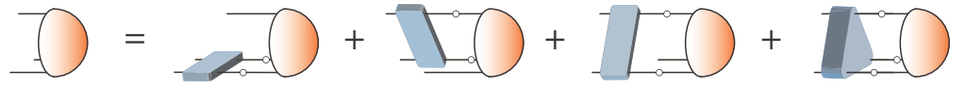
IV) Beyond rainbow-ladder (bRL): *solve DSEs for quark, gluon, vertex*

Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein
Binosi, Chang, Papavassiliou, Qin, Roberts PRD95 (2017) 031501 and refs. therein

DSE/BSE/Faddeev landscape (2015)

level of sophistication





	I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)	III) DSE (RL)	IV) DSE (bRL)
N, Δ masses	✓	✓	✓	✓	✓
N, Δ em. FFs	✓	✓	✓	✓	
$N \to \Delta\gamma$	✓	✓	✓		
Roper,...	✓	✓			
$N \to N^*\gamma$	✓	✓			
$N^*(1535), \ldots$		✓			
$N \to N^*\gamma$					
Σ, Ξ, Ω excited strange		✓			
Σ, Ξ, Ω em. FFs					
	Cloet, Thomas, Roberts, Segovia et al.	Oettel, Alkofer, Roberts, Bloch, Segovia et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF	Sanchis-Alepuz, Williams, CF

Eichmann, N*-Workshop, Trento 2015

DSE/BSE/Faddeev landscape

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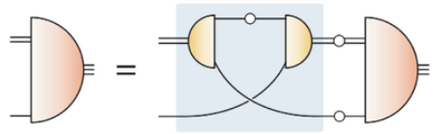

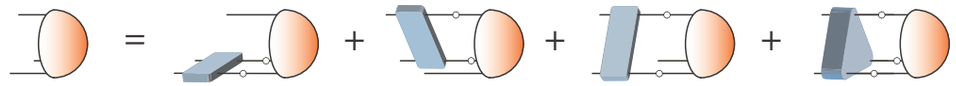

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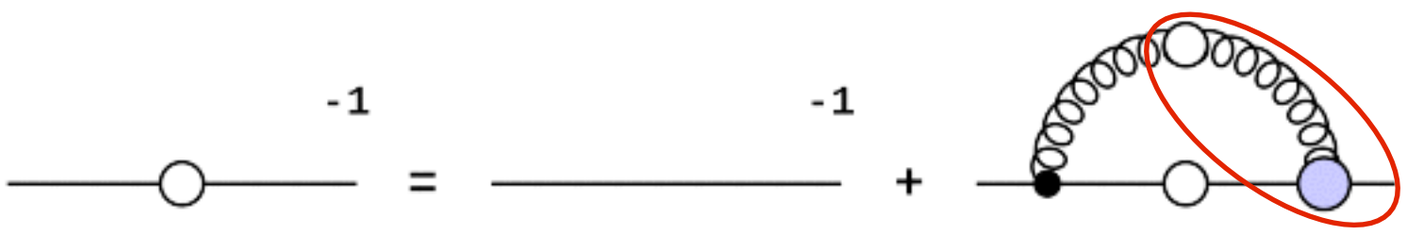
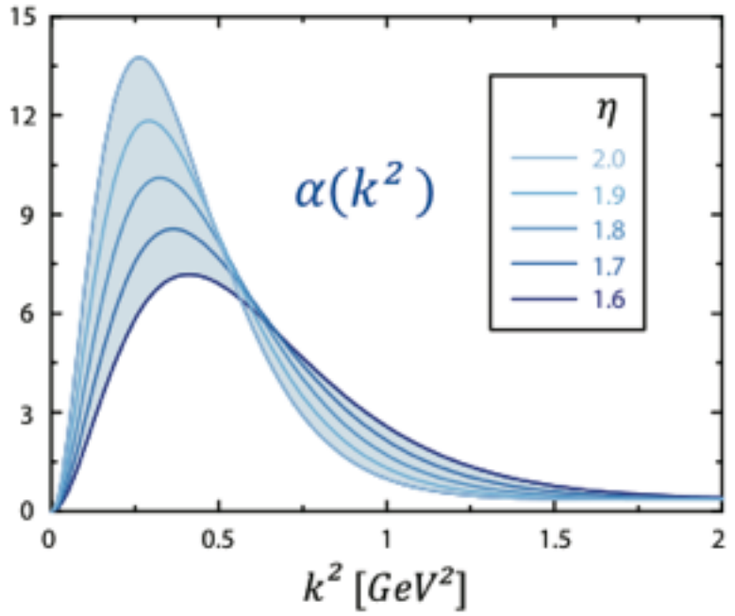
Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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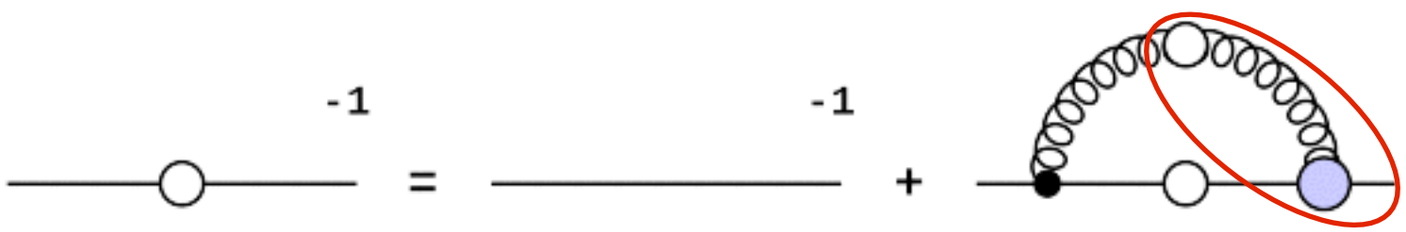

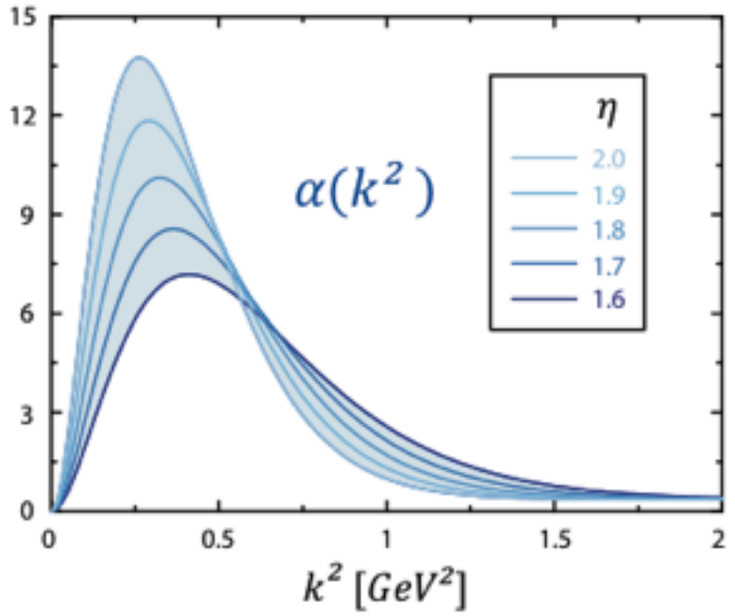

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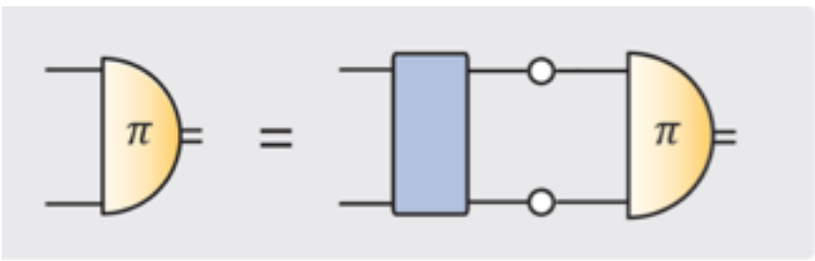

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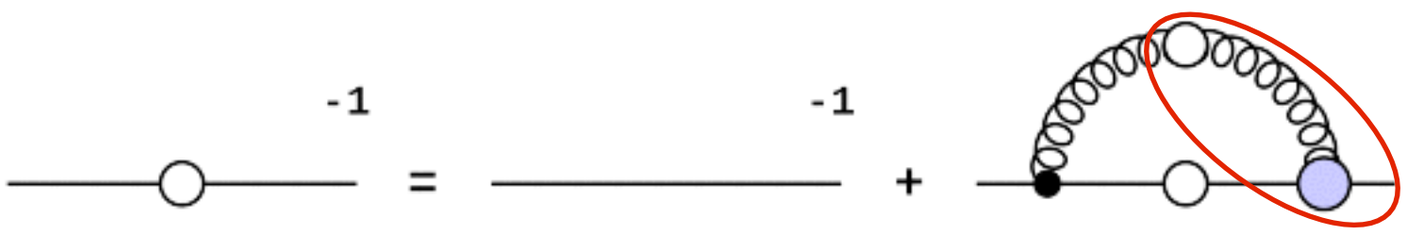

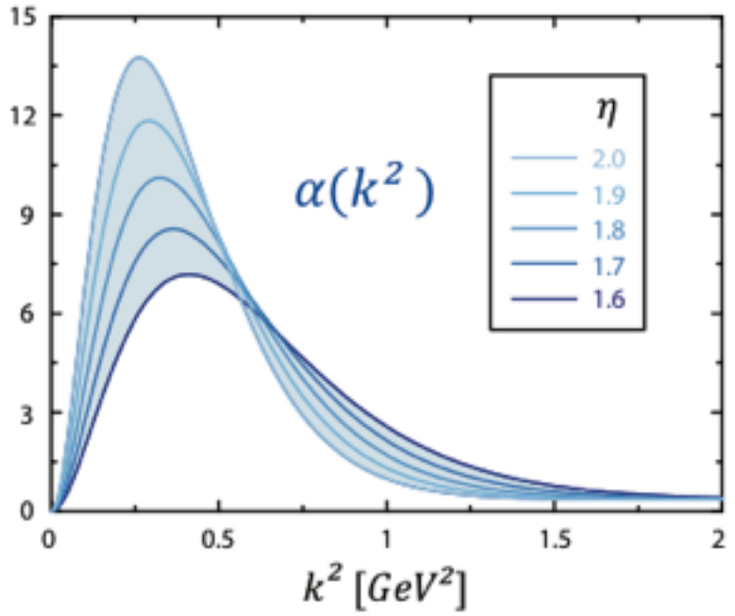

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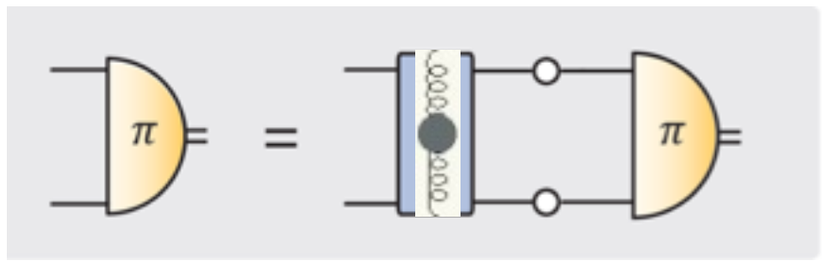


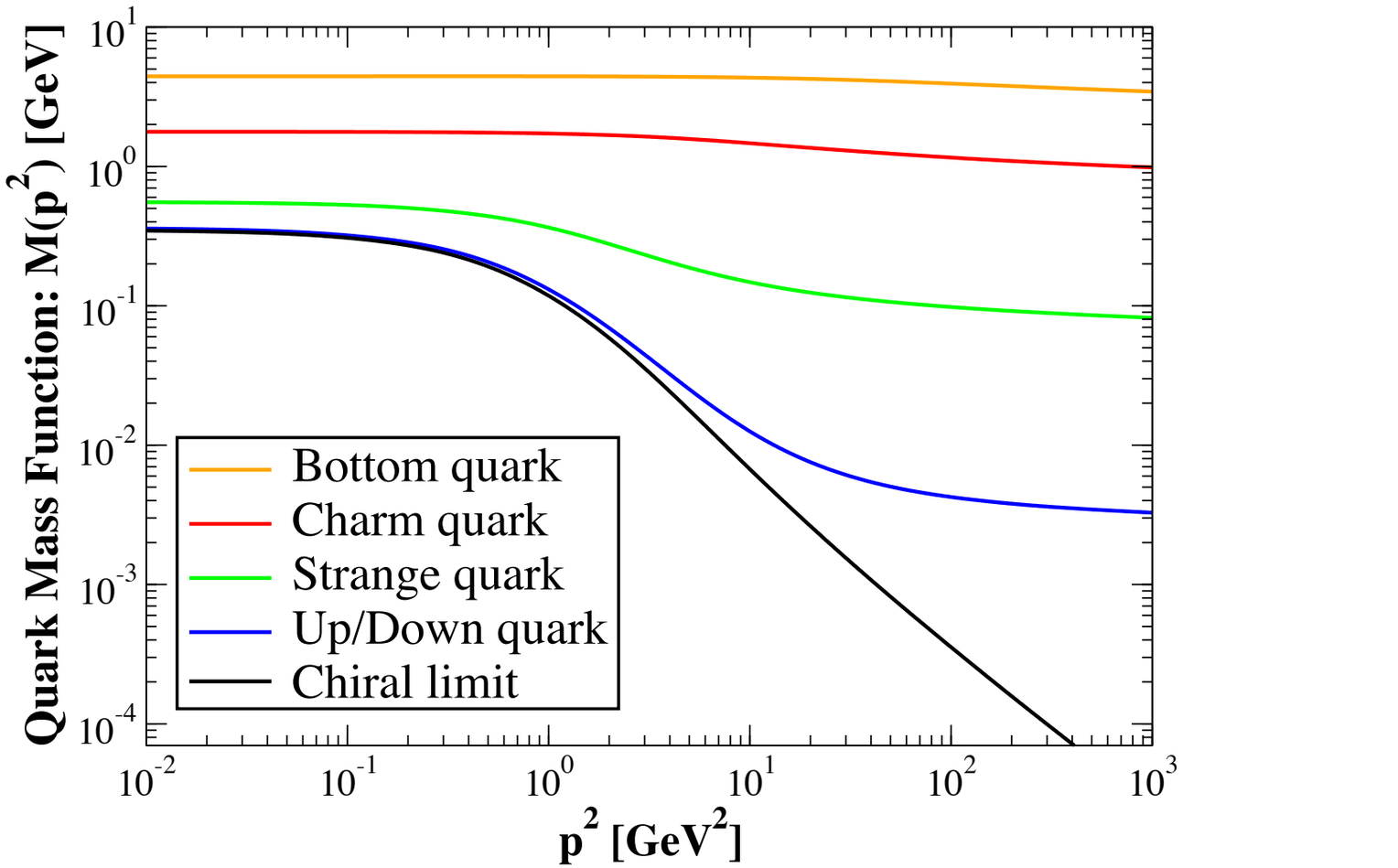
Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

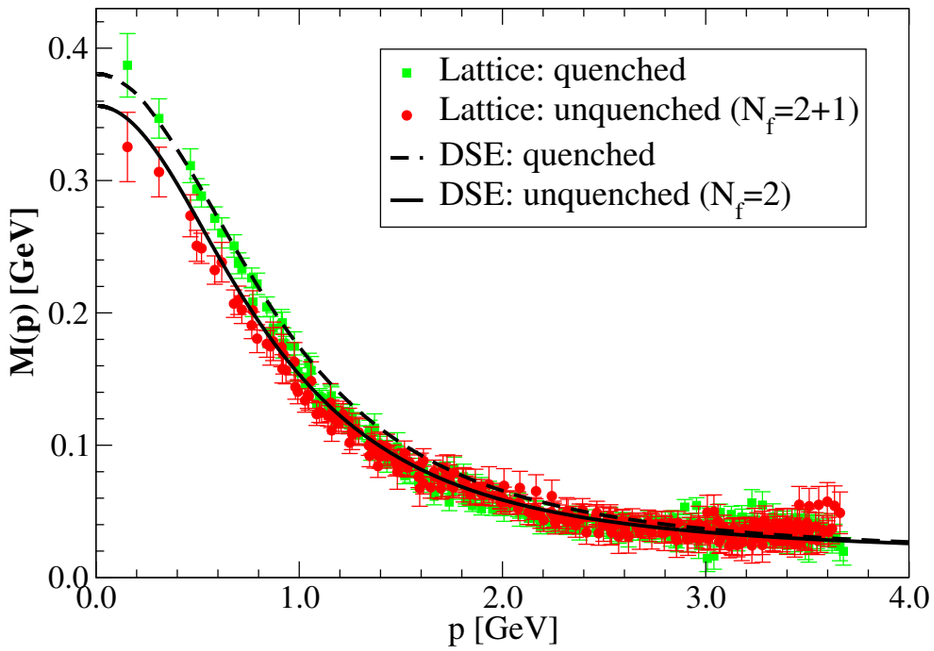
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

Quark mass: flavor dependence

Typical solution:

$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$





CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

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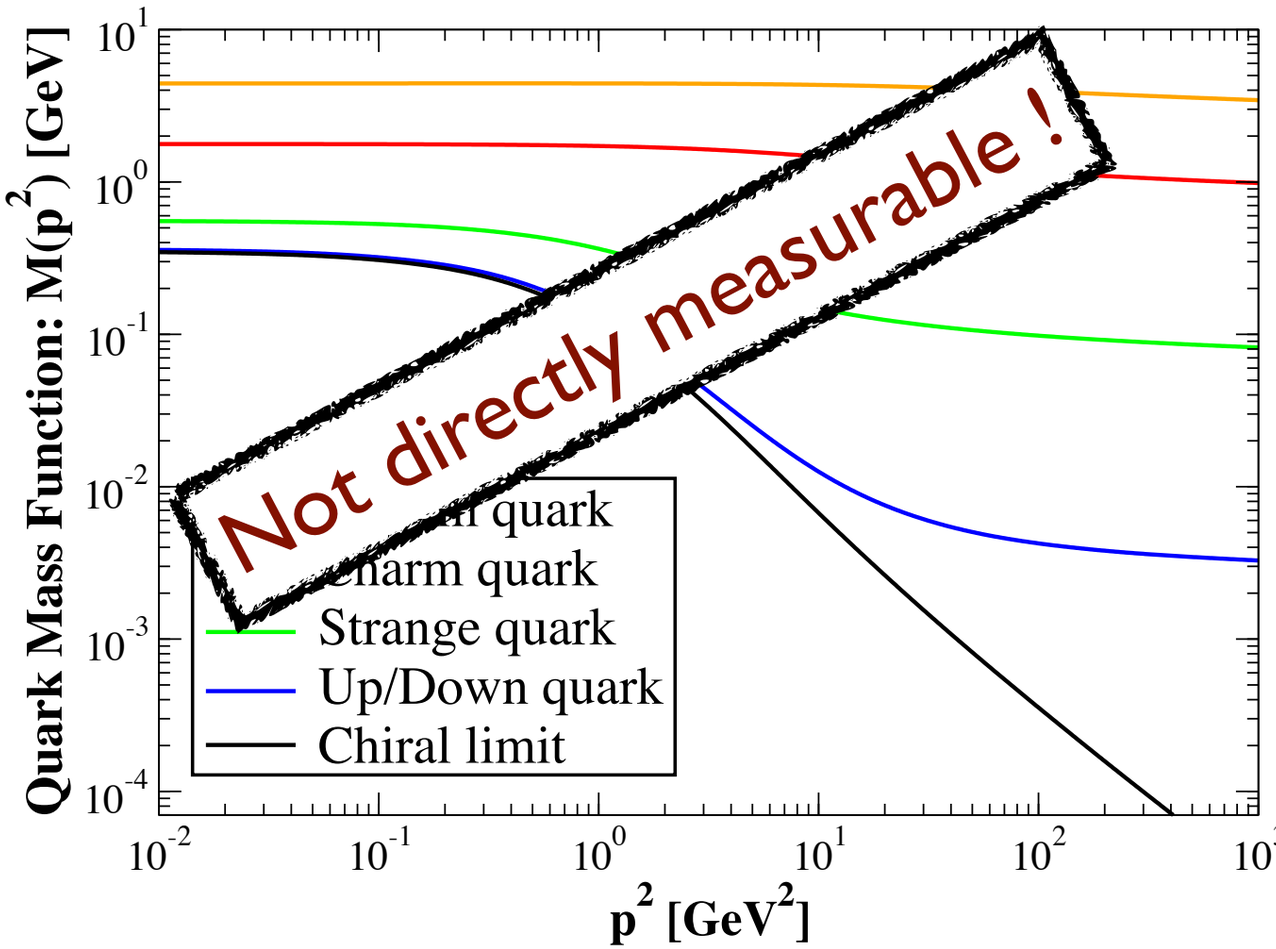

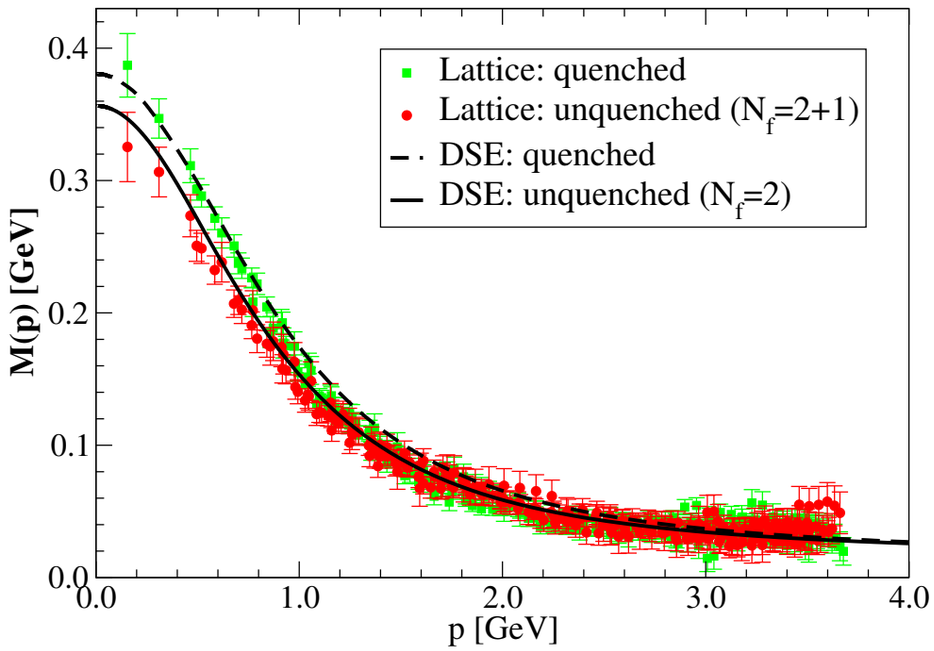

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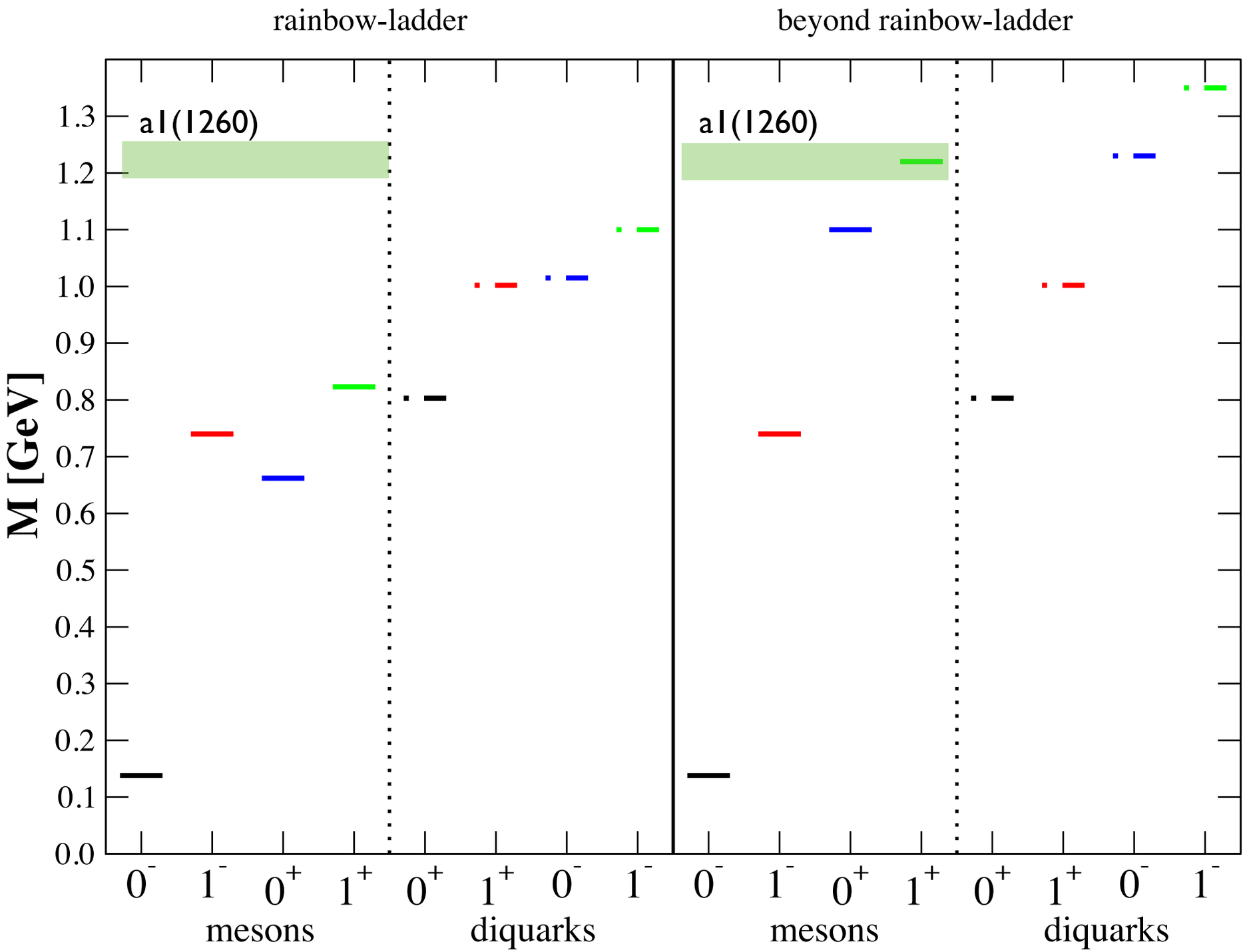
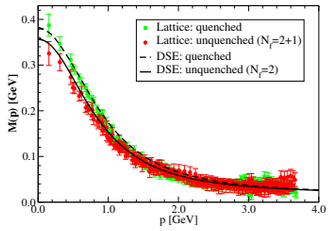
Diquarks with modified rainbow-ladder

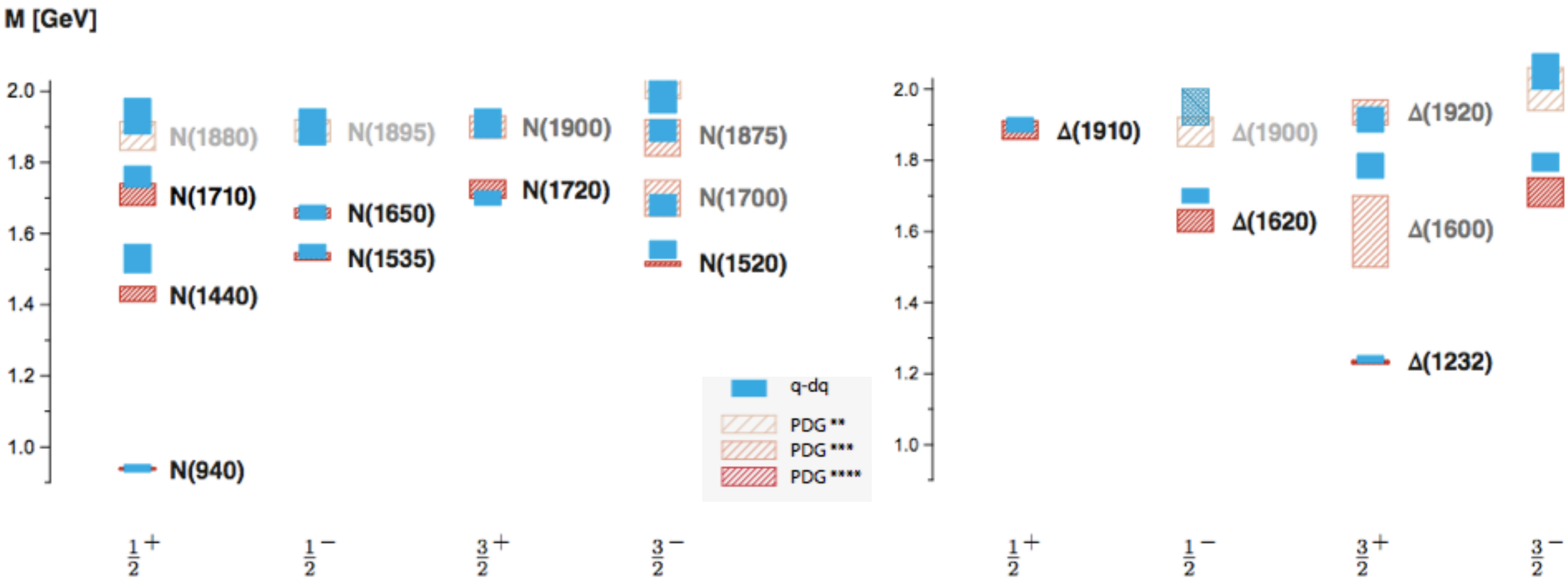


- α multiplied with 0.35 in 'bad' channels

see also: Williams, CF, Heupel, PRD93 (2016) 034026

Light baryon spectrum: DSE-RL



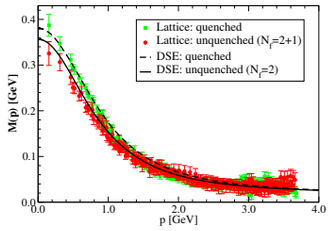


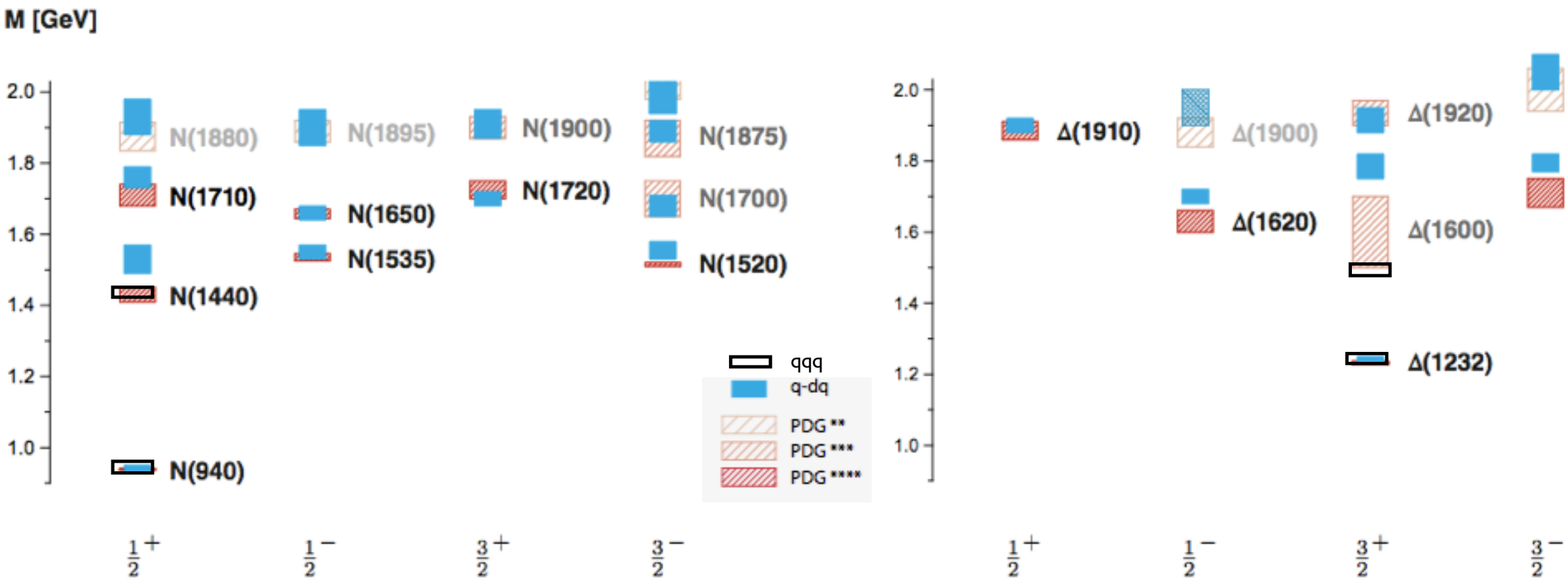
sc+av ps+v

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, in preparation

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)

Light baryon spectrum: DSE-RL





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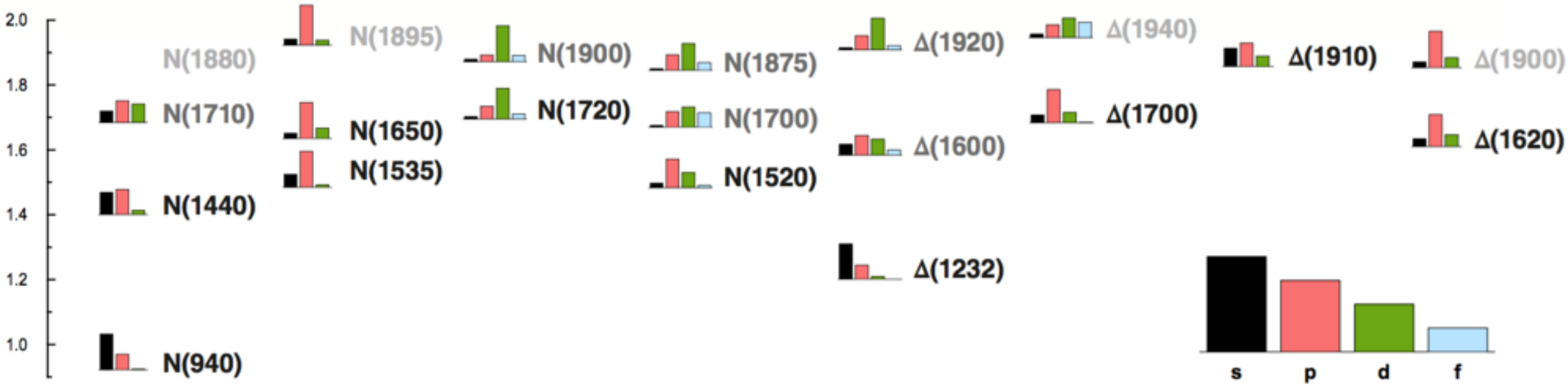
Eichmann, CF, in preparation

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- three-body agrees with diquark-quark where applicable

Angular momentum



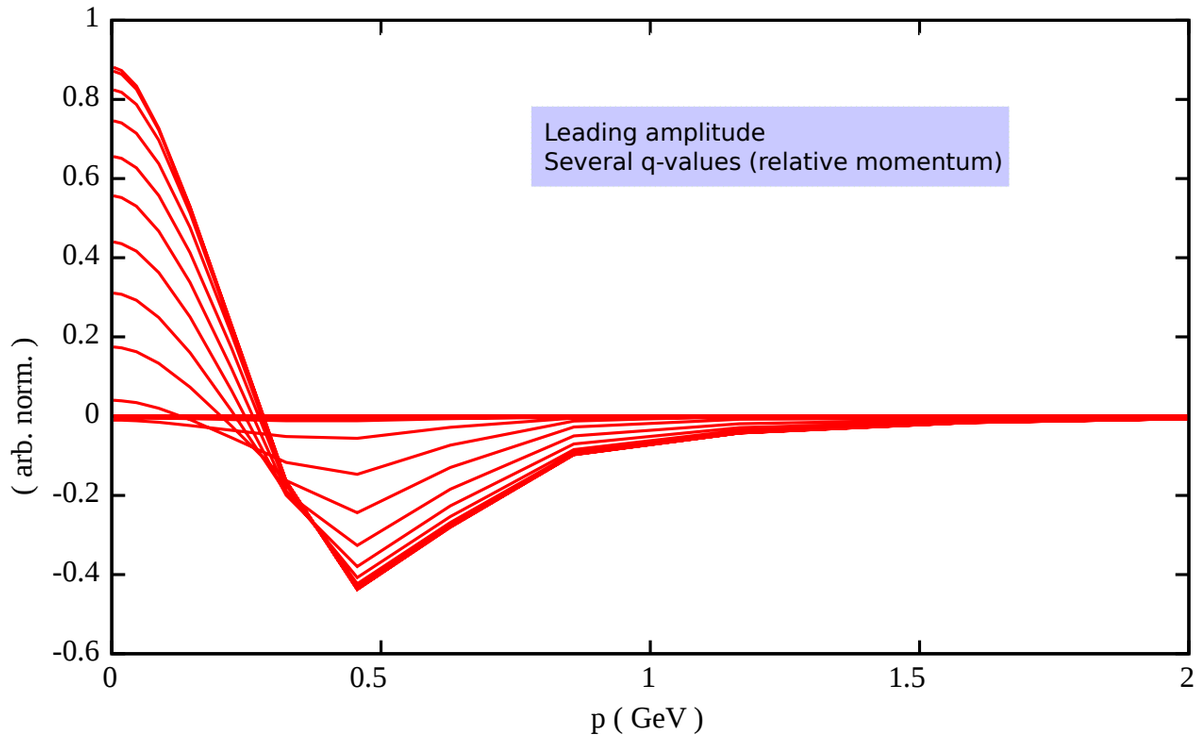
$J^P = \frac{1}{2}^+ \quad \frac{1}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{1}{2}^+ \quad \frac{1}{2}^-$

- non-relativistic quark model: restriction to certain ang. mom.
- here: quark-model forbidden contributions always present

Properties of the Roper

angular mom. decomposition

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

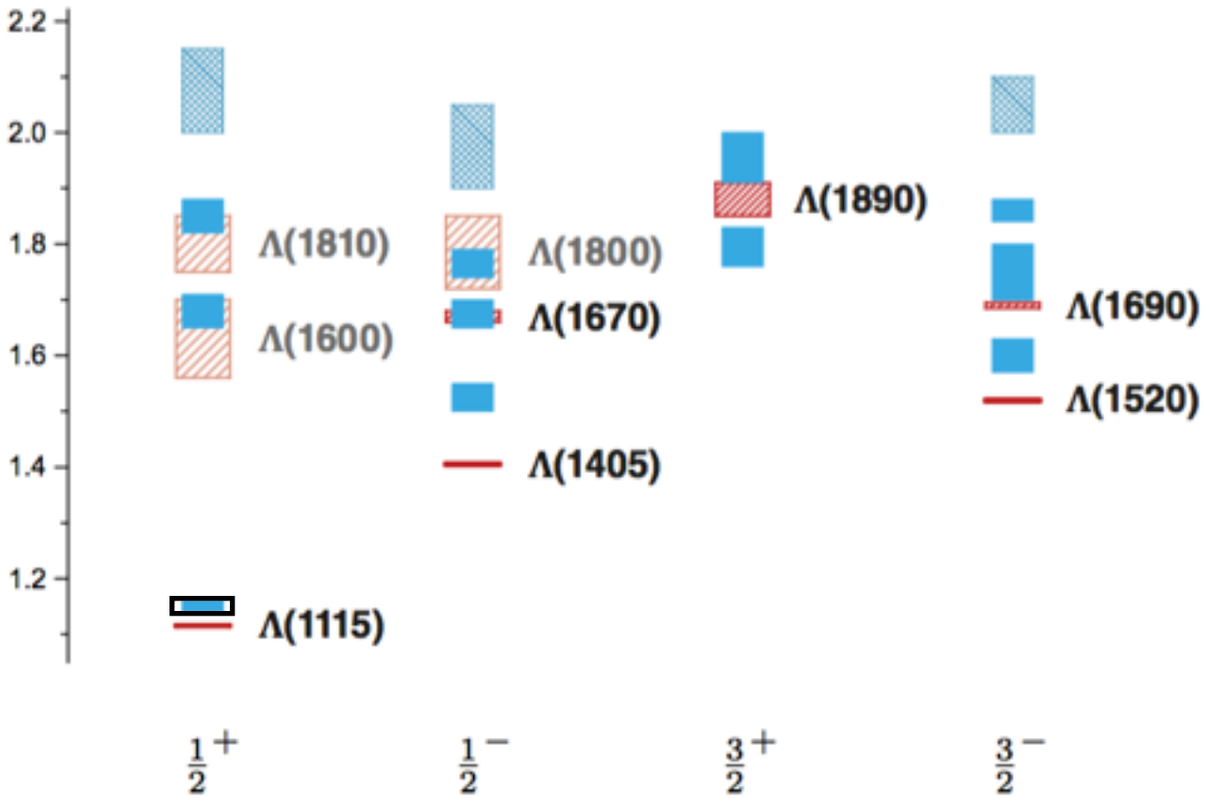


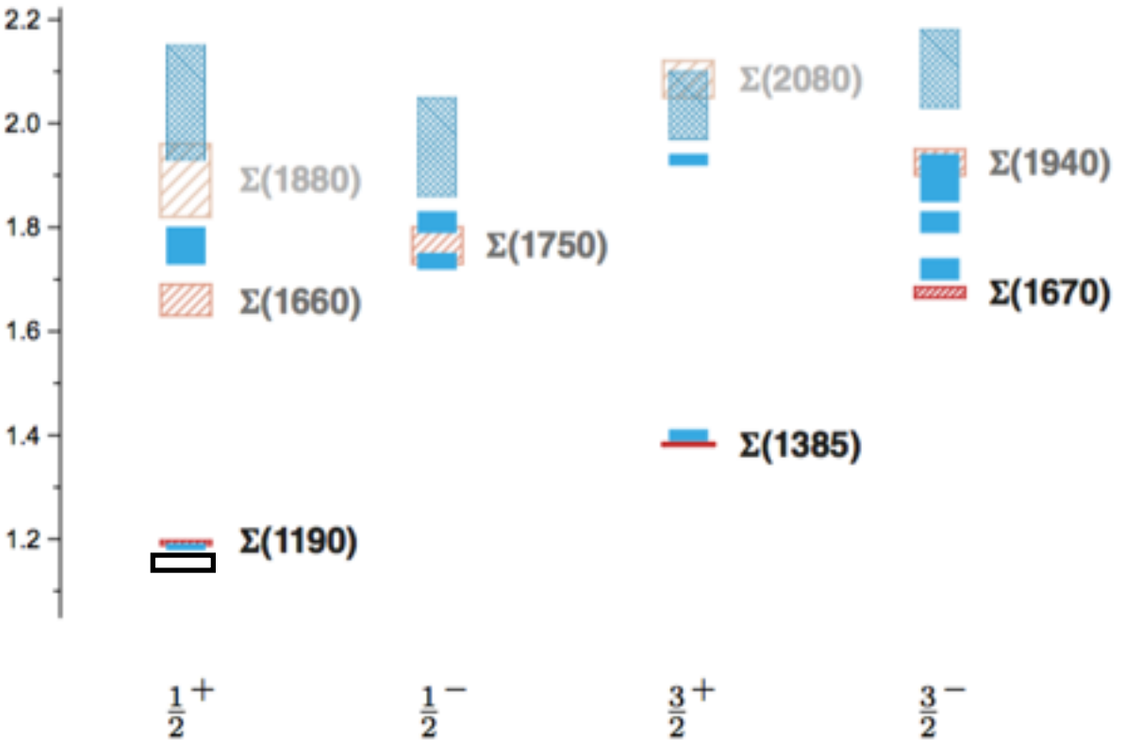
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)

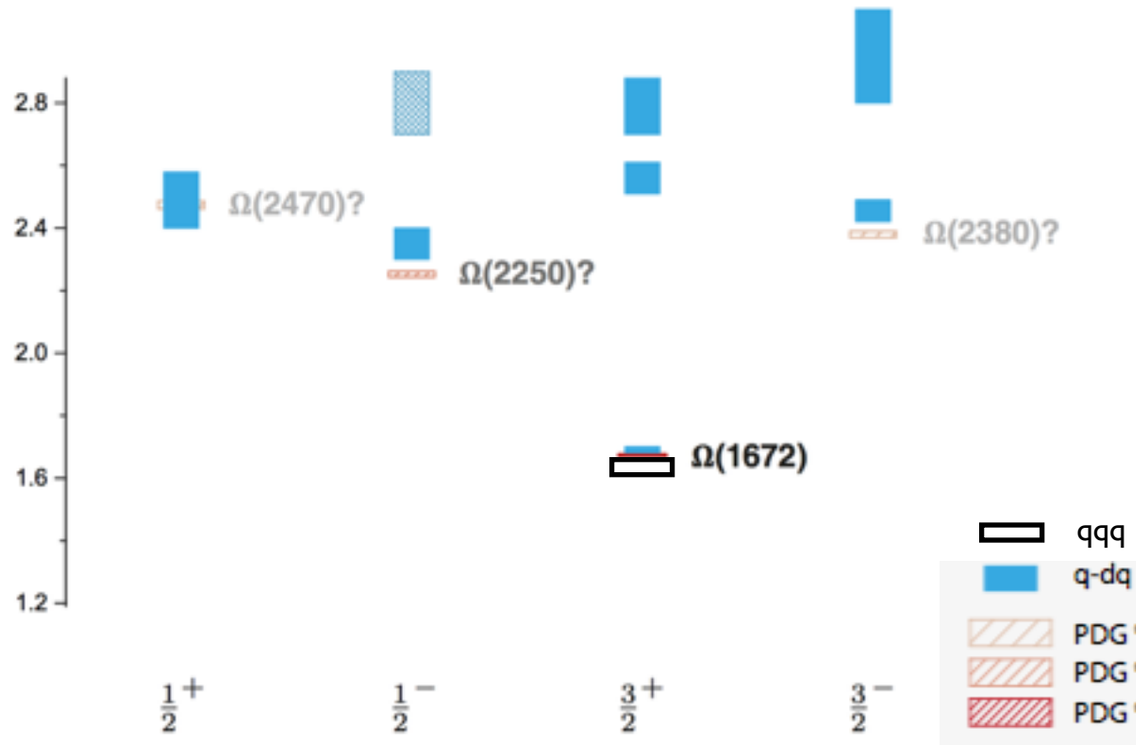
- zero crossing of wave function: 2s-state
- every state is mixture of several partial waves !
- different internal structure of radial excitations

tension with simpler calculations ('contact interaction', 'QCD based model'):
Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17
Lu, Chen, Roberts et al., PRC 96 (2017) 015208

Strange baryon spectrum: DSE-RL (preliminary !)







Eichmann, CF, in preparation
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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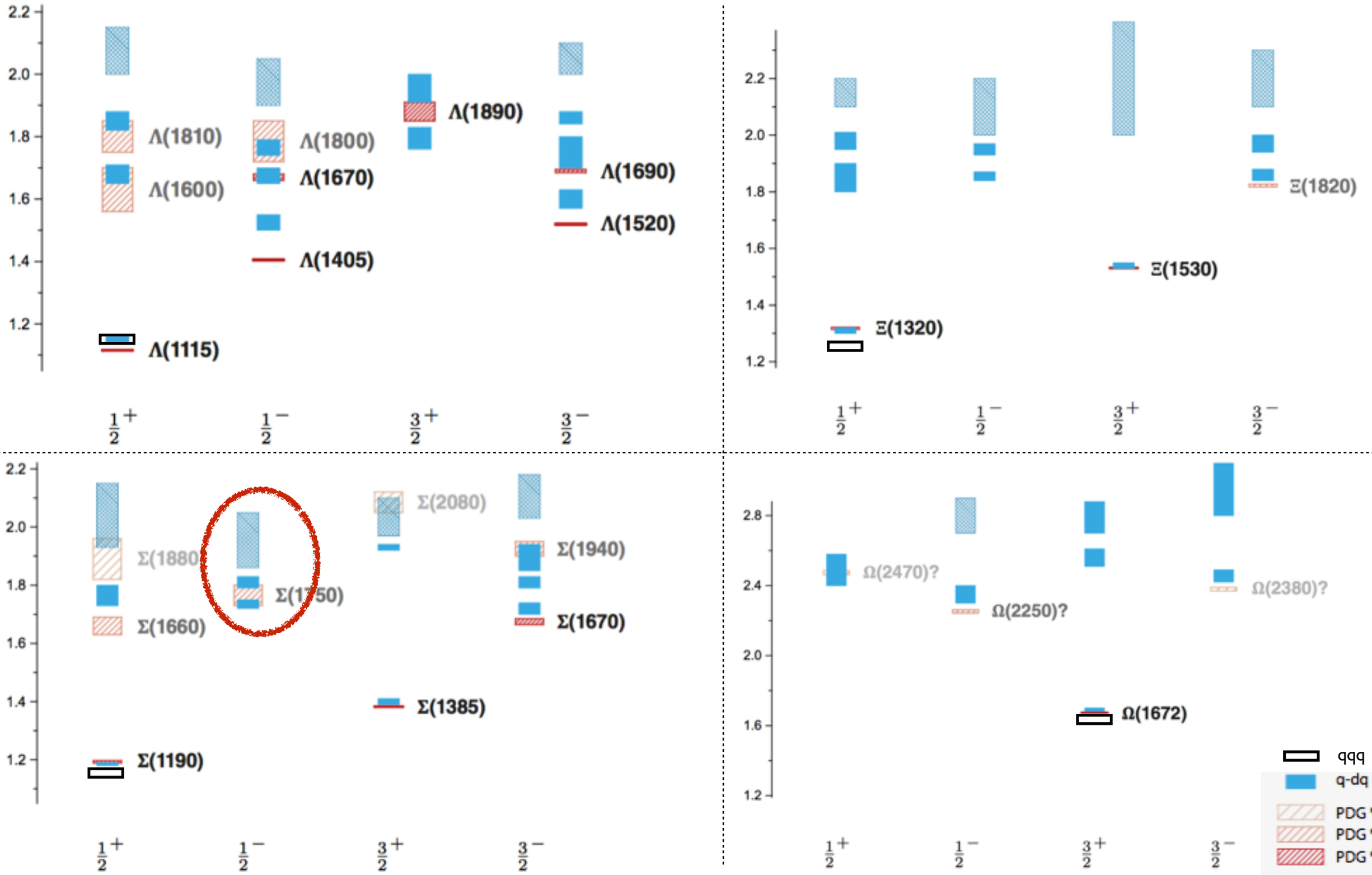

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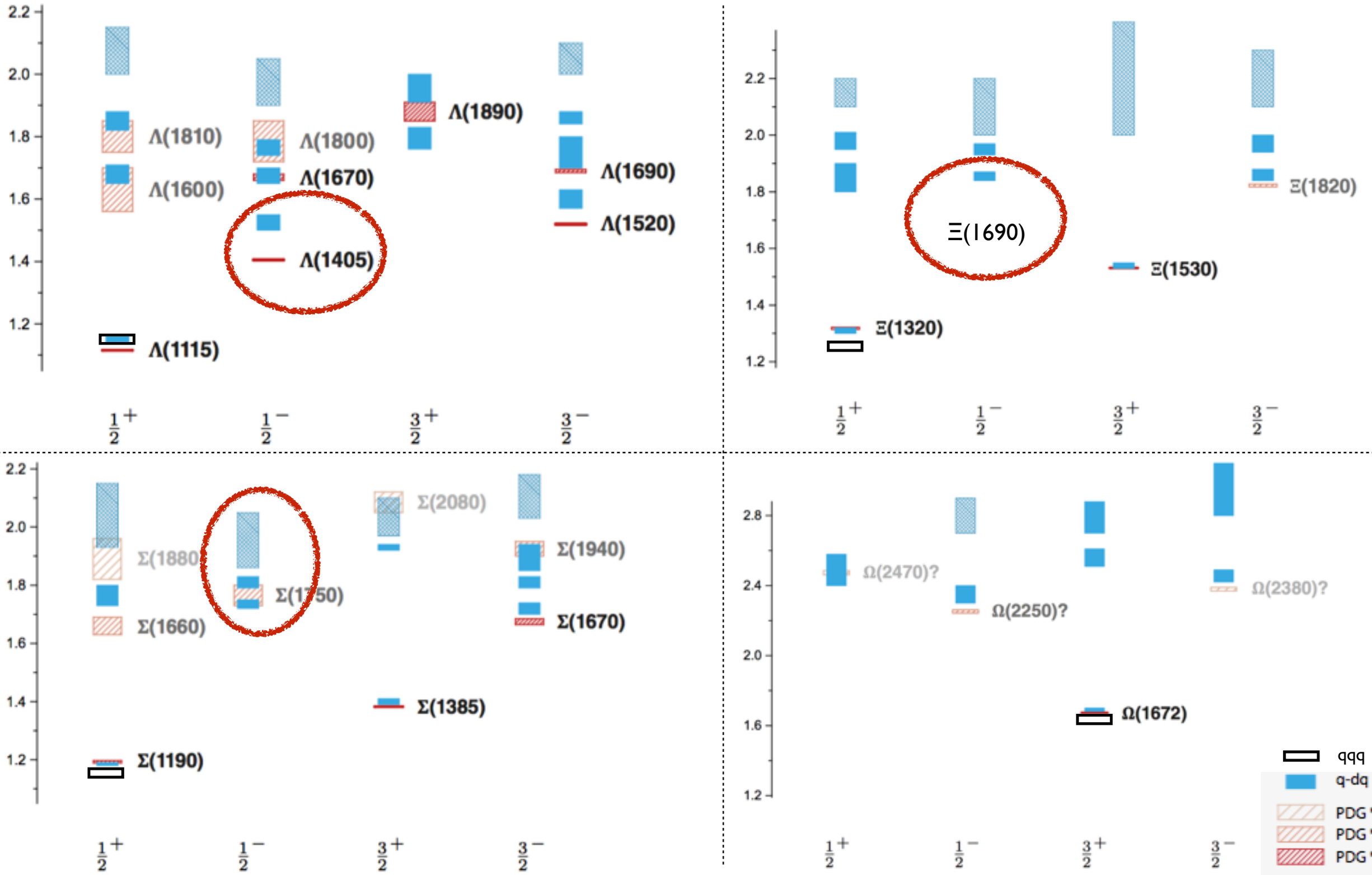


Eichmann, CF, in preparation
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Summary and outlook

Summary

- Baryon spectrum: good agreement with experiment!
- Three-body vs diquark-quark: fair agreement

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

- 'forbidden' angular momenta always present
- prediction for strange baryons

Further results:

- Baryons: form factors

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

- Tetraquarks: light scalar nonet done
  explore heavy-light systems

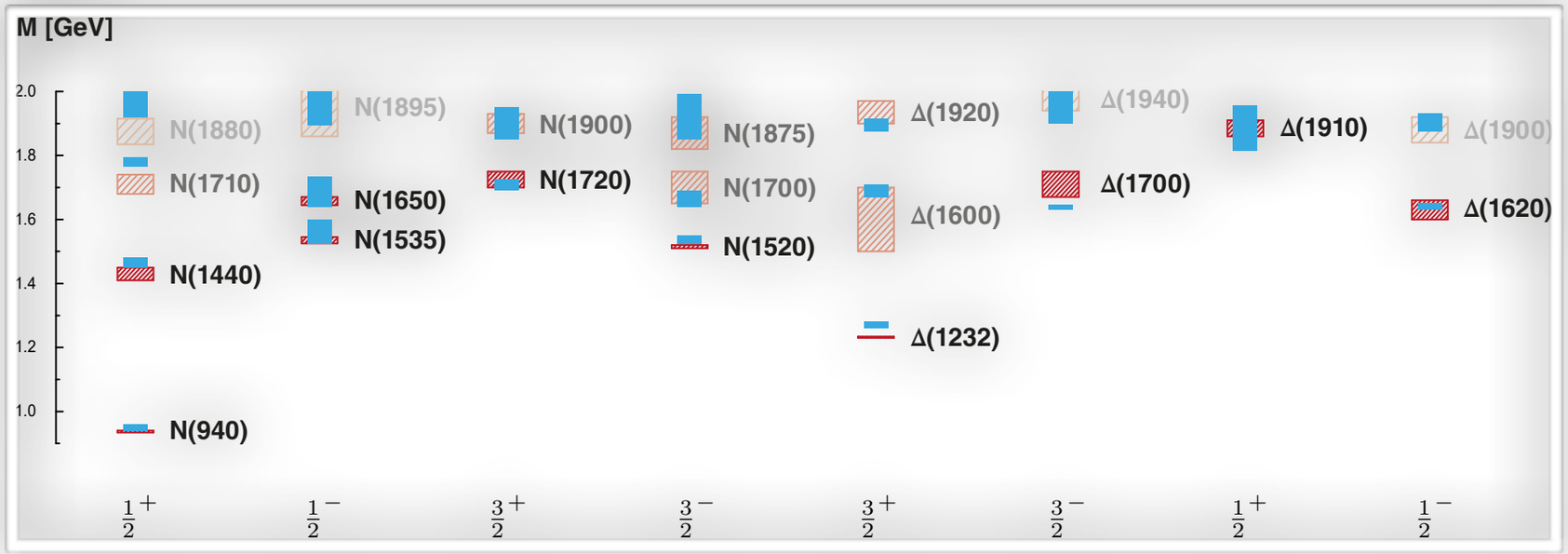
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Backup

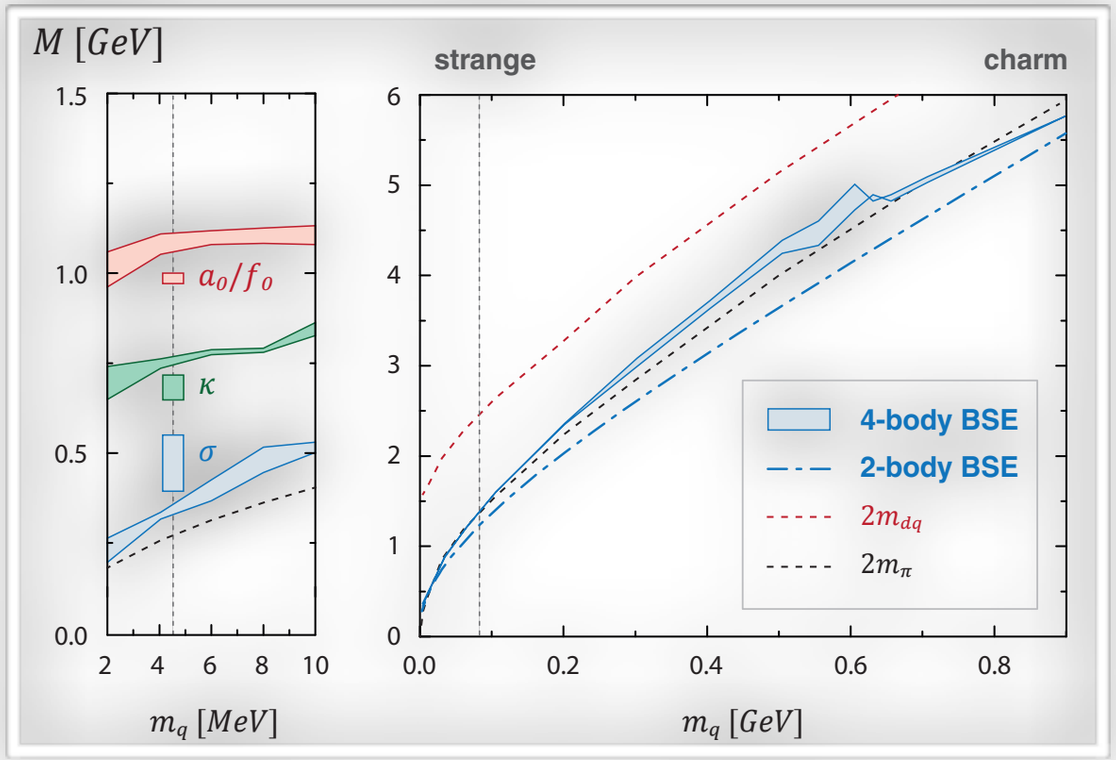
Overview - Take home messages

- Light and strange baryon spectrum:

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

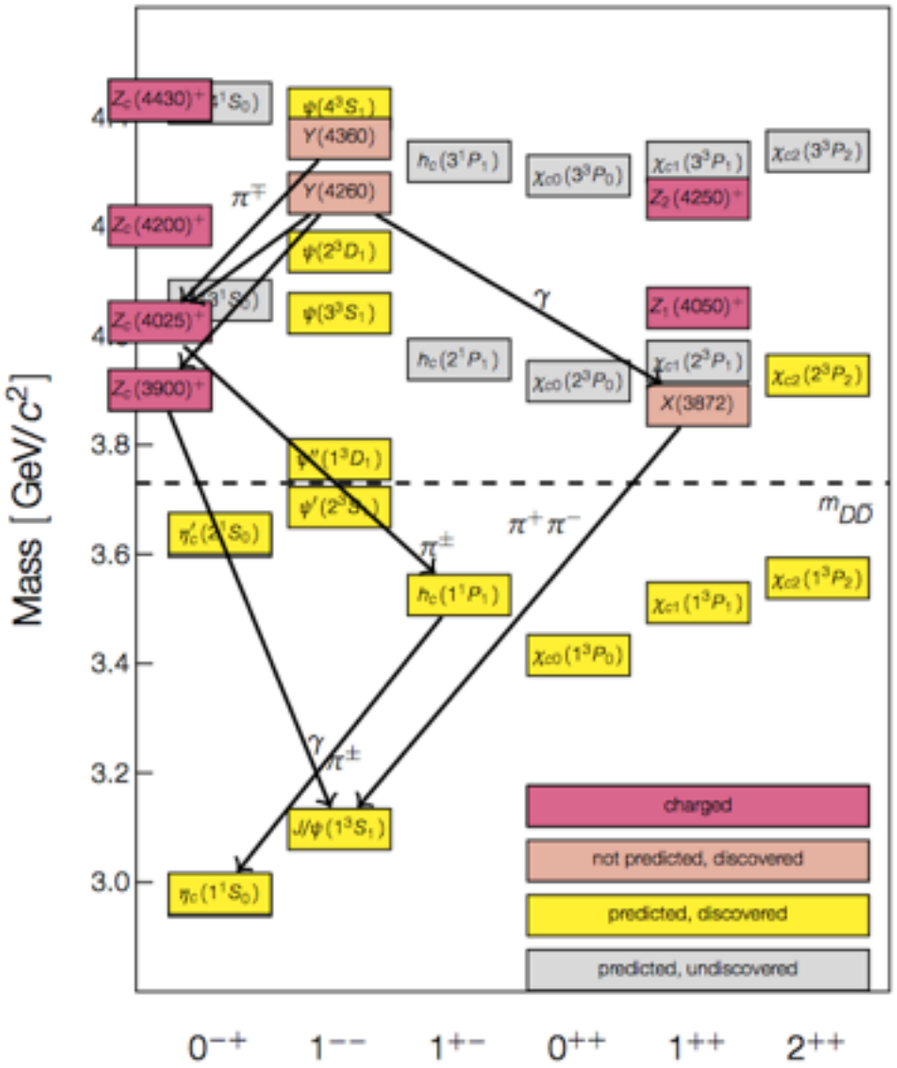


- Light tetraquarks:



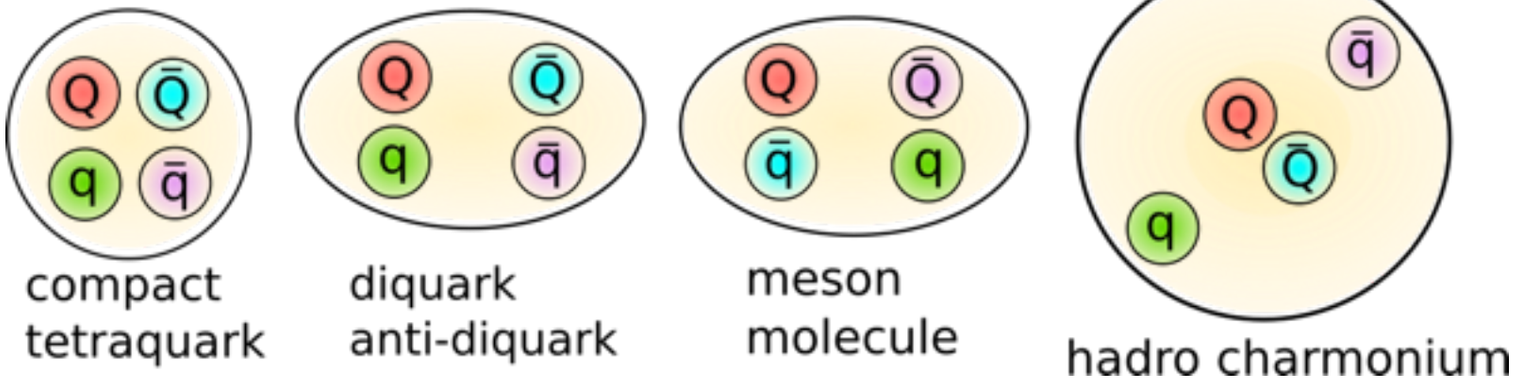
Eichman, CF, Heupel, PLB 753 (2016) 282-287

Heavy and light tetraquark



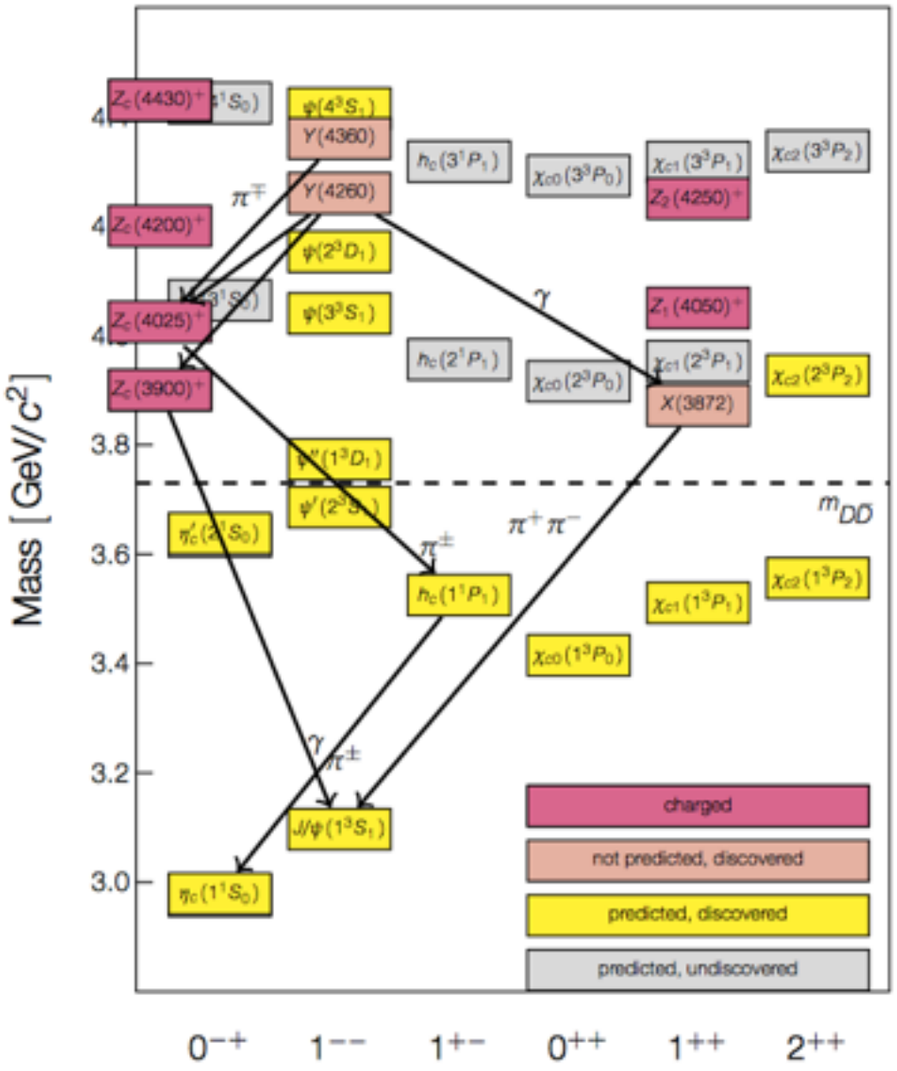
Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



Related to details of underlying QCD forces between quarks

Heavy and light tetraquark



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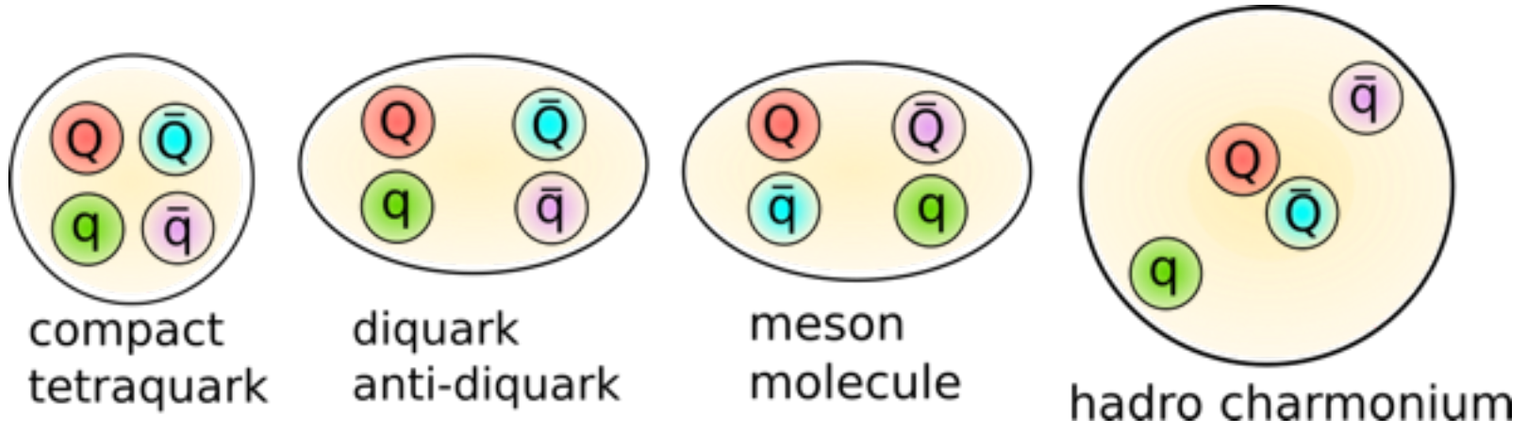
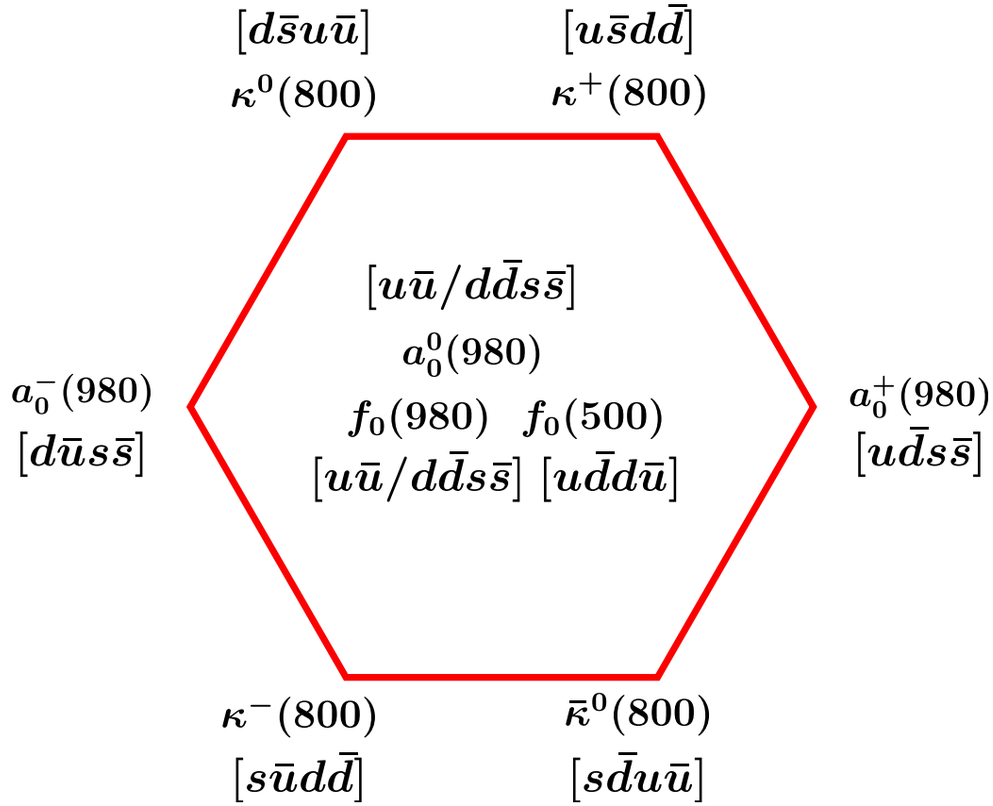
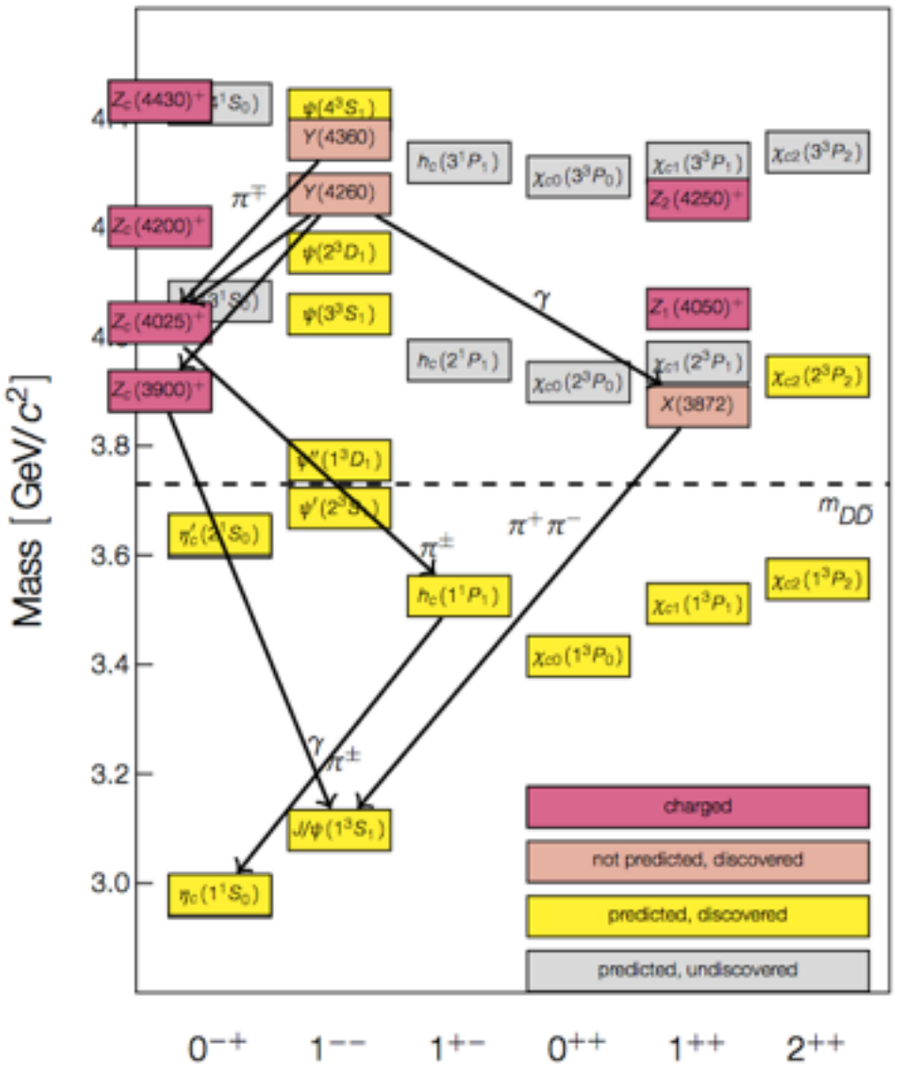




Heavy and light tetraquark



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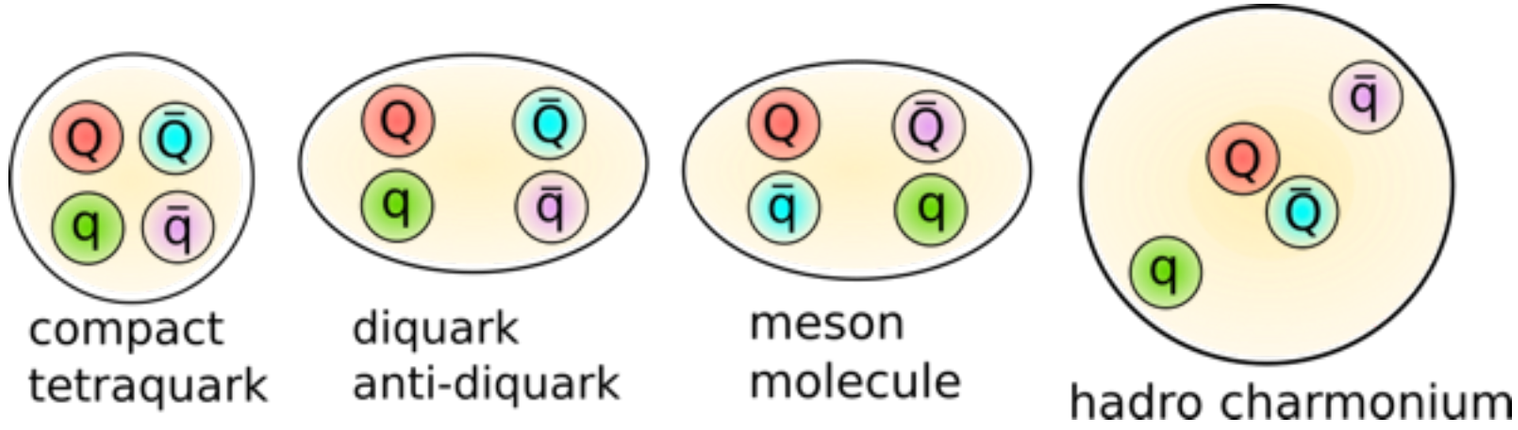
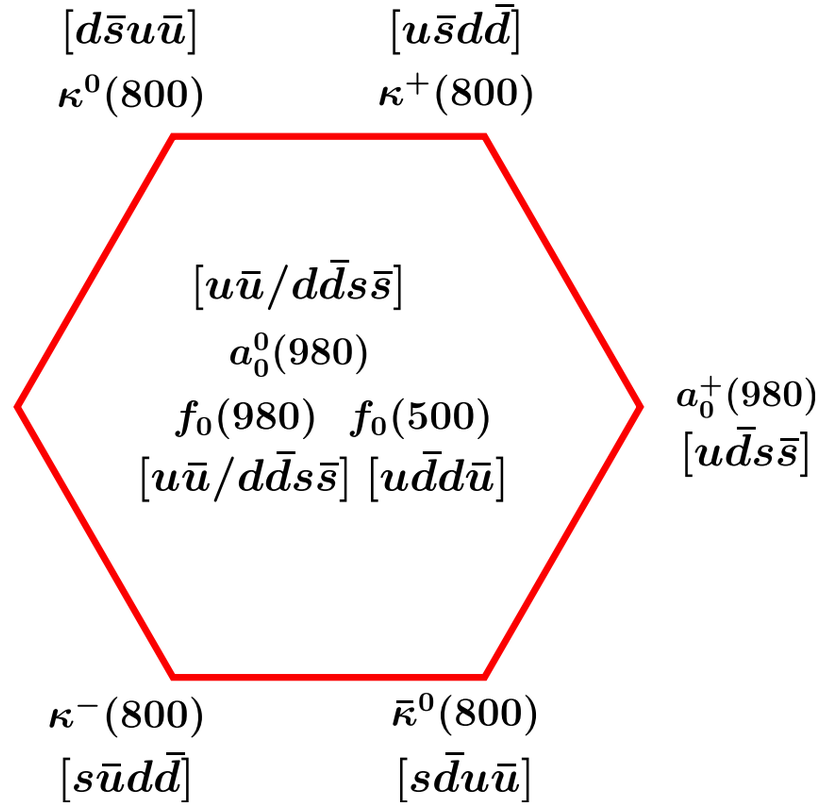


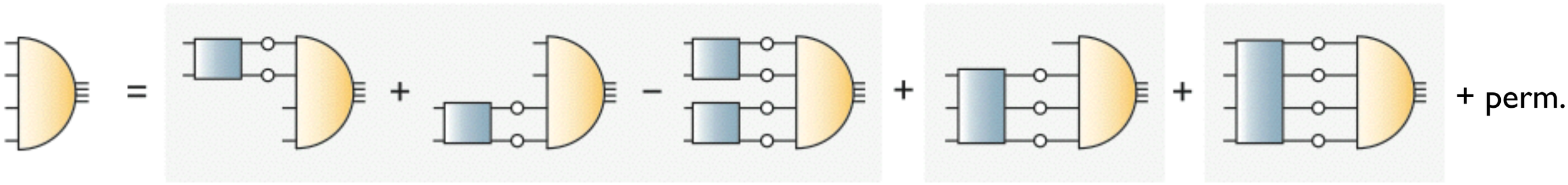
Relate QCD

Lattice: → S. Prelovsek
A. Martinez Torres
Y. Ikeda, G. Cheung, …



Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

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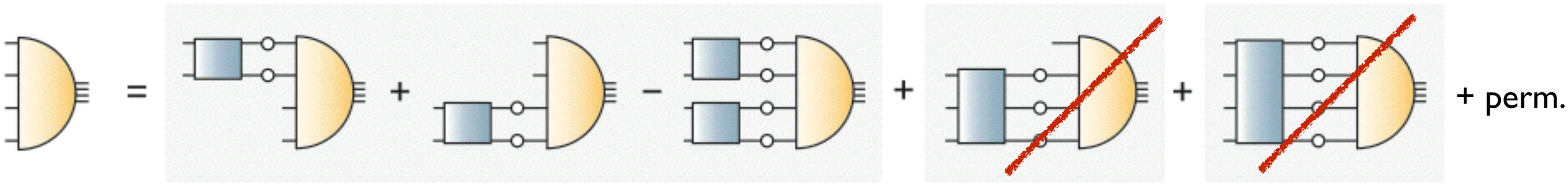

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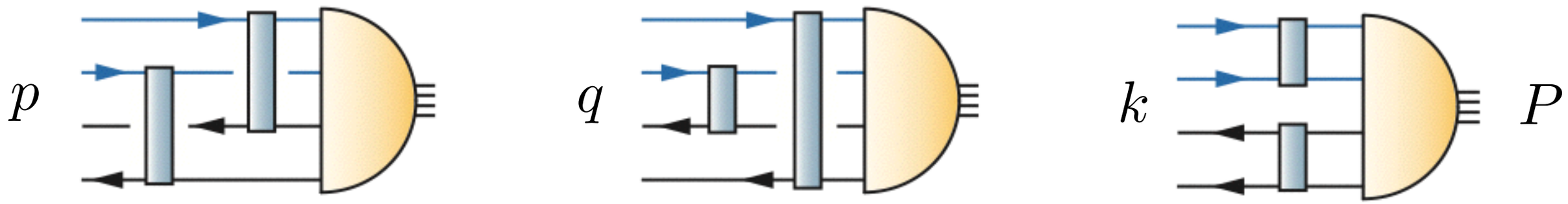
Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

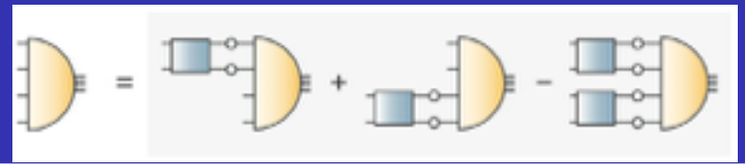
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

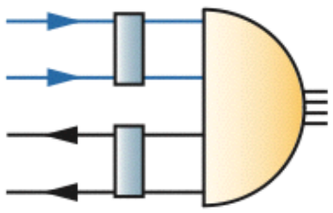
- reasonable approximation: keep s-waves only;
  → 16 tensor structures

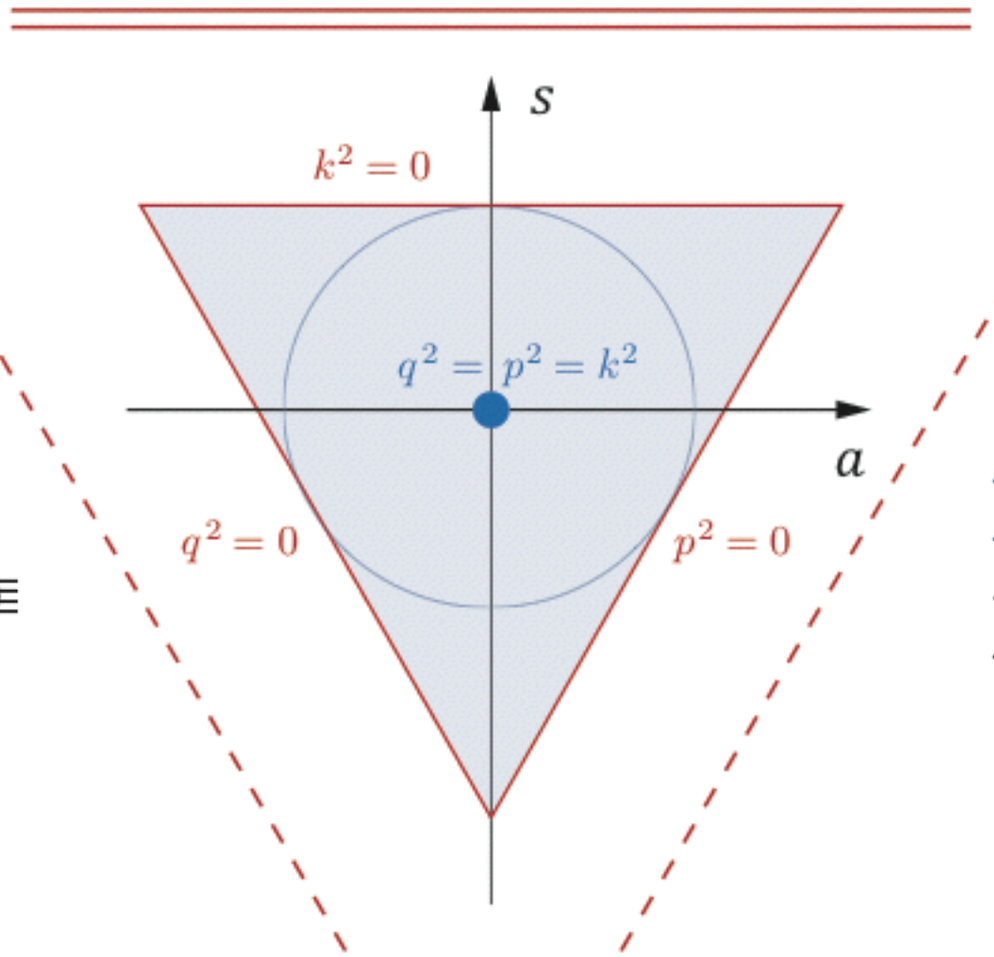
Four-body equation:



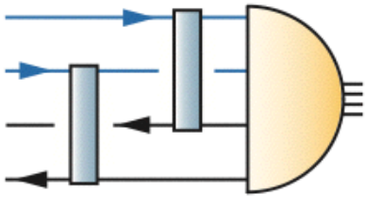
Organise Dirac-Lorentz-tensors into multiplets of **S4**

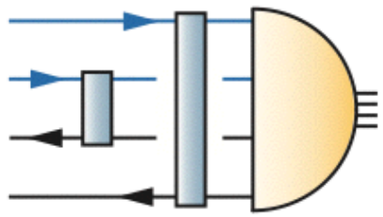
- Singlet, carries overall scale
- Doublet





diquark pole



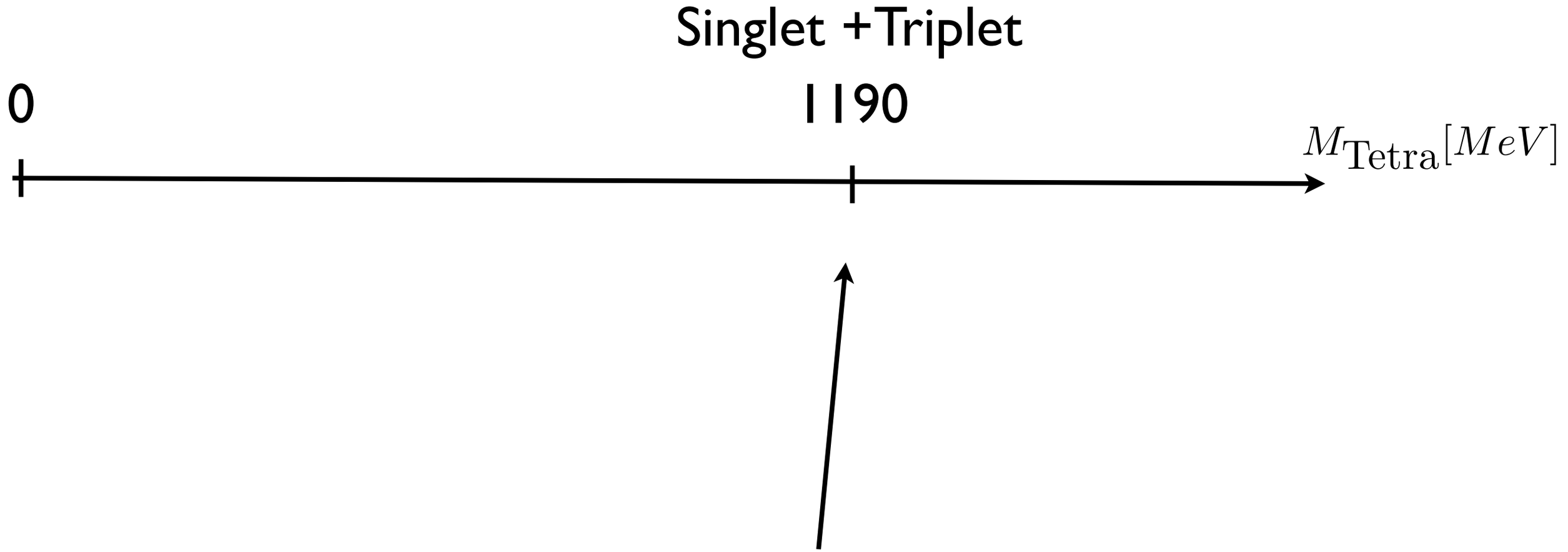


meson poles

- Two triplets

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance



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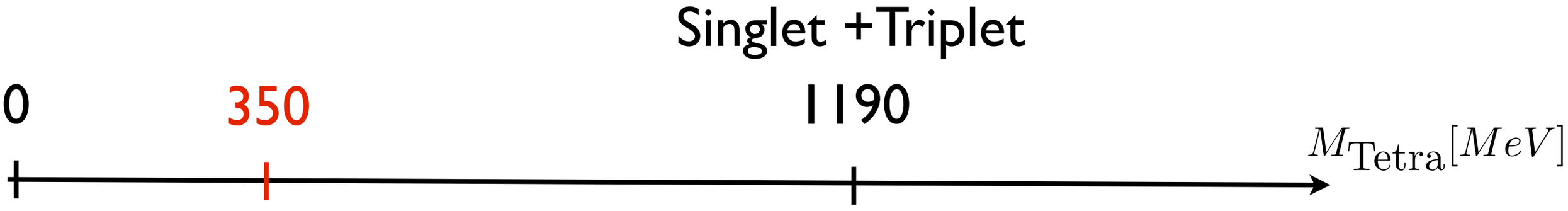
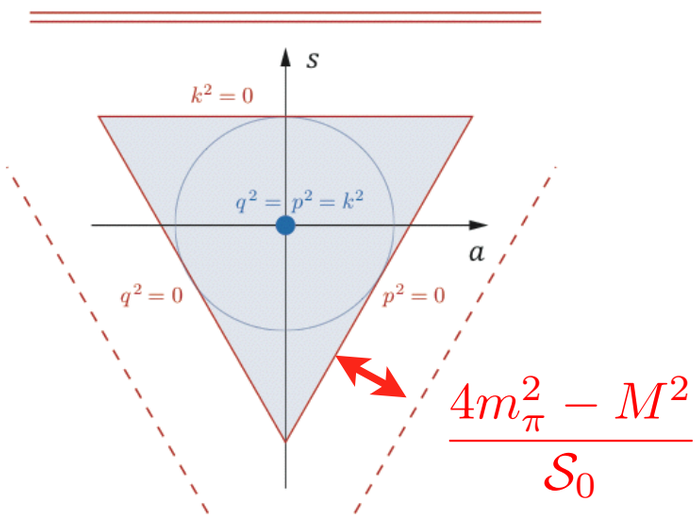


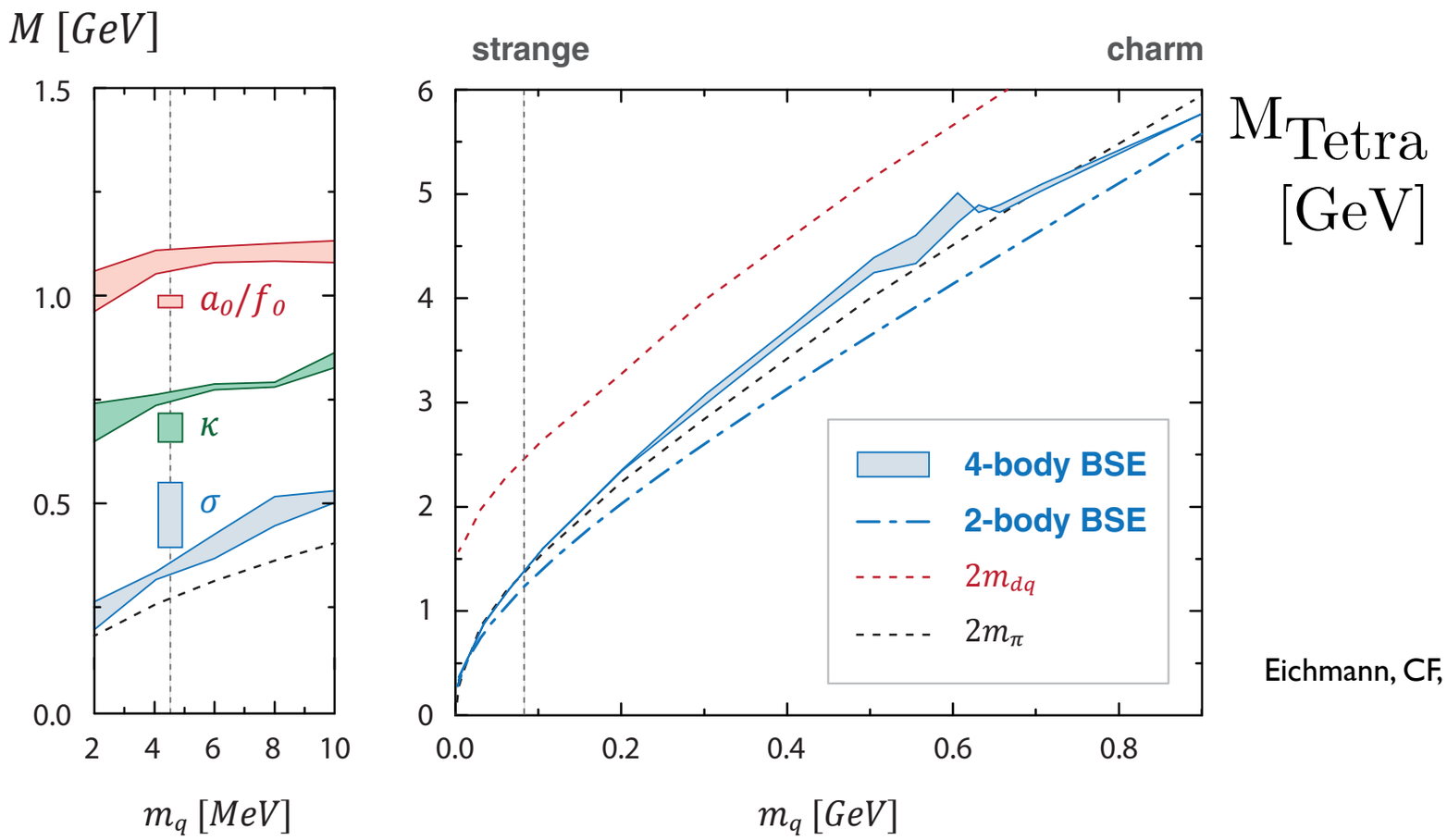
Singlet + Doublet



Two-pion resonance

Bound state of
four massive quarks

Mass evolution of tetraquark



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

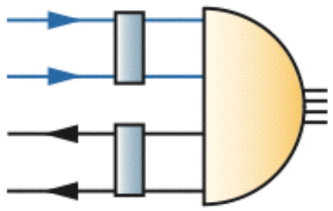
$$m_\sigma \sim 350\,\mathrm{MeV}$$
$$m_\kappa \sim 750\,\mathrm{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$
$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

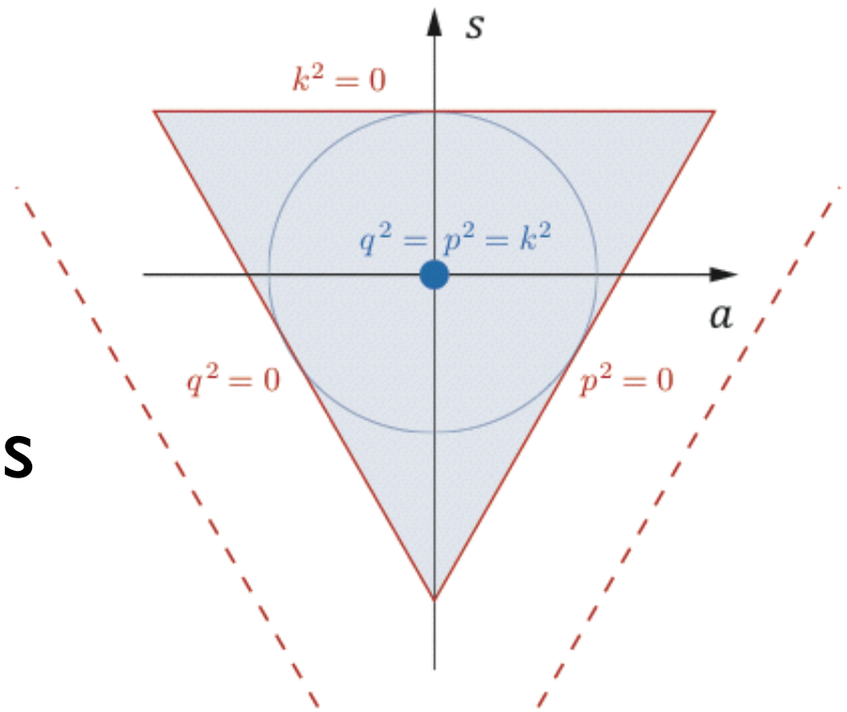
Outlook: heavy-light systems

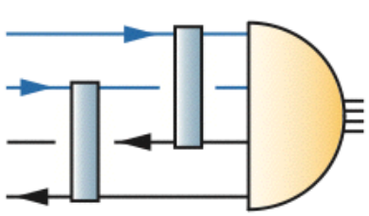
Dynamical situation in **S4**-doublet:



heavy-light diquarks

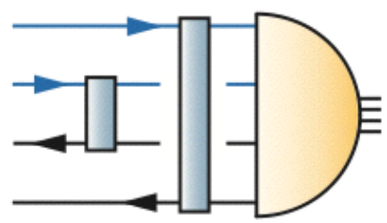
$$cq - \bar{c}\bar{q}$$





meson-molecules

$$c\bar{q} - \bar{c}q$$



hadro-charmonium

$$c\bar{c} - q\bar{q}$$

Dynamical decision of most important clustering!

Summary and outlook

Summary

- Baryon spectrum: good agreement with experiment!
- Three-body vs diquark-quark: fair agreement

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

- Tetraquarks dominated by internal meson-meson configurations
- Dynamical description of σ as π-π resonance

Outlook

- Baryons: transition form factors
- Tetraquarks: explore heavy-light systems
- Hybrids