Heavy Tetraquarks

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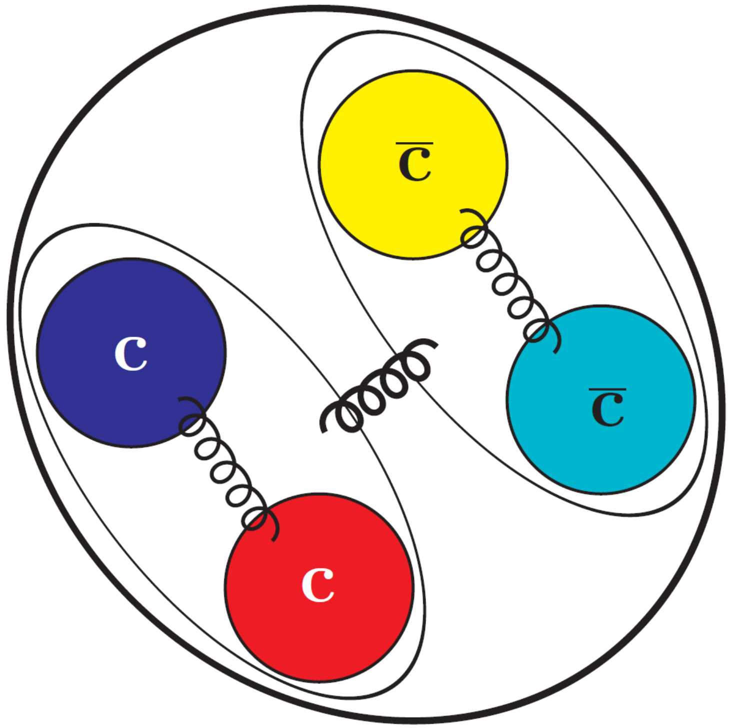
T_{4c} : The All-Charm Tetraquark



V. R. Debastiani and F. S. Navarra,
A non-relativistic model for the $[cc][\bar{c}\bar{c}]$ *tetraquark,*
arXiv:1706.07553 [*hep-ph*].

T_{4c} : The choice

- Few works on literature
  - None using diquark-antidiquark potential model including P-wave systematic treatment of spin-dependent interactions
- Mass $\sim$ 6 GeV
  - Energy Gap between $c\bar{c}$ and X Y Z ($\sim$3-4.5 GeV)
- Could be detected with current technology
  - Might decay mainly in $c\bar{c}$ pairs or leptons
- Restrictions due to Pauli Exclusion Principle
  - Less possibilities on quantum numbers
- Equal masses in 2-body systems
  - Simplifies spin-dependent interactions

T_{4c} : History

- First proposed by Y. Iwasaki in 1975
  - Y. Iwasaki, "A Possible Model for New Resonances-Exotics and Hidden Charm," Prog. Theor. Phys. 54, 492 (1975).
  - Y. Iwasaki, "How to Find $\eta_c$and a Possible State $c\bar{c}\,c\bar{c}$," Phys. Rev. D 16, 220 (1977).

- First study with diquark-antidiquark by K. T. Chao in 1981
  - K. T. Chao, "The (cc)-($\overline{cc}$) (Diquark - anti-Diquark) States in e^+e^- Annihilation," Z. Phys. C 7, 317 (1981).

- Four-body model in 1982
  - J. P. Ader, J. M. Richard and P. Taxil, "Do Narrow Heavy Multi-Quark States Exist?," Phys. Rev. D 25, 2370 (1982).
  - J. l. Ballot and J. M. Richard, "Four Quark States In Additive Potentials," Phys. Lett. B 123, 449 (1983).

- Potential model and MIT bag in 1985
  - L. Heller and J. A. Tjon, "On Bound States of Heavy $Q^2\bar{Q}^2$ Systems," Phys. Rev. D 32, 755 (1985).

- Chromomagnetic interaction in 1992
  - B. Silvestre-Brac, "Systematics of $Q^2\bar{Q}^2$ systems with a chromomagnetic interaction," Phys. Rev. D 46, 2179 (1992).
  - B. Silvestre-Brac and C. Semay, "Systematics of L = 0 $q^2\bar{q}^2$systems," Z. Phys. C 57, 273 (1993).

T_{4c} : Recent Works

- Four-body harmonic oscilator Hamiltonian:
  - R. J. Lloyd and J. P. Vary, "All charm tetraquarks," Phys. Rev. D 70, 014009 (2004).
- Hyperspherical harmonic formalism:
  - N. Barnea, J. Vijande and A. Valcarce, "Four-quark spectroscopy within the hyperspherical formalism," Phys. Rev. D 73, 054004 (2006).
- Lattice: tetraquark and meson molecule with $J^{PC} = 1^{--}$ around 6.4 GeV
  - T. W. Chiu et al. [TWQCD Collaboration], "Y(4260) on the lattice," Phys. Rev. D 73, 094510 (2006).
- Production (Single Parton Scattering):
  - A. V. Berezhnoy, A. K. Likhoded, A. V. Luchinsky, A. A. Novoselov, "Double J/psi-meson Production at LHC and 4c-tetraquark state", Phys. Rev. D 84, 094023 (2011),
  - A. V. Berezhnoy, A. V. Luchinsky, A. A. Novoselov, "Heavy tetraquarks production at the LHC", Phys. Rev. D 86, 034004 (2012).
- Bethe-Salpeter formalism
  - W. Heupel, G. Eichmann, C. S. Fischer, "Tetraquark Bound States in a Bethe-Salpeter Approach", Phys. Lett. B 718, 545 (2012).

T_{4c} : 2016 and 2017

- Production: (Double Parton Scattering):
  - F. Carvalho, E. R. Cazaroto, V. P. Gonçalves, F. S. Navarra, "Tetraquark Production in Double Parton Scattering", Phys. Rev. D 93, no. 3, 034004 (2016).
- QCD SUM RULES:
  - W. Chen, H. X. Chen, X. Liu, T. G. Steele, S. L. Zhu, "Hunting for exotic doubly hidden-charm/bottom tetraquark states", Phys. Lett. B 773, 247 (2017).
  - Z. G. Wang, ``Analysis of the $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules,'' Eur. Phys. J. C 77, no. 7, 432 (2017).
- Color-Magnetic model (S-wave):
  - J. Wu, Y. R. Liu, K. Chen, X. Liu and S. L. Zhu, "Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration", arXiv:1605.01134 [hep-ph].
- Estimates from meson and baryon masses, production and decays
  - M. Karliner, S. Nussinov and J. L. Rosner $QQ\bar{Q}\bar{Q}$ states: masses, production, and decays, Phys. Rev. D 95, no. 3, 034011 (2017).
- Stability check

J. M. Richard, A. Valcarce and J. Vijande, ``String dynamics and metastability of all-heavy tetraquarks,'' Phys. Rev. D 95, no. 5, 054019 (2017).

Diquarks

- Internal clusters, color state:
  - SU(3) color symmetry of QCD

$$|q\overline{q}\rangle : 3 \otimes \overline{3} = 1 \oplus 8$$
$$|qq\rangle : 3 \otimes 3 = \overline{3} \oplus 6$$
$$|\overline{qq}\rangle : \overline{3} \otimes \overline{3} = 3 \oplus \overline{6}$$

- Factorization of many-body problem:
  - Baryons: diquark-quark
    - ➢ Explain missing resonances
  - Tetraquarks: diquark-antidiquark
    - ➢ Heavy-light systems for X Y Z exotics:

$$[Qq] - [\bar{Q}\bar{q}], \quad Q \in (c, b), \quad q \in (u, d, s)$$

The Model

- Nonrelativistic:
  - Cornell-like Potential
  - Schrödinger equation

$$(\hbar = c = 1)$$

$$\left[\frac{1}{2\mu}\left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2}\right) + V^{(0)}(r)\right] y(r) = E\, y(r)$$

- Allows clear manipulation of:
  - Wavefunctions
  - Quantum numbers
  - Spin-dependent interactions
  - Radial and orbital excitations

$$V^{(0)} = V_V^{(0)} + V_S^{(0)}$$

$$V^{(0)}(r) = \kappa_s \frac{\alpha_s}{r} + br$$

- Successfull reproduction of Charmonium spectrum

$$3 \otimes \overline{3} \rightarrow 1 \;:\; \kappa_s = -4/3$$

$$3 \otimes 3 \rightarrow \overline{3} \;:\; \kappa_s = -2/3$$

- First step to guide:
  - Experimental searches
  - More precise models

$$M(c\bar{c}) = 2m_c + E_{c\bar{c}} + \langle V_{Spin}^{(1)} \rangle_{c\bar{c}}$$

A good description of charmonium spectrum can be obtained with

Spin-spin as zeroth-order smeared by a Gaussian (new parameter σ).

$$V^{(0)}(r) = \kappa_s \frac{\alpha_s}{r} + br - \frac{8\pi\kappa_s\alpha_s}{3m_c^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\mathbf{S}_1\cdot\mathbf{S}_2$$

Spin-orbit and Tensor as first-order pertubative corrections.

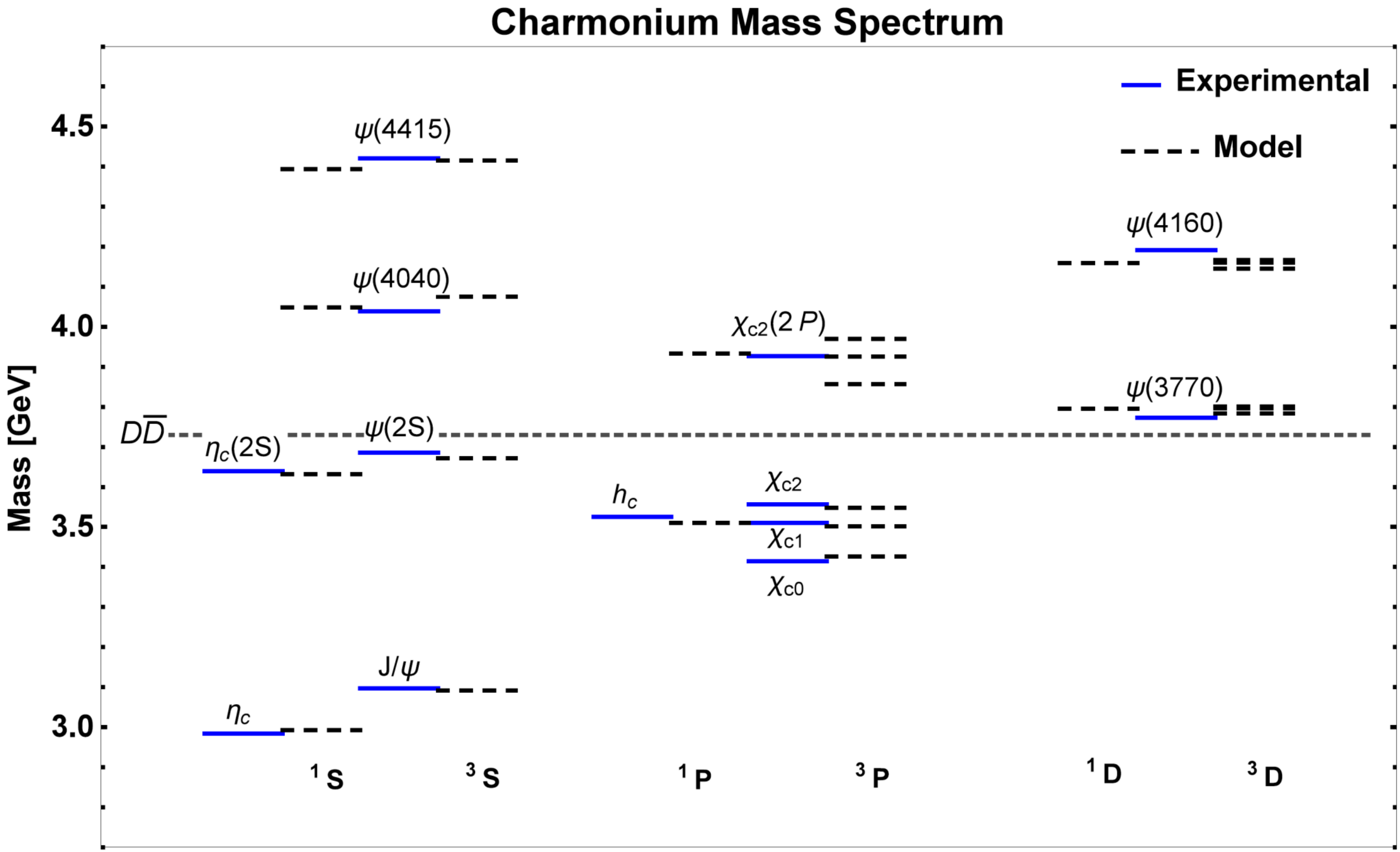
$$V^{(1)}(r) = \left(-\frac{3\kappa_s\alpha_s}{2m^2}\frac{1}{r^3} - \frac{b}{2m^2}\frac{1}{r}\right)\mathbf{L}\cdot\mathbf{S} + -\frac{\kappa_s\alpha_s}{4m^2}\frac{1}{r^3}\mathbf{S}_{12}$$

$$\mathbf{S}_{12} = 4[3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - \mathbf{S}_1\cdot\mathbf{S}_2]$$

We have made a FIT in all charmonium well-stablished states, including $h_c(1P)$ and $\chi_{c2}(2P)$, obtaining:

$$m_c = 1.4622\ \mathrm{GeV},\ \alpha_s = 0.5202,\ b = 0.1463\ \mathrm{GeV}^2,\ \sigma = 1.0831\ \mathrm{GeV}$$

T. Barnes, S. Godfrey and E. S. Swanson, "*Higher charmonia*," Phys. Rev. D **72**, 054026 (2005).



$$m_c = 1.4622 \text{ GeV}, \, \alpha_s = 0.5202, \, b = 0.1463 \text{ GeV}^2, \, \sigma = 1.0831 \text{ GeV}$$

Nice agreement!

Diquark

$$V_{cc} = \frac{1}{2} V_{c\bar{c}}$$

- We use the same parameters, but color factor -2/3 instead of -4/3, and also half of the string tension: $b_{cc} = b_{c\bar{c}}/2$
- Identical fermions cc → anti-symmetric wavefuncion
- S-wave : $\begin{cases} \bar{\mathbf{3}} \rightarrow \mathbf{S} = \mathbf{1} \\ \mathbf{6} \rightarrow \mathbf{S} = \mathbf{0} \end{cases}$ $\quad M(cc) = 2m_c + E_{cc} + \langle V^{(1)}_{Spin} \rangle_{cc}$
- Ground state Antitriplet Color diquark → Spin 1 (1^3S_1), like J/ψ

$N^{2S+1}\ell$	$M^{(0)}$ [GeV]	$\vert R(0)\vert^2$ [GeV^3]	$\langle r^2 \rangle^{1/2}$ [fm]	$\left\langle \frac{v^2}{c^2} \right\rangle$
$\mathbf{1^3S}$	**3.1334**	**0.3296**	**0.593**	**0.123**
1^1P	3.3530	0	0.906	0.141

$N^{2S+1}\ell_J$	$\langle T \rangle$	$\langle V_V^{(0)} \rangle$	$\langle V_S^{(0)} \rangle$	$\langle V_{SS}^{(0)} \rangle$	$E^{(0)}$	$M^{(0)}$ [MeV]	$\langle V_{LS}^{(1)} \rangle$	$\langle V_T^{(1)} \rangle$	M^f [MeV]
$\mathbf{1^3S_1}$	**180.4**	**−173.9**	**197.9**	**4.7**	**209.0**	**3133.4**	**0**	**0**	**3133.4**
1^1P_1	206.7	−93.3	316.2	−0.9	428.7	3353.0	0	0	3353.0

Tetraquark

- Same parameters of charmonium, with **diquark mass**
- Two-body problem $M(T_{4c}) = m_{cc} + m_{\bar{c}\bar{c}} + E_{[cc][\bar{c}\bar{c}]} + \langle V^{(1)}_{Spin} \rangle_{[cc][\bar{c}\bar{c}]}$
- Only diquark $\mathbf{1^3 S_1}$ are used to build the tetraquark
- Spin 1 x Spin 1 →New quantum numbers!
- We also include between diquark-antidiquark:
  - ✓ **Orbital** excitation (negative Parity!)
  - ✓ **Radial** excitation

$$C_T = (-1)^{L_T + S_T}$$

$$P_T = (-1)^{L_T}$$

S_T	L_T	J_T	J^{PC}
0	0	0	0^{++}
1	0	1	1^{+-}
2	0	2	2^{++}
0	1	1	1^{--}
1	1	2	2^{-+}
1	1	1	1^{-+}
1	1	0	0^{-+}
2	1	3	3^{--}
2	1	2	2^{--}
2	1	1	1^{--}

- We can get one EXOTIC quantum number
  - ✓ 1^{-+}

- Spin 1 x Spin 1 → **Spin-Spin** and **Spin-Orbit** easy!

$\mathbf{S_1} \cdot \mathbf{S_2}$ $\quad\quad$ $\mathbf{L} \cdot \mathbf{S}$

→ What about the **Tensor**? → *Spherical Representation of the Tensor*

$$\mathbf{S_{12}} = 4[3(\mathbf{S_1} \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) - \mathbf{S_1} \cdot \mathbf{S_2}] = 4[T_0 + T_0' + T_1 + T_{-1} + T_2 + T_{-2}]$$

$$\begin{aligned}
T_0 &= (3\cos^2\theta - 1)\; S_{1z}S_{2z} \\
T_0' &= -\frac{1}{4}(3\cos^2\theta - 1)\;(S_{1+}S_{2-} + S_{1-}S_{2+}) \\
T_1 &= \frac{3}{2}\sin\theta\cos\theta \mathrm{e}^{-i\varphi}\;(S_{1z}S_{2+} + S_{1+}S_{2z}) \\
T_{-1} &= \frac{3}{2}\sin\theta\cos\theta \mathrm{e}^{i\varphi}\;(S_{1z}S_{2-} + S_{1-}S_{2z}) \\
T_2 &= \frac{3}{4}\sin^2\theta \mathrm{e}^{-2i\varphi}\; S_{1+}S_{2+} \\
T_{-2} &= \frac{3}{4}\sin^2\theta \mathrm{e}^{2i\varphi}\; S_{1-}S_{2-}
\end{aligned}$$

$$\begin{aligned}
Y_2^0 &= \sqrt{\frac{5}{4\pi}}(\frac{3}{2}\cos^2\theta - \frac{1}{2}) \\
Y_2^{-1} &= +\sqrt{\frac{15}{8\pi}}(\sin\theta\cos\theta)\mathrm{e}^{-i\varphi} \\
Y_2^{1} &= -\sqrt{\frac{15}{8\pi}}(\sin\theta\cos\theta)\mathrm{e}^{i\varphi} \\
Y_2^{-2} &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta \mathrm{e}^{-2i\varphi} \\
Y_2^{2} &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta \mathrm{e}^{2i\varphi}
\end{aligned}$$

$$\int Y_{\ell'}^{m'*}(\theta,\varphi)Y_2^q(\theta,\varphi)Y_\ell^m(\theta,\varphi)\mathrm{d}\Omega$$

$$\begin{aligned}
\ell' &= \ell,\ \ell-2,\ \ell+2 \\
m' &= m+q
\end{aligned}$$

The Tetraquark Wavefunction

$$[(S_d = 1) \otimes (S_{\bar{d}} = 1) \to (S_T = 1)] \otimes (L_T = 1) \longrightarrow |J_T, M_{J_T}\rangle = |0,0\rangle_{J_T}$$

$$= \frac{1}{\sqrt{3}}|1,1\rangle_{S_T} \otimes |1,-1\rangle_{L_T} - \frac{1}{\sqrt{3}}|1,0\rangle_{S_T} \otimes |1,0\rangle_{L_T} + \frac{1}{\sqrt{3}}|1,-1\rangle_{S_T} \otimes |1,1\rangle_{L_T}$$

$$= \frac{1}{\sqrt{3}}\Big(\frac{1}{\sqrt{2}}|1,1\rangle_{12} \otimes |1,0\rangle_{34} - \frac{1}{\sqrt{2}}|1,0\rangle_{12} \otimes |1,1\rangle_{34}\Big)Y_1^{-1}(\theta,\varphi)$$

$$- \frac{1}{\sqrt{3}}\Big(\frac{1}{\sqrt{2}}|1,1\rangle_{12} \otimes |1,-1\rangle_{34} - \frac{1}{\sqrt{2}}|1,-1\rangle_{12} \otimes |1,1\rangle_{34}\Big)Y_1^{0}(\theta,\varphi)$$

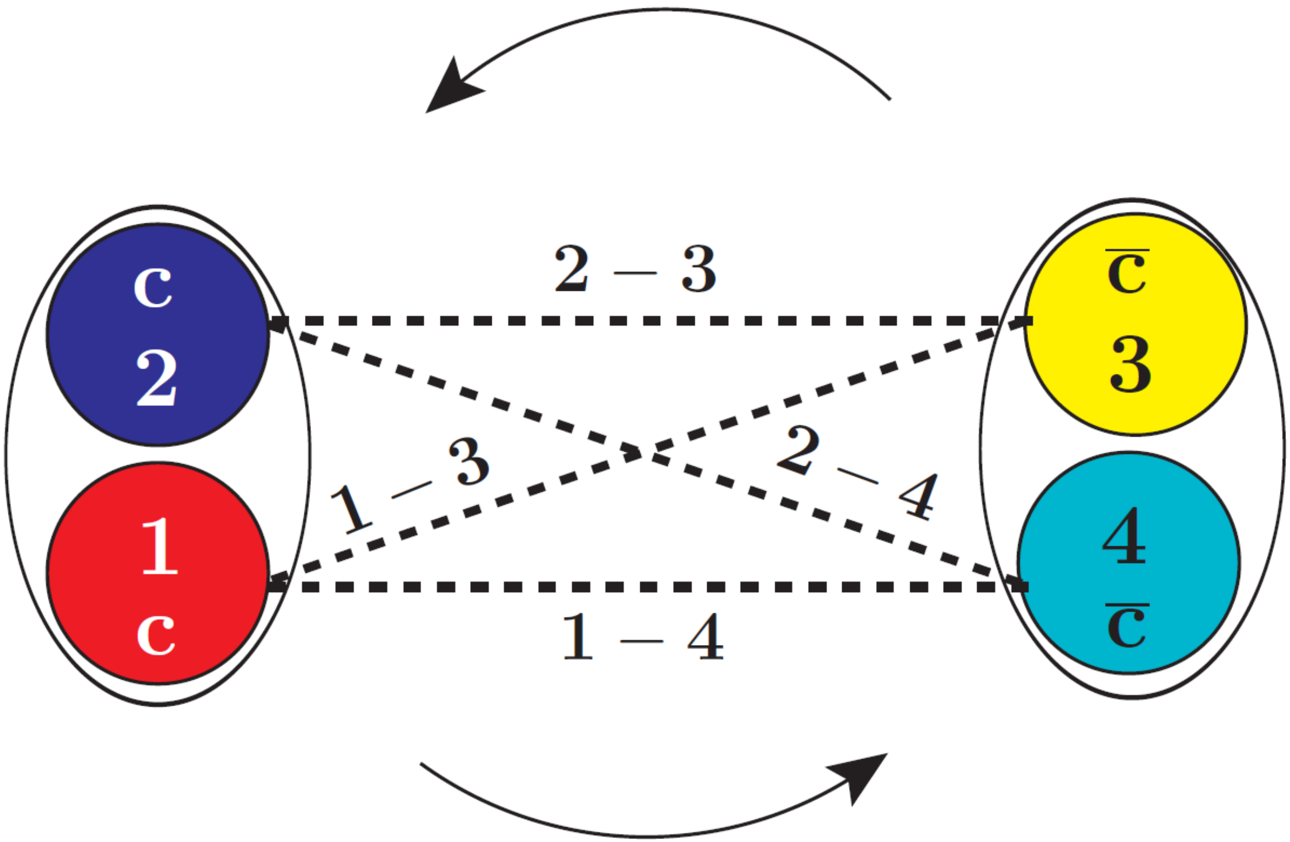
$$+ \frac{1}{\sqrt{3}}\Big(\frac{1}{\sqrt{2}}|1,0\rangle_{12} \otimes |1,-1\rangle_{34} - \frac{1}{\sqrt{2}}|1,-1\rangle_{12} \otimes |1,0\rangle_{34}\Big)Y_1^{1}(\theta,\varphi)$$

$$= \frac{1}{\sqrt{3}}\Big(\frac{1}{2}(\uparrow\uparrow\uparrow\downarrow + \uparrow\uparrow\downarrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow)\Big)Y_1^{-1}(\theta,\varphi)$$

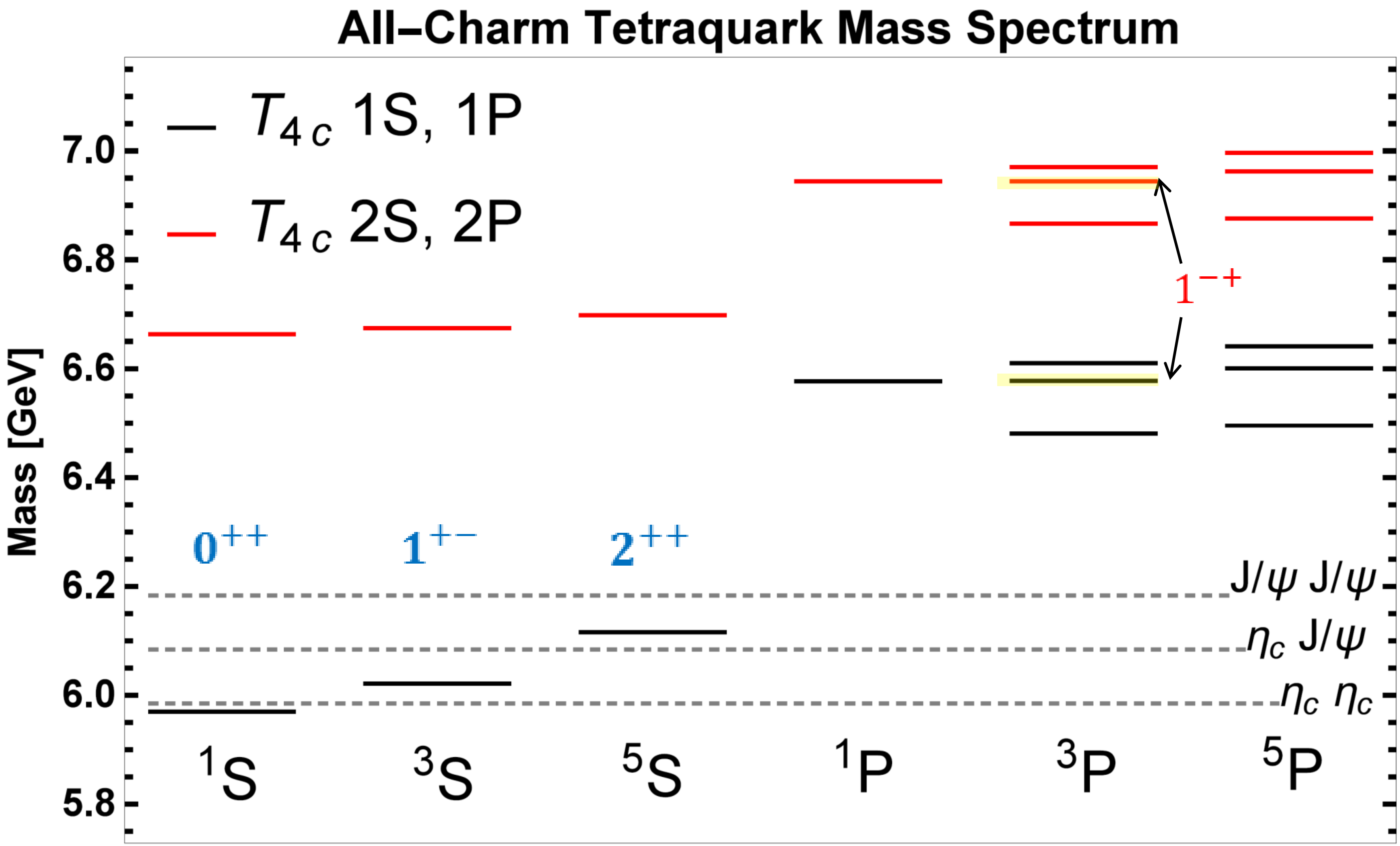
$$- \frac{1}{\sqrt{3}}\Big(\frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow\uparrow)\Big)Y_1^{0}(\theta,\varphi)$$

$$+ \frac{1}{\sqrt{3}}\Big(\frac{1}{2}(\uparrow\downarrow\uparrow\uparrow + \downarrow\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow\downarrow - \downarrow\downarrow\downarrow\uparrow)\Big)Y_1^{1}(\theta,\varphi)$$

The tensor acting in the diquark-antidiquark system with relative orbital angular momentum



- Within the 2-body approximation of point-like diquarks, it is equivalent:
  - One tensor acting between two spin 1 constituents: $[cc] - [\bar{c}\bar{c}]$
  - Four tensors acting between two spin ½ constituents: each $c\bar{c}$ pair
  - The four $c\bar{c}$ pairs yield the same result due to the symmetry of the angular wavefunctions



For consistence, $c\bar{c}$ pair thresholds are calculated with model predictions for $c\bar{c}$ mass.

- 0^{++} might be just a few MeV under the $\eta_c\ \eta_c$ threshold.
- 1^{+-} is below the $\eta_c\ J/\psi$ threshold, and can't decay in $\eta_c\ \eta_c$
- 2^{++} is below the $J/\psi\ J/\psi$ threshold, but can decay to $\eta_c\ \eta_c$ in *D*-wave.

TABLE IX: Results for T_{4c} masses using ground state (1^3S_1) diquarks. Parameters are $m_{cc} = 3133.4$ GeV, $\alpha_s = 0.5202$, $b = 0.1463$ GeV^2, $\sigma = 1.0831$ GeV.

$N^{2S+1}\ell_J$	$\langle T\rangle$	$\langle V_V^{(0)}\rangle$	$\langle V_S^{(0)}\rangle$	$\langle V_{SS}^{(0)}\rangle$	$E^{(0)}$	$M^{(0)}$ [MeV]	$\langle V_{LS}^{(1)}\rangle$	$\langle V_T^{(1)}\rangle$	M^f [MeV]	J^{PC}
1^1S_0	624.0	−966.6	151.1	−106.0	−297.3	5969.4	0	0	5969.4	0^{++}
1^3S_1	574.8	−928.0	157.6	−50.2	−245.8	6020.9	0	0	6020.9	1^{+-}
1^5S_2	479.4	−847.5	172.5	44.3	−151.3	6115.4	0	0	6115.4	2^{++}
1^1P_1	372.6	−371.8	325.3	−15.8	310.3	6577.1	0	0	6577.1	1^{--}
1^3P_0	358.9	−364.3	330.7	−7.4	318.0	6584.7	−59.4	−44.8	6480.4	0^{-+}
1^3P_1	358.9	−364.3	330.7	−7.4	318.0	6584.7	−29.7	22.4	6577.4	1^{-+}
1^3P_2	358.9	−364.3	330.7	−7.4	318.0	6584.7	29.7	−4.5	6609.9	2^{-+}
1^5P_1	335.4	−350.8	340.7	6.4	331.7	6598.4	−75.9	−27.2	6495.4	1^{--}
1^5P_2	335.4	−350.8	340.7	6.4	331.7	6598.4	−25.3	27.1	6600.2	2^{--}
1^5P_3	335.4	−350.8	340.7	6.4	331.7	6598.4	50.6	−7.7	6641.2	3^{--}
2^1S_0	410.8	−397.0	404.6	−21.8	396.6	6663.3	0	0	6663.3	0^{++}
2^3S_1	408.7	−398.2	408.7	−11.4	407.8	6674.5	0	0	6674.5	1^{+-}
2^5S_2	403.0	−400.7	416.8	12.3	431.4	6698.1	0	0	6698.1	2^{++}
2^1P_1	414.9	−262.9	537.5	−12.0	677.4	6944.1	0	0	6944.1	1^{--}
2^3P_0	407.8	−260.0	541.2	−5.7	683.3	6950.0	−47.9	−35.6	6866.5	0^{-+}
2^3P_1	407.8	−260.0	541.2	−5.7	683.3	6950.0	−23.9	17.8	6943.9	1^{-+}
2^3P_2	407.8	−260.0	541.2	−5.7	683.3	6950.0	23.9	−3.6	6970.4	2^{-+}
2^5P_1	394.5	−254.2	548.7	5.2	694.3	6961.0	−63.1	−22.2	6875.6	1^{--}
2^5P_2	394.5	−254.2	548.7	5.2	694.3	6961.0	−21.0	22.2	6962.1	2^{--}
2^5P_3	394.5	−254.2	548.7	5.2	694.3	6961.0	42.1	−6.3	6996.7	3^{--}

S-wave

P-wave

S-wave

P-wave

Radial Excitations

Comparison with Thresholds

TABLE II. Lowest S-wave two-meson thresholds (MeV) for all J^{PC} quantum numbers.

		J^{PC}	$M_1 M_2$	Model	Exp.
~Below?	**5969.4**	0^{++}	$\eta_c(1S)\eta_c(1S)$	5984.8	5966.8
Below	**6020.9**	1^{+-}	$J/\psi(1S)\eta_c(1S)$	6084.1	6080.3
Below	**6115.4**	2^{++}	$J/\psi(1S)J/\psi(1S)$	6183.4	6193.8
Above	**6480.4**	0^{-+}	$\eta_c(1S)\chi_{c0}(1P)$	6418.2	6398.1
Above	**6577.4**	1^{-+}	$\eta_c(1S)\chi_{c1}(1P)$	6494.2	6494.1
Above / *~Below?*	**6577.1** / **6495.4**	1^{--}	$\eta_c(1S)h_c(1P)$	6502.9	6508.8
Above	**6609.9**	2^{-+}	$\eta_c(1S)\chi_{c2}(1P)$	6540.5	6539.6
Above/Below?	**6600.2**	2^{--}	$J/\psi(1S)\chi_{c1}(1P)$	6593.5	6607.6

Phys. Rev. D 73, 054004 (2006).

Errors

- We can roughly estimate the error in charmonium fit by taking $\frac{\sqrt{\chi^2}}{N}$ where N=13 states (input). This yields **6.1 MeV**, too little.
- If we take $\sqrt{\chi^2/N}$ we get **22 MeV**, maybe too much.

- If instead of fitting 13 states, we fit only the 8 states below the $D\overline{D}$ threshold, the accuracy improves, and the parameters change just a little.

- With this other set of parameters, the tetraquark results are different by about **10 MeV**, but still qualitatively the same.

T_{4c} wavefunctions properties

$N^{2S+1}\ell$	$M^{(0)}$ [GeV]	$\|R(0)\|^2$ [GeV^3]	$\langle r^2\rangle^{1/2}$ [fm]	$\left\langle \frac{v^2}{c^2} \right\rangle$
1^1S	5.9694	8.4219	0.232	0.199
1^3S	6.0209	7.8384	0.241	0.183
1^5S	6.1154	6.6727	0.264	0.153
1^1P	6.5771	0	0.471	0.119
1^3P	6.5847	0	0.478	0.115
1^5P	6.5984	0	0.491	0.107
2^1S	6.6633	2.8414	0.588	0.131
2^3S	6.6745	2.8528	0.595	0.130
2^5S	6.6981	2.8616	0.610	0.129
2^1P	6.9441	0	0.785	0.132
2^3P	6.9500	0	0.790	0.130
2^5P	6.9610	0	0.800	0.126

Is the linear confinement faking a bound state?

- **NO**. If we turn off the linear confinement and repeat the calculation with the same parameters, we get:

- Diquark $\mathbf{1^3 S_1}$ = **2881.36** MeV

(lower since we don't the positive contribution of the linear term)

- $\mathbf{0^{++}}$ 5328.4 MeV
- $\mathbf{1^{+-}}$ 5374.9 MeV
- $\mathbf{2^{++}}$ 5452.3 MeV
- All S-wave below $\eta_c\ \eta_c$!

- Close to the results using the Bethe-Salpeter formalism
  - W. Heupel, G. Eichmann, C. S. Fischer, "Tetraquark Bound States in a Bethe-Salpeter Approach", Phys. Lett. B 718, 545 (2012).

T_{4c} : Comparisons

S-wave

J^{PC}	M_1^f [GeV]	M_{1-LS}^f [GeV]	M_2^f [GeV]	ref. [1]	ref. [2]	ref. [3]
0^{++}	5.8359	5.9073	5.9694	5.966	5.617 – 6.254	6.44 – 7.15
1^{+-}	6.0023	6.0122	6.0209	6.051	5.720 – 6.137	6.37 – 6.51
2^{++}	6.3352	6.2221	6.1154	6.223	5.777 – 6.194	6.51 – 6.37

P-wave

J^{PC}	$N^{2S+1}\ell_J$	M_1^f [GeV]	M_{1-LS}^f [GeV]	M_2^f [GeV]	ref. [4]	ref. [3]	ref. [5]
1^{--}	1^1P_1	6.5954	6.5447	6.5771	6.55 – 6.82	6.83 – 6.84	6.420
1^{--}	1^5P_1	6.5250	6.4676	6.4954	6.39		

- Ref. [1] : A. V. Berezhnoy, A. V. Luchinsky, A. A. Novoselov, "Heavy tetraquarks production at the LHC", Phys. Rev. D 86, 034004 (2012).
- Ref. [2] : J. Wu, Y. R. Liu, K. Chen, X. Liu and S. L. Zhu, "Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration," arXiv:1605.01134 [hep-ph] (2016).
- Ref. [3] : W. Chen, H. X. Chen, X. Liu, T. G. Steele, S. L. Zhu, "Hunting for exotic doubly hidden-charm/bottom tetraquark states", arXiv:1605.01647 [hep-ph] (2016).
- Ref [4] : K. T. Chao, "The (cc)-($\overline{cc}$) (Diquark - anti-Diquark) States in e^+e^- Annihilation," Z. Phys. C 7, 317 (1981).
- Ref [5] : T. W. Chiu et al. [TWQCD Collaboration], "Y(4260) on the lattice," Phys. Rev. D 73, 094510 (2006).

Conclusions for T_{4c}

- **Nonrelativistic** treatment + diquark-antidiquark **2-body** factorization:
  - Wavefunctions, **Orbital** and **Radial** excitations.
  - Contribution of each term of the Potential and Spin-dependent interactions

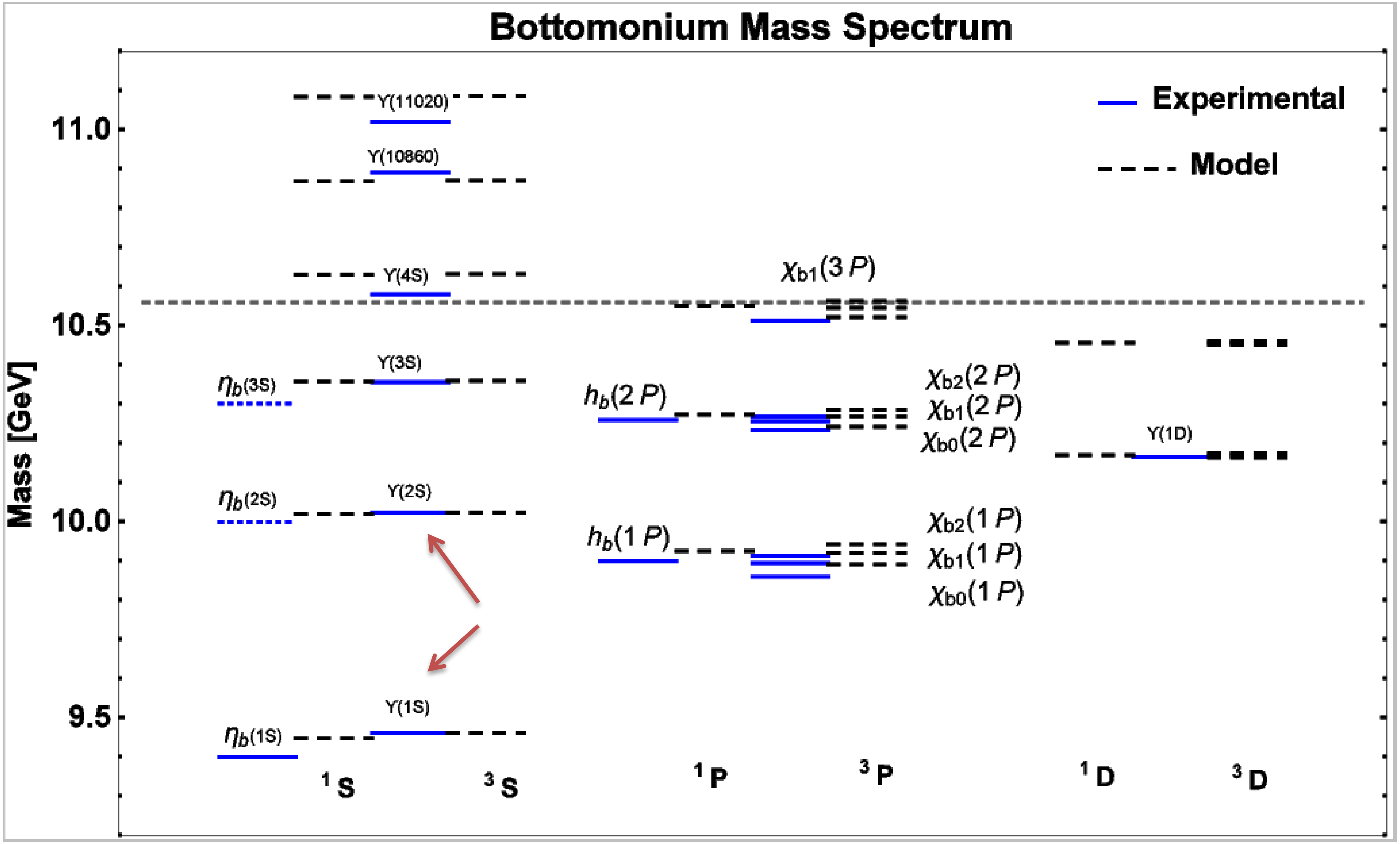
- Most of the **excited** states are **above** the thresholds for dissociation into $c\bar{c}$ pairs
- The lowest states **(1S) might be stable** against decay into $c\bar{c}$ pairs → Decay into **Leptons**

- Tetraquarks $[cc] - [\overline{cc}]$ are very **compact** and ~**Nonrelativistic** ($< v^2/c^2 > \sim \leq 0.2$)
  - $\sim$0.3 fm for 1S, $\sim$0.5 fm for 1P and 2S and $\sim$0.8 fm for 2P
  - **Smaller than the diquarks** $\sim$0.6 fm → 2-body factorization OK?

- The **P-wave** states might be better approximations, since the centrifugal barrier contributes to the **separation between diquark and antidiquark**
- In **S-wave** tetraquarks:
  - The One Gluon Exchange is very strong and dominates over confinement.
  - The Spin-Spin interaction is also very strong even though it has a factor $1/{m_{cc}}^2$
- **Tetraquarks also bind without the linear confinement! Only OGE → Lower masses!**

T_{4b} : The All-**Bottom** Tetraquark

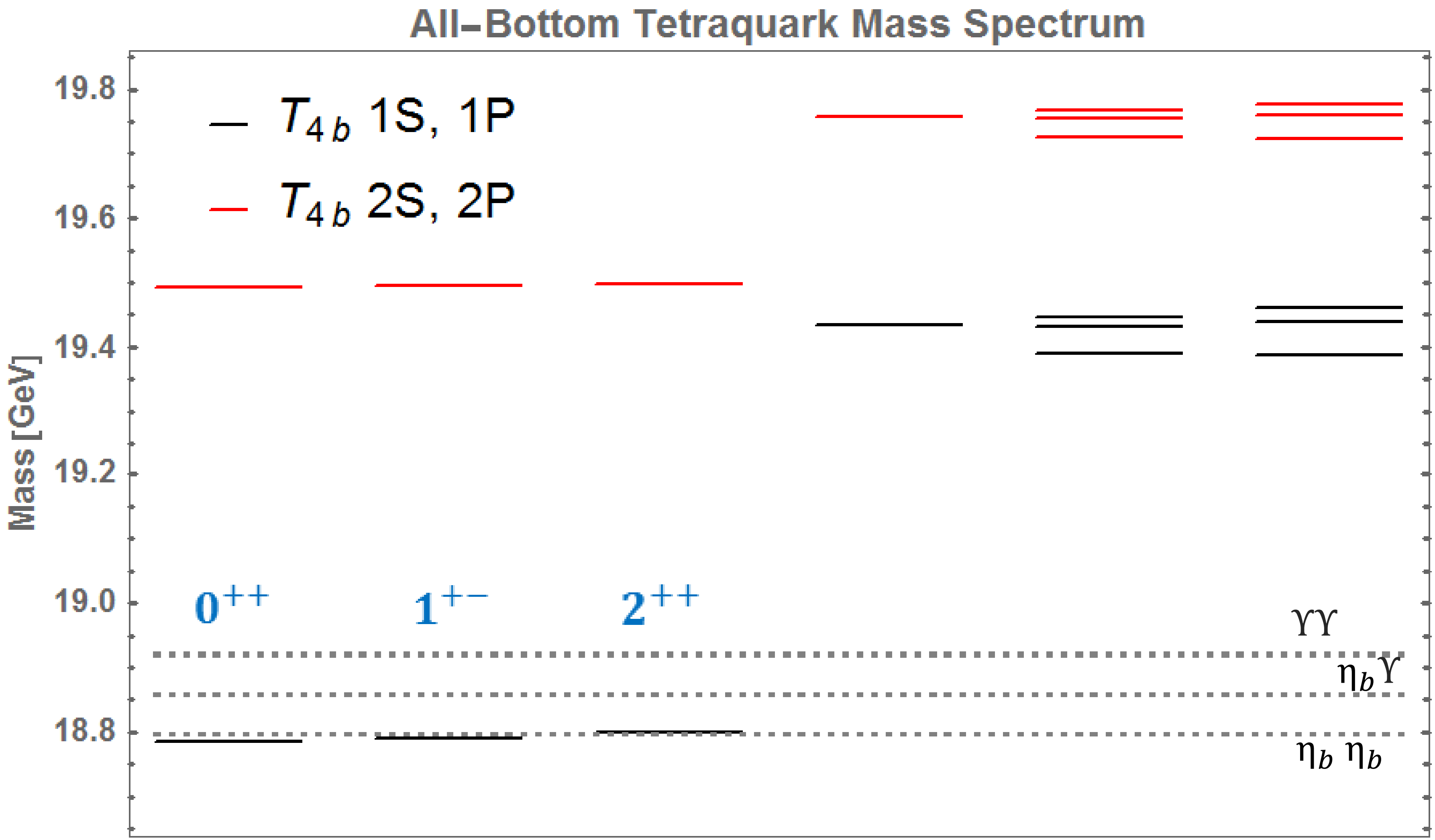
- We have some *preliminary* results for the T_{4b}
- Now we fit the bottomonium spectrum to get the parameters
- And repeat the same procedure to get the diquark and tetraquark



- Here we fit **only** $\Upsilon(1S)$ and $\Upsilon(2S)$. Notice that $\Upsilon(3S)$ comes for free!
- To fine-tune the spin-spin splitting and orbital excitations more states need to be included.

$$m_b = 4.7940 \text{ GeV},\ \alpha_s = 0.3845,\ b = 0.1729 \text{ GeV}^2,\ \sigma = 0.9200 \text{ GeV}$$

We find the $\mathbf{1^3 S_1}$ diquark mass to be $\boldsymbol{m_{bb}} = \mathbf{9.6646}$ GeV



Since our $b\bar{b}$ estimates aren't very accurate, we show thresholds with experimental $b\bar{b}$ mass.

→All S-wave T_{4b} tetraquarks could be below the $\eta_b\ \eta_b$ threshold!

Final remarks: Tcc and Tbb

- The $cc\bar{u}\bar{d}$ and $\bar{b}\bar{b}ud$ seem to be great candidates for manifestely exotic tetraquarks.
- Even tough the diquark-antidiquark assumption can be used, one *cannot* solve the **Schödinger** equation to get the **light diquark**.
- Also, in a two-body problem with *different masses* one should be carefull with the **spin-dependent** terms.
- Adjusting the **parameters** is also a delicate task since it involves three *different* energy scales: heavy-heavy, light-light and heavy-light.

- The **light and heavy diquarks** might **behave differently**. If so, maybe the tetraquark could also rearrenge as: antidiquark-quark-quark, like a baryon 3 body-problem.
- A different strategy is necessary.

Tcc

Their cc diquark = 3306.2 MeV

Our cc diquark = 3133.4 MeV

M. Karliner and J. Rosner arXiv:1707.07666

Contribution	Value (MeV)
$2m_c^b$	3421.0
$2m_q^b$	726.0
$a_{cc}/(m_c^b)^2$	14.2
$-3a/(m_q^b)^2$	−150.0
cc binding	−129.0
Total	3882.2 ± 12

2924.4

4.7

204.3

3709.4

Below the D0 D*+ threshold at 3875 MeV and also below D0D+gamma at 3734 MeV !

Tbb

Their bb diquark = 10087.0MeV

M. Karliner and J. Rosner arXiv:1707.07666

Our bb diquark = 9664.6 MeV

Preliminary!!

Contribution	Value (MeV)	
$2m_b^b$	10087.0	**9588.0**
$2m_q^b$	726.0	
$a_{bb}/(m_b^b)^2$	7.8	**0.9**
$-3a/(m_q^b)^2$	-150.0	
bb binding	-281.4	**75.7**
Total	10389.4 ± 12	**10240,6**

Still Below the BB* threshold at 10604 MeV and B- B0bar gamma at 10559 MeV.

Backup Slides

Comparison with Thresholds

Possible decay modes of the $cc\bar{c}\bar{c}$ states by spontaneous dissociation into two charmonium mesons.

J^{PC}	S-wave	P-wave
0^{++}	$\eta_c(1S)\eta_c(1S)$, $J/\psi J/\psi$	$\eta_c(1S)\chi_{c1}(1P)$, $J/\psi h_c(1P)$
0^{-+}	$\eta_c(1S)\chi_{c0}(1P)$, $J/\psi h_c(1P)$	$J/\psi J/\psi$
0^{--}	$J/\psi \chi_{c1}(1P)$	$J/\psi \eta_c(1S)$
1^{++}	–	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$, $\eta_c(1S)\chi_{c0}(1P)$
1^{+-}	$J/\psi \eta_c(1S)$	$J/\psi \chi_{c0}(1P)$, $J/\psi \chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$
1^{-+}	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$	–
1^{--}	$J/\psi \chi_{c0}(1P)$, $J/\psi \chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$	$J/\psi \eta_c(1S)$

Phys. Lett. B 773, 247 (2017).

Spin-dependent interactions $\rightarrow$ first-order perturbative corrections

- Splitting of states with different quantum numbers

$$V_{SS}^{(1)} = C_{SS}(r)\, \mathbf{S_1} \cdot \mathbf{S_2}$$ Spin-Spin

$$V_{LS}^{(1)} = C_{LS}(r)\, \mathbf{L} \cdot \mathbf{S}$$ Spin-Orbit

$$V_{T}^{(1)} = C_{T}(r) \left(\frac{(\mathbf{S_1} \cdot \mathbf{r})(\mathbf{S_2} \cdot \mathbf{r})}{\mathbf{r}^2} - \frac{1}{3}(\mathbf{S_1} \cdot \mathbf{S_2}) \right)$$ Tensor

Radial-dependent coefficients calculated with the wavefunction $y(r)$

$$C_{SS}(r) = \frac{2}{3m^2}\nabla^2 V_V(r) = -\frac{8\kappa_s \alpha_s \pi}{3m^2}\delta^3(r)$$

$$C_{LS}(r) = \frac{1}{2m^2}\frac{1}{r}\left[3\frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr}\right] = -\frac{3\kappa_s \alpha_s}{2m^2}\frac{1}{r^3} - \frac{b}{2m^2}\frac{1}{r}$$

$$C_{T}(r) = \frac{1}{m^2}\left[\frac{1}{r}\frac{dV_V(r)}{dr} - \frac{d^2 V_V(r)}{dr^2}\right] = -\frac{12\kappa_s \alpha_s}{4m^2}\frac{1}{r^3}$$

$$\langle \mathbf{S_{12}} \rangle_{1\otimes 1 \to S=2,\ 1,\ 0,\ \ell=1} \quad \to$$

$$\langle \mathbf{S_{12}} \rangle_{\frac{1}{2}\otimes\frac{1}{2}\to S=1,\ \ell\neq 0}$$

$$= \begin{cases} -\dfrac{2\ell}{(2\ell+3)}, & \text{if} \quad J = \ell+1, \\ +2, & \text{if} \quad J = \ell, \\ -\dfrac{2(\ell+1)}{(2\ell-1)}, & \text{if} \quad J = \ell-1. \end{cases}$$

$$\langle \mathbf{S_{12}} \rangle = -\frac{2}{5},\ +2,\ -4,$$

for $J = 2,\ 1,\ 0\ ,\ \ell = 1$

S_T	L_T	J_T	$\langle \mathbf{S_{d-\bar{d}}} \rangle$
0	1	1	0
1	1	2	$-\frac{4}{5} = -0,8$
1	1	1	4
1	1	0	-8
2	1	3	$-\frac{8}{5} = -1,6$
2	1	2	$\frac{28}{5} = 5,6$
2	1	1	$-\frac{561}{100} = -5,61$

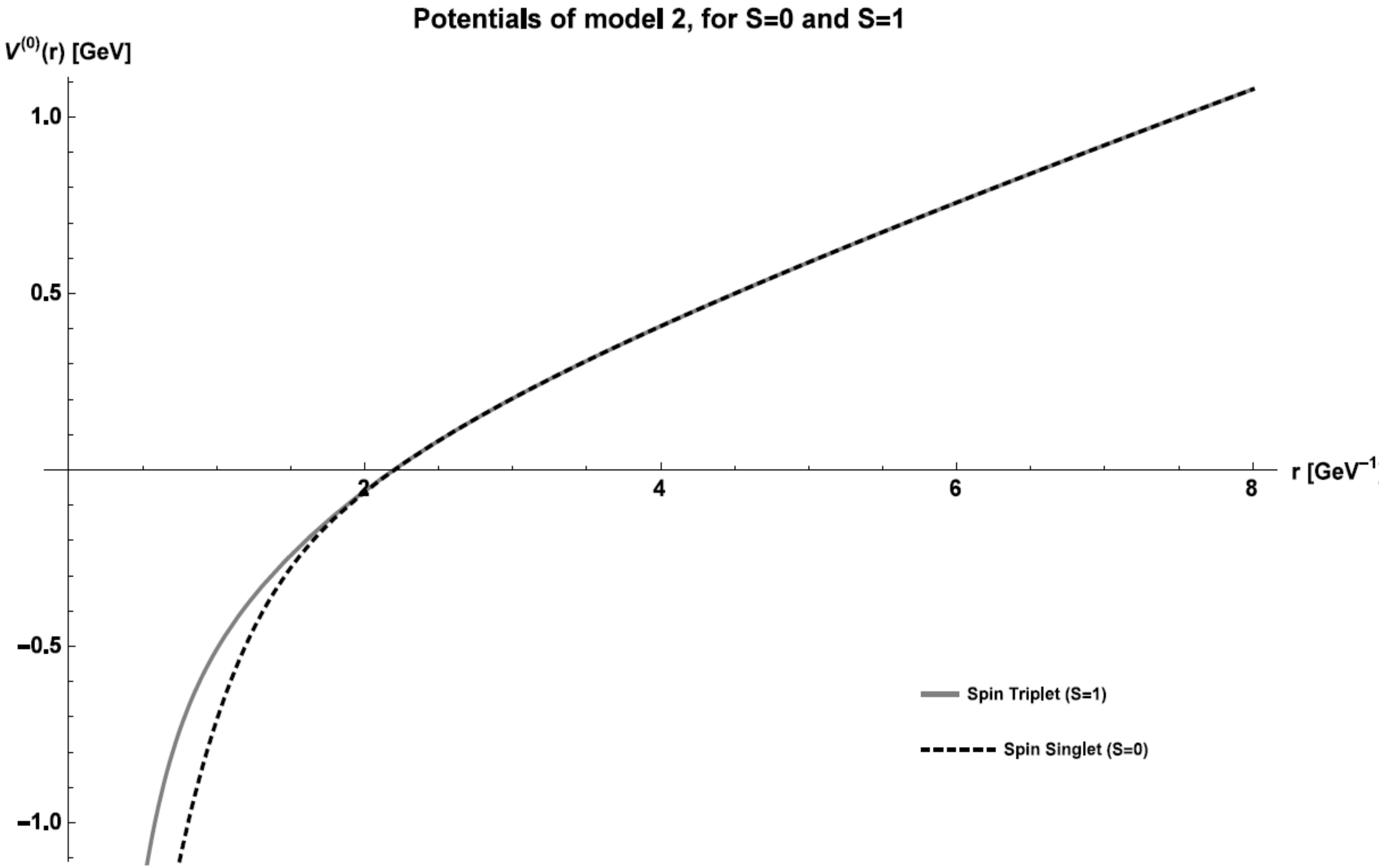
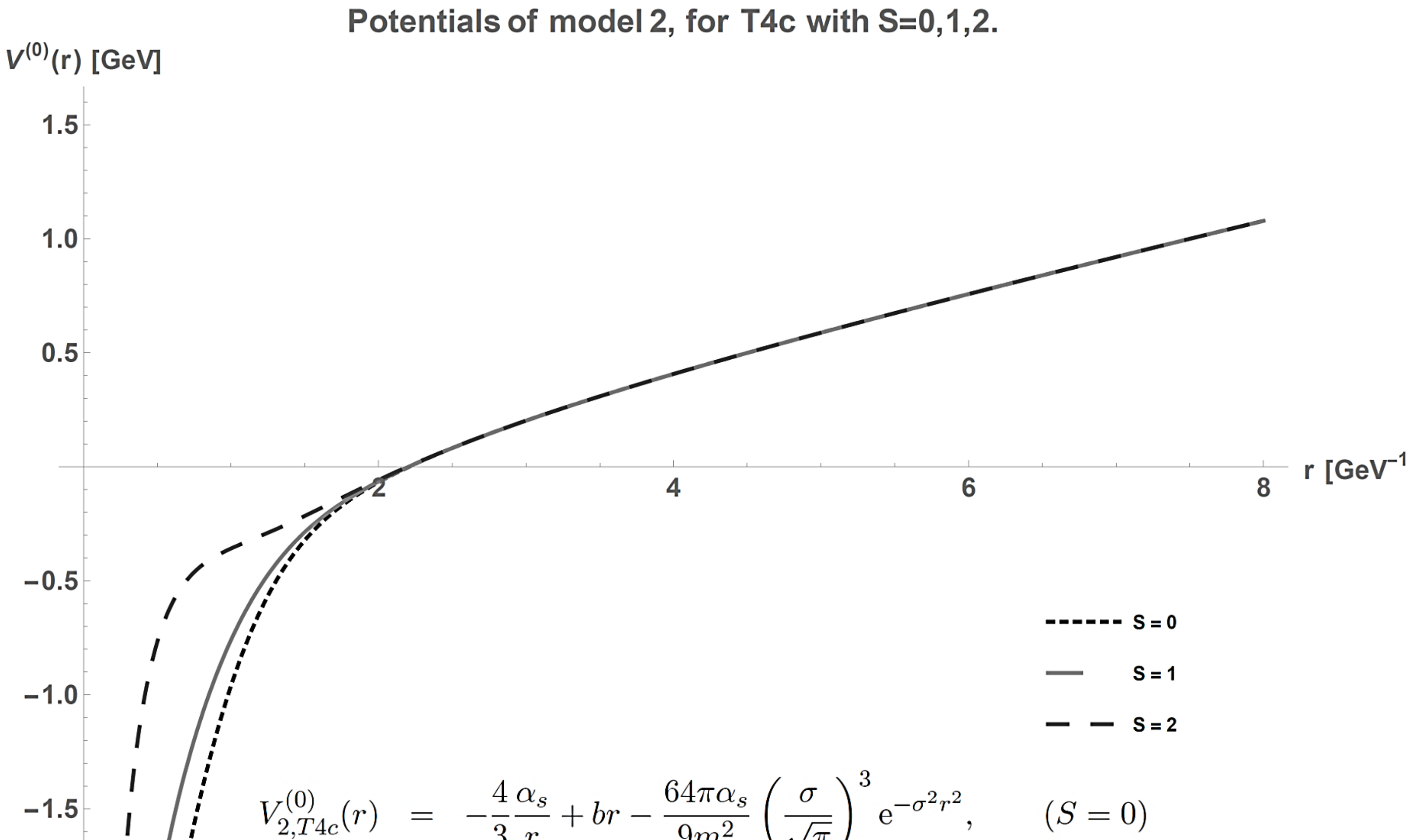


Figure: Potentials of model 2: *Coulomb plus linear plus smeared spin-spin*, for $S = 0$ and $S = 1$. Parameters are $\alpha_s = 0.5285$, $b = 0.1458$ GeV^2, $\sigma = 1.1779$

T_{4c}



$$V^{(0)}_{2,T4c}(r) \;=\; -\frac{4}{3}\frac{\alpha_s}{r} + br - \frac{64\pi\alpha_s}{9m^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 \mathrm{e}^{-\sigma^2 r^2}, \qquad (S=0)$$

$$V^{(0)}_{2,T4c}(r) \;=\; -\frac{4}{3}\frac{\alpha_s}{r} + br - \frac{32\pi\alpha_s}{9m^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 \mathrm{e}^{-\sigma^2 r^2}, \qquad (S=1)$$

$$V^{(0)}_{2,T4c}(r) \;=\; -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 \mathrm{e}^{-\sigma^2 r^2}, \qquad (S=2)$$

Charmonium

$N^{2S+1}\ell$	$M^{(0)}$ [GeV]	$\|R(0)\|^2$ [GeV^3]	$\langle r^2\rangle^{1/2}$ [fm]	$\left\langle \frac{v^2}{c^2} \right\rangle$
1^1S	2.9924	1.5405	0.375	0.336
1^3S	3.0917	1.1861	0.421	0.253
1^1P	3.5105	0	0.678	0.257
1^3P (c.o.g.)	3.5191	0	0.689	0.246
2^1S	3.6317	0.7541	0.839	0.308
2^3S	3.6714	0.7092	0.867	0.293
1^1D	3.7951	0	0.899	0.280
1^3D (c.o.g.)	3.7958	0	0.901	0.278
2^1P	3.9334	0	1.071	0.324
2^3P (c.o.g.)	3.9427	0	1.082	0.315
3^1S	4.0481	0.6088	1.210	0.364
3^3S	4.0755	0.5914	1.230	0.357
2^1D	4.1591	0	1.258	0.350
2^3D (c.o.g.)	4.1604	0	1.261	0.348
4^1S	4.3933	0.5430	1.531	0.424
4^3S	4.4150	0.5340	1.547	0.419

TABLE III: Results for charmonium $c\bar{c}$ masses from the model. Parameters are $m_c = 1.4622$ GeV, $\alpha_s = 0.5202$, $b = 0.1463$ GeV^2, $\sigma = 1.0831$ GeV.

$N^{2S+1}\ell_J$	$\langle T\rangle$	$\langle V_V^{(0)}\rangle$	$\langle V_S^{(0)}\rangle$	$\langle V_{SS}^{(0)}\rangle$	$E^{(0)}$	$M^{(0)}$ [MeV]	$\langle V_{LS}^{(1)}\rangle$	$\langle V_T^{(1)}\rangle$	M^f [MeV]
1^1S_0	491.9	−584.4	246.2	−85.6	68.1	2992.4	0	0	2992.4
1^3S_1	370.6	−504.0	279.4	21.4	167.4	3091.7	0	0	3091.7
1^3P_0	359.5	−246.6	480.0	2.0	594.8	3519.1	−63.9	−29.4	3425.8
1^3P_1	359.5	−246.6	480.0	2.0	594.8	3519.1	−32.0	14.7	3501.8
1^1P_1	375.2	−253.1	471.1	−7.0	586.2	3510.5	0	0	3510.5
1^3P_2	359.5	−246.6	480.0	2.0	594.8	3519.1	32.0	−2.9	3548.1
2^1S_0	450.6	−287.3	573.8	−29.7	707.4	3631.7	0	0	3631.7
2^3S_1	428.5	−281.7	590.4	9.8	747.1	3671.4	0	0	3671.4
1^3D_1	407.0	−175.4	639.7	0.2	871.5	3795.8	−8.8	−3.9	3783.1
1^3D_2	407.0	−175.4	639.7	0.2	871.5	3795.8	−2.9	3.9	3796.7
1^1D_2	408.8	−175.9	638.5	−0.6	870.8	3795.1	0	0	3795.1
1^3D_3	407.0	−175.4	639.7	0.2	871.5	3795.8	5.9	−1.1	3800.6
2^3P_0	460.4	−186.2	742.1	2.2	1018.4	3942.7	−59.9	−26.1	3856.7
2^3P_1	460.4	−186.2	742.1	2.2	1018.4	3942.7	−29.9	13.0	3925.8
2^1P_1	474.4	−190.8	733.1	−7.5	1009.1	3933.4	0	0	3933.4
2^3P_2	460.4	−186.2	742.1	2.2	1018.4	3942.7	29.9	−2.6	3970.0
3^1S_0	532.8	−215.4	826.5	−20.1	1123.8	4048.1	0	0	4048.1
3^3S_1	521.9	−215.3	837.7	6.9	1151.2	4075.5	0	0	4075.5
2^3D_1	508.6	−145.8	873.0	0.3	1236.1	4160.4	−11.6	−3.7	4145.1
2^3D_2	508.6	−145.8	873.0	0.3	1236.1	4160.4	−3.9	3.7	4160.2
2^1D_2	511.3	−146.5	871.0	−1.0	1234.8	4159.1	0	0	4159.1
2^3D_3	508.6	−145.8	873.0	0.3	1236.1	4160.4	7.7	−1.1	4167.1
4^1S_0	620.4	−179.5	1044.0	−15.8	1469.0	4393.3	0	0	4393.3
4^3S_1	613.2	−180.6	1053.0	5.6	1490.7	4415.0	0	0	4415.0

TABLE IV: Comparison of charmonium $c\bar{c}$ experimental data and results from the model.

$N^{2S+1}\ell_J$	M^{final} [MeV] Tab. III	Exp [76] [MeV]	Γ [MeV]	Meson	J^{PC}
1^1S_0	2992.4	2983.4 ± 0.5	31.8 ± 0.8	$\eta_c(1S)$	0^{-+}
1^3S_1	3091.7	3096.900 ± 0.006	0.0929 ± 0.0028	$J/\psi(1S)$	1^{--}
1^3P_0	3425.8	3414.75 ± 0.31	10.5 ± 0.6	$\chi_{c0}(1P)$	0^{++}
1^3P_1	3501.8	3510.66 ± 0.07	0.84 ± 0.04	$\chi_{c1}(1P)$	1^{++}
1^1P_1	3510.5	3525.38 ± 0.11	0.7 ± 0.4	$h_c(1P)^{\dagger}$	1^{+-}
1^3P_2	3548.1	3556.20 ± 0.09	1.93 ± 0.11	$\chi_{c2}(1P)$	2^{++}
$1P$ (c.o.g.)	(3519.1)	(3525.303)	–	–	–
2^1S_0	3631.7	3639.2 ± 1.2	$11.3^{+3.2}_{-2.9}$	$\eta_c(2S)$	0^{-+}
2^3S_1	3671.4	3686.097 ± 0.025	0.296 ± 0.008	$\psi(2S)$	1^{--}
1^3D_1	3783.1	3773.13 ± 0.35	27.2 ± 1.0	$\psi(3770)$	1^{--}
1^3D_2	3796.7	–	–	–	2^{--}
1^1D_2	3795.1	–	–	–	2^{-+}
1^3D_3	3800.6	–	–	–	3^{--}
$1D$ (c.o.g.)	(3795.8)	–	–	–	–
2^3P_0	3856.7	–	–	*	0^{++}
2^3P_1	3925.8	–	–	–	1^{++}
2^1P_1	3933.4	–	–	–	1^{+-}
2^3P_2	3970.0	3927.2 ± 2.6	24 ± 6	$\chi_{c2}(2P)$	2^{++}
$2P$ (c.o.g.)	(3942.7)	–	–	–	–
3^1S_0	4048.1	–	–	–	0^{-+}
3^3S_1	4075.5	4039 ± 1	80 ± 10	$\psi(4040)$	1^{--}
2^3D_1	4145.1	4191 ± 5	70 ± 10	$\psi(4160)$	1^{--}
2^3D_2	4160.2	–	–	–	2^{--}
2^1D_2	4159.1	–	–	–	2^{-+}
2^3D_3	4167.1	–	–	–	3^{--}
$2D$ (c.o.g.)	(4158.9)	–	–	–	–
4^1S_0	4393.3	–	–	–	0^{-+}
4^3S_1	4415.0	4421 ± 4	62 ± 20	$\psi(4415)$	1^{--}

Diquarks

$N^{2S+1}\ell$	$M^{(0)}$ [GeV]	$\|R(0)\|^2$ [GeV^3]	$\langle r^2\rangle^{1/2}$ [fm]	$\left\langle \frac{v^2}{c^2} \right\rangle$
$\mathbf{1^3S}$	**3.1334**	**0.3296**	**0.593**	**0.123**
1^1P	3.3530	0	0.906	0.141
2^3S	3.4560	0.2370	1.147	0.167
2^1P	3.6062	0	1.395	0.190

$N^{2S+1}\ell_J$	$\langle T\rangle$	$\langle V_V^{(0)}\rangle$	$\langle V_S^{(0)}\rangle$	$\langle V_{SS}^{(0)}\rangle$	$E^{(0)}$	$M^{(0)}$ [MeV]	$\langle V_{LS}^{(1)}\rangle$	$\langle V_T^{(1)}\rangle$	M^f [MeV]
$\mathbf{1^3S_1}$	**180.4**	**−173.9**	**197.9**	**4.7**	**209.0**	**3133.4**	**0**	**0**	**3133.4**
1^1P_1	206.7	−93.3	316.2	−0.9	428.7	3353.0	0	0	3353.0
2^3S_1	244.8	−105.7	389.8	2.9	531.7	3456.0	0	0	3456.0
2^1P_1	277.5	−72.3	477.9	−1.2	681.9	3606.2	0	0	3606.2

Diquarks: Comparisons

TABLE VII: Results for diquark cc from literature.

$N\ell$	M_{cc} [GeV]	$\|R(0)\|^2$ [GeV^3]	$\langle r^2\rangle^{1/2}$ [fm]	Ref.
1S	**3.13**	$\mathbf{(0.523)^2 = 0.2735}$	**0.58**	[43]
$2S$	3.47	$(0.424)^2 = 0.1798$	1.12	[43]
$2P$	3.35	–	0.88	[43]
1S	**3.226**	–	–	[39]
1S	**3.067**	–	–	[2] mod. I
1S	**3.082**	–	–	[2] mod. II
$1P$	3.523	–	–	[2] mod. I
$1P$	3.513	–	–	[2] mod. II
1S	**3204.1**	–	–	[63]

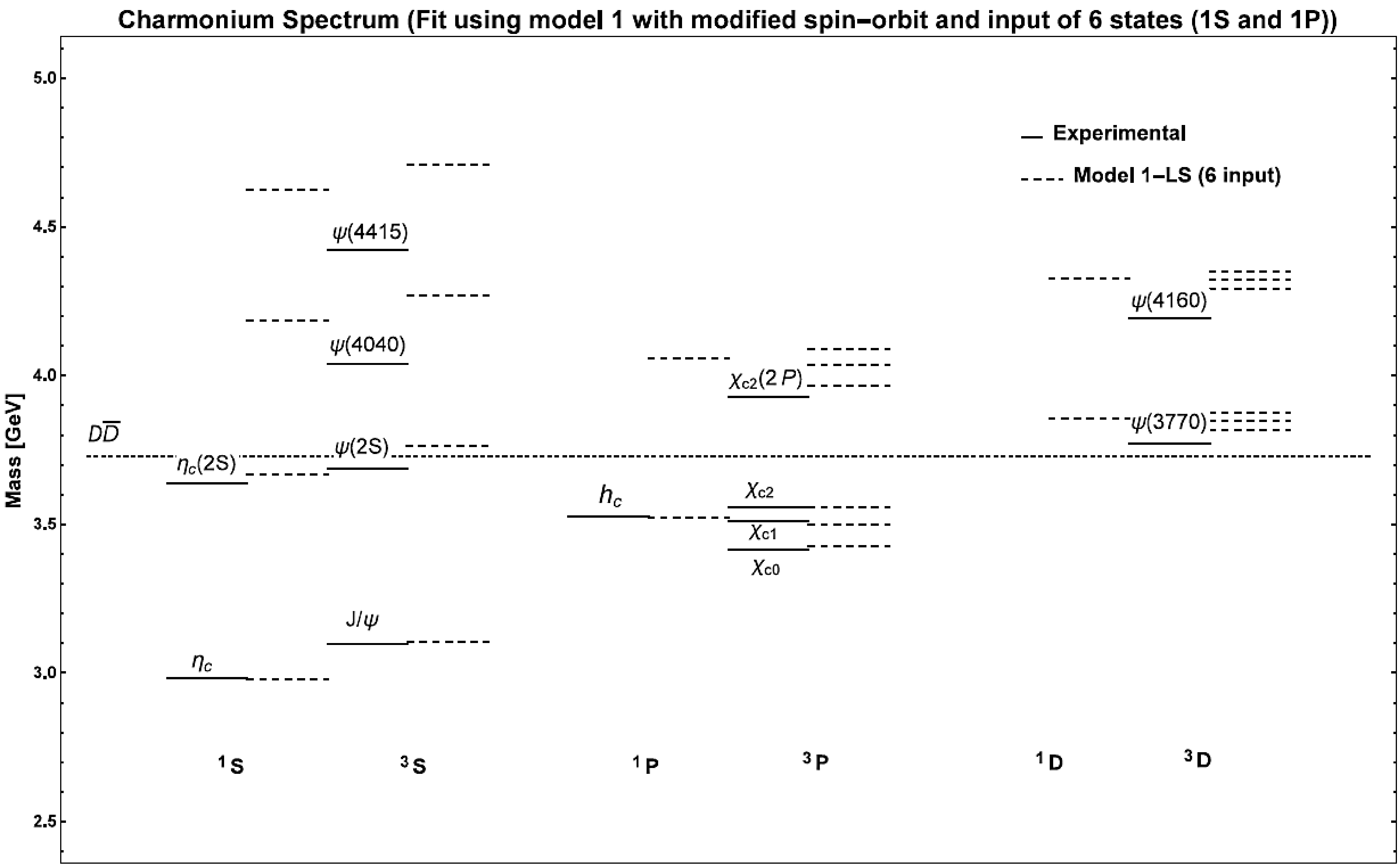


Figure: Spectrum of charmonium with model 1, input of 6 $c\bar{c}$ and modified spin-orbit. Parameters are $m_c = 1.2819$ GeV, $\alpha_s = 0.3289$, $b = 0.2150$ GeV^2.