Introduction to EWSB – Part 3

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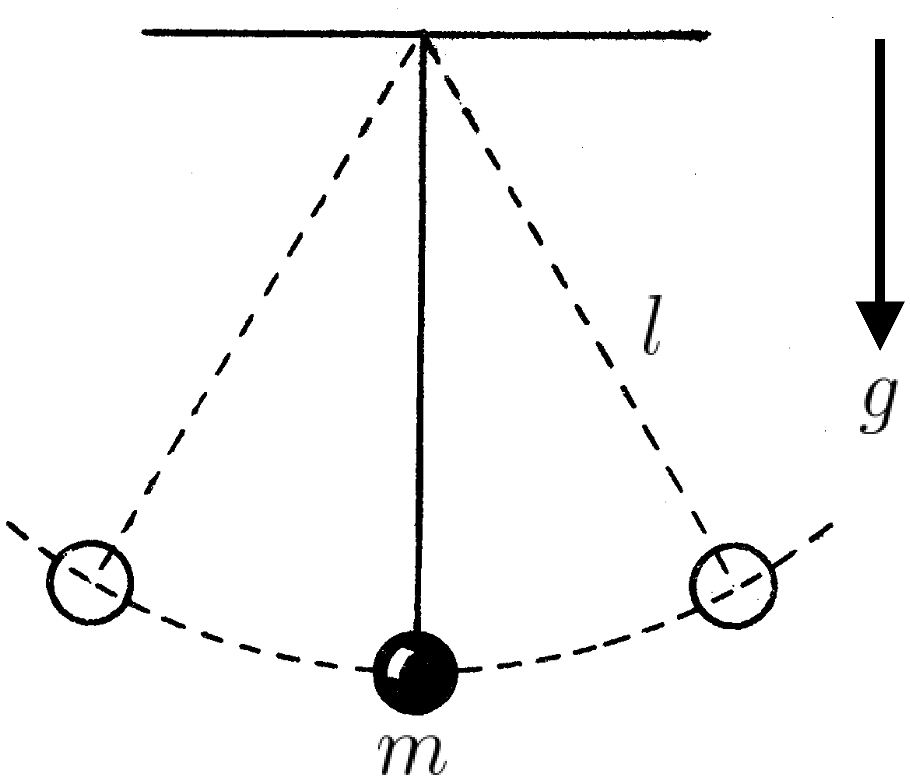
Outline

- Part 1: Standard Model 101 — ~ 1 hour
  - Particle content
  - Gauge invariance
  - SM Lagrangian before EWSB
  - Masses and the no-lose theorem for new physics
- Part 2: Spontaneous symmetry breaking — ~ 1 hour
  - SSB of global symmetries (Goldstone Theorem)
  - Linear sigma model (Wigner-Weyl vs Nambu-Goldstone realization)
  - Non-linear sigma model
  - Scalar QED (Abelian Higgs Model)
  - EWSB in the SM
- **Part 3: Applications — ~ 1 hour**
  - Naturalness problem
  - Equivalence theorem
  - Effective Vector Boson Approximation

Naïve Dimensional Analysis

- The Naturalness Problem is such a big issue that we do not even exactly know how to formulate it
- Certainly it can be phrased just as a question about the origin of the Higgs mechanism (and the Higgs potential)
- However, it is useful to see it in a more general way, as an issue of naive dimensional analysis



Under **scale transformations**:

$$l \to \alpha_l l \qquad m \to \alpha_m m$$
$$t \to \alpha_t t \qquad g \to \frac{\alpha_l}{\alpha_t^2} g$$

Let us guess a formula for the frequency of the pendulum:

$$\omega \to \frac{1}{\alpha_t}\omega \quad \Longrightarrow \quad \omega = \sqrt{g/l}$$

- The requirement of the correct transformation under our scale transformations gives us the correct parametric formula, almost without knowing any physics
- This exercise is usually done using dimensional units
- NDA is a statement on the transformation properties under scale transformations

Naturalness problem

- What does this have to do with the Higgs potential?
- Notice that the Higgs mass operator is the only operator with dimension less than 4 in the SM (relevant operator)
- In fact it is the only one that requires the introduction of a dimensionful parameter

$$\mathcal{L}_H = (D_\mu H)^\dagger D^\mu H - V(H), \qquad V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

- If we switch off the Higgs mass term, then there are no scales in the SM (but for the QCD scale, which is dynamically generated, i.e. we did not require it, QCD dynamics just generates it)
- Then, if any dimensionful scale is present in the UV, NDA would set the Higgs mass parameter equal to this scale

$$\mathcal{L}_{\rm NDA} = c\Lambda^2 H^\dagger H$$

- We do have some indication of the existence of fundamental high scales

$$M_{\rm GUT} = 10^{16}\ {\rm GeV}$$

Grand Unification Scale: is the scale where gauge couplings (almost) converge toward a single value

$$M_{\rm Planck} = 10^{19}\ {\rm GeV}$$

Planck Scale: the scale of quantum gravity

Naturalness problem

- If any of these new physics scales exist (and is associated to the propagation of new states), then NDA tells us that the Higgs mass should be of the order of these scales
- Of course it is not, and indeed all our statements were classical, i.e. did not include any radiative (loop) corrections
- Radiative corrections should then be responsible for the Higgs mass we observe

$$m_h^2 = c\Lambda^2 + \delta m_h^2$$

- However, this would require an extremely finely tuned c constant in order to cancel very big numbers (notice the square) and give rise to the observed Higgs mass
- This does not seem natural and it is therefore referred to as a problem of **naturalness**
- There are several directions to address this problem:
  - New physics at TeV scale: is the most studied solution and it aims at lowering Λ to the TeV scale. Then the required cancellation becomes much smaller
  - Anthropics: assumes the existence of many different universes (multiverse) with different values of m_h and aims at proving that our universe is possible only for the observed value
  - No UV scale: one can try to construct theories that do not have any high energy scale associated to new scales, so that $\Lambda = m_h$ is just the NDA prediction

Towards new physics

Examples

Supersymmetry

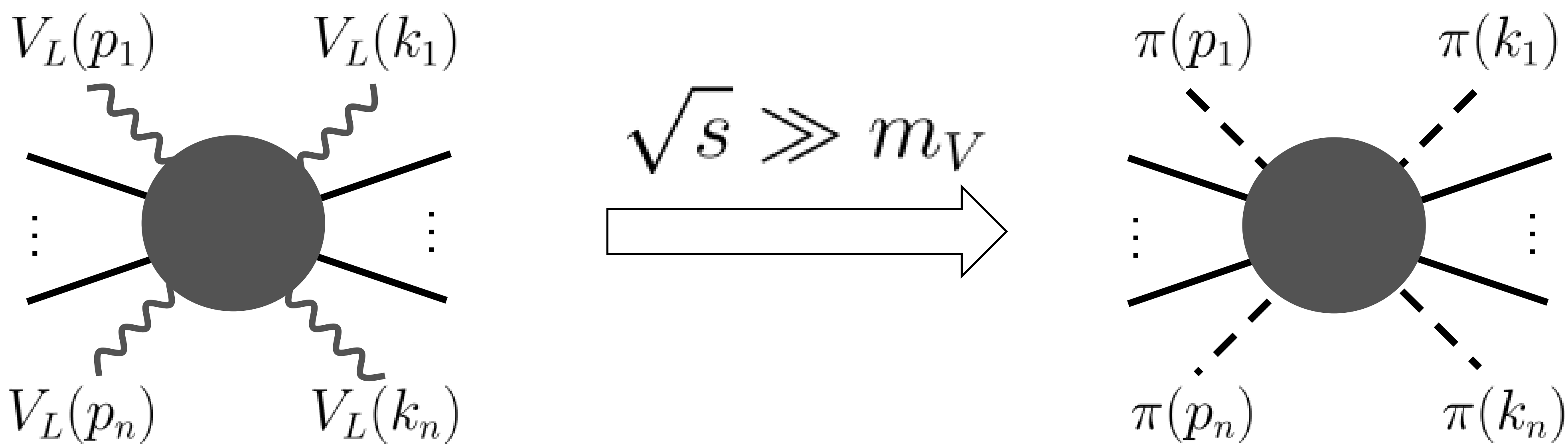
Extra-dimensions

Strong dynamics

- Great ideas in the last 40 years, but no evidence yet for any of this new physics
- People start wondering about our understanding of Naturalness
- This remains one of the biggest puzzles in fundamental physics

Equivalence Theorem

- Let us go back to the SM Lagrangian and fix any gauge different than the unitary gauge
- We have seen that Goldstone Bosons are related to one of the degrees of freedom of massive vector fields
- In fact, at high enough energies, the EW symmetry is "restored" so that the gauge bosons become effectively massless d.o.f. (2 transverse polarizations) and the Goldstone boson carries all the information on the longitudinal polarization
- This leads us to the Equivalence Theorem

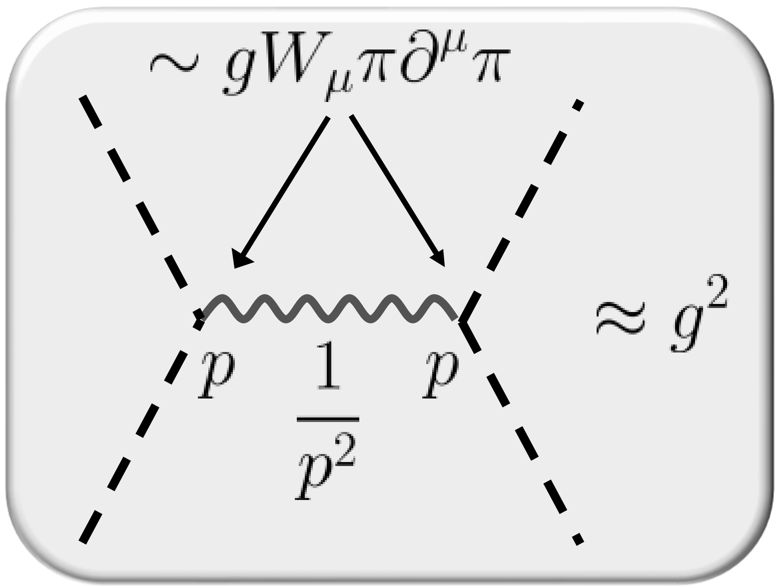


$$\mathcal{A}\left(V_L(p_1) + \ldots + V_L(p_n) + X \to V_L(k_1) + \ldots + V_L(k_m) + Y\right)$$
$$= (-i)^{n+m}\mathcal{A}\left(\pi(p_1) + \ldots + \pi(p_n) + X \to \pi(k_1) + \ldots + \pi(k_m) + Y\right)\left(1 + O\left(\frac{m_V}{\sqrt{s}}\right)\right)$$

- This allows to greatly simplify calculations

Longitudinal WW scattering

- Let's try and re-compute the scattering amplitude of four longitudinal W bosons using the equivalence theorem
- Feynman rules for Goldstone Bosons are entirely contained in the Higgs kinetic term (remember that they do not enter the potential)
- Diagrams involving gauge couplings do not grow with energy (simple power counting)

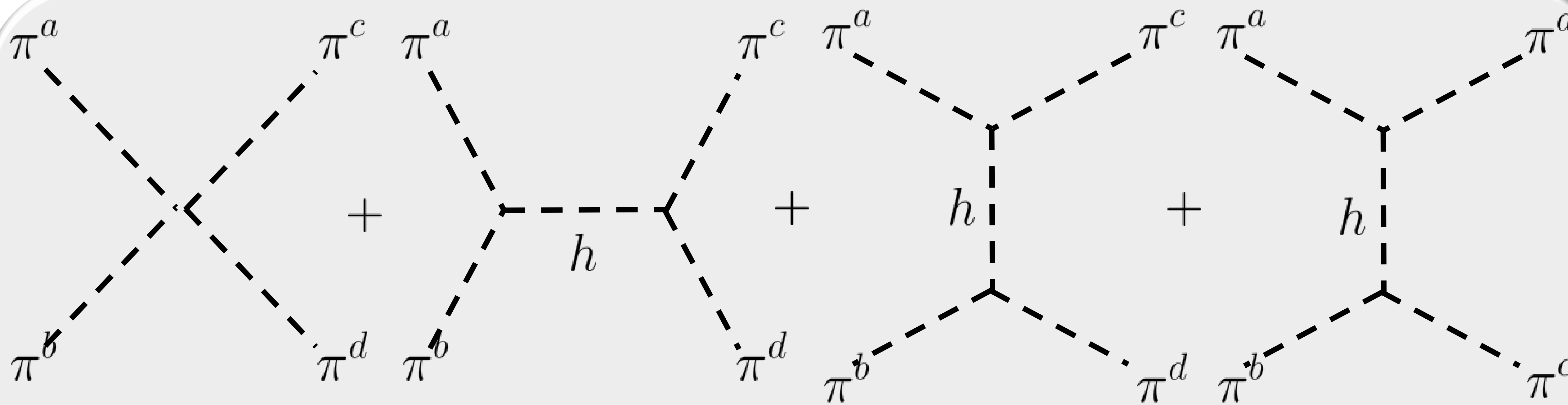


- To get the high energy behavior we can set the gauge couplings to zero (so-called gauge-less limit)
- We can also expand the exponential of the Goldstone bosons up to quartic Goldstone Boson interactions
- One can check that the relevant Lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{m_h^2}{2} h^2 + \frac{v^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right), \qquad U = e^{\frac{i\pi^a \sigma^a}{v}}$$

- We keep a and b free to see better what is the effect of the Higgs in longitudinal WW scattering

Longitudinal WW scattering



$$\mathcal{A}\left(\pi^a(p_1)+\pi^b(p_2)\to\pi^c(p_3)+\pi^d(p_4)\right)=\left[\frac{s}{v^2}-\frac{a^2}{v^2}\left(\frac{s^2}{s-m_h^2}\right)\right]\delta^{ab}\delta^{cd}+\text{perm.}$$

- We expand this amplitude in the limit $s \gg m_h^2$ and get

$$\mathcal{A}=\left(1-a^2\right)\frac{s}{v^2}-\frac{a^2m_h^2}{v^2}=\left(1-a^2\right)\frac{s}{v^2}-8a^2\lambda$$

- The SM predicts $a=1$ and therefore the amplitude is just a constant at high energy
- This makes the SM well behaved at high energies, so that it can be extrapolated to very high energy scales. $a\neq 1$ is often predicted in models BSM and would lead to a modification of Higgs couplings to gauge bosons, constrained by experiments at the 10% level

Before and after the Higgs

- We have seen that looking for a breakdown of perturbation theory in longitudinal WW scattering gives an handle on possible new physics
- Before Higgs discovery big accent was put on measuring WW scattering at LHC
- However, the Higgs discovery and the measurements of its couplings have set a strong constraint on the modification of Higgs coupling to gauge bosons
- Our argument on perturbativity taking into account modified Higgs couplings becomes

$$g(s)^2 = \left(1 - a^2\right)\frac{s}{v^2} \lesssim (4\pi^2) \quad \Longrightarrow \quad \sqrt{s} \lesssim \frac{4\pi v}{\sqrt{1-a^2}}$$

- If α is constrained to be equal to one within 10% level we get

$$\sqrt{s} \lesssim 7\ \mathrm{TeV}$$

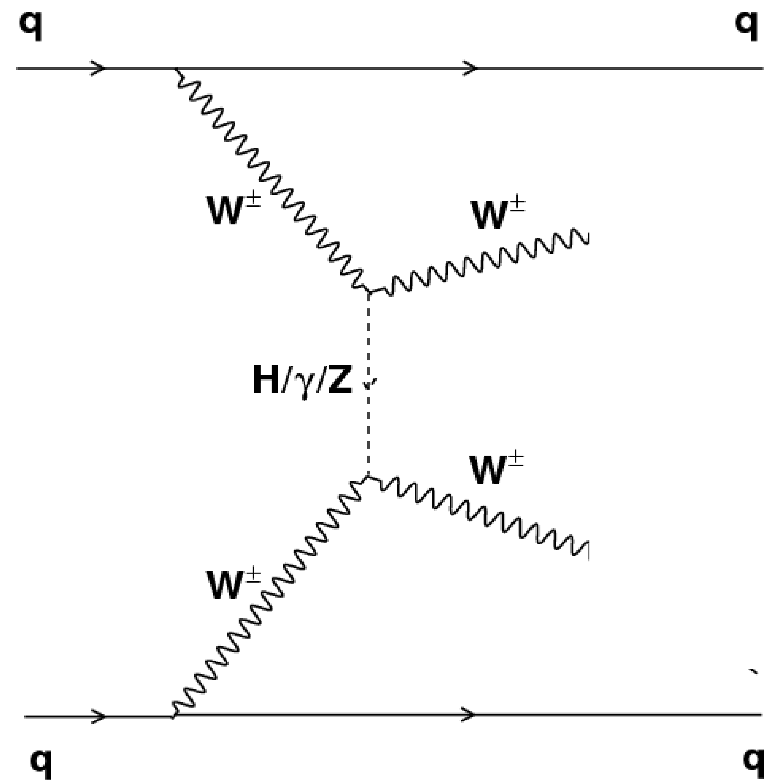
- Accessing this scale in WW scattering at LHC is extremely difficult
- Moreover it seems more likely that precision measurements of Higgs properties will be more sensitive
- However this is not a good reason not to look at WW scattering anymore
- This is a very important SM process that should be looked for at LHC

Summary

Result: We have seen that the equivalence theorem allows to compute high energy processes involving longitudinal vector bosons by replacing them with the corresponding Goldstone Bosons. This greatly simplifies calculations. Adding the Higgs to our calculation of longitudinal WW scattering we have shown that the SM Higgs makes the theory well behaved at all energies.

Vector Boson Fusion

- WW scattering at LHC can be studied looking at Vector Boson Fusion processes



- This is a complicated $2 \to 4$ scattering process
- In order to tag the VBF topology one usually applies cuts requiring the two jets to be forward and to have a large invariant mass
- The calculation of the inclusive rate can be done assuming that the vector bosons are "partons" inside the proton (without looking at the additional jets)
- The vector bosons pdfs can be computed analogously to the photon pdf inside the electron
- This approximation is known as effective vector boson (W) approximation

Effective Photon Approximation

- The calculation of the vector bosons pdfs proceeds in the same way as in the effective photon approximation (Weizsäcker-Williams approximation)
- Consider the process

$$eX \to eY$$

At small angles and through the exchange of a photon ($\gamma X \to Y$)

- The cross section for this process can be written as

$$\sigma^{eX\to eY} = \int_0^1 dz f_{e/\gamma}(z)\hat{\sigma}^{\gamma X\to Y}(\hat{s} = zs)$$

- Computing the splitting of an electron to and electron and a photon we get

$$\sigma^{eX\to eY} = \int_0^1 dz \int \frac{dp_T^2}{p_T^2}\frac{\alpha}{2\pi}\left(\frac{1+(1-z)^2}{z}\right)\hat{\sigma}^{\gamma X\to Y}(\hat{s} = zs)$$

- Performing the integration over the transverse momentum between the electron mass and the center of mass energy we get the photon pdf in the electron

$$f_{e/\gamma}(z) = \frac{\alpha}{2\pi}\ln\frac{s}{m_e^2}\left(\frac{1+(1-z)^2}{z}\right)$$

Effective W Approximation

- A very similar result is obtained in the effective W approximation for the transverse gauge bosons

$$f_{q/V_\pm}(x) = \frac{1}{16\pi^2}\sum_{q'}\left(\frac{|v_q^V \mp a_q^V|^2 + |v_q^V \pm a_q^V|^2(1-x)^2}{x}\right)\ln\frac{s}{M_V^2}$$

- The calculation for the longitudinal polarizations involves instead the splitting of a longitudinal boson and gives

$$f_{q/V_L}(x) = \frac{1}{4\pi^2}\frac{1-x}{x}\sum_{q'}\left(|v_q^V|^2 + |a_q^V|^2\right)$$

- The important thing to notice is that while the transverse pdf is log enhanced, the longitudinal one is not
- Even though the longitudinal amplitude grows more with energy the effect of the pdfs partially compensates
- Also taking into account the number of channels it turns out that longitudinal channels are suppressed compared to transverse ones up to high energies making it hard to extract information on high energy longitudinal WW scattering

Take home messages

- Symmetries are an incredibly powerful tool in physics
- SSB is not an actual breaking, rather a different realization of the symmetry
- SSB of global continuum symmetries implies the existence of as many massless scalar fields (Goldstone bosons) as the broken generators
- Gauge invariance does not correspond to a real symmetry (does not have associated conserved currents), and it is instead a redundancy in our description of the theory
- SSB of gauge invariance violates the Goldstone Theorem and the would be Goldstone bosons provide the missing degrees of freedom to move from a massless to a massive vector field (the Higgs mechanism)
- In the SM SSB of gauge invariance is responsible for all masses of elementary particles
- SSB in the SM is realized linearly, with the Higgs identified as the radial mode
- The microscopic origin of SSB in the SM is a big puzzle, and it is related to the Naturalness problem