(1) • Mathieu Pellen - First EWSB spring School - Maratea.

• Electroweak corrections

→ Many exercises taken from "Mécanique quantique relativiste", M. Klasen, Dunod

→ Exercise 1.

• $F = |\vec{v}_a - \vec{v}_b|\, 2E_a\, 2E_b$ (definition) ← $\vec{v}_j = \frac{\vec{p}_j}{E_j}$

$= 4\,(|\vec{p}_a E_b - \vec{p}_b E_a|)$

← $p_a = (E_a, 0, 0, E_a)$ and $p_b = (m_b, \vec{0})$

$= 4 E_a m_b$

• $$\int \frac{1}{(2\pi)^2}\, \delta^{(4)}(p_a + p_b - p_1 - p_2)\, \frac{d^3 p_1}{2E_1}\, \frac{d^3 p_2}{2E_2}$$

$$= \int \frac{1}{(2\pi)^2} \cdot \frac{|\vec{p}_1|^2\, d|\vec{p}_1|}{2E_1}\, d\Omega_1\, \delta\left[(p_a + p_b - p_1)^2 - m_2^2\right] \Theta(E_a + m_b - E_1)$$

$$= \int \frac{1}{(2\pi)^2}\, \frac{E_1\, dE_1}{2}\, \delta\left[2 m_b (E_a - E_1) - 2 E_a E_1 (1 - \cos\theta_1)\right] d\Omega_1$$

$$= \int \frac{1}{(2\pi)^2}\, \frac{E_1}{2\,|2 m_b + 2 E_a (1 - \cos\theta_1)|}\, d\Omega_1$$

$$= \int \frac{1}{(2\pi)^2}\, \frac{E_1^2}{2\,(2 m_b E_1 + 2 E_a E_1 (1 - \cos\theta_1))}\, d\Omega_1$$

$$= \int \frac{1}{(2\pi)^2}\, \frac{E_1^2}{4 m_b E_a}\, d\Omega_1$$

→ Exercise 2

• $\sqrt{s} = E_1 + E_2 = \sqrt{m_1^2 + p_f^2} + \sqrt{m_2^2 + p_f^2}$

$$\Rightarrow \frac{d\sqrt{s}}{dp_f} = p_f \left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$

② $\Rightarrow \quad dPS^{(2)} = (2\pi)^4 \, \delta^{(4)} \, (P_1 + P_2 - P_a - P_b) \, \frac{d^3 p_1}{(2\pi)^3 2E_1} \cdot \frac{d^3 p_2}{(2\pi)^3 2E_2}$

$$= \frac{1}{4\pi^2} \frac{d^3 p_1}{2E_1} \frac{1}{2E_2} \, \delta(E_1 + E_2 - E_a - E_b)$$

$$= \frac{1}{4\pi^2} \frac{p_f^2 \, dp_f \, d\Omega}{4 E_1 E_2} \cdot \delta(\sqrt{s} - E_1 - E_2) \qquad \frac{d\sqrt{s}}{dp_f} = p_f\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$

$$= \frac{1}{4\pi^2} \frac{p_f}{4} \left(\frac{1}{E_1 + E_2} \right) d\sqrt{s} \, d\Omega \, \delta(\sqrt{s} - E_1 - E_2)$$

$$= \frac{1}{4\pi^2} \frac{p_f}{4\sqrt{s}} \, d\Omega$$

- $$F = 4\sqrt{(P_a \cdot P_b)^2 - m_a^2 m_b^2}$$

$$= 4\sqrt{(E_a E_b - \vec{p_a} \cdot \vec{p_b})^2 - m_a^2 m_b^2}$$

$$= 4\sqrt{E_a^2 E_b^2 - 2E_a E_b \, \vec{p_a} \cdot \vec{p_b} + (\vec{p_a} \cdot \vec{p_b})^2 - m_a^2 m_b^2}$$

$$= 4\sqrt{E_a^2 \vec{p_b}^2 + E_b^2 \vec{p_a}^2 + 2E_a E_b |\vec{p_a}||\vec{p_b}|}$$

$|\vec{p_a}| = |\vec{p_b}| = p_i \rightarrow \quad = 4(p_i E_b + p_i E_a)$

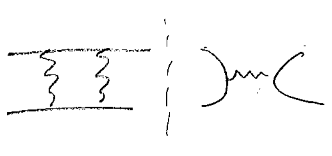
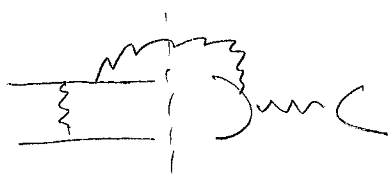
$$= 4 p_i \sqrt{s}$$

- $$\frac{d\sigma}{d\Omega} = \frac{1}{F} |T|^2 \frac{dPS^{(2)}}{d\Omega}$$

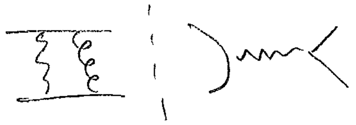
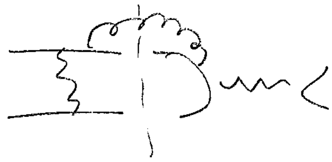
$$= \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |T|^2$$

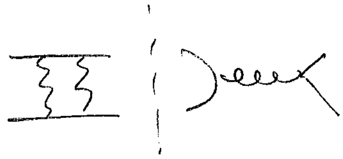
<u>Exercise 3:</u>

- LO : $O(\alpha^2)$, $O(\alpha_s \alpha)$, and $O(\alpha_s^2)$.
- NLO : $O(\alpha^3)$, $O(\alpha_s \alpha^2)$, $O(\alpha_s^2 \alpha)$, and $O(\alpha_s^3)$.
- ⓪ <u>$O(\alpha^3)$:</u>

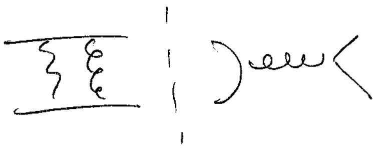
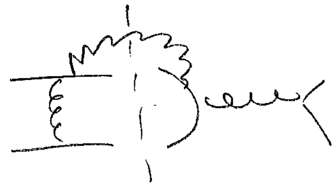
<u>virt:</u>  <u>real:</u> 

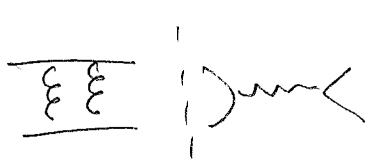
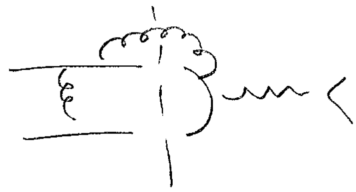
③ $\odot\ \mathcal{O}(\alpha_s \alpha^2)$ (mixed corrections)

virt:  real: 

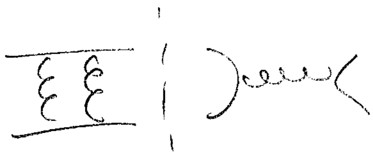
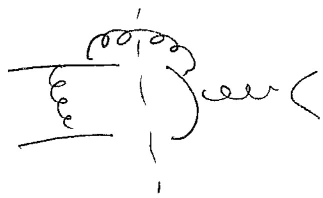
virt:  real: 

$\odot\ \mathcal{O}(\alpha_s^2 \alpha)$ (mixed corrections)

virt:  real: 

virt:  real: 

$\odot\ \mathcal{O}(\alpha_s^3)$

virt:  real: 

Exercise 4:

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	$pp \to e^+e^-$	$pp \to e^+e^-\gamma$	$pp \to e^+e^-\gamma\gamma$
$p_{T,\gamma+\gamma}$	/	LO	NLO
$m_{e^+\gamma}$	LO	NLO	NLO
Azi distance e^+e^-	LO	NLO	NLO.
N_{jet}	bin 2 NLO	bin 2 NLO	bin 2 NLO
n_γ	bin 1 LO	bin 1 NLO bin 2 LO	bin 2 NLO bin 3 LO.

- $pp \to e^+e^-$: LO $\mathcal{O}(\alpha^2)$ NLO EW $\mathcal{O}(\alpha^3)$
  $pp \to e^+e^-\gamma$: LO (α^3) NLO EW $\mathcal{O}(\alpha^4)$
  $pp \to e^+e^-\gamma\gamma$: LO(α^4) NLO EW $\mathcal{O}(\alpha^5)$

④ Exercise 6:

- $p_a = (p, \vec{p})$, $p_b = (p, -\vec{p})$, $p_1 = (p, \vec{p}\,')$, $p_2 (p, -\vec{p}\,')$

$q = (0, \vec{p} - \vec{p}\,')$

$$-iT = (-ie)\begin{pmatrix} 2p \\ \vec{p}+\vec{p}\,' \end{pmatrix}^{\mu} \frac{-ig_{\mu\nu}}{-(\vec{p}-\vec{p}\,')^2} (-ie)\begin{pmatrix} 2p \\ -\vec{p}-\vec{p}\,' \end{pmatrix}^{\nu}$$

$$= -ie^2 \frac{4p^2 + (\vec{p}+\vec{p}\,')^2}{(\vec{p}-\vec{p}\,')^2}$$

$$= -ie^2 \frac{3+\cos\theta_1^*}{1-\cos\theta_1^*}$$

- $$\frac{d\sigma}{d\Omega_1^*} = \frac{1}{64\pi^2 s} \cdot \frac{p_f}{p_i} e^4 \left(\frac{3+\cos\theta_1^*}{1-\cos\theta_1^*}\right)^2$$

Ex. 2 $$= \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta_1^*}{1-\cos\theta_1^*}\right)^2$$

- $$-iT = \frac{(p_a-p_b).(p_1-p_2)}{s} ie^2 - \frac{(p_a+p_1).(p_b+p_2)}{t} ie^2$$

$$= \frac{(2p_a - p_1 - p_2).(p_1-p_2)}{s} ie^2 - \frac{(2p_a + p_b - p_2).(p_b+p_2)}{t} ie^2$$

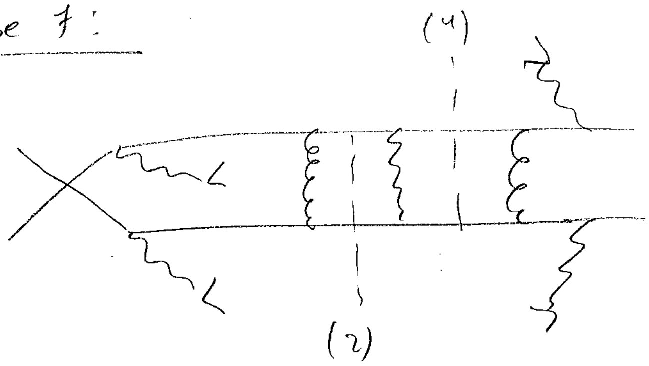
$$= \frac{2p_a(p_1-p_2)}{s} ie^2 - \frac{2p_a(p_b+p_2)}{t} ie^2$$

$$= \frac{-t+u}{s} ie^2 + \frac{-s+u}{t} ie^2$$

NB: Symmetric under $s \leftrightarrow t$ _

(5)

Exercise 7:



Two types of virtual correction according to (1) and (2).
Both are of the order $O(\alpha_s \alpha^6)$.

Exercise 8:

Use hep-ph/0010201 Denner, Pozzorini.

- $\log^2$ terms (3.7).

$$-\frac{1}{2} \underbrace{C^{ew}_{ik,ik'}} L(s)$$

C^{ew} for W (B.24).

$$= \underset{\text{virt.}}{2} \times \underset{4\,W \text{ in scattering}}{4} \times \frac{2}{s_w^2}$$

$$= -8 \cdot \frac{1}{s_w^2}$$

- Collinear and soft single logarithms.

$$\delta^C_{WW} = \delta_{\sigma\sigma'} \left[\frac{1}{2} b^{ew}_W l(s) + \cancel{Q_W^2 \, l^{em}(M_W^2)} \right] \quad (4.10).$$

neglected here.

$$= \delta_{\sigma\sigma'} \frac{1}{2} b^{ew}_W l(s)$$

b^{ew} for W (B.39, B.40)

$$= \frac{1}{2} \times \underset{\text{virt}}{2} \times \underset{4\,W}{4} \times \frac{19}{6 s_w^2}$$

$$= \frac{38}{3 s_w^2} l(s)$$

(6) • logarithms connected to parameter renormalisation

in high energy limit $\longrightarrow g_2^2 = \frac{e^2}{s_w^2}$

consider charge renormalisation only.

$$\delta^{\text{ferm}} = -\frac{1}{2}\,\hat{b}^{ew}_{w} \times 2 \times 2 \; \ell(s)$$

virt. g_2^2 and not g_2

(5.11) neglecting. δZ_e^{em}

$$= -2 \times \frac{19}{6 s_w^2} \times \ell(s) \; (\text{B.39, B.40}) \text{ as before.}$$

$$= -\frac{19}{3 s_w^2} \times \ell(s)$$

$$\Rightarrow \delta^{L}_{ww} + \delta^{\text{ferm}} = \frac{19}{3 s_w^2}\,\ell(s).$$

$\Rightarrow$ as in 1611.02951 Biedermann, Denner, Pellen:

$$\sigma_{LL} = \sigma_{L0}\left[1 - \frac{\alpha}{4\pi}\cdot 4\, C^{ew}_{w} \log^2\left(\frac{Q^2}{M_w^2}\right) + \frac{\alpha}{4\pi}\, 2\, b^{ew}_{w} \log\left(\frac{Q^2}{M_w^2}\right)\right]$$

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