Introduction to EWSB – Part 1

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Before starting let's know each other

How many of you have seen (raise your hand):

- An introductory course on the Standard Model?
- The topic of these lectures (i.e. EWSB)?
- Some lectures on Higgs physics?
- Some basic group theory (U(1), SU(2) and SU(3))?

How many of you know (shout the answer):

- The mass of the W boson? $\approx 80\ \mathrm{GeV}$
- The mass of the Higgs boson? $\approx 125\ \mathrm{GeV}$
- The mass of the muon? $\approx 105\ \mathrm{MeV}$
- The mass of the pion? $\approx 135\ \mathrm{MeV}$
- The mass of the gluon? 0
- The trace of the Pauli matrices? 0
- The adjoint representation of SU(3) octet

Some references

The topic of these lectures is very standard. So standard that it is part of the Standard Model

Being original on this topic is extremely hard and maximizes the risk of being incomprehensible

For this reason I decided to give standard lectures on a standard topic

Some standard references are:

- Peskin & Schroeder, An Introduction to Quantum Field Theory
- M. D. Schwartz, Quantum Field Theory and the Standard Model
- C. M. Becchi & G. Ridolfi, An Introduction to Relativistic Processes and the Standard Model of Electroweak Interactions
- …

Outline

- Part 1: Standard Model 101
  - Particle content
  - Gauge invariance
  - SM Lagrangian before EWSB
  - Masses and the no-lose theorem for new physics

~ 1 hour

- Part 2: Spontaneous symmetry breaking
  - SSB of global symmetries (Goldstone Theorem)
  - Linear sigma model (Wigner-Weyl vs Nambu-Goldstone realization)
  - Non-linear sigma model
  - Scalar QED (Abelian Higgs Model)
  - EWSB in the SM

~ 1 hour

- Part 3: Applications
  - Naturalness problem
  - Equivalence theorem
  - Effective Vector Boson Approximation

~ 1 hour

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The Standard Model of Particle Physics

- The Standard Model is a (Lorentz invariant) gauge theory aiming at describing all properties and interactions of the know elementary particles

Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

Matter sector:

- The matter sector of the SM consists of three families of leptons and quarks
- The SM is a chiral theory, i.e. left- and right-handed fermion components transform differently under the SM gauge group
- The transformation property is determined by the quantum numbers, that depend on the representation of the group to which particles belong
- The representation is denoted by $(\mathbf{r}_{\mathbf{SU(3)_C}}, \mathbf{r}_{\mathbf{SU(2)_L}})_Y$

Leptons

$$L^f = (\mathbf{1},\mathbf{2})_{-1/2} = \begin{pmatrix} \nu_L^f \\ l_L^f \end{pmatrix}$$

$$l_R^f = (\mathbf{1},\mathbf{1})_{-1}$$

$$f = e, \mu, \tau$$

Accidental symmetries: L_e, L_μ, L_τ

Quarks

$$Q_L^f = (\mathbf{3},\mathbf{2})_{1/6} = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}$$

$$u_R^f = (\mathbf{3},\mathbf{1})_{2/3} \quad d_R^f = (\mathbf{3},\mathbf{1})_{-1/3}$$

$$f = \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}$$

Accidental symmetries: B

The Standard Model of Particle Physics

- The Standard Model is a (Lorentz invariant) gauge theory aiming at describing all properties and interactions of the know elementary particles

<u>Gauge Group:</u> $\boxed{SU(3)_C \times SU(2)_L \times U(1)_Y}$

<u>Gauge sector:</u>

- The gauge fields of the Standard Model transform in the adjoint representation of the corresponding simple subgroup
- The $U(1)_Y$ subgroup is abelian (rotations in 2-dimensions commute with each other)
- The $SU(N)$ subgroups are non-abelian (two group transformations do not commute)

$$U(1)_Y : \; B_\mu$$

$$SU(2)_L : \; W_\mu^i\,, \quad i = 1, \ldots, 3$$

Electroweak interactions (here is where EWSB takes place and it is the focus of these lectures)

$$SU(3)_C : \; G_\mu^a\,, \quad a = 1, \ldots, 8$$

Color dynamics describing the strong force (this symmetry is unbroken and will be subject of other lectures)

- After EWSB (B_μ, W_μ^i) mix to give rise to the observed mass eigenstates: the massless photon and the massive W and Z bosons $(A_\mu, Z_\mu, W_\mu^\pm)$

Gauge invariance: The fermion Lagrangian

- Fermion fields are described by Dirac spinors (4-component complex objects): ψ
- All fermion indices (spinor and gauge indices) are understood
- The kinetic term for such a field is given by

$$\mathcal{L}_\psi^{\text{kin}} = i\overline{\psi}(x)\gamma^\mu\partial_\mu\psi(x)\,, \qquad \overline{\psi}(x) = \psi^\dagger(x)\gamma^0$$

<u>Global transformations:</u>

$$\psi(x) \to e^{iq^kT^k}\psi(x)$$
$$\overline{\psi}(x) \to e^{-iq^kT^k}\overline{\psi}(x)$$

Generators T^k belong to the representation where the fermion transforms and are Hermitian and q^k are a set of real parameters (therefore $U = \exp(iq^kT^k)$ is a unitary matrix $UU^\dagger = 1$)

- Under these transformations the Lagrangian is invariant

$$\mathcal{L}_\psi^{\text{kin}} \to i\overline{\psi}(x)U^\dagger\gamma^\mu\partial_\mu U\psi(x) = \mathcal{L}_\psi^{\text{kin}}$$

- Thanks to the unitarity of the matrix U and to the fact that it is space-time independent

Gauge invariance: The fermion Lagrangian

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<u>Gauge (local) transformations:</u>

$$\boxed{\begin{aligned}\psi(x) &\to e^{iq^{k}(x)T^{k}}\psi(x)\\ \overline{\psi}(x) &\to e^{-iq^{k}(x)T^{k}}\overline{\psi}(x)\end{aligned}}$$

Generators T^{k} belong to the representation where the fermion transforms and are Hermitian and q^{k} are a set of real parameters (now $U(x) = e^{iq^{k}(x)T^{k}}$ is a unitary, **space-time dependent** matrix)

- Under these transformations the Lagrangian is not invariant anymore

$$\boxed{\mathcal{L}_{\psi}^{\text{kin}} \to i\overline{\psi}(x)U^{\dagger}(x)\gamma^{\mu}\partial_{\mu}\left(U(x)\psi(x)\right) = \mathcal{L}_{\psi}^{\text{kin}} + i\overline{\psi}(x)\gamma^{\mu}U^{\dagger}(x)\partial_{\mu}U(x)\psi(x)}$$

- The problem is with the transformation law of the derivative of the fields:

$$\boxed{\partial_{\mu}\psi(x) \to \partial_{\mu}(U(x)\psi(x)) = U(x)\partial_{\mu}\psi(x) + \partial_{\mu}U(x)\psi(x)}$$

Gauge invariance: The fermion Lagrangian

- One can introduce a "covariant derivative" such that

$$\boxed{D_\mu\psi(x) = \partial_\mu\psi(x) + igA_\mu^k(x)T^k\psi(x) \to U(x)D_\mu\psi(x)}$$

Provided that new vector "gauge" fields are introduced and transform according to

$$\boxed{A_\mu^k(x)T^k \to U(x)A_\mu^k(x)T^kU^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x)}$$

- One can simply verify that this procedure makes the Lagrangian invariant under the local (gauge) transformation

- Notice that, since left and right fermion components transform differently under the gauge group, a mass term of the form

$$\boxed{m\overline{\psi}\psi = m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)}$$

is not invariant under gauge transformations

$$\boxed{\begin{array}{l}\psi_L \to U_L\psi_L \\ \psi_R \to U_R\psi_R\end{array} \quad \Longrightarrow \quad m\overline{\psi}\psi \to m\left(\overline{\psi}_L U_L^\dagger U_R\psi_R + \text{h.c.}\right)}$$

- The left- and right-U matrices are different unitary transformations so that the mass term is not compatible with gauge invariance

Gauge invariance: The fermion Lagrangian

- We end up with the gauge-invariant kinetic term for all SM fermions

$$\mathcal{L}_\psi^{\text{kin}} = \sum_f \sum_{\psi=L,e_R,Q_L,u_R,d_R} i\overline{\psi}^f \gamma^\mu D_\mu \psi^f$$

$$D_\mu \psi = \left(\partial_\mu - igW_\mu^i T^i - ig'YB_\mu - ig_S G_\mu^a T^a\right)\psi$$

Result: we have constructed gauge invariant kinetic terms describing the propagation and the interactions of massless fermions with gauge vector bosons

Gauge invariance: The gauge Lagrangian

- In order to write an invariant kinetic term for fermions we had to introduce gauge fields
- We now need to write down a kinetic term for these fields
- The gauge fields do not transform covariantly under the gauge group

$$A^k_\mu(x)T^k \to U(x)A^k_\mu(x)T^kU^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x)$$

- However, we can construct a covariant quantity out of the gauge fields by noticing that

$$[D_\mu, D_\nu]\psi(x) = \left(D_\mu D_\nu - D_\nu D_\mu\right)\psi(x) \to U(x)[D_\mu, D_\nu]\psi(x)$$

- Therefore, the commutator transforms as

$$[D_\mu, D_\nu] \to U(x)[D_\mu, D_\nu]U^\dagger(x)$$

- This quantity is not a differential operator acting on fermions (all derivatives on the fermion fields cancel) and can be written as

$$\begin{aligned}
&[D_\mu, D_\nu] = -igF^k_{\mu\nu}T^k \\
&F^k_{\mu\nu}T^k = \left(\partial_\mu A^k_\nu - \partial_\nu A^k_\mu\right)T^k - ig[A^k_\mu T^k, A^l_\nu T^l]
\end{aligned}$$

- In the abelian case everything simplifies since A_μ has no index and the generators are just equal to one, so that the commutator in the last term also vanishes

Gauge invariance: The gauge Lagrangian

- We found the gauge covariant operators that allow us to write down kinetic terms for the gauge fields

<u>Abelian Field-Strength:</u>

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

<u>Non-Abelian Field-Strength:</u>

$$F^k_{\mu\nu} = \partial_\mu A^k_\nu - \partial_\nu A^k_\mu + g f^{klm} A^l_\mu A^m_\nu$$

- With the proper normalization of the kinetic terms we finally find

$$\mathcal{L}^{\text{kin}}_V = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{\mu\nu\, i} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}$$

- Notice that the non-covariant transformation law of the gauge fields does not allow any gauge invariant bilinear in the fields without derivatives
- Therefore no gauge boson mass is allowed by gauge invariance

<u>Result:</u> we have constructed gauge invariant kinetic terms describing the propagation only for the abelian gauge fiels and the propagation and the self-interactions for non-abelian gauge fields

The SM Lagrangian before EWSB

- We have succeeded in constructing a theory of propagation and interactions of all known elementary particles (by now the Higgs is still assumed to be unknown)

$$\mathcal{L} = \mathcal{L}_V^{\text{kin}} + \mathcal{L}_\psi^{\text{kin}}$$

- Notice that all we did was to try and write down gauge-invariant kinetic terms for fermions and gauge bosons
- We never attempted at writing down interactions

<u>Result:</u> from the requirement of gauge invariance of kinetic terms we automatically determined all interactions. Gauge invariance gave us interactions “for free”.

<u>Problem:</u> our theory does not describe observations at all! Most of the particles we observe have masses and weak interactions have short-range (the mediators are massive). The gauge invariance that proved so powerful seems to be now our main problem!

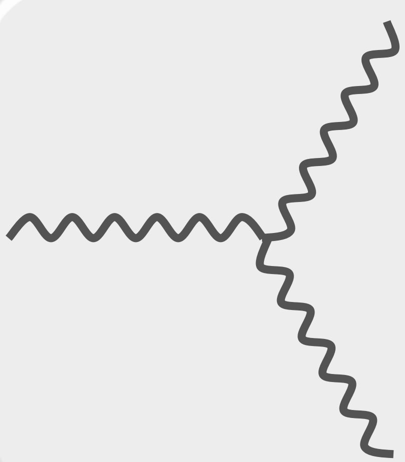
Towards masses

- The simplest way to understand where we are going is to actually ignore gauge invariance for a moment and write down masses anyway
- Let us see what happens with an example
- We consider the Lagrangian for a massive vector (Proca Lagrangian) given by

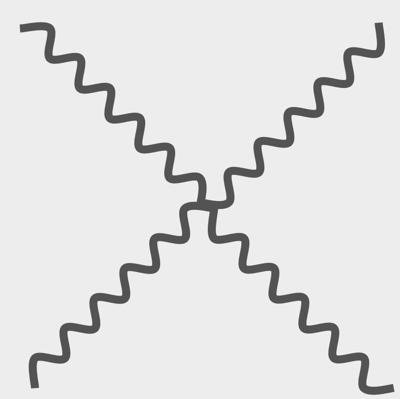
$$\mathcal{L} = -\frac{1}{4} V^a_{\mu\nu} V^{\mu\nu\, a} + \frac{m^2}{2} V^a_\mu V^{\mu\, a}$$

- This Lagrangian (that is not gauge invariant) describes propagation and interactions of a massive vector boson (3 degrees of freedom)

$$-\frac{1}{2}\partial_\mu V^a_\nu \left(\partial^\mu V^{\nu\, a} - \partial^\nu V^{\mu\, a}\right) + \frac{m^2}{2} V^a_\mu V^{\mu\, a}$$



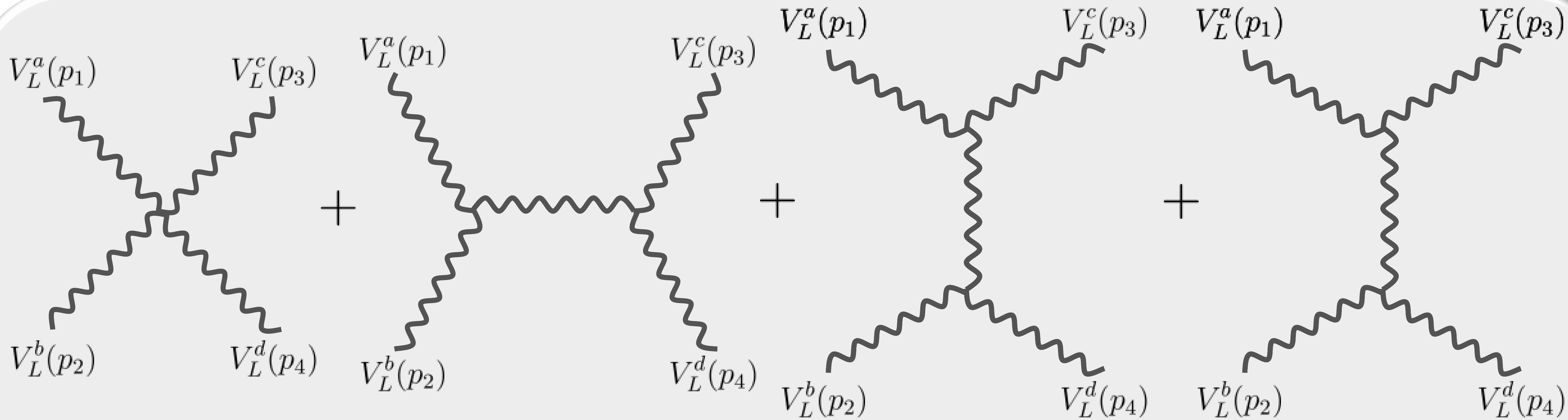
$$-g f^{abc} \left(\partial_\mu A^a_\nu\right) A^{\mu\, b} A^{\nu\, c}$$



$$-\frac{g}{4} f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu\, d} A^{\nu\, e}$$

Scattering amplitudes

- We can compute, in this theory, the scattering of four longitudinal vector bosons



$$\mathcal{A}^{\text{tree}}\left(V_L^a(p_1)V_L^b(p_2)\to V_L^c(p_3)V_L^d(p_4)\right)=\frac{g^2 s}{4m^2}\delta^{ab}\delta^{cd}+\frac{g^2 t}{4m^2}\delta^{ac}\delta^{bd}+\frac{g^2 u}{4m^2}\delta^{ad}\delta^{bc}$$

Mandelstam variables:

$$s=(p_1+p_2)^2 \qquad t=(p_1-p_3)^2 \qquad u=(p_1-p_4)^2$$

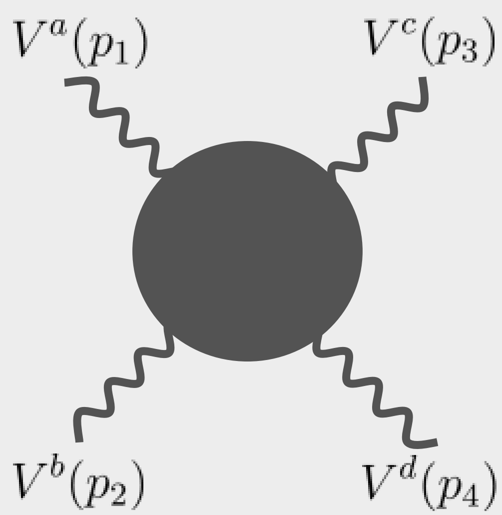
- The tree-level amplitude grows with energy
- We can interpret this with a coupling strength varying with the energy

$$\boxed{\mathcal{A}^{\text{tree}}\approx g(s)^2}$$

- But this coupling constant cannot become arbitrarily large!

Breakdown of perturbation theory

- Ordinary perturbation theory is an expansion in the coupling constant



$$\mathcal{A} = \mathcal{A}^{\text{tree}}(s)\left(1 + c_1\frac{g(s)^2}{(4\pi)^2} + c_2\frac{g(s)^4}{(4\pi)^4} + \dots\right)$$

- When the coupling $g(s)$ becomes of order 4π all orders in the perturbative expansion become of the same order and perturbation theory breaks down
- This problem is usually referred to as "unitarity violation", but this is incorrect, since unitarity, that is related to the probabilistic interpretation of the scattering matrix, here is only violated in perturbation theory, just because perturbation theory does not hold anymore
- The scattering matrix, if it exists, it is unitary by construction
- However not always it can be computed perturbatively

New physics no-lose theorem

- The constraint on perturbativity can tell us something about the electroweak theory
- If we interpret the massive vector as the W boson, then knowing its mass and coupling strength we can get information on what to expect for new physics
- The constraint reads

$$g(s) = \frac{g\sqrt{s}}{2m_W} \lesssim 4\pi \quad \Longrightarrow \quad \sqrt{s} \lesssim \frac{8\pi m_W}{g} = \frac{4\pi}{\sqrt{\sqrt{2}G_F}} \approx 3\ \mathrm{TeV}$$

- This is a clear indication that the existence of massive vector bosons in the Standard Model requires new phenomena at a scale almost fully accessible at collider experiments
- This was the strongest motivation to believe in the existence of the Higgs boson or, at least, to expect a discovery at the Large Hadron Collider (LHC)

Result: trying to introduce masses for the weak gauge bosons we discovered that no-matter what the mechanism generating these masses is, new phenomena are expected at the TeV scale. If these new phenomena enter at low scales, where $g(s)$ is still small (perturbative) we define the new dynamics "weak", while if they enter close to our bound we refer to them as a new "strong" dynamics (non-perturbative).

Interlude: d.o.f. of vector fields

- Let us go back to our massive vector field

$$\boxed{\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{m^2}{2}V_\mu V^\mu}$$

- The vector field V_μ has four components, however, we have mentioned that a massive vector has only 3 propagating degrees of freedom (polarizations)
- In fact the 4-dimensional vector representation of the Lorentz group decomposes into a scalar (1 d.o.f.) and a spin-1 (3 d.o.f.) particle under 3-dimensional rotations
- The scalar degree of freedom (due to the non-compact nature of the Lorentz group) has a negative kinetic term (negative norm)
- If it were physical it would spoil the probabilistic interpretation of quantum mechanics
- To avoid this problem it is necessary to project away the scalar (time-like) component of the vector field
- This is done by requiring all vector polarizations to be orthogonal to the four momentum

$$\boxed{\partial_\mu V^\mu = 0 \quad \Longrightarrow \quad p_\mu \epsilon_i^\mu = 0}$$

- This is not imposed by hand, but arises as a constraint from the e.o.m

Interlude: d.o.f. of vector fields

- The three physical polarizations of our vector field (in the rest frame) can be simply chosen aligned to the three axes (x,y,z)

$$k_\mu = (m,0,0,0) \qquad \epsilon^1_\mu = (0,1,0,0) \quad \epsilon^2_\mu = (0,0,1,0) \quad \epsilon^3_\mu = (0,0,0,1)$$

- To identify the components transverse and longitudinal with respect to the three momentum, we need to make a boost, say in the z-direction. The first two polarizations remain unchanged and represent the transverse ones, while the third becomes the longitudinal one

$$k_\mu = (\sqrt{k_z^2+m^2},0,0,k_z) \qquad \epsilon^3_\mu = \frac{1}{m}\left(k_z,0,0,\sqrt{k_z^2+m^2}\right)$$

- The longitudinal polarization vector grows with the energy
- This is at the origin of the growing with energy amplitude that we found before in the computation of the vector boson scattering for massive vectors
- As we have seen, growing with energy amplitudes were signaling the need, at some energy scale, of new dynamics and/or new degrees of freedom
- As we will see, in the SM, the solution to that problem will come exactly from the solution to the problem of generating masses through SSB (the Higgs mechanism)

Interlude: d.o.f. of vector fields

- We understood how a massive vector is described by three degrees of freedom
- A massless (gauge) vector has instead only two degrees of freedom
- If we try to take the limit of zero mass from the massive case we face the problem of the longitudinal polarization to blow up as $m \to 0$
- Let us try to understand what happens starting from the electromagnetism

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- We have shown before that this Lagrangian is invariant under gauge transformations

$$A_\mu \to A_\mu + \frac{1}{e} \partial_\mu q(x)$$

- We can use this guage invariance to impose the constraint we need to remove the time-like polarization (Lorentz gauge)

$$\partial_\mu A^\mu = 0$$

- However, now the rest frame four momentum is different and the three polarization vectors satisfying this relation become

$$k_\mu = (E, 0, 0, E) \quad \epsilon^1_\mu = (0, 1, 0, 0) \qquad \epsilon^2_\mu = (0, 0, 1, 0) \qquad \epsilon^3_\mu = (1, 0, 0, 1)$$

Interlude: d.o.f. of vector fields

- We can now show that the third polarization vector is unphysical
- In fact it is proportional to the four momentum and again describes a scalar degree of freedom

$$\boxed{A_\mu = \partial_\mu \phi}$$

- However, remember that A_μ is, by gauge invariance, arbitrary up to the divergence of a scalar function, we see that this configuration is gauge equivalent to $A_\mu = 0$ and is therefore not physical (is said to be a pure gauge configuration)

<u>**Result:**</u> massive vector fields have three propagating degrees of freedom thanks to the constraint $k_\mu V^\mu = 0$. The longitudinal polarization vector grows with energy and implies new dynamics/dof to enter at some energy scale. Massless vector fields have two polarization vectors. This can be seen by choosing a gauge and is then true in any other gauge by gauge invariance. Only polarizations transverse to the three momentum are physical.