Introduction to EWSB – Part 2

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Outline

- Part 1: Standard Model 101
  - Particle content
  - Gauge invariance
  - SM Lagrangian before EWSB
  - Masses and the no-lose theorem for new physics

  ~ 1 hour
- Part 2: Spontaneous symmetry breaking
  - SSB of global symmetries (Goldstone Theorem)
  - Linear sigma model (Wigner-Weyl vs Nambu-Goldstone realization)
  - Non-linear sigma model
  - Scalar QED (Abelian Higgs Model)
  - EWSB in the SM

  ~ 1 hour
- Part 3: Applications
  - Naturalness problem
  - Equivalence theorem
  - Effective Vector Boson Approximation

  ~ 1 hour

Spontaneous Symmetry Breaking

- We are back to the problem of generating masses
- The solution is provided by the mechanism of Spontaneous Symmetry Breaking of gauge invariance
- This is an unfortunate name since, as we will see, gauge invariance is not broken at all, it is just "realized" differently
- SSB could explain the generation of gauge boson masses both in the case of weak dynamics (in which case we expect new light degrees of freedom) and in the case of strong dynamics (in which case we expect to see resonances close to the cut-off scale)
- In the case of weak dynamics SSB is said to be linearly realized, while in the case of strong dynamics it is said to be non-linearly realized
- In the SM SSB is linearly realized through weak dynamics and the light degree of freedom is the Higgs boson
- Notice that this does not mean that the Higgs itself cannot be generated by some other strong dynamics responsible for EWSB (see Riva's lectures)

Example 0: discrete symmetry

- The simplest example of SSB: real scalar with discrete Z_2 symmetry

$$\mathcal{L}_\varphi = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\,, \qquad V(\varphi) = \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

- We need $\lambda \geq 0$ so that the potential is bounded from below
- This theory is invariant under the transformation

$$\varphi \to -\varphi$$

- The minimum of the potential is determined by its first derivative

$$V'(\varphi) = \left(\mu^2 + \frac{\lambda}{6}\varphi^2\right)\varphi = 0 \qquad V(\langle\varphi\rangle) = V_{\text{min}}$$

<u>Unbroken phase:</u> $\mu^2 > 0$

$$\langle\varphi\rangle = 0$$

$$m^2 = V''(\langle\varphi\rangle) = \mu^2$$

Single (symmetric) vacuum state

<u>Broken phase:</u> $\mu^2 < 0$

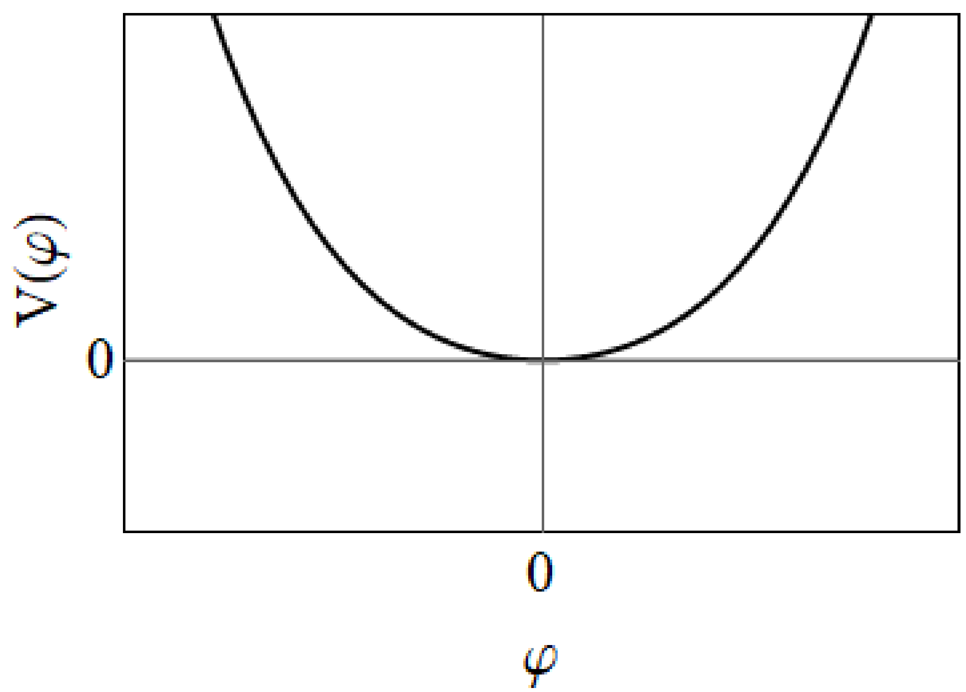
$$\langle\varphi\rangle = \pm\sqrt{\frac{-6\mu^2}{\lambda}} = \varphi_0$$

$$m^2 = V''(\langle\varphi\rangle) = -2\mu^2$$

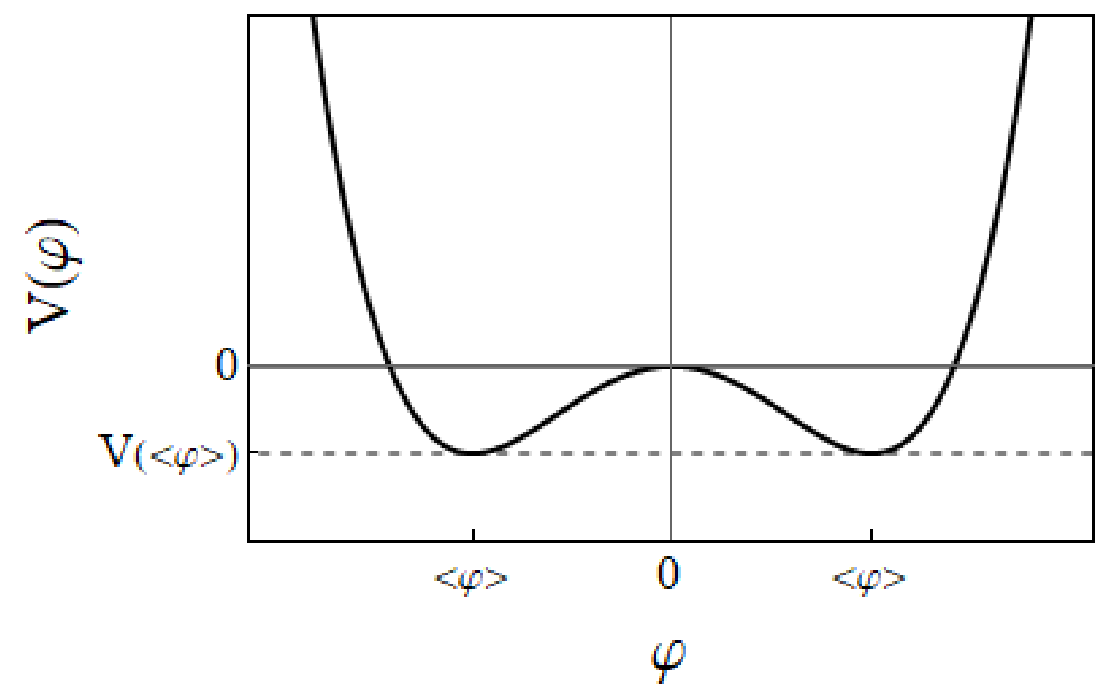
Two equivalent vacuum states

Example 0: discrete symmetry

Unbroken phase: $\mu^2 > 0$



Broken phase: $\mu^2 < 0$



- In the "broken" phase the symmetry seems lost

$$V(\varphi) = \frac{\mu^2}{2}(\varphi_0 + \varphi)^2 + \frac{\lambda}{4!}(\varphi_0 + \varphi)^4 = -\frac{3\mu^4}{2\lambda} - \mu^2\varphi^2 + \sqrt{\frac{-\lambda\mu^2}{6}}\varphi^3 - \frac{3\mu^4}{2\lambda}$$

- However it is only realized in a different way (it is not a symmetry of the vacuum)

Wigner-Weyl realization:

$$\varphi \to -\varphi$$

Nambu-Goldstone realization:

$$\varphi + \varphi_0 \to -(\varphi + \varphi_0)$$
$$\implies \quad \varphi \to -\varphi - 2\varphi_0$$

Example 1: continuous global symmetry

- A slightly more illustrative example is that of a global continuous symmetry
- Consider the following Lagrangian for a complex scalar field (2 real components)

$$\mathcal{L}_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)\,, \qquad V(\phi) = \mu^2 \phi^\dagger \phi + \frac{\lambda}{4}\left(\phi^\dagger \phi\right)^2$$

- Again $\lambda \geq 0$ is needed for the potential to be bounded from below
- This new theory is invariant under continuous $U(1) \sim SO(2)$ transformations

$$\phi \to U\phi\,, \quad \phi^\dagger \to U^\dagger \phi^\dagger\,, \quad U = e^{i\alpha}$$

- Looking again at the minimum of the potential we get

<u>Unbroken phase:</u> $\mu^2 > 0$

$$\langle \phi \rangle = 0$$

Single (symmetric) vacuum state

Two degenerate states with masses

$$m^2 = V''(\langle \varphi \rangle) = \mu^2$$

<u>Broken phase:</u> $\mu^2 < 0$

$$\langle \phi \rangle = \sqrt{\frac{-2\mu^2}{\lambda}} e^{i\theta}$$

Infinite class of equivalent vacuum states

The spectrum consists of a massive and a massless real scalar

Example 1: continuous global symmetry

- Let us give a closer look at the broken phase
- To do so we want again to expand the field as quantum fluctuations around a classical vacuum expectation value (where the potential is minimum)
- We can parametrize the complex field as an argument σ and a phase π and choose

$$\phi = \left(\sqrt{\frac{-2\mu^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma\right) e^{\frac{i\pi}{f_\pi}}$$

- The potential becomes

$$\begin{aligned} V(\phi) &= \mu^2 \left(\sqrt{\frac{-2\mu^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma\right)^2 + \frac{\lambda}{4}\left(\sqrt{\frac{-2\mu^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma\right)^4 \\ &= -\frac{\mu^4}{\lambda} - \mu^2\sigma^2 + \frac{\sqrt{-\lambda\mu^2}}{2}\sigma^3 + \frac{\lambda}{16}\sigma^4 \end{aligned}$$

- Only σ has a potential and has mass $m_\sigma^2 = -2\mu^2$
- But π cannot have disappeared, we should also look at the kinetic terms

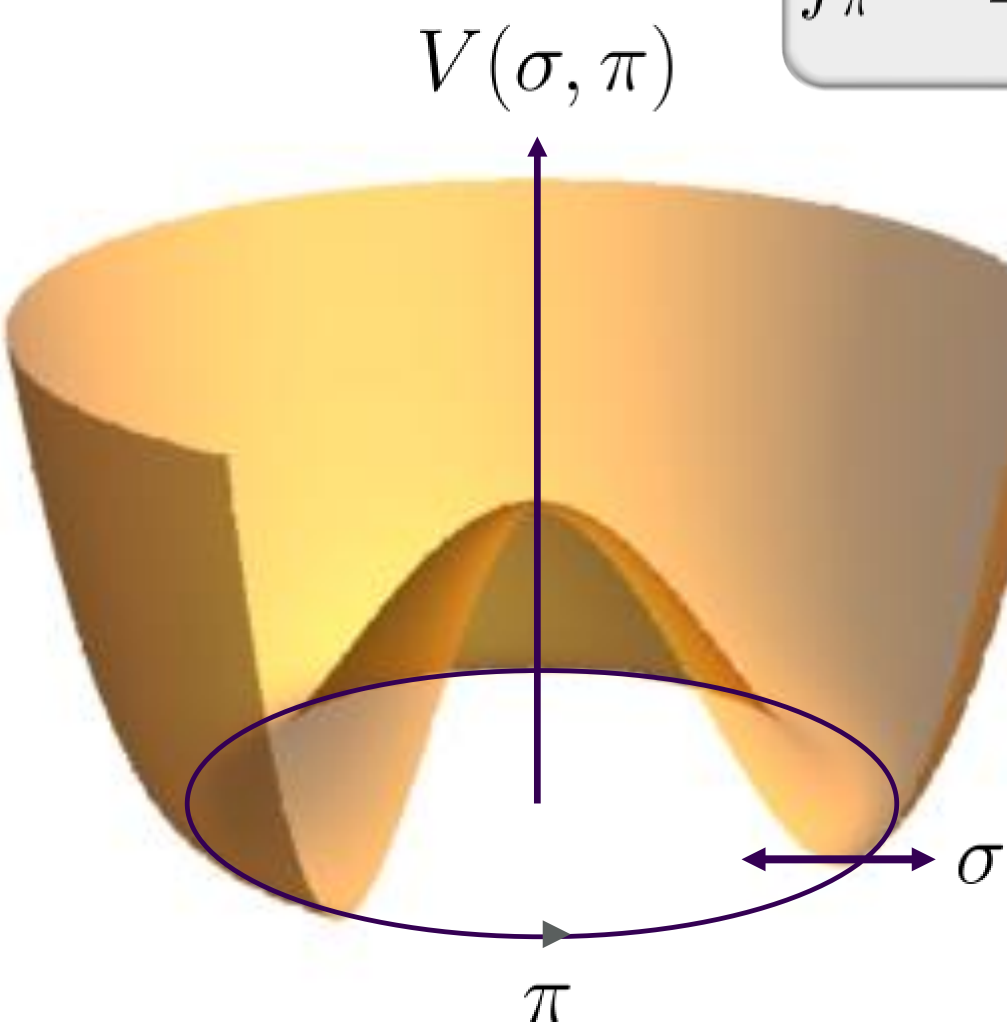
Example 1: continuous global symmetry

- The Lagrangian reads

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \left(\sqrt{\frac{-2\mu^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma\right)^2 \frac{1}{f_\pi^2}\partial_\mu\pi\partial^\mu\pi - V(\phi)$$

- The arbitrary dimensionful constant f_π can be fixed requiring π to have a normalized kinetic term

$$f_\pi = 2\sqrt{\frac{-\mu^2}{\lambda}} = \sqrt{2}|\langle\phi\rangle| = v$$



- Therefore the field π propagates and interacts with σ only with derivative couplings, but does not have any potential (it is massless)
- It is said to be a Goldstone Boson
- Notice how the original symmetry is now realized as a shift symmetry $\pi \to \pi + c$, that is the Nambu-Goldstone realization

The non-linear realization

- To identify the Goldstone boson and the radial excitation it is useful to consider the limit

$$\mu \to \infty\,, \quad \lambda \to \infty\,, \quad v = \text{const}$$

- In this limit the radial mode decouples and the Lagrangian reduces to the free theory of a scalar with a shift symmetry

$$\mathcal{L} = \frac{1}{2}\partial_\mu \pi \partial^\mu \pi$$

- This is a "trivial" example of what is known as a non-linear sigma model
- In this case SSB is said to be realized non-linearly and the radial degree of freedom is heavy and it is not part of the light spectrum of the theory
- The realization is said to be non-linear since now the ϕ field is just a non-linear function of the Goldstone bosons

$$\phi = \sqrt{\frac{-2\mu^2}{\lambda}} e^{\frac{i\pi}{v}}$$

- In an interacting theory the linear vs non-linear realizations can be distinguished looking at interactions

Goldstone Theorem

Goldstone Theorem: a spontaneously broken global symmetry implies the existence of massless scalars called Goldstone bosons. There are as many Goldstone bosons as broken symmetry generators.

Examples:

- QCD with 2 flavors (u, d quarks)

$$SU(2)_L \times SU(2)_R \to SU(2)_{L+R} = SU(2)_V$$

Each $SU(2)$ has three generators, therefore in total three generators are broken

⇨ 3 massless Goldstone bosons: $\pi^0, \pi^\pm$ triplet of $SU(2)_V$

The symmetry is approximate and is broken by light quark masses and therefore the pions are not exactly massless. Since the quark masses are much lighter than the QCD confinement scale ($m_u \approx 2 \text{ MeV}, m_d \approx 5 \text{ MeV}$), the symmetry is a good approximation and in fact pions are lighter ($m_\pi \approx 135 \text{ MeV}$) than the other QCD resonances

Goldstone Theorem

Goldstone Theorem: a spontaneously broken global symmetry implies the existence of massless scalars called Goldstone bosons. There are as many Goldstone bosons as broken symmetry generators.

Examples:

- QCD with 3 flavors (u, d, s quarks)

$$SU(3)_L \times SU(3)_R \to SU(3)_{L+R} = SU(3)_V$$

Each $SU(3)$ has eight generators, therefore in total eight generators are broken

⇒ 8 massless Goldstone bosons: $\pi^0, \pi^\pm, K^0, \overline{K}^0, K^\pm, \eta$
octet of $SU(3)_V$

While the symmetry is a very good approximation for the two light quarks it is less for the strange quark ($m_s \approx 100\ \mathrm{MeV}$) so that strange mesons are heavier ($m_K \approx 500\ \mathrm{MeV},\ m_\eta \approx 550\ \mathrm{MeV}$)

SSB of gauge invariance

- We have seen so far the implications of SSB of global discrete and continuous symmetries
- The Goldston Theorem is a crucial result in QFT and predicts the existence of massless, or approximately massless scalars
- SSB of gauge invariance is radically different
- We can understand this by just counting degrees of freedom

Unbroken phase

- Massless vector has 2 physical degrees of freedom
- These correspond for instance to the two transverse polarizations of photons

Broken phase

- Massive vector has 3 physical degrees of freedom
- There is a new Longitudinal polarization for massive vectors

Problem: how can SSB change the number of degrees of freedom, or, better, how can we reconcile the transition from the unbroken to the broken phase, while conserving the number of degrees of freedom?

SSB of gauge invariance

<u>Problem:</u> how can SSB change the number of degrees of freedom, or, better, how can we reconcile the transition from the unbroken to the broken phase, while conserving the number of degrees of freedom?

<u>Solution:</u> the Goldstone Theorem is violated by SSB of "local" (gauge) invariance. The Goldstone Bosons are not anymore new massless degrees of freedom, but provide the degrees of freedom corresponding to the longitudinal polarizations of gauge bosons. This is at the basis of the Higgs phenomenon (or Higgs mechanism).

SSB of gauge invariance: scalar QED

- SSB of gauge invariance is very simply understood in a model of scalar QED, that is the electrodynamics of a charged scalar (Abelian Higgs model)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger D_\mu\phi - V(\phi)\,, \quad V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

$$D_\mu\phi = (\partial_\mu + ieA_\mu)\,\phi \qquad (D_\mu\phi)^\dagger = (\partial_\mu - ieA_\mu)\,\phi^\dagger$$

- This theory is invariant under gauge transformations ($U(1)$ gauge group)

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu q(x) \qquad \phi \to e^{iq(x)}\phi \qquad \phi^\dagger \to e^{-iq(x)}\phi^\dagger$$

- We have chosen the sign of the mass term in such a way that for $\mu^2 > 0$ SSB occurs as in the case of the linear sigma model

$$|\langle\phi\rangle| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$

- Again, we expand the field around the vacuum configuration as

$$\phi = \left(\frac{v+\sigma}{\sqrt{2}}\right) e^{\frac{i\pi}{v}}$$

SSB of gauge invariance: scalar QED

- Expanding the Lagrangian we find

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(\frac{v+\sigma}{\sqrt{2}}\right)^2 \left(\frac{\partial_\mu \sigma}{v+\sigma} - i\frac{\partial_\mu \pi}{v} - ieA_\mu\right)\left(\frac{\partial^\mu \sigma}{v+\sigma} + i\frac{\partial^\mu \pi}{v} + ieA_\mu\right)$$
$$- \left(-\frac{\mu^4}{\lambda} + \mu^2\sigma^2 + \frac{\sqrt{\lambda}\mu}{2}\sigma^3 + \frac{\lambda}{16}\sigma^4\right)$$

- As expected interactions of the Goldstone boson are again only derivative interactions
- To understand the spectrum let us focus on the quadratic part of the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{e^2v^2}{2}A_\mu A^\mu + ev\partial_\mu\pi A^\mu - \mu^2\sigma^2$$

- We have successfully generated a mass term for the gauge boson, the kinetic term are canonical, but we have an unpleasant kinetic mixing between the Goldstone boson and the gauge field
- This can be removed by properly choosing a gauge
- We add to the original Lagrangian a gauge-fixing term (a generalization of the Lorentz gauge)

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi e v \pi\right)^2$$

SSB of gauge invariance: scalar QED

- With the introduction of this gauge-fixing, the quadratic part of the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{e^2v^2}{2}A_\mu A^\mu + \cancel{ev\partial_\mu\pi A^\mu}$$
$$- m^2\sigma^2 + \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + \frac{\xi}{2}e^2v^2\pi^2 + \cancel{ev\partial_\mu A^\mu\pi}$$

- Now the two kinetic mixing terms are equal up to an integration by parts, where the total derivative is irrelevant, so cancel out
- In order to understand the spectrum it is useful to write down the propagators of the three fields appearing in our Lagrangian

$$\frac{i}{k^2 - e^2v^2}\left(-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi e^2v^2}\right)$$

$$\frac{i}{k^2 - 2\mu^2}$$

$$\frac{i}{k^2 - \xi e^2v^2}$$

SSB of gauge invariance: scalar QED

$$\frac{i}{k^2 - e^2 v^2}\left(-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi e^2 v^2}\right)$$

$$\frac{i}{k^2 - 2\mu^2}$$

$$\frac{i}{k^2 - \xi e^2 v^2}$$

- The gauge boson has acquired a mass $m_A = ev$
- The sigma field, as before, has a mass $m_\sigma = \sqrt{2}\mu$
- The mass of the Goldstone boson π depends on the gauge parameter ξ signaling the fact that this field cannot be a physical independent degree of freedom anymore
- One can see that π can be completely removed from the spectrum by going in the so-called unitary gauge, which corresponds to $\xi \to \infty$
- In this gauge it becomes infinitely heavy and its propagator vanishes, while the boson propagator goes back to the propagator of the massive vector that we introduced previously
- This shows how the longitudinal polarization of the gauge field and the Goldstone boson are tight together (the GB is “eaten-up” by the corresponding Gauge boson)

SSB of gauge invariance: scalar QED

$$\frac{i}{k^2 - e^2v^2}\left(-g_{\mu\nu} + (1-\xi)\,\frac{k_\mu k_\nu}{k^2 - \xi e^2 v^2}\right)$$

$$\frac{i}{k^2 - 2\mu^2}$$

$$\frac{i}{k^2 - \xi e^2 v^2}$$

- Other known gauge choices are:

  $\xi = 0$ Landau Gauge

  $\xi = 1$ Feynman Gauge

All ξ gauges are called **renormalizable gauges** or **'t Hooft gauges**

<u>Result:</u> SSB of gauge invariance is the solution to the problem of generating gauge boson masses. As before gauge invariance is not actually broken, but it is realized a la Nambu-Goldstone. The GB is eaten up by the corresponding gauge field providing its longitudinal component and it is not anymore an independent degree of freedom.

SSB of in the SM

- We are not ready to build SSB in the SM ($SU(2)_L \times U(1)_Y \to U(1)_Q$)
- EWSB is the non-abelian generalization of the abelian case we have just seen
- We add to the SM Lagrangian a doublet H of $SU(2)_L$ with hypercharge $Y = 1/2$ described by the following Lagrangian

$$\mathcal{L}_H = (D_\mu H)^\dagger D^\mu H - V(H), \qquad V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$D_\mu H = \left(\partial_\mu - i\frac{g}{2} W_\mu^i \sigma^i - i\frac{g'}{2} B_\mu\right) H$$

- We choose to parametrize the field in a way analogous to the previous linear sigma model example through a radial excitation times a phase

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} e^{\frac{i\pi^a \sigma^a}{v}}$$

- From the kinetic term of H we immediately find the gauge boson spectrum

$$\mathcal{L}_{\text{mass}} = \frac{g^2 v^2}{8} \left((W_\mu^1)^2 + (W_\mu^2)^2 + \left(\frac{g'}{g} B_\mu - W_\mu^3\right)^2 \right)$$

Gauge boson masses

- The states W^i, B are neither mass not charge eigenstates
- The fields W^1_μ, W^2_μ do not have definite charge, but can be combined to give the charge eigenstates

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right)$$

- The fields W^3_μ, B instead are both neutral, but have a mass mixing, which needs to be diagonalized to determine the spectrum
- A simple 2x2 rotation is sufficient to identify the photon and the Z boson

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$
$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$

- The angle θ_W is called weak mixing angle (or Weinberg angle) and is related to the two gauge couplings g and g' by

$$\tan\theta_W = \frac{g'}{g} \qquad \sin\theta_W \approx 0.23$$

- The gauge boson masses are given by

$$m_W = \frac{gv}{2} \approx 80 \text{ GeV} \qquad m_Z = \frac{\sqrt{g^2 + g'^2}\,v}{2} = \frac{m_W}{\cos\theta_W} \approx 91 \text{ GeV}$$

Higgs interactions

- Let us look at the term in the Lagrangian involving the Higgs boson

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{m_h^2}{2}h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \left(m_W^2 W_\mu^+ W^{\mu -} + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)\left(\frac{h^2}{v^2} + \frac{2h}{v}\right)$$

- The Higgs field couples to gauge bosons proportionally to their masses
- This property will carry on fermions and it is a key prediction of EWSB
- This Lagrangian has several independent parameters to be determined from experiments

<u>**Parameters:**</u>

m_h	The Higgs mass (measured after 2012)
$m_{W,Z}$	The W and Z boson masses are known with very good accuracy and are said to be EW precision observables
v	The vev is usually traded for the Fermi constant $G_F = 1/(\sqrt{2}v^2)$ which is measured with very high accuracy from muon decay
λ	This governs Higgs self-interactions and it is related in the SM to the mass and the vev of the Higgs boson. It is still directly unmeasured and its direct measurement is a missing (challenging) test of the SM

Fermion masses

- We have successfully found a mechanism to generate gauge boson masses
- However, we know that also SM fermions have a mass
- Notice that now that we have introduced the Higgs field, we can write new renormalizable interactions between it and (some of) the SM fermions, for instance:

$$\mathcal{L}_{\text{Yuk}} = -y\bar{L}He_R + \text{h.c.}$$

- Expanding again on the Higgs vacuum this becomes

$$\mathcal{L}_{\text{Yuk}} = -y\frac{h+v}{\sqrt{2}}\overline{e}e = -m_e\left(1+\frac{h}{v}\right)\overline{e}e$$

- So that also the fermion acquired a mass, proportional to its Yukawa coupling

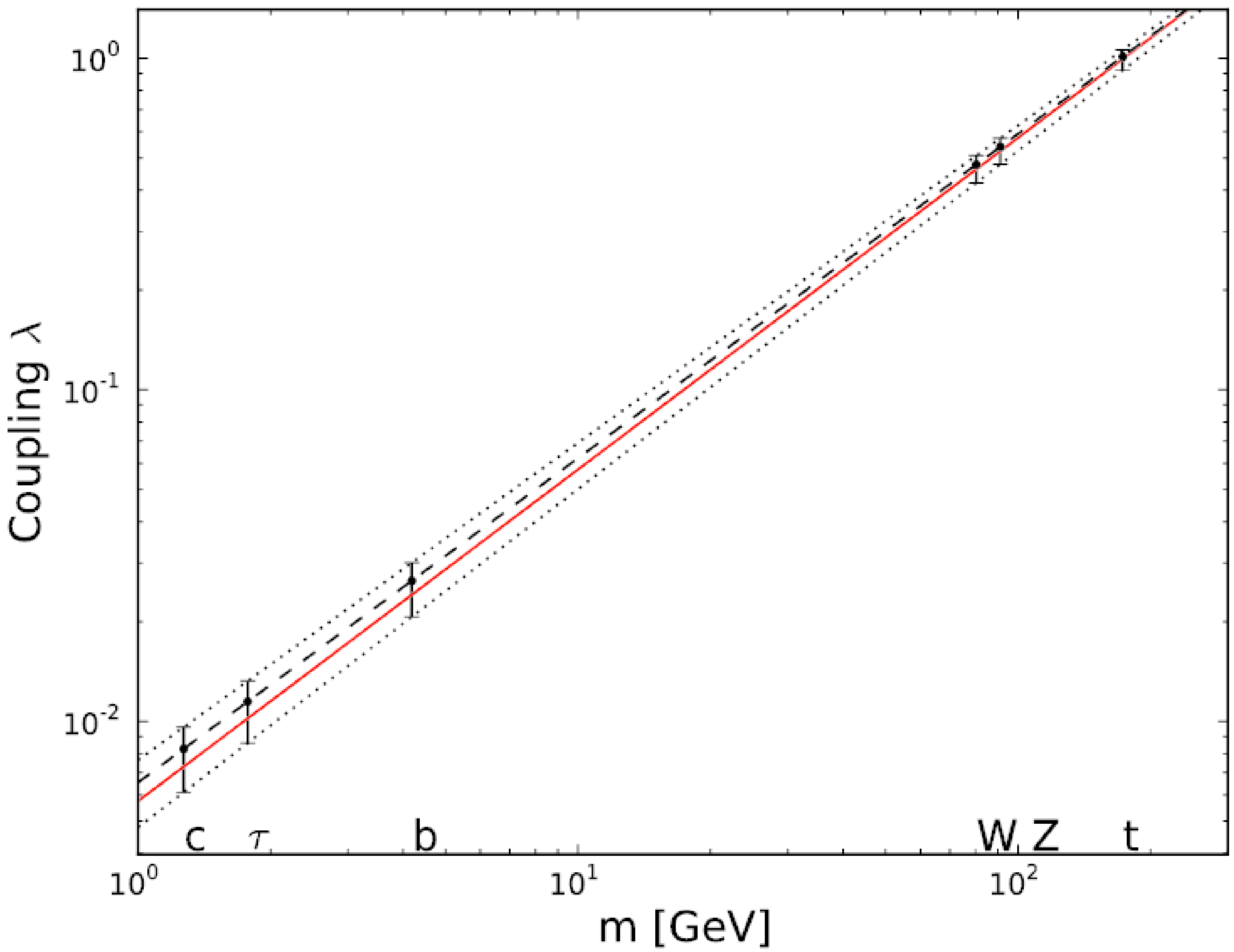
$$m_e = \frac{yv}{\sqrt{2}}$$

- Again, as we anticipated, Higgs interacts proportionally to the mass of the particle
- Notice that some Yukawa couplings would need the Higgs to have opposite hypercharge to be invariant
- Group theory allows us to construct a complex conjugate Higgs field as

$$\tilde{H} = i\sigma^2 H^*$$

The non-linear realization

- We have seen before that SSB can be realized non-linearly, with the radial mode decoupled from the light spectrum
- We have experimental evidence that the Higgs boson exists and is light, however we could ask ourself if is really the radial mode or if EWSB is realized non-linearly and the Higgs is just an accidentally light scalar (singlet under the SM)
- The rest of the Lagrangian would remain the same, but Higgs boson interactions would not be predicted anymore by EWSB (they would be free parameters)



- Measurements of the Higgs properties at the LHC reproduce very well the SM expectation
- The relation between the masses of the SM particles and their interactions with the Higgs is linear
- This suggests that the linear-realization is, at least to a very good level of accuracy, correct

Summary

<u>Result:</u> With the introduction of the Higgs field and the assumption that SSB occurs we have solved several problems at once. We have successfully generated gauge boson masses in a way compatible with gauge invariance. We have also generated fermion masses proportional to new Yukawa couplings. The Higgs is predicted to couple to all other particles proportionally to their masses. Most of these predictions have by now been verified. Important missing checks of the mechanism of EWSB are the measurement of the Higgs self-coupling and of the Yukawa couplings with fermions.

The SM Lagragian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ + i\bar{\Psi}\not{D}\psi \\ + D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi) \\ + \bar{\Psi}_L\hat{Y}\Phi\Psi_R + h.c.$$