Discussion: models and uncertainties on rho parameter

How to extract ρ from differential elastic cross section measurements…

…..an experimental point of view ….

….or my limited understanding

Historical outlook and reminder

All pre-LHC collider experiment:

Amaldi et al ISR
UA4 SppS collider
E811 at Fermilab

have used the Simplified West-Yennie formula (1968)
to fit the differential elastic cross section….
…and to extract ρ

where $\rho = \mathrm{Re}\, F_{el}(t=0)/\mathrm{Im}F_{el}(t=0)$

In SWY formula the interference is reduced to an additional phase φ between the coulomb amplitude and the nuclear amplitude

$$F^{\mathrm{C+N}}(s,t) = F^{\mathrm{C}}(s,t)\,\mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + F^{\mathrm{N}}(s,t);$$

$$\Phi(t) = -\left(\log\frac{b_1|t|}{2} + \gamma\right)$$

However ....

The Simplified West-Yennie formula is derived under the assumptions:

1) The strong amplitude (often called nuclear amplitude) can be described by a simple exp (bt/2)

2) The phase of the strong amplitude (often called nuclear phase) is constant i.e. independent of t

Advantage with SWY:

Easy to use.....interference reduced to additional phase φ between the coulomb amplitude and the nuclear amplitude

Has been used both at the ISR, SppS collider and the Tevatron to extract ρ .. Thus we have backward compatiblity

Disadvantage with SWY:

Is apriori not valid if the strong amplitude can not be described by exp(bt/2)

Is apriori not valid if the phase is not constant...(though in most models the phase is close to constant for small t)

The Cahn interference formula

Cahn 1982 –very general formula:

No imposed limit on the strong amplitude and phase (also including treatment of formfactor) but complicated and not easy to use …

…

…..and the interference formula can not be expressed as a simple additional phase between the Coulomb and Nuclear amplitude…

This approach was followed up among others by Kundrat and Locajcek in the 90th.

$$\frac{d\sigma^{C+N}}{dt} = \frac{\pi(\hbar c)^2}{sp^2}\left| -\frac{\alpha s}{q^2}\mathscr{F}^2 + \mathscr{A}^N\left[1 - i\alpha G(-q^2)\right]\right|^2 ,$$

$$G(-q^2) = -\int_0^\infty dq'^2 \log\frac{q'^2}{q^2}\frac{d}{dq'^2}\mathscr{F}^2(-q'^2) + \frac{1}{\pi}\int d^2q' \frac{\mathscr{F}^2(-q'^2)}{q'^2}\left[\frac{\mathscr{A}^N(-[\mathbf{q}-\mathbf{q}']^2)}{\mathscr{A}^N(-q^2)} - 1\right]$$

Other attempts to calculate the Coulomb phase have been done by e.g.
Oleg Selyugin
Kopeliovich and Tarasov
….

In the following I will compare the SWY with the Cahn approach as used by Kundrat -Lokajcek (I will call it CKL approach).
This choice is dictated by the fact that TOTEM uses the CKL approach to extract ρ and thus I have something to compare with.

Strong statements by Kundrat-Lokajcek in their publications
…the SWY approach must be abandoned…..
…inconsistent and leads to wrong conclusions

In spite of this I ask the question :

Using the SWY prescription with amplitudes deviating from exp(bt/2)and/ or phases not being constant… how important is the difference in the extracted ρ-value relative to the more general Cahn approach?

…I will try to answer it "experimentally"

Observe:

The CKL formula reduces to the SWY if the assumption of constant phase and amplitude exp(bt/2) are assumed.

That this is true is seen below from comparisons of fits to the TOTEM data using exp(bt/2) and constant phase with SWY and with Cahn/KL prescriptions

	SWY, constant	Cahn/KL, constant
step 1: χ^2/ndf	$48.0/27 = 1.78$	$48.1/27 = 1.78$
step 2: χ^2/ndf	$180.8/58 = 3.12$	$181.2/58 = 3.12$
a [mb/GeV^2]	533 ± 23	533 ± 23
b_1 [GeV^{-2}]	19.42 ± 0.05	19.42 ± 0.05
ρ	0.05 ± 0.02	0.05 ± 0.02
ζ_1		
κ		
ν [GeV^{-2}]		
σ_{tot} [mb]	102.0 ± 2.2	102.0 ± 2.2

Every thing is identical!!

Let us now see what happens with the difference between SWY and CKL if we deviate with the strong amplitude from the simple exp(bt/2) ??
TOTEM did not publish this for the case of SWY ....
I did my own fit from the published TOTEM data

Compare Cahn/KL with SWY for different assumptions of the amplitude
use TOTEM 8 TeV data in range 0.0006-0.19 GeV^2
(arXiv1610.00603 and Eur.Phys.J)

Strong amplitude Exp $(1/2 \sum b_n t^n)$	TOTEM published Cahn/KL Const. Phase ρ	My fit with SWY Const. Phase ρ
N=1	0.05 +- 0.02	0.05 +- 0.02
N=2	0.10	0.10 +- 0.02
N=3	0.12 +- 0.03	0.11 +- 0.02

My fit only statistical error

Conclusion: It looks like the SWY prescription gives basically the same result as the Cahn/KL approach for amplitudes deviating slightly from the pure exponent

Observe :

- West-Yennie states in the original paper –footnote page 1418….
  a second term in the exponent "has essentially no effect on the conclusions"
  The table above seems to confirm this

What about a non-constant nuclear phase?

Compare Cahn/Kl with SWY for different assumptions of the phase
use ISR 52.8 GeV data in range 0.001-.02 GeV^2

arXiv:1710.10640v1 [hep-ph] 29 Oct 2017

$$\sigma^{tot} = (42.38 \pm 0.27)\ \text{mb},$$
$$\rho(t=0) = (0.078 \pm 0.010),$$
$$B(t=0) = (13.1 \pm 0.2)\ \text{GeV}^{-2};$$

← ISR data analyzed with SWY (1977)

Re-analized in 2017 by →

Eikonal model analysis of elastic proton-proton collisions at 52.8 GeV and 8 TeV

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Fit Case		1b central	2b peripheral	3b peripheral	4b peripheral
$\rho(t=0)$		0.0766 ± 0.0017	0.0822 ± 0.0017	0.0824 ± 0.0017	0.0825 ± 0.016
$B(t=0)$	[GeV^{-2}]	13.514 ± 0.050	13.414 ± 0.045	13.42 ± 0.41	13.431 ± 0.044
σ^{tot}	[mb]	42.71 ± 0.15	42.86 ± 0.10	42.860 ± 0.10	42.86 ± 0.095
σ^{el}	[mb]	7.472	7.541	7.544	7.539
σ^{inel}	[mb]	35.23	35.31	35.32	35.32
σ^{el}/σ^{tot}		0.1750	0.1761	0.1760	0.1759
$d\sigma^N/dt(t=0)$	[mb.GeV^{-2}]	93.74	94.49	94.49	94.49
$\sqrt{\langle b^2\rangle^{tot}}$	[fm]	1.027	1.022	1.022	1.023
$\sqrt{\langle b^2\rangle^{el}}$	[fm]	0.6764	1.676	1.794	1.994
$\sqrt{\langle b^2\rangle^{inel}}$	[fm]	1.086	0.8170	0.7621	0.6487
χ^2/ndf		323/205	274/203	275/203	276/203

Table 1: Results of the analysis of pp elastic scattering at energy of 52.8 GeV corresponding to central and three different peripheral distributions in the impact parameter space.

Conclusion:

Also for the different phases tried here with the Cahn/Kl approach…
….no major differences ..at least in the ISR case …
observed relative SWY

My conclusions:

The SWY formula is derived under the assumption
that the amplitude is an simple exp(1/2bt) and
the nuclear phase is constant.

If other more complex amplitudes or phases are used
the interference can not be described just with an additional phase
as is done with the SWY but more complicated integrals over the complex
amplitude has to be solved.

However it turns out that in practice the differences between the extracted
ρ-values are small.

Another way of formulating the conclusion....

For an experimentalist

who measures $d\sigma/dt$ for elastic scattering and want to do just a little bit more than publish the differential cross section i.e extract ρ, σ_{tot} and B (B, C,D...) the SWY seems to give a reasonable result.

HOWEVER for a theoritician

who wants to understand the data in terms of his preferred model and address question related to the nature of elastic scattering i.e. in terms of being central or peripheral the SWY approach is not adequate.

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