Transverse Beam Stability with Realistic Longitudinal Profiles and Contribution from Space Charge

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CERN – HL-LHC WP2 Meeting

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Motivation

Study based on Elena Shaposhnikova's Jan'17 WP2 presentation ↗:
after longitudinal blow-up (during ramp), we have

- *q*-Gaussian longitudinal bunch profile
- hole in synchrotron frequency distribution

central goals

Investigate impact on head-tail instabilities at HL-LHC by

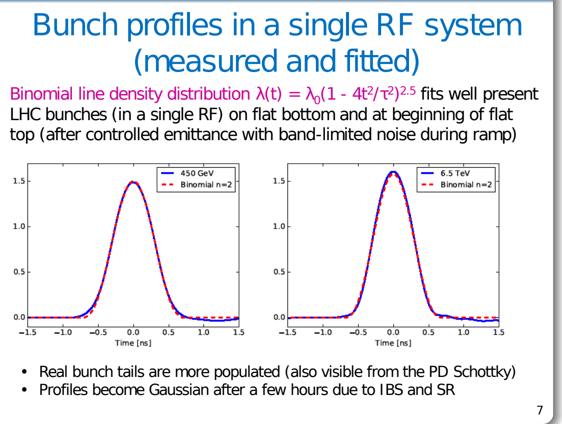
- longitudinal distribution
- direct space charge

Content of this talk:

1. stationary macro-particle distributions for longitudinal
   - *q*-Gaussian distribution (without hole)
   - hollow distribution from measured LHC tomogram
2. chromaticity scans for head-tail instabilities at flat-top
   ⟶ compare thermal, *q*-Gaussian and hollow distribution
3. contribution to stability from space charge (TMCI, head-tail)

Flat-bottom and initial flat-top: non-Gaussian longitudinal bunch profile

Elena Shaposhnikova: *q*-Gaussian bunch profile



⟹ consider equal **F**ull **W**idth at **H**alf **M**aximum
("[longitudinal] instability thresholds scale with the FWHM values")

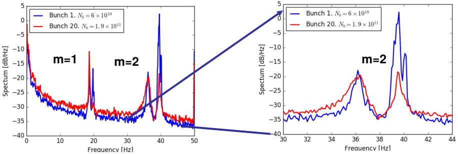
Hole in Synchrotron Frequency Distribution

Not only non-Gaussian bunch profile – also **depleted bunch centre**:

Elena Shaposhnikova: hole in spectrum

Particle distribution after controlled emittance blow-up in LHC

Particle distribution in synchrotron frequency Ω after blow-up from Peak Detected Schottky spectrum for low and high intensity bunches

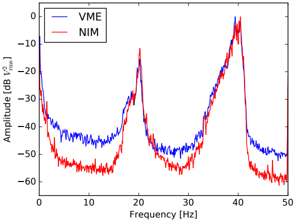


→ After controlled emittance blow-up the distribution function in synchrotron frequency F(Ω) (or in action F(J)) has a hole

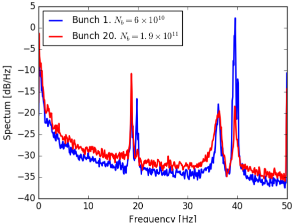
Particle distribution function F(Ω) = dN/dΩ = F(J)/Ω′(J)

8

Schottky Spectrum: "Hole"



(a) "usual" Schottky spectrum (image from Juan Müller's thesis)



(b) Schottky spectrum with hole (after longitudinal blow-up)

$\Longrightarrow$ slightly hollow phase space distribution can explain "missing" Schottky signal around linear synchrotron frequency

1. stationary longitudinal macro-particle distributions

FWHM Equivalence

Generate macro-particles matched to the RF bucket with

(i) thermal distribution: $f(\mathscr{H}) \propto \exp(-\mathscr{H}/\mathscr{H}_0)$

(ii) q-Gaussian distribution (for $q = 3/5$): $f(\mathscr{H}) \propto \left(1 - \frac{\mathscr{H}}{\mathscr{H}_0}\right)^2$

(iii) hollow distribution reconstructed from tomography

with equal FWHM via[1]

$$\sigma_z^{\mathrm{RMS}}\Big|_{q\text{-Gaussian}} \stackrel{!}{=} 0.846 \cdot \sigma_z^{\mathrm{RMS}}\Big|_{\mathrm{thermal}}$$

[1] cf. e.g. R. Tomás' & L. Medina's WP2 talk Mar'17 ↗ or their IPAC'17 paper ↗ or S. Papadopoulou's IPAC'17 paper ↗

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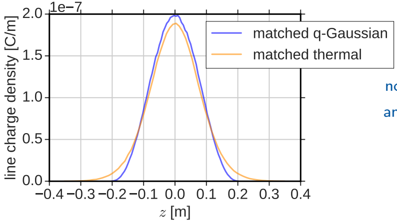
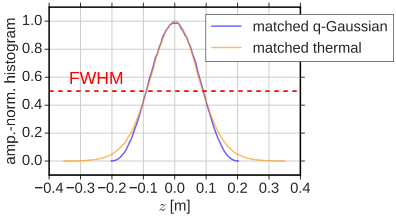


normalise $\Longrightarrow$ amplitude



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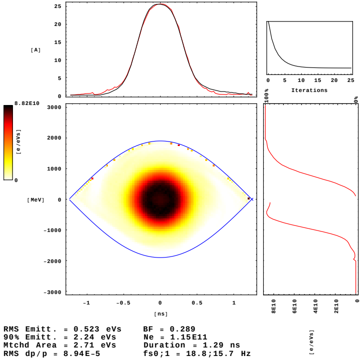
Tomography at 6.5 TeV

Measurement by Juan Müller et al., tomography by Steven Hancock:

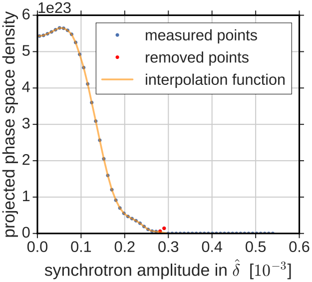
- context: first tomography experiences in LHC!
- turn-by-turn bunch profile data over 1000 turns
- $E_0 = 6.5\,\mathrm{TeV}$: flat-top immediately after the ramp
- $V_{\mathrm{RF}} = 10\,\mathrm{MV}$, $h = 35640$

⟶ by-product: nice measurement of the longitudinal distribution directly after the longitudinal blow-up



Details Macro-particle Generation

1. extract $f(\mathscr{H})$ from tomo: $\mathscr{H} = -\frac{1}{2}\eta\beta c\delta^2$



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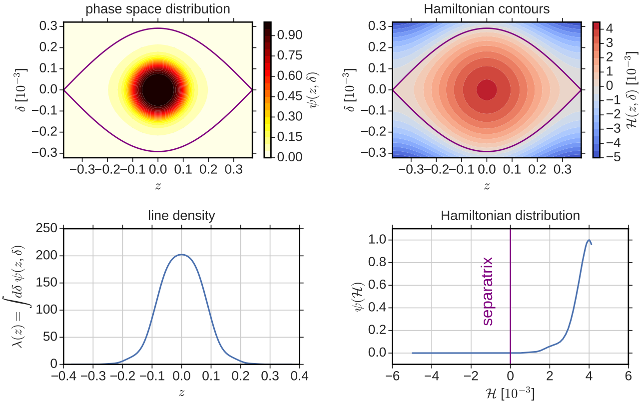

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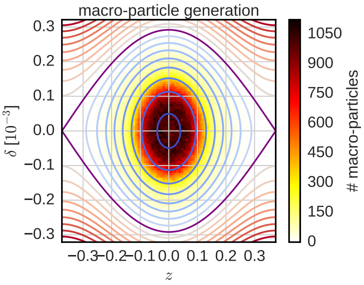
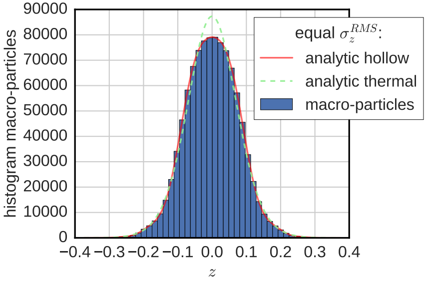
3. Markov chain Monte-Carlo sampling for macro-particles





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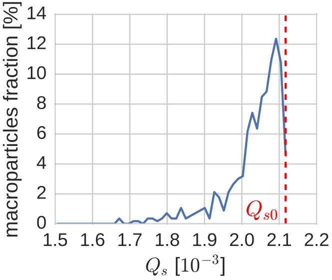
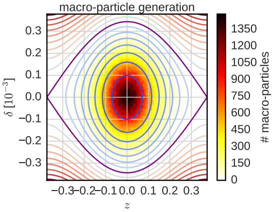
3. Markov chain Monte-Carlo sampling for macro-particles
4. indeed, “hole” in synchrotron frequency distribution reproduced



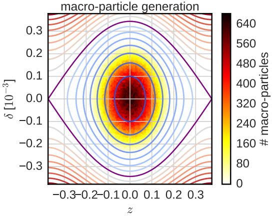
Longitudinal Macro-particle Distributions

thermal distribution



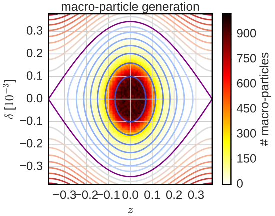
$$f(\mathscr{H}) \propto \exp\left(-\frac{\mathscr{H}}{\mathscr{H}_0}\right)$$

q-Gaussian distribution



$$f(\mathscr{H}) \propto \left(1 - \frac{\mathscr{H}}{\mathscr{H}_0}\right)^2$$

hollow distribution



from tomogram

All phase space distributions FWHM equivalent:

- thermal distribution ⟶ RMS bunch length: $\sigma_z = 8.1\,\text{cm}$
- q-Gauss distribution ⟶ RMS bunch length: $\sigma_z = 6.9\,\text{cm}$
- hollow distribution ⟶ RMS bunch length: $\sigma_z = 6.9\,\text{cm}$

2. chromaticity scans

keep in mind: 26 days per chromaticity scan (each plot!) on the GPU
(i.e. ≈ 8 months on CPU e.g. on lxplus)

PyHEADTAIL parameters

parameter	value
intensity	$N = 2.3 \times 10^{11}$
chromaticity	$-10 \leq Q'_{x,y} \leq 40$
damping rate	50 turns
RF voltage	$V_{RF} = 16\,\text{MV}$
flat-top energy	7 TeV
momentum compaction	$\alpha_c = 53.86^{-2}$
transverse tunes	$(Q_x, Q_y) = (62.31, 60.32)$
synchrotron tune	$Q_s \approx 2.12 \times 10^{-3}$
IP beta function	$\beta^* = 15\,\text{cm}$

⟶ stationary ('matched') longitudinal distributions

⟶ single bunch, non-linear synchrotron motion

⟶ ideal transverse damper model

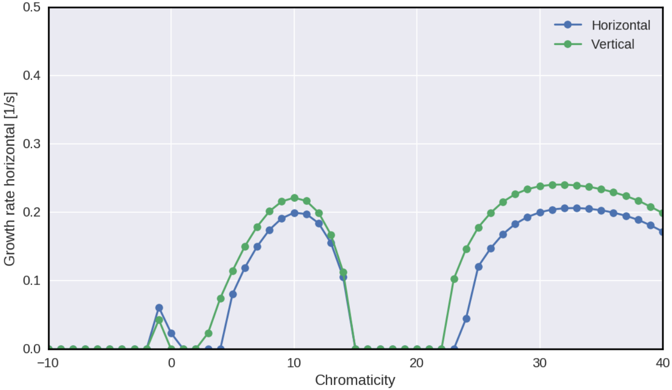
⟶ no octupole currents

⟶ current impedance model with crab cavities from gitlab (Mar'17) ↗

⟶ 600 kturns tracking

Details of Chromaticity Scans

σ_z^{Gauss}	**longitudinal distribution**
9 cm	thermal/Gaussian



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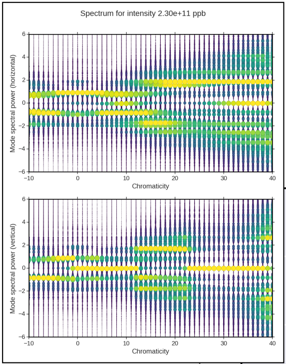

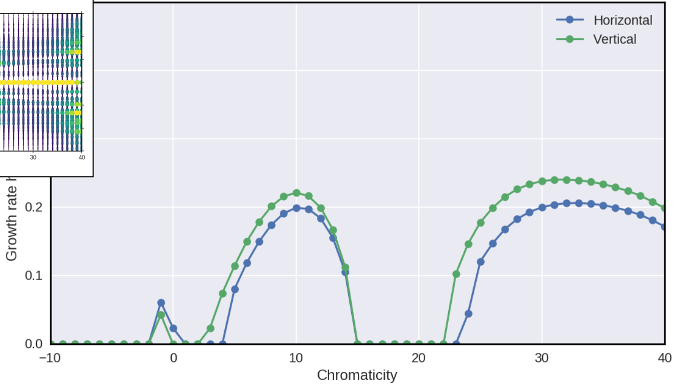

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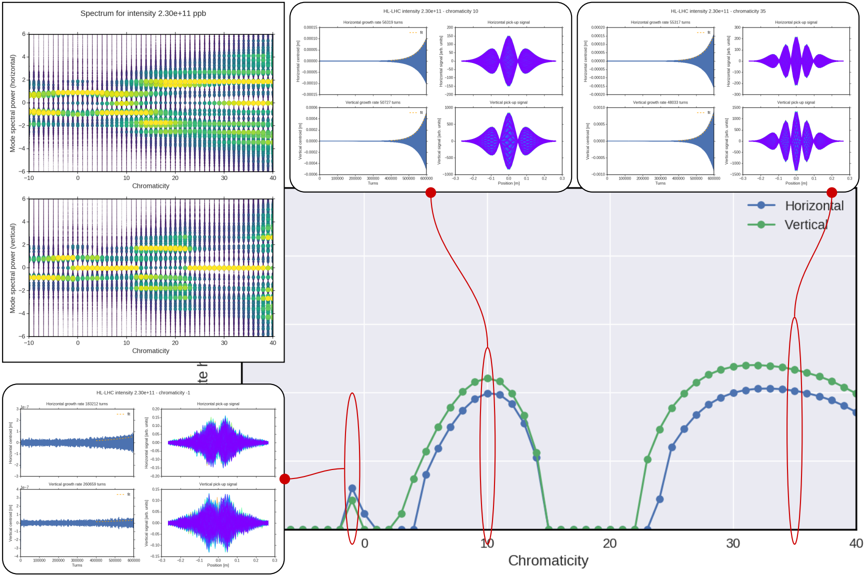
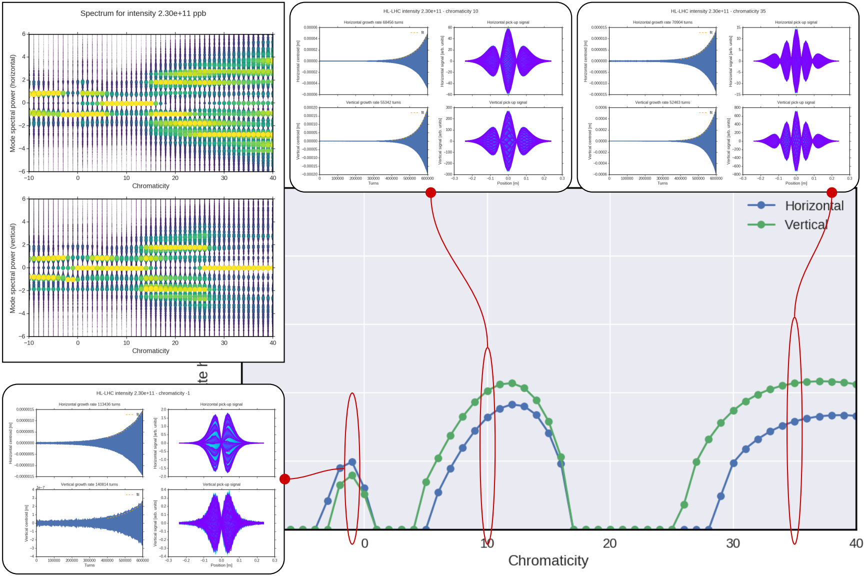


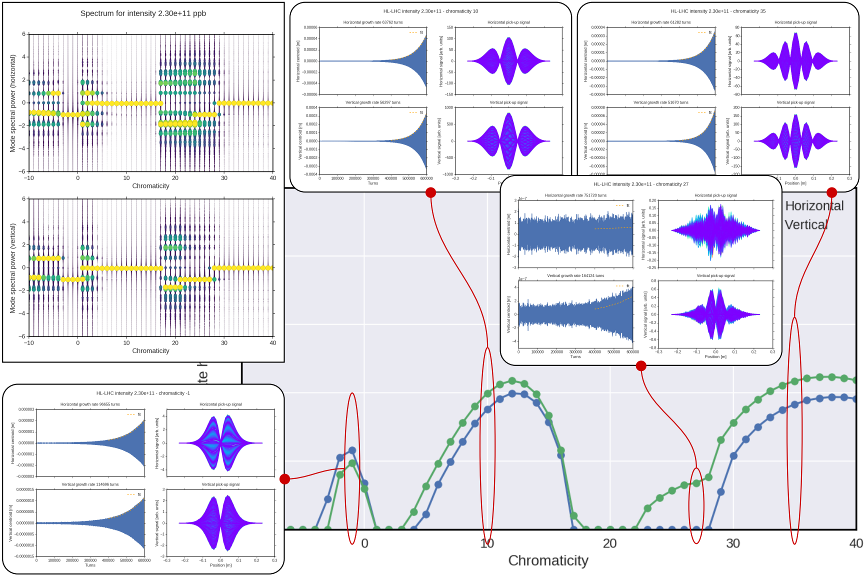
Details of Chromaticity Scans

σ_z^{Gauss}	**longitudinal distribution**
8.1 cm	thermal/Gaussian



Details of Chromaticity Scans

σ_z^{Gauss}	**longitudinal distribution**
9 cm	q-Gaussian



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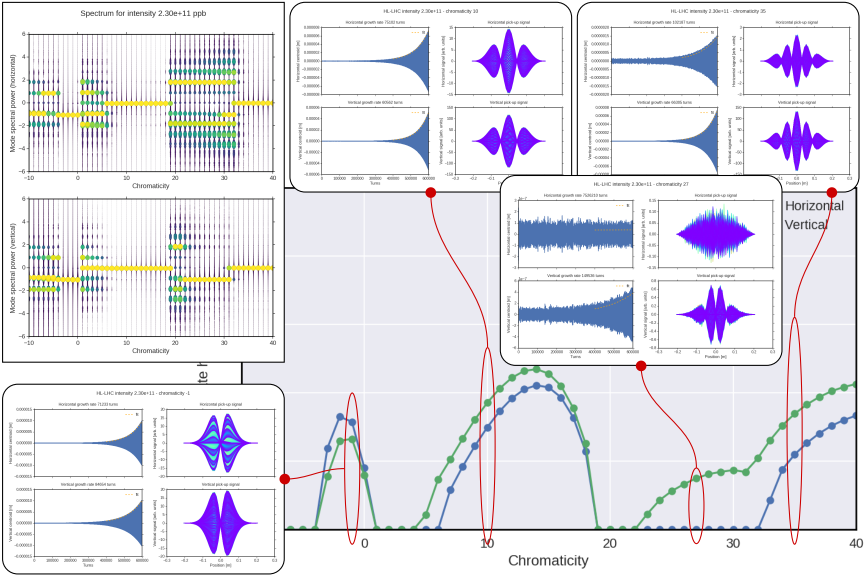
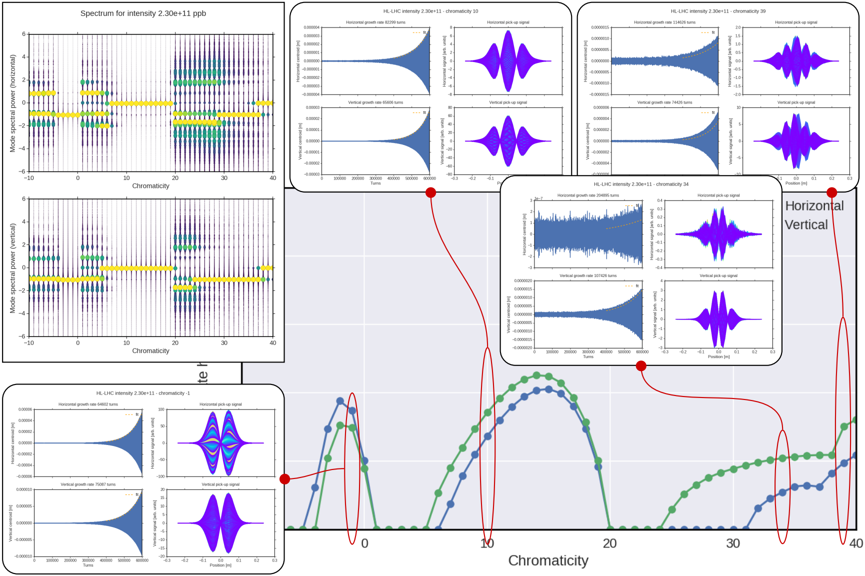
σ_z^{Gauss}	**longitudinal distribution**
8.1 cm	q-Gaussian



σ_z^{Gauss}	**longitudinal distribution**
8.1 cm	hollow



Observations

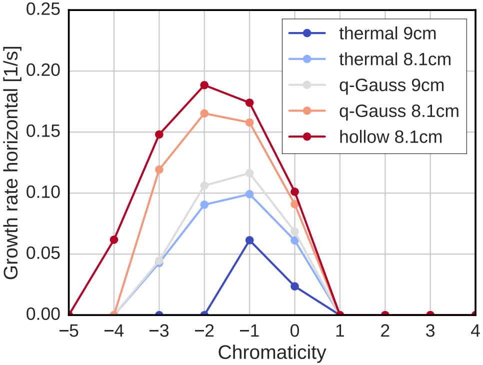
- $Q'_{x,y} \approx 0$ (radial mode 1):
  peak rise time grows from thermal via q-Gauss towards hollow
  (i.e. directly after ramp worse than later at flat-top,
  $\longrightarrow$ re-thermalisation of distribution)
- $Q'_{x,y} \approx 10$ (radial mode 2):
  no big change for peak rise time, but considerable shift in chroma
- $Q'_{x,y} > 20$ (radial mode 3):
  absent for thermal; q-Gauss smaller chroma range than hollow
- $Q'_{x,y} \gtrsim 30$ (radial mode 4):
  lower chroma region edge shifts higher from thermal via q-Gauss towards hollow

Conclusion: strong dependency of rise time on longitudinal distribution for

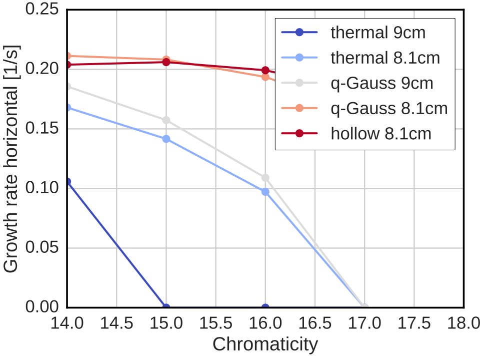
$\Longrightarrow$ destabilising effect of damper (head-tail instability around $Q'_{x,y} = 0$)

$\Longrightarrow$ fixed chromaticity in the operational area around $Q'_{x,y} \approx 10 \pm 5$

Destabilising Effect of Damper



Fixed Chromaticity in LHC Operational Area



Further Results

Further chromaticity scans for thermal distribution:

- **only dipolar wakes vs. incl. quadrupolar wakes**: additional unstable mode in horizontal plane at high chroma $Q'_{x,y} \approx 30$
- **impedance model update**: $\mathcal{O}(10\%)$ higher growth rates compared to before detailed crab cavity modelling[2]
- **pre-squeeze vs. full squeeze**: $\mathcal{O}(10\%)$ higher growth rates comparing $\beta^* = 48\,\text{cm}$ to $\beta^* = 15\,\text{cm}$, larger instability range

$\Longrightarrow$ for details cf. 110th HSC section meeting presentation (see last slide)

[2] cf. e.g. studies for RFQ or LIU-SPS Wide-band Feedback Review ↗

3. stability from space charge

Physical Parameters for PyHEADTAIL

parameter	value
normalised transverse RMS emittances	$\epsilon_{x,y} = 2.5\,\mathrm{mm\,mrad}$
longitudinal 4σ emittance	$\epsilon_z = 0.69\,\mathrm{eV\,s}$
RMS bunch length	$\sigma_z = 10.4\,\mathrm{cm}$
injection energy	$E_0 = 450\,\mathrm{GeV}$
transverse tunes	$(62.28, 60.31)$
synchrotron tune	$Q_s \approx 5.862 \times 10^{-3}$
momentum compaction	$\alpha_c = 53.83^{-2}$
RF voltage (200 MHz cavities)	$V_{\mathrm{RF}} = 8\,\mathrm{MV}$
direct space charge tune spread	$\mathcal{O}(10^{-3})$

⟶ single bunch with matched thermal longitudinal distribution

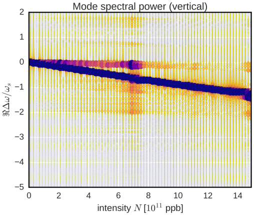
⟶ non-linear synchrotron motion in single-harmonic RF bucket

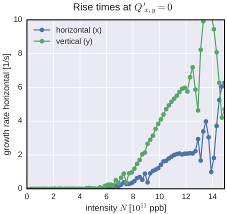
⟶ no transverse damper (ADT), no Landau octupole currents

⟶ linear betatron tracking, no machine non-linearities

⟶ current impedance model with crab cavities from gitlab (Mar'17) ↗

without space charge

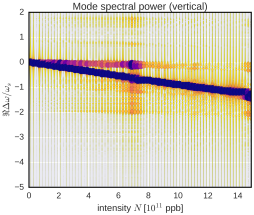


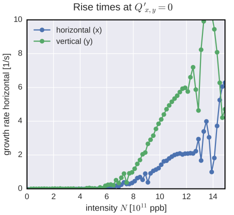


$\Longrightarrow$ modes 0 and -1 couple around TMCI threshold intensity $N_{\mathrm{TMCI,th}} \approx 6 \times 10^{11}$ ppb

Transverse 0 & −1 Mode Coupling Instability

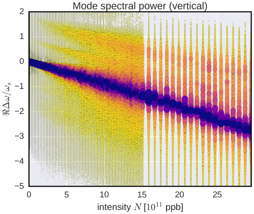
without space charge

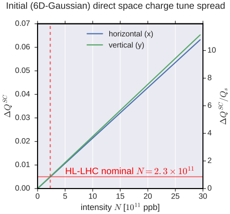




$\Longrightarrow$ modes 0 and -1 couple around TMCI threshold intensity $N_{\mathrm{TMCI,th}} \approx 6 \times 10^{11}$ ppb

with self-consistent space charge





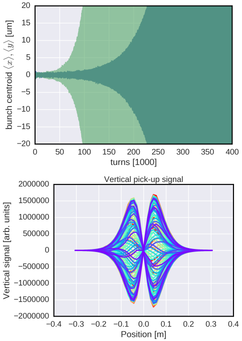
$\Longrightarrow$ stable over simulation run of 50 000 turns, no mode coupling

$\longrightarrow$ fixing emittances $\epsilon_{x,y}$ means increasing SC tune spread $\Delta Q^{\mathrm{SC}}_{x,y}$

Head-Tail Instability + Space Charge

Fix $Q'_{x,y} = 5$ and $N = 4 \times 10^{11}\,\mathrm{ppb} < N_{\mathrm{TMCI,th}}$:

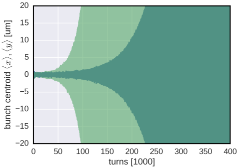
only wake field

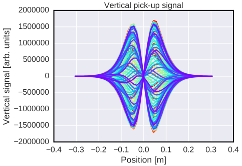


vert. head-tail instab.,
18.6 kturns rise time

Fix $Q'_{x,y} = 5$ and $N = 4 \times 10^{11}\,\text{ppb} < N_{\text{TMCI,th}}$:

only wake field

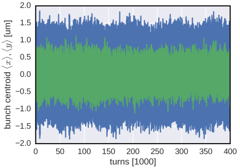


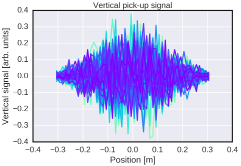


vert. head-tail instab., 18.6 kturns rise time

+

only space charge



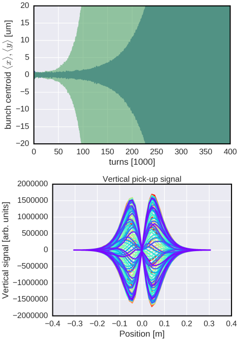


stable

Head-Tail Instability + Space Charge

Fix $Q'_{x,y} = 5$ and $N = 4 \times 10^{11}\,\text{ppb} < N_{\text{TMCI,th}}$:

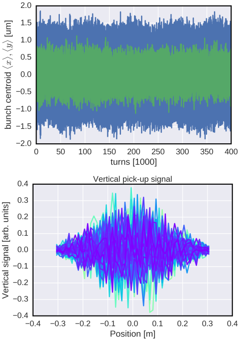
only wake field



vert. head-tail instab., 18.6 kturns rise time

+

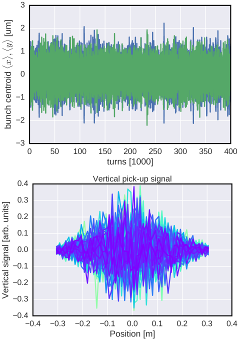
only space charge



stable

=

wake field + SC



stable!

$\Longrightarrow$ direct space charge stabilises mode $m = -1$ at HL-LHC injection

- fixing the intensity N and increasing the transverse emittances $\epsilon_{x,y}$
  - $\longrightarrow$ weaker space charge contribution $\Delta Q^{SC}_{x,y} \propto \frac{N}{\gamma^2 \epsilon_{x,y}}$
  - $\Longrightarrow$ between 20 and 40 mm mrad: head-tail instability $m = -1$ recovered
- first potential experimental evidence in 2010
  - no ADT, no Landau octupoles: stable beam at LHC injection
  - during ramp: $m = -1$ head-tail instability
  - $\Longrightarrow$ possible explanation: $\Delta Q^{SC}_{x,y} \propto 1/\gamma^2$ shrinks during ramp until Landau damping from space charge is lost

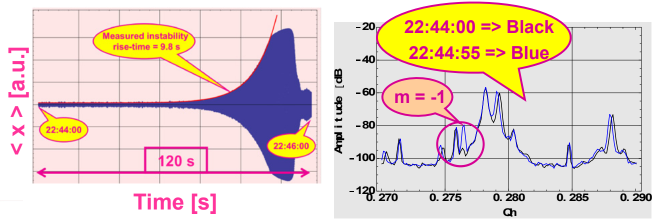


Figure: flat-top measurements of said head-tail instability at $E_0 = 3.5\,\mathrm{TeV}$ [1]

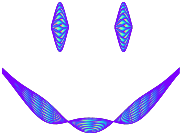
Take-home Messages

technical:

1. PyHEADTAIL macro-particle simulations on the GPU: massive and reliable head-tail instability studies possible for realistic scenarios (arbitrary longitudinal distributions, non-linear synchrotron motion, non-linear space charge)
2. now possible: generation of matched macro-particle distributions from tomography measurements (also first tomo for LHC :-)

physics:

3. long. distribution affects damper-related head-tail instability ($Q'_{x,y} \approx 0$)
4. long. distribution affects instability rise time for operational $Q'_{x,y} \approx 15$
5. space charge suppresses TMCI at HL-LHC injection
6. space charge possibly explains absence of (non-rigid) head-tail modes below certain energy



Further Resources

More detailed presentations:

- HL-LHC coherent stability studies with PyHEADTAIL,
  110th HSC section meeting, Jun'17 ↗
- can space charge stabilise head-tail instabilities at injection in the LHC? 89th LBOC meeting, Oct'17 ↗
- transverse stability for non-Gaussian bunches in HL-LHC,
  125th HSC section meeting, Oct'17 ↗

Further information:

- single-bunch stability with direct space charge,
  poster at impedance & instabilities workshop, Benevento (IT), Sep'17 ↗
- effect of space charge on the CERN LHC and SPS transverse instabilities: simulation vs. measurements,
  presentation by Elias Métral at space charge workshop, Darmstadt (DE), Oct'17 ↗

Thank you for your attention!

Acknowledgements:
Sergey Antipov, Nicolo Biancacci, Xavier Buffat, Steven Hancock, Elias Métral, Juan Müller

References I

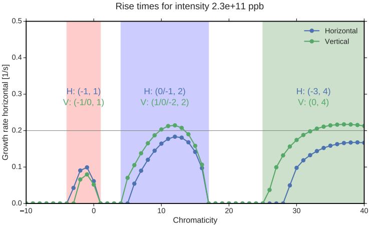
[1] Elias Metral et al. *Measurement and interpretation of transverse beam instabilities in the CERN large hadron collider (LHC) and extrapolations to HL-LHC*. Tech. rep. CERN-ACC-2016-0098. Geneva: CERN, July 2016. URL: http://cds.cern.ch/record/2199121.

Ingredients to generate macro-particles:

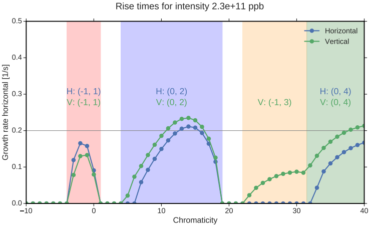
⟶ PyHEADTAIL RF bucket matching:
`https://github.com/PyCOMPLETE/PyHEADTAIL-playground/blob/master/RFBucket_Matching.ipynb`

⟶ create stationary macro-particle distribution $f(\mathscr{H})$ from tomogram:
`https://gitlab.cern.ch/oeftiger/tomo_to_sim/`

Thermal vs. *q*-Gaussian Distribution



equal FWHM $\Longrightarrow$ q-Gaussian



Gaussian σ_z^{RMS}

8.1 cm

$\Downarrow$

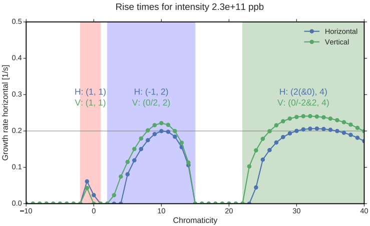
9 cm

q-Gaussian σ_z^{RMS}

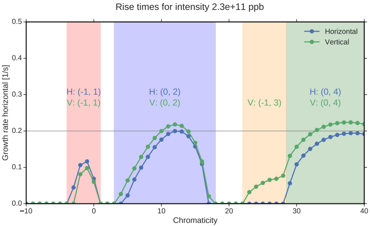
6.9 cm

$\Downarrow$

7.6 cm



equal FWHM $\Longrightarrow$ q-Gaussian



observe the emergence of radial mode 3 with the q-Gaussian distribution!

Without Space Charge: Head-tail Instabilities @Injection