Transverse Beam Stability with Realistic Longitudinal Profiles and Contribution from Space Charge

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CERN – HL-LHC WP2 Meeting
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Motivation

Study based on Elena Shaposhnikova’s Jan’17 WP2 presentation: after longitudinal blow-up (during ramp), we have

- $q$-Gaussian longitudinal bunch profile
- hole in synchrotron frequency distribution

Central goals

Investigate impact on head-tail instabilities at HL-LHC by

- longitudinal distribution
- direct space charge

Content of this talk:

1. stationary macro-particle distributions for longitudinal
   - $q$-Gaussian distribution (without hole)
   - hollow distribution from measured LHC tomogram
2. chromaticity scans for head-tail instabilities at flat-top
   → compare thermal, $q$-Gaussian and hollow distribution
3. contribution to stability from space charge (TMCI, head-tail)
Non-Gaussian Bunch Profile

Flat-bottom and initial flat-top: non-Gaussian longitudinal bunch profile

Elena Shaposhnikova: $q$-Gaussian bunch profile

Bunch profiles in a single RF system (measured and fitted)

Binomial line density distribution $\lambda(t) = \lambda_0(1 - 4t^2/\tau^2)^{2.5}$ fits well present LHC bunches (in a single RF) on flat bottom and at beginning of flat top (after controlled emittance with band-limited noise during ramp)

- Real bunch tails are more populated (also visible from the PD Schottky)
- Profiles become Gaussian after a few hours due to IBS and SR

$\implies$ consider equal **Full Width at Half Maximum**

("[longitudinal] instability thresholds scale with the FWHM values")
Not only non-Gaussian bunch profile – also **depleted bunch centre**:

Particle distribution after controlled emittance blow-up in LHC

→ After controlled emittance blow-up the distribution function in synchrotron frequency $F(\Omega)$ (or in action $F(J)$) has a **hole**

Particle distribution function $F(\Omega) = \frac{dN}{d\Omega} = \frac{F(J)}{\Omega'(J)}$
Schottky Spectrum: “Hole”

(a) “usual” Schottky spectrum
(image from Juan Müller’s thesis)

(b) Schottky spectrum with hole
(after longitudinal blow-up)

⇒ slightly hollow phase space distribution can explain “missing”
Schottky signal around linear synchrotron frequency
1. stationary longitudinal macro-particle distributions
Generate macro-particles matched to the RF bucket with

(i) thermal distribution: $f(H) \propto \exp(-H/H_0)$

(ii) $q$-Gaussian distribution (for $q = 3/5$): $f(H) \propto \left(1 - \frac{H}{H_0}\right)^2$

(iii) hollow distribution reconstructed from tomography with equal FWHM via\(^1\)

$$\sigma_{z,\text{RMS}}|_{\text{q-Gaussian}} = 0.846 \cdot \sigma_{z,\text{RMS}}|_{\text{thermal}}$$

\(^1\)cf. e.g. R. Tomás’ & L. Medina’s WP2 talk Mar’17 / or their IPAC’17 paper / or S. Papadopoulou’s IPAC’17 paper /
FWHM Equivalence

Generate macro-particles matched to the RF bucket with

(i) thermal distribution: \( f(H) \propto \exp(-H/H_0) \)

(ii) \( q \)-Gaussian distribution (for \( q = 3/5 \)): \( f(H) \propto \left(1 - \frac{H}{H_0}\right)^2 \)

(iii) hollow distribution reconstructed from tomography with equal FWHM via\(^1\)

\[
\sigma_z^{\text{RMS}}\left|_{\text{\(q\)-Gaussian}}\right. = 0.846 \cdot \sigma_z^{\text{RMS}}\left|_{\text{thermal}}\right.
\]

\(^1\) cf. e.g. R. Tomás’ & L. Medina’s WP2 talk Mar’17 / or their IPAC’17 paper / or S. Papadopoulou’s IPAC’17 paper
Tomography at 6.5 TeV

Measurement by Juan Müller et al., tomography by Steven Hancock:

- context: first tomography experiences in LHC!
- turn-by-turn bunch profile data over 1000 turns
- $E_0 = 6.5$ TeV: flat-top immediately after the ramp
- $V_{RF} = 10$ MV, $h = 35640$

→ by-product: nice measurement of the longitudinal distribution directly after the longitudinal blow-up
extract \( f(\mathcal{H}) \) from tomo: \( \mathcal{H} = -\frac{1}{2} \eta \beta c \delta^2 \)
Details Macro-particle Generation

1. extract \( f(\mathcal{H}) \) from tomo: \( \mathcal{H} = -\frac{1}{2} \eta \beta c \delta^2 \)
2. analytically match to \( \sigma_{z}^{\text{RMS}} \big|_{q\text{-Gaussian}} \stackrel{!}{=} 0.846 \cdot \sigma_{z}^{\text{RMS}} \big|_{\text{thermal}} = 8.1 \text{ cm} \)
extract \( f(\mathcal{H}) \) from tomo: \( \mathcal{H} = -\frac{1}{2} \eta \beta c \delta^2 \)

analytically match to \( \sigma_{z|q-Gaussian}^{RMS} = 0.846 \cdot \sigma_{z|thermal}^{RMS} = 8.1 \text{ cm} \)

Markov chain Monte-Carlo sampling for macro-particles
Details Macro-particle Generation

1. extract \( f(\mathcal{H}) \) from tomo: \( \mathcal{H} = -\frac{1}{2} \eta \beta c \delta^2 \)

2. analytically match to \( \sigma_{z}^{\text{RMS}} \big|_{q-\text{Gaussian}} = 0.846 \cdot \sigma_{z}^{\text{RMS}} \big|_{\text{thermal}} = 8.1 \text{ cm} \)

3. Markov chain Monte-Carlo sampling for macro-particles

4. indeed, “hole” in synchrotron frequency distribution reproduced
Longitudinal Macro-particle Distributions

thermal distribution

\[ f(H) \propto \exp\left(-\frac{H}{H_0}\right) \]

\begin{itemize}
  \item thermal distribution \quad \rightarrow \quad \text{RMS bunch length: } \sigma_Z = 8.1 \text{ cm}
  \item q-Gauss distribution \quad \rightarrow \quad \text{RMS bunch length: } \sigma_Z = 6.9 \text{ cm}
  \item hollow distribution \quad \rightarrow \quad \text{RMS bunch length: } \sigma_Z = 6.9 \text{ cm}
\end{itemize}
2. chromaticity scans

keep in mind: 26 days per chromaticity scan (each plot!) on the GPU (i.e. \(\approx\) 8 months on CPU e.g. on lxplus)
### PyHEADTAIL parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>intensity</td>
<td>( N = 2.3 \times 10^{11} )</td>
</tr>
<tr>
<td>chromaticity</td>
<td>(-10 \leq Q'_{x,y} \leq 40)</td>
</tr>
<tr>
<td>damping rate</td>
<td>50 turns</td>
</tr>
<tr>
<td>RF voltage</td>
<td>( V_{RF} = 16 \text{ MV} )</td>
</tr>
<tr>
<td>flat-top energy</td>
<td>7 \text{ TeV}</td>
</tr>
<tr>
<td>momentum compaction</td>
<td>( \alpha_c = 53.86^{-2} )</td>
</tr>
<tr>
<td>transverse tunes</td>
<td>((Q_x, Q_y) = (62.31, 60.32))</td>
</tr>
<tr>
<td>synchrotron tune</td>
<td>( Q_s \approx 2.12 \times 10^{-3} )</td>
</tr>
<tr>
<td>IP beta function</td>
<td>( \beta^* = 15 \text{ cm} )</td>
</tr>
</tbody>
</table>

→ stationary (‘matched’) longitudinal distributions
→ single bunch, non-linear synchrotron motion
→ ideal transverse damper model
→ no octupole currents
→ current impedance model with crab cavities from gitlab (Mar’17)
→ 600 kturns tracking
Details of Chromaticity Scans

\[ \sigma_{z}^{\text{Gauss}} \quad \text{longitudinal distribution} \]

9 cm \quad \text{thermal/Gaussian}

![Graph showing growth rate vs chromaticity for horizontal and vertical directions.](image_url)
Details of Chromaticity Scans

\[ \sigma_z^{\text{Gauss}} \quad \text{longitudinal distribution} \]

9 cm
thermal/Gaussian

Spectrum for intensity 2.30e+11 ppb

Mode spectral power (horizontal)

Mode spectral power (vertical)

Growth rate

Chromaticity

Horizontal
Vertical
Details of Chromaticity Scans

\[ \sigma_z^{\text{Gauss}} \quad \text{longitudinal distribution} \]

9 cm \quad \text{thermal/Gaussian}

Spectrum for intensity $2.30 \times 10^{11}$ ppb

Mode spectral power (horizontal)

Mode spectral power (vertical)

HL-LPC intensity $2.30 \times 10^{11}$ - chromaticity 10

HL-LPC intensity $2.30 \times 10^{11}$ - chromaticity 20

Horizontal growth rate 50937 turns

Vertical growth rate 50937 turns

Horizontal pick-up signal

Vertical pick-up signal

Horizontal

Vertical

Chromaticity

0 10 20 30 40

0 10 20 30 40
Details of Chromaticity Scans

$\sigma^\text{Gauss}_z \quad \text{longitudinal distribution}$

8.1 cm \hspace{1cm} \text{thermal/Gaussian}

$\sigma^\text{Gauss}_z \quad \text{longitudinal distribution}$

Spectrum for intensity 2.30e+11 ppb

Chromaticity

Mode spectral power (horizontal)

Mode spectral power (vertical)

Horizontal/Vertical signal (Hz)

Horizontal growth rate 656436 Hz

Vertical growth rate 656310 Hz

Horizontal pick-up signal

Vertical pick-up signal

Chromaticity

Horizontal

Vertical

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Details of Chromaticity Scans

\[ \sigma_z^{\text{Gauss}} \]  
6 cm

longitudinal distribution

\[ q\text{-Gaussian} \]
Details of Chromaticity Scans

$\sigma^\text{Gauss}_z \quad \text{longitudinal distribution} \quad q\text{-Gaussian}$

8.1 cm

Spectrum for intensity 2.30e+11 ppb

Mode spectral power (horizontal)

Mode spectral power (vertical)

Chromaticity

Horizontal Vertical

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Details of Chromaticity Scans

\[ \sigma_{Z}^{Gauss} = 8.1 \text{ cm} \]

longitudinal distribution hollow
Observations

- $Q'_{x,y} \approx 0$ (radial mode 1):
  peak rise time grows from thermal via $q$-Gauss towards hollow
  (i.e. directly after ramp worse than later at flat-top, \(\rightarrow\) re-thermalisation of distribution)

- $Q'_{x,y} \approx 10$ (radial mode 2):
  no big change for peak rise time, but considerable shift in chroma

- $Q'_{x,y} > 20$ (radial mode 3):
  absent for thermal; $q$-Gauss smaller chroma range than hollow

- $Q'_{x,y} \gtrsim 30$ (radial mode 4):
  lower chroma region edge shifts higher from thermal via $q$-Gauss towards hollow

Conclusion: strong dependency of rise time on longitudinal distribution for
\[ \implies \] destabilising effect of damper (head-tail instability around $Q'_{x,y} = 0$)
\[ \implies \] fixed chromaticity in the operational area around $Q'_{x,y} \approx 10 \pm 5$
Destabilising Effect of Damper

Growth rate horizontal [1/s] vs Chromaticity

- thermal 9cm
- thermal 8.1cm
- q-Gauss 9cm
- q-Gauss 8.1cm
- hollow 8.1cm
Further chromaticity scans for thermal distribution:

- **only dipolar wakes vs. incl. quadrupolar wakes:** additional unstable mode in horizontal plane at high chroma $Q'_{x,y} \approx 30$

- **impedance model update:** $\mathcal{O}(10\%)$ higher growth rates compared to before detailed crab cavity modelling\(^2\)

- **pre-squeeze vs. full squeeze:** $\mathcal{O}(10\%)$ higher growth rates comparing $\beta^* = 48\text{cm}$ to $\beta^* = 15\text{cm}$, larger instability range

$\implies$ for details cf. 110th HSC section meeting presentation (see last slide)

\(^2\text{cf. e.g. studies for RFQ or LIU-SPS Wide-band Feedback Review} \uparrow$
3. stability from space charge
<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>normalised transverse RMS emittances</td>
<td>$\epsilon_{x,y} = 2.5 \text{ mm mrad}$</td>
</tr>
<tr>
<td>longitudinal $4\sigma$ emittance</td>
<td>$\epsilon_z = 0.69 \text{ eVs}$</td>
</tr>
<tr>
<td>RMS bunch length</td>
<td>$\sigma_z = 10.4 \text{ cm}$</td>
</tr>
<tr>
<td>injection energy</td>
<td>$E_0 = 450 \text{ GeV}$</td>
</tr>
<tr>
<td>transverse tunes</td>
<td>$(62.28, 60.31)$</td>
</tr>
<tr>
<td>synchrotron tune</td>
<td>$Q_s \approx 5.862 \times 10^{-3}$</td>
</tr>
<tr>
<td>momentum compaction</td>
<td>$\alpha_c = 53.83^{-2}$</td>
</tr>
<tr>
<td>RF voltage (200 MHz cavities)</td>
<td>$V_{RF} = 8 \text{ MV}$</td>
</tr>
<tr>
<td>direct space charge tune spread</td>
<td>$\Theta(10^{-3})$</td>
</tr>
</tbody>
</table>

$\rightarrow$ single bunch with matched thermal longitudinal distribution
$\rightarrow$ non-linear synchrotron motion in single-harmonic RF bucket
$\rightarrow$ no transverse damper (ADT), no Landau octupole currents
$\rightarrow$ linear betatron tracking, no machine non-linearities
$\rightarrow$ current impedance model with crab cavities from gitlab (Mar'17)
Transverse 0 & −1 Mode Coupling Instability without space charge

→ modes 0 and -1 couple around TMCI threshold intensity $N_{\text{TMCI,th}} \approx 6 \times 10^{11}$ ppb

\[ \Delta Q_{\text{SC}} \text{HL}/\text{DLHC/nominal/} N = 2.3 \times 10^{11} \]
Transverse 0 & −1 Mode Coupling Instability

without space charge

⇒ modes 0 and -1 couple around TMCI threshold intensity $N_{\text{TMCI,th}} \approx 6 \times 10^{11}$ ppb

with self-consistent space charge

⇒ stable over simulation run of 50000 turns, no mode coupling

⇒ fixing emittances $\epsilon_{x,y}$ means increasing SC tune spread $\Delta Q_{x,y}^{\text{SC}}$
Fix $Q'_x, y = 5$ and $N = 4 \times 10^{11}$ ppb $< N_{\text{TMCL}, \text{th}}$:

only wake field

vert. head-tail instab., 18.6 kturns rise time
Only wake field

Vert. head-tail instab., 18.6 kturns rise time

Only space charge

stable

Fix $Q'_{x,y} = 5$ and $N = 4 \times 10^{11}$ ppb $< N_{TMCl,th}$:
Head-Tail Instability + Space Charge

Fix $Q'_{x,y} = 5$ and $N = 4 \times 10^{11}$ ppb $< N_{\text{TMCI,th}}$:

only wake field

only space charge

wake field + SC

vert. head-tail instab., 18.6 kturns rise time

stable

stable!

$\implies$ direct space charge stabilises mode $m = -1$ at HL-LHC injection
Space Charge Limit

- fixing the intensity $N$ and increasing the transverse emittances $\epsilon_{x,y}$
  - weaker space charge contribution $\Delta Q_{x,y}^{SC} \propto \frac{N}{\gamma^2 \epsilon_{x,y}}$
  - between 20 and 40 mm mrad: head-tail instability $m = -1$ recovered

- first potential experimental evidence in 2010
  - no ADT, no Landau octupoles: stable beam at LHC injection
  - during ramp: $m = -1$ head-tail instability
  - possible explanation: $\Delta Q_{x,y}^{SC} \propto 1/\gamma^2$ shrinks during ramp until Landau damping from space charge is lost

Figure: flat-top measurements of said head-tail instability at $E_0 = 3.5$ TeV [1]
Take-home Messages

technical:
1. PyHEADTAIL macro-particle simulations on the GPU: massive and reliable head-tail instability studies possible for realistic scenarios (arbitrary longitudinal distributions, non-linear synchrotron motion, non-linear space charge)
2. now possible: generation of matched macro-particle distributions from tomography measurements (also first tomo for LHC :-)

physics:
3. long. distribution affects damper-related head-tail instability ($Q'_{x,y} \approx 0$)
4. long. distribution affects instability rise time for operational $Q'_{x,y} \approx 15$
5. space charge suppresses TMCI at HL-LHC injection
6. space charge possibly explains absence of (non-rigid) head-tail modes below certain energy
Further Resources

More detailed presentations:

- HL-LHC coherent stability studies with PyHEADTAIL, 110th HSC section meeting, Jun’17
- can space charge stabilise head-tail instabilities at injection in the LHC? 89th LBOC meeting, Oct’17
- transverse stability for non-Gaussian bunches in HL-LHC, 125th HSC section meeting, Oct’17

Further information:

- single-bunch stability with direct space charge, poster at impedance & instabilities workshop, Benevento (IT), Sep’17
- effect of space charge on the CERN LHC and SPS transverse instabilities: simulation vs. measurements, presentation by Elias Métral at space charge workshop, Darmstadt (DE), Oct’17
Thank you for your attention!

Acknowledgements:
Sergey Antipov, Nicolo Biancacci, Xavier Buffat, Steven Hancock, Elias Métral, Juan Müller
Generating Macro-particles in PyHEADTAIL

Ingredients to generate macro-particles:

→ PyHEADTAIL RF bucket matching:
  https://github.com/PyCOMPLETE/PyHEADTAIL-playground/blob/master/RFBucket_Matching.ipynb

→ create stationary macro-particle distribution $f(H)$ from tomogram:
  https://gitlab.cern.ch/oeftiger/tomo_to_sim/
Thermal vs. $q$-Gaussian Distribution

\[ \text{Gaussian } \sigma_z^{\text{RMS}} = 8.1 \text{ cm} \]
\[ \Downarrow \]
\[ 9 \text{ cm} \]

\[ q\text{-Gaussian } \sigma_z^{\text{RMS}} = 6.9 \text{ cm} \]
\[ \Downarrow \]
\[ 7.6 \text{ cm} \]

observe the emergence of radial mode 3 with the $q$-Gaussian distribution!