New solutions of hydrodynamics for a hadronic resonance gas: with constant and with temperature dependent masses

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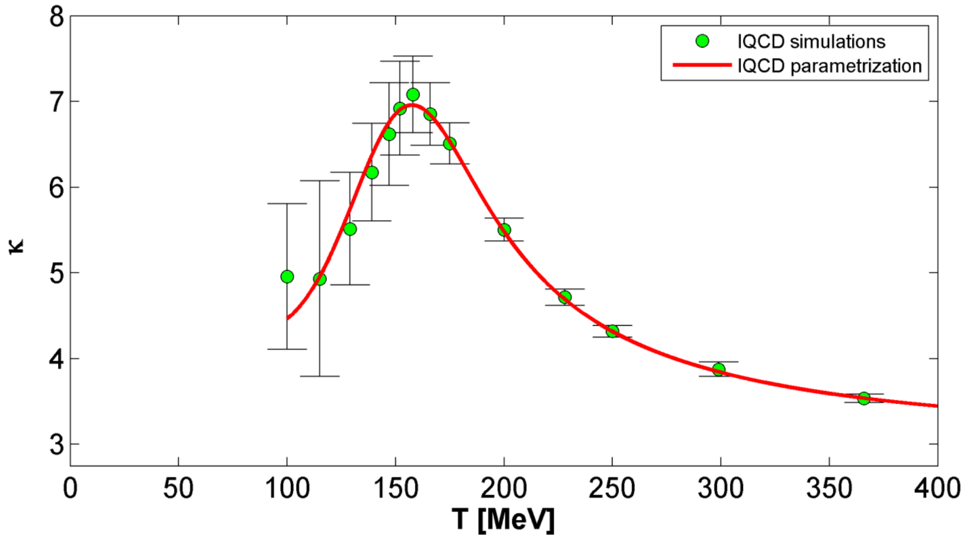
Outlook

- Lattice QCD EoS parametrization
- Quark-hadron transitions
  - Crossover
  - 2nd order PT
  - 1st order PT
- Multi-component hadronic solution
- Temperature dependent mass
- Observables
  - Scaling behaviour
  - Rotational angles

Motivation

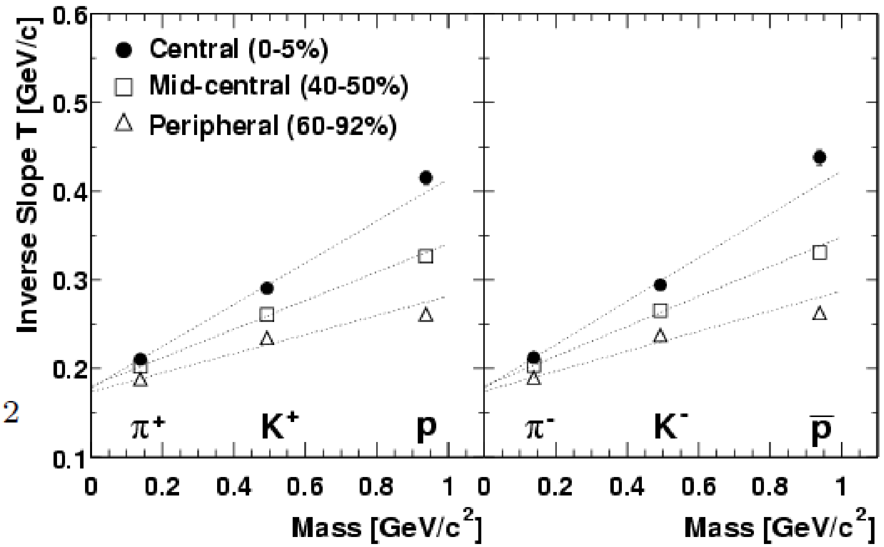


$\kappa(T) = \varepsilon/p$
from lattice QCD
arXiv:1007.2580

Can they be understood in a consistent picture?

Scaling behaviour of single particle spectra
nucl-ex/0307022

$$T = T_f + m\langle u_t\rangle^2 \Longrightarrow T_i = T_f + m_i\langle u_t\rangle^2$$



Non-relativistic hydrodynamics

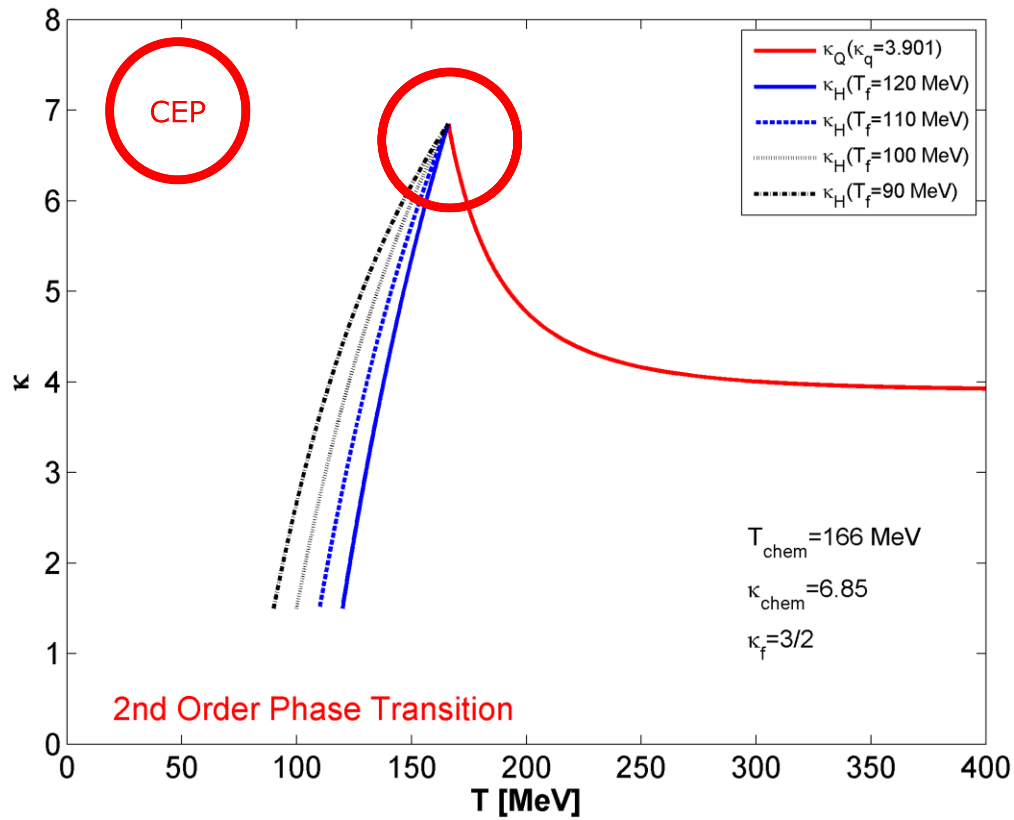
- Nonrelativistic approximation:

$$|\vec{v}|^2 \ll c^2$$

$\mathbf{QM}\ (T_i \geq T \geq T_{chem})$	$\mathbf{HM}\ (T_{chem} > T \geq T_f)$
$\partial_t \sigma + \nabla(\sigma \mathbf{v}) = 0$	$\partial_t n_i + \nabla(n_i \mathbf{v}) = 0, \quad \forall i$
$T\sigma(\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$	$\sum_i m_i n_i (\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p$
$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right](\partial_t + \mathbf{v}\nabla)T + \nabla\mathbf{v} = 0$	$\frac{1}{T}\left[\frac{d(\kappa T)}{dT}\right](\partial_t + \mathbf{v}\nabla)T + \nabla\mathbf{v} = 0$
$p = \sigma T/(1+\kappa)$	$p = \sum_i p_i = T\sum_i n_i$

$$\begin{aligned}
\varepsilon + p &= \sum_i \mu_i n_i + T\sigma, \\
\varepsilon + p &\approx T\sigma, && (T_i \geq T \geq T_{chem}), \\
\varepsilon + p &\approx \sum_i m_i n_i && (T_{chem} > T \geq T_f).
\end{aligned}$$

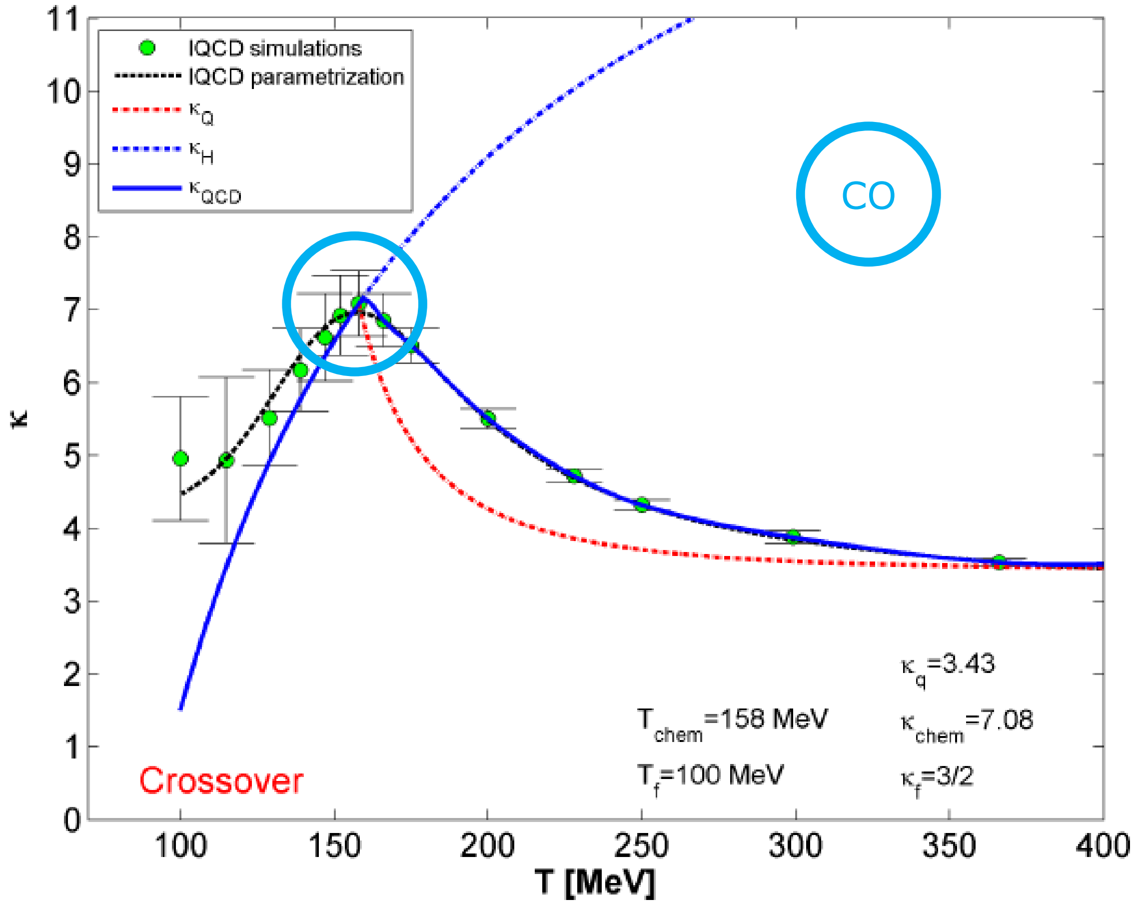
2nd Order Phase Transition (CEP)



- Combined EoS at T_{chem}:

$$\kappa(T) = \Theta(T_{chem} - T)\kappa_H(T) + \Theta(T - T_{chem})\kappa_Q(T)$$

Crossover (CO), lattice QCD EoS



- EoS of the whole $[T_f, T_0]$ interval:

$$\kappa(T) = \Theta(T_{chem} - T)\kappa_H(T) + \Theta(T - T_{chem})\kappa_{QCD}(T)$$

Multi-component (MC) HM solutions

QM $T > T_{chem}$	$\sigma(\vec{r},t) = \sigma_0 \frac{V_0}{V} e^{-s/2}$ $(1+\kappa)\left[\frac{d}{dT}\left(\frac{\kappa T}{1+\kappa}\right)\right]\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X\left(\ddot{X} - R\omega^2\right) = Y\ddot{Y} = Z\left(\ddot{Z} - R\omega^2\right) = \frac{1}{1+\kappa}$
MC HM $T < T_{chem}$	$n_i(\vec{r},t) = n_{i,0} \frac{V_0}{V} e^{-s/2}$ $\left[\frac{d}{dT}(\kappa T)\right]\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X\left(\ddot{X} - R\omega^2\right) = Y\ddot{Y} = Z\left(\ddot{Z} - R\omega^2\right) = \frac{T}{\langle m\rangle}$
SC HM $T < T_{chem}$	$n(\vec{r},t) = n_0 \frac{V_0}{V} e^{-s/2}$ $\left[\frac{d}{dT}(\kappa T)\right]\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X\left(\ddot{X} - R\omega^2\right) = Y\ddot{Y} = Z\left(\ddot{Z} - R\omega^2\right) = \frac{T}{m}$

$$m \Leftrightarrow \langle m\rangle = \frac{\sum_i m_i n_i}{\sum_i n_i}$$

Multi-component hadronic matter (HM):
Expands and rotates together

Dynamics at the QCD Critical Point

- Boundary conditions:

$$T_Q(t_{chem}) = T_H(t_{chem}) = T_{chem}$$
$$\vec{v}_Q(\vec{r}, t_{chem}) = \vec{v}_H(\vec{r}, t_{chem})$$
$$\kappa_Q(T_{chem}) = \kappa_H(T_{chem})$$

- From the equations of motion:

$$\frac{1}{1+\kappa_{chem}} \approx 0.13 < \frac{T_{chem}}{\langle m \rangle} \approx 0.59$$

- The 2nd derivative of the scales jump at t_{chem}
- Second explosion: starts just after the conversion to the HM

Second explosion for <m> ~ 280 MeV

T. Csörgő and G. Kasza: arXiv:1610.02197

Dynamics at a QCD Crossover

- Temperature equation at a crossover

$$(1+\kappa)\left[\frac{d}{dT}\left(\frac{\kappa T}{1+\kappa}\right)\right]\frac{\dot{T}}{T}+\frac{\dot{V}}{V}=0$$

- Equation of state

$$\varepsilon=\sum_i m_i n_i+\kappa p$$

- Infinitesimal relations

$$dp=\sigma dT$$
$$d\varepsilon=\sum_i m_i dn_i+\kappa\sigma dT+pd\kappa$$

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What if m is T dependent?

Dynamics at a QCD Crossover

- Temperature equation at a crossover

$$(1+\kappa)\left[\frac{d}{dT}\left(\frac{\kappa T}{1+\kappa}\right)+\frac{f_H}{1+\kappa}\langle\frac{dm}{dT}\rangle\right]\frac{\dot{T}}{T}+\frac{\dot{V}}{V}=0$$

- Equation of state

$$\varepsilon=\sum_i m_i(T)n_i+\kappa p$$

- Infinitesimal relations

$$dp=\sigma dT+\sum n_i dm_i$$

$$d\varepsilon=\sum_i m_i dn_i+\kappa\left(\sigma+\sum_i n_i\frac{dm_i}{dT}\right)dT+pd\kappa$$

What if m is T dependent?

Dynamics at a QCD Crossover

- Equations of motion:

$$X\left(\ddot{X}-R\omega^2\right)=Y\ddot{Y}=Z\left(\ddot{Z}-R\omega^2\right)=\frac{1}{1+\kappa+f_H\frac{\langle m\rangle}{T}}.$$

- Low temperature limit with $\kappa(T<T_f)=\kappa_f$ condition:

$$\lim_{T\to 0}\frac{1}{1+\kappa+f_H\frac{\langle m\rangle}{T}}=\frac{T}{\langle m\rangle}$$

- High temperature limit:

$$\lim_{T\to\infty}\frac{1}{1+\kappa+f_H\frac{\langle m\rangle}{T}}=\frac{1}{1+\kappa_Q}$$

Crossover smoothens explosion for <m> ~ 280 MeV

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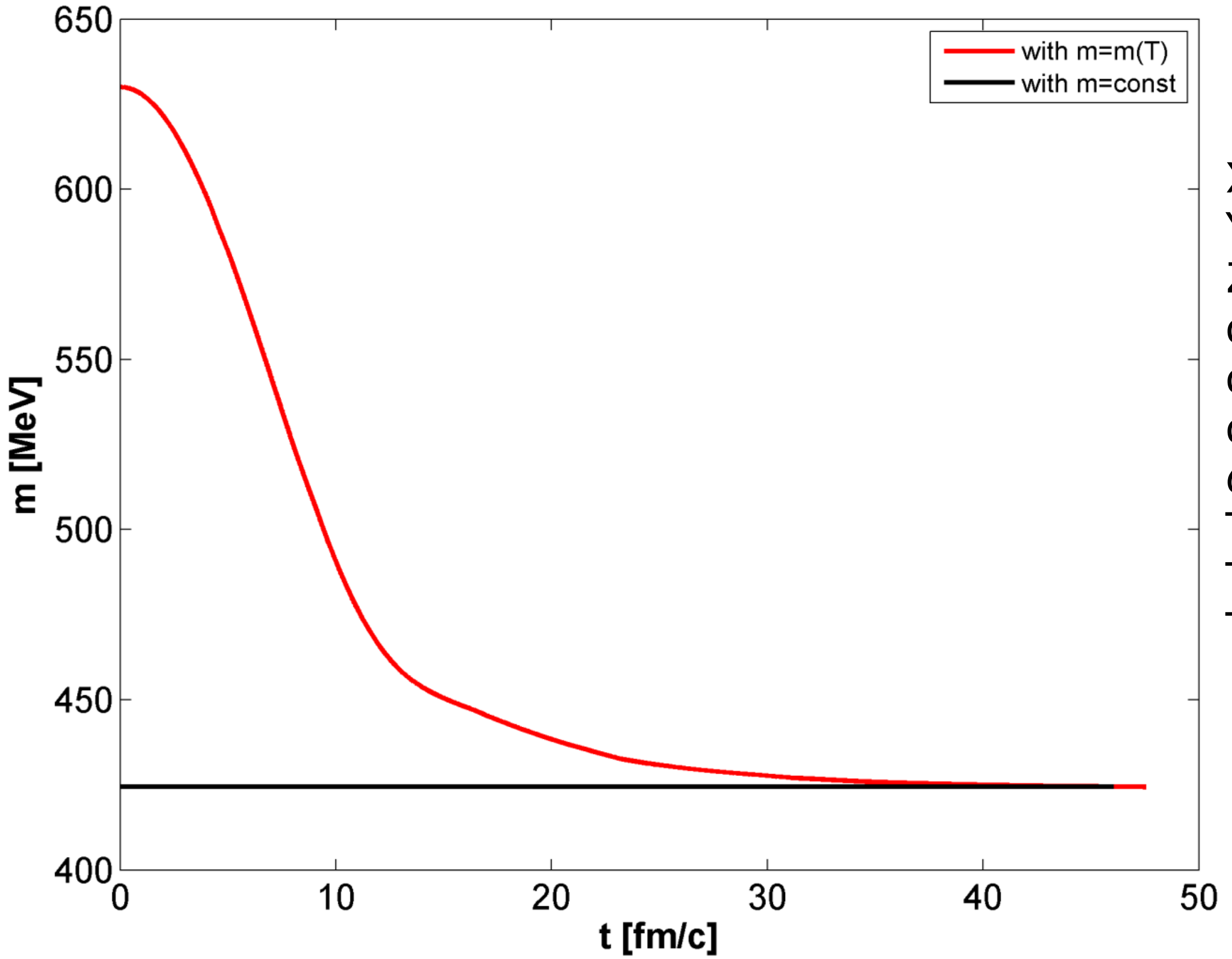
What if m is T dependent?

m(T) and m=const comparison



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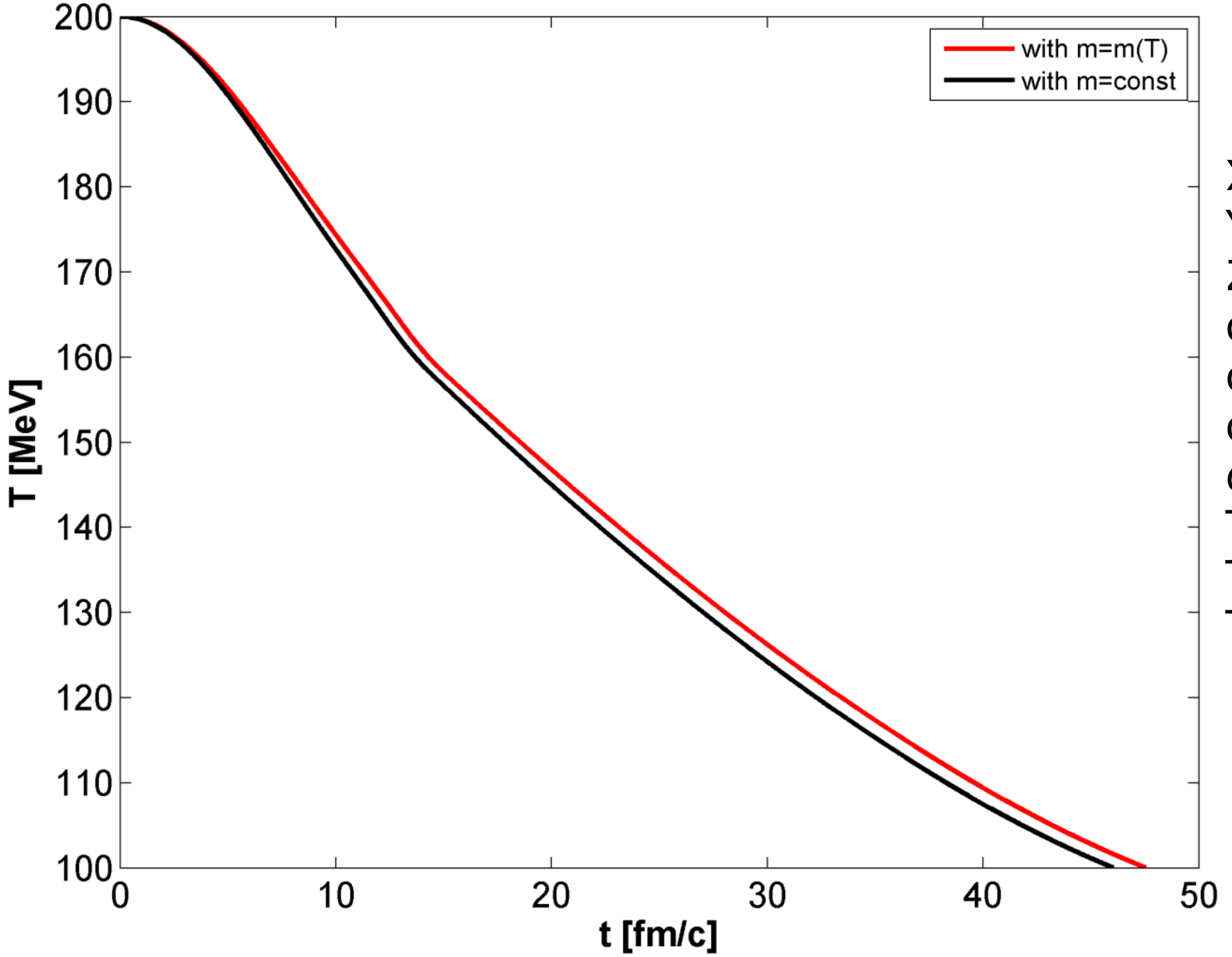
There is no significant difference!

m(T) and m=const comparison



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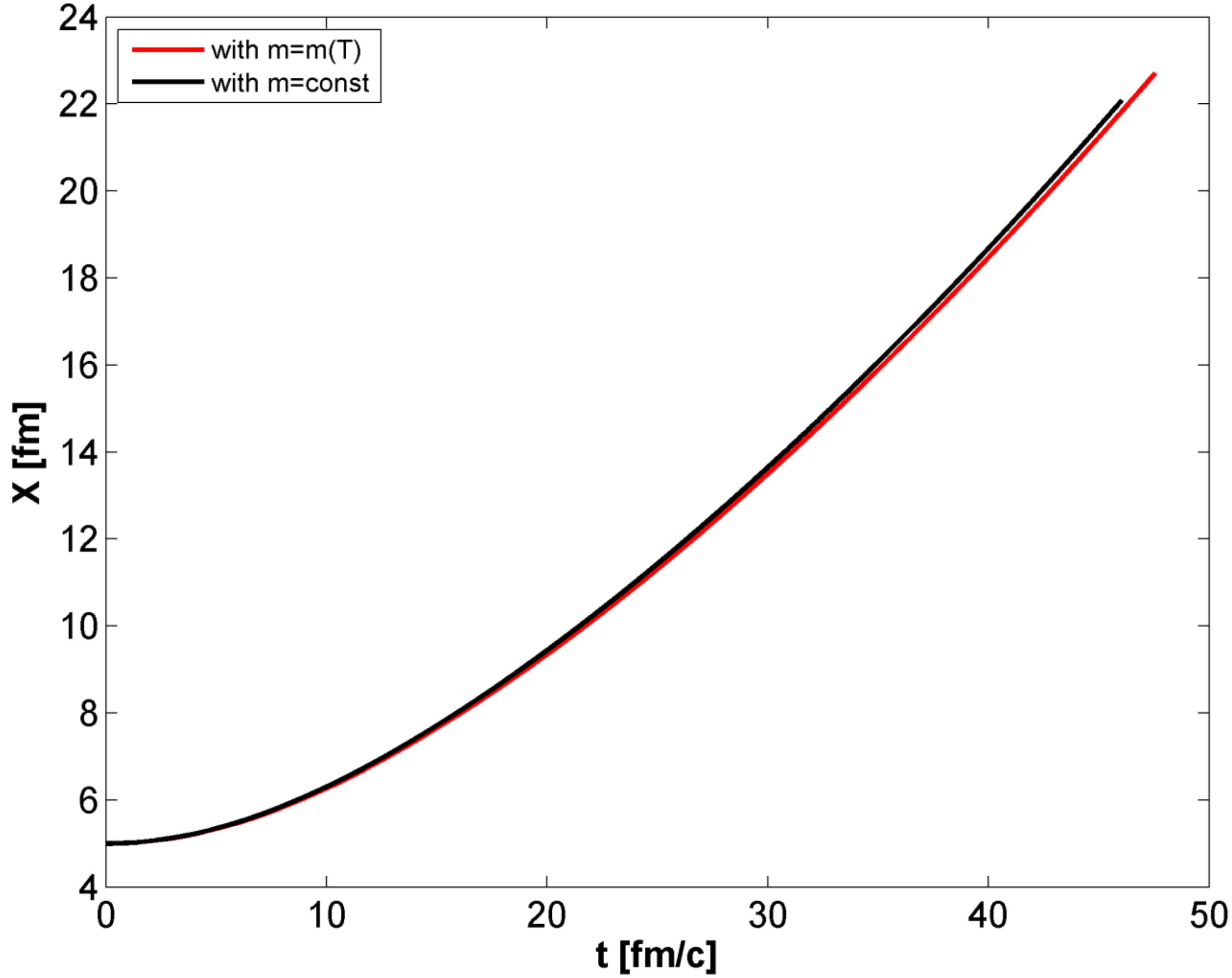

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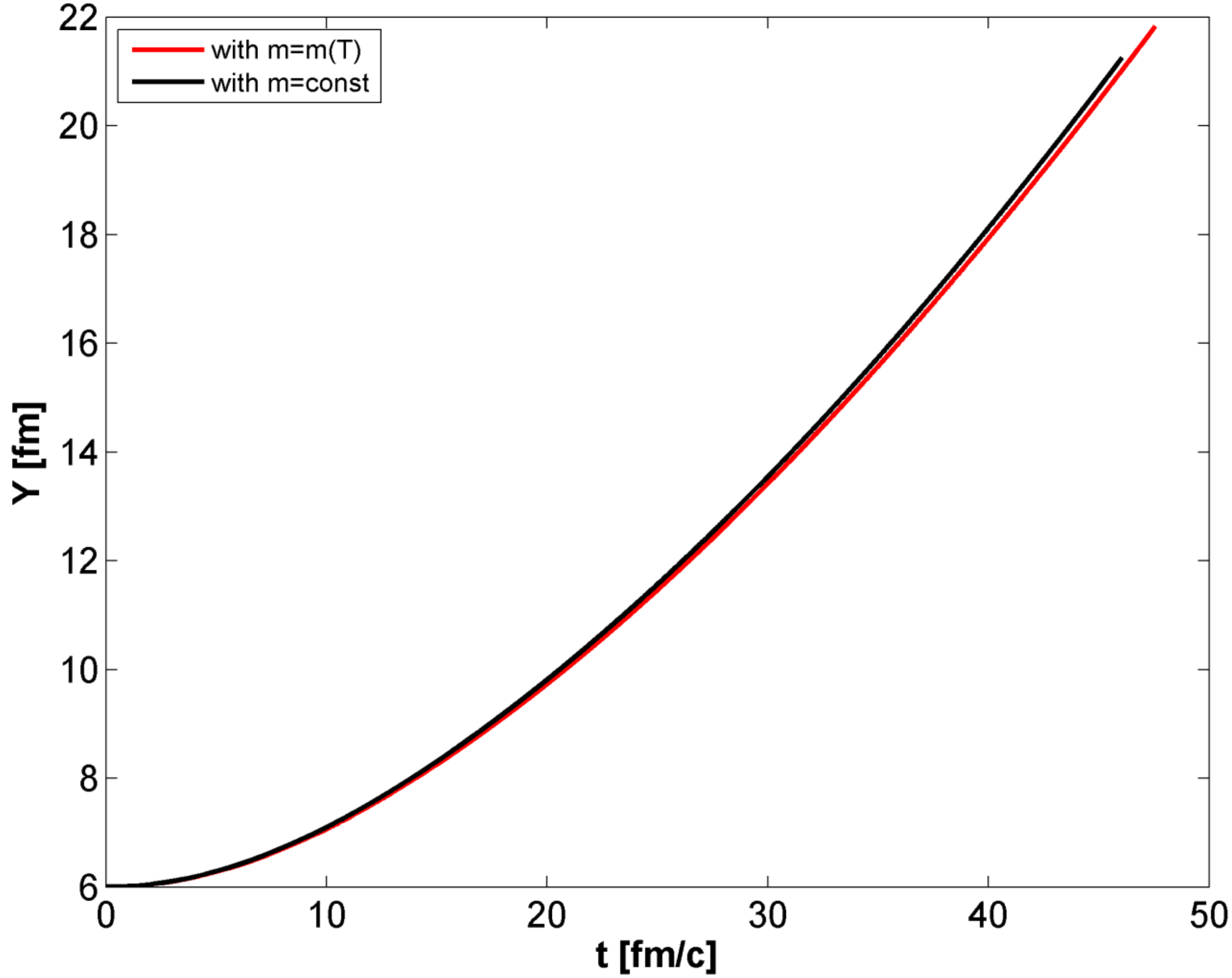

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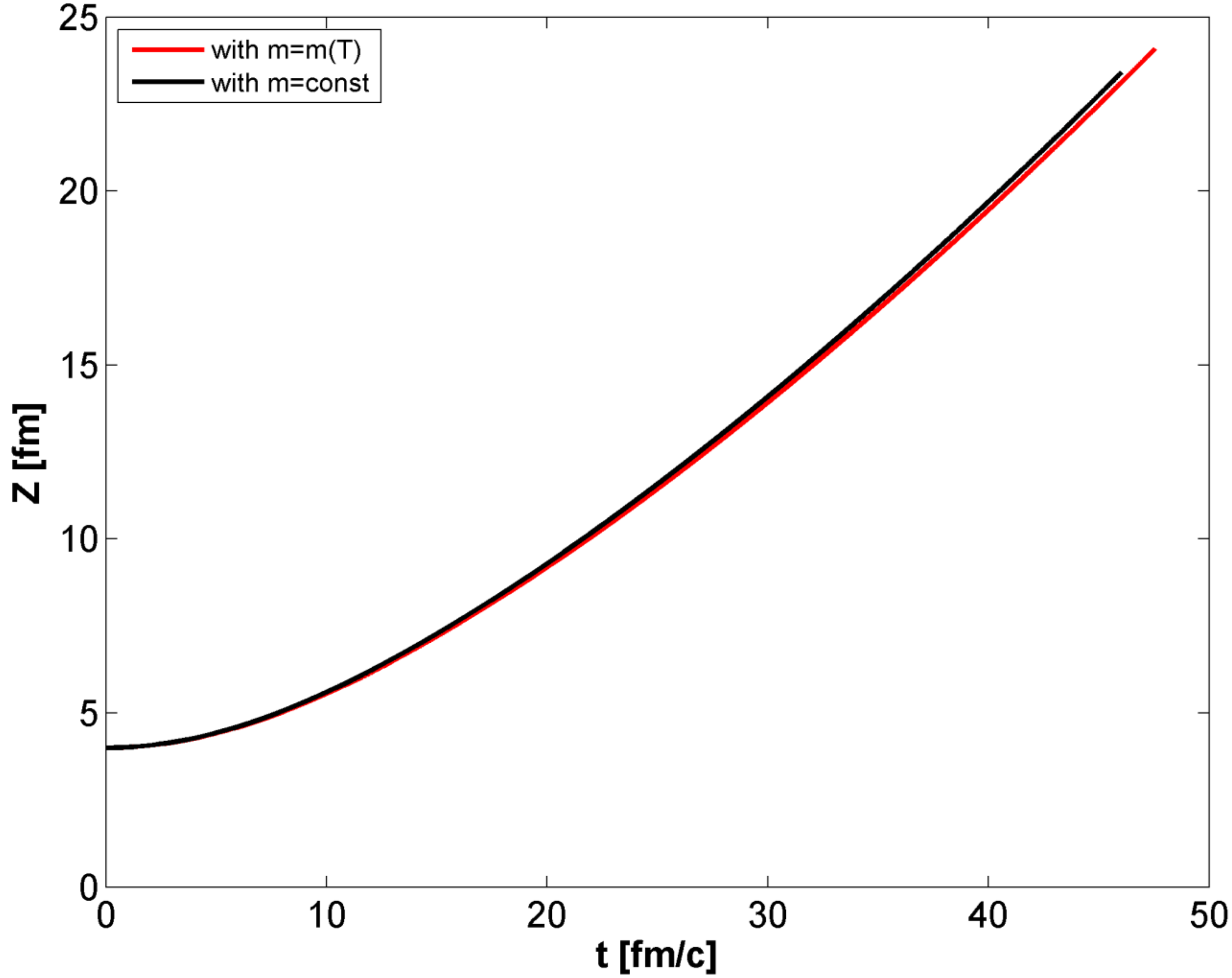

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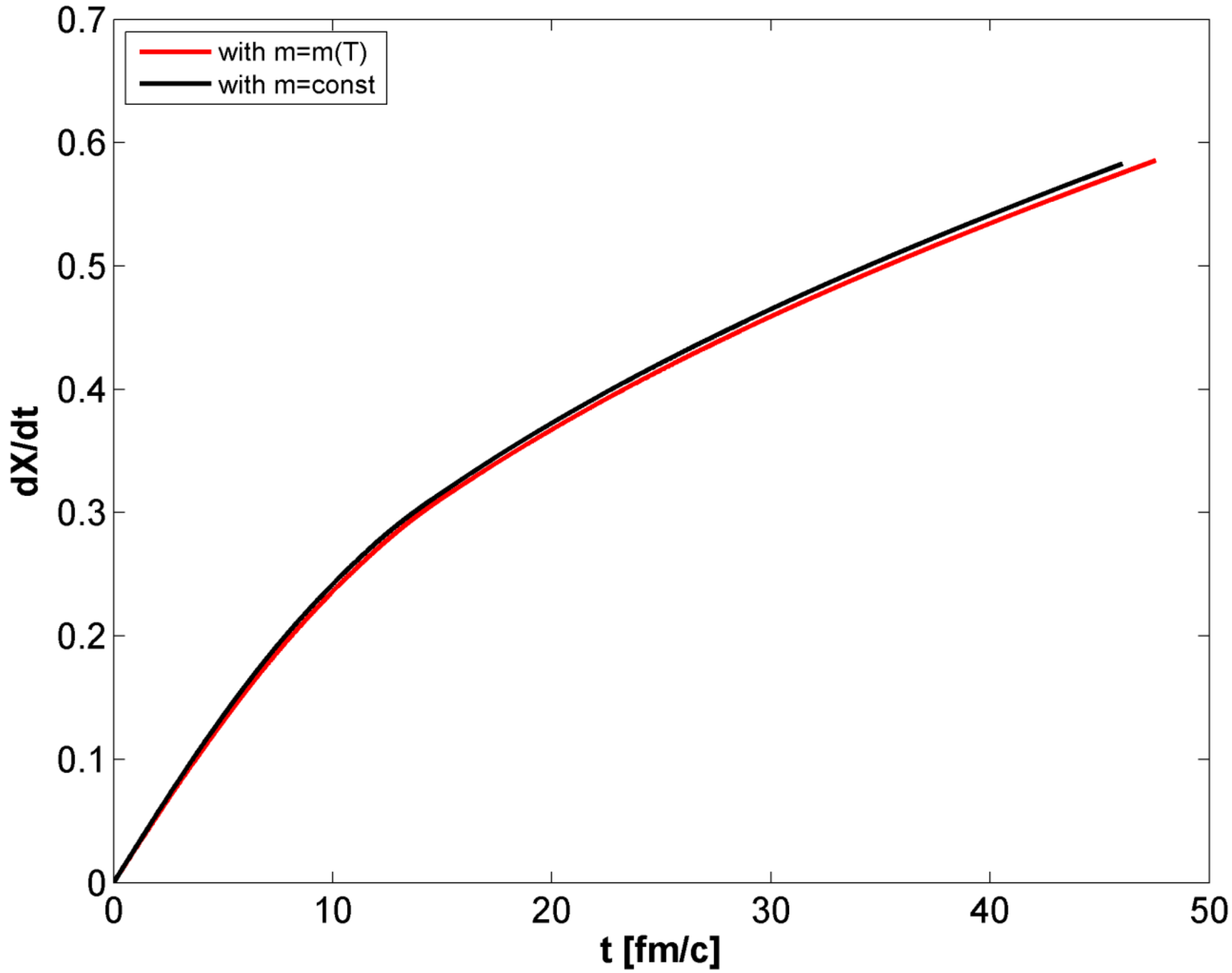

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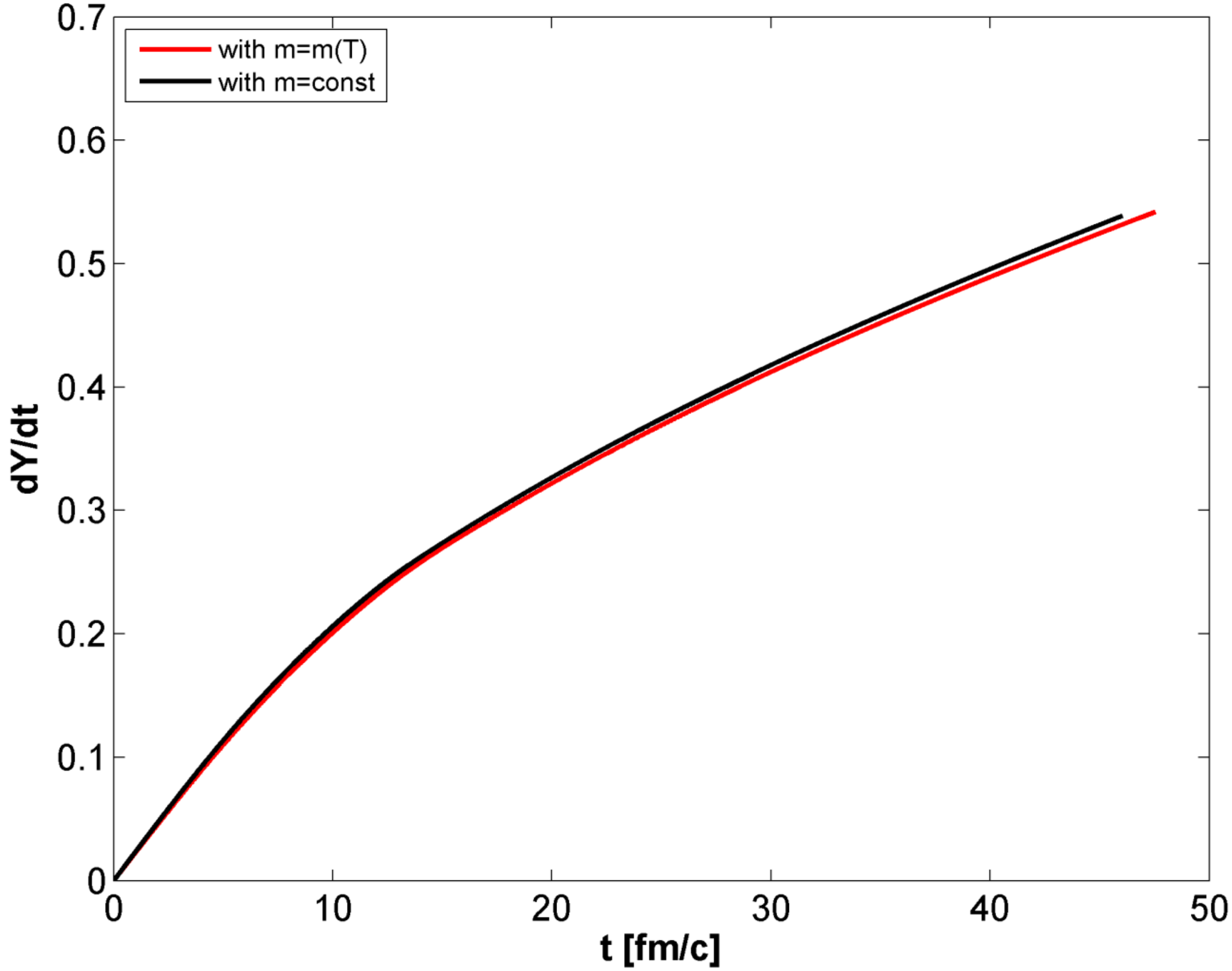

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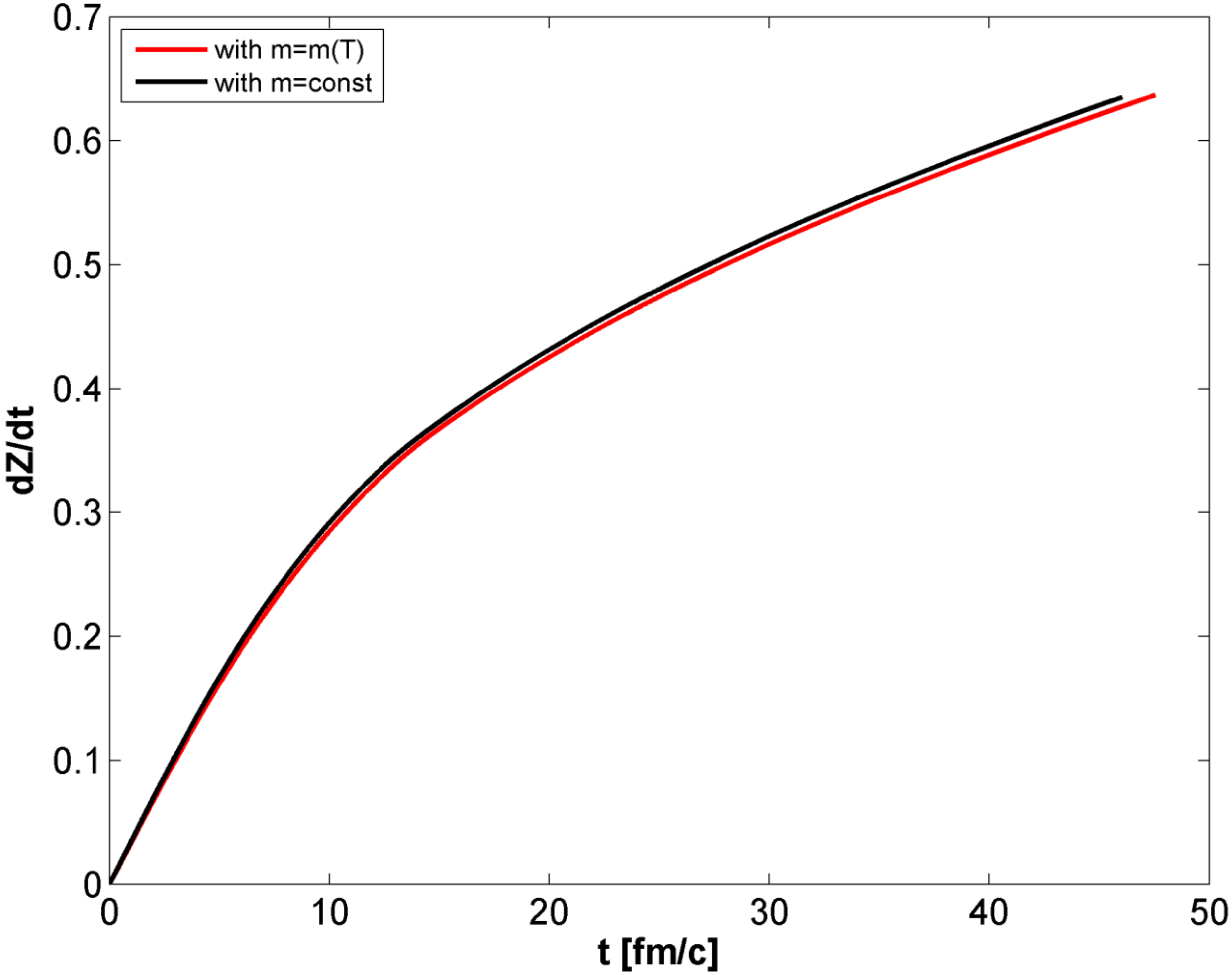

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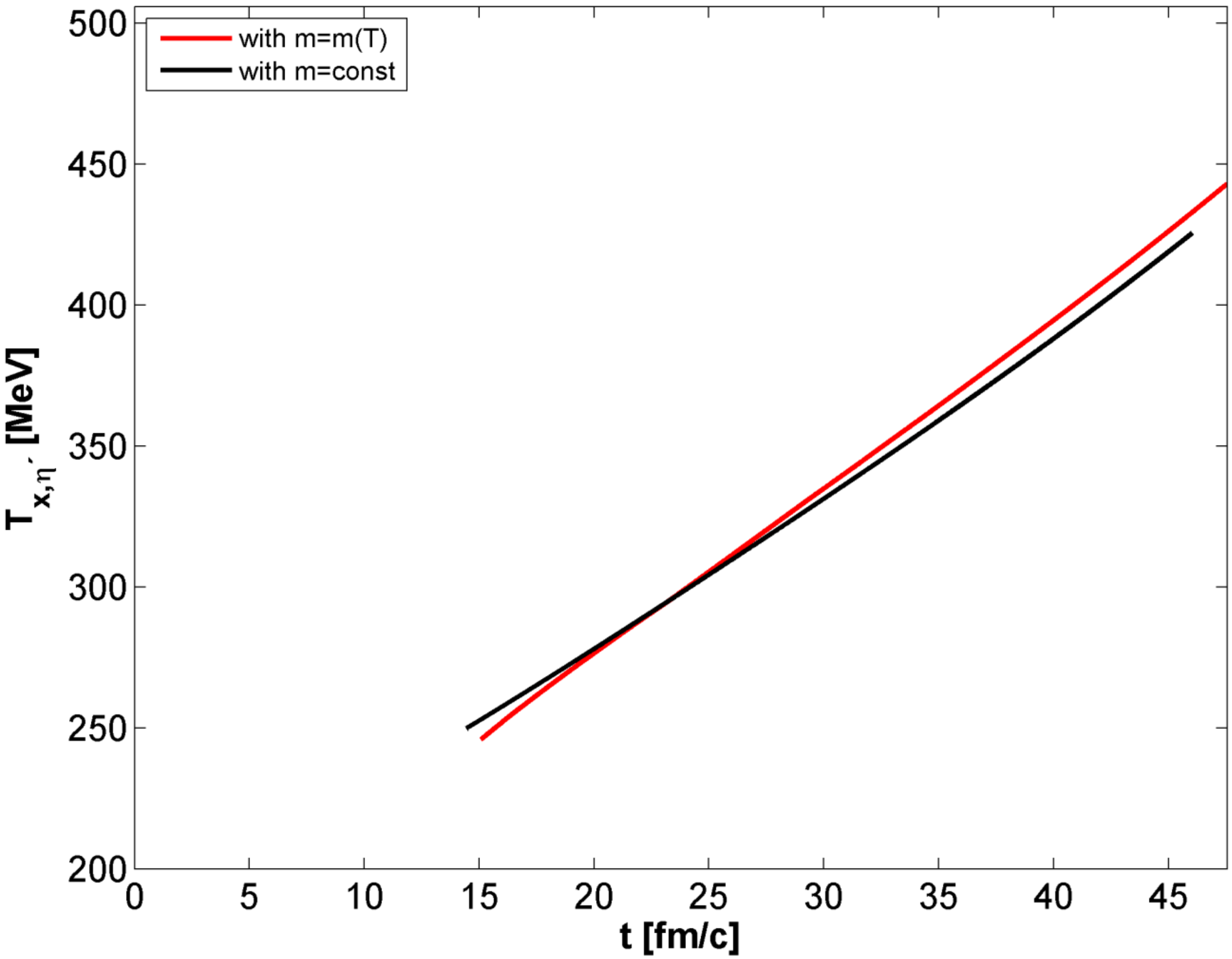

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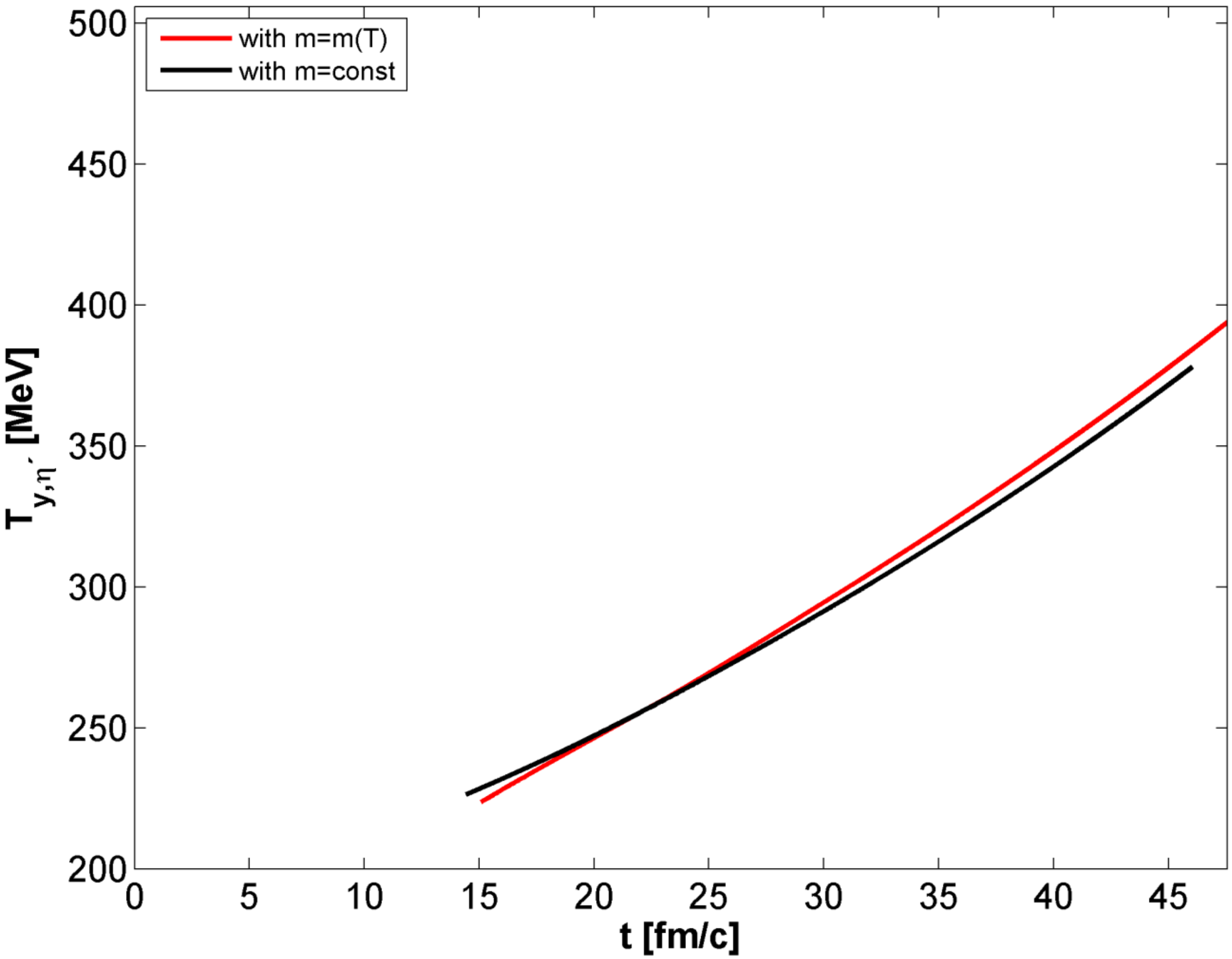

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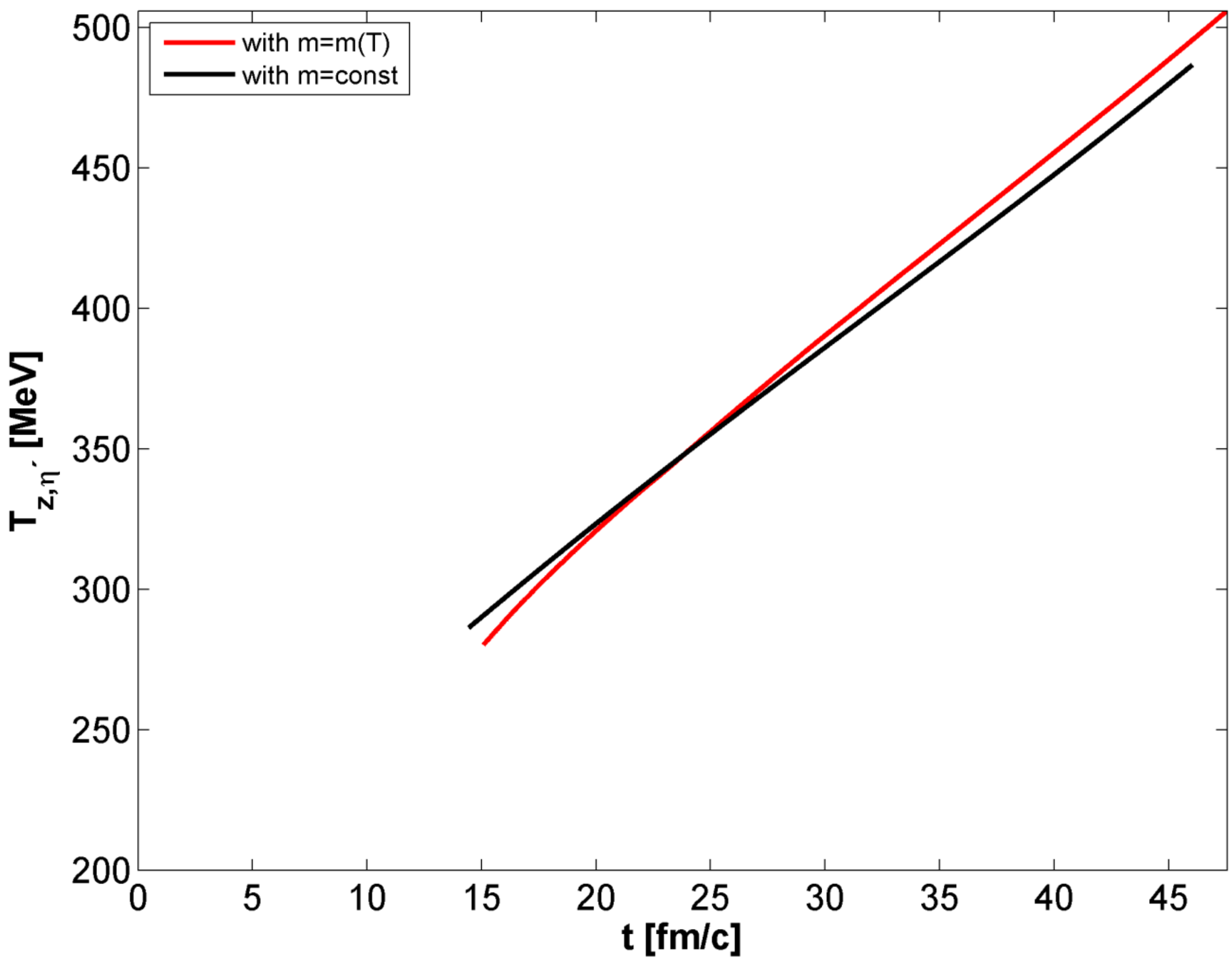

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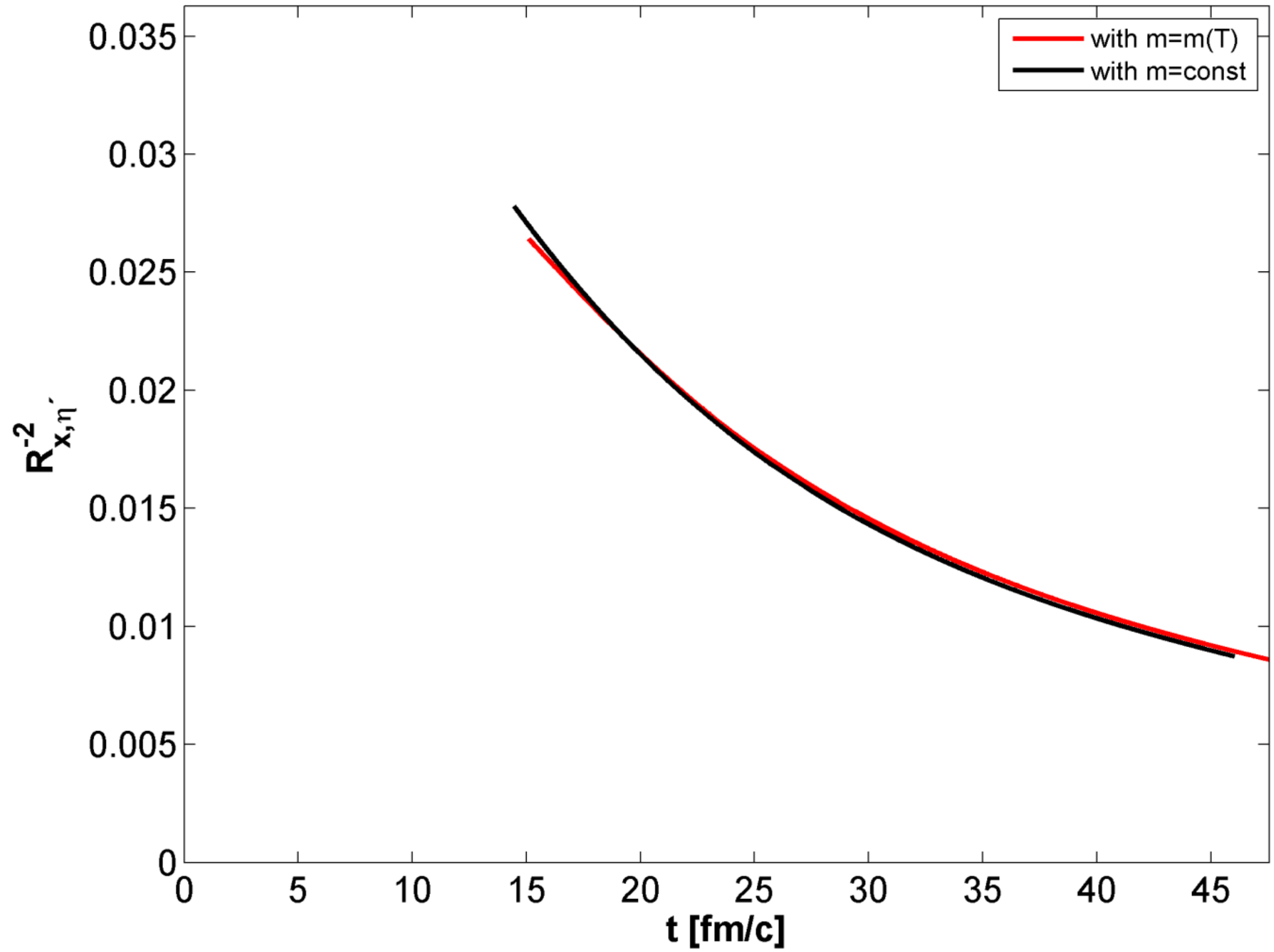

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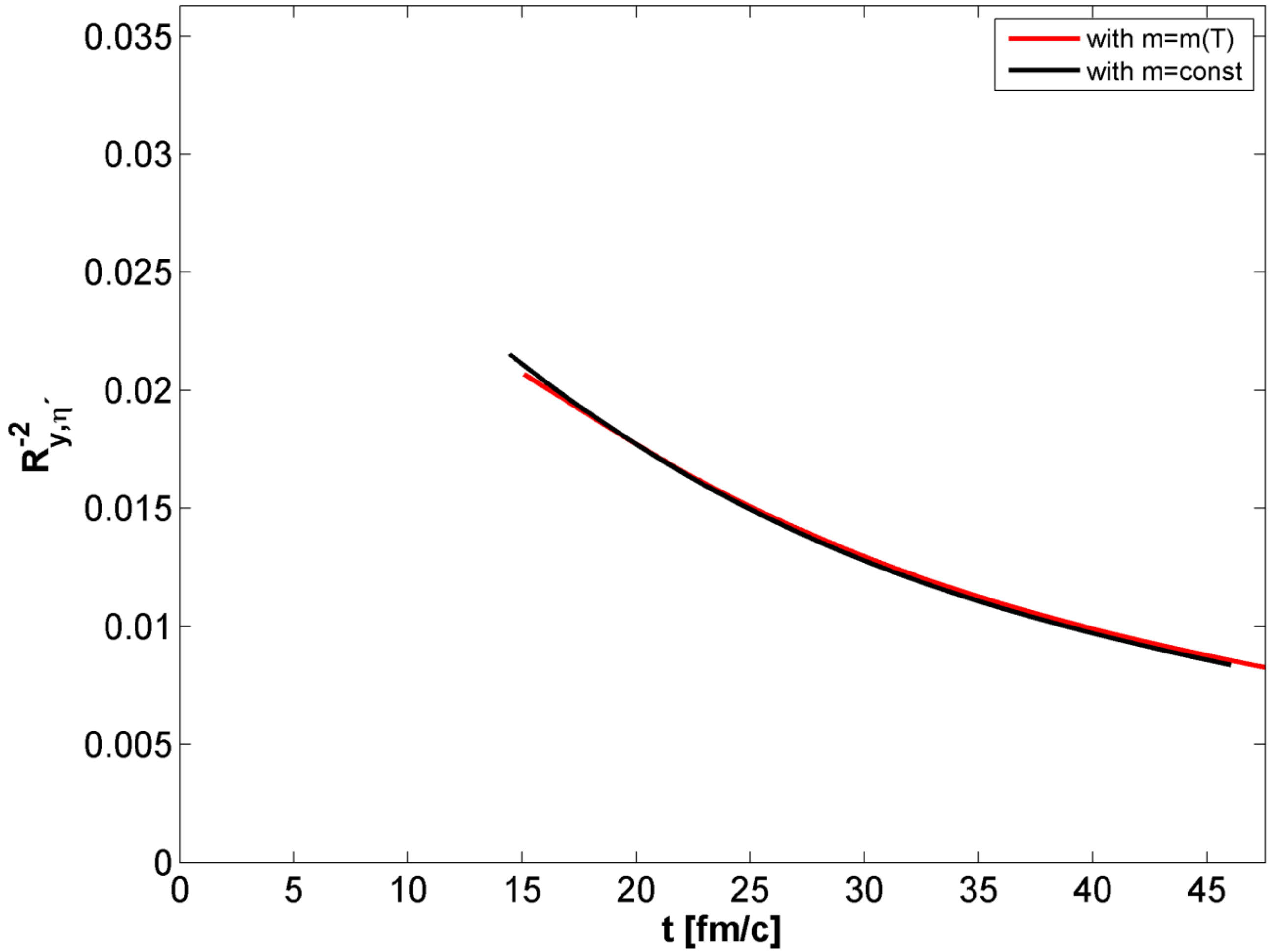

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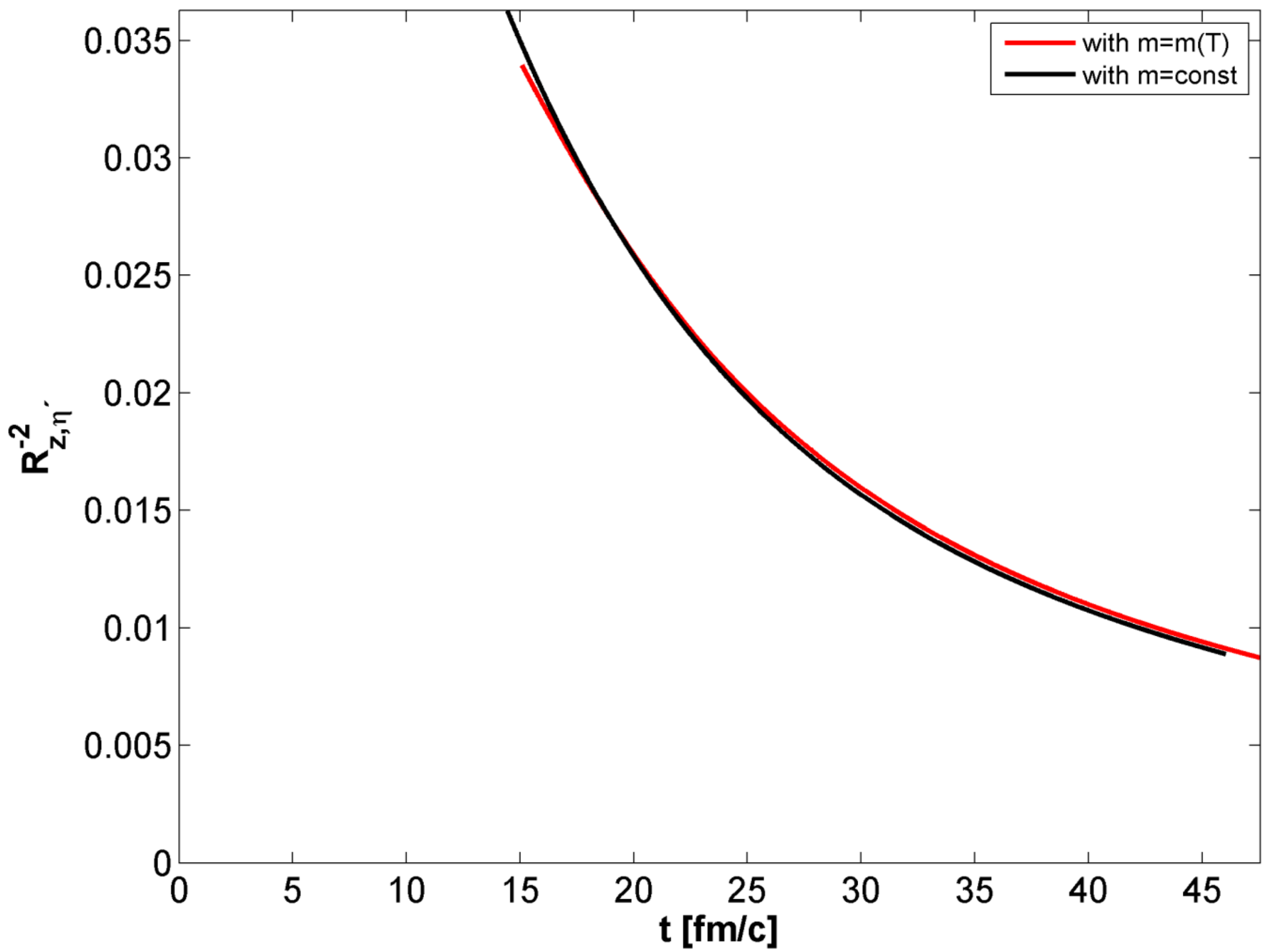

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Slopes: linear mass dependence

	SC HM	MC HM
$\omega_0 = 0$	$T_x = T_f + m\dot{X}_f^{\,2}$ $T_y = T_f + m\dot{Y}_f^{\,2}$ $T_z = T_f + m\dot{Z}_f^{\,2}$	$T_{x,i} = T_f + m_i\dot{X}_f^{\,2}$ $T_{y,i} = T_f + m_i\dot{Y}_f^{\,2}$ $T_{z,i} = T_f + m_i\dot{Z}_f^{\,2}$
$\omega_0 \neq 0$ (K' frame)	$T'_{xx} = T_f + m\left(\dot{X}_f^{\,2} + \omega_f^2 R_f^2\right)$ $T'_{yy} = T_f + m\dot{Y}_f^{\,2}$ $T'_{zz} = T_f + m\left(\dot{Z}_f^{\,2} + \omega_f^2 R_f^2\right)$	$T'_{xx,i} = T_f + m_i\left(\dot{X}_f^{\,2} + \omega_f^2 R_f^2\right)$ $T'_{yy,i} = T_f + m_i\dot{Y}_f^{\,2}$ $T'_{zz,i} = T_f + m_i\left(\dot{Z}_f^{\,2} + \omega_f^2 R_f^2\right)$

- Transform to the lab. frame ($\omega_0 \neq 0$) with **M** rotation matrix:

$$\mathbf{T}_i^{-1} = \mathbf{M}^{-1}\mathbf{T}_i'^{-1}\mathbf{M}$$

- Linear mass dependence
- Scaling behaviour

Hydro scaling: $T = T_f + m \langle u_t \rangle^2$

Slopes: linear mass dependence

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What if m is T dependent?

Slopes: linear mass dependence

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- Transform to the lab. frame ($\omega_0 \neq 0$) with **M** rotation matrix:

$$\mathbf{T}_i^{-1} = \mathbf{M}^{-1}\mathbf{T}_i'^{-1}\mathbf{M}$$

- Linear mass dependence
- Scaling behaviour

Hydro scaling: $T = T_f + m \langle u_t \rangle^2$

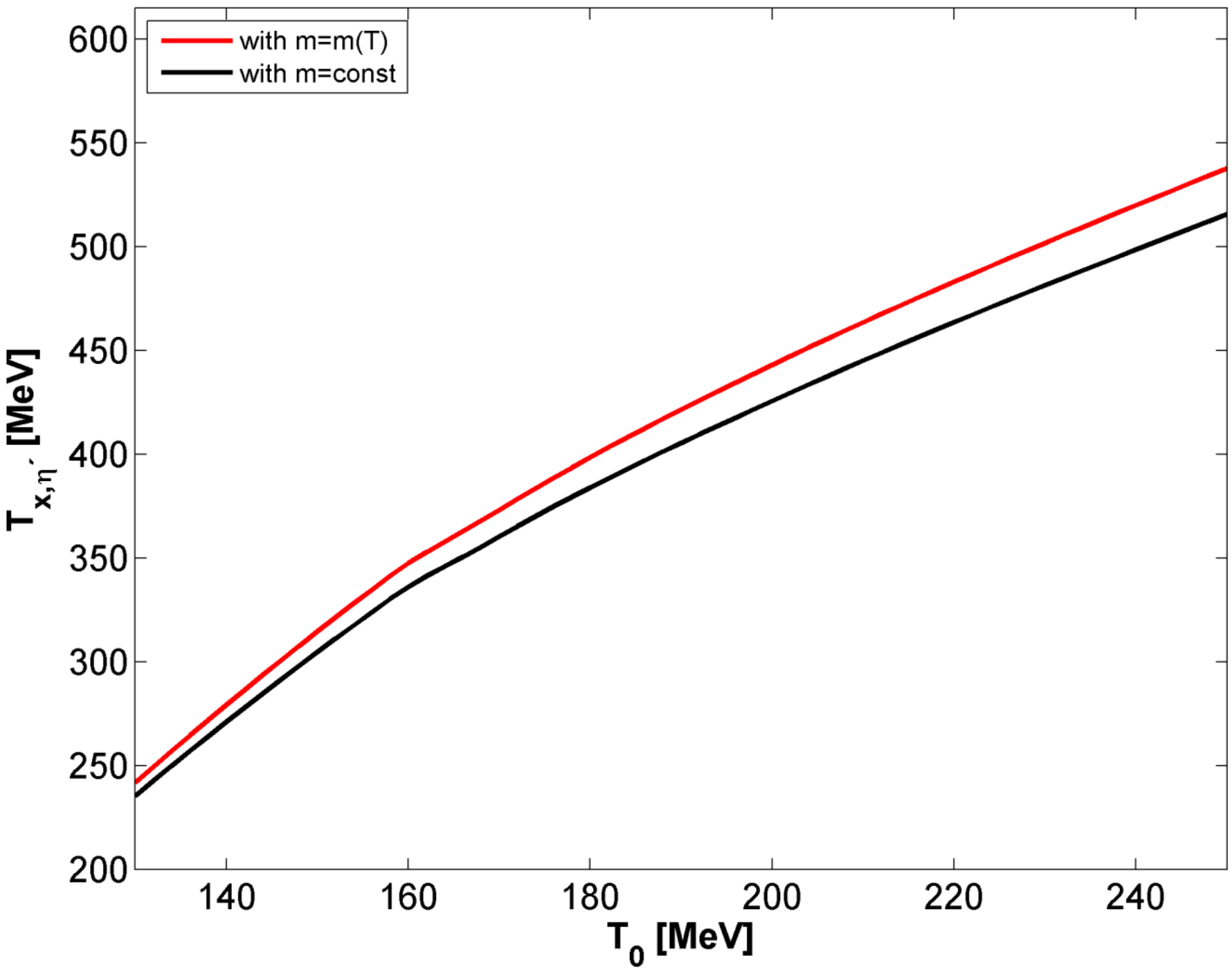
What if m is T dependent?

HBT results: Gaussian radii

	SC HM	MC HM
$\omega_0 = 0$	$R_x^{-2} = X_f^{-2}\frac{T_x}{T_f}$ $R_y^{-2} = Y_f^{-2}\frac{T_y}{T_f}$ $R_z^{-2} = Z_f^{-2}\frac{T_z}{T_f}$	$R_{x,i}^{-2} = X_f^{-2}\frac{T_{x,i}}{T_f}$ $R_{y,i}^{-2} = Y_f^{-2}\frac{T_{y,i}}{T_f}$ $R_{z,i}^{-2} = Z_f^{-2}\frac{T_{z,i}}{T_f}$
$\omega_0 \neq 0$ (K' frame)	$R_{xx}'^{-2} = X_f^{-2}\frac{T'_{xx}}{T_f}\left[1 - \frac{T'^2_{xz}}{T'_{xx}T'_{zz}}\right]$ $R_{yy}'^{-2} = Y_f^{-2}\frac{T_{yy}}{T_f}$ $R_{zz}'^{-2} = Z_f^{-2}\frac{T_{zz}}{T_f}\left[1 - \frac{T'^2_{xz,}}{T_{xx}T_{zz}}\right]$	$R_{xx,i}'^{-2} = X_f^{-2}\frac{T'_{xx,i}}{T_f}\left[1 - \frac{T'^2_{xz,i}}{T'_{xx,i}T'_{zz,i}}\right]$ $R_{yy,i}'^{-2} = Y_f^{-2}\frac{T_{yy,i}}{T_f}$ $R_{zz,i}'^{-2} = Z_f^{-2}\frac{T'_{zz,i}}{T_f}\left[1 - \frac{T'^2_{xz,i}}{T'_{xx,i}T'_{zz,i}}\right]$

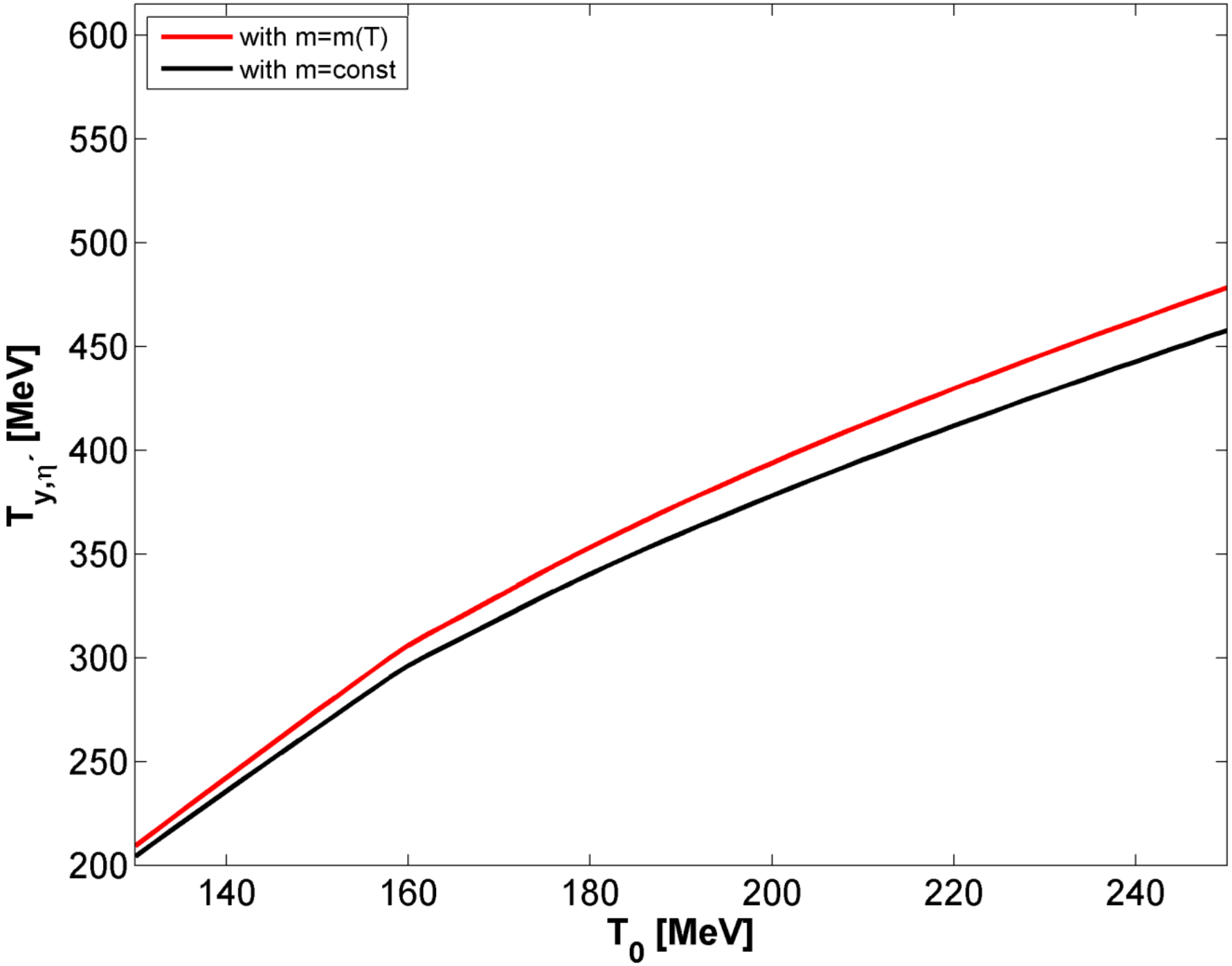
- Transform to the lab. frame ($\omega_0 \neq 0$): $\mathbf{R} = \mathbf{M}^{-1}\mathbf{R}\mathbf{M}$
- Linear mass dependence
- Scaling behaviour

$m_{\eta'}(T)$ and $m_{\eta'}$=const comparison



For every T_0:
X_0=5 fm
Y_0=6 fm
Z_0=4 fm
$dX/dt|_{t0}=0$
$dY/dt|_{t0}=0$
$dZ/dt|_{t0}=0$
ω_0=0 c/fm
T_{chem}=158 MeV
T_f=100 MeV

$m_{\eta'}(T)$ and $m_{\eta'}$=const comparison



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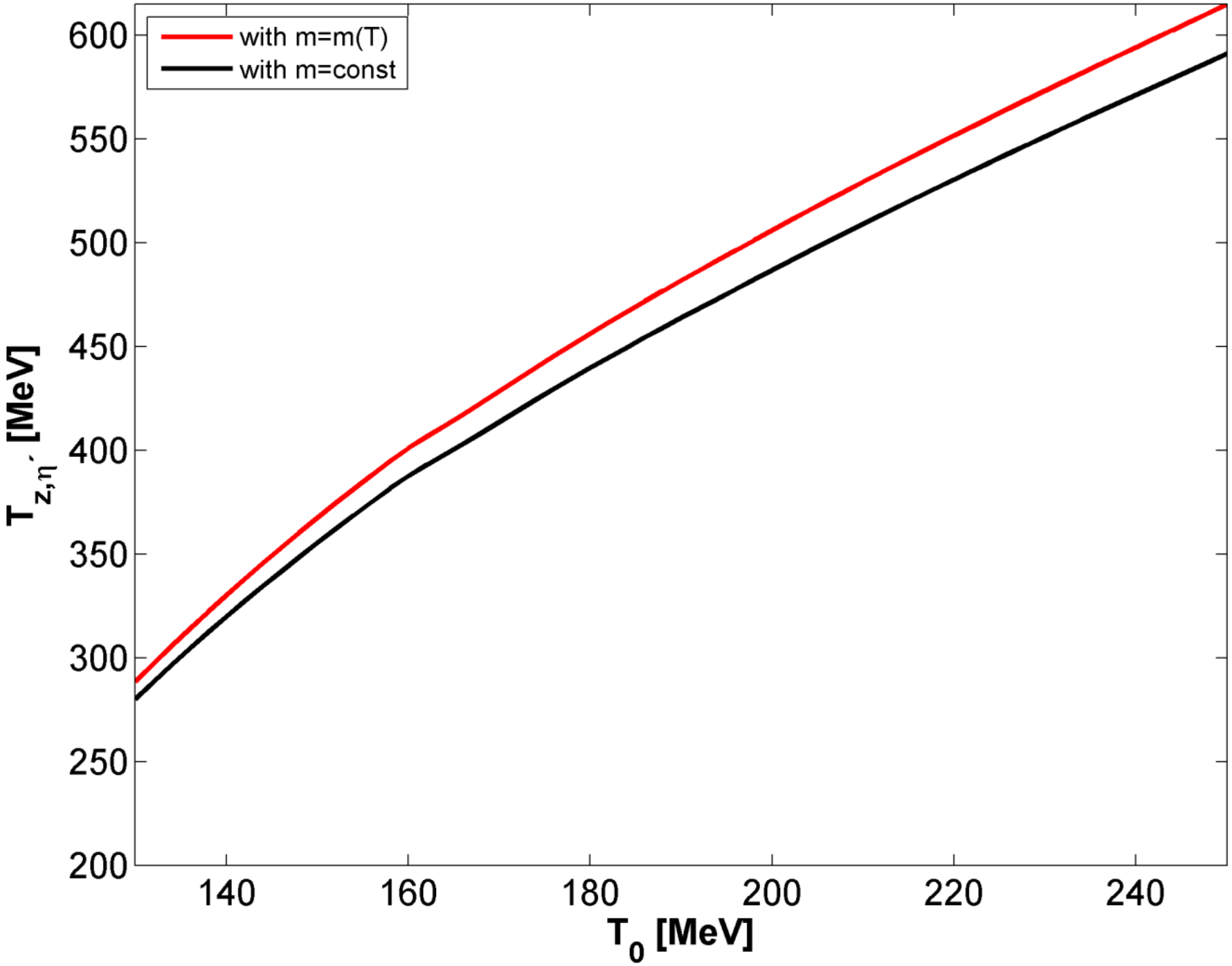

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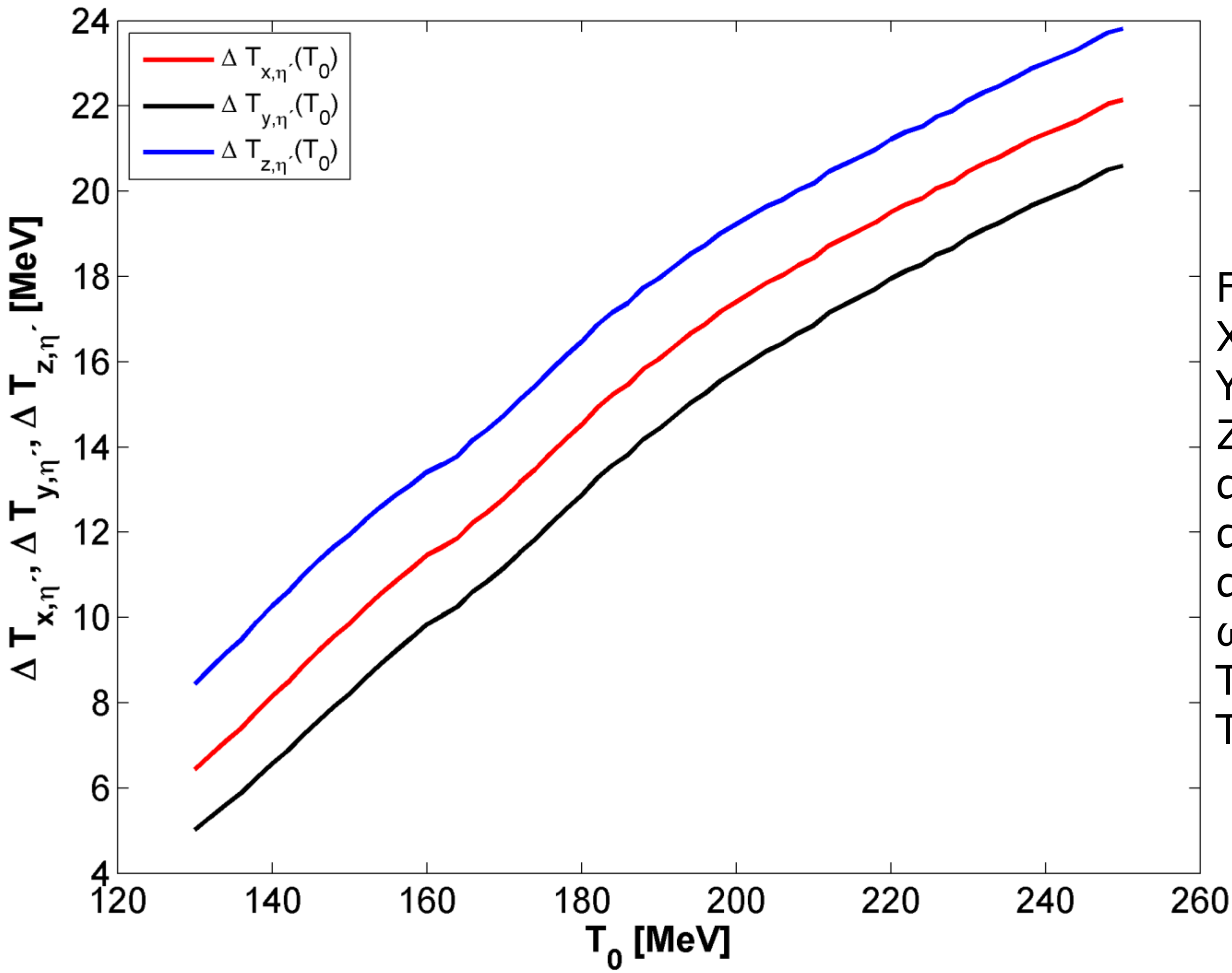

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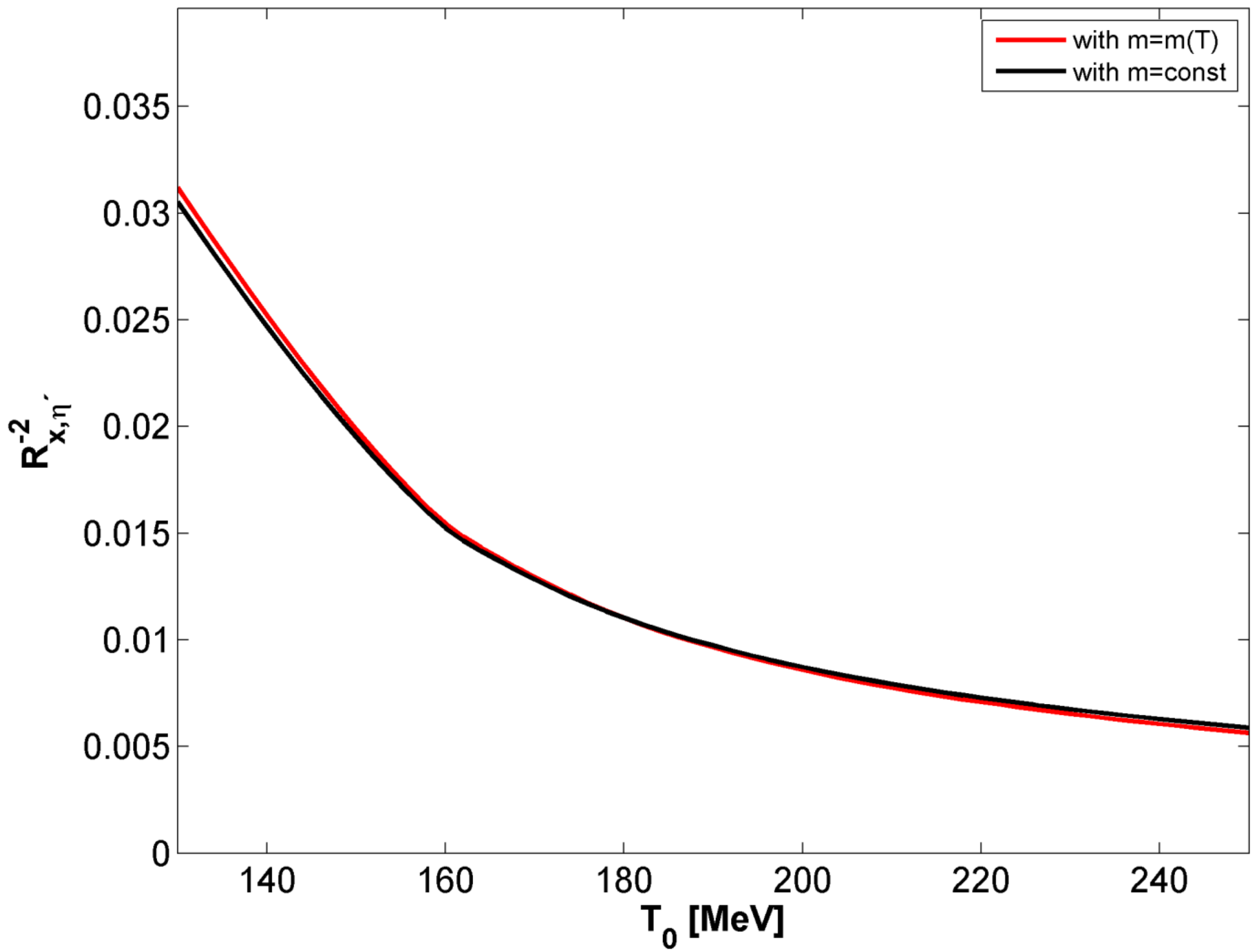

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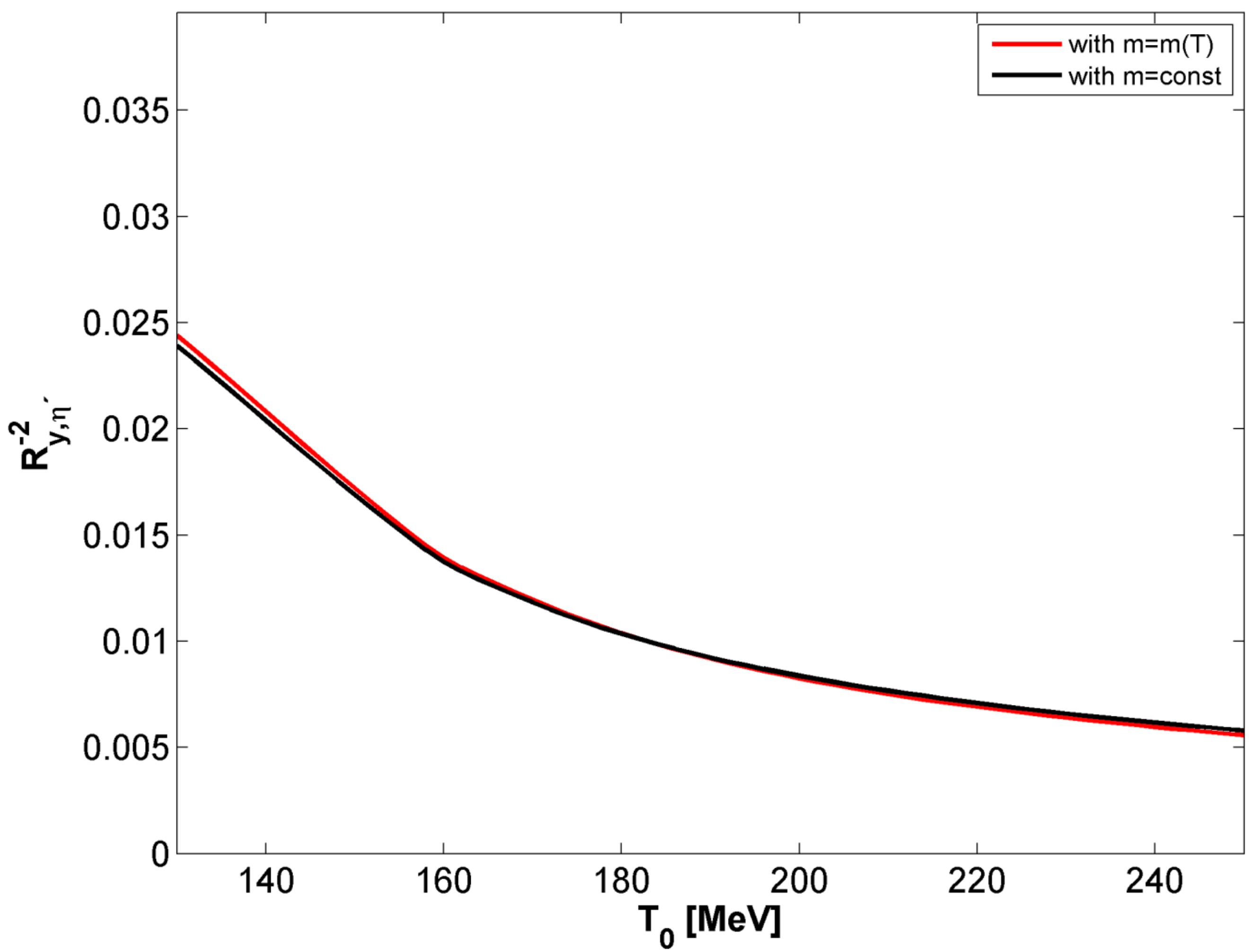

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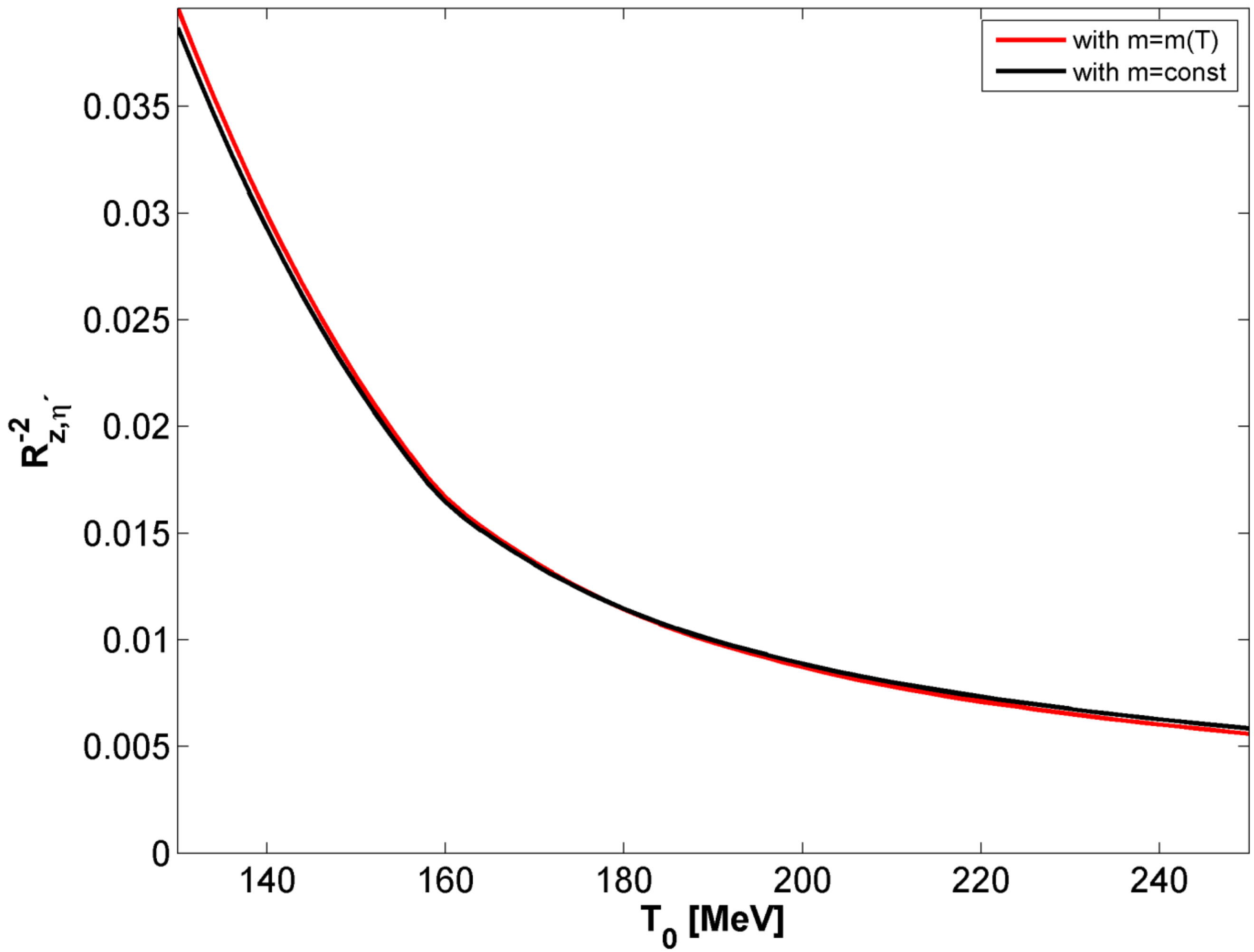

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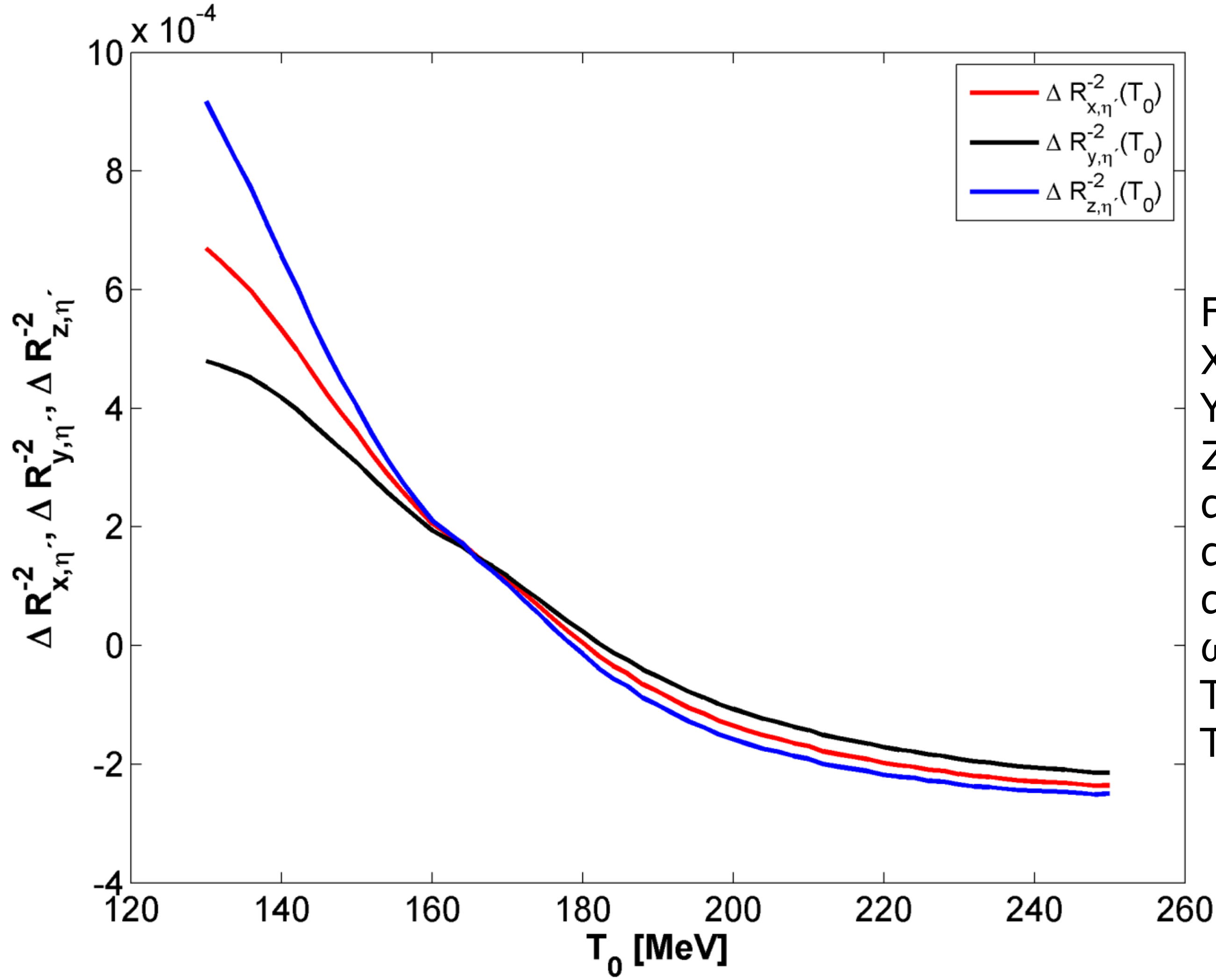

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Summary

- Lattice QCD EoS parametrization ☑
- Quark-hadron transitions
  - Crossover ☑
  - 2nd order PT ☑
  - 1st order PT ☒
- Multi-component hadronic solution ☑
- Temperature dependent mass ☒
- Observables
  - Scaling behaviour ☑
  - Rotational angles ☒

Backup slides

Relativistic hydrodynamics

QM $(T_i \geq T \geq T_{chem})$	**HM** $(T_{chem} > T \geq T_f)$
$\partial_\mu \sigma u^\mu = 0$	$\partial_\mu n_i u^\mu = 0, \quad \forall i$
$T\sigma u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu)\, p$	$\left(\sum_i m_i n_i\right) u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu)\, p$
$\frac{1+\kappa}{T}\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right] u^\mu \partial_\mu T + \partial_\mu u^\mu = 0$	$\frac{1}{T}\left[\frac{d(\kappa T)}{dT}\right] u^\mu \partial_\mu T + \partial_\mu u^\mu = 0$
$p = \sigma T/(1+\kappa)$	$p = \sum_i p_i = T \sum_i n_i$

$$
\begin{aligned}
\varepsilon + p &= \sum_i \mu_i n_i + T\sigma, \\
\varepsilon + p &\approx T\sigma, \qquad\qquad (T_i \geq T \geq T_{chem}), \\
\varepsilon + p &\approx \sum_i m_i n_i \qquad (T_{chem} > T \geq T_f).
\end{aligned}
$$

Rotation angles, p and q space

- Rotation angle of the momentum space (K' frame):

$$\theta'_p = \frac{1}{2}\arctan\left(\frac{2T'_{xz}}{T'_{xx} - T'_{zz}}\right) = \frac{1}{2}\arctan\left(\frac{2\omega R}{\dot{X} + \dot{Z}}\right)$$

  In laboratory frame: $\theta_p = \vartheta_f + \theta'_p$

- Rotation angle of the HBT space (K' frame):

$$\theta'_{q,i} = \frac{1}{2}\arctan\left(\frac{2XZT'_{xz,i}}{Z^2T'_{xx,i} - X^2T'_{zz,i}}\right) =$$

$$= \frac{1}{2}\arctan\left(\frac{2m_iXZ\omega R\left(\dot{X} - \dot{Z}\right)}{\left(T + m_i\omega^2R^2\right)\left(Z^2 - X^2\right) + m_i\left(Z^2\dot{X}^2 - X^2\dot{Z}^2\right)}\right)$$

  In laboratory frame: $\theta_{q,i} = \vartheta_f + \theta'_{q,i}$

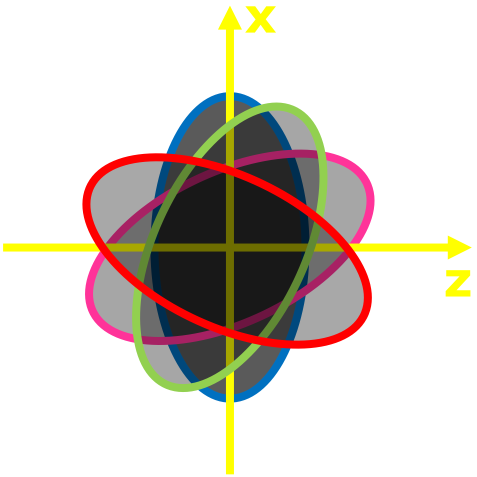
- $\theta_{q,i}$, θ_p correlation: it can be a new possibility to measure the type of the quark-hadron transition

Observables from the new solutions - triaxial, rotating and expanding

Coordinate-space ellipsoid at the beginning of time evolution
Final coordinate-space ellipsoid at freeze-out — ϑ_f
"Momentum-space ellipsoid" (eigenframe of single-particle spectrum) — θ_p
"HBT-space ellipsoid" (eigenframe of HBT correlations) — θ_q

$$\boxed{\theta'_p = \frac{1}{2}\arctan\left(\frac{2T'_{xz}}{T'_{xx}-T'_{zz}}\right) = \frac{1}{2}\arctan\left(\frac{2\omega R}{\dot{X}+\dot{Z}}\right)}$$

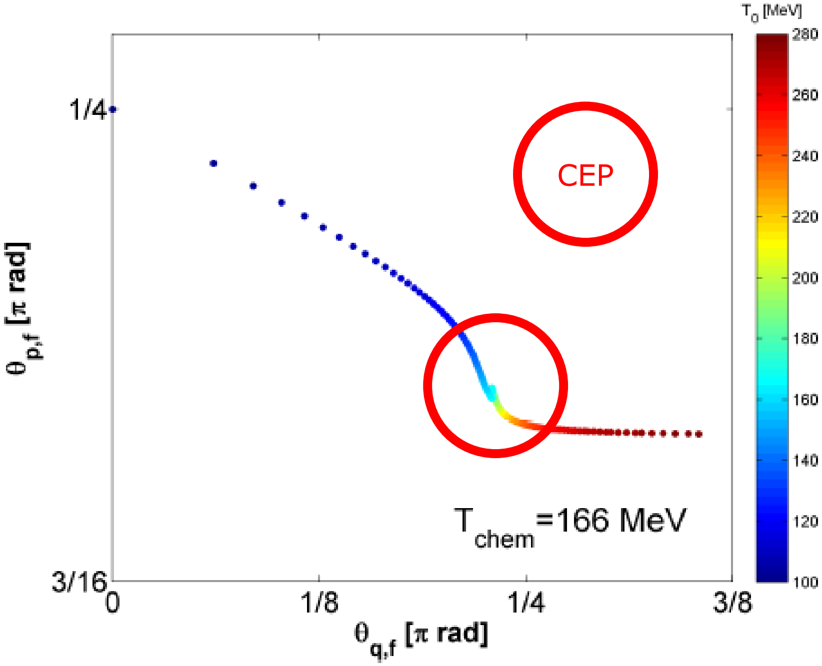
$$\boxed{\theta'_{q,i} = \frac{1}{2}\arctan\left(\frac{2XZT'_{xz,i}}{Z^2T'_{xx,i}-X^2T'_{zz,i}}\right)}$$

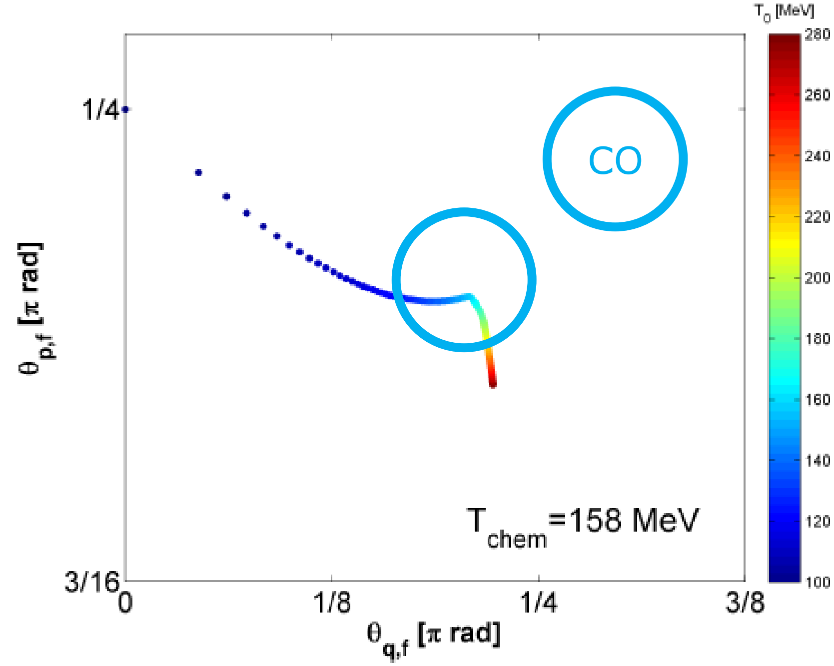


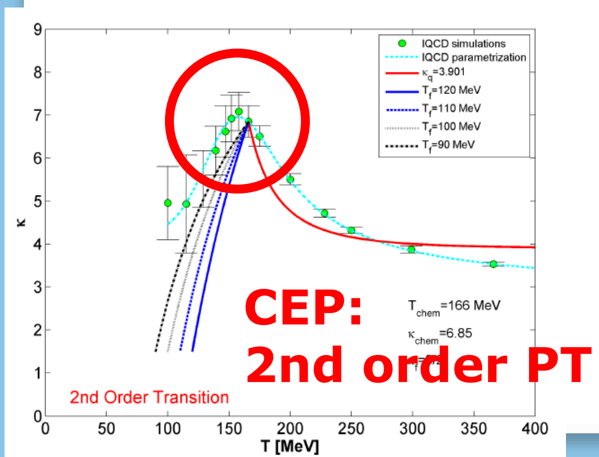
M.I. Nagy and T. Csörgő: **arXiv:1606.09160**

Correlation of p and q space

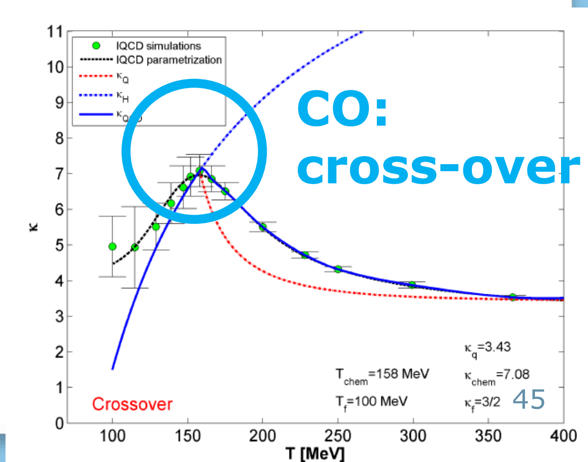
difference of dynamics at CEP and CO QCD transition





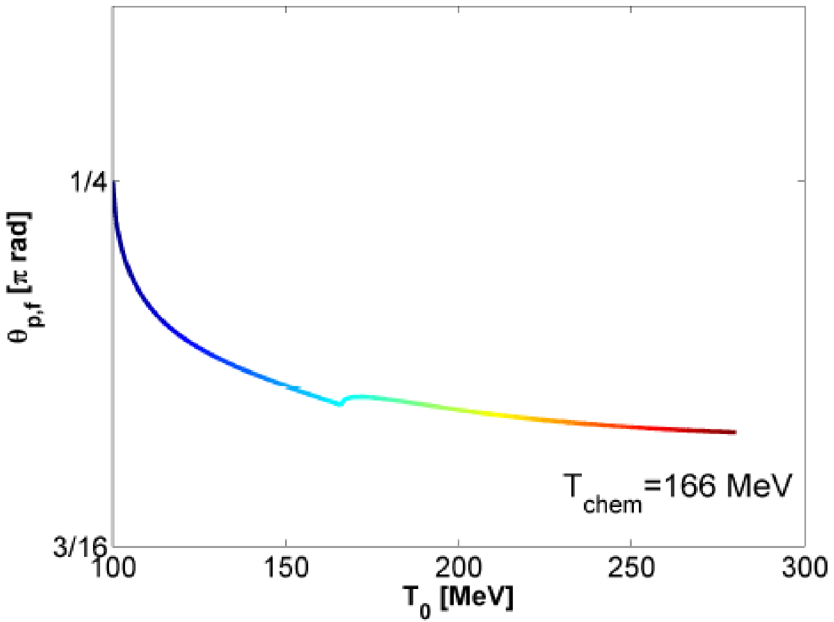


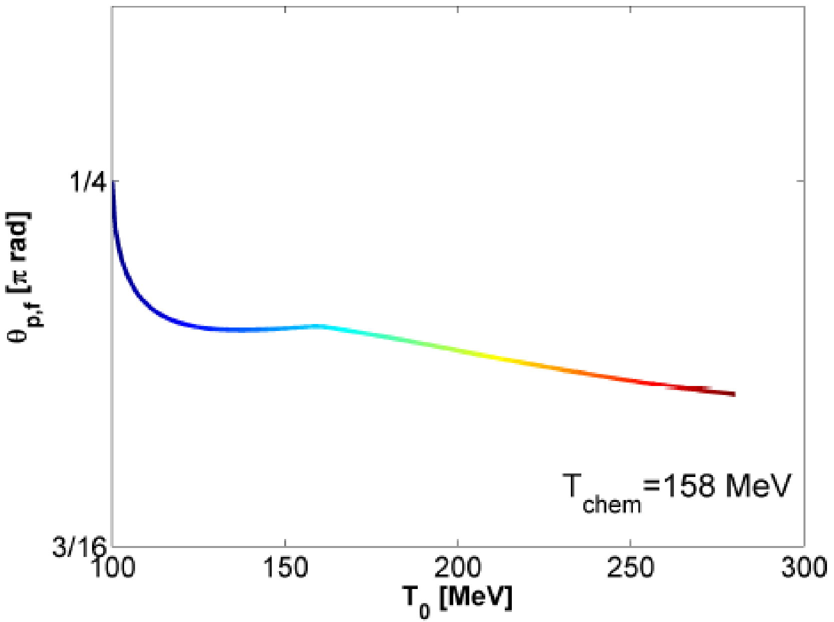
Correlation of angles in momentum and in HBT space: sensitive to CEP, CO, in general to QCD EoS



Rotation angles, p space

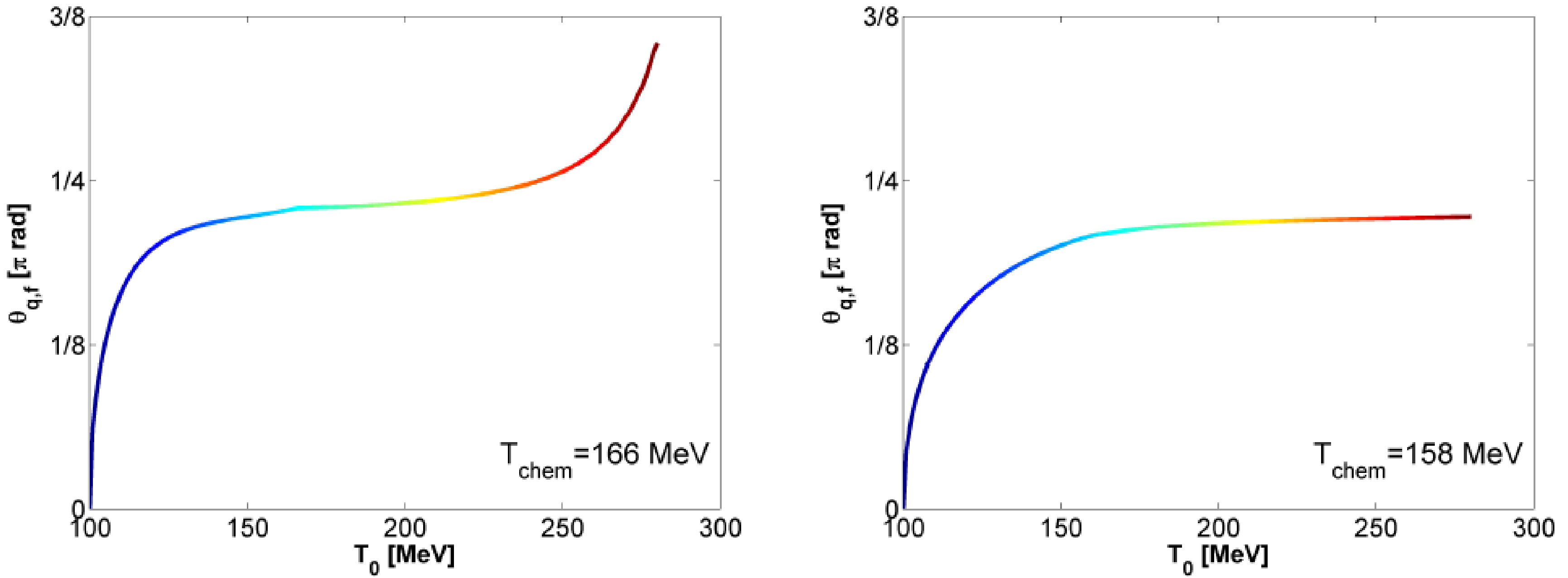
- θ_p as a function of the initial temperature
- Left: 2nd Order PT, right: crossover





Rotation angles, q space

- $\theta_{q,i}$ as a function of the initial temperature
- Left: 2nd Order PT, right: crossover



Triaxial, rotating exact hydro solutions

In both frames:

$$H_x = \frac{\dot X}{X}, \quad H_y = \frac{\dot Y}{Y}, \quad H_z = \frac{\dot Z}{Z}, \qquad \frac{\mathrm{d}[T\kappa(T)]}{\mathrm{d}T}\frac{\dot T}{T} + \frac{\dot V}{V} = 0 \quad \text{if} \quad \kappa(T) \neq const,$$

$$V = (2\pi)^{3/2} XYZ, \quad n = n_0 \frac{V_0}{V}\exp(-s/2), \qquad T = T_0\left(\frac{V_0}{V}\right)^{1/\kappa} \quad \text{if} \quad \kappa(T) = const$$

$$\dot\vartheta \equiv \frac{\omega}{2}, \quad \omega = \omega_0 \frac{R_0^2}{R^2}, \quad R = \frac{X+Z}{2}, \qquad X(\ddot X - \omega^2 R) = Y\ddot Y = Z(\ddot Z - \omega^2 R) = \frac{T}{m_0},$$

in laboratory frame K:

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2} + \left(\frac{1}{Z^2} - \frac{1}{X^2}\right)\left[(r_x^2 - r_z^2)\sin^2\vartheta + r_x r_z \sin(2\vartheta)\right]$$

$$\mathbf{v}(\mathbf{r},t) = \mathbf{v}_H(\mathbf{r},t) + \mathbf{v}_R(\mathbf{r},t)$$

$$\mathbf{v}_H(\mathbf{r},t) = \begin{pmatrix} (H_x\cos^2\vartheta + H_z\sin^2\vartheta) r_x \\ H_y r_y \\ (H_x\sin^2\vartheta + H_z\cos^2\vartheta) r_z \end{pmatrix} + (H_z - H_x)\frac{\sin(2\vartheta)}{2}\begin{pmatrix} r_z \\ 0 \\ r_x \end{pmatrix}$$

$$\mathbf{v}_R(\mathbf{r},t) = \dot\vartheta \begin{pmatrix} r_z \\ 0 \\ -r_x \end{pmatrix} + \dot\vartheta \begin{pmatrix} \left(\frac{X}{Z}\cos^2\vartheta + \frac{Z}{X}\sin^2\vartheta\right) r_z \\ 0 \\ -\left(\frac{X}{Z}\sin^2\vartheta + \frac{Z}{X}\cos^2\vartheta\right) r_x \end{pmatrix} + \dot\vartheta\left(\frac{X}{Z} - \frac{Z}{X}\right)\frac{\sin(2\vartheta)}{2}\begin{pmatrix} r_x \\ 0 \\ -r_z \end{pmatrix}$$

in the co-rotating frame K':

$$s = \frac{r_x'^2}{X^2} + \frac{r_y'^2}{Y^2} + \frac{r_z'^2}{Z^2}$$

$$\mathbf{v}'(\mathbf{r}',t) = \mathbf{v}'_H(\mathbf{r}',t) + \mathbf{v}'_R(\mathbf{r}',t)$$

$$\mathbf{v}'_H(\mathbf{r}',t) = \begin{pmatrix} H_x r'_x \\ H_y r'_y \\ H_z r'_z \end{pmatrix}$$

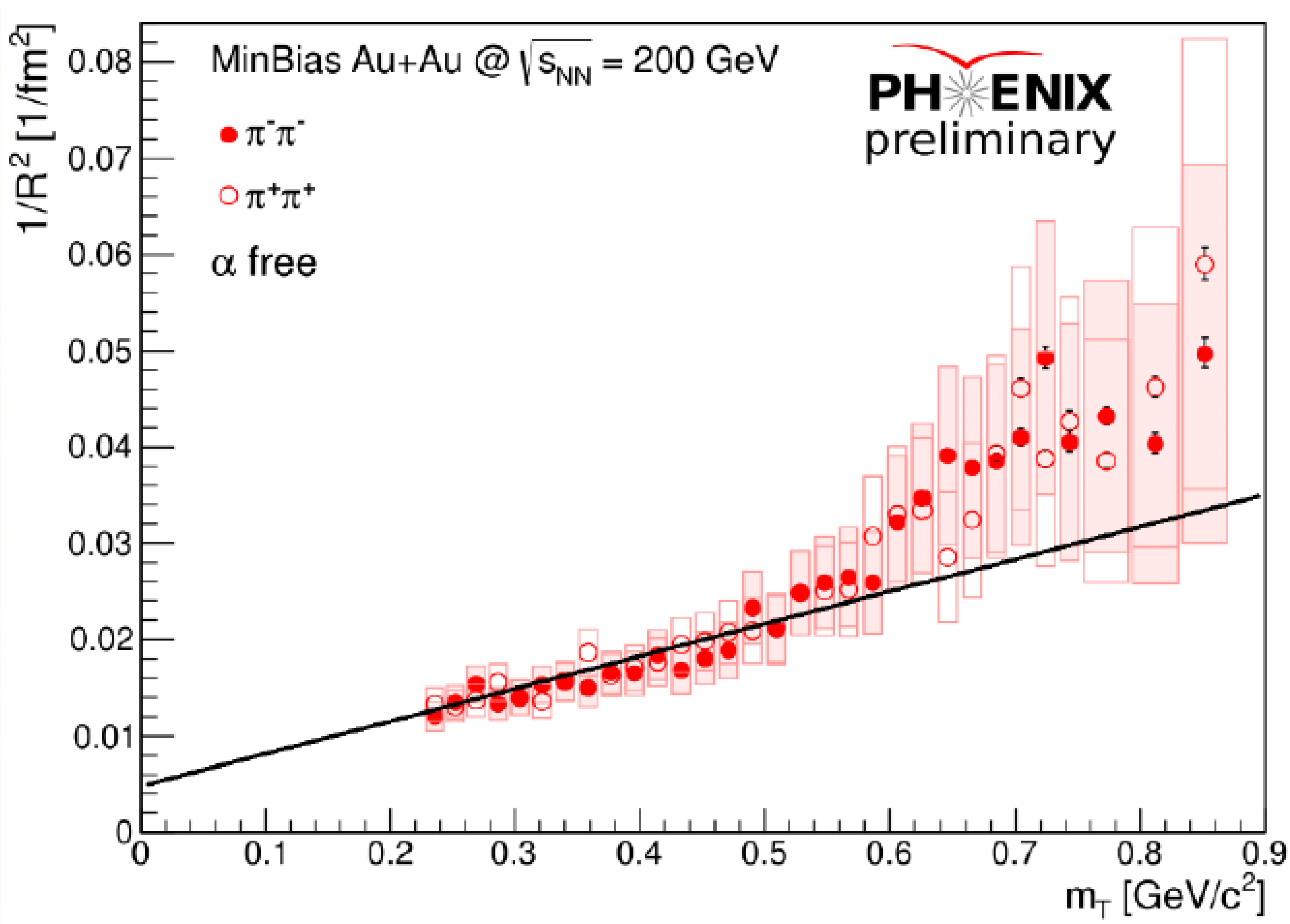
$$\mathbf{v}'_R(\mathbf{r}',t) = \dot\vartheta \begin{pmatrix} \frac{X}{Z} r'_z \\ 0 \\ -\frac{Z}{X} r'_x \end{pmatrix}$$

TABLE II: Summary of the new rotating solution of the hydrodynamical equations, written up both in the intertial, laboratory frame K and in the K' frame, where the coordinate axes rotate together with the (X, Z) axes of a triaxial ellipsoid.

M. Nagy and T. Csörgő,
arXiv:1610.02197

Hydro: mass systematics of HBT radii

- Linear mass dependence



Crossover (CO), lattice QCD EoS

- Energydensity and pressure:

$$\varepsilon = \varepsilon_Q + \varepsilon_H = g_Q \varepsilon + g_H \varepsilon$$
$$p = p_Q + p_H = f_Q p + f_H p$$

- EoS of the crossover:

$$\kappa_{QCD} = f_Q \kappa_Q + f_H \kappa_H$$

- Weights:

$$0 \leq g_Q = \varepsilon_Q/\varepsilon \leq 1 \qquad 0 \leq g_H = \varepsilon_H/\varepsilon \leq 1$$
$$0 \leq f_Q = p_Q/p \leq 1 \qquad 0 \leq f_H = p_H/p \leq 1$$

- Connection between g_i and f_i:

$$g_Q = \frac{\kappa_Q}{\kappa_{QCD}} f_Q \qquad g_H = \frac{\kappa_H}{\kappa_{QCD}} f_H$$

Method of multi-component solutions

- Suppose we know the u^μ velocity field
- The scale variable (s) of the fireball satisfies

$$u^\mu \partial_\mu s = 0$$

- We assume that T and n are known, and a connection between n and n_i:

$$n_i = \frac{n_{i,0}}{n_0} n$$

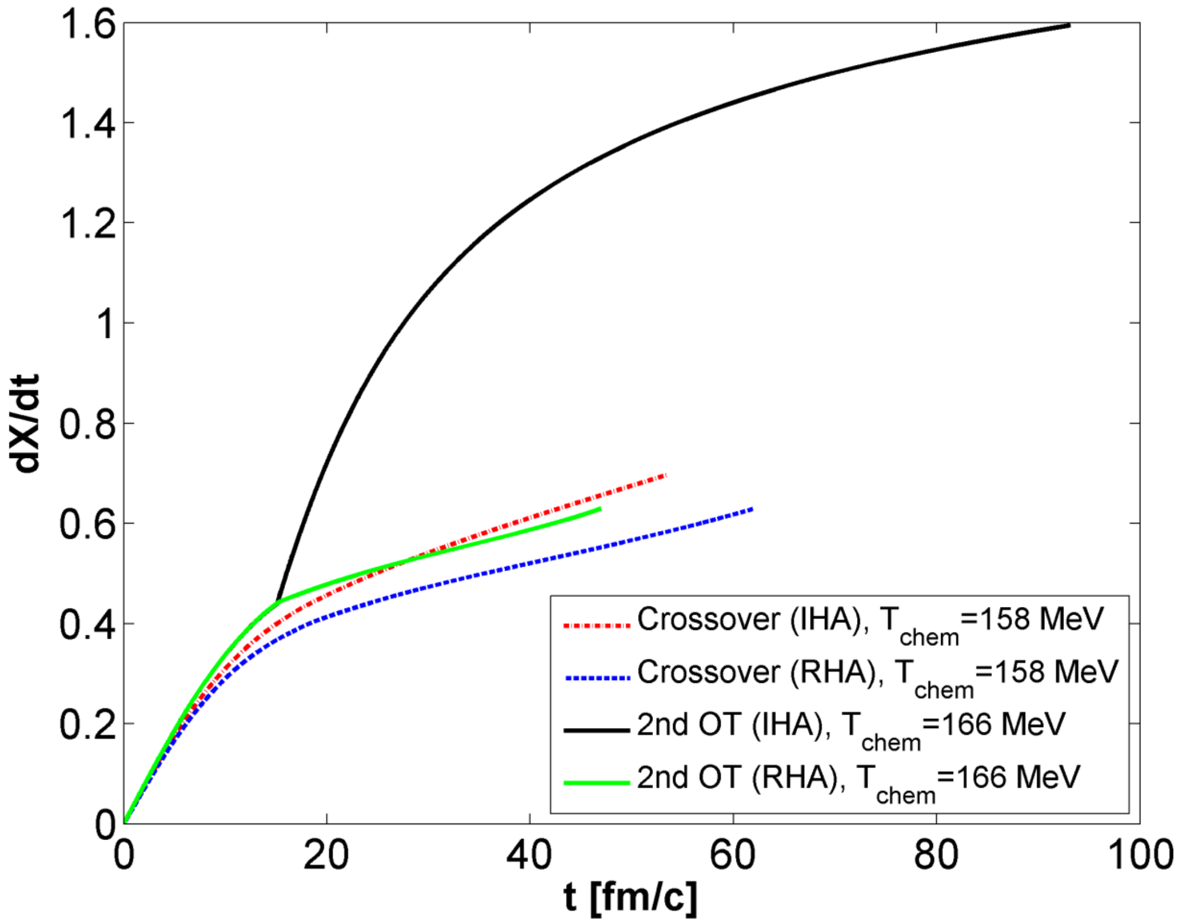
- If $p \propto n$ then the pressure of the MC scenario is

$$p_{MC} = \sum_i p_i = p \frac{n_{i,0}}{n_0}$$

- In this way, if we know a single component solution then we get the multi component generalization with this trivial method

It works for relativistic and nonrelativistic!

Second explosion



Three classes of analytic solutions

Class A: T = T(t) and κ = κ(T)

Gaussian integrals: all observables are analytic.

QM $(T_i \geq T \geq T_{chem})$	**HM** $(T_{chem} > T \geq T_f)$
$\mathbf{v} = (\frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z)$ $\sigma = \sigma_0 \frac{V_0}{V} \exp\left(-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2}\right)$	$\mathbf{v} = (\frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z)$ $n_i = n_{i,c} \frac{V_c}{V} \exp\left(-\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2}\right)$
$(1+\kappa)\left[\frac{d}{dT}\frac{\kappa T}{1+\kappa}\right]\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{1}{1+\kappa(T)}$	$\frac{d(\kappa T)}{dT}\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0$ $X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T}{\langle m \rangle}$

Class B: T = T(t,r), but κ ≠κ(T)

Allows to handle init. temp. inhomogeneities analytically.
But class B excludes the use of lattice QCD EoS.

What about class C?

Three classes of analytic solutions

Class C: T = T(t,r) AND κ = κ (T) but …

κ (T) has to have special form! ***Is it QCD compatible?***

- Analytic solution - QM ($T > T_c$):

$$\kappa_Q(T) = \frac{\kappa_q \left(\frac{T}{T_{chem}}\right)^{1+\kappa_q} + \frac{\kappa_{chem}-\kappa_q}{\kappa_{chem}+1}}{\left(\frac{T}{T_{chem}}\right)^{1+\kappa_q} - \frac{\kappa_{chem}-\kappa_q}{\kappa_{chem}+1}}$$

- Analytic solution - HM ($T < T_c$)

$$\kappa_H(T) = \frac{\kappa_{chem}T_{chem} - \kappa_f T_f}{T_c - T_f} - \frac{\kappa_{chem} - \kappa_f}{T_{chem} - T_f}\frac{T_{chem}T_f}{T}$$

- $\kappa_Q(T)$: EoS of the quark matter
- $\kappa_H(T)$: EoS of the hadronic matter
- $\kappa_{chem} = \kappa(T_{chem})$
- $\kappa_f = \kappa(T_f)$