# New perturbative solutions for relativistic hydrodynamics

Ze-fang Jiang[1], Tamás Csörgő [2, 3],
Máté Csanád[4], Chun-bin Yang[1] .

[1] Central China Normal University, Wuhan, China
[2] Wigner RCP, Budapest, Hungary
[3] EKU KRC, Gyöngyös, Hungary
[4] Eötvös University, Budapest, Hungary

**Parallel Talks Based on**

T. Csörgő, Z. F Jiang, M. Csanád, new perfect fluid solutions presented in the previous talk

# Motivation

the next phase … will focus on detailed investigations of the QGP, "both to quantify its properties and to understand precisely how they emerge from the fundamental properties of QCD"

--The frontiers of nuclear science, a long range plan

*-What is the initial temperature and thermal evolution of the produced matter?*

*-What is the viscosity of the produced matter? ...* http://www.bnl.gov/physics/rhiciiscience/

## Outline

- Accelerating hydrodynamic solution and results (perfect flow).

  T.Cs+M. Csanád+M.I.Nagy, arXiv:0605070, arXv: 0709.3677,
  arXv: 0710.0327, arXiv: 0805.1562...
  Csanád, et. arXiv:1609.07176.

- Accelerating solution with viscosity, and final state discussion.
- Outlook.

# Accelerating solution of relativistic perfect fluid hydrodynamics

The possible non-trivial solution in above case, which are 4 different sets of the parameters $\lambda$, $\kappa$, $d$ and K the possible cases as follows ($\lambda = 1$ is the Hwa-Bjorken solution in 1+1 dimensions.):

CsT+Csanád+Nagy, arXiv:0605070, arXv: 0709.3677, arXv: 0710.0327, arXiv: 0805.1562...

| Case | λ | d | κ | ϕ |
|---|---|---|---|---|
| a.) | 2 | **R** | d | 0 |
| b.) | 1/2 | **R** | 1 | (κ+1)/κ |
| c.) | 3/2 | **R** | (4d-1)/3 | (κ+1)/κ |
| d.) | 1 | **R** | **R** | 0 |
| e.) | **R** | **R** | 1 | 0 |

⬅ accelerating, *d* dimension

⬅ ⬅ *d* dimensional ( T. S. Biró)

⬅ **Hwa-Bjorken, Buda-Lund type**

⬅ Special EoS, but general velocity

In all ideal cases, the velocity field and the pressure is expressed as :

$$v = \tanh \lambda\eta_s, \quad p = p_0\left(\frac{\tau_0}{\tau}\right)^{\lambda d\frac{(\kappa+1)}{\kappa}}\left(\cosh\frac{\eta_s}{2}\right)^{-(d-1)\phi}$$

# The initial energy density estimation

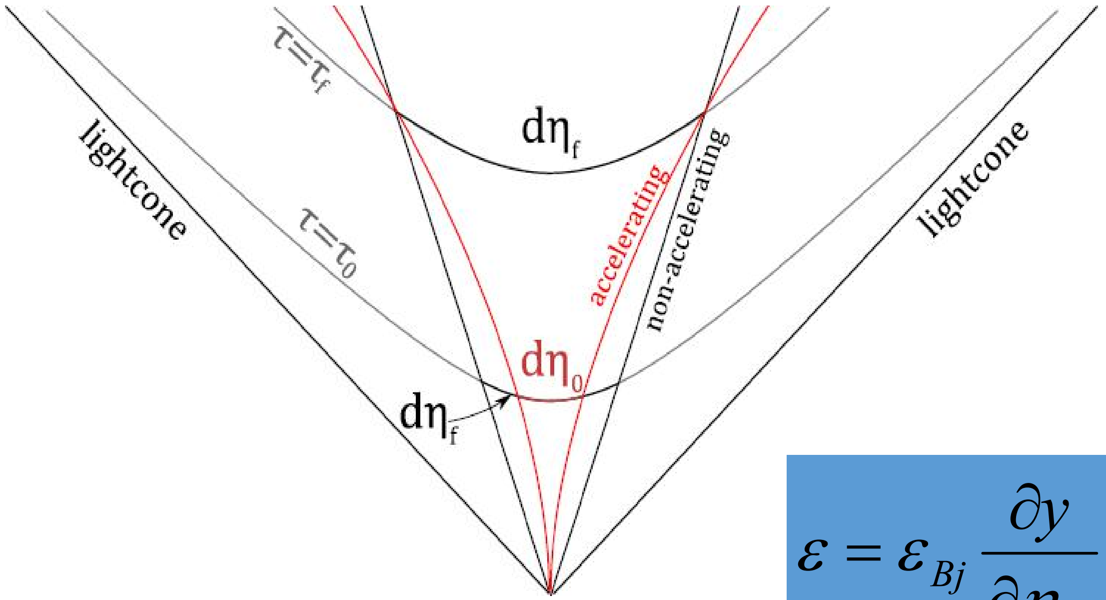
## Rapidity distribution:

$$\frac{dN}{dy} \approx \frac{dN}{dy}\bigg|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}\left[\cosh^{\alpha}\left(\frac{y}{\alpha}\right)-1\right]} \Leftrightarrow \frac{dN}{d\eta} \approx \frac{\overline{p}}{\overline{E}}\frac{dn}{dy} = \frac{\overline{p_T}\cosh\eta}{\sqrt{m^2+\overline{p_T}^2}}\frac{dN}{dy}$$

$$\alpha = \frac{2\lambda - 1}{\lambda - 1}.$$

$$\varepsilon_{Bj} = \frac{1}{R^2\pi\tau_0}\frac{dE}{d\eta} = \frac{\langle E\rangle}{R^2\pi\tau_0}\frac{dn}{d\eta}$$

J. D. Bjroken, Phys. Rev. D 27, 140 (1983)




$$\varepsilon = \varepsilon_{Bj}\frac{\partial y}{\partial \eta_f}\frac{\partial \eta_f}{\partial \eta_i}$$

For an accelerating flow, two modifications for **Initial conditions:** $y \neq \eta$; $\eta_{final} \neq \eta_{initial}$ .

$$\lambda = 1.18 \pm 0.01, \quad \tau_f/\tau_0 = 8 \pm 2 \text{fm}/c,$$

$$\varepsilon_{corr} = (2.0 \pm 0.1)\varepsilon_{Bj} = 10.0 \pm 0.5 \text{GeV/fm}^3.$$

BRAHMS 200 GeV Au+Au data

# Equations of relativistic viscosity hydrodynamic

**The energy-momentum tensor ( viscous hydro):**

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu} u^{\nu}$$

**Shear viscosity tensor:** $\pi^{\mu\nu}$

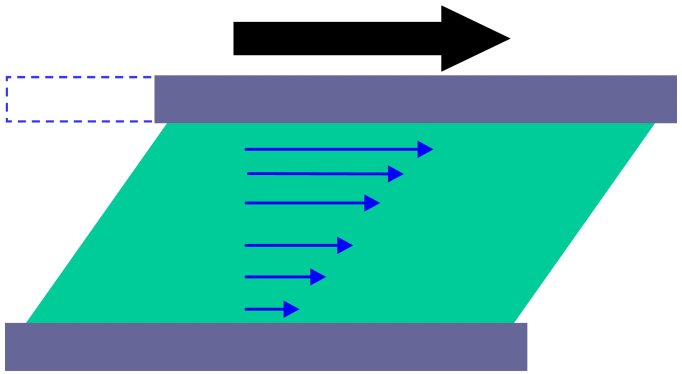
**Bulk viscosity:** $\Pi$

**The fundamental equations of viscous fluid:**

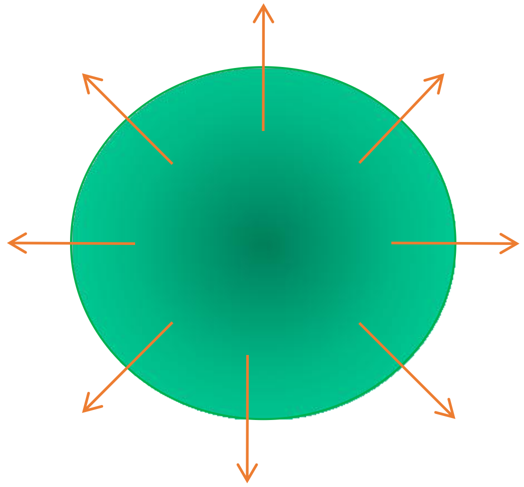
$$\varepsilon = \kappa p \quad \partial_{\nu} T^{\mu\nu} = 0$$

**Assume:** $\mathrm{n} \approx 0 \quad \partial(\mathrm{n}u^{\mu}) = 0$

shear viscosity



bulk viscosity



# Equations of relativistic viscosity hydrodynamic

The **second law of thermodynamics:** $\partial_\mu S^\mu \geq 0$

$$\tau_\pi \Delta^{\alpha\mu} \Delta^{\beta\nu} \dot{\pi}_{\alpha\beta} + \boxed{\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$\tau_\Pi \dot{\Pi} + \boxed{\Pi = -\zeta(\partial \cdot u)} - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

- Israel-Stewart equations.

viscous hydro: near-equilibrium system

The **Navier-Stokes** approximation,

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad \Pi = -\zeta\left(\partial_\rho \mu^\rho\right)$$

The **shear viscosity and bulk viscosity**,

Strongly coupled AdS/CFT prediction: $\eta / s \geq 1/4\pi \approx 0.08$ D.T. Son, et,al. 05

Via lattice calculation: $\zeta / s < 0.015$ H.B. Meyer, et,al. 07 10.3717

# Solutions of viscosity hydrodynamic equation

In Rindler coordinate, the energy equation and Euler equation reduce to:

$$\tau\frac{\partial T}{\partial \tau}+\tanh(\Omega-\eta_s)\frac{\partial T}{\partial \eta_s}+\frac{\Omega'}{\kappa}T=\frac{\Pi_d}{\kappa}\frac{\Omega'^2}{\kappa}\cosh(\Omega-\eta_s),$$

$$\begin{cases}\Omega'=\dfrac{\partial\Omega}{\partial\eta_s}\\ \Pi_d=\left(\dfrac{\zeta}{s}+\dfrac{2\eta}{s}(1-\dfrac{1}{d})\right)\end{cases}$$

$$\tanh(\Omega-\eta_s)\left[\tau\frac{\partial T}{\partial \tau}+T\Omega'\right]+\frac{\partial T}{\partial \eta_s}=\frac{\Pi_d}{\kappa}[2\Omega'(\Omega'-1)+\Omega''\coth(\Omega-\eta_s)]\sinh(\Omega-\eta_s)$$

Bjorken approximation:

$$\Pi_d=0,\ \Omega(\eta_s)=\eta_s$$

The perturbative case ,

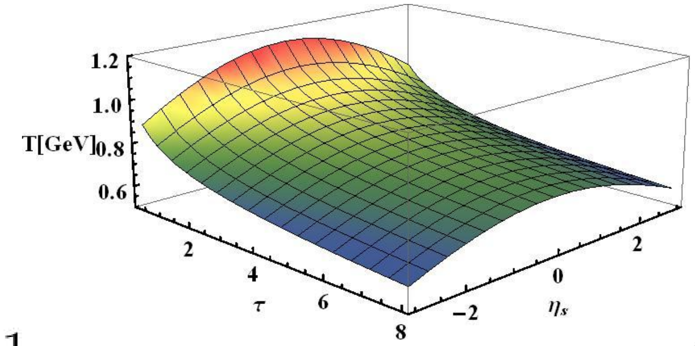
$$\Omega=\lambda\eta_s=(1+\varepsilon)\eta_s,\qquad |\varepsilon|<<1.$$

Up to $\mathcal{O}(\epsilon)$ ,

$$\begin{cases}\tau\dfrac{\partial T}{\partial\tau}+\dfrac{\epsilon+1}{\kappa}T=\dfrac{\Pi_d}{\kappa}\dfrac{2\epsilon+1}{\tau}+\mathcal{O}(\epsilon^2).\\ T_1(\eta_s)\left(1-\dfrac{1}{\kappa}\right)\epsilon\eta_s+\dfrac{\epsilon\Pi_d}{(\kappa-1)\tau_0}\left(1-\dfrac{1}{\kappa}\right)\eta_s+\dfrac{\partial T_1(\eta_s)}{\partial\eta_s}+\mathcal{O}(\epsilon^2)=0\end{cases}$$

# Solutions of viscosity hydrodynamic equation

## The temperature solution:




$$T(\tau,\eta_s) = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{1+\epsilon}{\kappa}}$$
$$\times\left[\exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2] + \frac{R_0^{-1}}{\kappa-1}\left(2\epsilon+\exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2]-(2\epsilon+1)\left(\frac{\tau_0}{\tau}\right)^{\frac{\kappa-\epsilon-1}{\kappa}}\right)\right]$$

One special case of our recent work, presented at BGL 17.

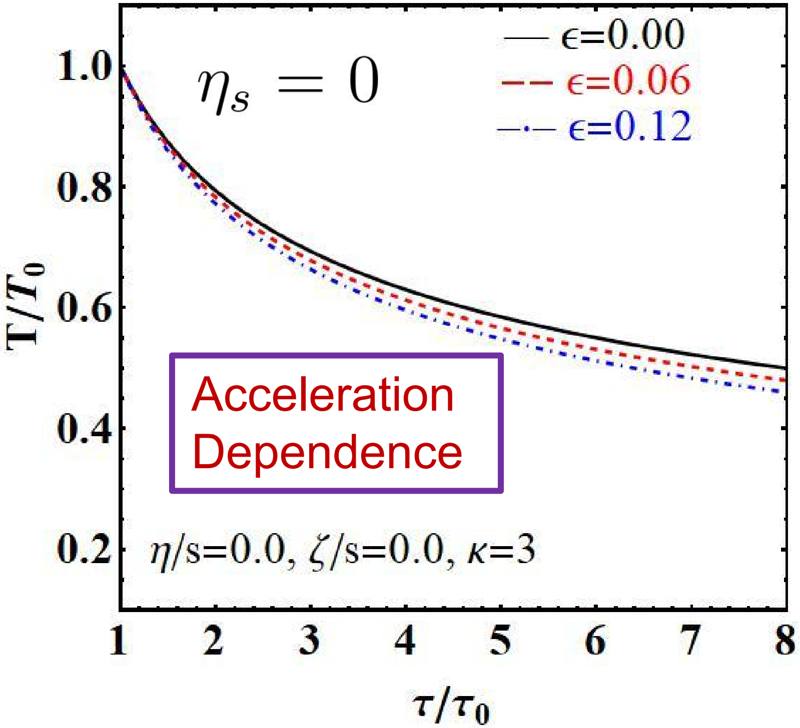
*Contribution from viscous effect*

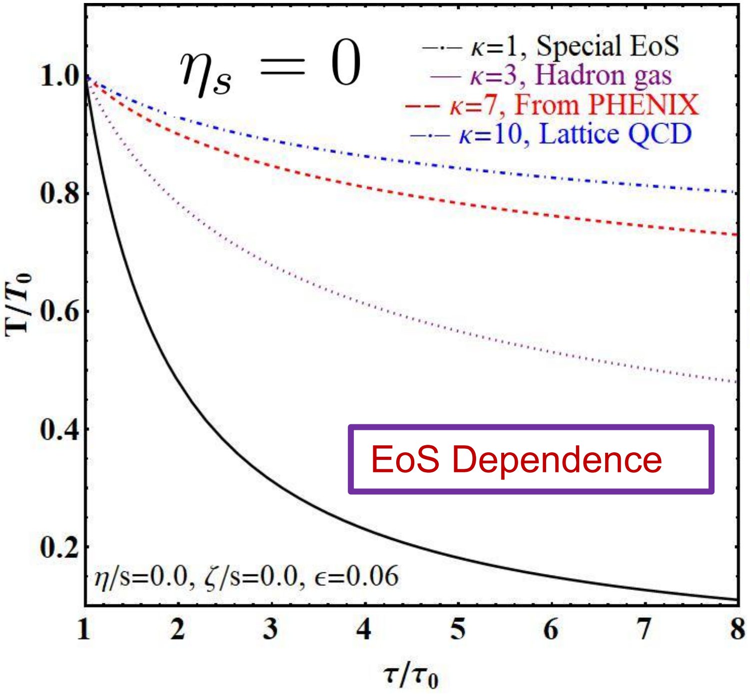
$$R_0^{-1} = \frac{\Pi_d}{T_0\tau_0}$$

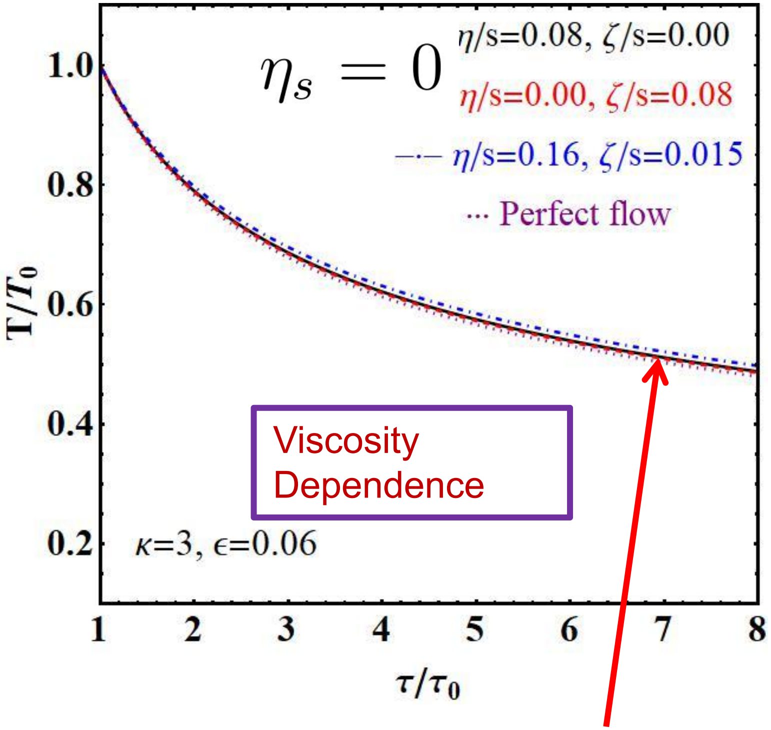
Reyonlds number
[A. Muronga, arxiv: 0309055 ]

A non-zero Reyonlds numbers $R_0^{-1}$ makes cooling rate smaller,
A non-vanishing acceleration $\epsilon$ makes the colling rate is larger.

# Temperature evolution




The acceleration effect is almost fully **compensated** by the viscous contribution.

- Acceleration comes from the pressure gradient, make the cooling ratio larger. [M. I. Nagy, T.Cs., M. Csanád: arXiv:0709.3677v1]
- EOS is one of the important factors that influence the expansion of system. κ=7 comes from [PHENIX, arXiv:nucl-ex/0608033v1 ]
- Viscosity effect make the cooling rate samller. [H. Song, S. Bass, U. Heinz. et, PRL2011]

# The final state observables

The thermal spectrum of particles:

$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{g}{(2\pi)^3}\int p_\mu d\Sigma^\mu f$$

$$p^\mu = (m_T \cosh y, p_T \cos\phi, p_T \sin\phi, m_T \sinh y)$$

For a system out of equilibrium (Boltzmann approximation):

$$f = f_0 + \delta f \quad \begin{cases} f_0 = \exp\left(\frac{\mu(x)}{T} - \frac{p_\mu u^\mu}{T}\right) \\ \delta f = \frac{1}{2(e+p)T^2} f_0 p^\mu p^\nu \left[\pi_{\mu\nu} + \frac{2}{5}\Pi\Delta_{\mu\nu}\right] \end{cases}$$

[K.Dusling and D. Teaney: arXiv: 0710.5932]

Freeze-out condition (Cooper-Frye formula):

$$\left(\frac{\tau_f}{\tau}\right)^{\Omega'-1} \cosh((\Omega'-1)\eta_s) = 1$$

[M. I. Nagy, T.Cs., M. Csanád: arXiv:0709.3677v1]

# The final state observables

## Freeze-out hypersurface:

$$p_\mu d\Sigma^\mu = m_T \tau_f \cosh^{\frac{2-\Omega'}{\Omega'-1}}((\Omega'-1)\eta_s)\cosh(\Omega - y) r dr d\phi$$

[M. I. Nagy, T.Cs., M. Csanád: arXiv:0709.3677v1]

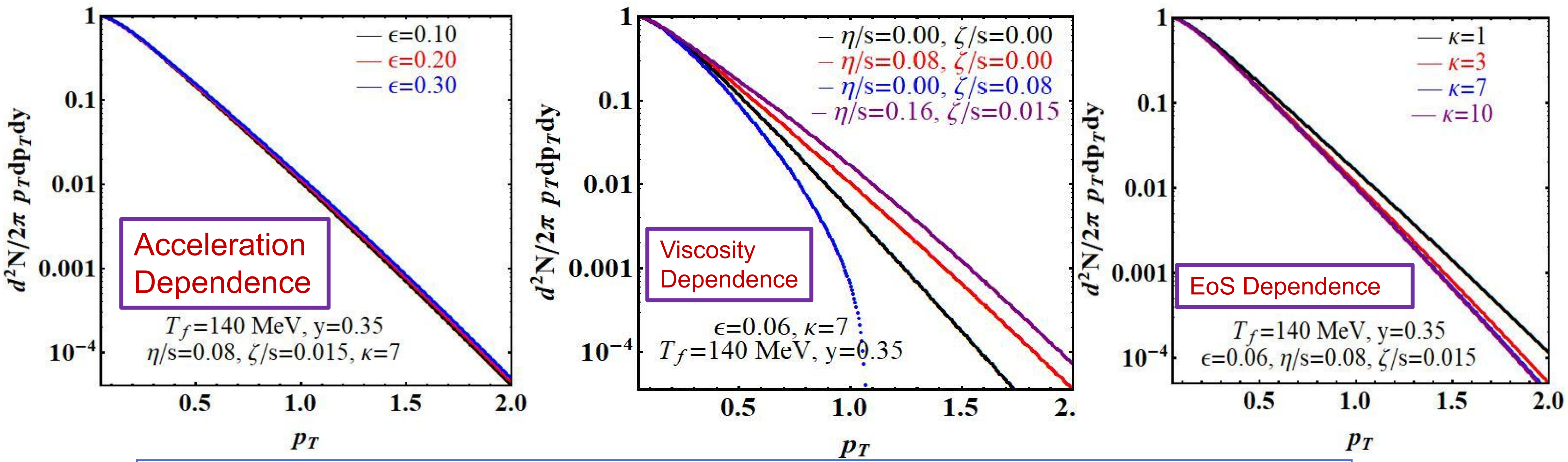
## The transverse momentum distribution:

$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{\pi R_0^2}{(2\pi)^3}\int_{-\infty}^{+\infty} m_T \cosh((\epsilon+1)\eta_s - y)\exp\left[-\frac{m_T}{T(\tau,\eta_s)}\cosh((\epsilon+1)\eta_s - y)\right]$$
$$\times\left(\tau_f \cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) + \frac{1+\epsilon}{T^3(\tau,\eta_s)}\left[\frac{1}{3}\frac{\eta}{s}(p_T^2 - 2m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right.\right.$$
$$\left.\left. - \frac{1}{5}\frac{\zeta}{s}(p_T^2 + m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right]\right) d\eta_s$$

-Temperature solution,
-Boltzmann approximation,
-viscosity, acceleration, mass, rapidity, space-time rapidity...

# The final state observables

## Numerical Results:




Transverse momentum distribution (academic study),
- the first viscous correction play a improtant role,
- EoS and acceleration's effect is not obvious.

# The final state observables

$$p_T \approx m_T \qquad \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

Rapidity distribution

Contribution from perfect fluid

$$\frac{dN}{dy} = \frac{\pi R_0^2}{(2\pi)^3} \int_0^{+\infty} \left\{ \cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) \frac{4\tau_f T^3(\tau,\eta_s)}{\cosh^2((\epsilon+1)\eta_s - y)} + \frac{48(1+\epsilon)T^2(\tau,\eta_s)}{\cosh^4((\epsilon+1)\eta_s - y)} \right.$$
$$\left. \times \left[ \frac{1}{3}\frac{\eta}{s}(1 - 2\sinh^2((\epsilon+1)\eta_s - y)) - \frac{1}{5}\frac{\zeta}{s}\cosh^2((\epsilon+1)\eta_s - y) \right] \right\} d\eta_s$$

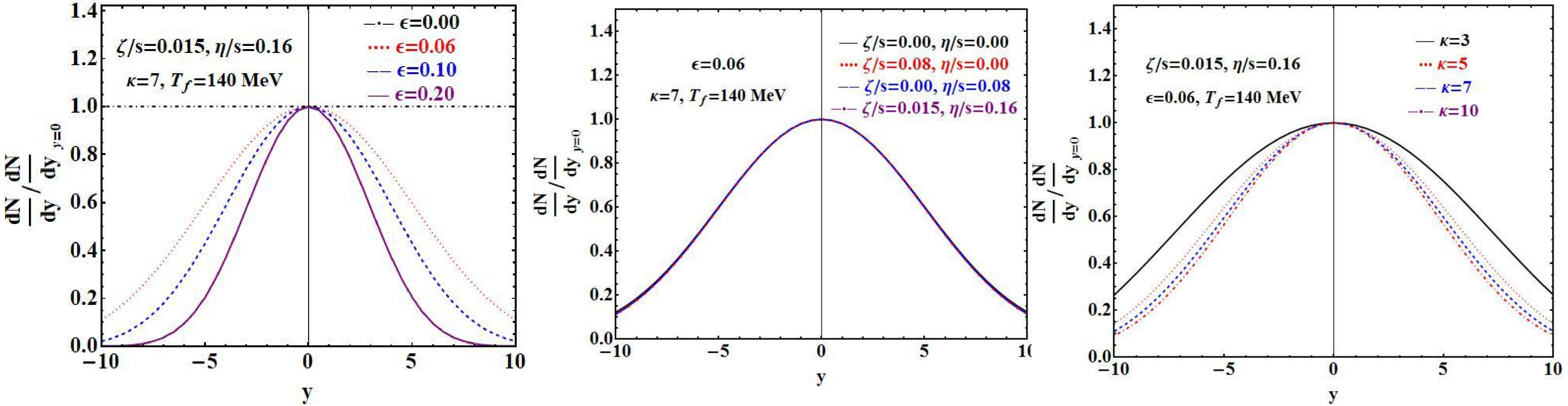
Rapidity distribution (academic study),
- the integral value $error \propto m^3$, this is a good approximation for the particle that mass m is little.

# The final state observables

$$p_T \approx m_T$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

Numerical Results:




Rapidity distribution (academic study),
- at finial state, the dn/dy is effected sensitively by the acceleration parameter and EoS.

# The final state observables

## Pseudo-rapidity distribution

$$\frac{d^2N}{d\eta p_T dp_T} = \frac{p}{E}\frac{d^2N}{dyp_T dp_T} = \sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\frac{d^2N}{dyp_T dp_T}$$

$$m_T^2 = p_T^2 + m^2 \qquad y = \frac{1}{2}\ln\frac{\sqrt{m^2+p_T^2\cosh^2\eta}+p_T\sinh\eta}{\sqrt{m^2+p_T^2\cosh^2\eta}-p_T\sinh\eta}$$

$$\frac{dN}{d\eta} = \frac{\pi R_0^2}{(2\pi)^3}\int_{-\infty}^{+\infty} d\eta_s \int_0^{+\infty} dp_T \sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}} m_T p_T \cosh((\epsilon+1)\eta_s - y)\exp\left[-\frac{m_T}{T(\tau,\eta_s)}\cosh((\epsilon+1)\eta_s - y)\right]$$
$$\times\left(\tau_f\cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) + \frac{1+\epsilon}{T^3(\tau,\eta_s)}\left[\frac{1}{3}\frac{\eta}{s}(p_T^2 - 2m_T^2\sinh^2((\epsilon+1)\eta_s - y)) - \frac{1}{5}\frac{\zeta}{s}(p_T^2 + m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right]\right)$$

Contribution from perfect fluid

Contribution from viscous effect

# a brief summary and outlook

1. We obtained exact accelerating solutions of prefect flow.
2. The perturbative solution with viscous correction are obtained. We obtained the final state distribution.

outlook:
1. 2rd Israel-Stewart problem, Rotation , CME/CVE...
2. Jet/heavy quarkonium disturbance evolution on such a medium background...

# Thank you for your attention