MONET code: evaluation of the dose in Hadrontherapy

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MONET\textsuperscript{1} is a fast and accurate model for the computation of the energy deposition of protons and $^4$He ions in water. MONET\textsubscript{\alpha} is validated with Dev version 2017.0.

\textsuperscript{1} Embriaco, A model for the fast and accurate dose evaluation in hadrontherapy, \textit{PhD thesis}

A. Embriaco | MONET code: evaluation of the dose in Hadrontherapy

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Helium ions

From a physical point of view, $^4\text{He}$ ions seem to fill the gap between protons and $^{12}\text{C}$.

The most important advantages of $^4\text{He}$ beams are a reduction of:

- **multiple Coulomb scattering** compared to protons,
- **energy straggling** compared to protons,
- **projectile fragmentation** than Carbon ions.
Lateral profile

The lateral distribution is calculated as the sum of multiple Coulomb scattering and nuclear interactions:\(^2\):

\[
f_x(x) = W_p f_M(x) + (1 - W_p) \frac{t(x)}{\int t(u) du}
\]

- \(f_M\) Molière distribution
- \(W_p\) primary particle weight
- \(t(x)\) is a Cauchy-Lorentz function:
  \[
t(x) = \frac{1 - A \exp \left[-\frac{x^2}{2b^2\sigma^2}\right]}{\pi b \left(\frac{x^2}{b^2} + 1\right)}
  \]
- \(1 - W_p\) nuclear contribution

\(^2\)Bellinzona et al. 2016 Physics in Medicine and Biology 61 N102
\(^3\)Embriaco et al. 2017 Physica Medica 40 51–58

\(^4\)He ions of 150 MeV/u at \(z=15\) cm
Attenuation of $^4\text{He}$ ions

The attenuation curves of $^4\text{He}$ ions are fitted using an error function multiplied by a linear parametrization:

$$W_p = (\alpha z + \beta) \times \text{erf} \left( \frac{R - z}{\gamma} \right)$$

The energy analyzed are left: $E=100$ MeV/u, middle: $E=150$ MeV/u and right: $E=200$ MeV/u.

For energy of 200 MeV/u, the experimental data are added for the validation of the curve. The black dots are the results of new development version of FLUKA 2018 (Thanks to Giulia Aricò).

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4 Embriaco, A model for the fast and accurate dose evaluation in hadrontherapy, PhD thesis

5 Rovituso et al. 2017 Physics in Medicine and Biology 62(4):1310
The longitudinal profile is characterized by the sum of three effects\textsuperscript{6}:

1. Average energy loss
2. Straggling
3. Nuclear interaction

Average energy loss

We have implemented a new formula of the average energy loss:

$$E_K(z) = -m + \frac{F(z)}{2} + \sqrt{m^2 + \frac{F^2(z)}{4}}$$

where $F(z)$ is:

$$F(z) = p\beta \left(1 - \frac{z}{R}\right)^{k/2}$$

and $k = 1.07$. 
Straggling

The energy deposed at the actual depth $z$ can be obtained by the convolution of the energy loss with a Gaussian function:

$$\hat{E}_K(z) = \int_0^R E_K(\bar{z}) \frac{e^{-(z-\bar{z})^2/2\sigma^2}}{\sqrt{2\pi\sigma}} d\bar{z}$$

Approximation for $\sigma^6^7^8$:

$$\sigma \approx 0.012R^s$$

<table>
<thead>
<tr>
<th>E(MeV/u)</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>0.55</td>
<td>0.71</td>
<td>0.76</td>
</tr>
</tbody>
</table>

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8 Embriaco, A model for the fast and accurate dose evaluation in hadrontherapy, *PhD thesis*
Including the nuclear interaction, the total expression of the deposited energy becomes:

\[ f_z(z) = W_p \hat{E}_K(z) + (1 - W_p) E_N(z) \]

where \( W_p \) is the primary particle weight and \( E_N(z) \) is a **linear parametrization** for the nuclear contribution:

\[ E_N(z) = az + b \]

with the coefficients \( a \) and \( b \) are obtained by fitting FLUKA simulations.
The depth dose curve from FLUKA simulations are analyzed, focusing in the few centimeters after the peak.

Practical range\footnote{Kempe et al. 2008 Medical Physics 35(1) 159–70}: the depth corresponding to the intersection point between the tangent at the inflection point of the descending part of the depth dose profile and the exponentially extrapolated tail.
After Bragg peak

Therefore the **total energy deposition** for $^4$He ions is given by:

$$f_z(z) = \begin{cases} 
W_p \hat{E}_k(z) + (1 - W_p)(a'z + b') & 0 < z < R \\
mz + q & R < z < R_p \\
\exp(p_0 + p_1z) & z > R_p 
\end{cases}$$

where $m$ and $q$ are the parameter of the tangent at the inflection point and the $p_0$ and $p_1$ the parameters of the exponential fit of the tails.
Total energy deposition

Bragg curves including nuclear contributions of $^4\text{He}$ ions calculated with MONET$^{\alpha}$ compared with FLUKA simulations for 200 MeV/u.

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3-dimensional dose distribution

\[ D(x, y, z) = f(z) f(r) \]

\(^4\text{He}\) ions of 150 MeV/u
4He Single Gaussian beam

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Energy 100 MeV/u at depth z=4 cm (Bragg peak at 7.8 cm).
$^4\text{He}$ Single Gaussian beam

Energy 150 MeV/u at depth $z=15$ cm (Bragg peak at 15.9 cm).

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He Single Gaussian beam

Energy 200 MeV/u at depth z=25 cm (Bragg peak at 26.1 cm).

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The field size factor is defined as:

\[
FSF(f) = \frac{D_f}{D_{10}}
\]

where \( f \) assumes the values 4, 6, 8, 10, 12 cm.

Field size factor for \(^4\text{He}\) ions: (left) 100 Mev/u at \( z = 6 \) cm, (middle) 150 MeV/u at \( z = 15 \) cm and (right) 200 MeV/u at \( z = 25 \) cm.
Field size factor test

FSF differences between the model and FLUKA for four fields, three depths and three energies:

<table>
<thead>
<tr>
<th>E(MeV)</th>
<th>100/u</th>
<th>150/u</th>
<th>200/u</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 4 6</td>
<td>5 10 15</td>
<td>10 15 25</td>
</tr>
<tr>
<td>4</td>
<td>0.4% 0.6% 0.5%</td>
<td>0.9% 1.2% 0.9%</td>
<td>1.8% 1.9% 1.3%</td>
</tr>
<tr>
<td>6</td>
<td>0.1% 0.1% 0.1%</td>
<td>0.3% 0.6% 0.3%</td>
<td>1.1% 1.2% 0.4%</td>
</tr>
<tr>
<td>8</td>
<td>0.0% 0.1% 0.0%</td>
<td>0.1% 0.3% 0.2%</td>
<td>0.6% 0.7% 0.3%</td>
</tr>
<tr>
<td>12</td>
<td>0.0% -0.0% -0.0%</td>
<td>-0.1% -0.2% -0.2%</td>
<td>-0.4% -0.6% -0.3%</td>
</tr>
</tbody>
</table>

The FSF tests show a **good evaluation of the low dose contributions** with MONETα code.

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The results of the model for Helium ions are validated with FLUKA.

**Advantages**

- Physical foundation
- only 4 free parameters for energy
- accuracy
- fast calculation time
  (for each depth 2/4 s for single beam/lateral scan)
The results of the model for Helium ions are validated with FLUKA.

**Advantages**
- Physical foundation
- only 4 free parameters for energy
- accuracy
- fast calculation time (for each depth 2/4 s for single beam/lateral scan)

**Developments**
- dose database of interest for the clinical use
- online/in-room fast dose evaluation tool
Backup Slides
Lateral profile of $^4$He ions

Lateral profile: $E=100$ MeV/u $z=7$ cm, middle: $E=150$ MeV/u $z=7$ cm and right: $E=200$ MeV/u $z=25$ cm.
The projected distribution $f_x$ and $f_y$ are uncorrelated, but not independent.\textsuperscript{8}

\[ f(x, y) \neq f_x(x) \times f_y(y) \]

In case of \textit{cylindrical symmetry}:\textsuperscript{9}

Transform $f_x(x)$ into $f(r)$:

\[
h(r) = 2 \int_{0}^{+\infty} f_x(\sqrt{r^2 + y^2}) dy
\]

\[
f(r) = \frac{1}{2\pi r} \frac{dh(r)}{dr}
\]

\textsuperscript{9} Papoulis 1968 \textit{IEEE Transactions on Information Theory}
Let us consider a distribution \( f(x) \) in the xy plane with circular symmetry\(^4\):

\[
f(x, y) = f(r) \quad r = \sqrt{x^2 + y^2}
\]

The marginal distribution \( f_x(x) \) corresponding to \( f(x, y) \) is:

\[
f_x(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \int_{-\infty}^{+\infty} f(r) dy = 2 \int_{0}^{+\infty} f(\sqrt{x^2 + y^2}) dy
\]

Transform \( f_x(x) \) into \( f(r) \):

\[
h(r) = 2 \int_{0}^{+\infty} f_x(\sqrt{r^2 + y^2}) dy
\]

\[
f(r) = \frac{1}{2\pi r} \frac{dh(r)}{dr}
\]

\(^4\)Papoulis 1968 IEEE Transactions on Information Theory
Bragg curves including nuclear contributions of $^4$He ions calculated with MONET$_\alpha$ compared with FLUKA simulations for 100 MeV/u. The bullet point represents the practical range $R_p$. In the inset plot, fragmentation tail is reported in log scale. The practical range as the intersection between tangent at inflection point and the exponential tail is reported in linear scale.

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6 Embriaco, A model for the fast and accurate dose evaluation in hadrontherapy, PhD thesis
Bragg curves including nuclear contributions of $^4\text{He}$ ions calculated with MONET$\alpha$ compared with FLUKA simulations for 150 MeV/u.

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Helium lateral profile

Energy of 100 MeV/u in water at a depth $z = 7$ cm.

Lateral distribution of Helium beam with the different models. The plotted residuals are calculated between the FLUKA simulation and the model.
Helium lateral profile

Energy of 150 MeV/u in water at a depth $z = 10$ cm.

Lateral distribution of Helium beam with the different models. The plotted residuals are calculated between the FLUKA simulation and the model.
Helium lateral profile

Energy of 200 MeV/u in water at a depth $z = 15$ cm.

Lateral distribution of Helium beam with the different models. The plotted residuals are calculated between the FLUKA simulation and the model.