Towards an effective model for low-energy deuteron interactions in FLUKA

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Deuteron interactions

▶ Of interest both from a fundamental and from an applied point of view.
▶ Li(d,xn), Be(d,xn), C(d,xn), etc.: effective source of high-intensity n beams.
▶ Applications:

High n yield from d-induced reactions → fissionable material.

Li(d,xn) as n source for material-damage studies.

d-induced production of $^{99}$Mo for medical applications.

▶ FLUKA: missing a dedicated model for low-energy deuteron interactions ($\lesssim 100 – 150$ MeV/n).

In this talk: brief summary of efforts towards including deuteron interactions in FLUKA’s radiation transport model.
Interaction mechanisms at low energy ($\lesssim 150$ MeV/n)

Deuteron-nucleus interaction

Elastic break-up

Stripping to bound state

Stripping to continuum

$Z, A$

$Z, A$

$Z + 1, A + 1$

g.s., 1$^{st}$, 2$^{nd}$

$Z', A'$
Typical neutron spectrum from (d,xn)

Fig. 4. (Color online) Comparison of the calculated DDXs of $^7\text{Li}(d, xn)$ at 25 MeV with the experimental data for different neutron emission angles. The experimental data (dot) is from Ref. 20.

Model requirements for MC

▶ Activation / material damage: we need to keep track of the final state of the recoiling target nucleus.

▶ Strict control of kinematics: ensure energy and momentum conservation interaction by interaction. Interesting for break-up processes (2 → 3 particles).

▶ The simulation will ask for deuteron cross sections at arbitrary deuteron energy (≲ 100 – 150 MeV/n) and target combination $Z, A$.

▶ We cannot afford to have tabulated $n$-differential cross sections for just this process: parametrize the cross section as far as possible (unavoidably sacrificing a bit of detail).
Elastic break-up


Elastic break-up as a direct process

- d splits into n and p, leaving the recoiling target nucleus in its ground state.
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- Owing to the deuteron’s low binding energy (2.225 MeV), one may describe elastic break-up as a direct single-step process (first-order perturbation theory).
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Before elastic break-up

\[ \hat{H}_i = \hat{T}_{d-A} + \hat{V}_{d-A} \]

After elastic break-up

\[ \hat{H}_f = \hat{T}_{p-A} + \hat{V}_{p-A} + \hat{T}_{n-pA} + \hat{V}_{n-pA} + \hat{V}_{n-p} \]

\[ \mathcal{T} = \langle \chi_p^{(-)}(\mathbf{p}_{p-A}, \mathbf{r}_{p-A}) \chi_n^{(-)}(\mathbf{p}_{n-pA}, \xi \mathbf{r}_{n-pA}) | \hat{V}_{n-p} | \phi_d(\mathbf{r}) \chi_d^{(+)}(\mathbf{p}_{d-A}, \mathbf{r}_{d-A}) \rangle \]
Elastic break-up as a direct process

- *d* splits into *n* and *p*, leaving the recoiling target nucleus in its ground state.
- Owing to the deuteron’s low binding energy (2.225 MeV), one may describe elastic break-up as a direct single-step process (first-order perturbation theory).

Before elastic break-up

\[ \hat{H}_i = \hat{T}_{d-A} + \hat{V}_{d-A} \]

After elastic break-up

\[ \hat{H}_f = \hat{T}_{p-A} + \hat{V}_{p-A} + \hat{T}_{n-pA} + \hat{V}_{n-pA} + \hat{V}_{n-p} \]

\[ T_{ZR} = D_0 \langle \chi_p^- (p_{p-A}, r) \chi_n^- (p_{n-pA}, \xi r) | \chi_d^+ (p_{d-A}, r) \rangle \]

\[ D_0 \sim 124 \text{ MeV fm}^{3/2} \]
Elastic break-up as a direct process

▶ d splits into n and p, leaving the recoiling target nucleus in its ground state.
▶ Owing to the deuteron’s low binding energy (2.225 MeV), one may describe elastic break-up as a direct single-step process (first-order perturbation theory).

\[
\begin{align*}
\hat{H}_i &= \hat{T}_{d-A} + \hat{V}_{d-A} \\
\hat{H}_f &= \hat{T}_{p-A} + \hat{V}_{p-A} + \hat{T}_{n-pA} + \hat{V}_{n-pA} + \hat{V}_{n-p}
\end{align*}
\]

\[
T^{\text{DWBA}}_{\text{post}} = \langle e^{i\mathbf{k} \cdot \mathbf{r}} | \hat{V}_{n-p}(\mathbf{r}) | \phi_d(\mathbf{r}) \rangle \langle \chi_{p_{p-A}}^{(-)} \chi_{p_{n-pA}}^{(-)} | \chi_{p_{d-A}}^{(+)} \rangle
\]

Vertex func. / form fact. [Zadro2002] Dynamics
DWBA differential cross section

The differential cross section for elastic deuteron break-up is:

\[
\frac{d^4 \sigma}{dE_n d\Omega_n d\Omega_p} = J \frac{2\pi}{\hbar^2 k_d A} \rho(E_n, \Omega_n, \Omega_p) |T_{\text{DWBA}}| ^2
\]

Routine:

- Scattering geometry determined by 4 degrees of freedom: \( E_n, \theta_n, \theta_p, \varphi_p \rightarrow \) close kinematics.
- Specify \( V(r) \) felt by p, n, d \( \rightarrow \) find unbound solutions for 3 respective uncoupled Schrödinger eqs, giving a conceptually simpler description than more elaborate CDCC approaches.
- Evaluate matrix element and differential cross section.
Elastic Coulomb break-up

- $V_{d-A}$ and $V_{p-A}$ are Coulomb potentials, while $V_{n-A} = 0$.
- Permissible for deuteron energies $\ll$ Coulomb barrier.
- Wave functions are analytical (Coulomb functions) and so is the matrix element (!).
- Useful in practice:
  - Transition amplitudes for more realistic OPM require use of partial-wave series, the convergence of which is greatly accelerated by subtraction of Coulomb amplitude.
  - Even at higher energies, elastic break-up in forward emission appears Coulomb dominated (Tostevin1998, Okamura1998 140-MeV measurements).

Own code (dnr.f90) to evaluate all shown expressions
Coulomb elastic break-up at 12 MeV

Why tipped towards “fast” protons?

- Deuteron “climbs” Coulomb potential.
- Velocity reduced.
- Break-up.
- Neutron carries on with low energy.
- Proton repelled (accelerated) as it moves away from the nucleus.

Coulomb elastic break-up for 140-MeV d on $^{12}\text{C}$ ($\theta_n = 0^\circ$)

- Various dashed curves: calculations at given scatt. geometry.
- Dots: experimental data (Okamura1998 RIKEN)
- Solid curve: integration over detector solid angles.
- Coulomb break-up unexpectedly appears to dominate forward emission (140 MeV $\gg$ typical Coulomb barriers).

Tostevin1998
Coulomb elastic break-up for 140-MeV d on $^{12}\text{C}$ ($\theta_n = 0^\circ$)

Why double peak at fwd emission $\theta_n = \theta_p = 0$? Okamura1998:

Dipole approx:

$$\mathcal{T}_{\text{DWBA prior}} \sim \left\langle \chi_A^* \left| \frac{k_{np} r_{dA}} {r^3_{dA}} \right| \chi_A \right\rangle$$

Vanishes at fwd emission for $E_n \approx E_p \approx E_d / 2$ because $k_{np} \approx 0$. 
Coulomb elastic break-up for 140-MeV d on $^{12}$C ($\theta_n = 0^\circ$)

**Tostevin1998**

\[ \frac{d^3\sigma}{dE_p d\Omega_p d\Omega_n} (\text{mb MeV}^{-1} \text{sr}^{-2}) \]

\[ E_p (\text{MeV}) \]

\[ \theta_p = 0^\circ, 1^\circ, 2^\circ \]

**dnr.f90 (Tostevin1998 dashed)**

140-MeV deuterons on $^{12}$C

\[ \frac{d\sigma}{dE_p d\Omega_p d\Omega_n} (\text{mb MeV}^{-1} \text{sr}^{-2}) \]

\[ \theta_p = 0^\circ, 1^\circ, 2^\circ \]

- Right: our numbers (solid), Tostevin’s digitized (dashed).
- Good agreement.
- We have the 3-body kinematics reasonably under control.
Coulomb+nuclear optical potential model (OPM)

OPM param. for d-nucleus interaction: Han2006*.
OPM param. for nucleon-nucleus interaction: Koning2003**.

\[ V(r) = V_{\text{sphere}}(r) + V_{\text{WS}}(r) + i[W_{\text{WS}}(r) + S_{\text{WS}}(r)] + V_{\text{SO}}(r) \]

Wavefunctions no longer known analytically.
Distorted waves (partial-wave expansion)

- Since the potential is central: partial-wave expansion

\[
\chi_{p-A}^{(\pm)}(r) = \frac{4\pi}{k_{p-A}r} \sum_{\ell=0}^{\infty} i^{\ell} e^{\pm i\delta_{\ell}} P_{E_{p-A}\ell}(r) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{r}) Y_{\ell m}^{*}(\hat{k}_{p-A})
\]

\[
\chi_{n-pA}^{(\pm)}(r) = \frac{4\pi}{k_{n-pA}r} \sum_{\ell=0}^{\infty} i^{\ell} e^{\pm i\delta_{\ell}} P_{E_{n-pA}\ell}(r) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{r}) Y_{\ell m}^{*}(\hat{k}_{n-pA})
\]

\[
\chi_{d-A}^{(\pm)}(r) = \frac{4\pi}{k_{d-A}r} \sum_{\ell=0}^{\infty} i^{\ell} e^{\pm i\delta_{\ell}} P_{E_{d-A}\ell}(r) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{r}) Y_{\ell m}^{*}(\hat{k}_{d-A})
\]

- \(P(r)\) are unbound solutions of the respective Schrödinger eqs:

\[
\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + U_{i-A}(r) + \frac{\hbar^2}{2\mu} \frac{\ell(\ell + 1)}{r^2} \right] P_{E_{i\ell}}(r) = E_{i} P_{E_{i\ell}}(r), \quad i = \{n, p, d\}
\]
We had at hand the state-of-the-art RADIAL2017:

RADIAL: a Fortran subroutine package for the solution of the radial Schrödinger and Dirac wave equations

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For a Coulomb + finite range central potential:

- Radial functions $P_{E\ell}(r)$,
- Phase shifts $\tilde{\delta}_\ell = \Delta_\ell + \delta_\ell$.

Strongly tempted to evaluate the transition amplitude:

$$\mathcal{T}^{\text{DWBA}}_{\text{post}} \sim \langle \chi_{p_{p-A}}^{(-)} \chi_{p_{n-pA}}^{(-)} | \chi_{p_{d-A}}^{(+)} \rangle$$
The DWBA matrix element for elastic break-up

After a bit of algebra/patience:

\[ T_{\text{post}}^{\text{DWBA}} = (4\pi)^2 \frac{D_0}{k dk_n k_p} \sum_{\ell \ell' \ell''} \sqrt{(2\ell + 1)(2\ell' + 1)} \]

\[ \times i^{\ell'' - \ell - \ell'} e^{i(\delta_\ell + \delta_\ell' + \delta_\ell'')} e^{i(\Delta_\ell + \Delta_\ell' + \Delta_\ell'')} \]

\[ \times \langle \ell\ell'00|\ell''0 \rangle \left[ \sum_m \langle \ell\ell'm, -m|\ell''0 \rangle Y_{\ell m}(\hat{k}_n)Y_{\ell' - m}(\hat{k}_p) \right] \]

\[ \times \int_0^\infty dr \frac{1}{r} P_{E_{n\ell}}(r) P_{E_{p\ell'}}(r) P_{E_d\ell''}(r). \]

▸ A term describing angular dependency

▸ An integral over unbound radial functions

... to be added over \( \ell_n, \ell_p, \ell_d \).
Radial integrals (Vincent1970, Davies1988)

Radial integral:

$$\int_{0}^{\infty} \frac{dr}{r} P_{E_n\ell}(r) P_{E_p\ell'}(r) P_{E_d\ell''}(r).$$

Considering $P_{E\ell}(r) \sim H^{\pm}(r)$ (exp decay in complex plane),

$$\oint dz \ f(z) = 2\pi i \text{Res}[f]$$

(0 in our case!)
Matrix element involves sum over $\ell_p, \ell_n, \ell_d$.

Coulomb $\rightarrow$ long range $\rightarrow$ large $\ell$ contribute (fig a).

Slow convergence.

Trick: subtract Coulomb amplitude (fig b).

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**Okamura1998**

![Amplitudes](image)

FIG. 13. Amplitudes of the partial $T$ matrices summed over the deuteron partial waves plotted as a function of $\ell_p$ and $\ell_n$ for the $^{118}$Sn$(d,pn)^{118}$Sn$_{gs}$ reaction at $E_d=56$ MeV, $\theta_p^L=+15^\circ$, $\theta_p^L=-15^\circ$ (opposite side of the proton in a coplanar geometry), and $E_p^L=30$ MeV. (a) $T_{2R}$, (b) $T_{2R}^c$, and (c) $T_{2R}^d$ correspond to Eqs. (A1), (A2), and (A3), respectively.
Coulomb vs Coulomb+nuclear break-up


12-MeV deuterons on $^{197}$Au

\[
\frac{d\sigma}{dE_p d\Omega_p d\Omega_n} \text{ (mb MeV}^{-1} \text{ sr}^{-2})
\]

Proton lab kinetic energy (MeV)

$\theta_p = 50^\circ$

$\theta_n = 20^\circ$

$\phi_n = 0^\circ$

exp
Coulomb vs Coulomb+nuclear break-up


Why the drop?

- Potential less repulsive.

12-MeV deuterons on $^{197}$Au

\[
\frac{d\sigma}{dE_p d\Omega_p d\Omega_n} \text{ (mb MeV}^{-1} \text{ sr}^{-2})
\]

- Proton lab kinetic energy (MeV)
- Distance to nuclear center (fm)

\[\frac{d\sigma}{dE_p d\Omega_p d\Omega_n} \text{ (mb MeV}^{-1} \text{ sr}^{-2})\]

\[\theta_p = 50^\circ, \theta_n = 20^\circ, \varphi_n = 0^\circ\]

\[E_d = 12 \text{ MeV}\]

V(r) (MeV)

Coulomb

Coulomb+nuclear (OPM)
56-MeV deuterons on various targets

Okamura1998 - Integrated over proton energies

![Graphs showing the differential cross sections for \( ^{12}\text{C}(d,pn)^{12}\text{g.s.} \), \( ^{51}\text{V}(d,pn)^{51}\text{g.s.} \), and \( ^{118}\text{Sn}(d,pn)^{118}\text{g.s.} \) reactions.]
56-MeV deuterons on various targets

Okamura1998 - Integrated over proton energies

$^{12}$C

$^{51}$V

$^{118}$Sn

A Coulomb+nuclear potential is of course more realistic than a naked point-charge Coulomb potential.
DWBA elastic break-up cross section (1/3)

- Reasonable agreement at 4-differential level.
- Encouraged to integrate over $E_n, \theta_n, \theta_p, \varphi_p$ to obtain the integrated elastic break-up cross section.
Reasonable agreement at 4-differential level.

Encouraged to integrate over $E_n, \theta_n, \theta_p, \varphi_p$ to obtain the integrated elastic break-up cross section.


Reasonable agreement with CDCC calculations.
Reasonable agreement with CDCC calculations.

Experimental data to gauge against (?).
DWBA elastic break-up cross section (3/3)

- $\sigma_{\text{EBU}}$ tabulated for various target nuclei and various $E_d$:

![Graph showing elastic break-up cross section vs. deuteron lab kinetic energy for different nuclei.

- Good agreement for targets lighter than $\sim^{27}\text{Al}$.
- $\sim$Disagreement for heavier targets at 20-60 MeV.
- Finite-range corrections, non-locality corrections (?).
Preparing for use in FLUKA

- We cannot carry a huge database of $d\sigma/(dE_nd\Omega_n d\Omega_p)$ for various $E_d, Z, A$.
- 20 abscissas for each: $\sim 0.5$ Gb.

...so we are forced to parametrize:

- $\sigma_{EBU}$ for a grid of $Z, E_d$
- $d\sigma/dE_n$ for a grid of $Z, E_d$
- $d\sigma/(dE_n d\Omega_n)$ for a grid of $Z, E_d, E_n$
- $d\sigma/(dE_n d\Omega_n d\Omega_p)$ for a grid of $Z, E_d, E_n, \theta_n$
\[ \frac{d\sigma_{EBU}}{dE_n} \] for \(^{211}\text{Rn}\)

As a parametrization example:
Unit-height-normalized distribution of neutron energies \(T_n\) in units of maximum kinematically allowed \(T_{n,max}\):

\[ Z=86, \ E=10 \text{ MeV} \]
\[ d\sigma_{EBU}/dE_n \] for \(^{211}\text{Rn}\)

As a parametrization example:

Unit-height-normalized distribution of neutron energies \( T_n \) in units of maximum kinematically allowed \( T_{n,\text{max}} \):

\[ Z=86, \ E=15 \ \text{MeV} \]
\( \frac{d\sigma_{EBU}}{dE_n} \) for \(^{211}\text{Rn}\)

As a parametrization example:
Unit-height-normalized distribution of neutron energies \( T_n \) in units of maximum kinematically allowed \( T_{n,\max} \):

\[ \frac{T_n}{T_{n,\text{max}}} \]

\[ Z=86, \ E=25 \ \text{MeV} \]
\( \frac{d\sigma_{EBU}}{dE_n} \) for \(^{211}\text{Rn}\)

As a parametrization example:
Unit-height-normalized distribution of neutron energies \( T_n \) in units of maximum kinematically allowed \( T_{n,max} \):

![Graph showing unit-height normalized distribution of neutron energies](image)

- \( Z=86, E=50 \text{ MeV} \)
- ebudtn.f
- DWBA (norm)
\( \frac{d\sigma}{dE_n} \) for \(^{211}\)Rn

As a parametrization example:
Unit-height-normalized distribution of neutron energies \( T_n \) in units of maximum kinematically allowed \( T_{n,max} \):

\[
\text{Unit-height normalized distribution} \quad \frac{T_n}{T_{n,max}} \\
Z=86, E=75 \text{ MeV}
\]
$d\sigma_{EBU}/dE_n$ for $^{211}$Rn

As a parametrization example:
Unit-height-normalized distribution of neutron energies $T_n$ in units of maximum kinematically allowed $T_{n,max}$:

![Graph showing unit-height normalized distribution of neutron energies for $^{211}$Rn]
$d\sigma_{EBU}/dE_n$ for $^{211}\text{Rn}$

As a parametrization example:
Unit-height-normalized distribution of neutron energies $T_n$ in units of maximum kinematically allowed $T_{n,max}$:
\( \frac{d\sigma_{EBU}}{dE_n} \) for \(^{211}\text{Rn}\)

As a parametrization example:

Unit-height-normalized distribution of neutron energies \( T_n \) in units of maximum kinematically allowed \( T_{n,max} \):

![Graph showing unit-height-normalized distribution of neutron energies for Z=86, E=200 MeV]
Sampling of deuteron elastic break-up events

In the simulation, for a deuteron with given energy and a target $Z, A$:

- Determine elastic break-up cross section.
- Sample neutron energy from parametrized $d\sigma/dE_n$.
- Sample neutron polar angle from $d^2\sigma/dE_n/d\theta_n$.
- Sample proton polar angle from $d^3\sigma/dE_n/d\theta_n/d\theta_p$.
- Sample proton azimuthal angle from $d^4\sigma/dE_n/d\theta_n/d\Omega_p$.
- Construct $p_n, p_p, p_t$ and sample a global rotation angle homogeneously around direction of incidence.
Summary

- We have set up a simple DWBA scheme to calculate the 4-differential cross section for elastic deuteron break-up (Coulomb+nuclear OPM).
- Comparisons at 4-differential level have been encouraging.
- Integrated cross section in reasonable agreement with similar calculations.
- Have had to disregard spin-orbit coupling terms in the potential.
- Have had to suppress nuclear transition amplitude at fwd emission to reproduce the apparent Coulomb dominance seen experimentally.
- We have parametrized partial integrals of the differential cross section and adapted them for sampling of elastic break-up events in FLUKA.
- Ahead: stripping to bound states, stripping to the continuum.
Outlook

Stripping to bound states:

- Involves nuclear structure (spectroscopic factors).
- We will probably not have a systematic approach, but at least for selected relevant reactions.

\[ P = \frac{\sigma_R - \sigma_{bnd} - \sigma_{ebu}}{\sigma_R} \]

Evaluate \( \sigma_R, \sigma_{ebu}, \sigma_{bnd,j} \) for \( E_d \) between threshold and 200 MeV

Decide channel with \( P \)

\[ P = \frac{\sigma_{bnd}}{\sigma_R} \]

Stripping to bound state

\[ P = \frac{\sigma_{ebu}}{\sigma_R} \]

Elastic break-up

Sample \( \Omega \sim d\sigma_j/d\Omega \)

Sample \( \Omega_p \) and close kinematics

Boost back to lab

Serber/INC

Sample \( E_n, \theta_n \)
Additional material
The vertex function $F(x)$

- **Zero-range approximation (ZRA):**

$$V_{nc}(r) \Phi_p^{lm}(r) = D_0 \delta(r), \quad D_0 \sim 123.5 \text{ MeV fm}^{3/2}$$

$$F(x) = D_0$$

- **Baur-Trautmann approx (BTA):** $x = q_{n-pA}$ (wrt CM of p-d)

$$F(x) = -V_0 \frac{(\alpha + \mu)^{1/2}(2\alpha + \mu)^{3/2}}{(\alpha + \mu)^2 + x^2}$$

- **Asymptotic-momentum approximation (AMA) [1], AKA adiabatic model [2]:** $x = |q_v - q_pm_v/(m_v + m_c)|$

Zadro2002: ZRA gives best results, BTA underestimates. We find BTA underestimates too, ZRA gives a bit too high cross sections compared to CDCC, AMA is more reasonable.

Finite-range correction

\[ \mathcal{T}_{\text{post}}^{\text{DWBA}} \sim \langle \chi_{p_{-A}}(r) \chi_{p_{n-pA}}(r) | \Lambda(r) \chi_{p_{d-A}}(r) \rangle \]  \hspace{1cm} (1)

\[ \Lambda(r) = \left[ 1 - \frac{2\mu_{pn}}{\hbar^2} R^2 (V_p(r) + V_n(\xi r) - V_d(r) + \epsilon_d) \right]^{-1} \]  \hspace{1cm} (2)

\[ R = 0.667 \text{ fm} \]  \hspace{1cm} (3)
Non-locality correction

\[ \chi(r) \rightarrow \chi(r) \exp \left[ \frac{\beta_n^2}{8} \frac{2\mu}{\hbar^2} V_{\text{nuc}}(r) \right], \quad \beta_{n,p} = 0.85 \text{ fm}, \quad \beta_d = 0.54 \text{ fm} \]
Effect of FRC+NLC

12-MeV d on $^{197}$Au

![Graph showing the effect of FRC+NLC on 12-MeV deuteron on $^{197}$Au. The graph plots the variation of a parameter $P(r)$ with distance $r$ in femtometers (fm), showing two curves: one for FRC+NLC and another for $P(r)$, $L=0$. The axes range from 0 to 100 fm along the x-axis and from -1.5 to 2 on the y-axis.](image-url)
DWBA prior form matrix element for elastic break-up

\[ \mathcal{T} = \langle \chi_d^- \phi_{np} | U_n(r_n) + U_p(r_p) | \phi d \chi_d^+ \rangle \]  

(5)
Reaction cross section systematics

Nordsieck fortunately solved the matrix-element integral:

\[ T_{ba,0}^{DWBA+,C} = 2\pi D_0 e^{\pi(\eta_d-\eta_p)/2} \Gamma(1+i\eta_p)\Gamma(1+i\eta_d)\alpha^{-2} \left( \frac{\gamma}{\alpha} \right)^{i\eta_d} \left( \frac{\gamma + \delta}{\gamma} \right)^{i\eta_p} \]

\[
\left\{ \begin{array}{l}
\phantom{-}2F_1(1+i\eta_d, -i\eta_p, 1, z)\alpha \left[ \gamma^{-1} k_d (\eta_d - \eta_p) + (\gamma + \delta)^{-1} \eta_p (k_d + k_p) \right] \\
- \phantom{-}2F_1(2 + i\eta_d, 1 - i\eta_p, 2, z)(1 + i\eta_d)\eta_p (\gamma + \delta)^{-1} [k_p (\gamma + \alpha) - k_d \beta - (k_d + k_p) \alpha z] 
\end{array} \right. 
\]

the worldwide technical progress, high ranking advisory boards have evaluated strategy scenarios to meet as early as possible the growing need of environmentally acceptable electricity [1]. A major result of these advisory boards was, that a properly organized and funded fusion development program could lead to a prototype fusion power station putting electricity into the grid within 30 years, with commercial fusion power following 10 years later [2,3]. A common feature of the developed strategy scenarios is that they include as indispensable elements ITER as major next step machine, the dedicated intense neutron source International Fusion Materials Irradiation Facility (IFMIF) for materials qualification, and a DEMO.

IFMIF will achieve this using two 40 MeV deuteron continuous-wave (CW) linear accelerators each with 125 mA beam current striking a single thick, flowing Li-target under a 20° impinging angle, thus providing an intense neutron flux of about $10^{18}$ n/m$^2$/s with a broad peak near 14 meV. As the neutrons produced within the common beam footprint of 5 cm $\times$ 20 cm are mainly collided in forward direction, the test modules housing the specimens to be irradiated are positioned immediately adjacent of the Li-target. IFMIF is presently with a beam power of 10 MW on a single target the worlds’ most powerful accelerator driven neutron source.

Fig. 1 shows a bird’s eye view of the cost-optimized...
Mechanism of the forward-angle \((d,pn)\) reaction at intermediate energies

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The elastic breakup of the deuteron has been measured in a kinematical condition of \(\theta_p = \theta_n = 0^\circ\) at \(E_d = 140\) MeV and 270 MeV on seven targets ranging from \(Z = 6\) to 82. A double-peaked structure with its minimum at \(E_p = E_n\) is observed for all the measured triple-differential cross sections, which indicates a large contribution from Coulomb breakup. While a symmetric shape is expected for an \(E1\) transition, the observed proton-energy distribution is asymmetric, showing considerable dependence on both the target and the incident energy. These data and previous data at 56 MeV have been analyzed using the post-form of the distorted-wave Born approximation (DWBA) theory. Although the calculations account reasonably well for the asymmetric shape, by including the nuclear potential, they consistently overestimate the magnitudes of the cross sections over the whole measured region. Limitations of the post-form DWBA are discussed. [S0556-2813(98)03910-7]

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FIG. 2. (a) Two-dimensional and (b) summed energy spectra of the coincidence protons and neutrons from the deuteron breakup on the $^{12}$C target at 270 MeV. The effects of accidental coincidence are corrected for.
Form factor

Fig. 2. The form factor $F(q_n^2)$ using a Hulthén wave function for the deuteron.
Form factor - various approximations

Zadro2002

TABLE II. Momenta $q_{nc}$ which appear in the vertex function $F(q_{nc})$ for different approximation methods: the zero-range approximation (ZRA), the Baur-Trautmann approximation (BTA), the local momentum approximations (ILMA and FLMA), and the asymptotic momentum approximations (IAMA and FAMA).

<table>
<thead>
<tr>
<th></th>
<th>ZRA</th>
<th>BTA</th>
<th>ILMA</th>
<th>FLMA</th>
<th>IAMA</th>
<th>FAMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$k + \alpha K_f$</td>
<td>$k - \alpha (K'_i - K_f)$</td>
<td>$k - \alpha (k'_c - k_c)$</td>
<td>$k - \alpha (K_i - K_f)$</td>
<td>$k$</td>
<td></td>
</tr>
</tbody>
</table>
Switch function

\[ f(E_d, \theta_n, \theta_p) = 1 - \left[ \frac{1}{1 + e^{(\theta_n - t_n)/a_n}} \frac{1}{1 + e^{(\theta_p - t_p)/a_p}} \frac{1}{1 + e^{(E_n - u_n)/b_n}} \right] \]

\[ u_n = 120\text{MeV}, \quad b_n = 20\text{ MeV} \]

\[ t_p = t_n = \text{from 25 deg (light) to 5 deg (heavy target) degrees} \]

\[ a_n = a_p = 3\text{ deg} \]
Neutron polar angle parametrization

(red: DWBA, blue: fit)

$Z=18$, $A=40$, $E=100$ MeV
Proton polar angle parametrization

Z=13, A=27, E_{deut}=100 \text{ MeV}, T_n/T_{nmax}=0.200

\text{arxiu} \quad \text{blue}

\text{f}(x,y) \quad \text{red}
Proton azimuthal angle parametrization

(at centroid of $\theta_n, \theta_p$)

$Z=44$, $A=102$, $E_{\text{deo}}=150$ MeV, $T_n/T_{n\max}=0.400$