Towards an effective model for low-energy deuteron interactions in FLUKA

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On behalf of the FLUKA Collaboration

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Deuteron interactions

- Of interest both from a fundamental and from an applied point of view.
- Li(d,xn), Be(d,xn), C(d,xn), etc.: effective source of high-intensity n beams.
- Applications:

  IAEA Tech. Rep. Ser. **435** (2004):
  High n yield from d-induced reactions $\rightarrow$ fissionable material.

  Moeslang A. *et al.*, *Fus. Eng. Des.* **81** 863-871 (2006):
  Li(d,xn) as n source for material-damage studies.

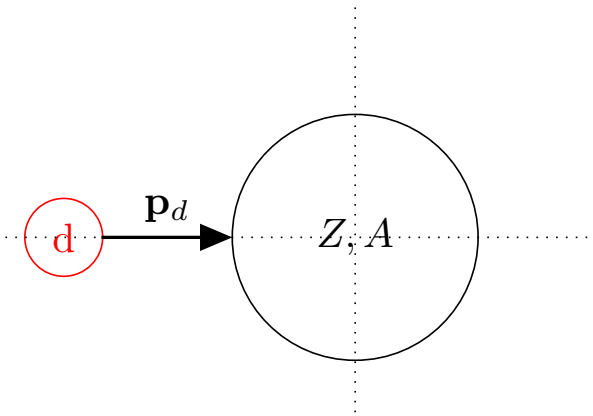
  Nagai Y. *et al.*, *J. Phys. Soc. Jpn.* **82** 064201 (2013):
  d-induced production of ^{99}Mo for medical applications.

- FLUKA: missing a dedicated model for low-energy deuteron interactions ($\lesssim 100 - 150$ MeV/n).

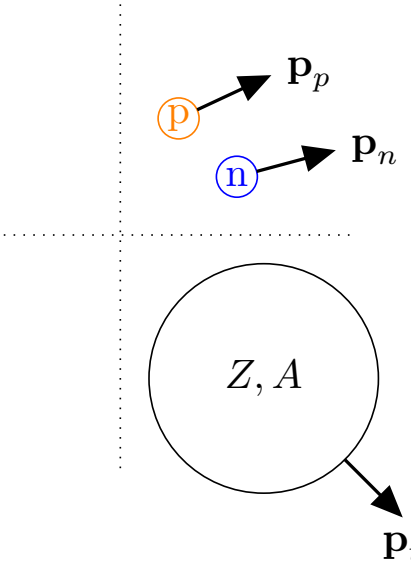
In this talk: brief summary of efforts towards including deuteron interactions in FLUKA's radiation transport model.

Interaction mechanisms at low energy ($\lesssim 150$ MeV/n)

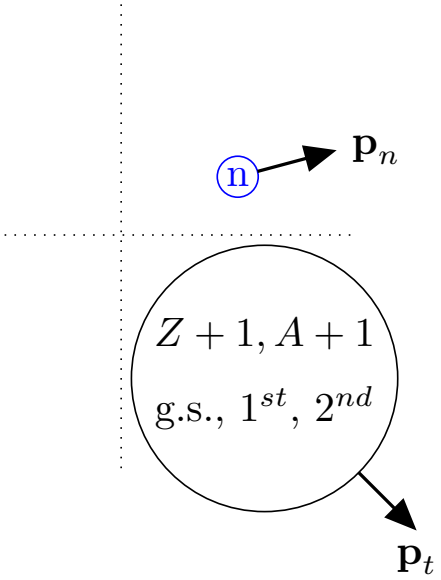
Deuteron-nucleus interaction



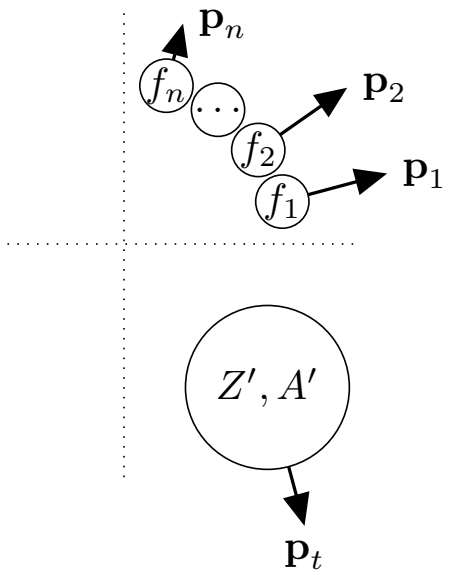
Elastic break-up



Stripping to bound state



Stripping to continuum



Typical neutron spectrum from (d,xn)

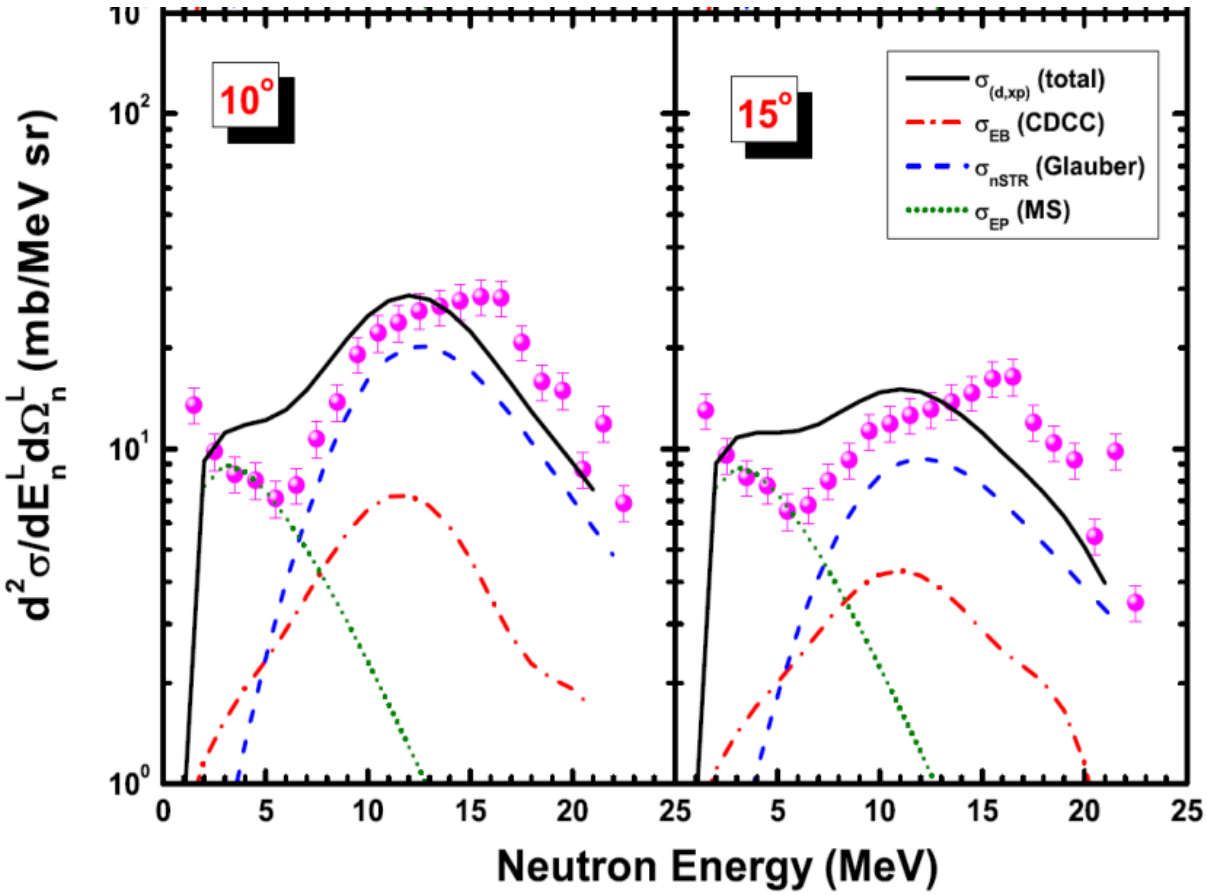


Fig. 4. (Color online) Comparison of the calculated DDXs of ^{7}Li(d, xn) at 25 MeV with the experimental data for different neutron emission angles. The experimental data (dot) is from Ref. 20.

T. Ye. *et al.*, *J. Kor. Phys. Soc.* **59** 2 871-874 (2011)

Model requirements for MC

- Activation / material damage: we need to keep track of the final state of the recoiling target nucleus.
- Strict control of kinematics: ensure energy and momentum conservation interaction by interaction. Interesting for break-up processes ($2 \to 3$ particles).
- The simulation will ask for deuteron cross sections at arbitrary deuteron energy ($\lesssim 100 - 150$ MeV/n) and target combination Z, A.
- We cannot afford to have tabulated n-differential cross sections for just this process: parametrize the cross section as far as possible (unavoidably sacrificing a bit of detail).

Elastic break-up

G. Baur *et al.*, *Nucl. Phys.* **A191** 321–331 (1972)
J. Lang *et al.*, *Nucl. Phys.* **A204** 97–109 (1973)
G. Baur *et al.*, *Nucl. Phys. A* **265** 101–112 (1976)
H. Fuchs, *Nucl. Instrum. Meth.* **200** 261–368 (1982)
N. Matsuoka *et al.*, *Nucl. Phys. A* **391** 357–376 (1982)
H. Okamura *et al.*, *Phys. Rev. C* **58** 4 2180–2191 (1998)
J.A. Tostevin *et al.*, *Phys. Rev. C* **57** 6 3225–3236 (1998)
M. Zadro, *Phys. Rev. C* **66** 034603 (2002)
M. Zadro *Phys. Rev. C* **70** 044605 (2004)
T. Ye *et al.*, *Phys. Rev. C* **80** 014604 (2009)
M. Avrigeanu *et al.*, *Phys. Rev. C* **82** 037601 (2010)
T. Ye *et al.*, *J. Kor. Phys. Soc.* **59** 2 871–874 (2011)
Y.S. Neoh *et al.*, *Phys. Rev. C* **94** 044619 (2016)
M. Avrigeanu *et al.*, *Phys. Rev. C* **95** 024607 (2017)
C. Kalbach, *Phys. Rev. C* **95** 014606 (2017)

C.M. Vincent *et al.*, *Phys. Rev. C* **2** 3 782–792 (1970)
K.T.R. Davies *et al.*, *J. Phys. G Nucl. Phys.* **14** 961-972 (1988)
Koning A.J. *et al.*, *Nucl. Phys. A* **713** 231–310 (2003)
An H. *et al.*, *Phys. Rev. C* **73** 054605 (2006)

Elastic break-up as a direct process

- d splits into n and p, leaving the recoiling target nucleus in its ground state.

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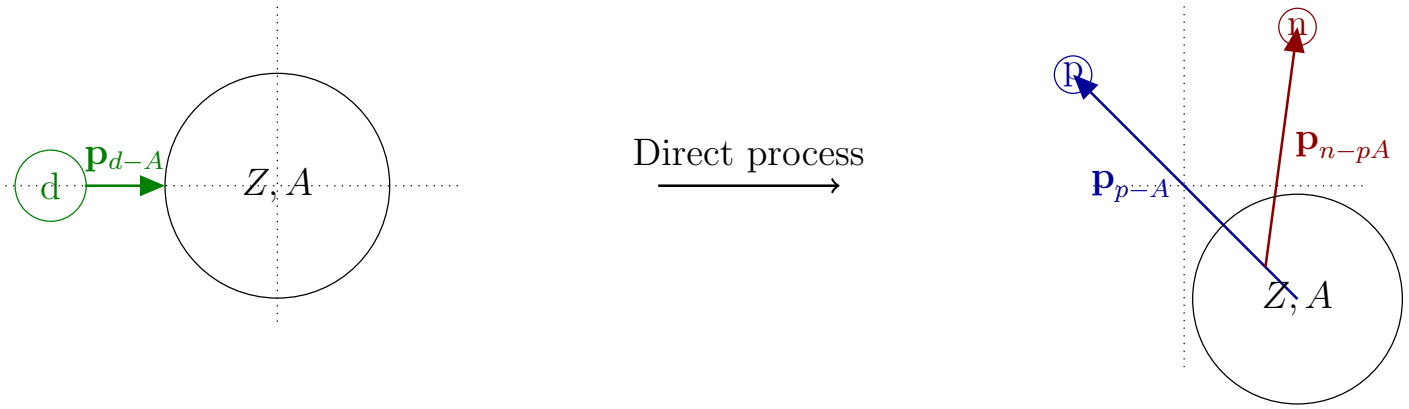
Before elastic break-up **After elastic break-up**



$$\hat{\mathcal{H}}_i = \hat{T}_{d-A} + \hat{V}_{d-A}$$

$$\hat{\mathcal{H}}_f = \hat{T}_{p-A} + \hat{V}_{p-A} + \hat{T}_{n-pA} + \hat{V}_{n-pA} + \hat{V}_{n-p}$$

$$\mathcal{T} = \langle \chi_p^{(-)}(\mathbf{p}_{p-A}, \mathbf{r}_{p-A}) \chi_n^{(-)}(\mathbf{p}_{n-pA}, \xi \mathbf{r}_{n-pA}) | \hat{V}_{n-p} | \phi_d(\mathbf{r}) \chi_d^{(+)}(\mathbf{p}_{d-A}, \mathbf{r}_{d-A}) \rangle$$

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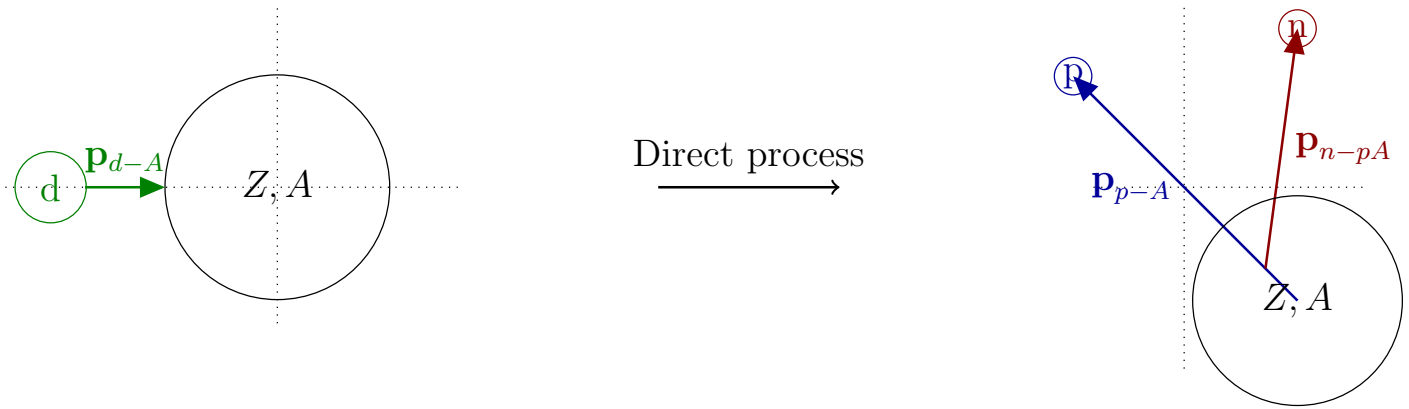

$$\hat{\mathcal{H}}_i = \hat{T}_{d-A} + \hat{V}_{d-A} \qquad \hat{\mathcal{H}}_f = \hat{T}_{p-A} + \hat{V}_{p-A} + \hat{T}_{n-pA} + \hat{V}_{n-pA} + \hat{V}_{n-p}$$

$$\mathcal{T}_{ZR} = D_0 \langle \chi_p^{(-)}(\mathbf{p}_{p-A}, \mathbf{r}) \chi_n^{(-)}(\mathbf{p}_{n-pA}, \xi\mathbf{r}) | \chi_d^{(+)}(\mathbf{p}_{d-A}, \mathbf{r}) \rangle$$

$$D_0 \sim 124 \text{ MeV fm}^{3/2}$$

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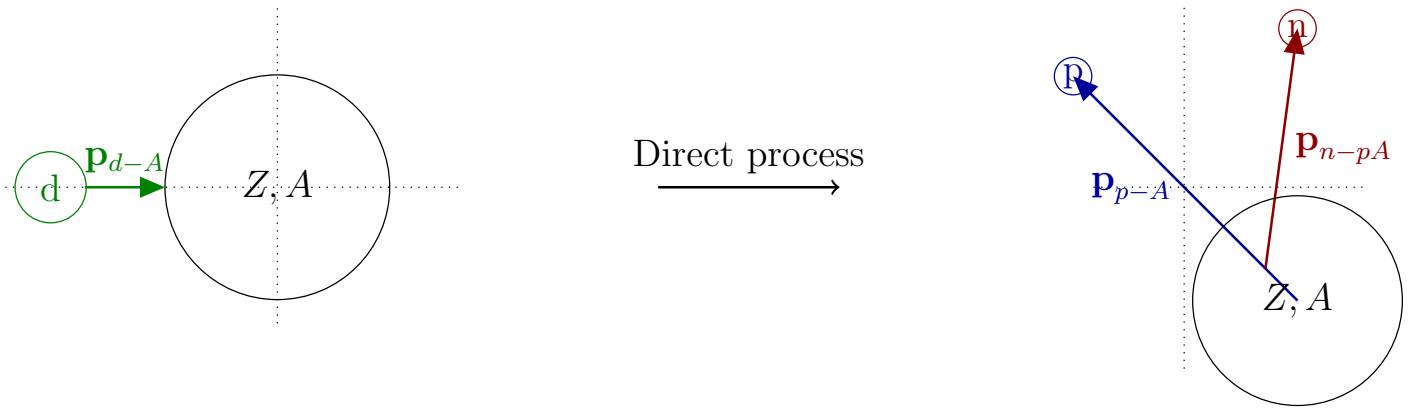

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$$\mathcal{T}^{\text{DWBA}}_{\text{post}} = \underbrace{\langle \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} | \hat{V}_{n-p}(\mathbf{r}) | \phi_d(\mathbf{r}) \rangle}_{\text{Vertex func. / form fact.}} \underbrace{\langle \chi^{(-)}_{\mathbf{p}_{p-A}} \chi^{(-)}_{\mathbf{p}_{n-pA}} | \chi^{(+)}_{\mathbf{p}_{d-A}} \rangle}_{\text{Dynamics}}$$

Zadro2002

DWBA differential cross section

The differential cross section for elastic deuteron break-up is:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}E_n \mathrm{d}\Omega_n \mathrm{d}\Omega_p} = J \frac{2\pi}{\hbar^2 k_{dA}} \rho(E_n, \Omega_n, \Omega_p) |\mathcal{T}_{post}^{\mathsf{DWBA}}|^2$$

Routine:

- Scattering geometry determined by 4 degrees of freedom: $E_n, \theta_n, \theta_p, \varphi_p \rightarrow$ close kinematics.
- Specify $V(r)$ felt by p, n, d $\rightarrow$ find unbound solutions for 3 respective uncoupled Schrödinger eqs, giving a conceptually simpler description than more elaborate CDCC approaches.
- Evaluate matrix element and differential cross section.

Elastic Coulomb break-up

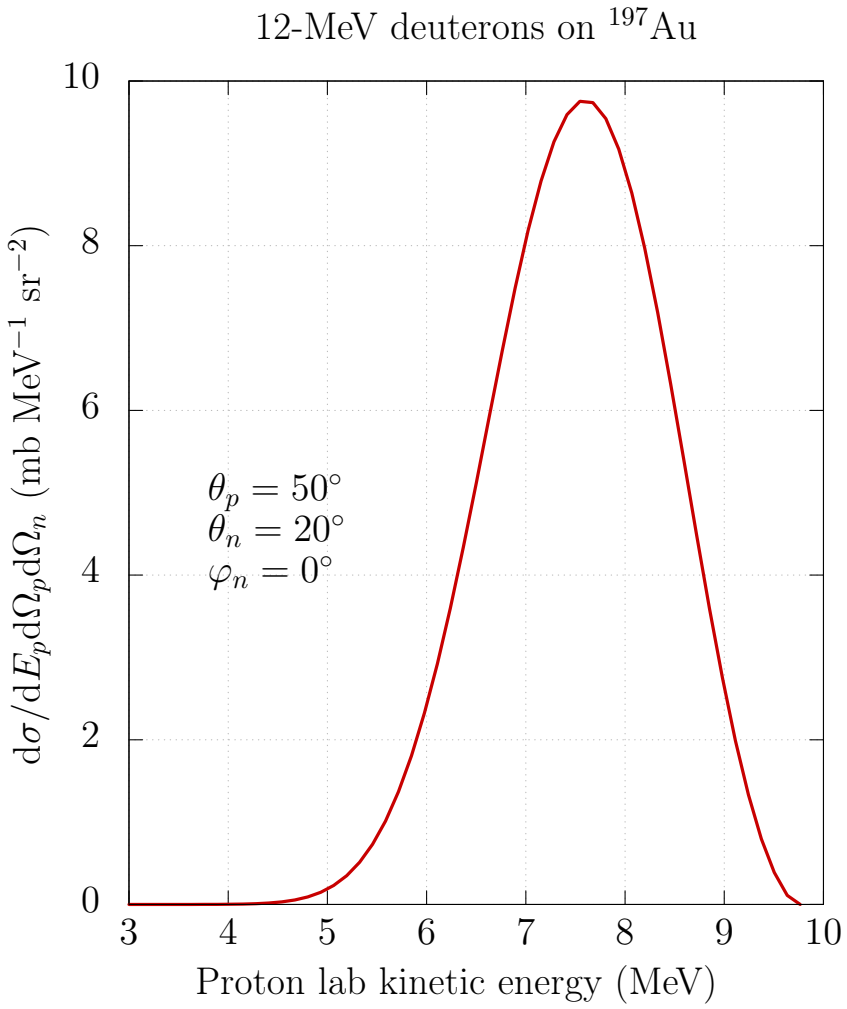
- V_{d-A} and V_{p-A} are Coulomb potentials, while $V_{n-A} = 0$.
- Permissible for deuteron energies $\ll$ Coulomb barrier.
- Wave functions are analytical (Coulomb functions) and so is the matrix element (!).
- Useful in practice:
  - Transition amplitudes for more realistic OPM require use of partial-wave series, the convergence of which is greatly accelerated by subtraction of Coulomb amplitude.
  - Even at higher energies, elastic break-up in forward emission appears Coulomb dominated (Tostevin1998, Okamura1998 140-MeV measurements).

Own code (dnr.f90) to evaluate all shown expressions

Coulomb elastic break-up at 12 MeV

Why tipped towards "fast" protons?

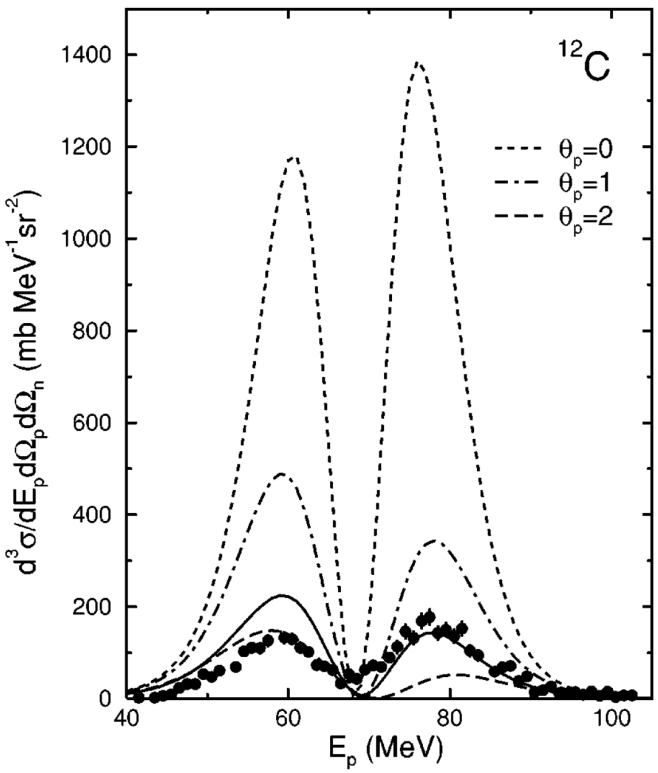
- Deuteron "climbs" Coulomb potential.
- Velocity reduced.
- Break-up.
- Neutron carries on with low energy.
- Proton repelled (accelerated) as it moves away from the nucleus.



G. Baur *et al.*, *Nucl. Phys. A* **265** 101–112 (1976)

Coulomb elastic break-up for 140-MeV d on ^{12}C ($\theta_n = 0°$)

Tostevin1998



- Various dashed curves: calculations at given scatt. geometry.
- Dots: experimental data (Okamura1998 RIKEN)
- Solid curve: integration over detector solid angles.
- Coulomb break-up unexpectedly appears to dominate forward emission (140 MeV $\gg$ typical Coulomb barriers).

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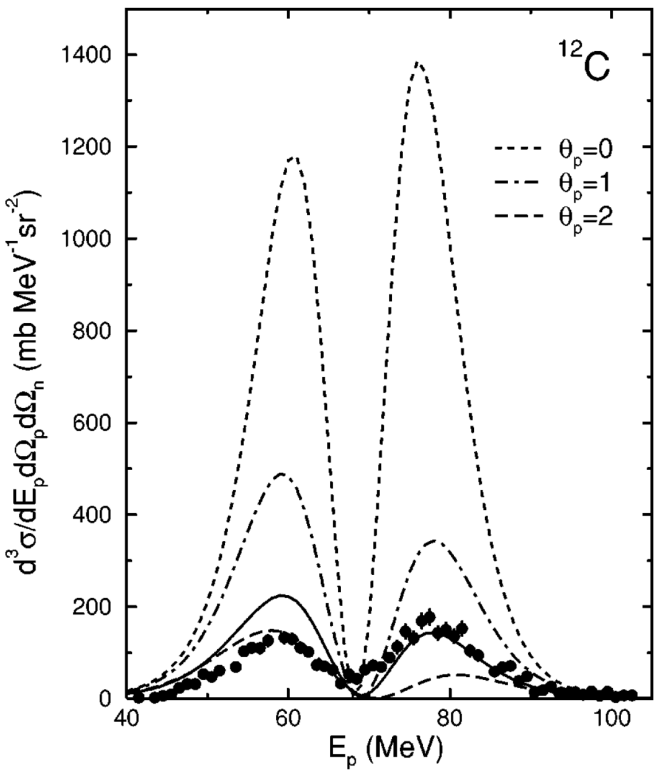
Tostevin1998



Why double peak at fwd emission $\theta_n = \theta_p = 0$? Okamura1998:

$$\text{Dipole approx :} \qquad \mathcal{T}^{\text{DWBA}}_{\text{prior}} \sim \left\langle \chi^*_{dA} \left| \frac{\mathbf{k}_{np}\mathbf{r}_{dA}}{r^3_{dA}} \right| \chi_{dA} \right\rangle$$

Vanishes at fwd emission for $E_n \approx E_p \approx E_d/2$ because $\mathbf{k}_{np} \approx 0$.

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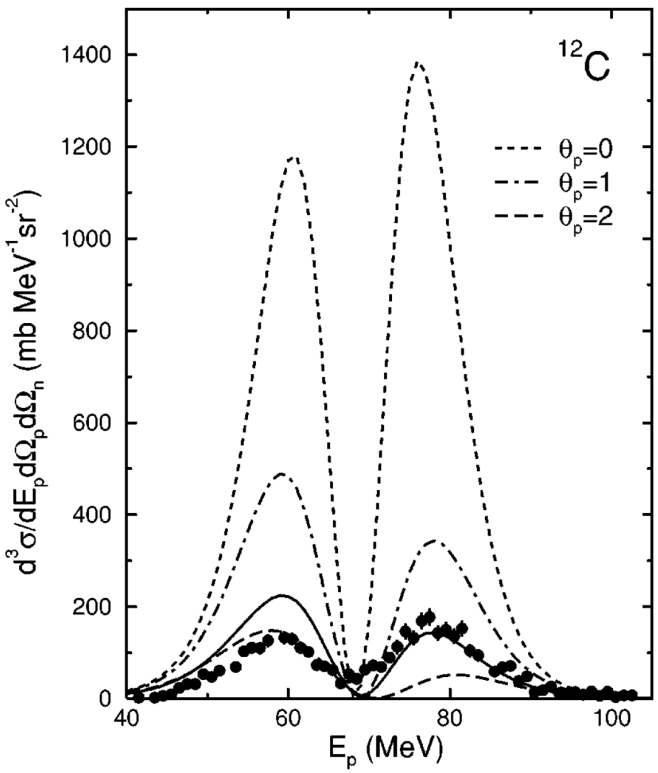
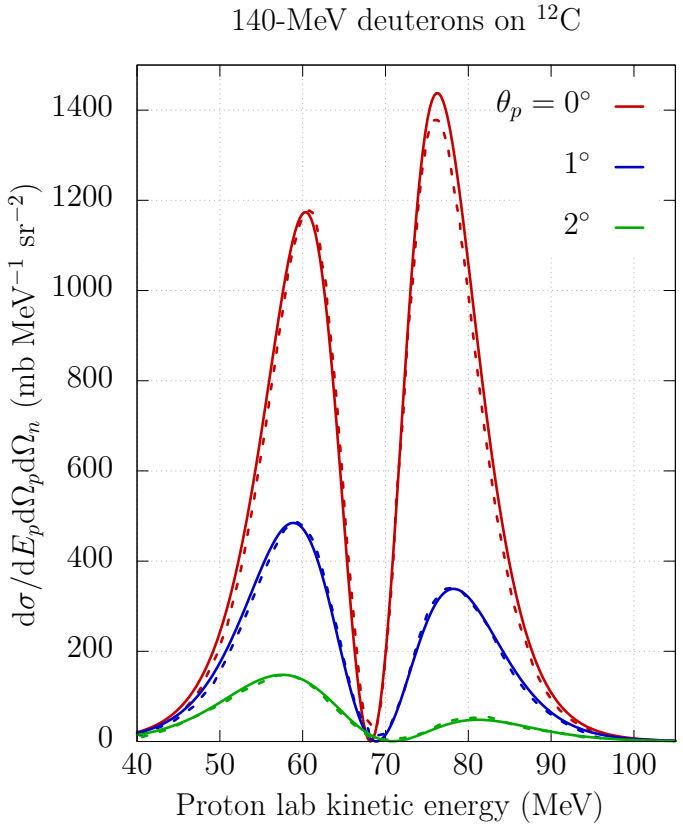


`dnr.f90` (Tostevin1998 dashed)

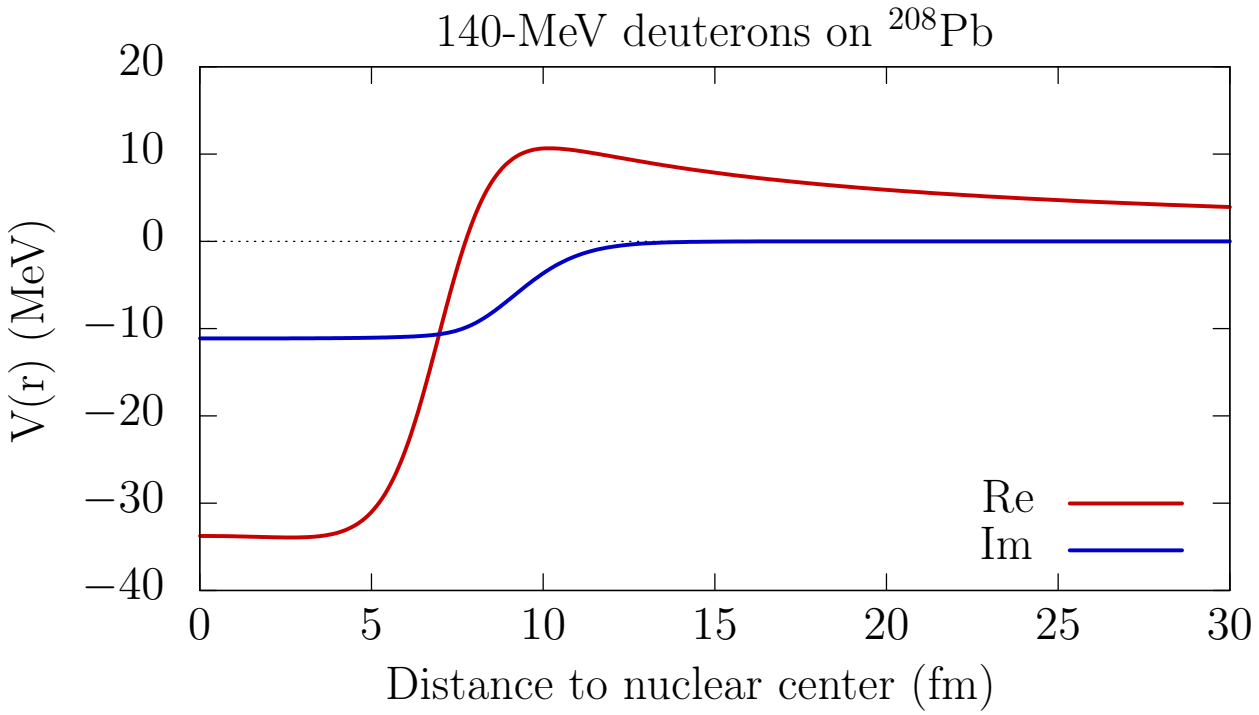


- Right: our numbers (solid), Tostevin's digitized (dashed).
- Good agreement.
- We have the 3-body kinematics reasonably under control.

Coulomb+nuclear optical potential model (OPM)

OPM param. for d-nucleus interaction: Han2006*.
OPM param. for nucleon-nucleus interaction: Koning2003**.

$$V(r) = V_{\text{sphere}}(r) + V_{\text{WS}}(r) + \mathrm{i}[W_{\text{WS}}(r) + S_{\text{WS}}(r)] + V_{SO}(r)$$



- Wavefunctions no longer known analytically.

Distorted waves (partial-wave expansion)

- Since the potential is central: partial-wave expansion

$$\chi^{(\pm)}_{p-A}(\mathbf{r}) = \frac{4\pi}{k_{p-A}r}\sum_{\ell=0}^{\infty} \mathrm{i}^{\ell}\mathrm{e}^{\pm\mathrm{i}\delta_\ell} P_{E_{p-A}\ell}(r) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{\boldsymbol{r}}) Y^*_{\ell m}(\hat{\boldsymbol{k}}_{p-A})$$

$$\chi^{(\pm)}_{n-pA}(\mathbf{r}) = \frac{4\pi}{k_{n-pA}r}\sum_{\ell=0}^{\infty} \mathrm{i}^{\ell}\mathrm{e}^{\pm\mathrm{i}\delta_\ell} P_{E_{n-pA}\ell}(r) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{\boldsymbol{r}}) Y^*_{\ell m}(\hat{\boldsymbol{k}}_{n-pA})$$

$$\chi^{(\pm)}_{d-A}(\mathbf{r}) = \frac{4\pi}{k_{d-A}r}\sum_{\ell=0}^{\infty} \mathrm{i}^{\ell}\mathrm{e}^{\pm\mathrm{i}\delta_\ell} P_{E_{d-A}\ell}(r) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{\boldsymbol{r}}) Y^*_{\ell m}(\hat{\boldsymbol{k}}_{d-A})$$

- $P(r)$ are unbound solutions of the respective Schrödinger eqs:

$$\left[-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + U_{i-A}(r) + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right] P_{E_i\ell}(r) = E_i P_{E_i\ell}(r), \qquad i = \{n, p, d\}$$

RADIAL

We had at hand the state-of-the-art RADIAL2017:

> RADIAL: **a Fortran subroutine package for the solution of the radial Schrödinger and Dirac wave equations**
>
> **Francesc Salvat and José M. Fernández-Varea**
>
> Facultat de Física (FQA and ICC). Universitat de Barcelona.
> Diagonal 645, 08028 Barcelona, Spain

For a Coulomb + finite range central potential:

- Radial functions $P_{E\ell}(r)$,
- Phase shifts $\tilde{\delta}_\ell = \Delta_\ell + \delta_\ell$.

Strongly tempted to evaluate the transition amplitude:

$$\mathcal{T}_{\text{post}}^{\text{DWBA}} \sim \langle \chi^{(-)}_{\mathbf{p}_{p-A}} \chi^{(-)}_{\mathbf{p}_{n-pA}} | \chi^{(+)}_{\mathbf{p}_{d-A}} \rangle$$

The DWBA matrix element for elastic break-up

After a bit of algebra/patience:

$$\mathcal{T}_{\mathsf{post}}^{DWBA} = (4\pi)^2 \frac{D_0}{k_d k_n k_p} \sum_{\ell\ell'\ell''} \sqrt{(2\ell+1)(2\ell'+1)}$$
$$\times\, \mathrm{i}^{\ell''-\ell-\ell'} \mathrm{e}^{\mathrm{i}(\delta_\ell+\delta_{\ell'}+\delta_{\ell''})} \mathrm{e}^{\mathrm{i}(\Delta_\ell+\Delta_{\ell'}+\Delta_{\ell''})}$$
$$\times\, \langle \ell\ell' 00 | \ell'' 0 \rangle \left[\sum_m \langle \ell\ell' m, -m | \ell'' 0 \rangle Y_{\ell m}(\hat{\boldsymbol{k}}_n) Y_{\ell' -m}(\hat{\boldsymbol{k}}_p) \right]$$
$$\times \int_0^\infty \mathrm{d}r\, \frac{1}{r} P_{E_{\mathsf{n}}\ell}(r) P_{E_{\mathsf{p}}\ell'}(r) P_{E_{\mathsf{d}}\ell''}(r).$$

- A term describing angular dependency
- An integral over unbound radial functions

... to be added over ℓ_n, ℓ_p, ℓ_d.

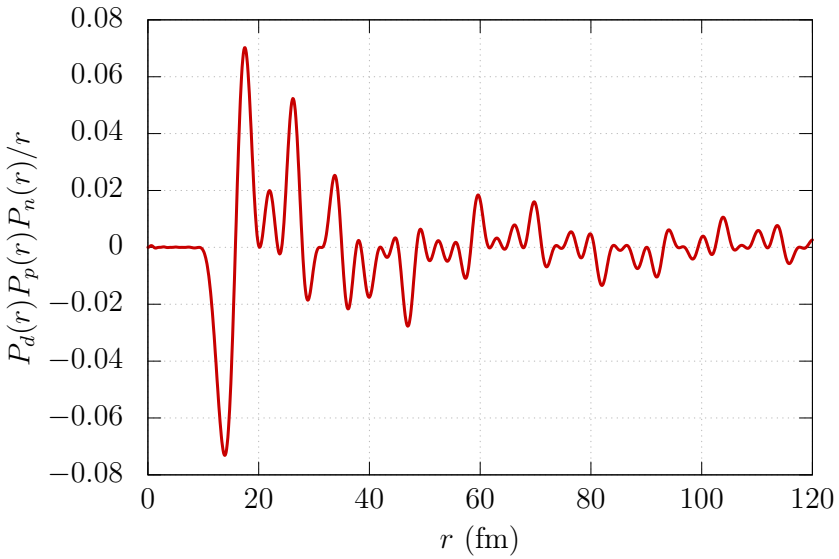
Radial integrals (Vincent1970, Davies1988)

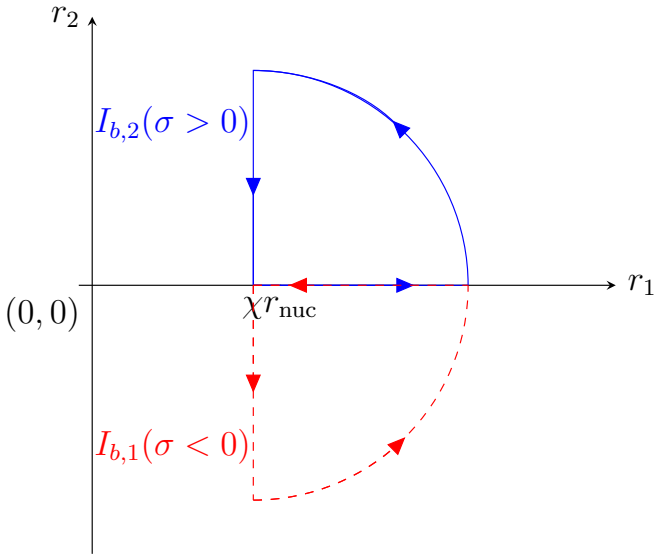
Radial integral:

$$\int_0^\infty \mathrm{d}r\ \frac{1}{r} P_{E_\mathsf{n}\ell}(r) P_{E_\mathsf{p}\ell'}(r) P_{E_\mathsf{d}\ell''}(r).$$

Considering $P_{E\ell}(r) \sim H^{\pm}(r)$ (exp decay in complex plane),

$$\oint \mathrm{d}z\ f(z) = 2\pi \mathrm{i} \mathsf{Res}[f] \qquad (0 \text{ in our case!})$$





Matrix-element convergence - Subtract Coulomb amplitude

- Matrix element involves sum over ℓ_p, ℓ_n, ℓ_d.
- Coulomb $\rightarrow$ long range $\rightarrow$ large ℓ contribute (fig a).
- Slow convergence.
- Trick: subtract Coulomb amplitude (fig b).

Okamura1998

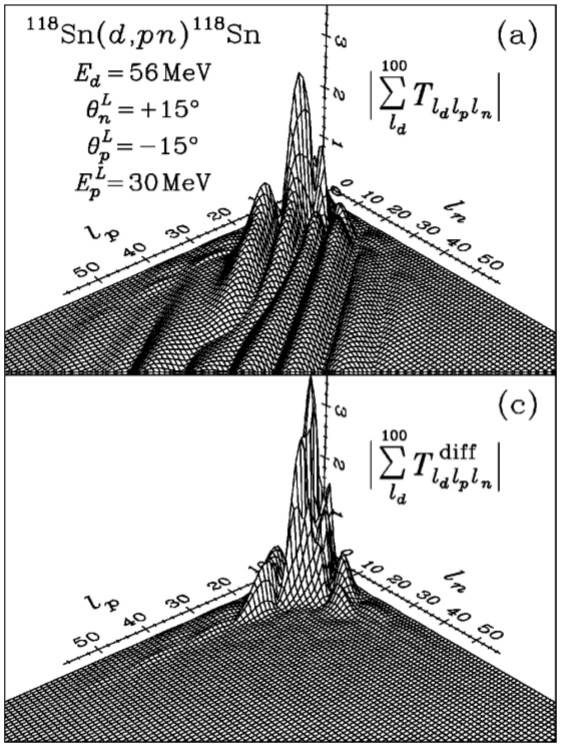
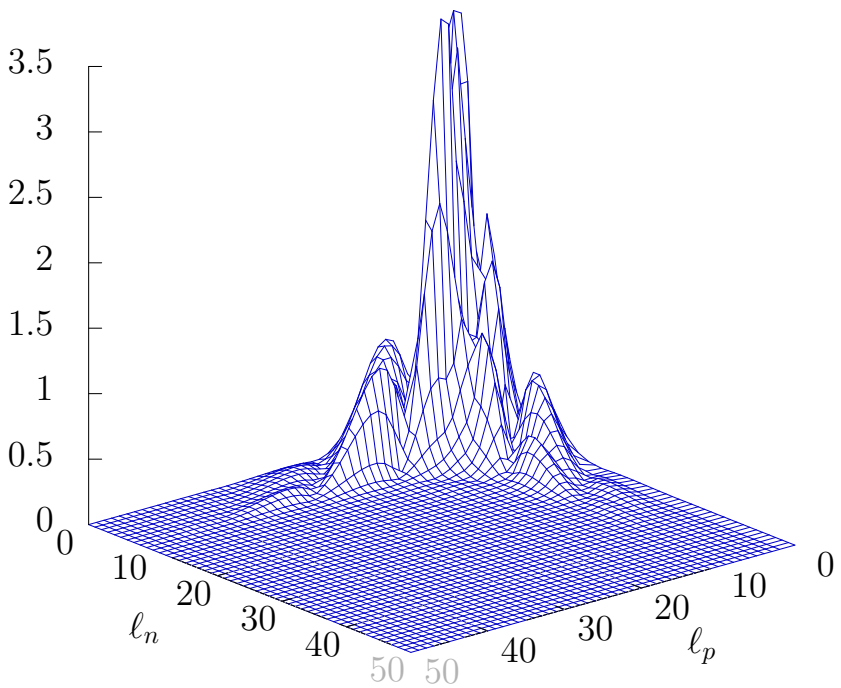
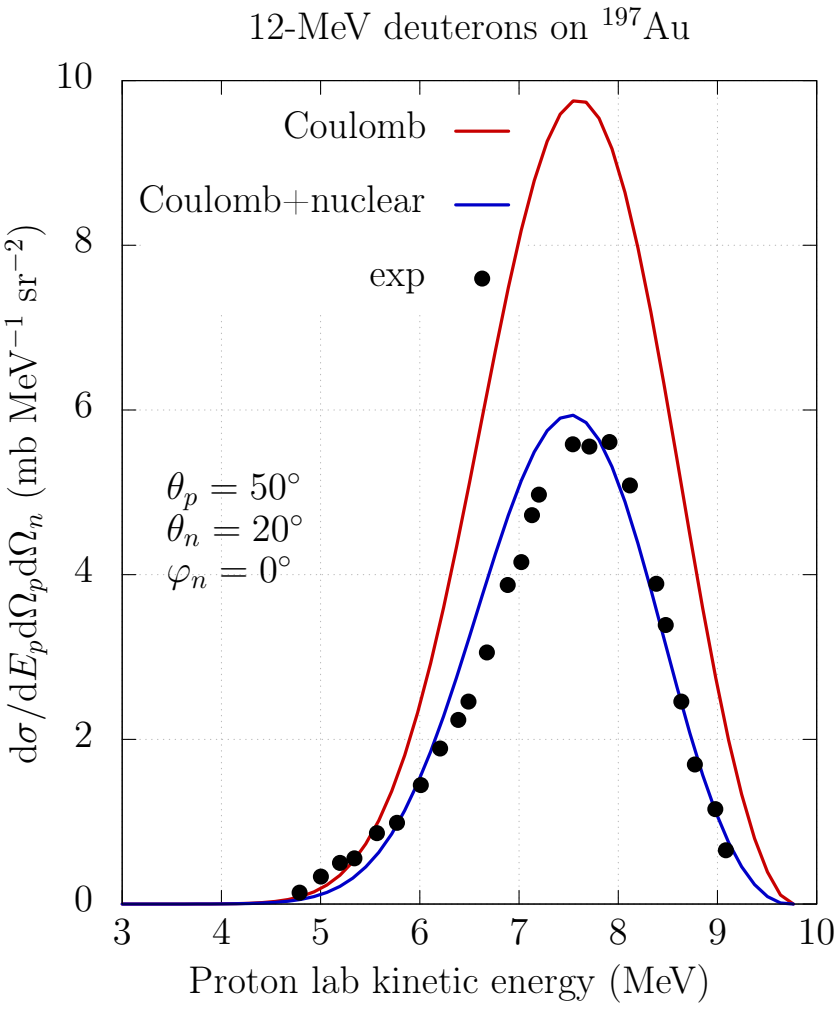


FIG. 13. Amplitudes of the partial T matrices summed over the deuteron partial waves plotted as a function of l_p and l_n for the ^{118}Sn$(d,pn)^{118}$Sn$_{\rm g.s.}$ reaction at $E_d = 56$ MeV, $\theta_p^L = +15°$, $\theta_n^L = -15°$ (opposite side of the proton in a coplanar geometry), and $E_p^L = 30$ MeV. (a) $T_{\rm ZR}$, (b) $T_{\rm ZR}^{\rm pC}$, and (c) $T_{\rm ZR}^{\rm diff}$ correspond to Eqs. (A1), (A2), and (A3), respectively.



Coulomb vs Coulomb+nuclear break-up

G. Baur *et al.*, *Nucl. Phys. A* **265** 101–112 (1976)



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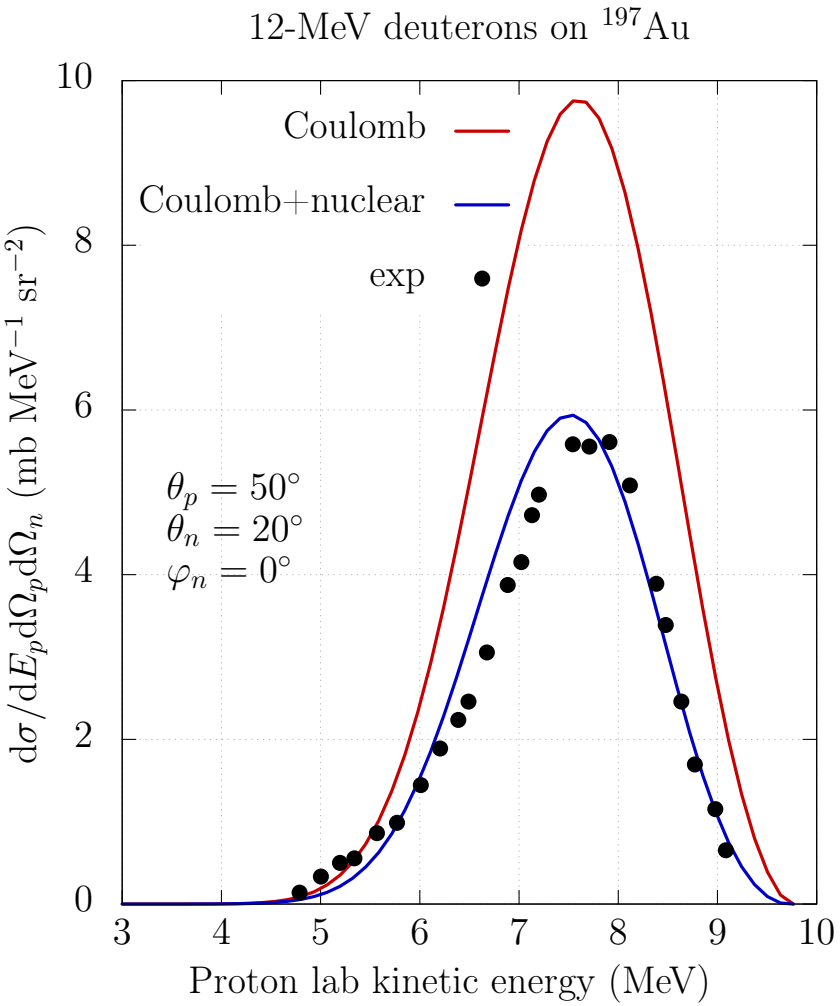
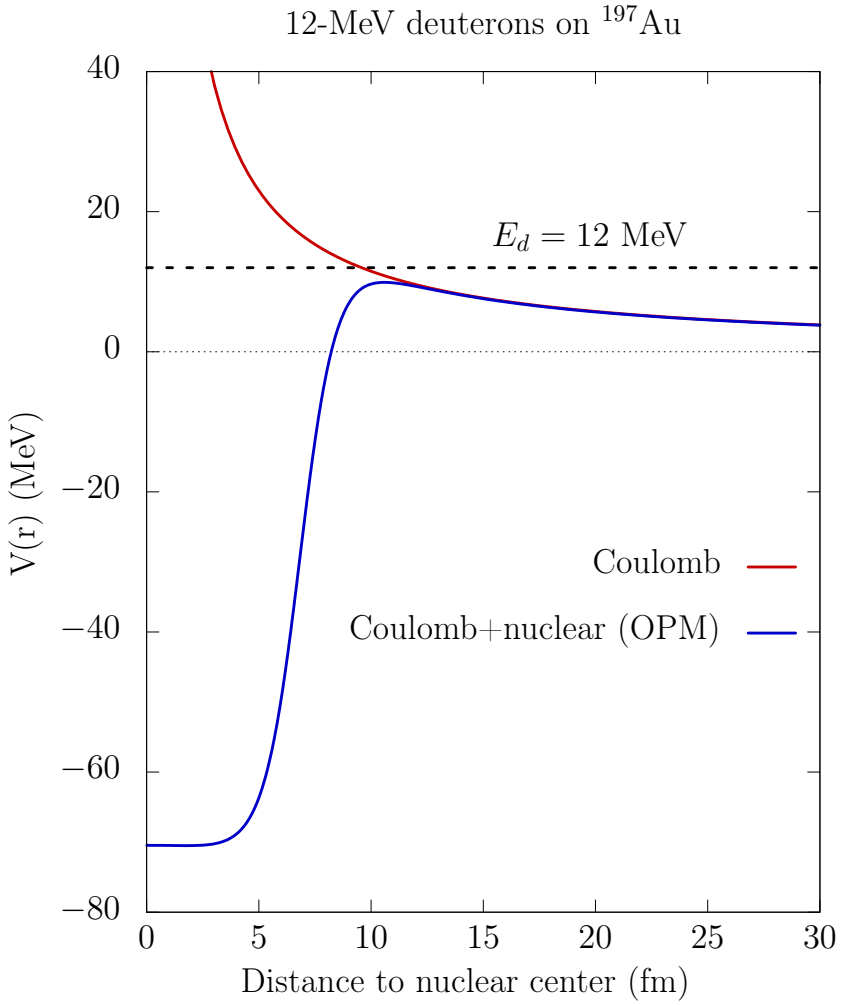


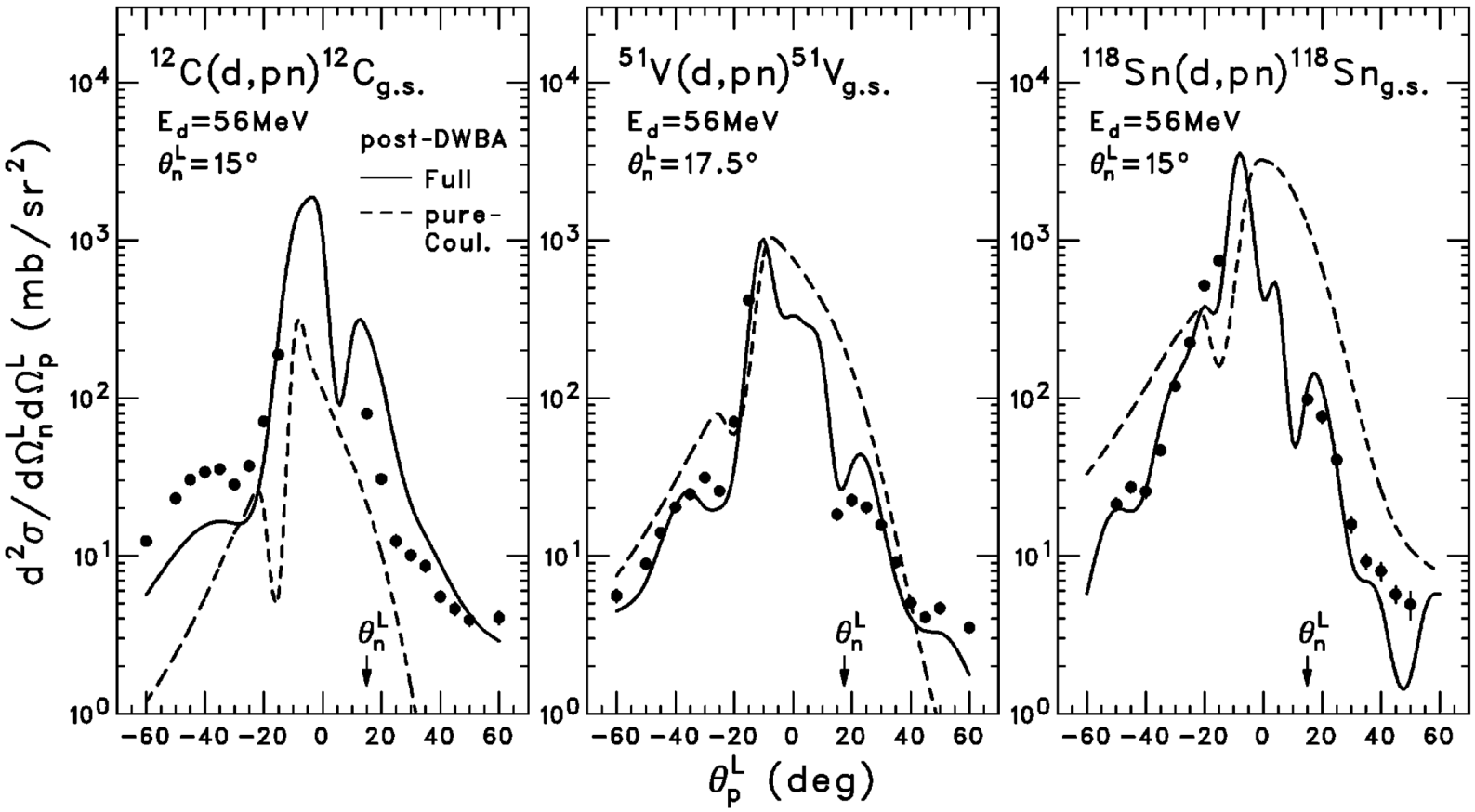
Why the drop?

- Potential less repulsive.



56-MeV deuterons on various targets

Okamura1998 - Integrated over proton energies



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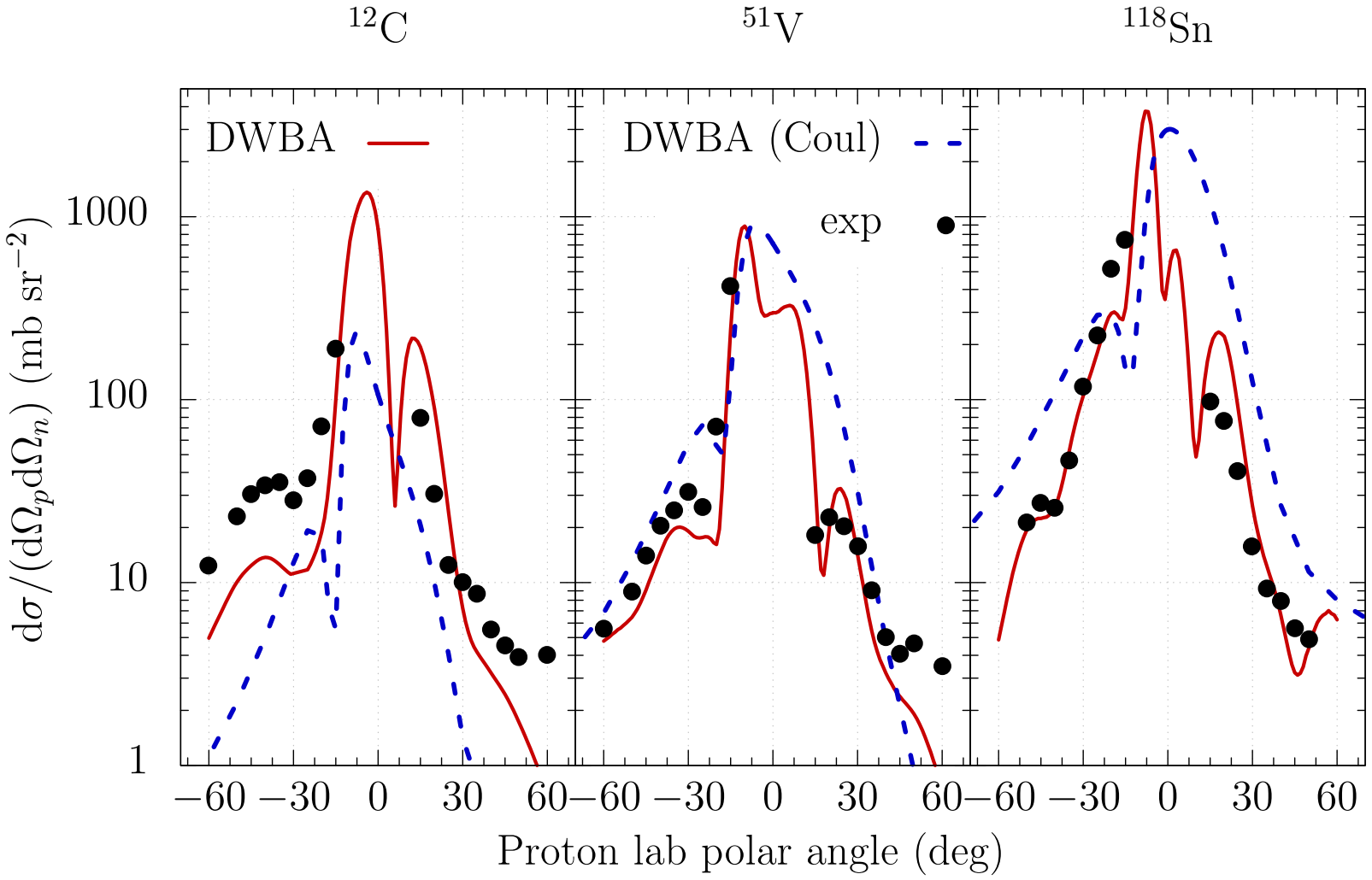

A Coulomb+nuclear potential is of course more realistic than a naked point-charge Coulomb potential.

DWBA elastic break-up cross section (1/3)

- Reasonable agreement at 4-differential level.
- Encouraged to integrate over $E_n, \theta_n, \theta_p, \varphi_p$ to obtain the integrated elastic break-up cross section.

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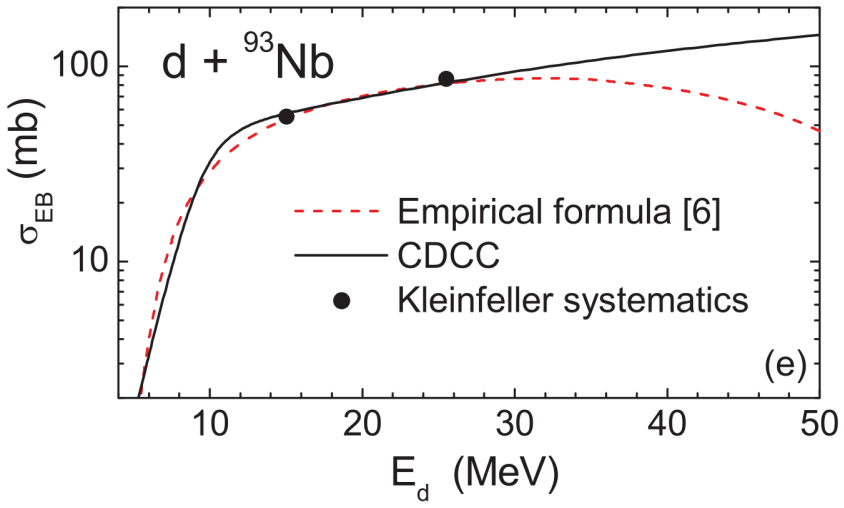
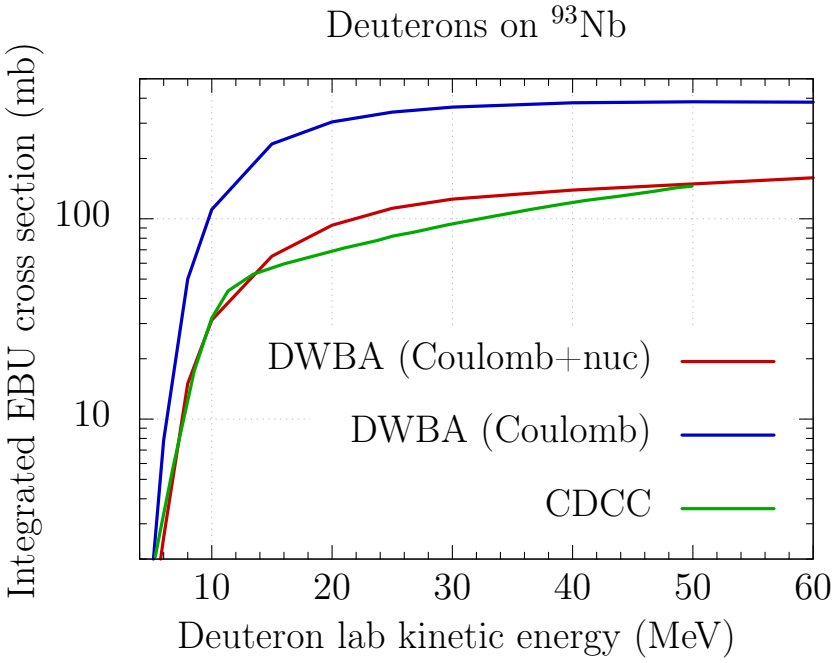
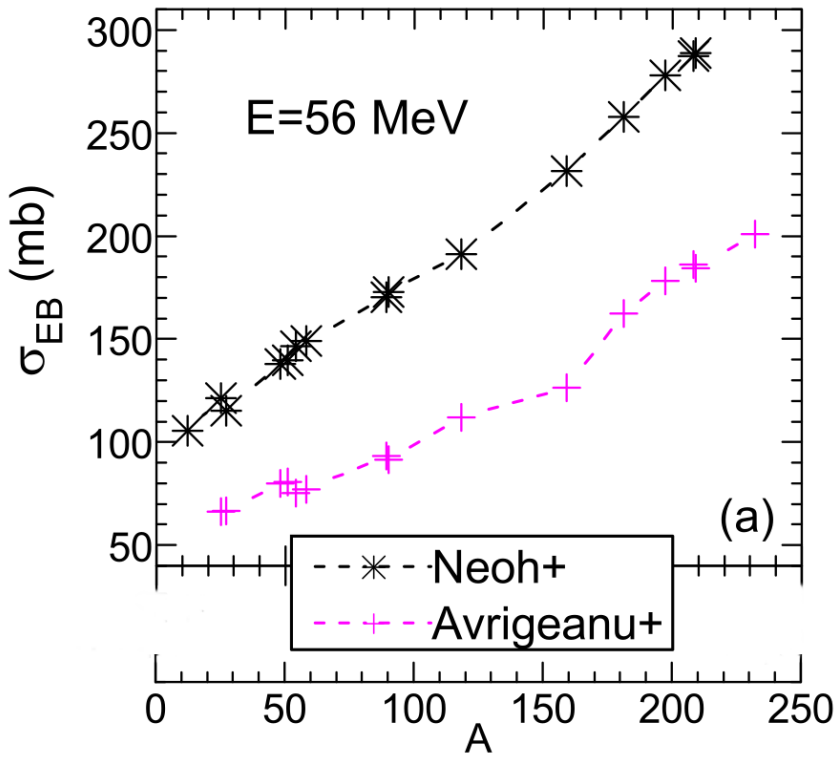
M. Avrigeanu and A. Moro, *Phys. Rev. C* **82** 037601 (2010)



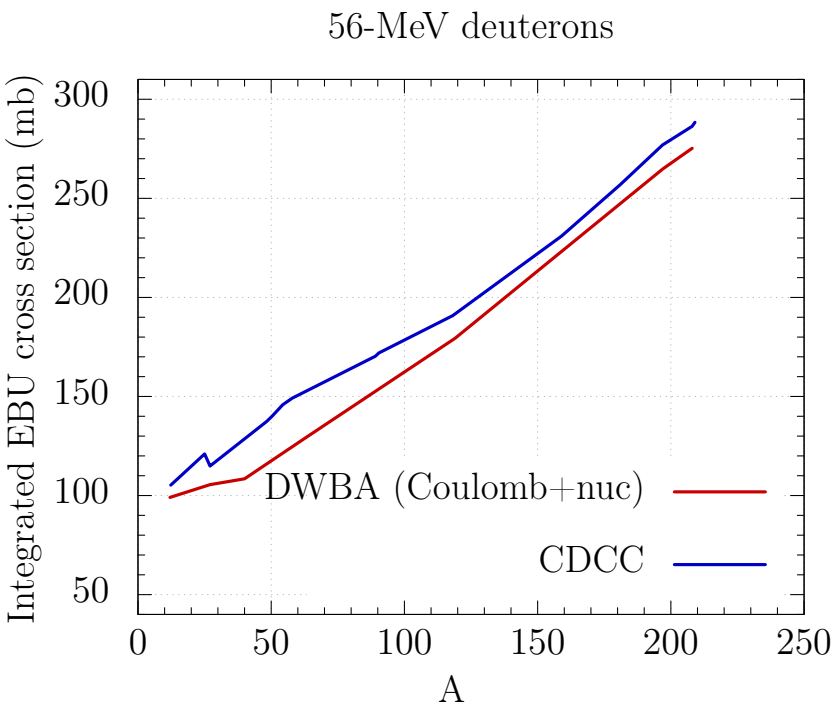


- Reasonable agreement with CDCC calculations.

DWBA elastic break-up cross section (2/3)



Avrigeanu2017

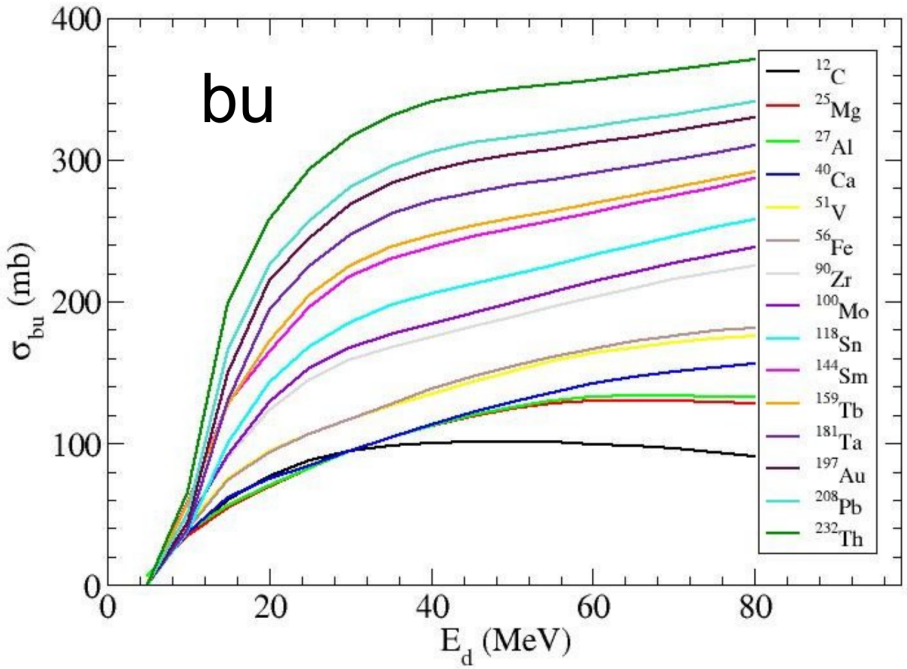


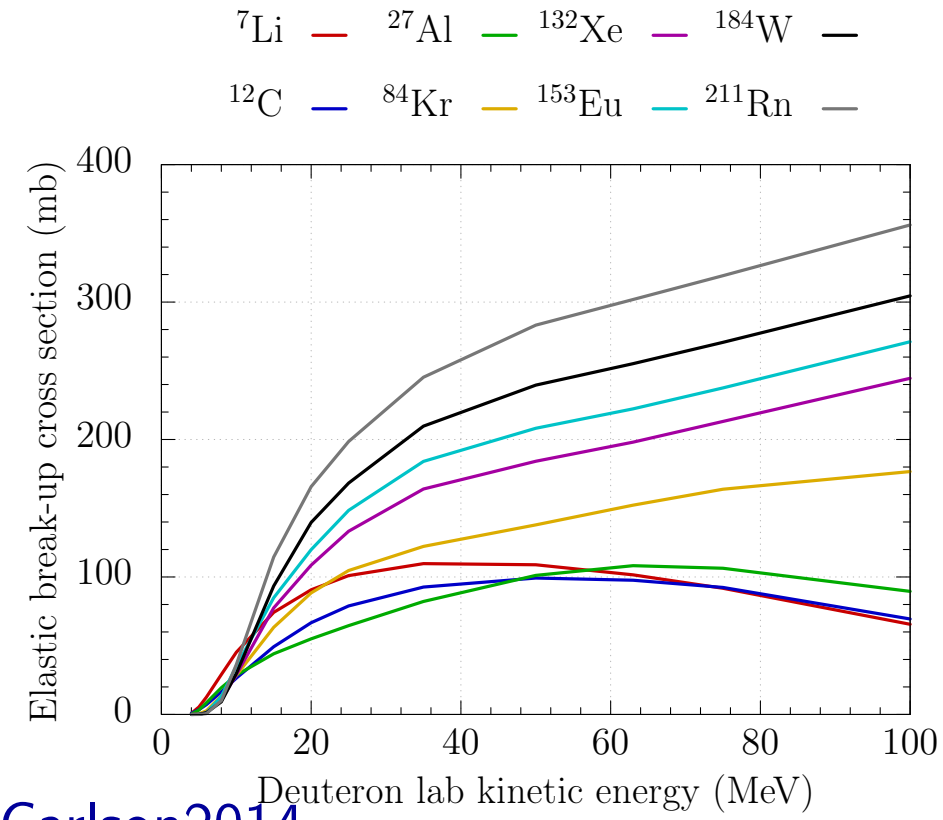
`dnr.f90`

- Reasonable agreement with CDCC calculations.
- Experimental data to gauge against (?).

DWBA elastic break-up cross section (3/3)

- σ_{EBU} tabulated for various target nuclei and various E_d:





Carlson2014

`dnr.f90`

- Good agreement for targets lighter than $\sim$ ^{27}Al.
- $\sim$Disagreement for heavier targets at 20-60 MeV
- Finite-range corrections, non-locality corrections (?).

Preparing for use in FLUKA

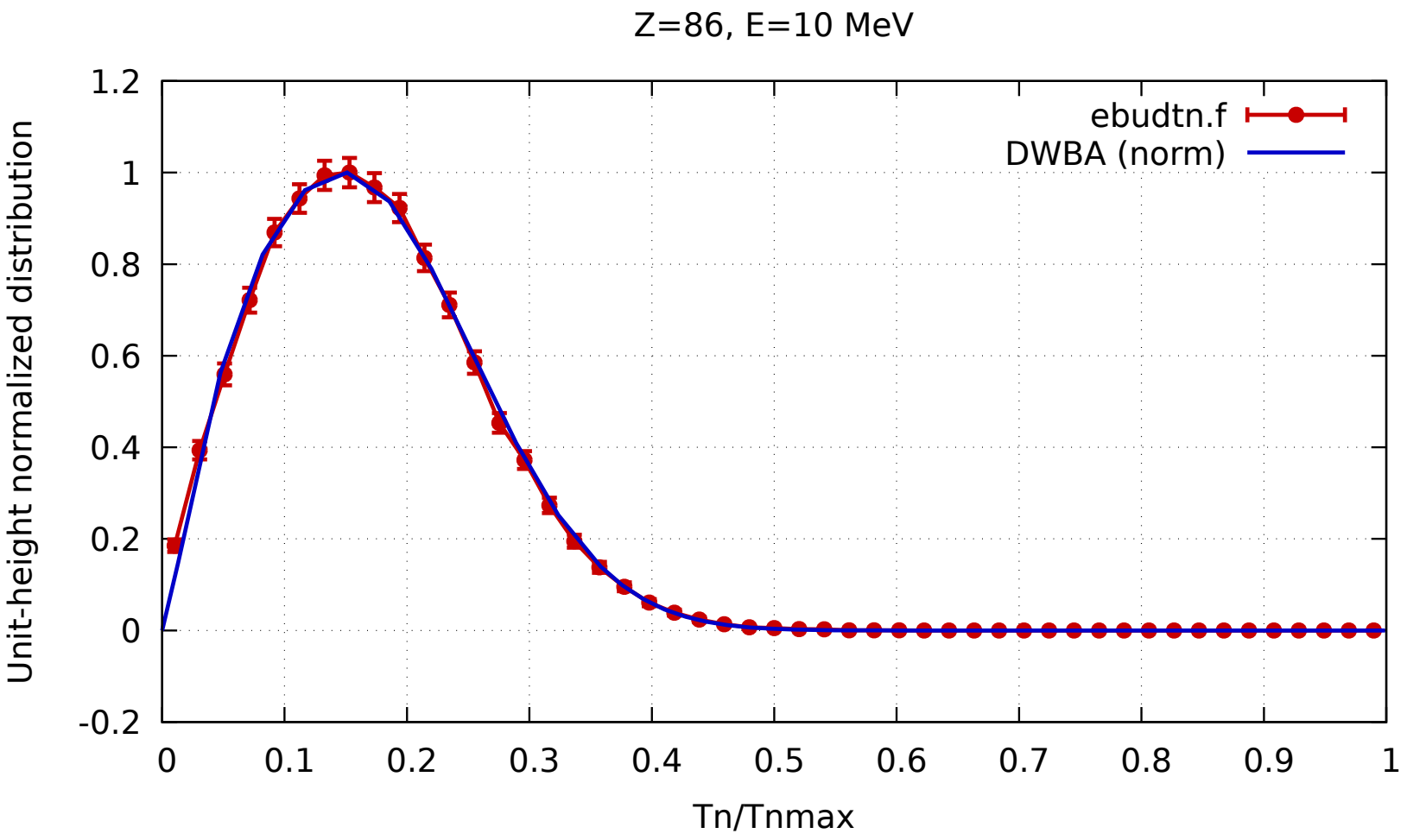
- We cannot carry a huge database of $\mathrm{d}\sigma/(\mathrm{d}E_n\mathrm{d}\Omega_n\mathrm{d}\Omega_p)$ for various E_d, Z, A.
- 20 abscissas for each: $\sim$0.5 Gb.

...so we are forced to parametrize:

- σ_{EBU} for a grid of Z, E_d
- $\mathrm{d}\sigma/\mathrm{d}E_n$ for a grid of Z, E_d
- $\mathrm{d}\sigma/(\mathrm{d}E_n\mathrm{d}\Omega_n)$ for a grid of Z, E_d, E_n
- $\mathrm{d}\sigma/(\mathrm{d}E_n\mathrm{d}\Omega_n\mathrm{d}\Omega_p)$ for a grid of Z, E_d, E_n, θ_n

$\mathrm{d}\sigma_{EBU}/\mathrm{d}E_n$ for ^{211}Rn

As a parametrization example:
Unit-height-normalized distribution of neutron energies T_n in units of maximum kinematically allowed $T_{n,max}$:



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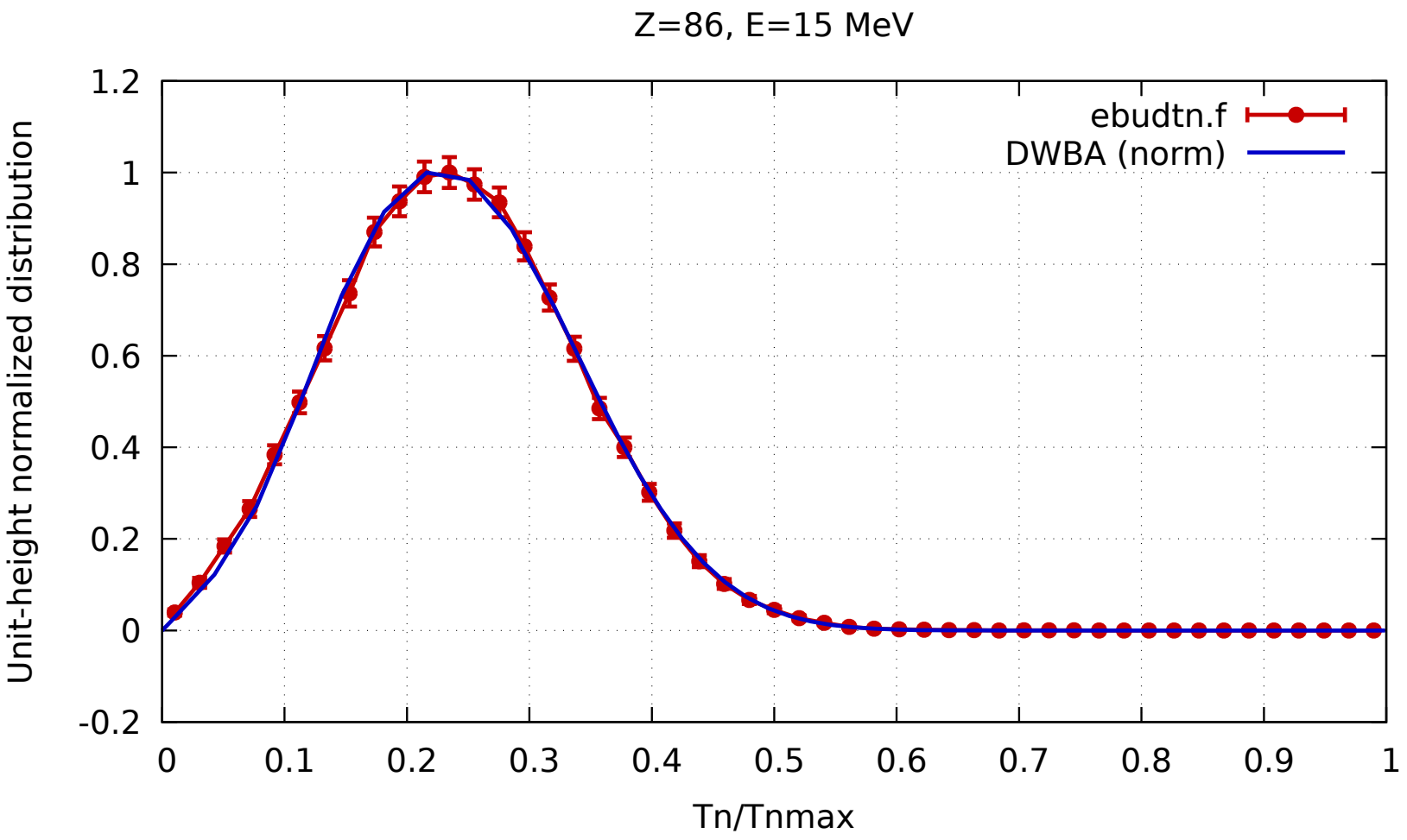

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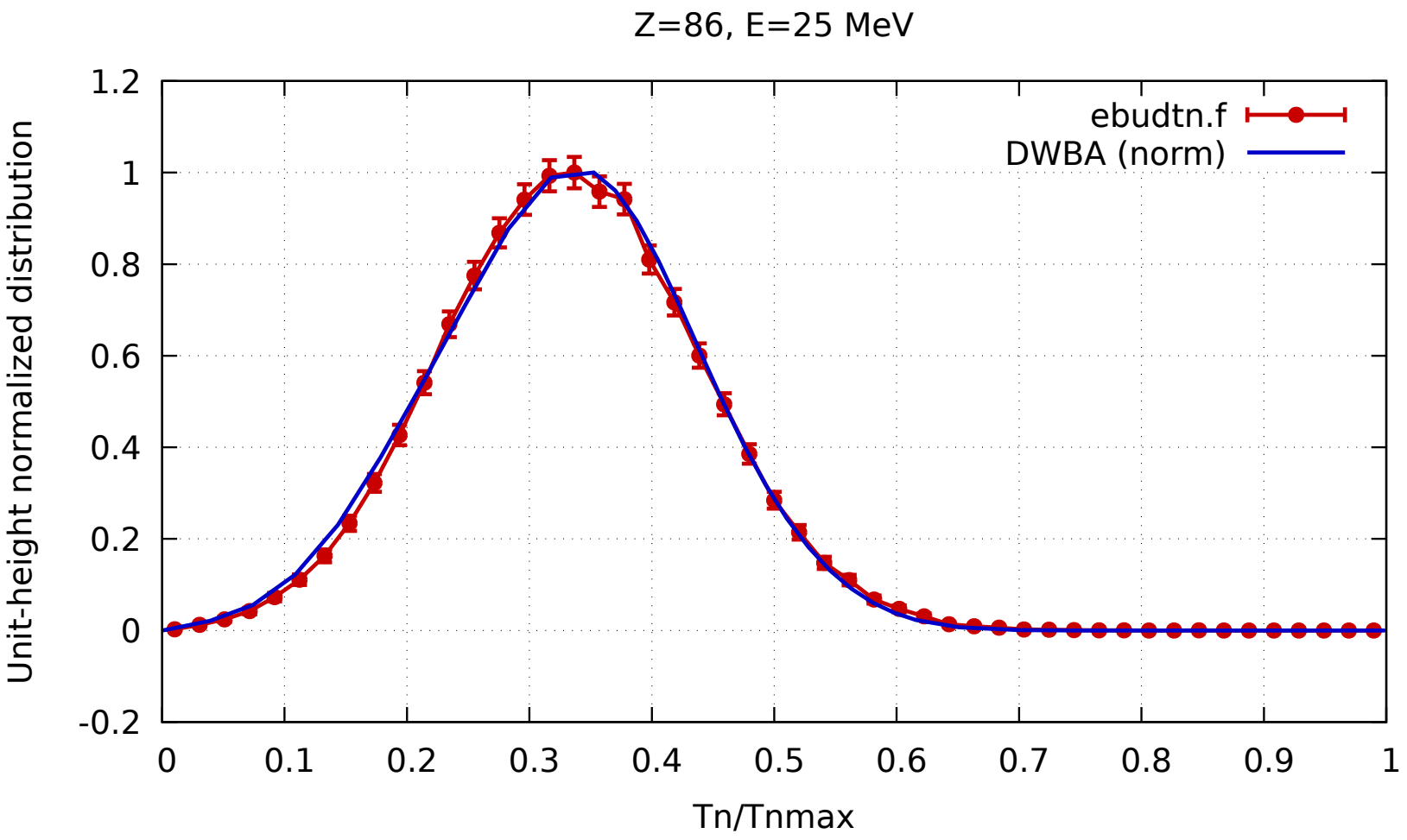

$\mathrm{d}\sigma_{EBU}/\mathrm{d}E_n$ for ^{211}Rn

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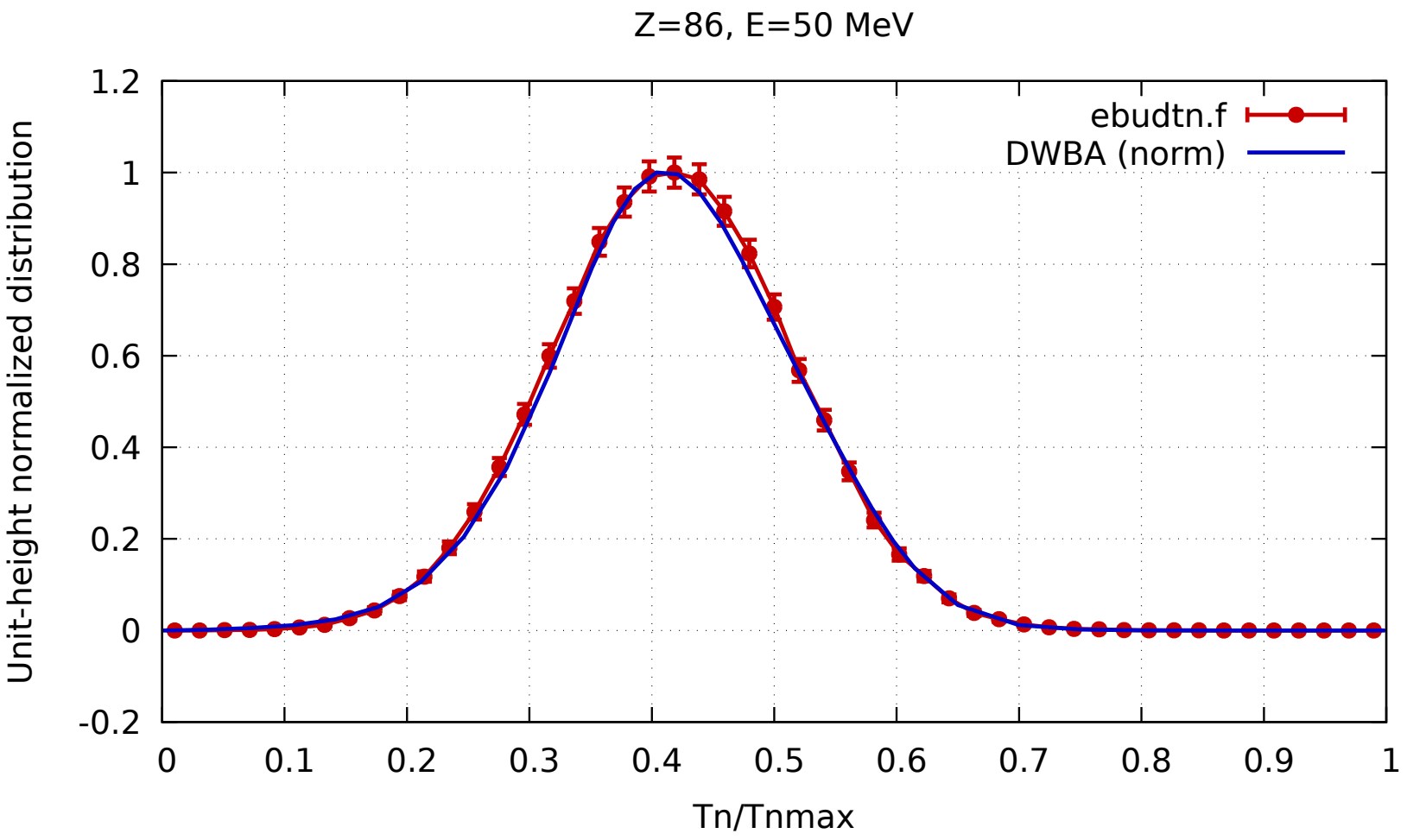
Unit-height-normalized distribution of neutron energies T_n in units of maximum kinematically allowed $T_{n,max}$:



$\mathrm{d}\sigma_{EBU}/\mathrm{d}E_n$ for ^{211}Rn

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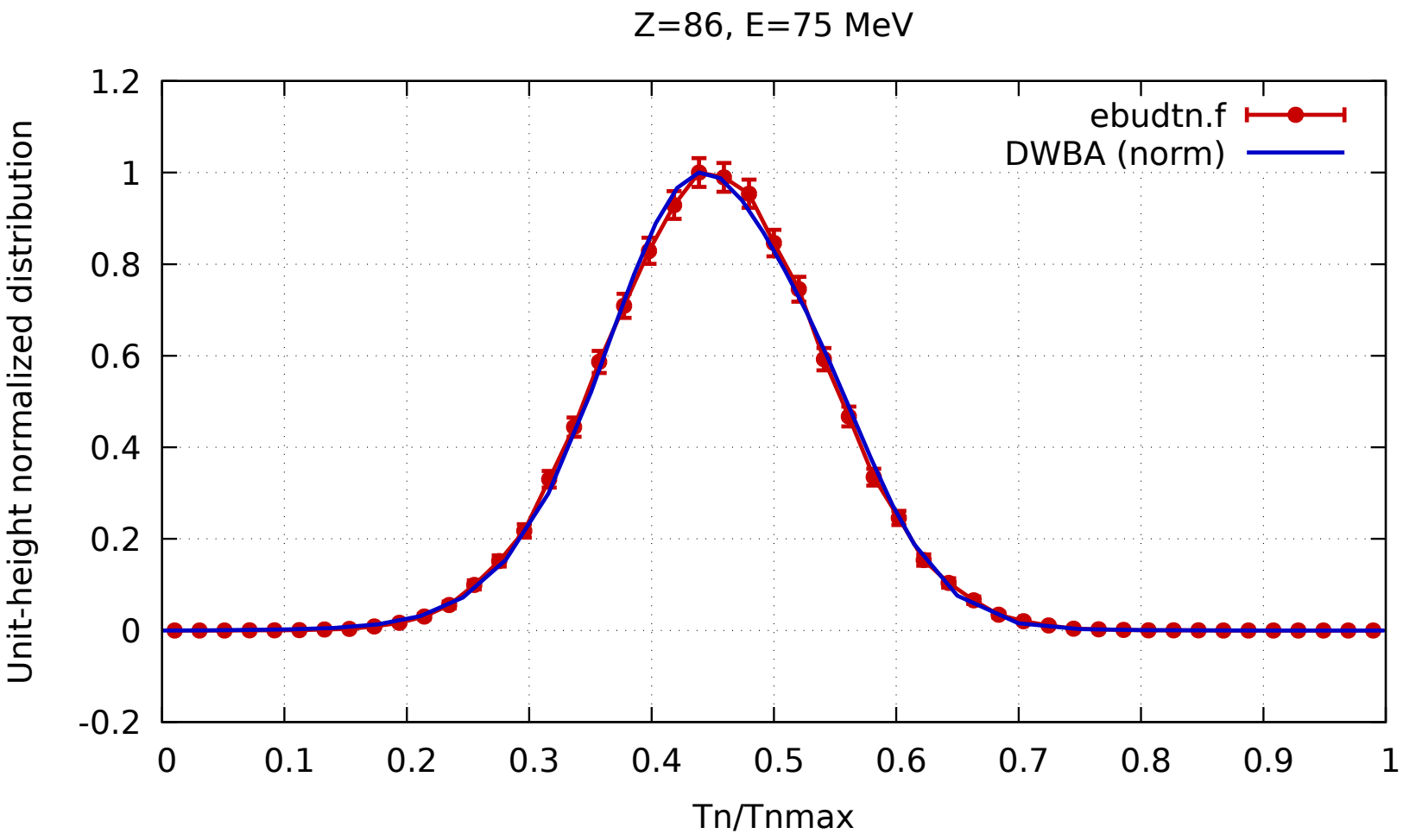
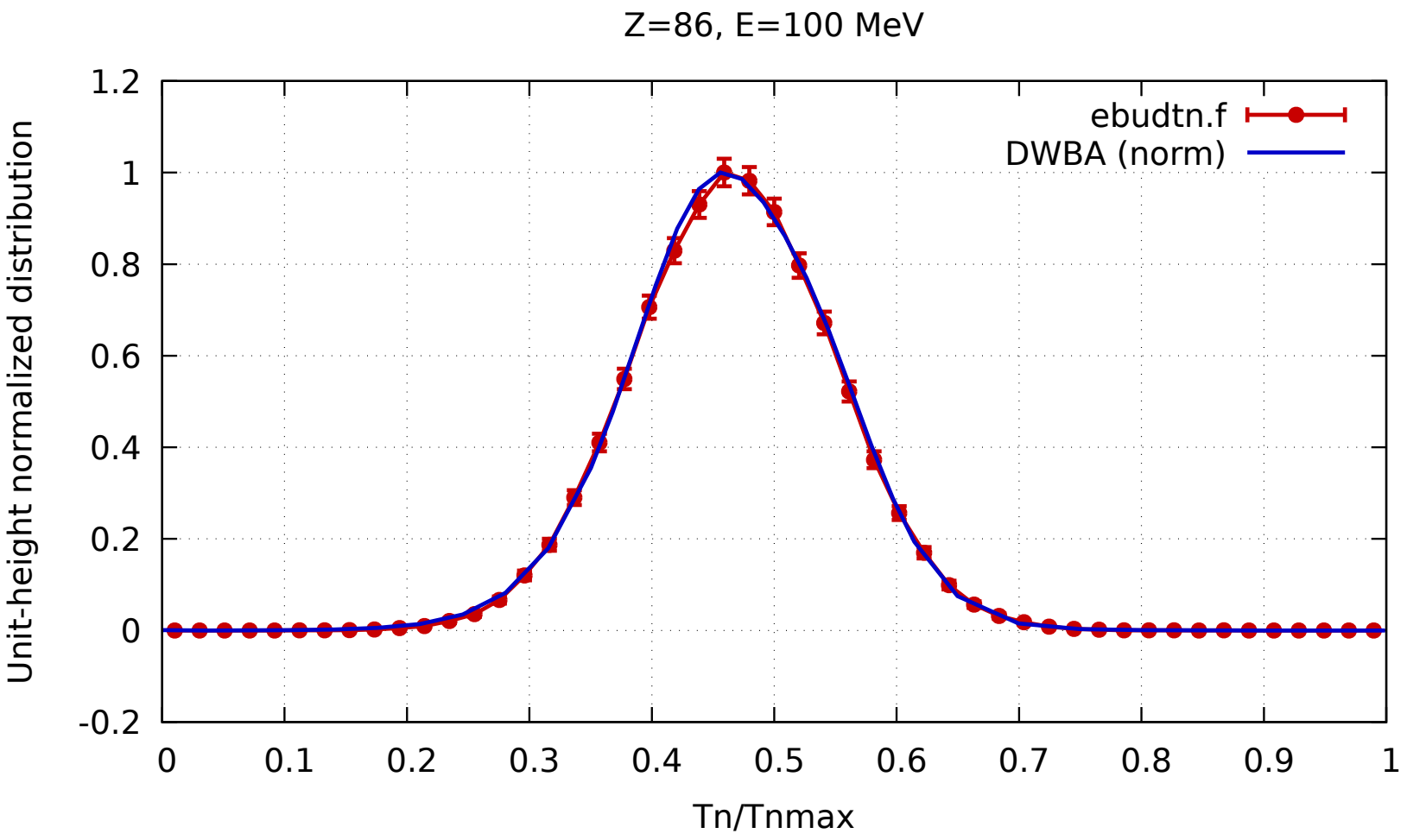


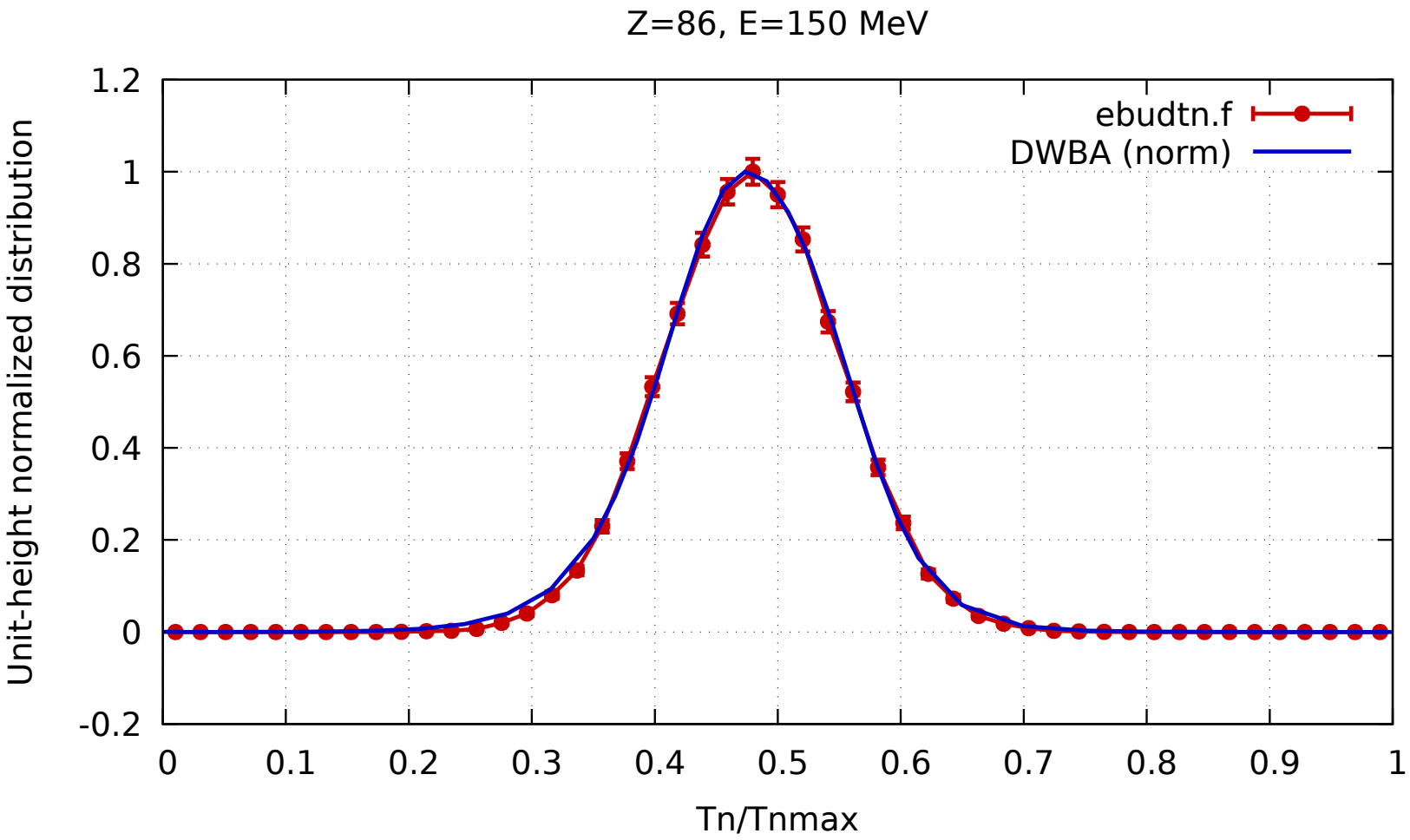
$\mathrm{d}\sigma_{EBU}/\mathrm{d}E_n$ for ^{211}Rn

As a parametrization example:
Unit-height-normalized distribution of neutron energies T_n in units of maximum kinematically allowed $T_{n,max}$:



$\mathrm{d}\sigma_{EBU}/\mathrm{d}E_n$ for ^{211}Rn

As a parametrization example:
Unit-height-normalized distribution of neutron energies T_n in units of maximum kinematically allowed $T_{n,max}$:



$\mathrm{d}\sigma_{EBU}/\mathrm{d}E_n$ for ^{211}Rn

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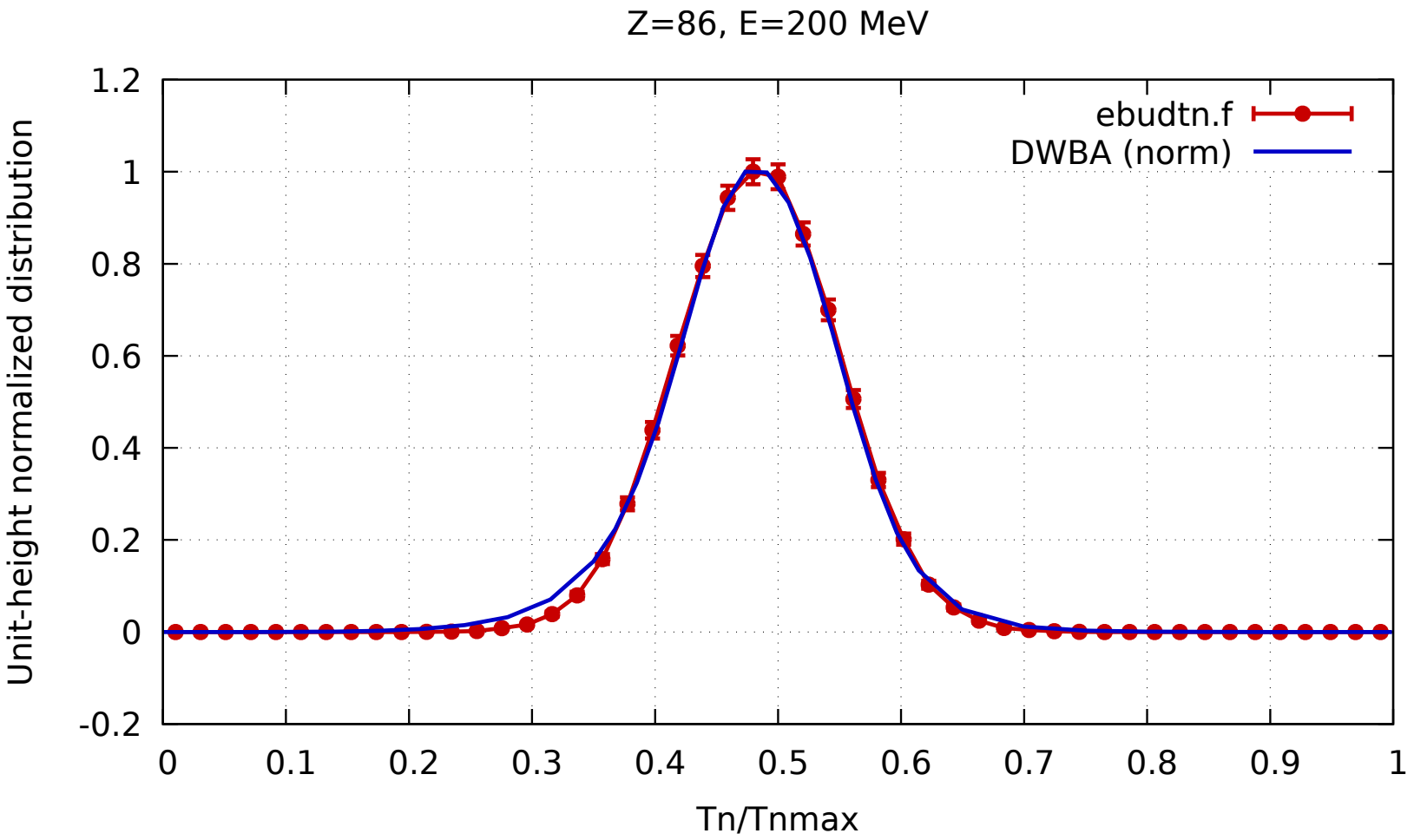
Unit-height-normalized distribution of neutron energies T_n in units of maximum kinematically allowed $T_{n,max}$:



Sampling of deuteron elastic break-up events

In the simulation, for a deuteron with given energy and a target Z, A:

- Determine elastic break-up cross section.
- Sample neutron energy from parametrized $\mathrm{d}\sigma/\mathrm{d}E_n$.
- Sample neutron polar angle from $\mathrm{d}^2\sigma/\mathrm{d}E_n/\mathrm{d}\theta_n$.
- Sample proton polar angle from $\mathrm{d}^3\sigma/\mathrm{d}E_n/\mathrm{d}\theta_n/\mathrm{d}\theta_p$.
- Sample proton azimuthal angle from $\mathrm{d}^4\sigma/\mathrm{d}E_n/\mathrm{d}\theta_n/\mathrm{d}\Omega_p$.
- Construct $\mathbf{p}_n$, $\mathbf{p}_p$, $\mathbf{p}_t$ and sample a global rotation angle homogeneously around direction of incidence.

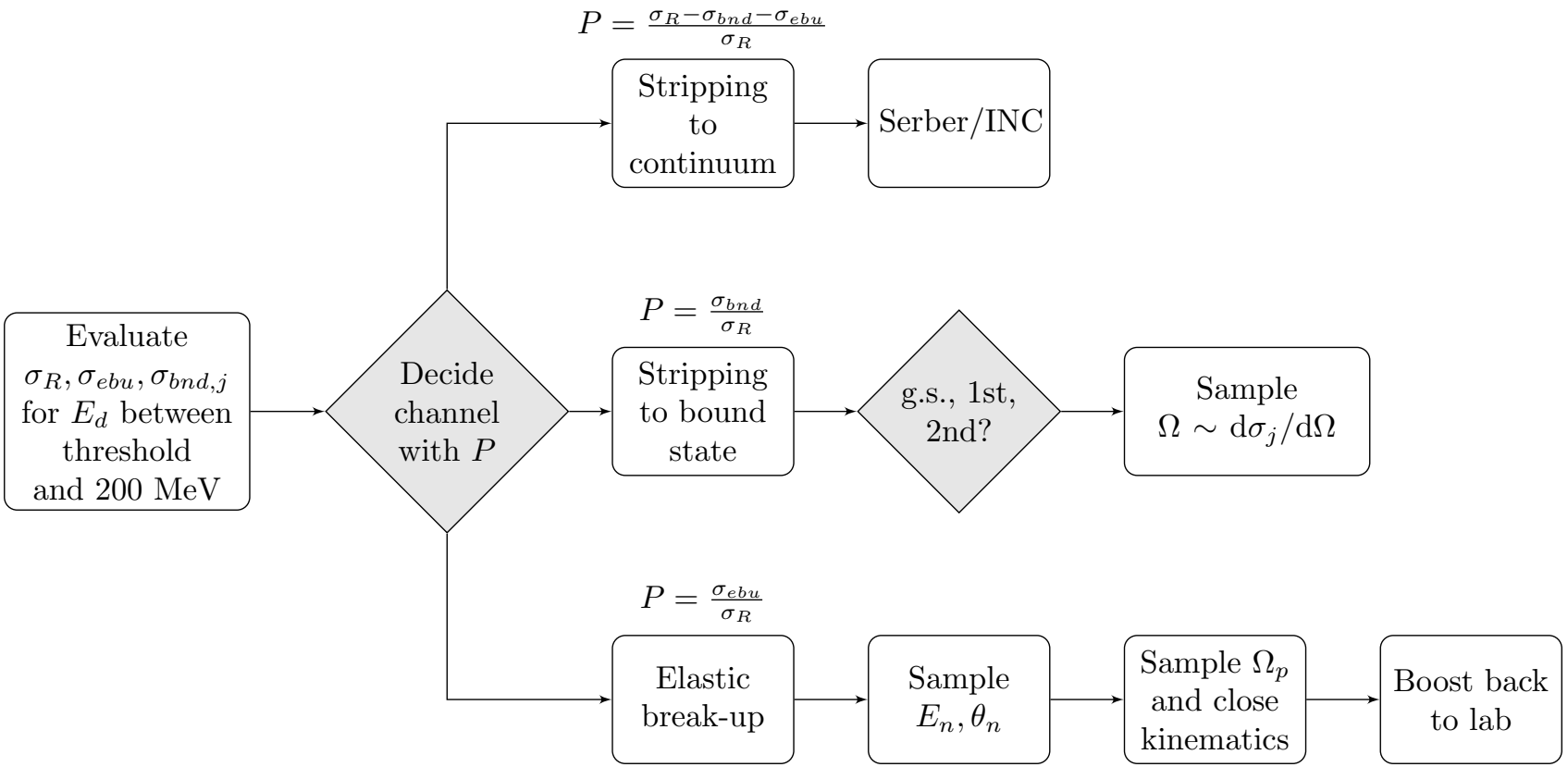
Summary

- We have set up a simple DWBA scheme to calculate the 4-differential cross section for elastic deuteron break-up (Coulomb+nuclear OPM).
- Comparisons at 4-differential level have been encouraging.
- Integrated cross section in reasonable agreement with similar calculations.
- Have had to disregard spin-orbit coupling terms in the potential.
- Have had to suppress nuclear transition amplitude at fwd emission to reproduce the apparent Coulomb dominance seen experimentally.
- We have parametrized partial integrals of the differential cross section and adapted them for sampling of elastic break-up events in FLUKA.
- Ahead: stripping to bound states, stripping to the continuum.

Outlook

Stripping to bound states:

- Involves nuclear structure (spectroscopic factors).
- We will probably not have a systematic approach, but at least for selected relevant reactions.



Additional material

The vertex function F(x)

- Zero-range approximation (ZRA):

$$V_{nc}(r)\Phi_p^{lm}(r) = D_0\delta(\mathbf{r}), \qquad D_0 \sim 123.5 \text{ MeV fm}^{3/2}$$
$$F(x) = D_0$$

- Baur-Trautmann approx (BTA): $x = q_{n-pA}$ (wrt CM of p-d)

$$F(x) = -V_0 \frac{(\alpha+\mu)^{1/2}(2\alpha+\mu)^{3/2}}{(\alpha+\mu)^2 + x^2}$$

- Asymptotic-momentum approximation (AMA) [1], AKA adiabatic model [2]: $x = |\mathbf{q}_v - \mathbf{q}_p m_v/(m_v + m_c)|$

Zadro2002: ZRA gives best results, BTA underestimates. We find BTA underestimates too, ZRA gives a bit too high cross sections compared to CDCC, AMA is more reasonable.

[1] Zadro2002, [2] Tostevin1998

Finite-range correction

$$\mathcal{T}_{\text{post}}^{\text{DWBA}} \sim \langle \chi^{(-)}_{\mathbf{p}_{p-A}}(\mathbf{r}) \chi^{(-)}_{\mathbf{p}_{n-pA}}(\mathbf{r}) | \Lambda(r) \chi^{(+)}_{\mathbf{p}_{d-A}}(\mathbf{r}) \rangle \tag{1}$$

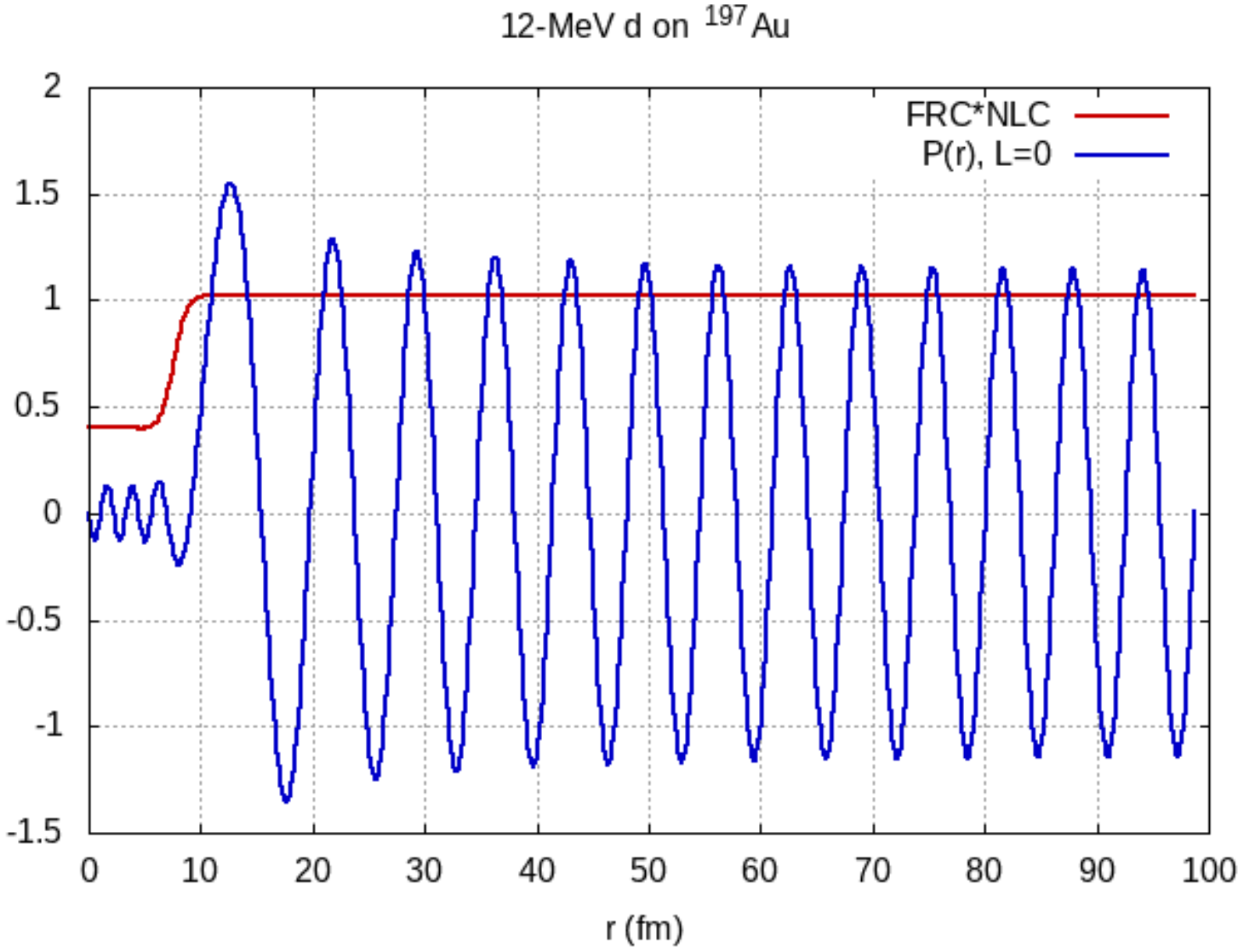
$$\Lambda(r) = \left[1 - \frac{2\mu_{pn}}{\hbar^2} R^2 \left(V_p(r) + V_n(\xi r) - V_d(r) + \epsilon_d \right) \right]^{-1} \tag{2}$$

$$R = 0.667 \text{ fm} \tag{3}$$

Non-locality correction

$$\chi(\mathbf{r}) \to \chi(\mathbf{r})\exp\left[\frac{\beta_i^2}{8}\frac{2\mu}{\hbar^2}V_{\mathsf{nuc}}(r)\right], \qquad \beta_{n,p} = 0.85 \text{ fm}, \qquad \beta_d = 0.54 \text{ fm} \tag{4}$$

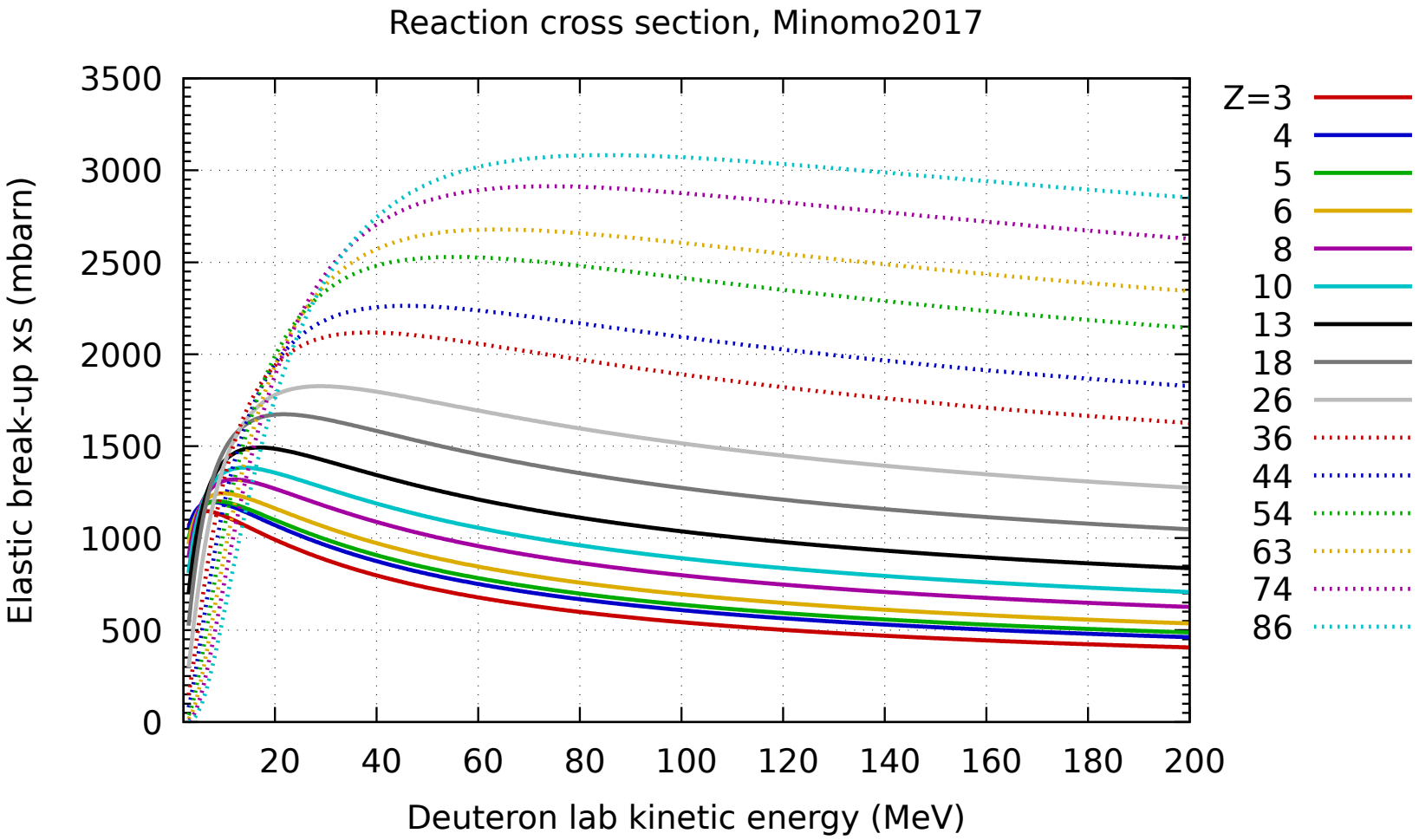
Effect of FRC+NLC



DWBA prior form matrix element for elastic break-up

$$\mathcal{T} = \langle \chi_d^- \phi_{np} | U_n(r_n) + U_p(r_p) | \phi_d \chi_d^+ \rangle \tag{5}$$

Reaction cross section systematics



K. Minomo, *J. Nucl. Sci. Technol* **54** 1 127-130 (2017)

Cross section for Coulomb dissociation

Nordsieck fortunately solved the matrix-element integral:

PHYSICAL REVIEW VOLUME 93, NUMBER 4 FEBRUARY 15, 1954

Reduction of an Integral in the Theory of Bremsstrahlung

A. Nordsieck

After some (a lot of) patience:

$$\mathcal{T}_{ba,0}^{DWBA+,C} = 2\pi D_0 \mathrm{e}^{\pi(\eta_\mathsf{d}-\eta_\mathsf{p})/2}\Gamma(1+\mathrm{i}\eta_\mathsf{p})\Gamma(1+\mathrm{i}\eta_\mathsf{d})\alpha^{-2}\left(\frac{\gamma}{\alpha}\right)^{\mathrm{i}\eta_\mathsf{d}}\left(\frac{\gamma+\delta}{\gamma}\right)^{\mathrm{i}\eta_\mathsf{p}}$$
$$\Big\{ \ {}_2F_1(1+\mathrm{i}\eta_\mathsf{d},-\mathrm{i}\eta_\mathsf{p},1,z)\alpha\left[\gamma^{-1}k_\mathsf{d}(\eta_\mathsf{d}-\eta_\mathsf{p})+(\gamma+\delta)^{-1}\eta_\mathsf{p}(k_\mathsf{d}+k_\mathsf{p})\right]$$
$$- \ {}_2F_1(2+\mathrm{i}\eta_\mathsf{d},1-\mathrm{i}\eta_\mathsf{p},2,z)(1+\mathrm{i}\eta_\mathsf{d})\eta_\mathsf{p}(\gamma+\delta)^{-1}\left[k_\mathsf{p}(\gamma+\alpha)-k_\mathsf{d}\beta-(k_\mathsf{d}+k_\mathsf{p})\alpha z\right] \Big\}$$

N. Michel, *Comput. Phys. Commun.* **178** 535–551 (2008).

Current...

the worldwide technical progress, high ranking advisory boards have evaluated strategy scenarios to meet as early as possible the growing need of environmentally acceptable electricity [1]. A major result of these advisory boards was, that a properly organized and funded fusion development program could lead to a prototype fusion power station putting electricity into the grid within 30 years, with commercial fusion power following 10 years later [2,3]. A common feature of the developed strategy scenarios is that they include as indispensable elements ITER as major next step machine, the dedicated intense neutron source International Fusion Materials Irradiation Facility (IFMIF) for materials qualification, and a DEMO

IFMIF will achieve this using two 40 MeV deuteron continuous-wave (CW) linear accelerators each with 125 mA beam current striking a single thick, flowing Li-target under a 20° impinging angle, thus providing an intense neutron flux of about 10^{18} n/m^2 s with a broad peak near 14 meV. As the neutrons produced within the common beam footprint of 5 cm × 20 cm are mainly collided in forward direction, the test modules housing the specimens to be irradiated are positioned immediately adjacent of the Li-target. IFMIF is presently with a beam power of 10 MW on a single target the worlds' most powerful accelerator driven neutron source.

Fig. 1 shows a bird's eye view of the cost-optimized

(d,pn) coincidence measurement

PHYSICAL REVIEW C VOLUME 58, NUMBER 4 OCTOBER 1998

Mechanism of the forward-angle (d,pn) reaction at intermediate energies

H. Okamura,[1,*] S. Ishida,[1,†] N. Sakamoto,[1,‡] H. Otsu,[1,§] T. Uesaka,[1,‡] T. Wakasa,[1,‖] H. Sakai,[1] T. Niizeki,[2,¶] H. Toyokawa,[2,**] Y. Tajima,[2,††] H. Ohnuma,[2,‡‡] M. Yosoi,[3] K. Hatanaka,[4] and T. Ichihara[5]

[1]*Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan*

[2]*Department of Physics, Tokyo Institute of Technology, Meguro-ku, Tokyo 152-8551, Japan*

[3]*Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan*

[4]*Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*

[5]*The Institute of Physical and Chemical Research (RIKEN), Saitama 351-0198, Japan*

(Received 23 March 1998)

The elastic breakup of the deuteron has been measured in a kinematical condition of $\theta_p = \theta_n = 0°$ at $E_d = 140$ MeV and 270 MeV on seven targets ranging from $Z=6$ to 82. A double-peaked structure with its minimum at $E_p \simeq E_n$ is observed for all the measured triple-differential cross sections, which indicates a large contribution from Coulomb breakup. While a symmetric shape is expected for an $E1$ transition, the observed proton-energy distribution is asymmetric, showing considerable dependence on both the target and the incident energy. These data and previous data at 56 MeV have been analyzed using the post-form of the distorted-wave Born approximation (DWBA) theory. Although the calculations account reasonably well for the asymmetric shape, by including the nuclear potential, they consistently overestimate the magnitudes of the cross sections over the whole measured region. Limitations of the post-form DWBA are discussed. [S0556-2813(98)03910-7]

PACS number(s): 24.10.−i, 24.50.+g, 25.45.De

(d,pn) coincidence measurement

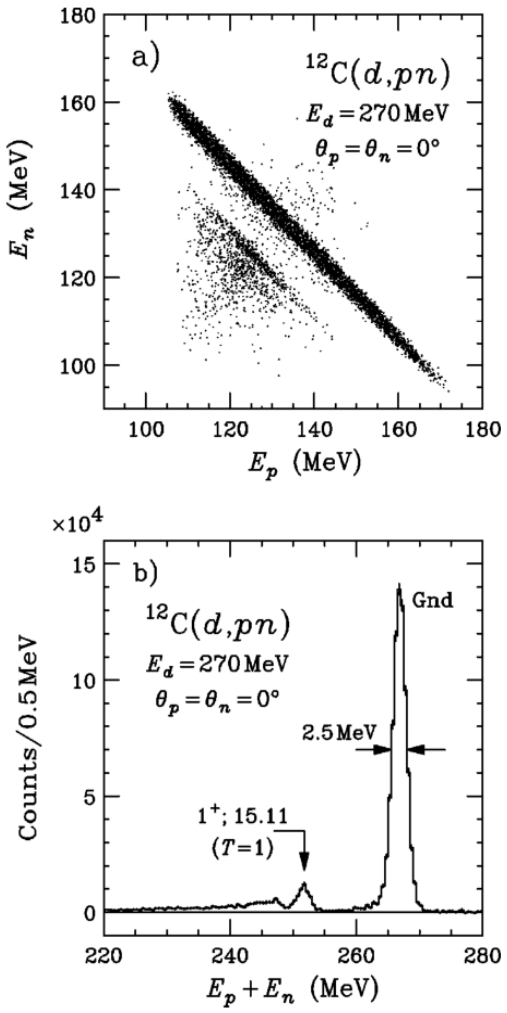


FIG. 2. (a) Two-dimensional and (b) summed energy spectra of the coincidence protons and neutrons from the deuteron breakup on the ^{12}C target at 270 MeV. The effects of accidental coincidence are corrected for.

Form factor

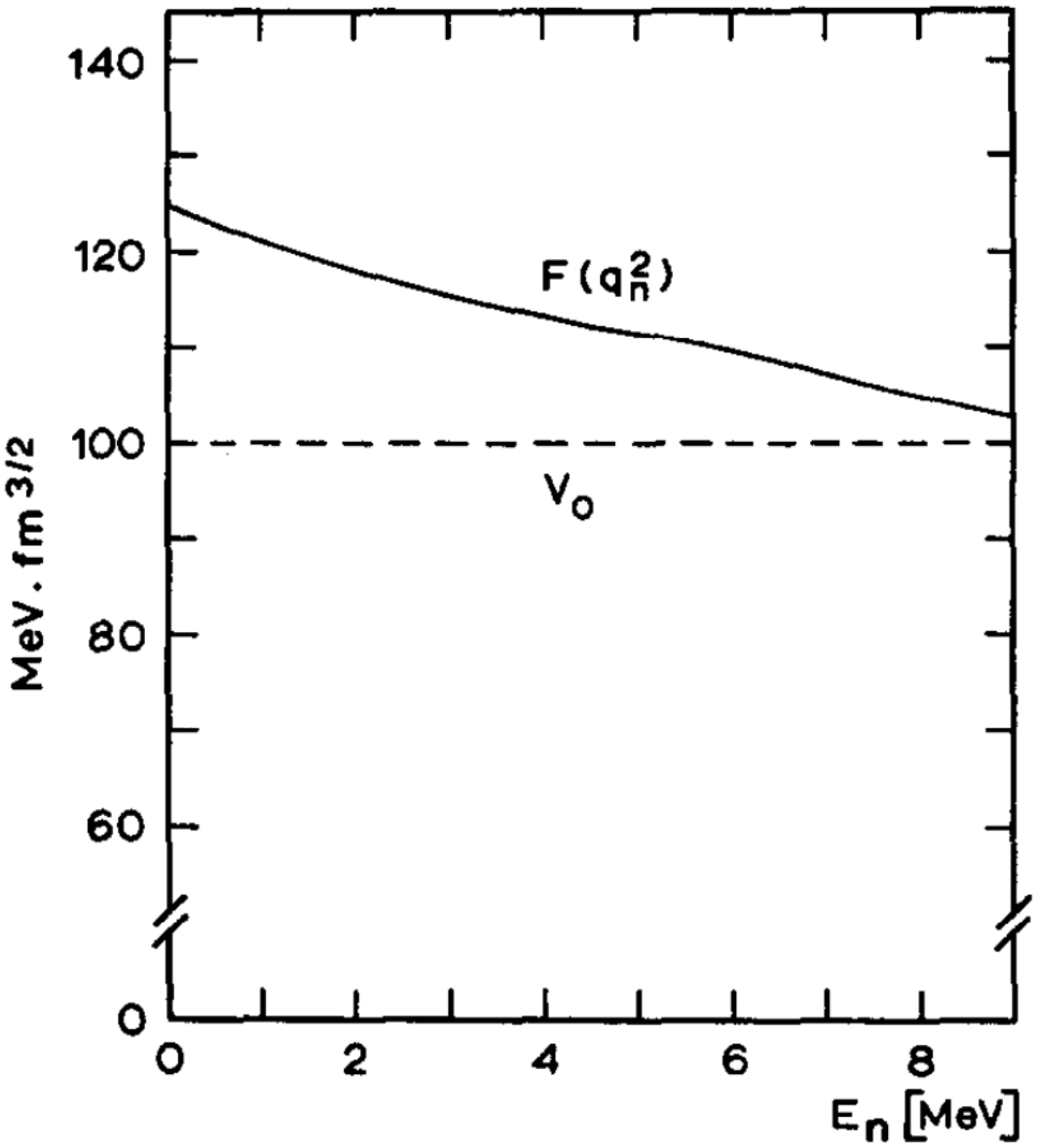


Fig. 2. The form factor $F(q_n{}^2)$ using a Hulthén wave function for the deuteron.

Form factor - various approximations

Zadro2002

TABLE II. Momenta $\mathbf{q}_{nc}$ which appear in the vertex function $F(\mathbf{q}_{nc})$ for different approximation methods: the zero-range approximation (ZRA), the Baur-Trautmann approximation (BTA), the local momentum approximations (ILMA and FLMA), and the asymptotic momentum approximations (IAMA and FAMA).

ZRA	BTA	ILMA	FLMA	IAMA	FAMA
$\mathbf{0}$	$\mathbf{k}+\alpha\mathbf{K}_f$	$\mathbf{k}-\alpha(\mathbf{K}_i'-\mathbf{K}_f)$	$\mathbf{k}-\alpha(\mathbf{k}_{ct}'-\mathbf{k}_{ct})$	$\mathbf{k}-\alpha(\mathbf{K}_i-\mathbf{K}_f)$	$\mathbf{k}$

Switch function

$$f(E_d, \theta_n, \theta_p) = 1 - \left[\frac{1}{1 + \mathrm{e}^{(\theta_n - t_n)/a_n}} \frac{1}{1 + \mathrm{e}^{(\theta_p - t_p)/a_p}} \frac{1}{1 + \mathrm{e}^{(E_n - u_n)/b_n}} \right]$$

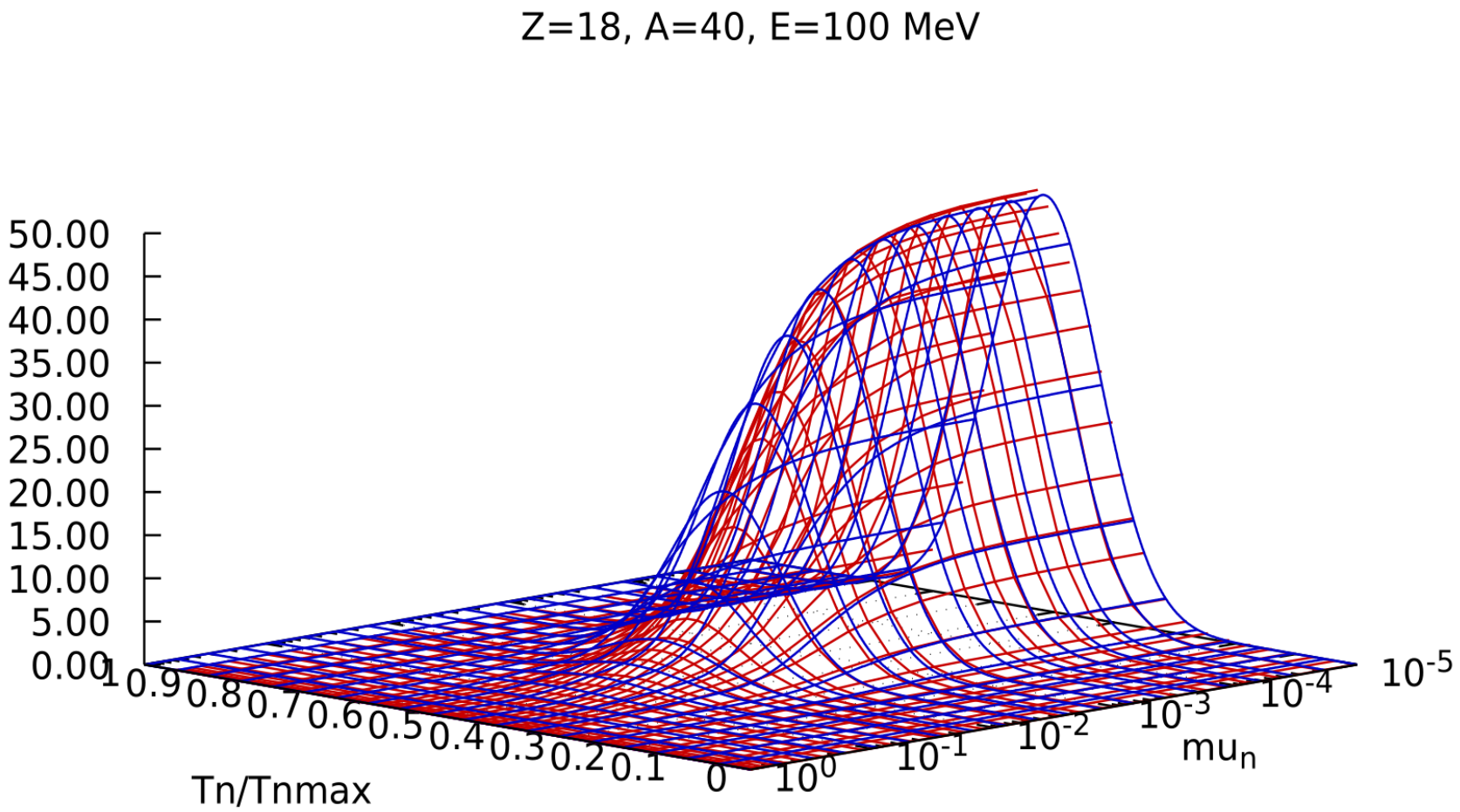
$$u_n = 120\text{MeV}, \qquad b_n = 20 \text{ MeV}$$

$$t_p = t_n = \text{ from 25 deg (light) to 5 deg (heavy target) degrees}$$

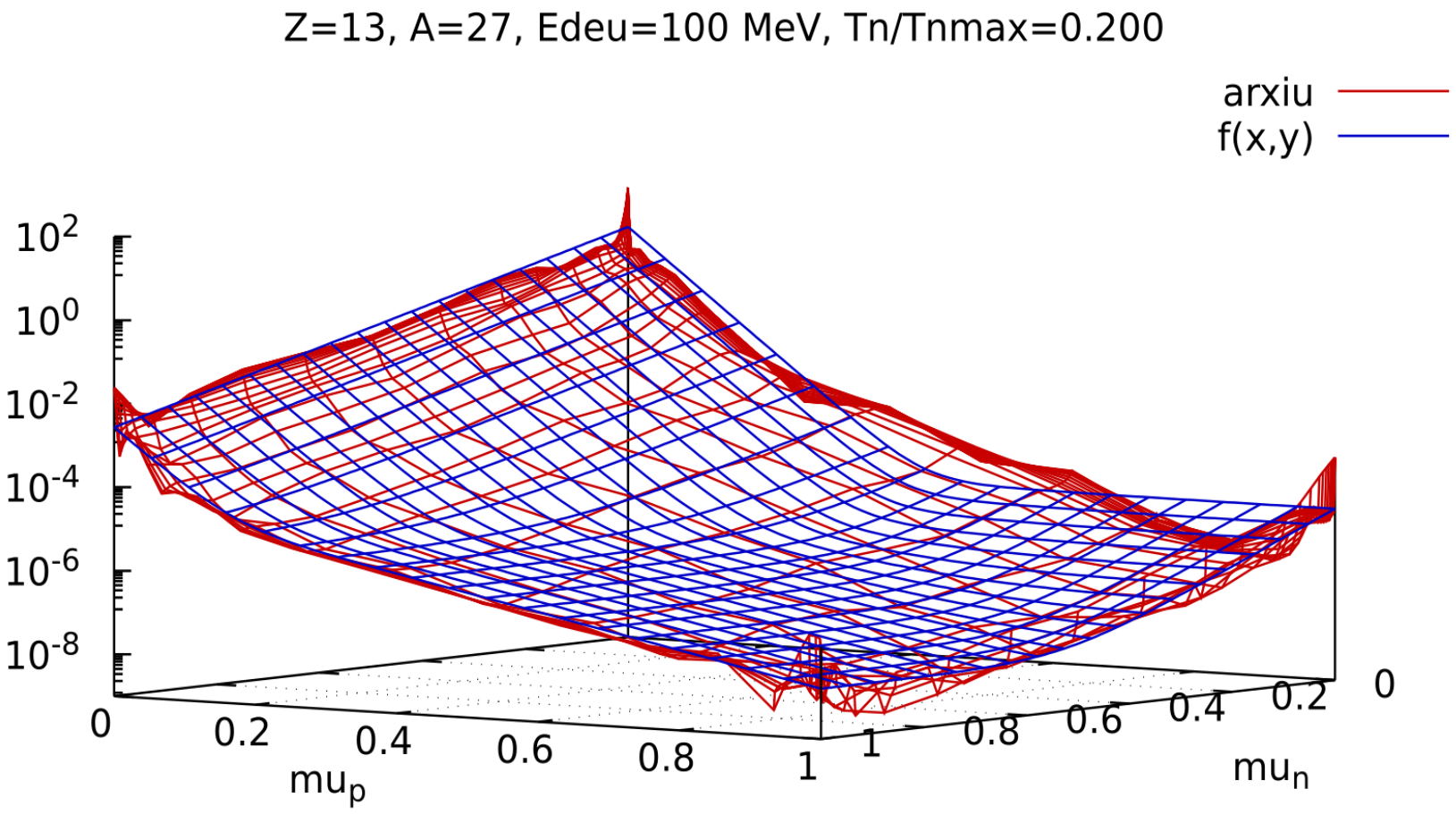
$$a_n = a_p = 3 \text{ deg}$$

Neutron polar angle parametrization

(red: DWBA, blue: fit)



Proton polar angle parametrization



Proton azimuthal angle parametrization

(at centroid of θ_n, θ_p)