Probing nuclear structure with neutron transfer reactions

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Varenna, June 11th 2018

Outline

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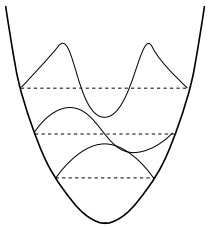
Extracting the structure information: a standard approach

STRUCTURE

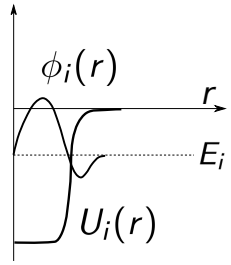
$(H-E)|\Psi(\xi_A)\rangle = 0$ many-body Hamiltonian



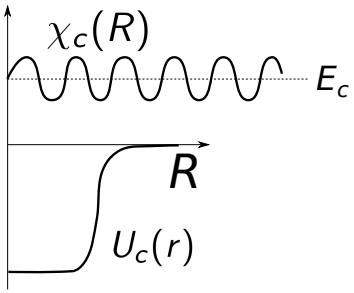
$|\Psi_i(\xi_A)\rangle$, E_i many-body wfs and energies

$S_i = \langle \varphi_i(r)\psi_0(\xi_{A-1})|\Psi\rangle$ "spectroscopic amplitudes"

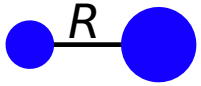
REACTIONS



s-p wfs $\phi_i(r)$
interactions $U_i(r)$



scattering wfs $\chi_c(R)$
optical potentials $U_c(r)$



$$\tilde{\sigma} \sim \left|\langle \chi_{c'}^{(-)}(R)\phi_f(r)|U_i(r)|\chi_c^{(+)}(R)\phi_i(r)\rangle\right|^2$$

$$\sigma = S_i^2 \tilde{\sigma}$$

- factorization of structure and reactions
- can suffer from inconsistency between the two schemes

Extracting the structure information: an alternative program for (d, p)

1. Describe the structure of the 2–body subsystems in some given framework of choice.
2. Employ the same quantum many–body methods to work out the interactions V_{An}, V_{Ap}, V_{pn}.
3. Write down the resulting 3–body Hamiltonian H.
4. Obtain cross sections from H using controlled approximations.

3–body Hamiltonian

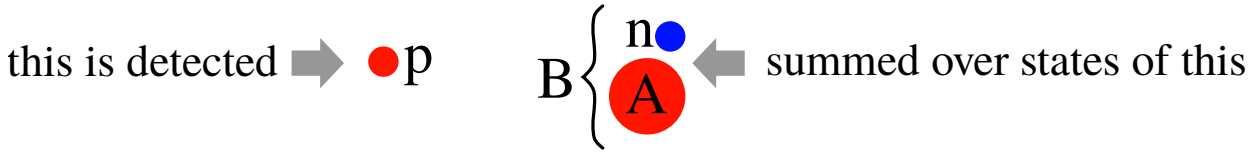
$$H = T + V_{An}(r_n, r'_n, E_n, J_n, \pi_n) + V_{Ap}(r_p, r'_p, E_p, J_p, \pi_p) + V_{pn}(r_{pn}, r'_{pn}, E_{pn}, J_{pn}, \pi_{pn})$$

Disclaimer

Still not the end of the story! 3–body forces V_{Anp} not taken into account at this stage.

Outline

Framework for inclusive (d, p) experiments



$$\frac{d\sigma(E)}{dE_p} \sim \sum_n |\langle \phi_p \phi_B^n | V | \Psi \rangle|^2 \, \delta(E - E_p - E_B^n)$$

$$= \sum_n \langle \Psi | V^\dagger | \phi_p \phi_B^n \rangle \delta(E - E_p - E_B^n) \, \langle \phi_B^n \phi_p | V | \Psi \rangle$$

$$= \lim_{\eta \to 0} \mathrm{Im} \langle \Psi | V^\dagger | \phi_p \rangle \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_p - H_B + i\eta} \langle \phi_p | V | \Psi \rangle$$

Main challenges

- describe propagator $G_B = \lim_{\eta \to 0} \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_A - H_B + i\eta}$
- describe wave function Ψ

Optical reduction of G

Making use of the approximation $\Psi \approx \chi_d(r_d)\phi_d(r_{pn})\phi_A(\xi_A)$,

$$\begin{aligned}\langle \chi_d\phi_d\phi_A| V |\chi_p\rangle G_B \langle\chi_p| V |\phi_A\chi_d\phi_d\rangle \\ &= \langle \chi_d\phi_d| V |\chi_p\rangle \langle\phi_A|G_B|\phi_A\rangle \langle\chi_p| V |\chi_d\phi_d\rangle \\ &= \langle \chi_d\phi_d| V |\chi_p\rangle G_B^{sp} \langle\chi_p| V |\chi_d\phi_d\rangle ,\end{aligned}$$

where G_B^{sp} is the optical reduction of G_B

$$G_B^{sp} = \lim_{\eta\to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}, r'_{An}, E_n, J, \pi) + i\eta},$$

now $U_{An} = V_{An} + iW_{An}$ and G_B^{sp} are single–particle, tractable operators (though both non–local!).

U_{An} is the target–neutron optical potential/self–energy/effective interaction→ non–local, energy–spin–parity–dependent. The role of structure theory is to provide it.

Factorization revisited

The strength at given $E_n, \mathbf{r}_n, \mathbf{r}'_n, J, \pi \rightarrow \mathrm{Im}\, G_B^{sp}(E_n, \mathbf{r}_n, \mathbf{r}'_n, J, \pi)$
inclusive cross section $\rightarrow$ folding of strength with "breakup probability density"

$$\rho_{bu}(\mathbf{r}_n, \mathbf{k}_i, \mathbf{k}_f) = |\big(\chi_p(\mathbf{r}_p, \mathbf{k}_f)|U_{An}|\chi_d(\mathbf{r}_d, \mathbf{k}_i)\phi_d(\mathbf{r}_{pn})\rangle|^2$$

$$\frac{d\sigma}{dEd\Omega} \sim \int \underbrace{\mathrm{Im}\, G_B^{sp}(E_n, \mathbf{r}_n, \mathbf{r}'_n, J, \pi)}_{structure} \underbrace{\rho_{bu}(\mathbf{r}'_n, \mathbf{k}_i, \mathbf{k}_f)}_{reaction} d\mathbf{r}_n d\mathbf{r}'_n$$

- Provided χ_d is derived from V_{An} and V_{Ap} (work in progress), all elements come from a single 3–body Hamiltonian.
- No spectroscopic factors needed.

Non–elastic and elastic breakup cross sections

the imaginary part of G_B^{sp} splits in two terms

$$\mathrm{Im}\, G_B^{sp} = \overbrace{-\pi \sum_{k_n} |\chi_n\rangle \delta\left(E - E_p - \frac{k_n^2}{2m_n}\right) \langle\chi_n|}^{\text{elastic breakup}} + \overbrace{G_B^{sp\dagger} W_{An}\, G_B^{sp}}^{\text{non elastic breakup}} ,$$

we define the neutron wavefunction $|\psi_n\rangle = G_B^{sp} \langle\chi_p| V \,|\chi_d \phi_d\rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{NEB} = -\frac{2}{\hbar v_d} \rho(E_p) \langle\psi_n| W_{An} |\psi_n\rangle ,$$

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{EB} = -\frac{2}{\hbar v_d} \rho(E_p)\rho(E_n) \left|\langle\chi_n\chi_p| V |\chi_d\phi_d\rangle\right|^2 ,$$

Outline

Choosing the potential/propagator

Dispersive optical model (DOM)

- Elastic scattering and reaction cross section fit $\rightarrow$ dispersion $\rightarrow$ negative energies. Local and non–local versions.
- Limited availability. Good reproduction of scattering and low–energy observables. Satisfy sum rules (dispersive).

Coupled cluster (CC)

- Ab–initio calculation $\rightarrow$ predictive power. Non–local. Dispersive.

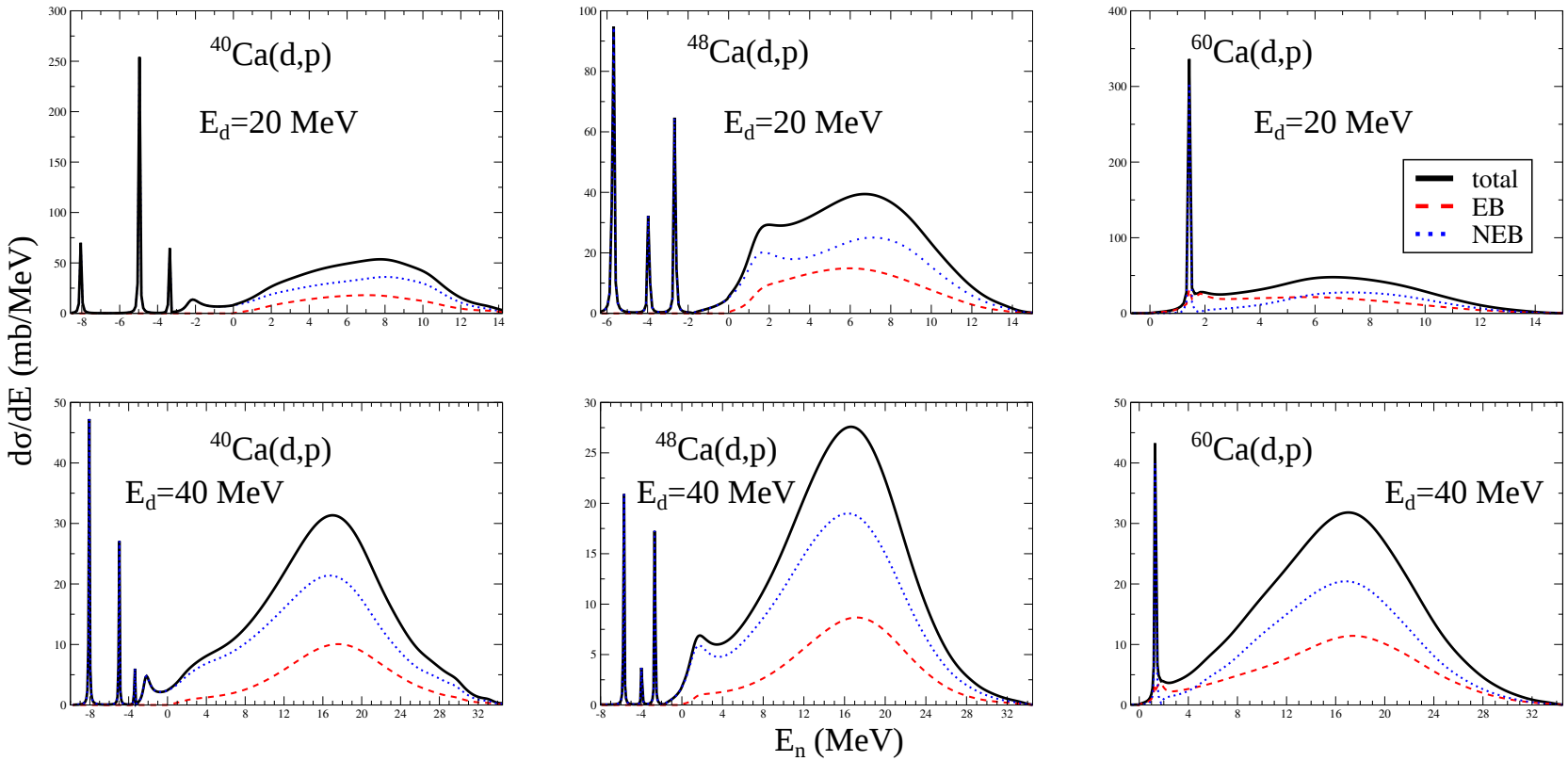
Nuclear Field Theory (NFT)

- Describes the interweaving of collective (surface and pairing) modes and single–particle degrees of freedom. Non–local. Dispersive.

Others

Any structure calculation providing overlaps $\langle A+1|a^\dagger|A\rangle$ can be implemented in the reaction formalism.

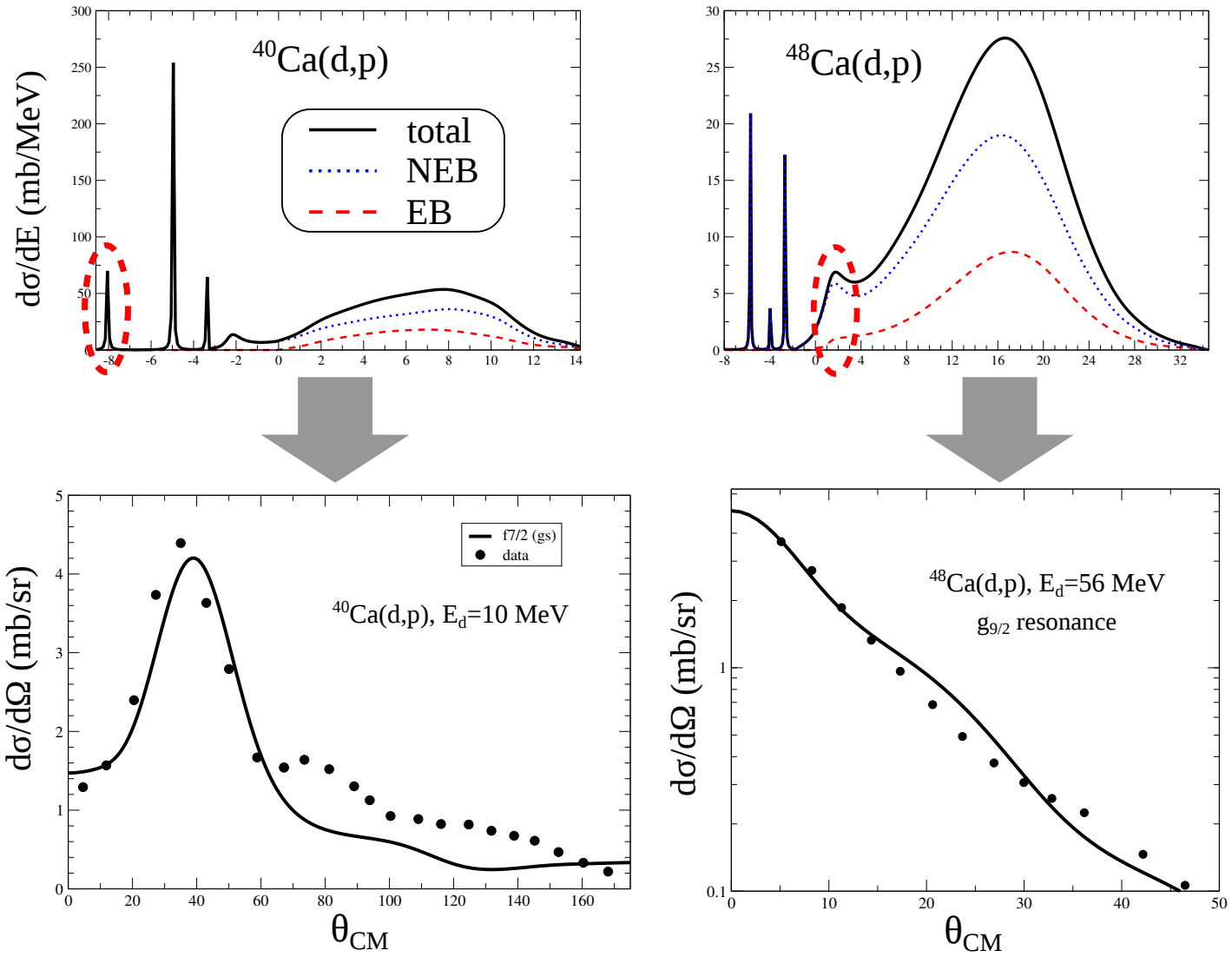
DOM: Calcium isotopes



GP *et al.*, Eur. Phys. J. A **53** (2017) 178.

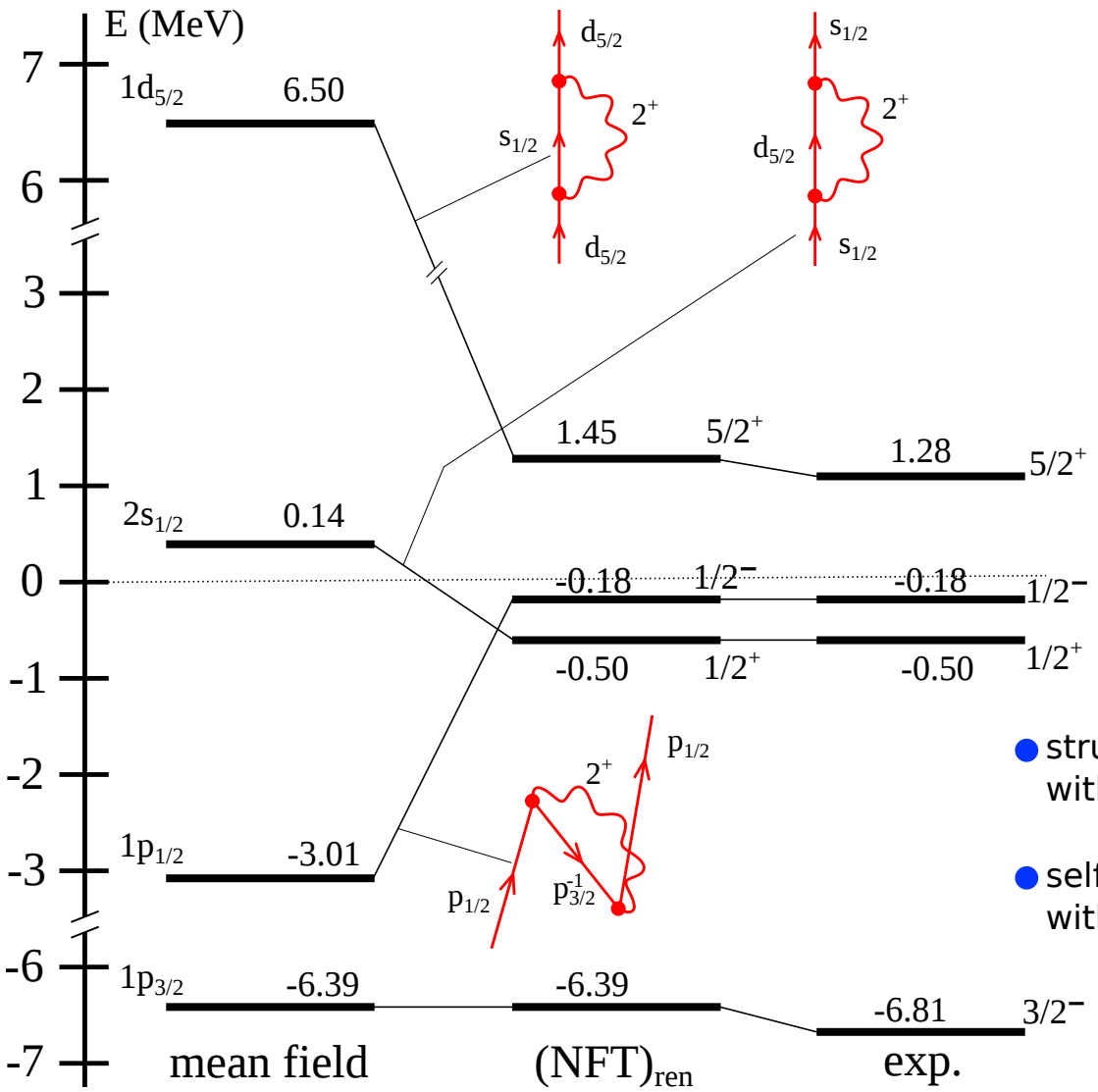
- DOM used to compute (d, p) cross sections on Ca isotopes.
- Both bound and continuum neutron states described.
- EB and NEB contributions to the proton singles are disentangled.
- DOM can be extrapolated to unknown territory (^{60}Ca).

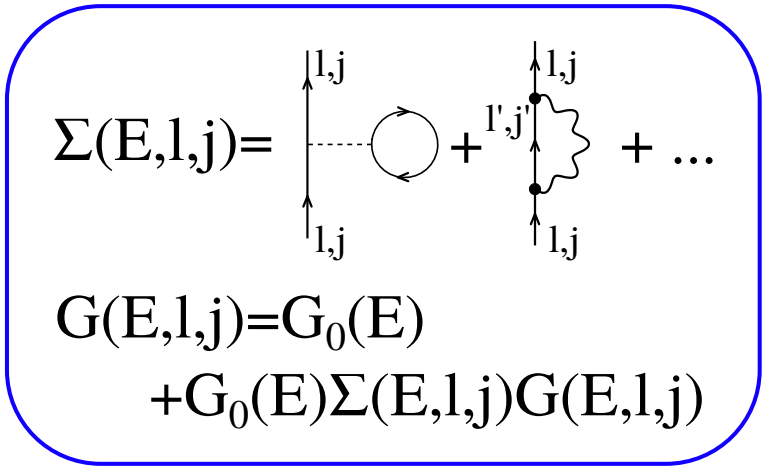
DOM: Calcium isotopes



Absolute transfer cross sections without spectroscopic factors.

^{11}Be: Structure and reactions of a halo nucleus in NFT

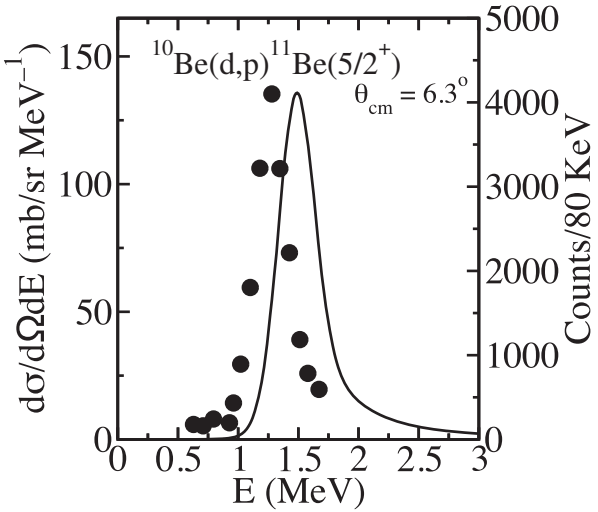


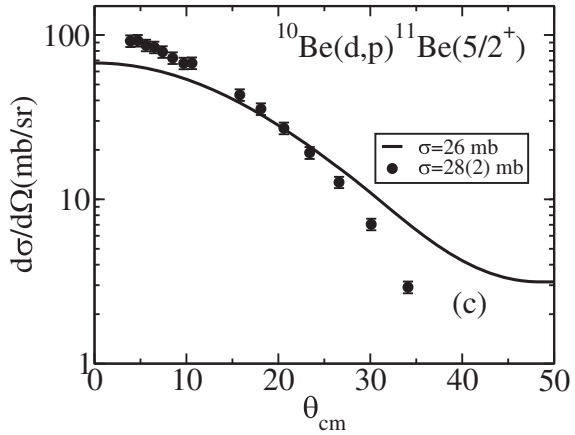


- structure (energies, widths) reproduced within an EFT-like framework (NFT)
- self energies and Green's functions obtained within the same framework

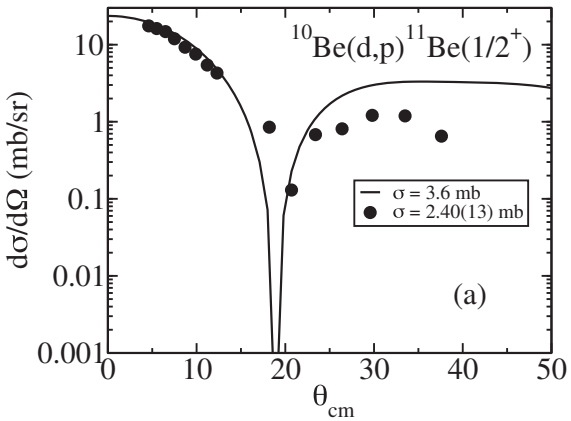
^{11}Be: Structure and reactions of a halo nucleus in NFT

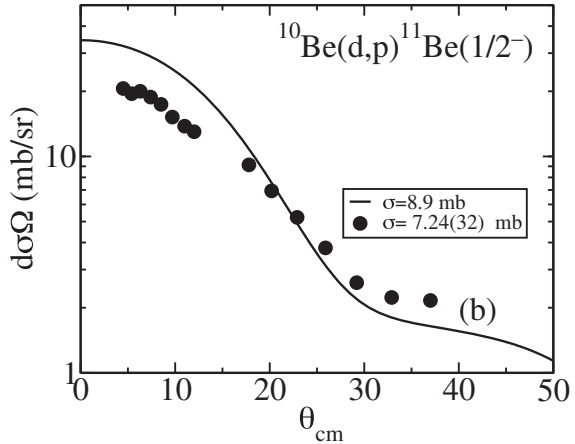
Transfer cross sections and strength functions well reproduced with the structure ingredients $\Sigma(E, l, j)$ and $G(E, l, j)$.





continuum
(5/2$^+$ resonance)



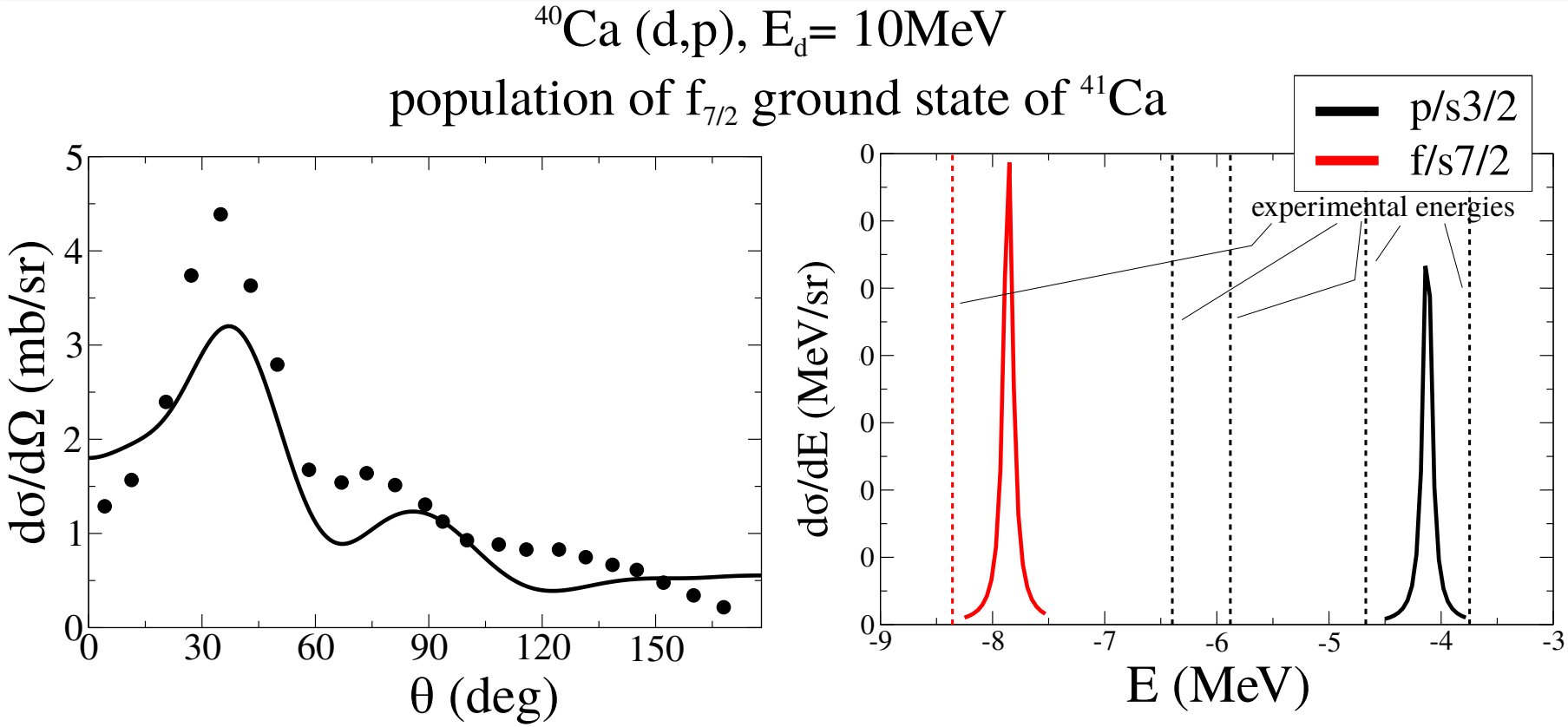


bound states
(1/2$^+$ and 1/2$^-$)

Barranco, GP, Broglia, Vigezzi, Phys. Rev. Lett. **119**, 082501 (2017)

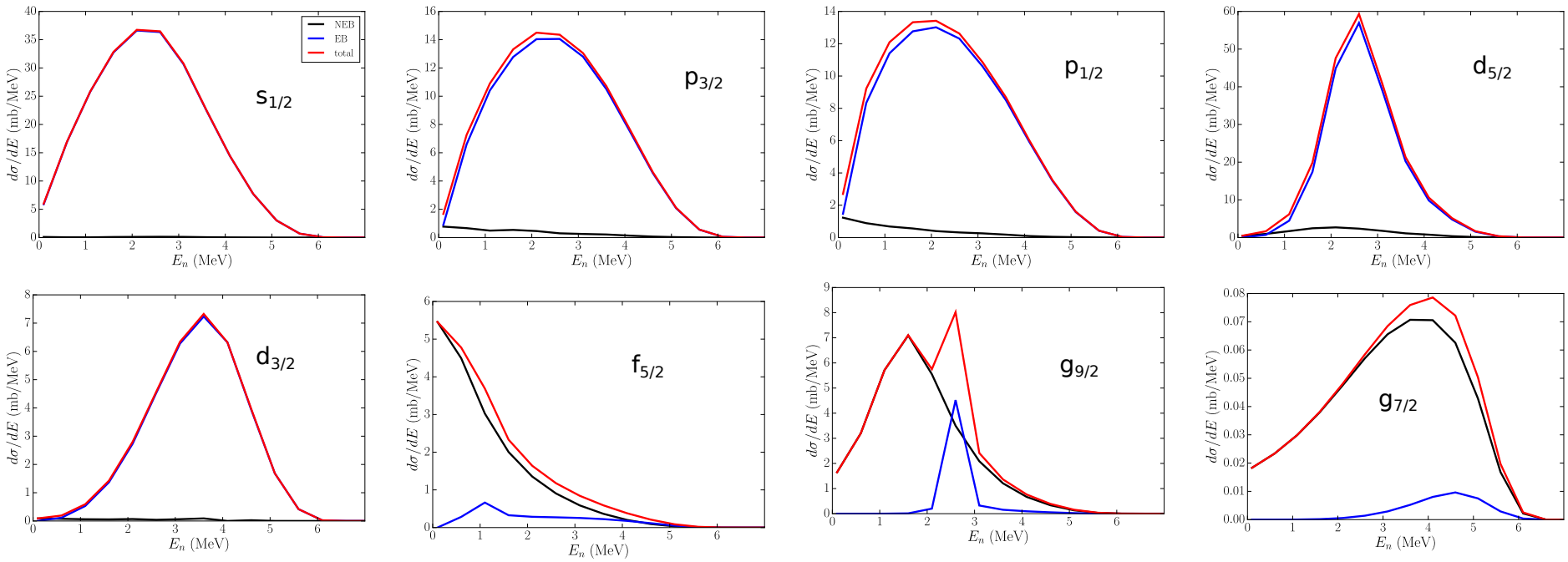
CC: preliminary results (bound states)

first results using a propagator and self–energy generated within the coupled–cluster formalism (J. Rotureau, MSU, G. Hagen, G. R. Jansen ORNL)



CC: preliminary results (continuum states)

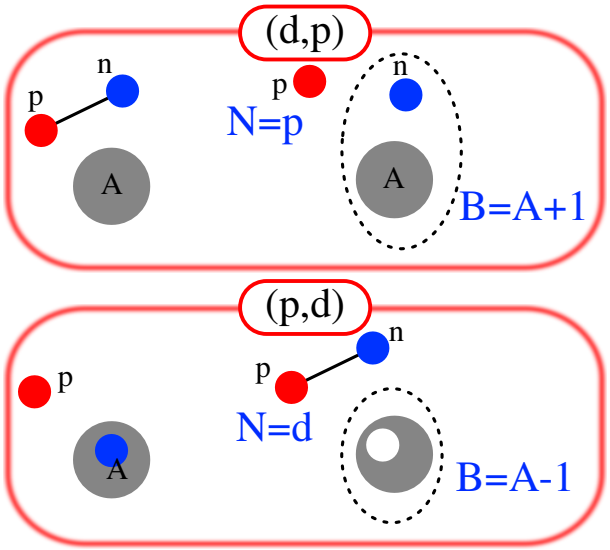
$^{40}\mathrm{Ca}(d,p)$, $E_d = 10$ MeV

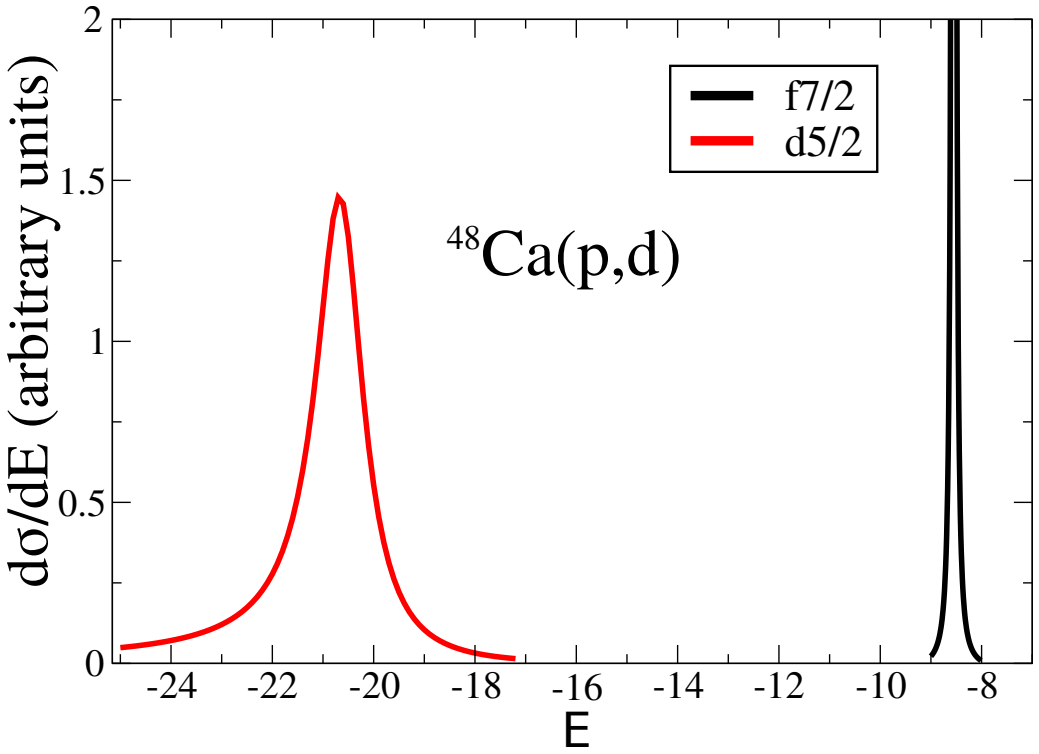


- Population of resonant and non–resonant continuum for different spins and parities.
- Contributions from EB and NEB disentangled.
- CC formalism can take us to the Ca drip line and compared with DOM extrapolations.

First steps towards (p, d) calculations

- Description of (p, d) reactions $\rightarrow$ propagation of the hole+target system.
- First very preliminary calculations using non–local version of the DOM hole propagator (W. Dickhoff, M. Atkinson (WUSTL)).
- Coupled cluster hole propagator also available.





Outline

Summary, conclusions and some prospectives

- Integrated structure+reactions framework for (d, p) reactions.
- Flexible formalism: allows for integration of general structure approaches.
- Provides relevant observables in both bound and continuum regions.
- Can be generalized to other three–body problems$\rightarrow$ (p, d) reactions (hole states)$\rightarrow$ work in progress.

We include state–of–the–art structure, adapted to the specific system under study, in the description of transfer reactions. In particular, can be used far from stability.

work in progress

Develop a consistent description of the deuteron channel.

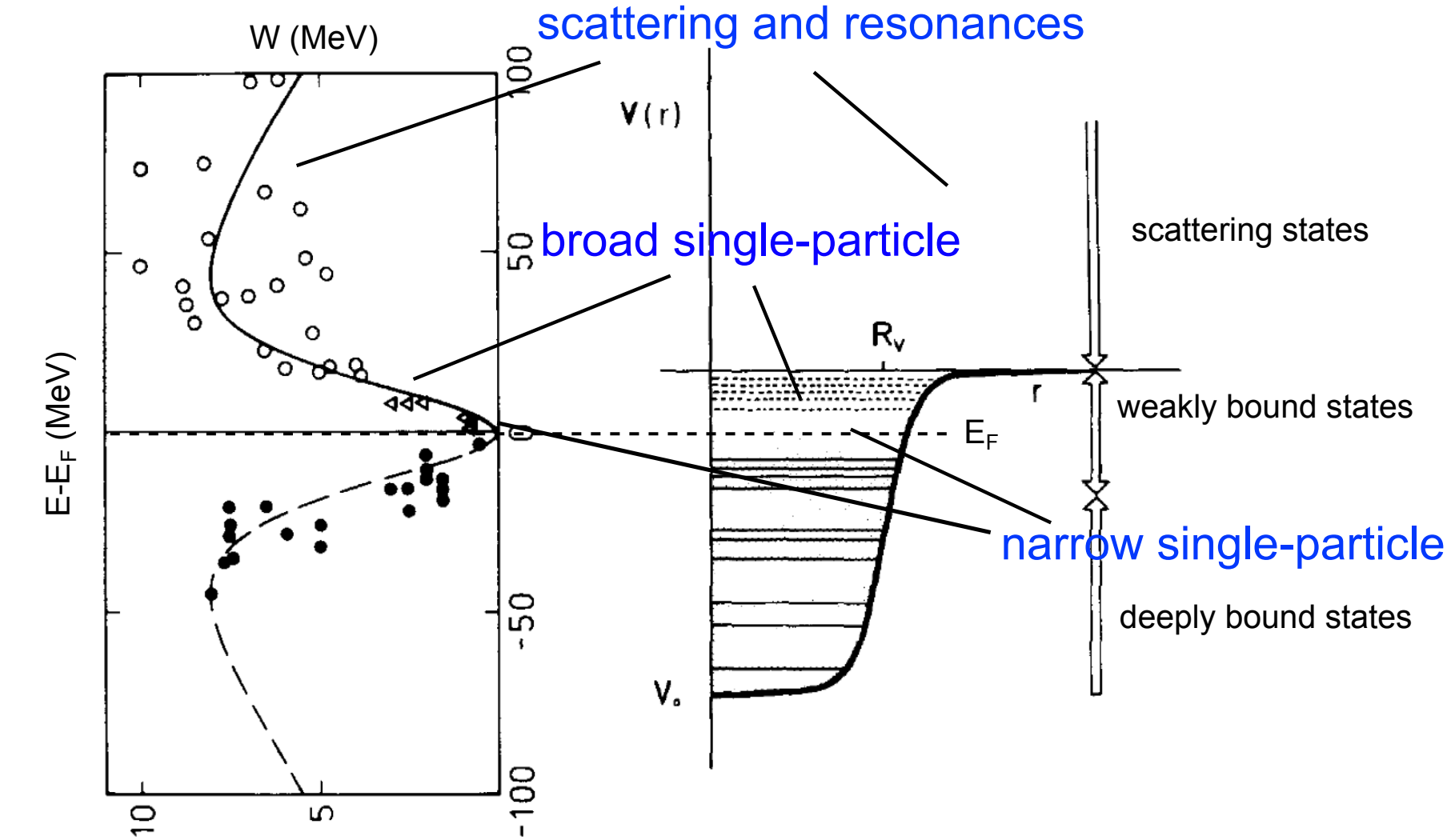
Collaborators

F. Nunes, W. Li, J. Rotureau (MSU, FRIB)
I. Thomson (LLNL)
G. Hagen, G. Jansen (ORNL)
F. Barranco (U of Seville)
E. Vigezzi (INFN Milano)
R. A. Broglia (U of Milano, Niels Bohr Institute)
M. Atkinson, W. Dickhoff (WUSL)

Thank you!

BACKUP SLIDES

Neutron states in nuclei



Imaginary part of optical potential neutron states

Mahaux, Bortignon, Broglia and Dasso Phys. Rep. **120** (1985) 1

Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final ϕ_B^c

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\left|\left\langle \chi_p \phi_B^c | V | \Psi^{(+)}\right\rangle\right|^2 .$$

Sum over all channels, with the approximation $\Psi^{(+)} \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d}\rho(E_p) \times \sum_c \langle \chi_d \phi_d \phi_A | V | \chi_p \phi_B^c \rangle \, \delta(E - E_p - E_B^c) \, \langle \phi_B^c \chi_p | V | \phi_A \chi_d \phi_d \rangle$$

$\chi_d \rightarrow$ deuteron incoming wave, $\phi_d \rightarrow$ deuteron wavefunction,
$\chi_p \rightarrow$ proton outgoing wave $\phi_A \rightarrow$ target core ground state.

Sum over final states

the imaginary part of the Green's function G is an operator representation of the δ–function,

$$\pi\delta(E - E_p - E_B^c) = \lim_{\epsilon\to 0} \Im \sum_c \frac{|\phi_B^c\rangle \langle\phi_B^c|}{E - E_p - H_B + i\epsilon} = \Im G$$

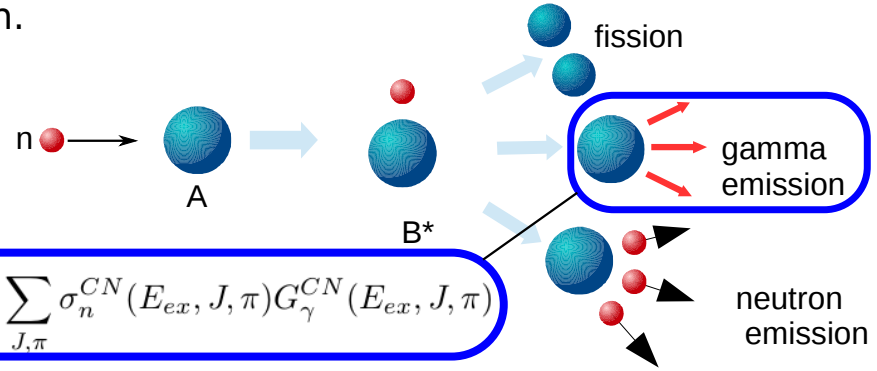
$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d}\rho(E_p)\Im \langle \chi_d\phi_d\phi_A| V |\chi_p\rangle G \langle\chi_p| V |\phi_A\chi_d\phi_d\rangle$$

- We got rid of the (infinite) sum over final states,
- but G is an extremely complex object!
- We still need to deal with that.

Application to surrogate reactions

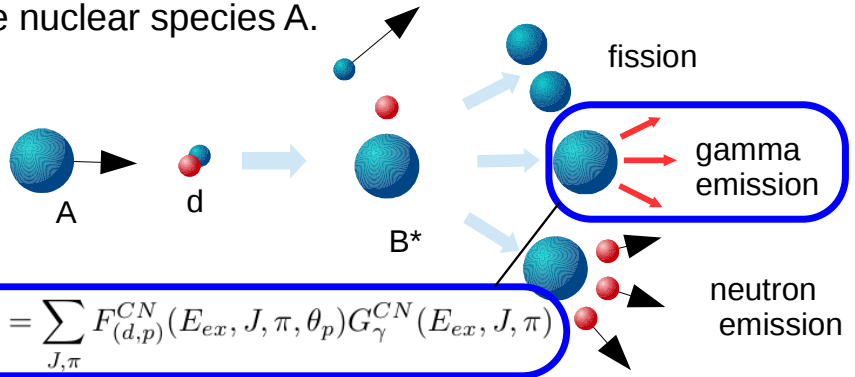
Surrogate for neutron capture

* Desired reaction: absorption of the neutron and gamma emission.



$$\sigma_{n,\gamma}(E_n) = \sum_{J,\pi} \sigma_n^{CN}(E_{ex}, J, \pi) G_\gamma^{CN}(E_{ex}, J, \pi)$$

* The surrogate method consists in producing the same compound nucleus B* by bombarding a deuteron target with a radio active beam of the nuclear species A.



$$P_{(d,p);\gamma}(E_{ex}, \theta_p) = \sum_{J,\pi} F_{(d,p)}^{CN}(E_{ex}, J, \pi, \theta_p) G_\gamma^{CN}(E_{ex}, J, \pi)$$

* **A theoretical reaction formalism that describes the production of all open channels B* is needed.**

neutron transfer limit (isolated–resonance, first–order approximation)

Let's consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy E such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An} \to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \to 0} \langle \chi_d \phi_d | V | \chi_p \rangle \times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle \chi_p | V | \chi_d \phi_d \rangle ,$$

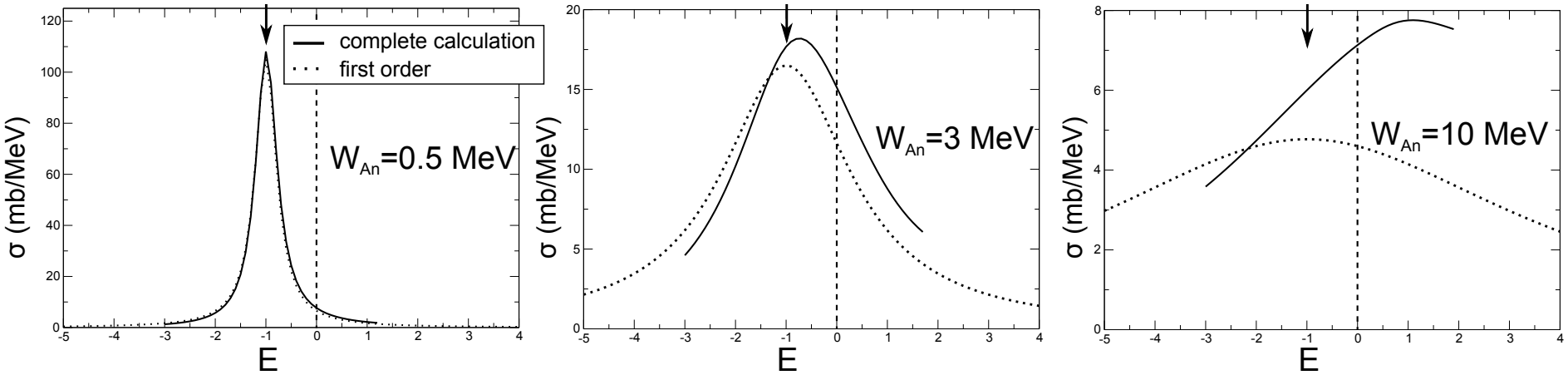
we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\langle \chi_p \phi_n | V | \chi_d \phi_d \rangle|^2 \delta(E - E_p - E_n)$$

Validity of first order approximation

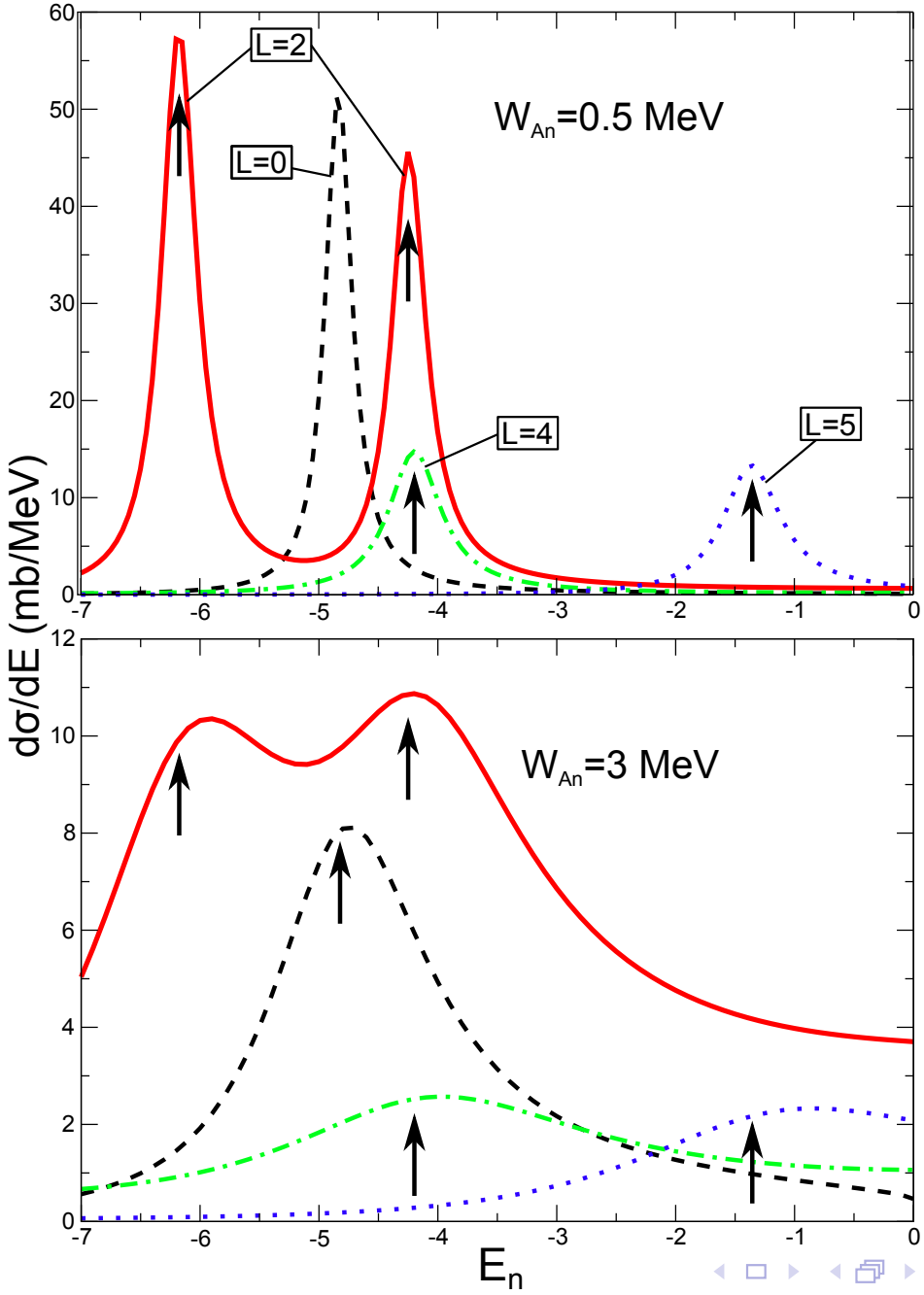
For W_{An} small, we can apply first order perturbation theory,

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}(E,\Omega)\right]^{NEB} \approx \frac{1}{\pi}\frac{\langle\phi_n|W_{An}|\phi_n\rangle}{(E_n-E)^2+\langle\phi_n|W_{An}|\phi_n\rangle^2}\left.\frac{d\sigma_n}{d\Omega}(\Omega)\right]^{transfer}$$

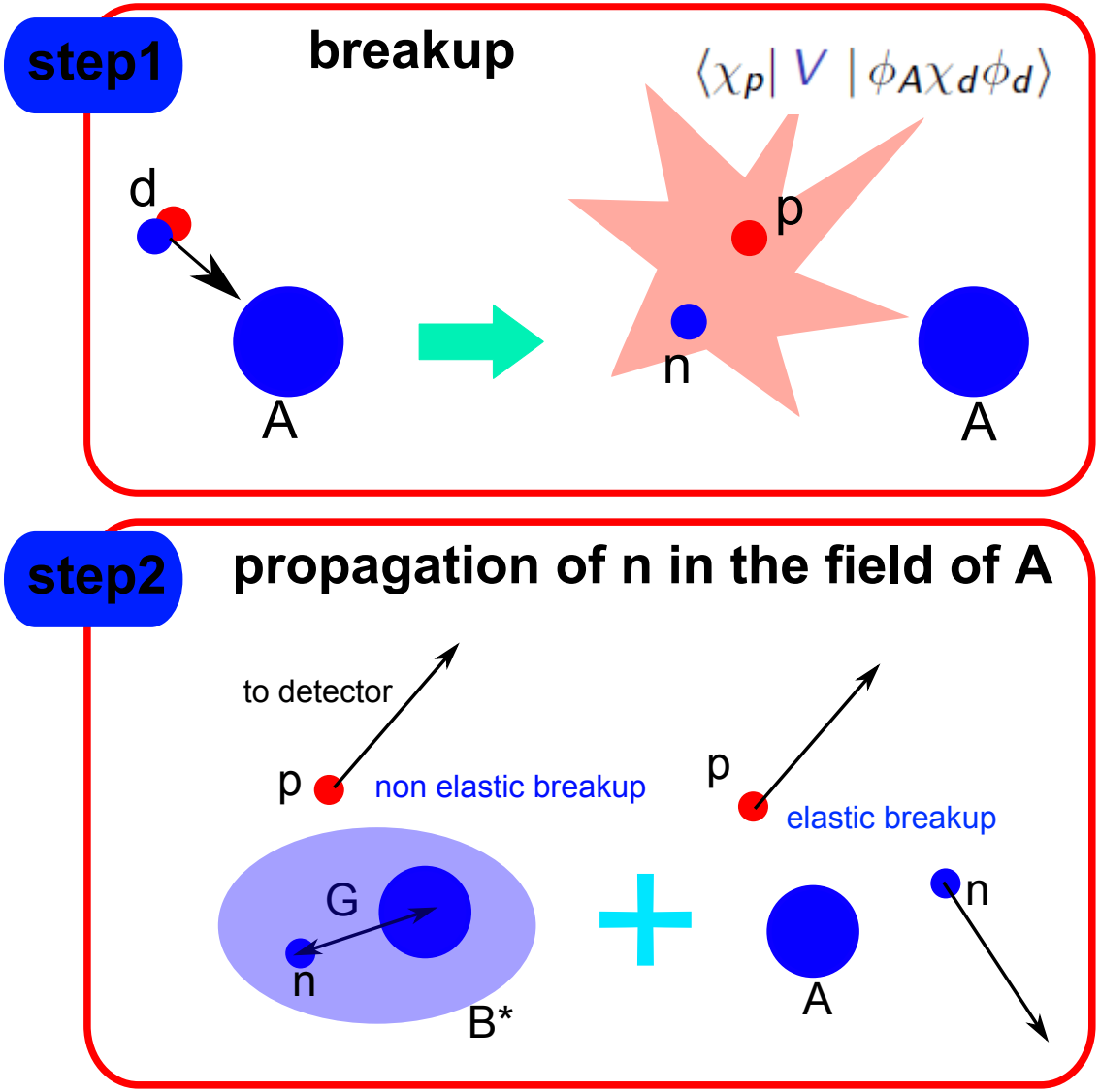


we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV

Spectral function and absorption cross section



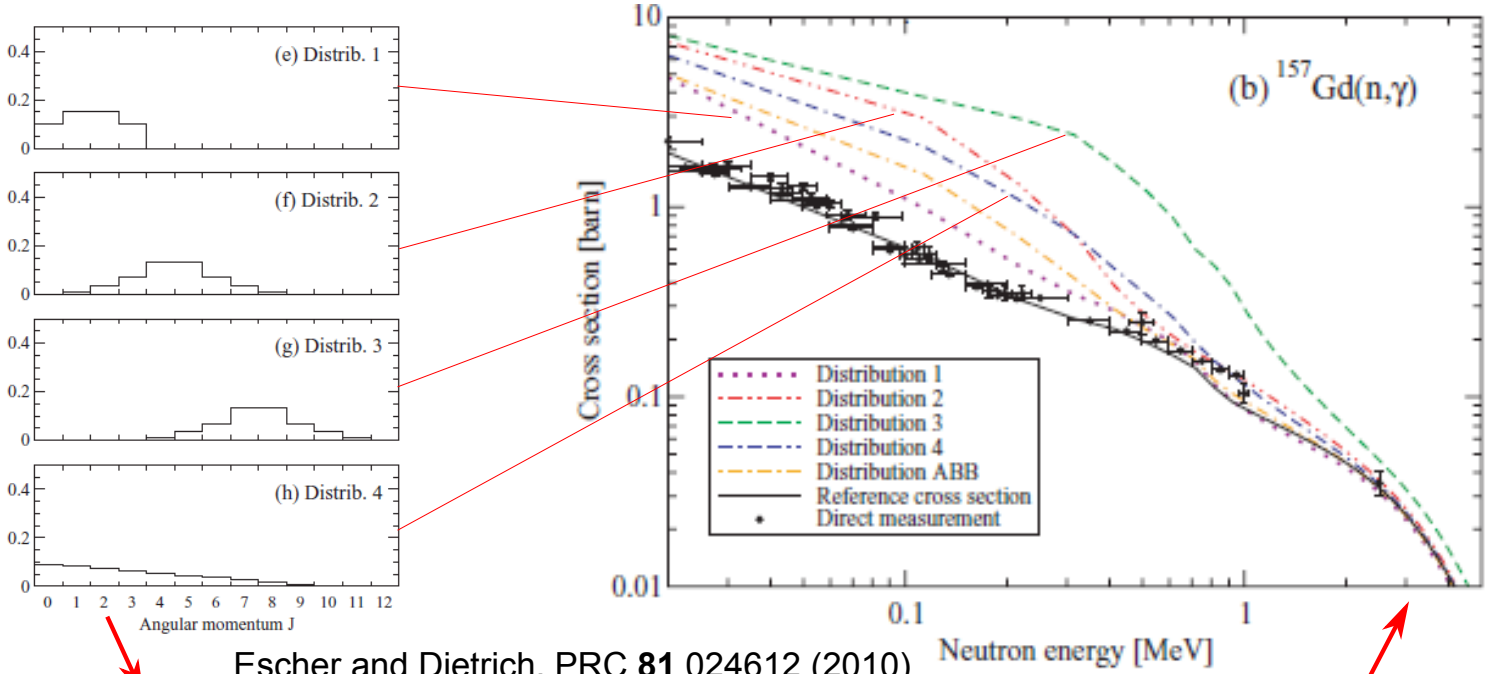
2–step process (post representation)



Weisskopf–Ewing approximation

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{\mathrm{CN}}(E_{\mathrm{ex}}, J, \pi) G_\chi^{\mathrm{CN}}(E_{\mathrm{ex}}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{\mathrm{WE}}(E_a) = \sigma_\alpha^{\mathrm{CN}}(E_{\mathrm{ex}}) \mathcal{G}_\chi^{\mathrm{CN}}(E_{\mathrm{ex}})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



Escher and Dietrich, PRC **81** 024612 (2010)

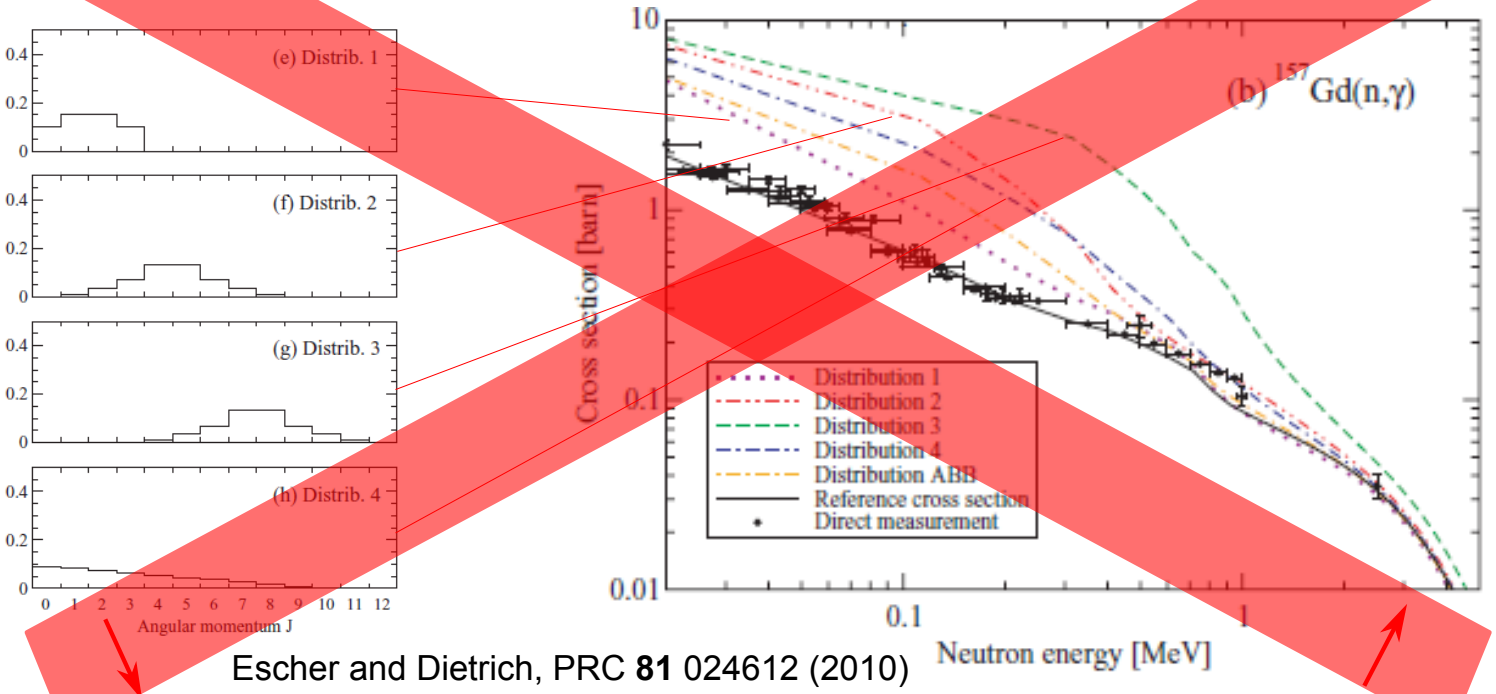
Different J,π → Different cross section for γ emission

Weisskopf–Ewing is inaccurate for (n, γ)

Weisskopf–Ewing approximation

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{\mathrm{CN}}(E_{\mathrm{ex}}, J, \pi) G_\chi^{\mathrm{CN}}(E_{\mathrm{ex}}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{\mathrm{WE}}(E_a) = \sigma_\alpha^{\mathrm{CN}}(E_{\mathrm{ex}}) \mathcal{G}_\chi^{\mathrm{CN}}(E_{\mathrm{ex}})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



Escher and Dietrich, PRC **81** 024612 (2010)

Different J,π → Different cross section for γ emission

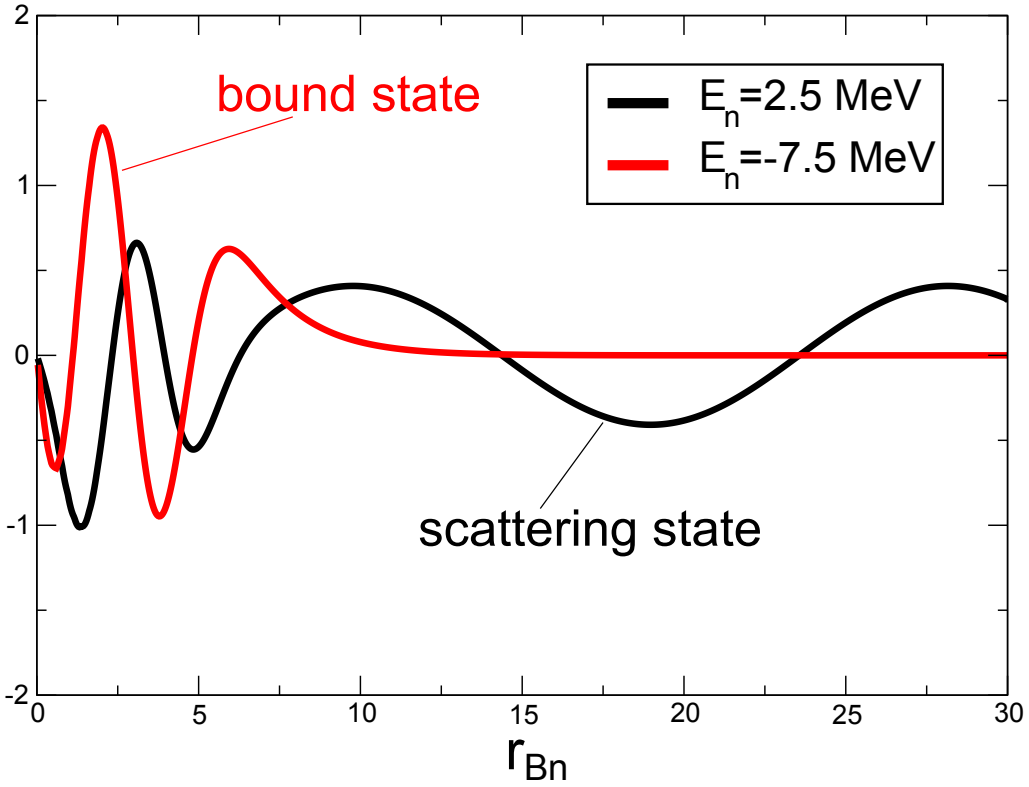
We need theory to predict J, π distributions

neutron wavefunctions

the neutron wavefunctions

$$|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$$

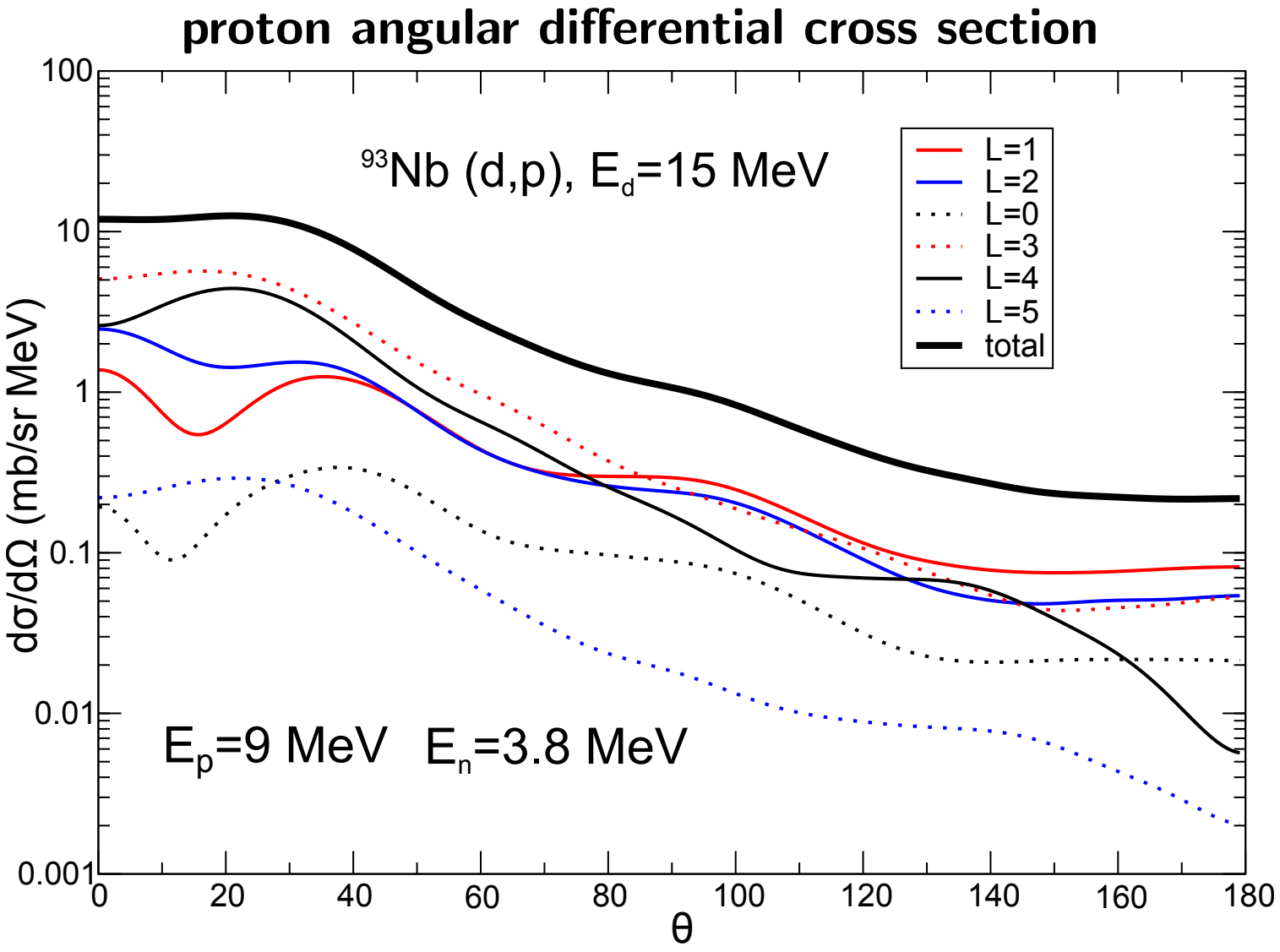
can be computed for **ANY** neutron energy, positive or negative



transfer to resonant and non-resonant continuum well described

$|\psi_n\rangle$ are the solutions of an inhomogeneous Schrödinger equation
$(H_{An} - E_{An}) |\psi_n\rangle = \langle \chi_p | V | \chi_d \phi_d \rangle$

Breakup above neutron–emission threshold



Austern (post)–Udagawa (prior) controversy

The interaction V can be taken either in the *prior* or the *post* representation,

- Austern (post)$\rightarrow V \equiv V_{post} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, University of Sevilla)
- Udagawa (prior) $\rightarrow V \equiv V_{prior} \sim V_{An}(r_{An}, \xi_{An})$

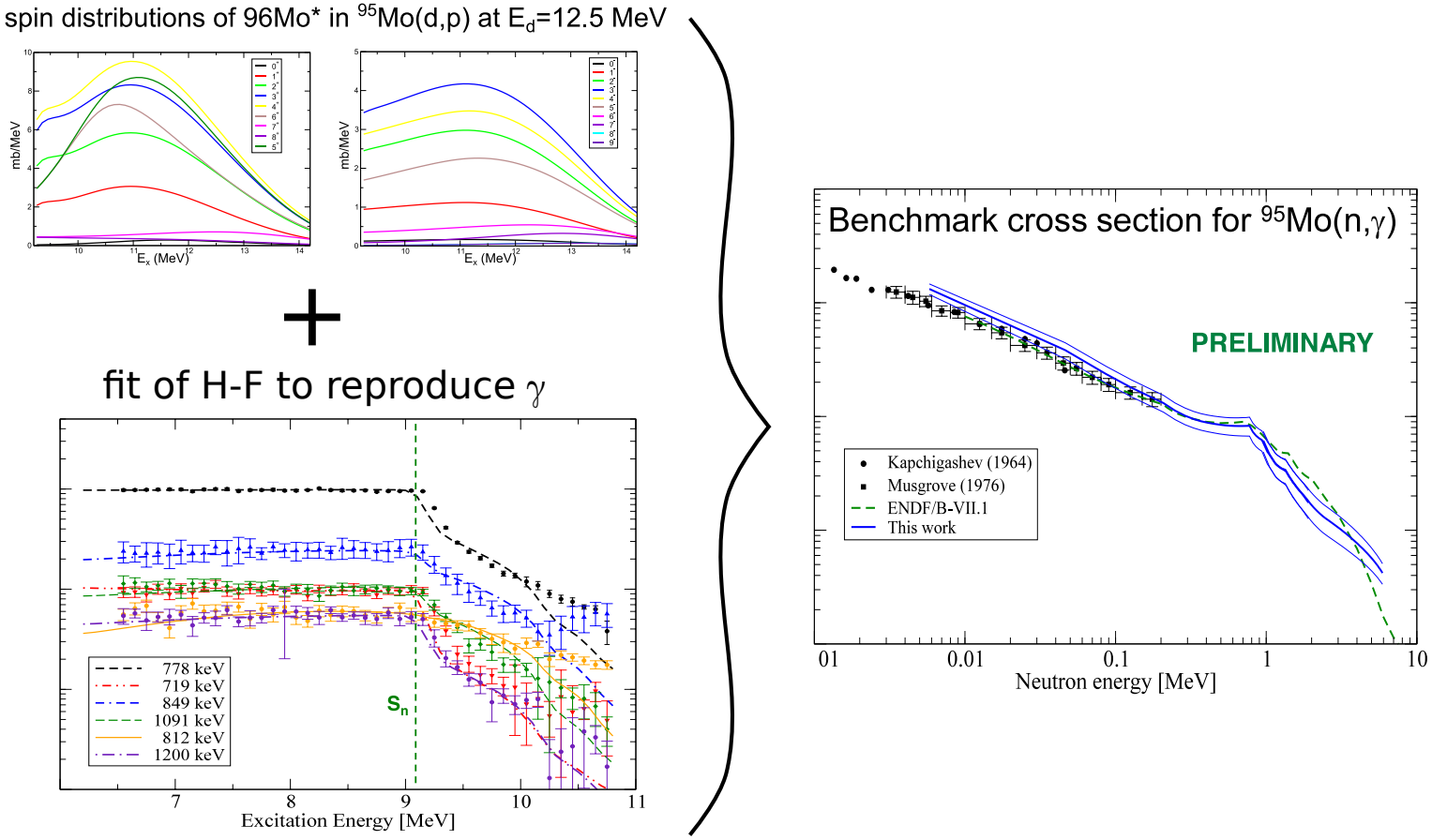
in the prior representation, V can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{post} = -\frac{2}{\hbar v_d}\rho(E_p)\left[\Im\left\langle \psi_n^{prior}|W_{An}|\psi_n^{prior}\right\rangle + 2\Re\left\langle \psi_n^{NON}|W_{An}|\psi_n^{prior}\right\rangle + \left\langle \psi_n^{NON}|W_{An}|\psi_n^{NON}\right\rangle\right],$$

where $\psi_n^{NON} = \langle \chi_p|\ \chi_d\phi_d\rangle$.

The nature of the 2–step process depends on the representation

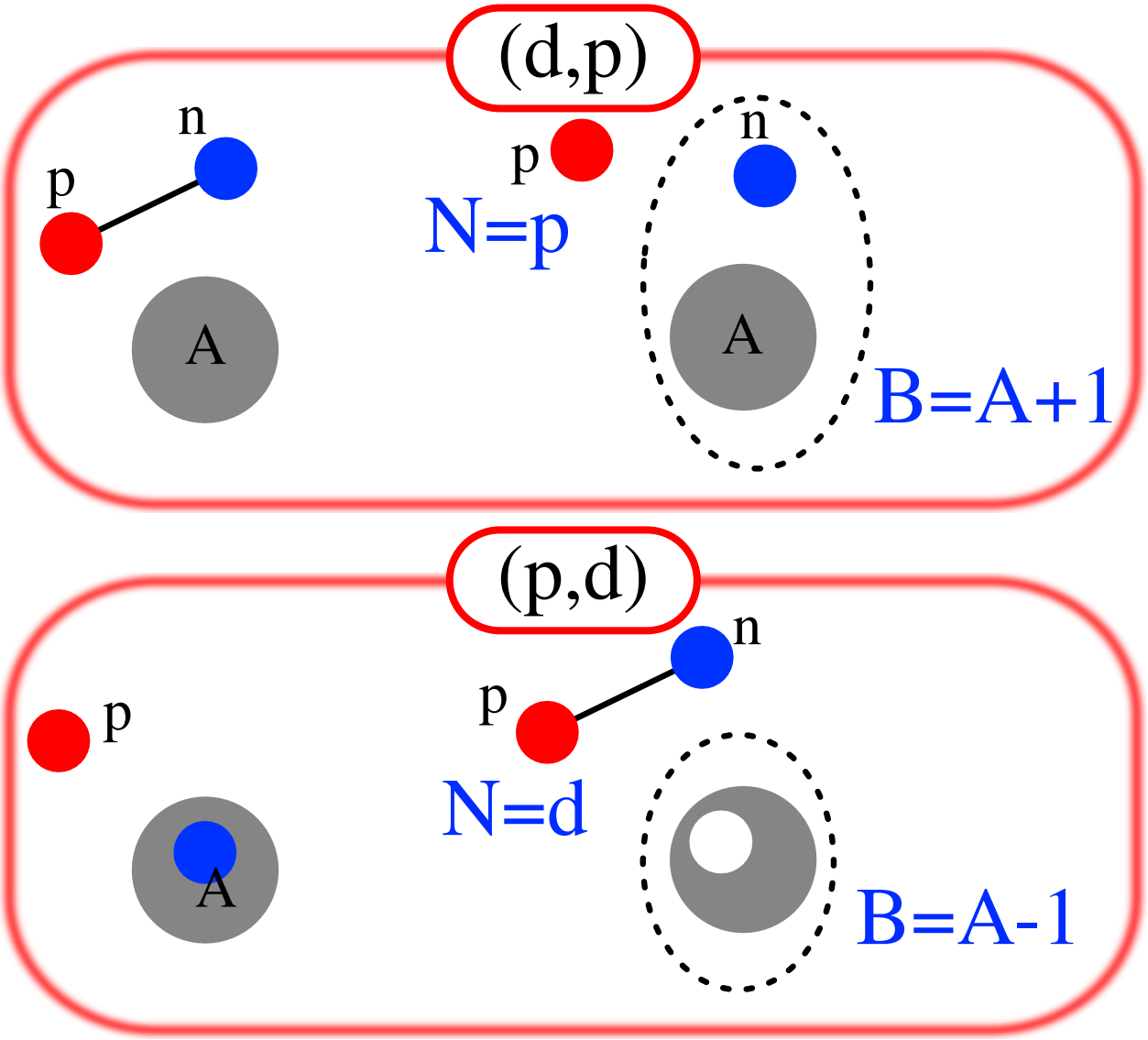
Obtaining (N, γ) cross sections from $(d, N\gamma)$



- In collaboration with A. Ratkiewicz, J. Escher (LLNL) and J. Cizewski (Rutgers).

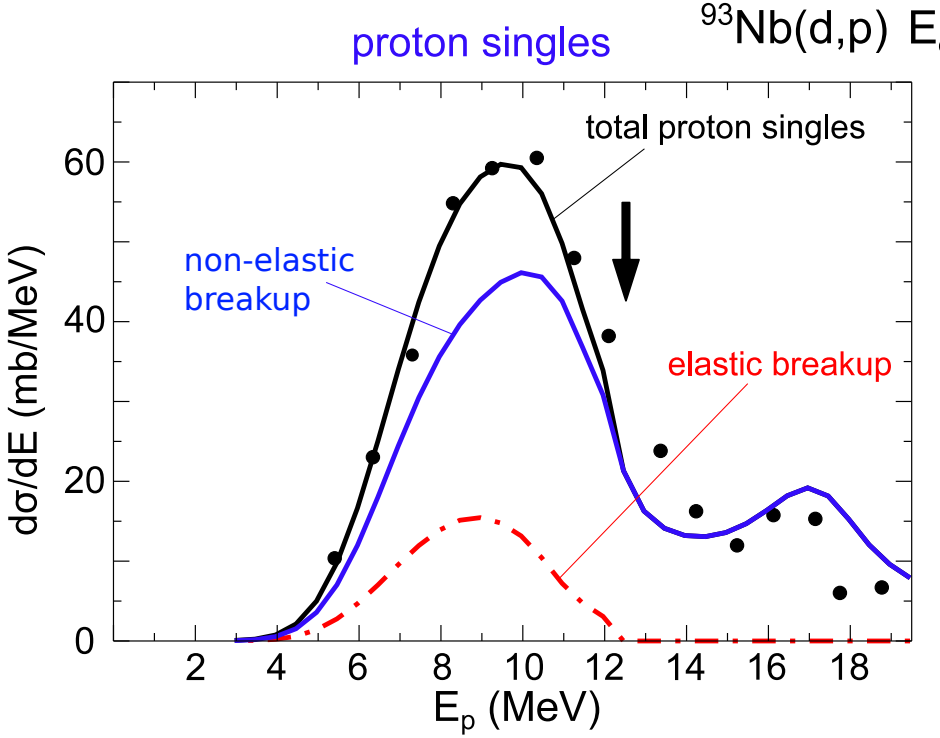
FRIB physics: measure $(d, N\gamma)$ and extract (N, γ) relevant for astrophysics.

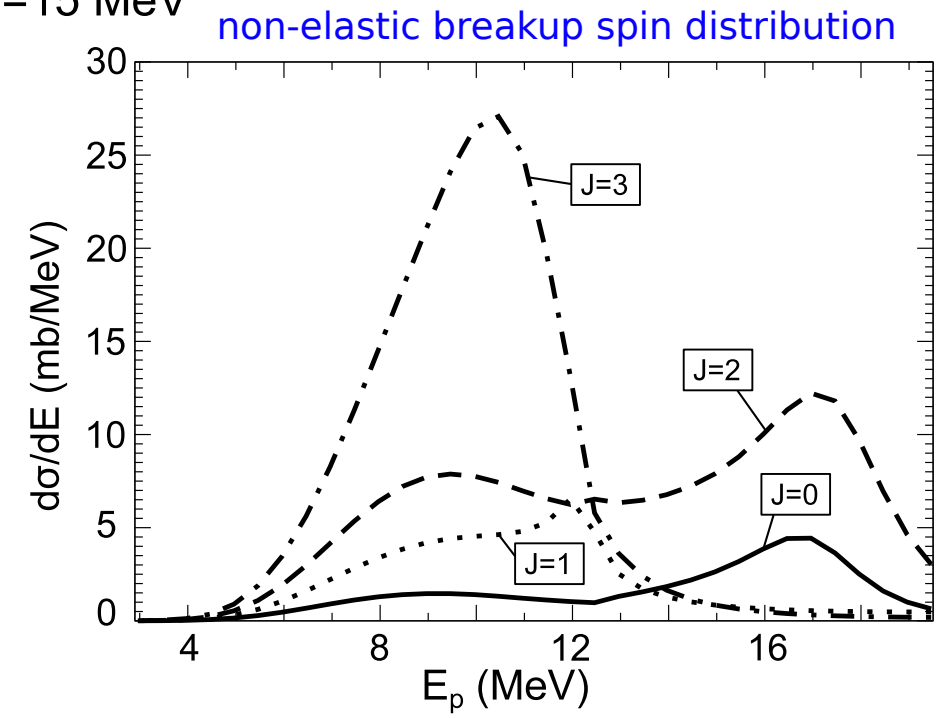
Some applications



KD: Spin–parity distributions of compound nuclei

^{93}Nb(d,p) (Data from Mastroleo *et al.*, Phys. Rev. C **42** (1990) 683)

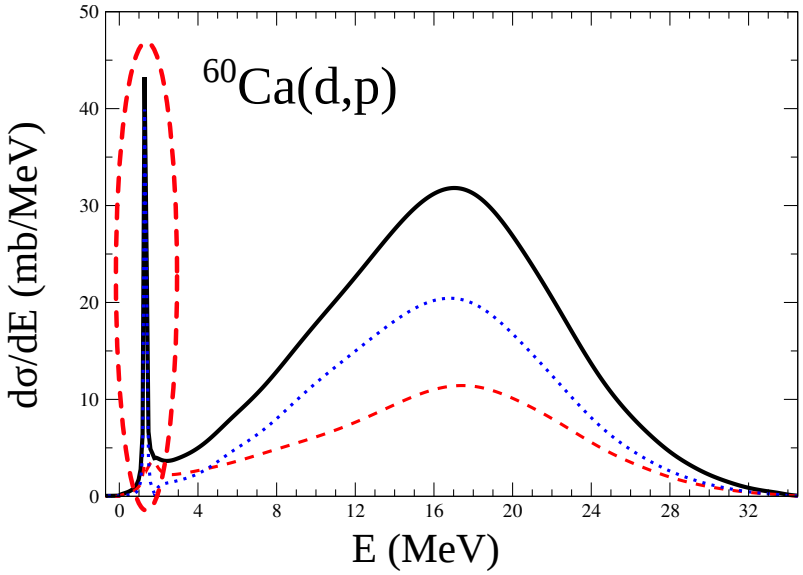


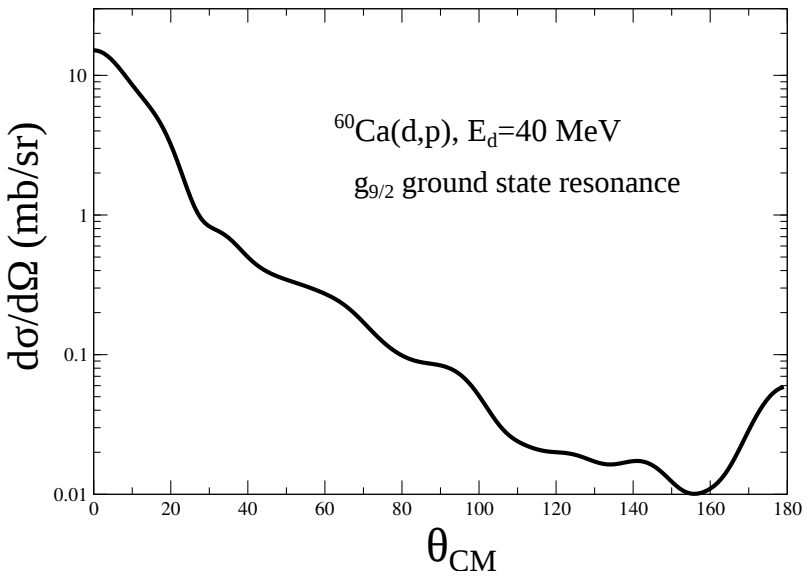


GP, Nunes, Thompson PRC **92** 034611 (2015)

- We obtain spin–parity distributions for the compound nucleus→ application for (n,γ) surrogates.
- Contributions from elastic and non elastic breakup disentangled.

Looking for the Ca dripline

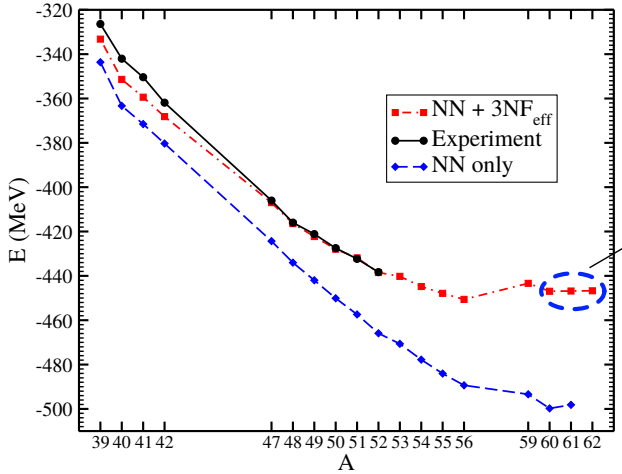




- The DOM can be extrapolated along an isotopic chain based on the fit on stable isotopes.
- The n–^{60}Ca DOM optical potential describes the ^{61}Ca structure.
- ^{61}Ca predicted to be unbound, with a $9/2^+$ resonance as the ground state.
- The absolute angular differential (d, p) cross section is predicted.

Looking for the Ca dripline

is ^{61}Ca bound?

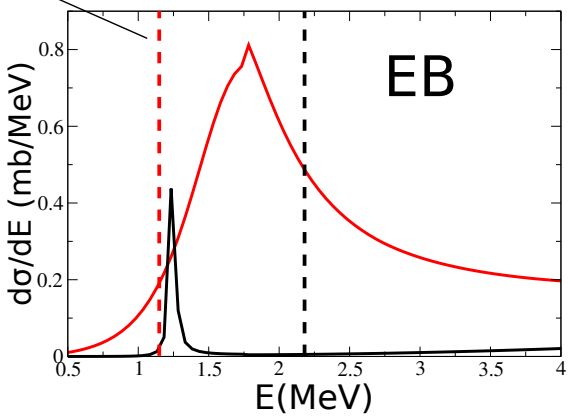


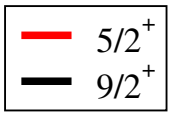
NO (but too close to be conclusive)

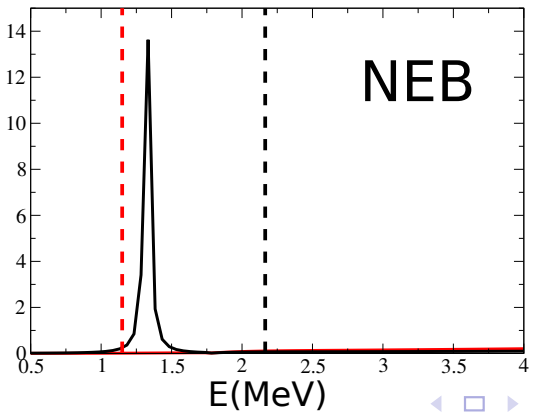
	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

NO



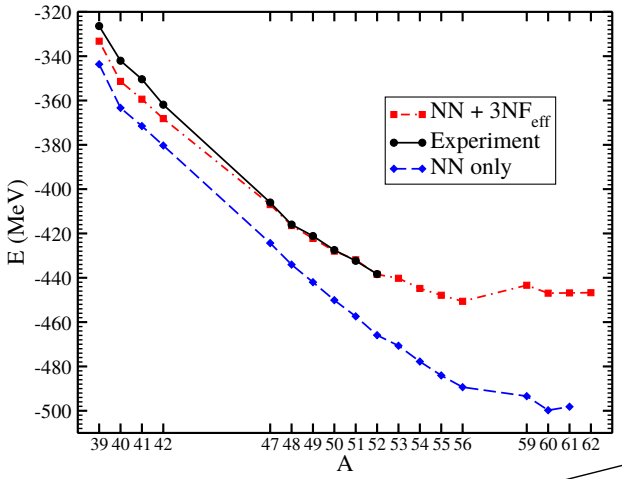




^{60}Ca(d,p) with DOM

Looking for the Ca dripline

inversion of 9/2+ and 5/2+ orbitals?



YES

NO

	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
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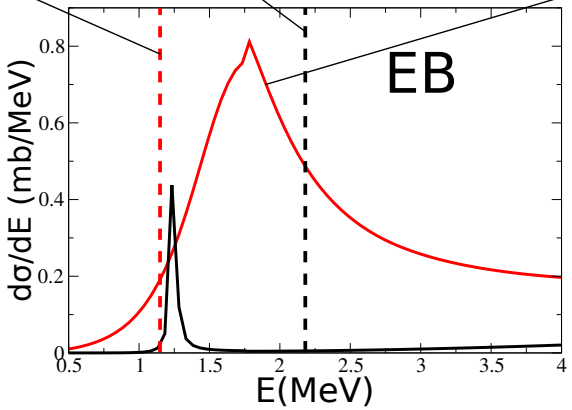
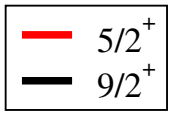
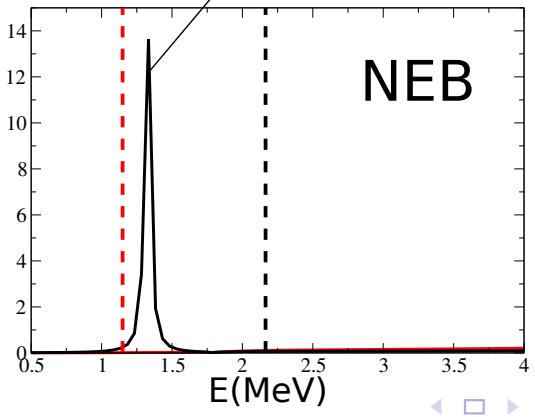
coupled cluster
Hagen et al., PRL
109, 032502 (2012)

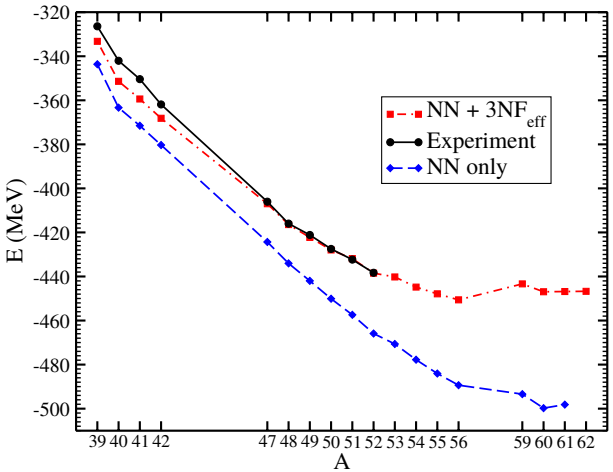






^{60}Ca(d,p) with DOM

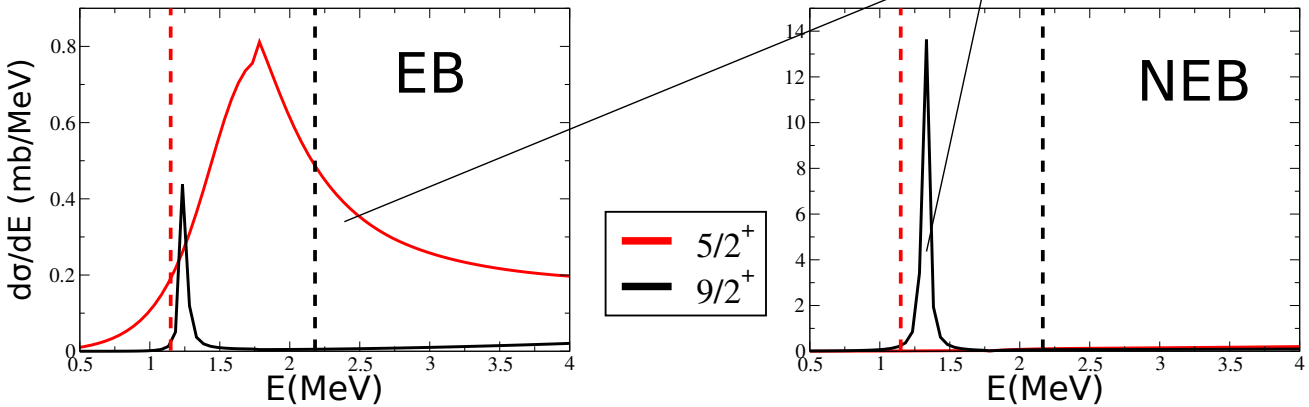
Looking for the Ca dripline



	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

transfer strength function informs about widths

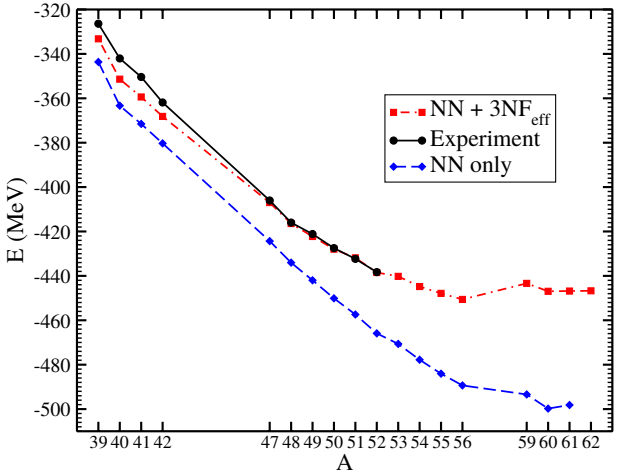
coupled cluster Hagen et al., PRL **109**, 032502 (2012)



^{60}Ca(d,p) with DOM

Looking for the Ca dripline

is ^{61}Ca bound?



coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

5/2$^+$ seen in EB

9/2$^+$ seen in NEB

	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
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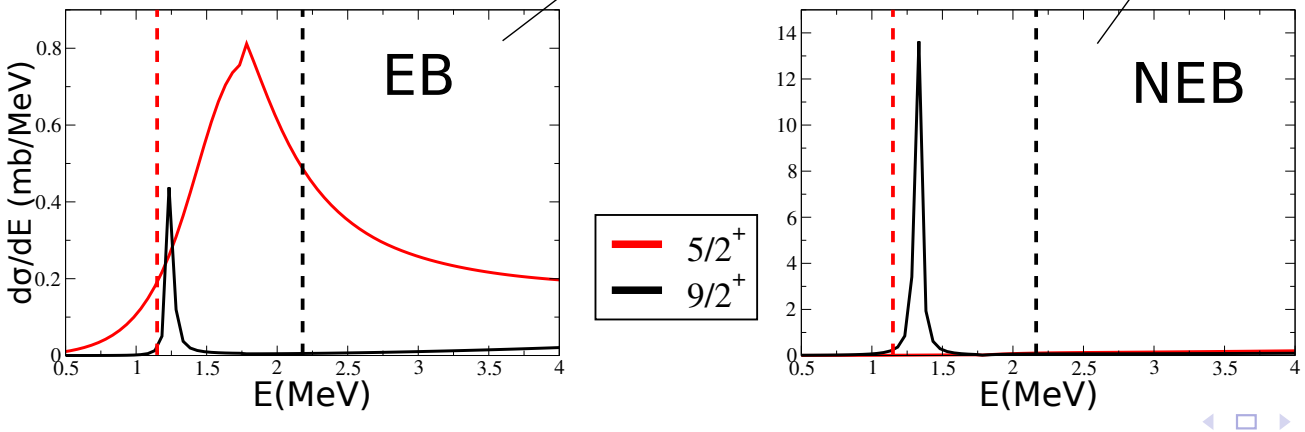
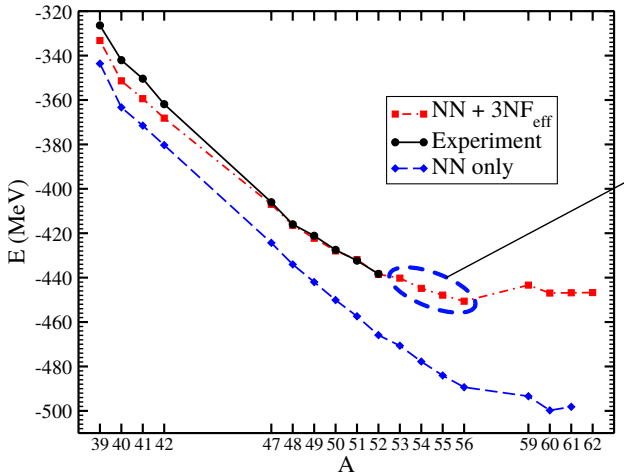


^{60}Ca(d,p) with DOM

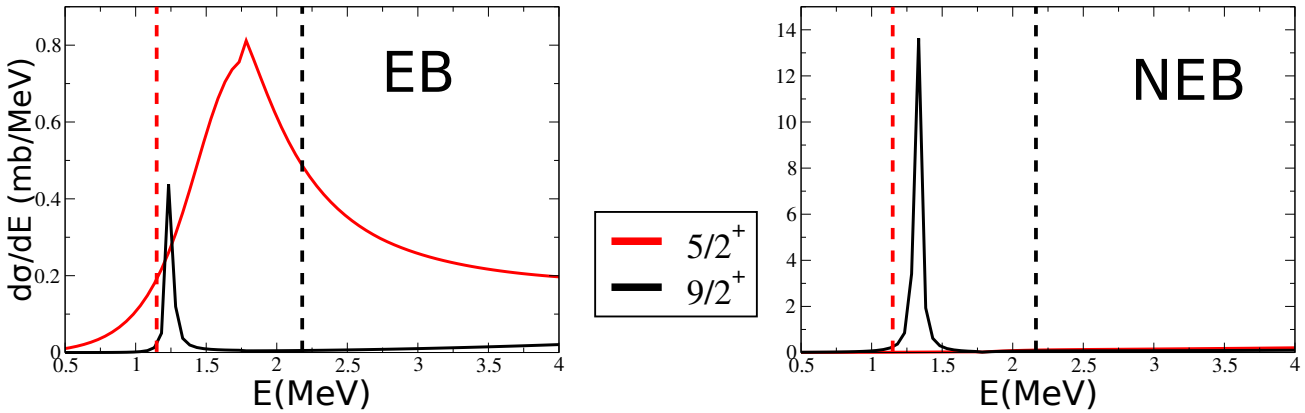
Looking for the Ca dripline



very interesting region might be accessible for (d,p) in FRIB Day 1

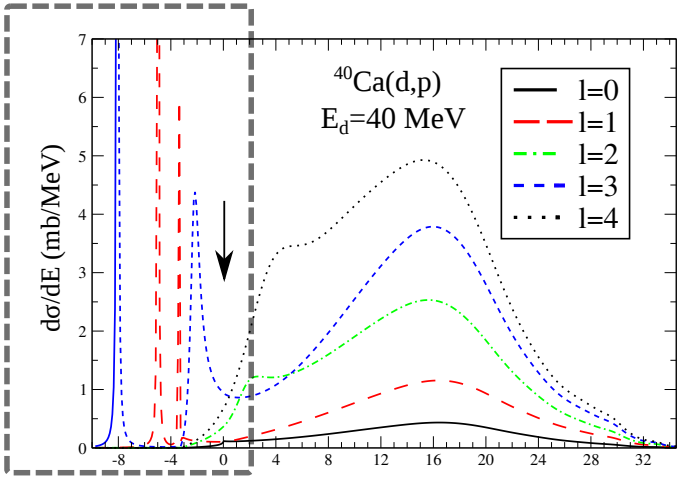
	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

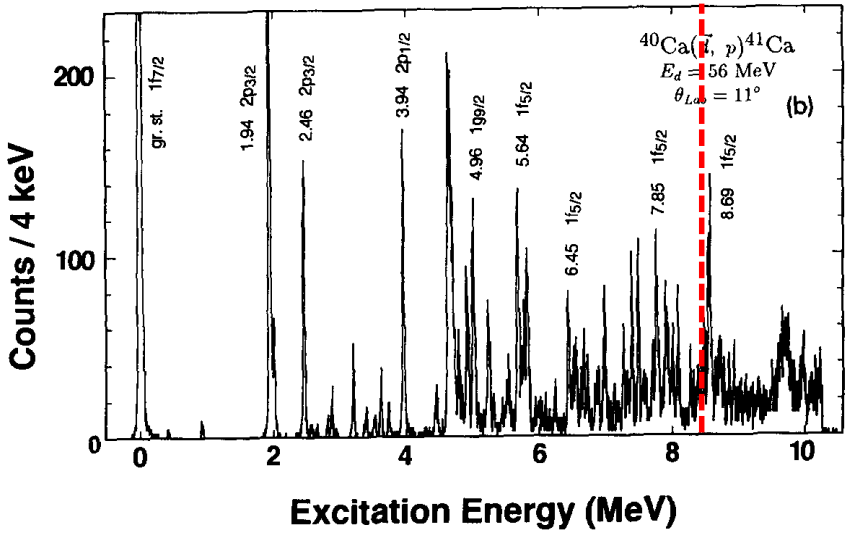
coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

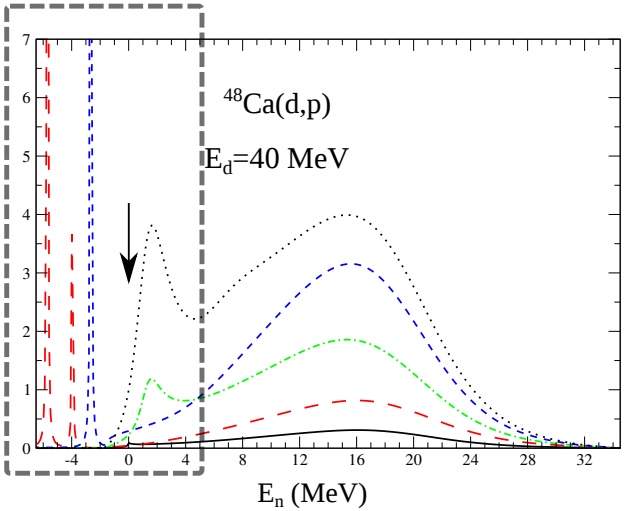


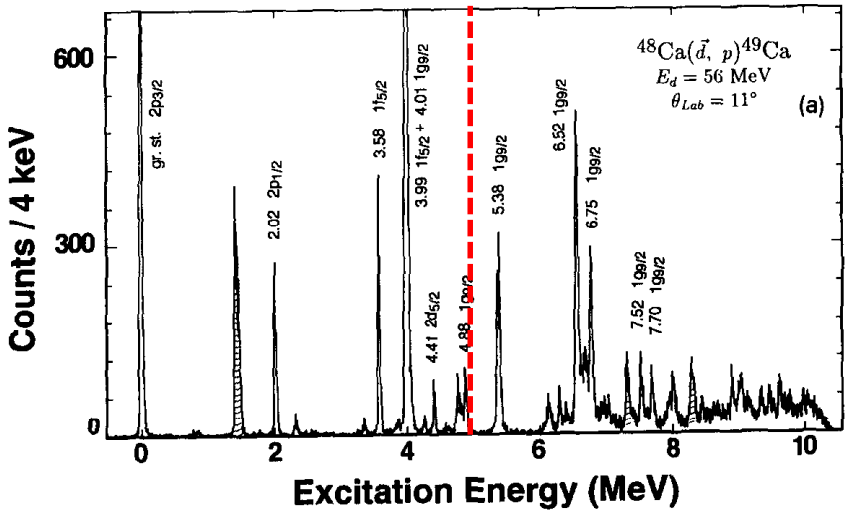
^{60}Ca(d,p) with DOM

DOM: Calcium isotopes





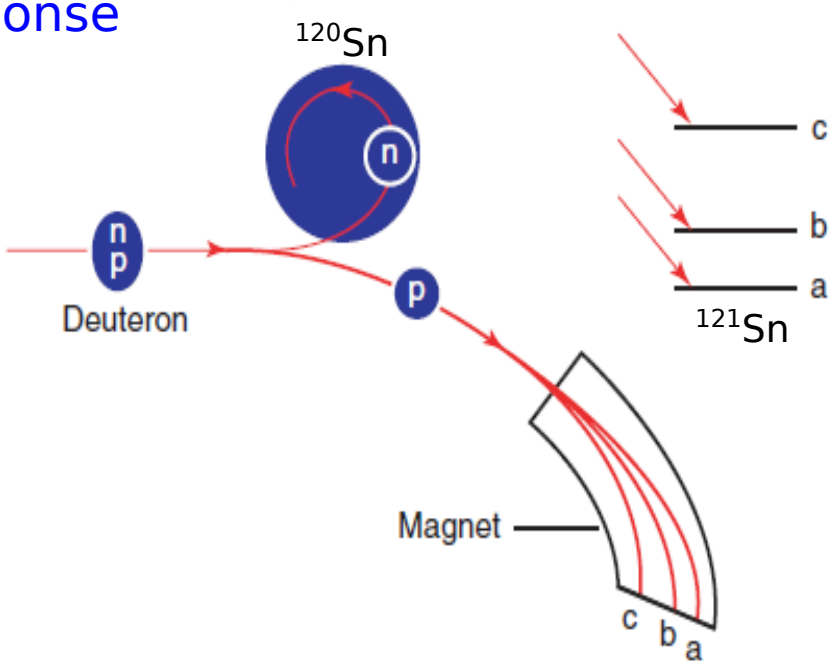




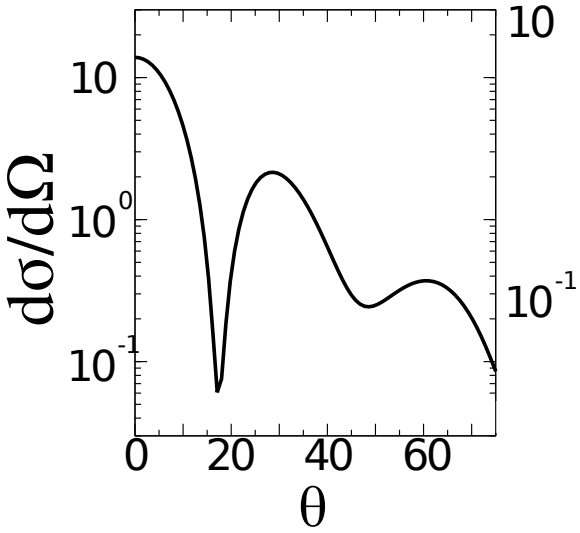
Data from Uozimi *et al.*, NPA **576** (1994) 123

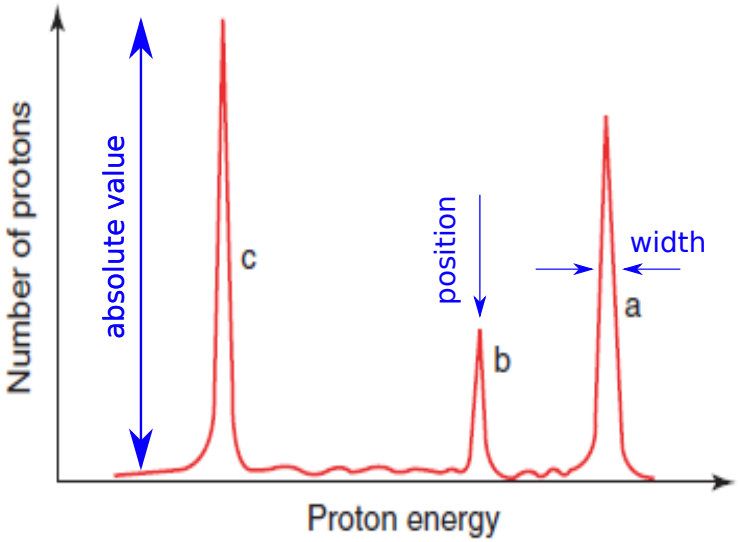
Probing nuclear structure with transfer reactions

- transfer reactions probe nuclear response to the addition of a nucleon
- a variety of observables provide rich information about nuclear structure:
  - angular differential cross section
  - absolute value
  - position
  - width (when in the continuum)

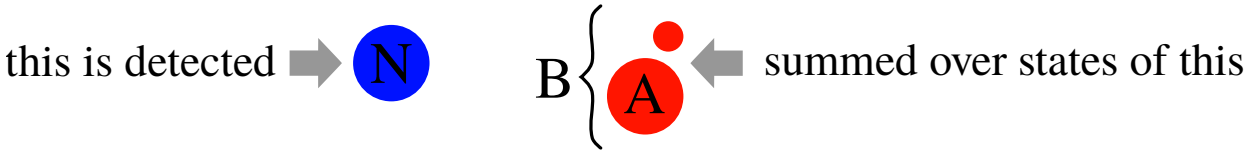


angular differential cross section





General framework for inclusive experiments



$$\frac{d\sigma(E)}{dE_N} \sim \sum_n |\langle \phi_N \phi_B^n | V | \Psi \rangle|^2 \, \delta(E - E_N - E_B^n)$$

$$= \sum_n \langle \Psi | V^\dagger | \phi_N \phi_B^n \rangle \delta(E - E_N - E_B^n) \, \langle \phi_B^n \phi_N | V | \Psi \rangle$$

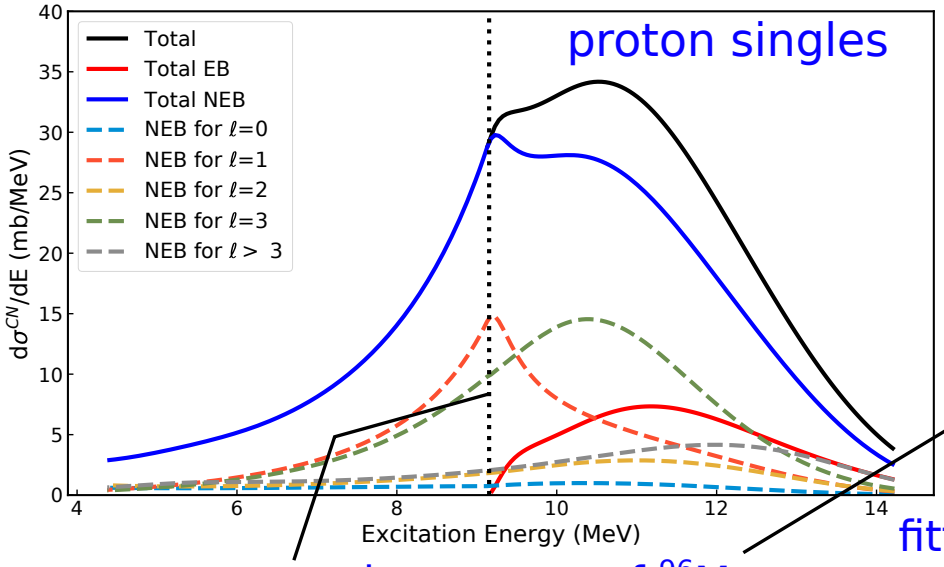
$$= \lim_{\eta \to 0} \mathrm{Im} \langle \Psi | V^\dagger | \phi_N \rangle \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_N - H_B + i\eta} \langle \phi_N | V | \Psi \rangle$$

Main challenges

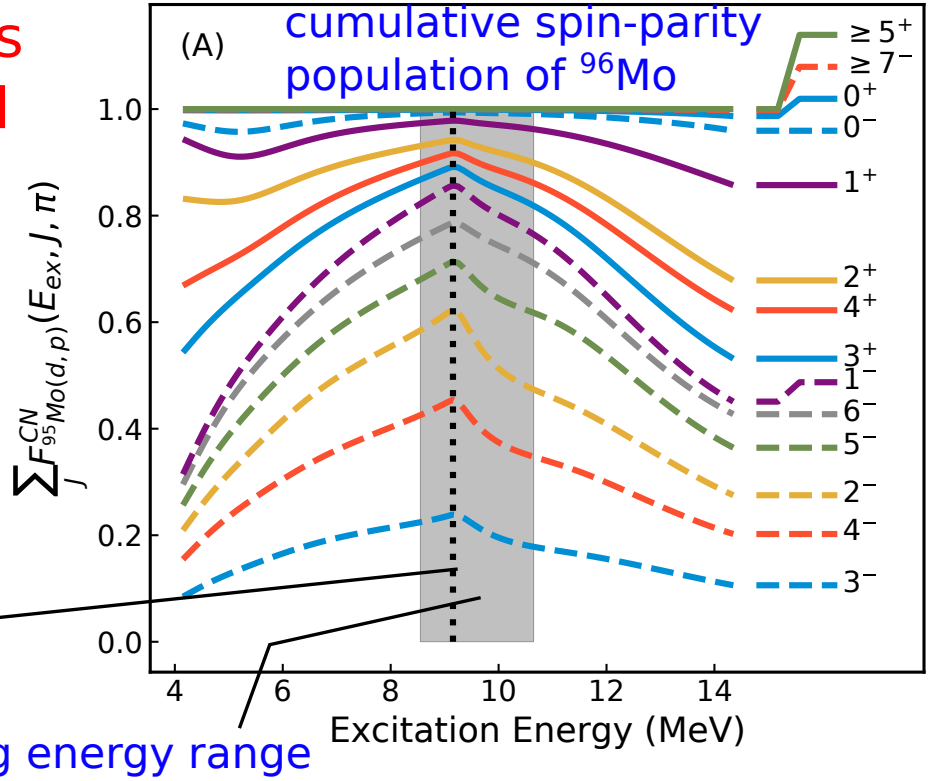
- describe propagator $G_B = \lim_{\eta \to 0} \sum_n \frac{|\phi_B^n\rangle\langle\phi_B^n|}{E - E_A - H_B + i\eta}$
- describe wave function Ψ

Benchmarking the method: the ^{95}Mo$(d, p\gamma)$ experiment

^{95}Mo(d,pγ) with 12.5 MeV deuterons at Texas A&M. Protons are detected in coincidence with γ rays

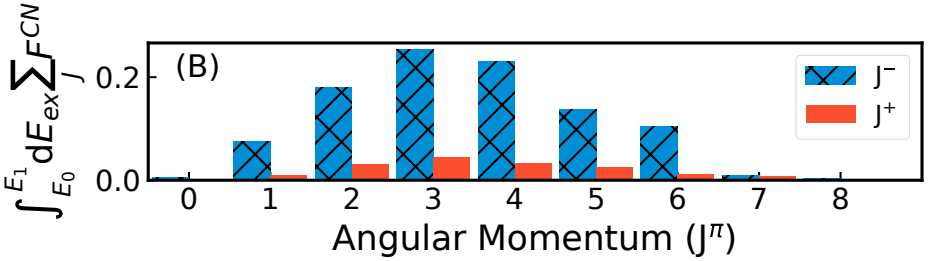


neutron separation energy of ^{96}Mo

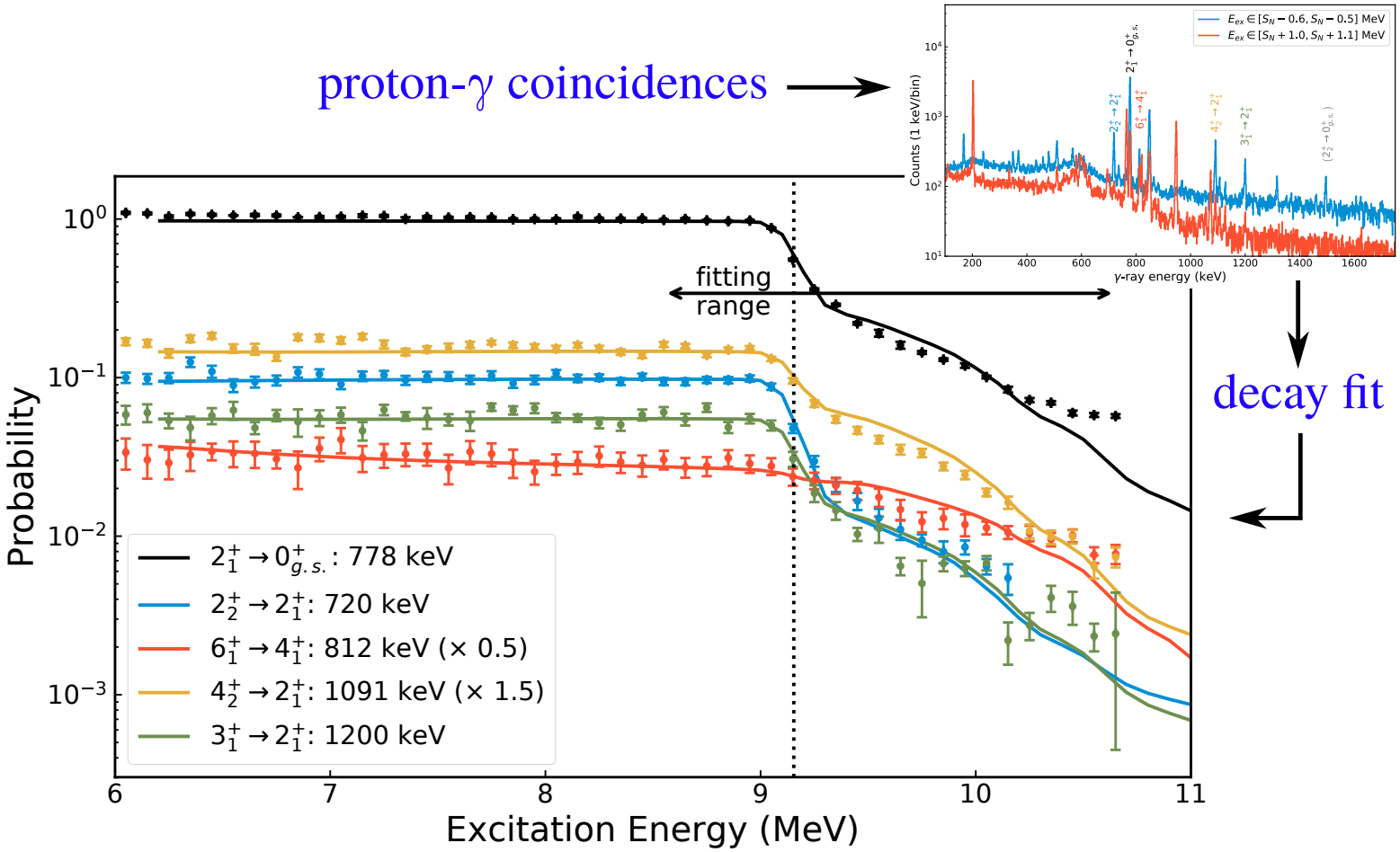


fitting energy range

spin-parity histogram of compound nucleus ^{96}Mo

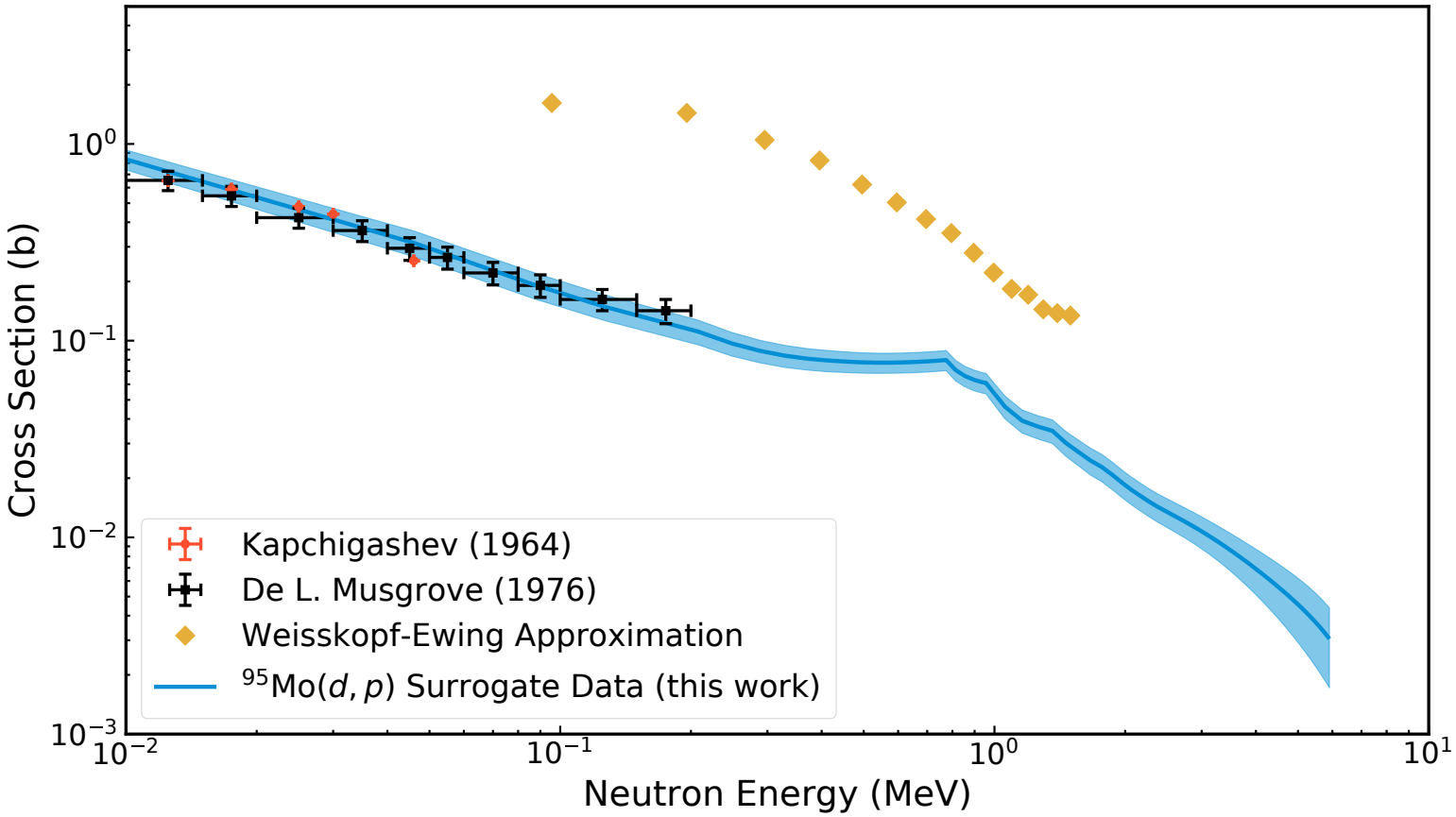


the ^{95}Mo$(d, p\gamma)$ experiment: fitting the γ decay



- The information about the predicted spin–parity distribution is used as input for a Hauser–Feshbach description of the γ decay.
- The parameters of the γ decay (NLD and γSF) are fitted over the indicated energy range for chosen transitions.

the ^{95}Mo$(d, p\gamma)$ experiment: obtaining (n, γ) rates



- The fitted NLD and γSF are used to infer (n, γ) rates.
- Error band from Bayesian analysis of the fit.

No previous knowledge of D_0, and/or $\langle \Gamma_\gamma \rangle$ is needed.