Probing nuclear structure with neutron transfer reactions

Grégory Potel Aguilar (FRIB)

Varenna, June 11th 2018
Outline

1. Introduction and motivation

2. Reaction formalism

3. Structure models and applications
   - Ca isotopes with the Dispersive Optical Model (DOM)
   - $^{11}$Be in Nuclear Field Theory (NFT)
   - Coupled–cluster propagators
   - Extension to ($p, d$) reactions

4. Conclusions
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4. Conclusions
Extracting the structure information: a standard approach

\[ (H - E)|\psi(\xi_A)\rangle = 0 \]

many-body Hamiltonian

\[ |\psi_i(\xi_A)\rangle, E_i \]

many-body wfs and energies

\[ S_i = \langle \varphi_i(r)\psi_0(\xi_{A-1})|\psi\rangle \]

"spectroscopic amplitudes"

\[ \sigma = S_i^2 \tilde{\sigma} \]

factorization of structure and reactions

can suffer from inconsistency between the two schemes
1. Describe the structure of the 2–body subsystems in some given framework of choice.

2. Employ the same quantum many–body methods to work out the interactions $V_{An}$, $V_{Ap}$, $V_{pn}$.

3. Write down the resulting 3–body Hamiltonian $H$.

4. Obtain cross sections from $H$ using controlled approximations.

3–body Hamiltonian

$$H = T + V_{An}(r_n, r'_n, E_n, J_n, \pi_n) + V_{Ap}(r_p, r'_p, E_p, J_p, \pi_p) + V_{pn}(r_{pn}, r'_{pn}, E_{pn}, J_{pn}, \pi_{pn})$$

Disclaimer

Still not the end of the story! 3–body forces $V_{Anp}$ not taken into account at this stage.
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Framework for inclusive \((d, p)\) experiments

\[
\frac{d\sigma(E)}{dE_p} \sim \sum_n |\langle \phi_p \phi_B^n | V | \Psi \rangle|^2 \delta(E - E_p - E_B^n)
\]

\[
= \sum_n \langle \Psi | V^\dagger | \phi_p \phi_B^n \rangle \delta(E - E_p - E_B^n) \langle \phi_B^n \phi_p | V | \Psi \rangle
\]

\[
= \lim_{\eta \to 0} \text{Im} \langle \Psi | V^\dagger | \phi_p \rangle \sum_n \frac{|\phi_B^n \rangle \langle \phi_B^n|}{E - E_p - H_B + i\eta} \langle \phi_p | V | \Psi \rangle
\]

**Main challenges**

- describe propagator \(G_B = \lim_{\eta \to 0} \sum_n \frac{|\phi_B^n \rangle \langle \phi_B^n|}{E - E_A - H_B + i\eta}\)
- describe wave function \(\Psi\)
Optical reduction of $G$

Making use of the approximation $\Psi \approx \chi_d(r_d)\phi_d(r_{pn})\phi_A(\xi_A)$,

$$
\langle \chi_d\phi_d\phi_A | V | \chi_p \rangle G_B \langle \chi_p | V | \phi_A\chi_d\phi_d \rangle \\
= \langle \chi_d\phi_d | V | \chi_p \rangle \langle \phi_A | G_B | \phi_A \rangle \langle \chi_p | V | \chi_d\phi_d \rangle \\
= \langle \chi_d\phi_d | V | \chi_p \rangle G_B^{sp} \langle \chi_p | V | \chi_d\phi_d \rangle,
$$

where $G_B^{sp}$ is the optical reduction of $G_B$

$$
G_B^{sp} = \lim_{\eta \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}, r'_{An}, E_n, J, \pi) + i\eta},
$$

now $U_{An} = V_{An} + iW_{An}$ and $G_B^{sp}$ are single–particle, tractable operators (though both non–local!).

$U_{An}$ is the target–neutron optical potential/self–energy/effective interaction $\rightarrow$ non–local, energy–spin–parity–dependent. The role of structure theory is to provide it.
The strength at given $E_n, r_n, r'_n, J, \pi \rightarrow \text{Im} G^\text{sp}_B (E_n, r_n, r'_n, J, \pi)$ inclusive cross section $\rightarrow$ folding of strength with "breakup probability density"

$$\rho_{bu}(r_n, k_i, k_f) = | \chi_p(r_p, k_f) U_{An} \chi_d(r_d, k_i) \phi_d(r_{pn}) |^2$$

$$\frac{d\sigma}{dE d\Omega} \sim \int \text{Im} G^\text{sp}_B (E_n, r_n, r'_n, J, \pi) \rho_{bu}(r'_n, k_i, k_f) dr_n dr'_n$$

- Provided $\chi_d$ is derived from $V_{An}$ and $V_{Ap}$ (work in progress), all elements come from a single 3–body Hamiltonian.
- No spectroscopic factors needed.
Non–elastic and elastic breakup cross sections

the imaginary part of $G_B^{sp}$ splits in two terms

$$\text{Im} G_B^{sp} = -\pi \sum_{k_n} |\chi_n\rangle \delta \left( E - E_p - \frac{k_n^2}{2m_n} \right) \langle \chi_n | + G_B^{sp} \dagger W_{An} G_B^{sp},$$

we define the neutron wavefunction $|\psi_n\rangle = G_B^{sp} \langle \chi_p | V | \chi_d \phi_d \rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\frac{d^2\sigma}{d\Omega_p dE_p} \bigg|_{\text{NEB}} = -\frac{2}{\hbar \nu_d} \rho(E_p) \langle \psi_n | W_{An} | \psi_n \rangle,$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \bigg|_{\text{EB}} = -\frac{2}{\hbar \nu_d} \rho(E_p) \rho(E_n) |\langle \chi_n \chi_p | V | \chi_d \phi_d \rangle|^2,$$
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### Choosing the potential/propagator

#### Dispersive optical model (DOM)
- Elastic scattering and reaction cross section fit → dispersion → negative energies. Local and non-local versions.

#### Coupled cluster (CC)

#### Nuclear Field Theory (NFT)
- Describes the interweaving of collective (surface and pairing) modes and single-particle degrees of freedom. Non-local. Dispersive.

#### Others
- Any structure calculation providing overlaps $\langle A + 1 | a^\dagger | A \rangle$ can be implemented in the reaction formalism.
DOM: Calcium isotopes


- DOM used to compute \((d, p)\) cross sections on Ca isotopes.
- Both bound and continuum neutron states described.
- EB and NEB contributions to the proton singles are disentangled.
- DOM can be extrapolated to unknown territory \((^{60}\text{Ca})\).
DOM: Calcium isotopes

Absolute transfer cross sections without spectroscopic factors.
\[ \Sigma(E,l,j) = \sum_{l,j} G(E,l,j) \]

\[ G(E,l,j) = G_0(E) + G_0(E) \Sigma(E,l,j) G(E,l,j) \]

- Structure (energies, widths) reproduced within an EFT-like framework (NFT)
- Self energies and Green's functions obtained within the same framework
Transfer cross sections and strength functions well reproduced with the structure ingredients $\Sigma(E, l, j)$ and $G(E, l, j)$.

In the plots:
- **(a)**: $^{10}\text{Be}(d,p)^{11}\text{Be}(1/2^+)$
  - $\sigma = 3.6\text{ mb}$
  - $\sigma = 2.40(13)\text{ mb}$

- **(b)**: $^{10}\text{Be}(d,p)^{11}\text{Be}(1/2^-)$
  - $\sigma = 8.9\text{ mb}$
  - $\sigma = 7.24(32)\text{ mb}$

- **(c)**: $^{10}\text{Be}(d,p)^{11}\text{Be}(5/2^+)$
  - $\sigma = 26\text{ mb}$
  - $\sigma = 28(2)\text{ mb}$

Continuum (5/2$^+$ resonance)

Bound states (1/2$^+$ and 1/2$^-$)

first results using a propagator and self–energy generated within the coupled–cluster formalism (J. Rotureau, MSU, G. Hagen, G. R. Jansen ORNL)

\[ ^{40}\text{Ca} (d,p), E_d = 10\text{MeV} \]

population of \( f_{7/2} \) ground state of \( ^{41}\text{Ca} \)
Population of resonant and non–resonant continuum for different spins and parities.

Contributions from EB and NEB disentangled.

CC formalism can take us to the Ca drip line and compared with DOM extrapolations.
First steps towards \((p, d)\) calculations

- Description of \((p, d)\) reactions → propagation of the hole+target system.

- First very preliminary calculations using non–local version of the DOM hole propagator (W. Dickhoff, M. Atkinson (WUSTL)).

- Coupled cluster hole propagator also available.

\[
\begin{align*}
&\text{\(48\text{Ca}(p,d)\)} \\
&\text{\(E\)} \\
&\text{\(\frac{d\sigma}{dE}\) (arbitrary units)} \\
&\text{(d,p)} \\
&\text{(p,d)} \\
&\text{\(B = A+1\)} \\
&\text{\(B = A-1\)} \\
&\text{\(N = p\)} \\
&\text{\(N = d\)} \\
&\text{\(A\)} \\
&\text{\(p\)} \\
&\text{\(n\)} \\
&\text{\(A\)} \\
&\text{\(n\)} \\
&\text{\(A\)} \\
&\text{\(p\)} \\
&\text{\(A\)}
\end{align*}
\]
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Summary, conclusions and some prospectives

- Integrated structure+reactions framework for \((d, p)\) reactions.
- Flexible formalism: allows for integration of general structure approaches.
- Provides relevant observables in both bound and continuum regions.
- Can be generalized to other three-body problems \(\rightarrow (p, d)\) reactions (hole states) \(\rightarrow\) work in progress.

We include state-of-the-art structure, adapted to the specific system under study, in the description of transfer reactions. In particular, can be used far from stability.

work in progress
Develop a consistent description of the deuteron channel.
F. Nunes, W. Li, J. Rotureau (MSU, FRIB)
I. Thomson (LLNL)
G. Hagen, G. Jansen (ORNL)
F. Barranco (U of Seville)
E. Vigezzi (INFN Milano)
R. A. Broglia (U of Milano, Niels Bohr Institute)
M. Atkinson, W. Dickhoff (WUSL)

Thank you!
Neutron states in nuclei

Imaginary part of optical potential

scattering and resonances

broad single-particle

weakly bound states

deeply bound states

scattering states

narrow single-particle

E-E_F (MeV)

W (MeV)

V(r)

R_v

E_F

Mahaux, Bortignon, Broglia and Dasso Phys. Rep. 120 (1985) 1
the double differential cross section with respect to the proton energy and angle for the population of a specific final $\phi^c_B$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \left| \left\langle \chi_p \phi^c_B \left| V \right| \Psi^{(+)} \right\rangle \right|^2.$$ 

Sum over all channels, with the approximation $\Psi^{(+)} \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d} \rho(E_p) \times \sum_c \left\langle \chi_d \phi_d \phi_A \left| V \right| \chi_p \phi^c_B \right\rangle \delta(E - E_p - E^c_B) \left\langle \phi^c_B \chi_p \left| V \right| \phi_A \chi_d \phi_d \right\rangle$$

$\chi_d \to$ deuteron incoming wave, $\phi_d \to$ deuteron wavefunction,
$\chi_p \to$ proton outgoing wave $\phi_A \to$ target core ground state.
the imaginary part of the Green’s function $G$ is an operator representation of the $\delta$–function,

$$
\pi \delta (E - E_p - E^c_B) = \lim_{\epsilon \to 0} \Im \sum_c \frac{|\phi^c_B \rangle \langle \phi^c_B|}{E - E_p - H_B + i\epsilon} = \Im G
$$

$$
\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d} \rho(E_p) \Im \langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle
$$

- We got rid of the (infinite) sum over final states,
- but $G$ is an extremely complex object!
- We still need to deal with that.
Application to surrogate reactions

**Surrogate for neutron capture**

* Desired reaction: absorption of the neutron and gamma emission.

\[
\sigma_{n,\gamma}(E_n) = \sum_{J,\pi} \sigma_{n,\gamma}^{CN}(E_{\text{ex}}, J, \pi) G_{\gamma}^{CN}(E_{\text{ex}}, J, \pi)
\]

* The surrogate method consists in producing the same compound nucleus B* by bombarding a deuteron target with a radio active beam of the nuclear species A.

\[
P_{(d,p):\gamma}(E_{\text{ex}}, \theta_p) = \sum_{J,\pi} F_{(d,p)}^{CN}(E_{\text{ex}}, J, \pi, \theta_p) G_{\gamma}^{CN}(E_{\text{ex}}, J, \pi)
\]

* A theoretical reaction formalism that describes the production of all open channels B* is needed.
neutron transfer limit (isolated–resonance, first–order approximation)

Let’s consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy $E$ such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An} \to 0} \frac{\langle \phi_n | \phi_n \rangle}{E - E_p - E_n - i\langle \phi_n | W_{An} | \phi_n \rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2 \sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \to 0} \langle \chi_d \phi_d | V | \chi_p \rangle \times \frac{|\phi_n\rangle \langle \phi_n | W_{An} | \phi_n \rangle \langle \phi_n |}{(E - E_p - E_n)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \langle \chi_p | V | \chi_d \phi_d \rangle,$$

we get the direct transfer cross section:

$$\frac{d^2 \sigma}{d\Omega_p dE_p} \sim | \langle \chi_p \phi_n | V | \chi_d \phi_d \rangle |^2 \delta(E - E_p - E_n).$$
Validity of first order approximation

For $W_{An}$ small, we can apply first order perturbation theory,

$$\frac{d^2\sigma}{d\Omega_p dE_p}(E, \Omega) \left[N_{EB}\right] \approx \frac{1}{\pi} \frac{\langle \phi_n | W_{An} | \phi_n \rangle}{(E_n - E)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \frac{d\sigma_n}{d\Omega}(\Omega) \left[\text{transfer}\right]$$

we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV
Spectral function and absorption cross section

$W_{A_n} = 0.5\,\text{MeV}$

$W_{A_n} = 3\,\text{MeV}$
2-step process (post representation)

**Step 1**: Breakup

- Interaction of nucleons $d$, $p$, and $n$ with an atomic nucleus $A$.

**Step 2**: Propagation of $n$ in the field of $A$

- Non-elastic breakup to nucleons $p$, $G$, and $B^*$.
- Elastic breakup to nucleons $p$ and $n$.

The process involves the propagation of the neutron $n$ in the field of the atomic nucleus $A$. The interaction results in a breakdown, leading to the formation of different nucleons. The final state shows the propagation of the neutron to the detector.
Weisskopf–Ewing approximation

\[ \sigma_{a\chi}^W(E_a) = \sum_{J,\pi} \sigma_{a}^{CN}(E_{ex}, J, \pi) G_{\chi}^{CN}(E_{ex}, J, \pi) \]

W-E approximation

\[ \sigma_{a\chi}^{W-E}(E_a) = \sigma_{a}^{CN}(E_{ex}) G_{\chi}^{CN}(E_{ex}) \]

Weisskopf-Ewing approximation: probability of \( \gamma \) decay independent of \( J,\pi \)

Different \( J,\pi \) Different cross section for \( \gamma \) emission

Escher and Dietrich, PRC 81 024612 (2010)

Weisskopf–Ewing is inaccurate for \((n, \gamma)\)
Weisskopf–Ewing approximation

\[ \sigma_{\alpha}(E_a) = \sum_{J, \pi} \sigma_{\alpha}^{CN}(E_{ex}, J, \pi) G_{\chi}^{CN}(E_{ex}, J, \pi) \]

W-E approximation

Weisskopf-Ewing approximation: probability of γ decay independent of J, π

Escher and Dietrich, PRC 81 024612 (2010)

Different J, π → Different cross section for γ emission

We need theory to predict J, π distributions
the neutron wavefunctions

\[ |\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle \]

can be computed for **ANY** neutron energy, positive or negative.

\[ |\psi_n\rangle \] are the solutions of an inhomogeneous Schrödinger equation

\[ (H_{An} - E_{An}) |\psi_n\rangle = \langle \chi_p | V | \chi_d \phi_d \rangle \]
Breakup above neutron–emission threshold

proton angular differential cross section

$^9\text{3}\text{Nb} (d,p)$, $E_d = 15$ MeV

$E_p = 9$ MeV, $E_n = 3.8$ MeV
The interaction $V$ can be taken either in the prior or the post representation,

- Austern (post) → $V \equiv V_{post} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, University of Sevilla)
- Udagawa (prior) → $V \equiv V_{prior} \sim V\text{An}(r_{An}, \xi_{An})$

in the prior representation, $V$ can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$
\frac{d^2 \sigma}{d\Omega_p dE_p} \bigg|^{\text{post}} = - \frac{2}{\hbar \nu_d} \rho(E_p) \left[ \Im \langle \psi_{n}^{\text{prior}} \mid W\text{An} \mid \psi_{n}^{\text{prior}} \rangle 
+ 2 \Re \langle \psi_{n}^{\text{NON}} \mid W\text{An} \mid \psi_{n}^{\text{prior}} \rangle + \langle \psi_{n}^{\text{NON}} \mid W\text{An} \mid \psi_{n}^{\text{NON}} \rangle \right],
$$

where $\psi_{n}^{\text{NON}} = \langle \chi_{p} \mid \chi_{d}\phi_{d} \rangle$.

The nature of the 2–step process depends on the representation.
Obtaining \((N, \gamma)\) cross sections from \((d, N\gamma)\)

- In collaboration with A. Ratkiewicz, J. Escher (LLNL) and J. Cizewski (Rutgers).

**FRIB physics:** measure \((d, N\gamma)\) and extract \((N, \gamma)\) relevant for astrophysics.
Some applications

\[ (d,p) \]
\[ (p,d) \]

\[ N = p \]
\[ B = A + 1 \]

\[ N = d \]
\[ B = A - 1 \]
We obtain spin–parity distributions for the compound nucleus → application for \((n, \gamma)\) surrogates.

Contributions from elastic and non elastic breakup disentangled.
The DOM can be extrapolated along an isotopic chain based on the fit on stable isotopes.

The $n-^{60}\text{Ca}$ DOM optical potential describes the $^{61}\text{Ca}$ structure.

$^{61}\text{Ca}$ predicted to be unbound, with a $9/2^+$ resonance as the ground state.

The absolute angular differential $(d, p)$ cross section is predicted.
Looking for the Ca dripline

is $^{61}$Ca bound?

NO (but too close to be conclusive)

coupled cluster
Hagen et al., PRL 109, 032502 (2012)

$^5_{3}$Ca

$^{5^3}_{5^5}$Ca

$^{6^1}_{1^1}$Ca

$J^\pi$ Re$[E]$ $\Gamma$ Re$[E]$ $\Gamma$ Re$[E]$ $\Gamma$

$5/2^+$ 1.99 1.97 1.63 1.33 1.14 0.62

$9/2^+$ 4.75 0.28 4.43 0.23 2.19 0.02

$^{60}$Ca(d,p) with DOM
Looking for the Ca dripline

inversion of 9/2+ and 5/2+ orbitals?

\( E \text{(MeV)} \) \( E \text{(MeV)} \) \( d\sigma/dE \text{(mb/MeV)} \)

\begin{tabular}{cccccc}
\( 39 \) & 40 & 41 & 42 & 47 & 48 & 49 & 50 & 51 & 52 & 53 & 54 & 55 & 56 & 59 & 60 & 61 & 62 \\
\end{tabular}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Graph showing energy levels and transition probabilities for Ca isotopes.}
\end{figure}

\begin{table}
\centering
\begin{tabular}{cccccc}
\( ^{53}\text{Ca} \) & \( ^{55}\text{Ca} \) & \( ^{61}\text{Ca} \) & & & \\
\hline
\( \Gamma \) & \( \Gamma \) & \( \Gamma \) & & & \\
\hline
5/2+ & 1.99 & 1.97 & 1.63 & 1.33 & 1.14 & 0.62 \\
9/2+ & 4.75 & 0.28 & 4.43 & 0.23 & 2.19 & 0.02 \\
\end{tabular}
\caption{Energy levels and transition strengths for Ca isotopes.}
\end{table}

\textbf{YES}

\textbf{NO}

\( ^{60}\text{Ca}(d,p) \) with DOM

Varenna, June 11th 2018
Looking for the Ca dripline

**Transfer strength function informs about widths**

<table>
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**$^{60}$Ca(d,p) with DOM**

Hagen et al., PRL 109, 032502 (2012)
Looking for the Ca dripline

is $^{61}$Ca bound?

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$^{60}$Ca(d,p) with DOM

$5/2^+$ seen in EB

$9/2^+$ seen in NEB

coupled cluster

Hagen et al., PRL 109, 032502 (2012)
Looking for the Ca dripline

very interesting region might be accessible for (d,p) in FRIB Day 1

coupled cluster
Hagen et al., PRL 109, 032502 (2012)

$$^{53}\text{Ca} \quad ^{55}\text{Ca} \quad ^{61}\text{Ca}$$

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$^{60}\text{Ca}(d,p)$ with DOM
Data from Uozimi et al., NPA 576 (1994) 123
transfer reactions probe nuclear response to the addition of a nucleon

a variety of observables provide rich information about nuclear structure:
- angular differential cross section
- absolute value
- position
- width (when in the continuum)
General framework for inclusive experiments

\[
\frac{d\sigma(E)}{dE_N} \sim \sum_n |\langle \phi_N \phi_B^n | V | \psi \rangle|^2 \delta(E - E_N - E_B^n)
\]

\[
= \sum_n \langle \psi | V^\dagger | \phi_N \phi_B^n \rangle \delta(E - E_N - E_B^n) \langle \phi_B^n \phi_N | V | \psi \rangle
\]

\[
= \lim_{\eta \to 0} \text{Im} \langle \psi | V^\dagger | \phi_N \rangle \sum_n \frac{|\phi_B^n \rangle \langle \phi_B^n|}{E - E_N - H_B + i\eta} \langle \phi_N | V | \psi \rangle
\]

Main challenges

- describe propagator \( G_B = \lim_{\eta \to 0} \sum_n \frac{|\phi_B^n \rangle \langle \phi_B^n|}{E - E_A - H_B + i\eta} \)
- describe wave function \( \psi \)
Benchmarking the method: the $^{95}\text{Mo}(d,p\gamma)$ experiment

$^{95}\text{Mo}(d,p\gamma)$ with 12.5 MeV deuterons at Texas A&M. Protons are detected in coincidence with $\gamma$ rays

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Varenna, June 11th 2018
The information about the predicted spin–parity distribution is used as input for a Hauser–Feshbach description of the \( \gamma \) decay.

- The parameters of the \( \gamma \) decay (NLD and \( \gamma \)SF) are fitted over the indicated energy range for chosen transitions.
The fitted NLD and $\gamma$SF are used to infer $(n, \gamma)$ rates.

Error band from Bayesian analysis of the fit.

No previous knowledge of $D_0$, and/or $\langle \Gamma_{\gamma} \rangle$ is needed.