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# Neutron capture cross sections and strength functions on $^{147}Sm$

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## Outline

1. Introduction

2. Theoretical background

3. Computer codes and calculations

4. Results and discussion

5. Conclusions

# 1. Introduction

**Fast neutron reactions**

**Fundamental research** – new data on nuclear reaction mechanisms and structure of nuclei

**Applicative researches** – precise nuclear data for nuclear fission and fusion reactors, reprocessing of *U* and *Th* for transmutation and energy projects and ADS, Fast Neutron Activation Analysis

**Samarium Isotopes** – investigated for a long time at LNF facilities

- Low (n,$\alpha$) cross section for fast neutrons

**-New data are necessary on $^{147}$Sm reactions with neutrons**

# 2. Theoretical background

**The cross section for (n,α) reaction (Hauser – Feshbach) (HF)**

**- without fluctuation correction factor**

$$\sigma_{n\alpha} = \pi \lambda_n^2 \frac{T_n T_\alpha}{\sum_c T_c}$$

**- with fluctuation correction factor**

$$\sigma_{n\alpha} = \pi \lambda_n^2 \frac{T_n T_\alpha}{\sum_c T_c} W_{n\alpha}$$

$T =$ **transmission coefficient**

$W_{n\alpha} =$ **fluctuation correction factor**

**Differential cross section**

$$\frac{d\sigma}{d\Omega} = \pi \lambda^2 (2l+1) T_l \sum_J \frac{A_J\left(l, j \mid l^{'}, j^{'} \mid \theta\right)}{1 + \sum_{p,q,r} \frac{T_p\left(E_q^{'}\right)}{T_{l'}\left(E^{'}\right)}}$$

$$A_J\left(l, j \mid l^{'}, j^{'} \mid \theta\right) = \sum_{m,m'} \left|\left(l, j; 0m \mid l, j; Jm\right)\right|^2 \left|\left(l', j'; m' m - m' \mid l', j'; Jm\right)\right|^2 \left|Y_{l'm'}(\theta, \varphi)\right|^2$$

**A contains the dependence on**

- quantum numbers in incident and emergent channels (l, j, l’, j’, J, m)
- solid angle (Ω (θ,ϕ))

# 2. Theoretical background

**Assumptions**

- The nuclear reaction induced by neutrons take place by formation of compound nucleus (CN)
- The CN has long time of life in comparison with the time necessary for the incident particle to traverses the nucleus
- The CN decays on one possible channel
- The possible ways of decaying not depend on how the CN was formed
- The CN and the residual nucleus are characterized by a great number of states
- The nuclear potential acts in finite range and is zero outside of this range

**Consequences**

- No interference terms in the cross section
- Differential cross section is symmetrical to $90^0$ in SCM

Gh. Vladuca, Nuclear Physics, v. 2., University of Bucharest Editing House, Bucharest, 1986 (in Romanian)

# 2. Theoretical background

**Transmission coefficients**

**Evaluation –** Semi - classical approach

$$T(l,E) = \exp\left\{ -\sqrt{\frac{8m}{\hbar^2}} \int_D \left[ V(r) + \frac{zZe^2}{r} + \frac{\hbar^2 l(l+1)}{2mr^2} - E \right]^{1/2} dr \right\}$$

$m$ = reduced mass

$V(r)$ = nuclear potential

$\frac{zZe^2}{r}$ = Coulomb potential, $z$ = particle charge, $Z$ = residual nucleus charge

$\frac{\hbar^2 l(l+1)}{2mr^2}$ = centrifugal potential

# 2. Theoretical background

**Quantum mechanical approach used**

$$T(l,E) = 1 - |U_l(E)|^2$$

$W_l^+(r) =$ **Ingoing wave function**

$W_l^-(r) =$ **Outgoing wave function**

**Reflection factor**

$$U_l = \left\{ \frac{D_l - R\left[\frac{1}{W_l^-}\frac{dW_l^-}{dr}\right]}{D_l - R\left[\frac{1}{W_l^+}\frac{dW_l^+}{dr}\right]} \frac{W_l^-}{W_l^+} \right\}_{r=R}$$

**Solution of Radial Schrodinger Equation**

$$W_l(r) \sim W_l^-(r) - U_l W_l^+(r)$$

**Logarithmic derivative**

$$D_l = R\left[\frac{1}{W_l}\frac{dW_l}{dr}\right]_{r=R}$$

**Radial Schrodinger Equation**

$$\frac{d^2 W_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left[E_l - V(r) - \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right] W_l(r) = 0$$

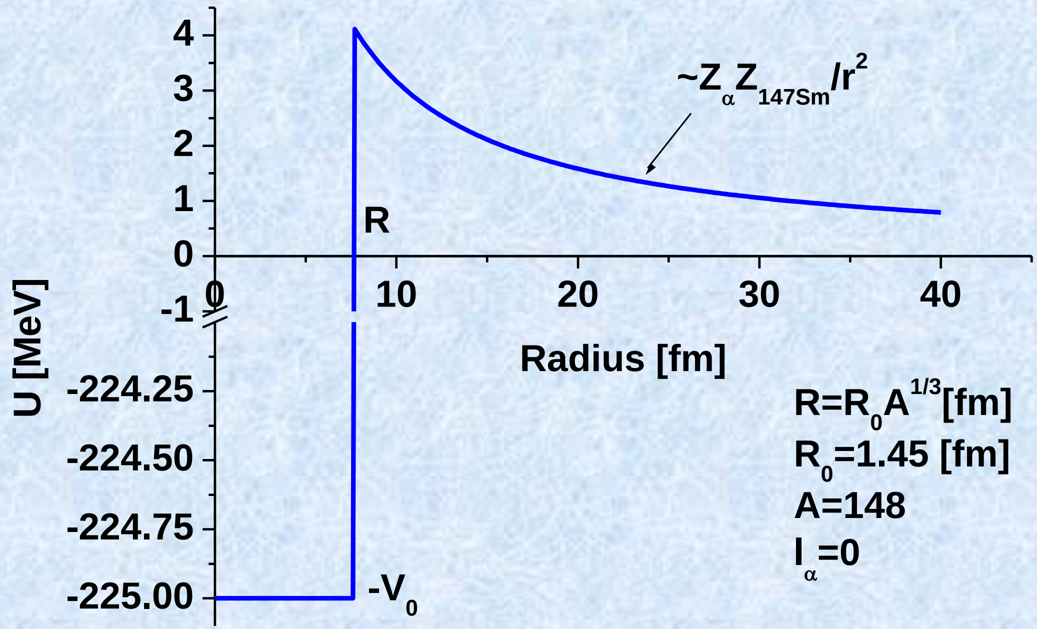
Antonio Foderaro, The Neutron Interaction Theory, The MIT Press, Cambridge, Massachusetts and London, England, 1971

## 2. Theoretical background. Nuclear Potential in alpha channel

$$V(r) = \begin{cases} -V_0, r \le R \\ 0, r > R, (neutrons) \\ \dfrac{zZe^2}{r}, r > R, (protons, \alpha, ....) \end{cases}$$

$$V_0 = 225 MeV$$

$$R = R_0 A^{\frac{1}{3}} [fm]$$




$^{147}Sm(n,\alpha)^{144}Nd$

Orbital momentum of alpha particles $l_\alpha=0$

In the figure – real part of the potential

Used in the soft realized by us

# 2. Theoretical background

**Quantum mechanical approach used**

**For neutrons - combination of Neumann (*n*) and Bessel (*j*) functions**

$$W_l^+(r) = kr\left[n_l(kr) + ij_l(kr)\right] \qquad W_l^-(r) = kr\left[n_l(kr) - ij_l(kr)\right]$$

For charged particles - combination of Regular (*F*) and Irregular (*G*) Coulomb functions

$$W_l^+(r) = kr\left[F_l(kr) + iG_l(kr)\right] \qquad W_l^-(r) = kr\left[F_l(kr) - iG_l(kr)\right]$$

**Widths Fluctuation Correction Factor (WFC)**

- Represents a correlation between incident and emergent channels
- At low energies WFC = 1
- Then slowly decreasing with energy
- Mainly three ways of evaluation
- Moldauer expression chosen

$$W_{ab} = \left(1 + \frac{2\delta_{ab}}{\nu_a}\right)\int_0^\infty \prod_c \left(1 + \frac{2T_c x}{\nu_c \sum_i T_i}\right)^{-\left(\delta_{ac} + \delta_{bc} + \frac{\nu_c}{2}\right)} dx$$

$$\nu_a = 1.78 + \left(T_a^{1.212} - 0.78\right)\cdot e^{-0.228\sum_c T_c}$$

P. A. Moldauer, Rev. Mod. Phys., v. 36, p. 1079, 1964

# 2. Theoretical background

**Strength functions**

$$S = \frac{T}{2\pi} = \left\langle \frac{\Gamma}{D} \right\rangle$$

$\Gamma =$ Width $\quad D =$ Level spacing

Gledenov et all. Phys Rev C **69**, 015803 (2004)

where T are the transmission coefficients which

- are related to strength functions;

- are dimensionless between 0 and 1;

- give the probability that a certain state will break-up into products of specified type.

Strength functions describe how widths are distributed in the nucleus

**Objective –** to evaluate alpha strength functions in $^{147}$Sm(n,$\alpha$)$^{144}$Nd reaction

# 3. Computer codes and calculations

**Own computer code**

**We implemented Hauser – Feshbach (HB) approach**

We realized a software in Mathematica able to compute:

- The regular and irregular Coulomb functions for neutral and charged particles and their derivatives

- For Coulomb functions no approximations were used

- The transmission coefficients for neutral and charged particles

- Implementation the quantum mechanical approach

- The cross section is obtained by taking into account the fluctuation factor and other open channels (n, n', p, $\gamma$)

- This software was used for the evaluation of the $^{147}Sm(n,\alpha)^{144}Nd$ cross section and alpha strength function

# 3. Computer codes and calculations

**TALYS Codes**

**-** free software working under Linux operating system in continue development

- friendly interface

- a large number of models for nuclear structure and nuclear reactions (direct, compound, pre - equilibrium) implemented

- data base on nuclear structure for a large number of nuclei

- allows to evaluate: nuclear structure data, inclusive and exclusive cross sections (XS)

- **Inclusive XS – Ex,** in a binary reaction A(a,b), b will be considered emergent particles from other possible open channels

- **Exclusive XS** – in a binary reaction b will be considered emergent particles from a well defined “b+B” exit channel

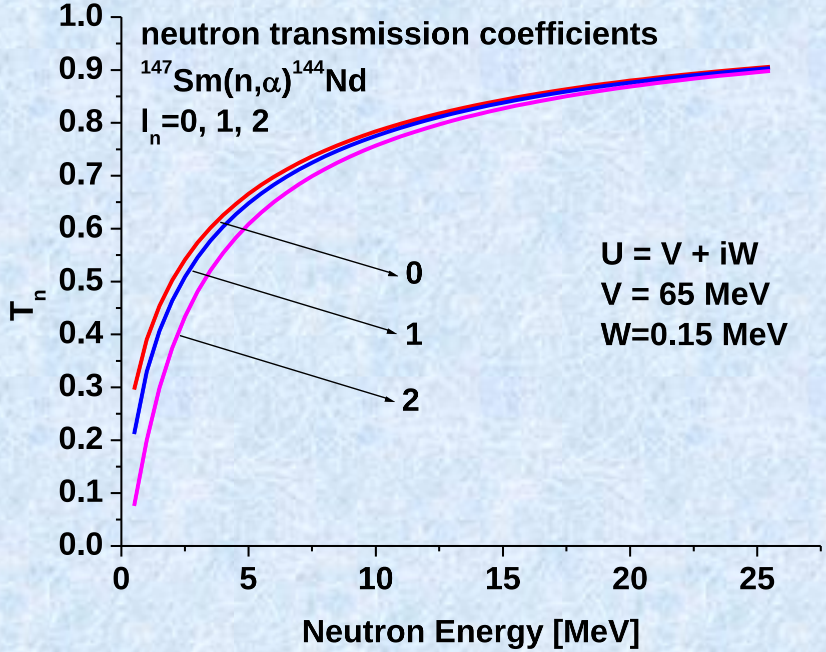
- **Talys** will be used in the XS calculations of fast neutron induced reactions with emission of alpha particles

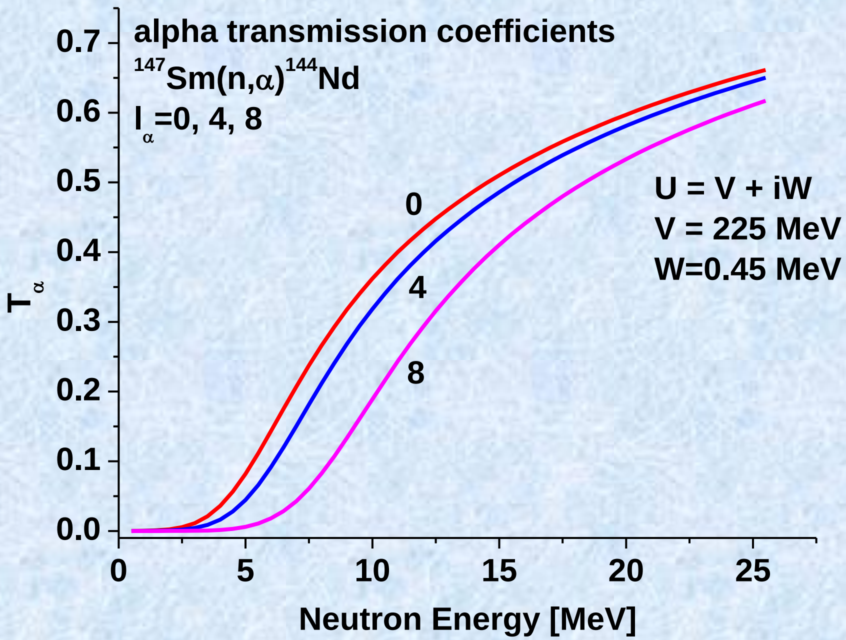
The results were compared between the two softwares for HB approach.

A.J. Koning, S. Hilaire and M.C. Duijvestijn, .TALYS-1.0., Proceedings of the International Conference on Nuclear Data for Science and Technology, April 22-27, 2007, Nice, France, editors O.Bersillon, F.Gunsing, E.Bauge, R.Jacqmin, S.Leray, EDP Sciences, p. 211 (2008)

# 4. Results and discussion - $^{147}Sm(n,\alpha)^{144}Nd$ – transmission coefficients

## Energy dependence of neutron and alpha transmission coefficients



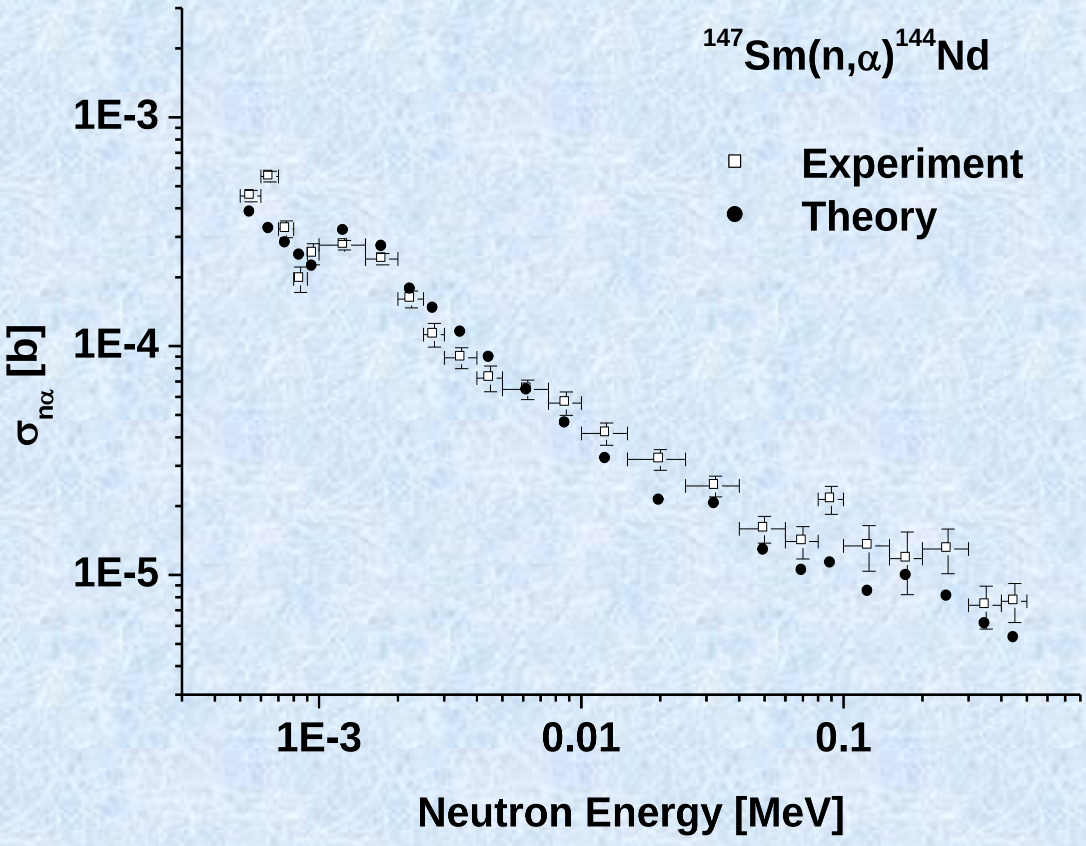




Orbital momentum: neutrons – $l_n$=0,1,2; alphas – $l_\alpha$=0,4,8

Calculated with our soft based quantum mechanical approach

# 4. Results and discussion - $^{147}Sm(n,\alpha)^{144}Nd$ – cross section up to 0.5 MeV

**Compound processes are dominant**




- From 1 keV up to 450 keV real part of the potential V was slowly modified from 0.85 and up to 1 in order to describe experimental data

**Calculations by our soft**

$^{147}$Sm - $J^{\Pi} = (7/2)^{-}$

$^{148}$Sm - $J^{\Pi} = 3^{-}, 4^{-}$

Rectangular potential

U = V + iW

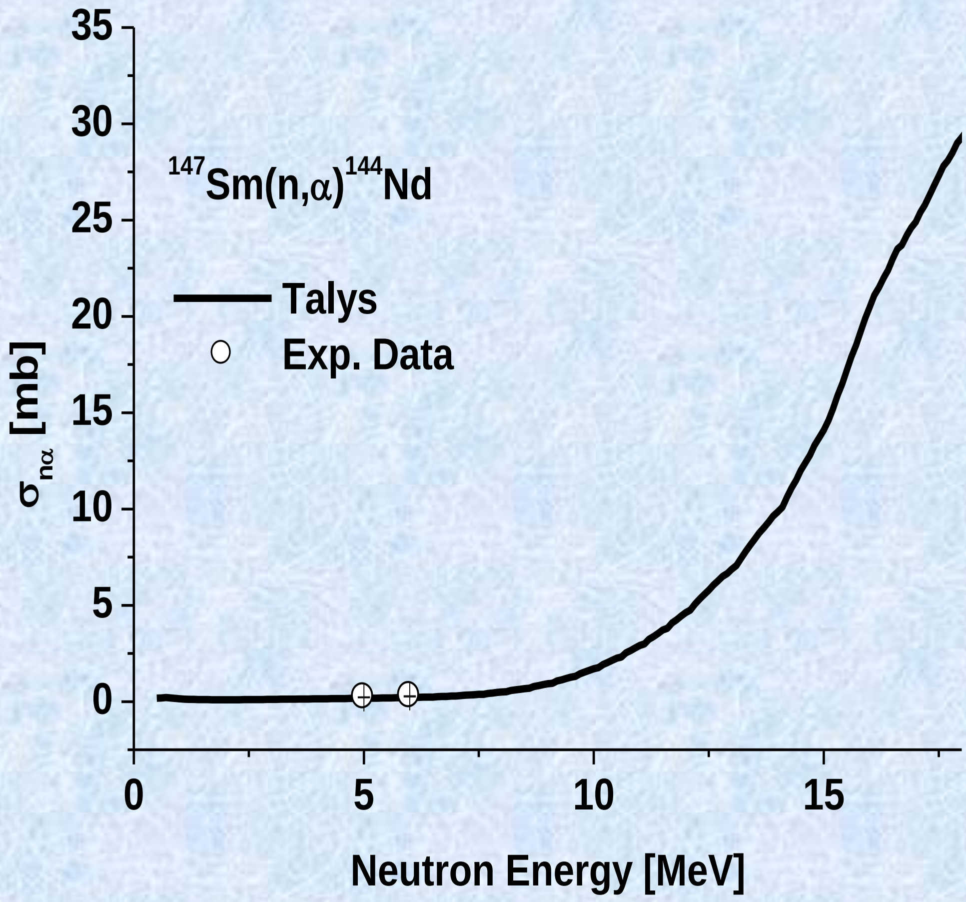
- neutron channel
V = 65 MeV, W = 0.15 MeV

- alpha channel

$^{147}Sm(n,\alpha)^{144}Nd$ with Q = 10.128 MeV
V = 225 MeV; W = 0.15 MeV

Gledenov et. all, Phys. Rev. C, 62, 042801R (2000)

# 4. Results and discussion - $^{147}Sm(n,\alpha)^{144}Nd$ with fast neutrons




Experimental Data – obtained using a double grid ionization chamber mainly at FLNP JINR Dubna basic facilities

**Talys evaluation**

- **from 0.5 up to 18 MeV**

Implemented all components of Wodd-Saxon potential – volume, surface, spin – orbit with real and imaginary part

- taking into account all nuclear reaction mechanism (compound, direct, pre-equilibrium)
- Compound processes are dominant

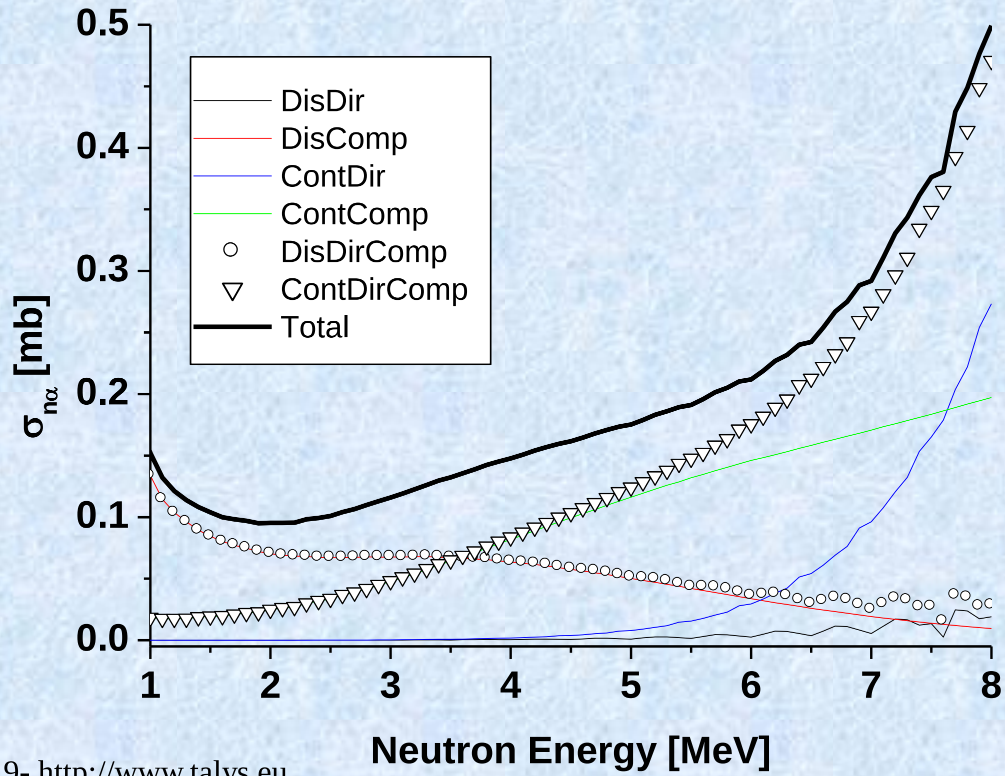
- pre-equilibrium – Two exciton model

- density levels model – Constant Temperature Fermi Gas Model

A. Oprea, C. Oprea, Rom. Rep. Phys, 63, 1, 107 (2011)

# 4. Results and discussion – $^{147}Sm(n,\alpha)^{144}Nd$ with fast neutrons

| $E_n$ [MeV] | Direct [mb] | | Compound [mb] | | $\sigma_{n\alpha}$ [mb] |
|---|---|---|---|---|---|
| | Discr | Cont | Discr | Cont | |
| 5 | 0.00097 | 0.00787 | 0.05023 | 0.11627 | 0.1754 |
| 6 | 0.00248 | 0.02951 | 0.03379 | 0.14606 | 0.2118 |
| 15 | 0.04970 | 1.57825 | 0.00156 | 0.26330 | 1.89201 |



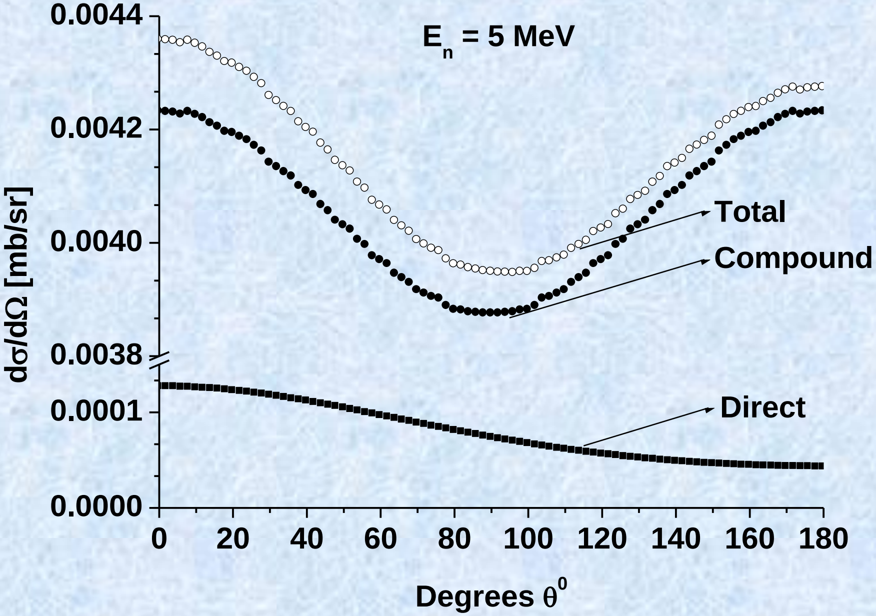

Talys 1.9- http://www.talys.eu

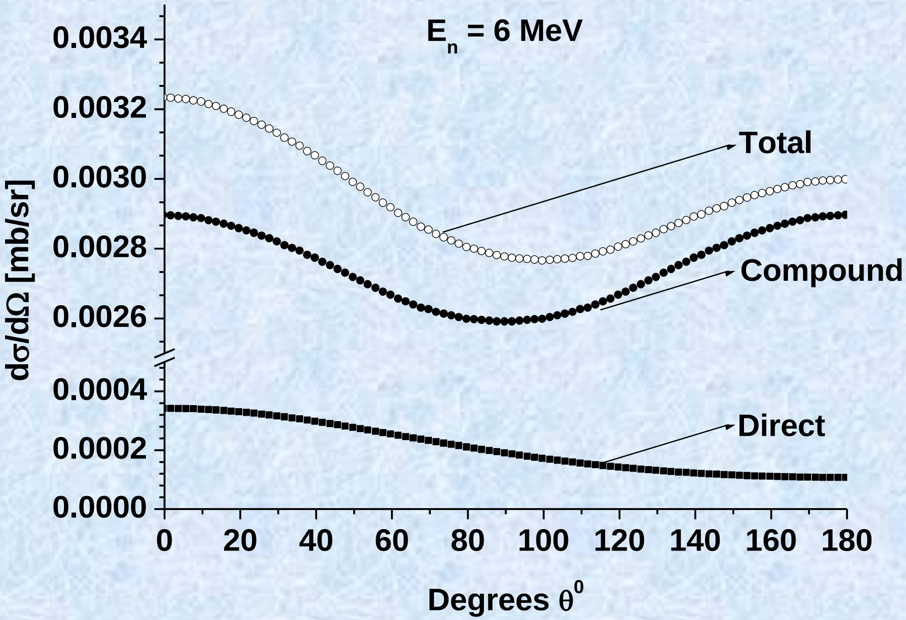
**Cross Section**
-Separation of different reaction mechanism and states of residual nucleus
**Mechanism**
– Direct, Compound
**States**
-Discrete, Continuum

At low energies compound processes are dominant but with the increasing of the energy the direct ones becomes more important

Necessary for differential cross sections analysis

# 4. Results – Differential cross section / Forward – Backward ratio / Talys




Forward / backward ratio

| $E_n$ [MeV] | $a_F/a_B$ (Talys) | $(a_F/a_B)_{exp}$ |
|---|---|---|
| 5 | 1.0122 ± 0.0096 | 1.65 ± 0.165 |
| 6 | 1.0436 ± 0.0127 | 2.54 ± 0.254 |

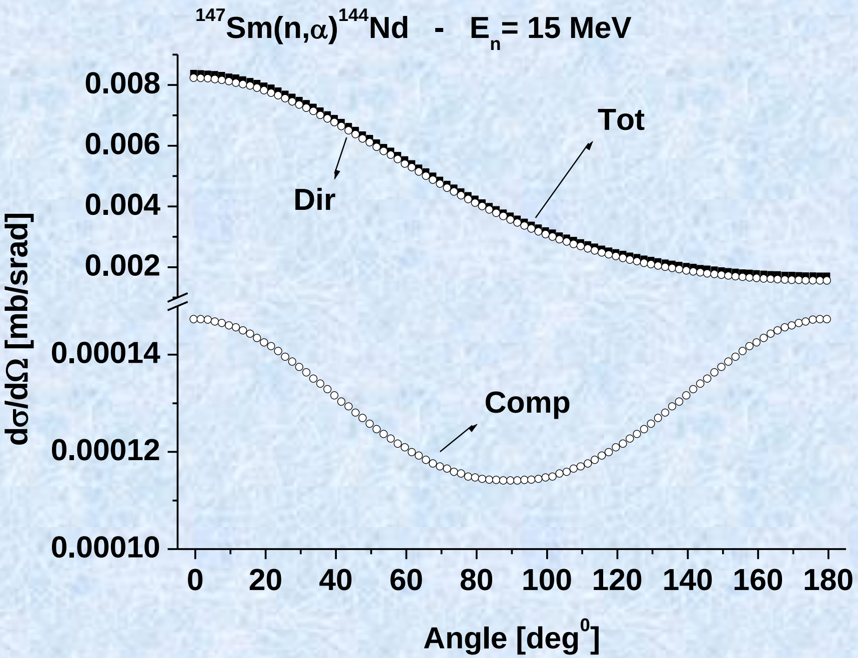
Experimental – all events in forward and backward directions

In evaluation – diff. XS integrated in:
FW – θ – >0-π/2 / BW - θ – >π/2 - π

Serious discrepancy between experiment and theory

- The evaluations show that the contribution of direct process, causing the asymmetries is very low in comparison with compound processes / Further experimental and theoretical investigations are necessary

# 4. Results – Differential cross section / Forward – Backward ratio / Talys




**Differential Cross Section**

$$\frac{d\sigma}{d\Omega}(\theta) = p_0 + p_1\cos(\theta) + p_2\cos^2(\theta)$$

| $E_n$ [MeV] | $p_0$ | $p_1$ | $p_2$ |
|---|---|---|---|
| 5 | 0.0038 | 0.00004 | 0.00036 |
| 6 | 0.00294 | 0.00012 | 0.00035 |
| 15 | 0.0063 | 0.00324 | 0.00136 |

| **$E_n$ [MeV]** | **$a_F/a_B$ (Talys)** | **$(a_F/a_B)_{exp}$** |
|---|---|---|
| 15 | 2.342 ± 0.008 | - |

At 15 MeV neutron incident energies direct processes are dominant -> asymmetries in diff. XS can be observed.

Using the Talys results on diff. XS (Table + expression) the evaluated asymmetry is much higher. For 15 MeV there are not experimental evaluation of asymmetry.

- The errors on the ratios – the integrals on solid angles are in fact sums on the θ angle with step $1^0$

## **4. Results and discussion** – Forward – Backward ratio. Alpha Spectra Simulation

$$\frac{d\sigma}{d\Omega}(\theta) = p_0 + p_1\cos(\theta) + p_2\cos^2(\theta)$$ Differential Cross Section

$$\frac{2\pi}{\sigma_{n\alpha}}\int_0^{\theta_c}\frac{d\sigma}{d\Omega}\sin(\theta)d\theta = r \Rightarrow \theta_c$$

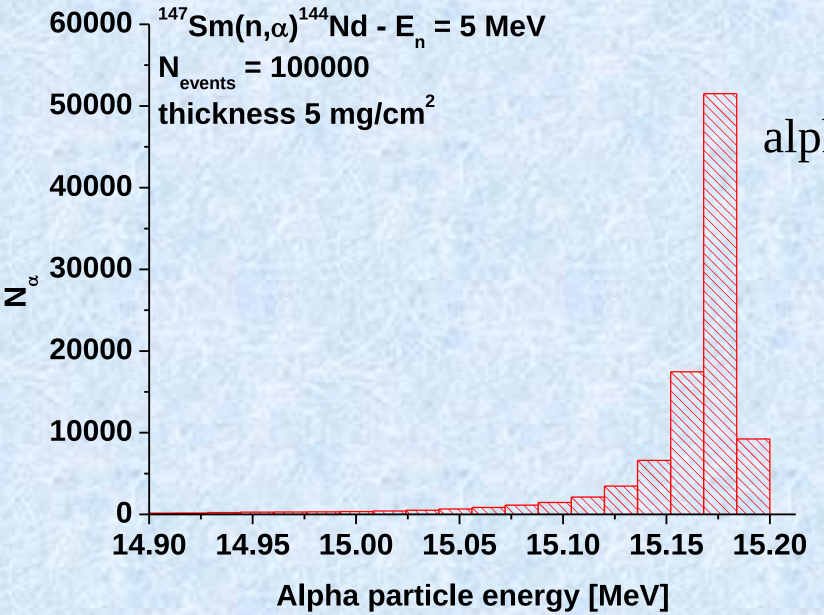
$$r \in [0,1), \theta \in [0,\pi)$$

Direct method

- solved numerically

FW-BW effect

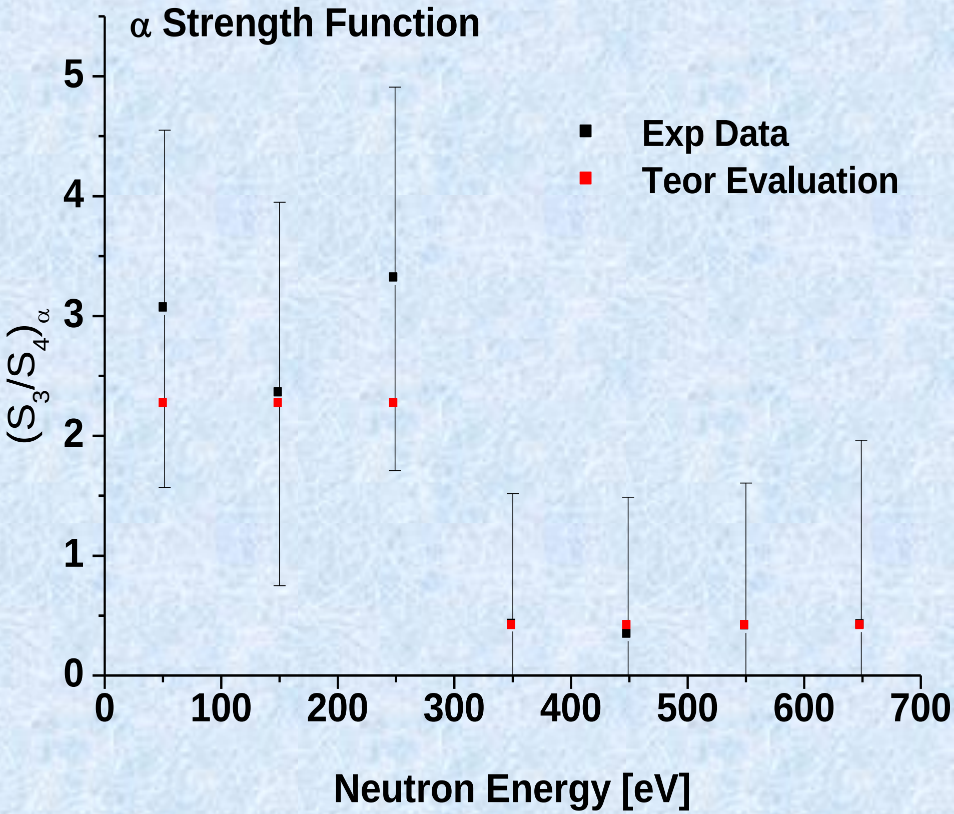
| $E_\alpha$ [MeV] | $(a_F/a_B)_{sim}$ |
|---|---|
| 5 | 1.02 ± 0.007 |
| 6 | 1.04 ± 0.009 |
| 15 | 2.31 ± 0.017 |




alpha spectra – tin target

| $E_n$ [MeV] | $a_F/a_B$ (Talys) | $(a_F/a_B)_{exp}$ |
|---|---|---|
| 5 | 1.0122 ± 0.0096 | 1.65 ± 0.165 |
| 6 | 1.0436 ± 0.0127 | 2.54 ± 0.254 |

# 4. Results and discussion – α Strength Function / Non statistical effects




Ratio of α strength functions

Corresponding to compound nucleus spin J = 3,4

- $S_3/S_4$ has a serious change at 300 eV

- usual expected to be constant

- described by our soft

$U = V + IW = (225 + I\ 0.45)$ MeV

$R = R_0 A^{(1/3)}$ – channel radius

$E_n < 300$ eV; $E_n > 300$ eV – 2 regions

Ratio observed by FLNP group – explained by the presence of "non statistical effects"

From evaluation – the ratio is not so sensible to the modification of Wood - Saxon potential in incident and emergent channels

- Description of experimental ratio -> radius in the emergent channel was modified with almost 20% -> one possible explanation – presence of particles higher than alpha

Gledenov et. all, Phys. Rev. C, 69, 015803 (2004)

# 4. Results and discussion – Nuclear Potential Parameters

Wood - Saxon Potential

Volume WS – Real Part

| | V [MeV] | $r_v$ [fm] | $a_v$ [$fm^{-1}$] |
|---|---|---|---|
| N_chann | 49.81 | 1.227 | 0.656 |
| $\alpha$_chann | 226.25 | 1.227 | 0.657 |

Volume WS – Imaginary

| W [MeV] | $r_w$ [fm] | $a_w$ [$fm^{-1}$] |
|---|---|---|
| 0.11 | 1.227 | 0.656 |
| 0.38 | 1.227 | 0.657 |

Spin orbit – Real Part

| | $V_{so}$ [MeV] | $r_{vso}$ [fm] | $a_{vso}$ [$fm^{-1}$] |
|---|---|---|---|
| N_chann | 6.963 | 0.590 | 0.01 |
| $\alpha$_chann | 1.071 | 0.590 | 0 |

Spin orbit – Imaginary

| $W_{so}$ [MeV] | $r_{wso}$ [fm] | $a_{wso}$ [$fm^{-1}$] |
|---|---|---|
| 1.063 | 0.590 | 0 |
| 1.071 | 0.590 | 1.300 |

# 6. Conclusions

**Cross sections**

Good description of experimental data using own codes and Talys
Differential cross sections were evaluated / no experimental data
Concurrence of different nuclear reaction mechanism were evidenced

**Strength functions**

In experiment determined with large errors; decreasing by energy can be considered
Few explanations were proposed by the present research based on the theoretical evaluations

## Future tasks

- New theoretical evaluations based on the new XS and diff. XS measurements in wide neutron energy interval

- To determine for each energy the contribution of nuclear reaction mechanisms based on future experimental data

- Improvements of Monte Carlo simulation

- Theoretical follow up of new nuclear experimental data on strength functions

Present work - proposal for new experiments at FLNP JINR Dubna Facilities


THANK YOU VERY MUCH
FOR YOUR ATTENTION! ☺