Neutron width statistics in a realistic resonance-reaction model

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- Statistical model of compound nucleus reactions.
- Violation of statistical model in n + Pt reaction and overview of proposed theoretical explanations.
- Novel realistic resonance-reaction model.
- Cross sections and neutron width fluctuations in the n + ^{194}Pt reaction.
- Summary and conclusions.

PF, G. F. Bertsch, and Y. Alhassid, arXiv:1710.00792, accepted in Phys. Rev. C

Statistical model of compound nucleus reactions

- Compound nucleus (CN): complex, equilibrated system of projectile + target.

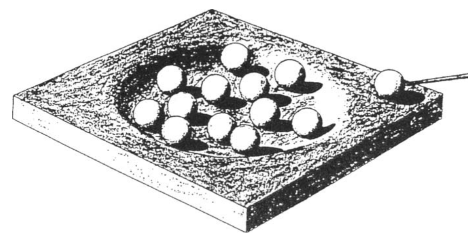
- Statistical model: CN Hamiltonian is a random matrix from the Gaussian Orthogonal Ensemble (GOE).

- CN resonance energies and widths exhibit local fluctuations that are independent of system-specific details.

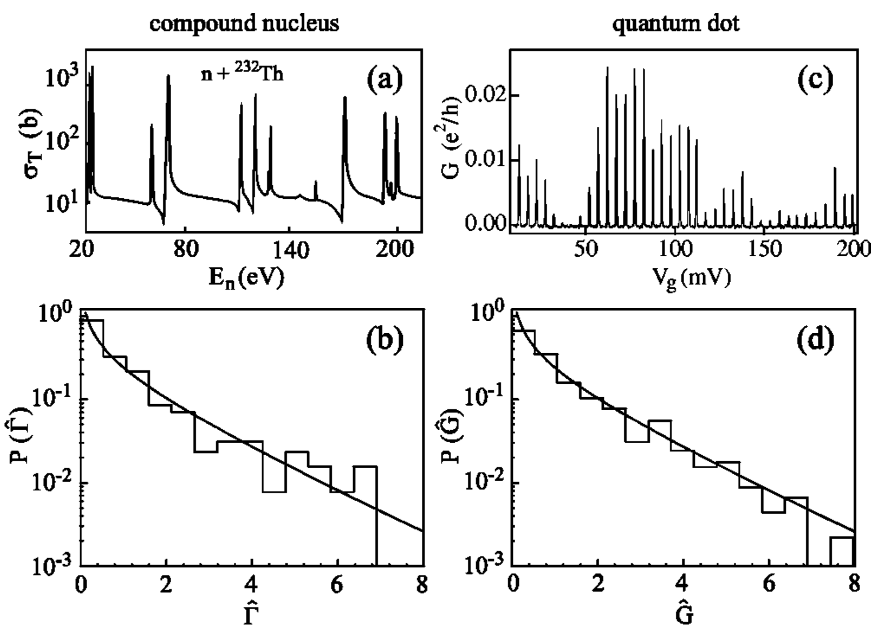
Mitchell, Weidenmüller, and Richter, RMP (2010).

- Modifies Hauser-Feshbach theory of CN reactions through width fluctuation correction factor.
  - Enhances elastic channels; decreases other open channels.

- Widely used in statistical reaction calculations.



Bohr, Nature (1936)

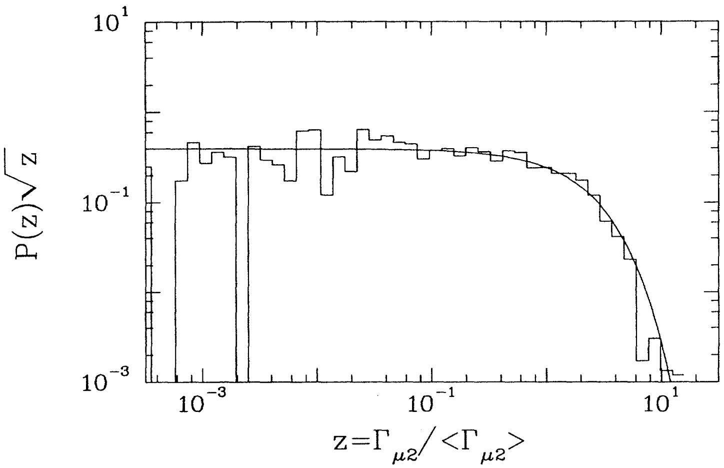


Alhassid, RMP (2000)

Violation of Porter-Thomas Distribution in Platinum

- For isolated resonances, widths for any reaction channel should fluctuate according to the Porter-Thomas distribution (PTD).

Porter and Thomas, Phys. Rev. (1956).



- Neutron resonance widths are considered compelling evidence for PTD, especially the Nuclear Data Ensemble.

    Haq, Bohigas, and Pandey, PRL (1982).

- Evidence in other quantum chaotic systems described by the GOE, e.g. microwave stadium billiard.

Alt *et al.* PRL (1995)

- Koehler *et al.* PRL (2010): neutron width fluctuations form neutron scattering off Pt isotopes are much broader than the PTD.

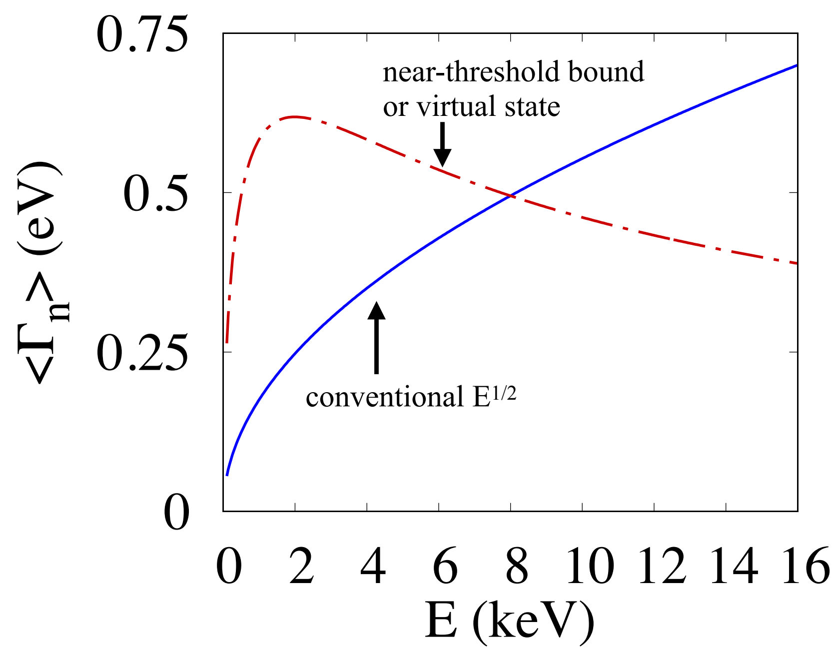
- Other statistical model violations observed in reanalysis of the NDE and in total gamma width distributions.

Koehler, PRC (2011).
Koehler *et al.* PRC (2013).

Statistical Model Explanations

1. Energy dependence of neutron strength function changed by near-threshold bound or virtual state of neutron channel potential [Weidenmüller PRL 2010].

- Evidence: peak in the NSF near Pt.



2. Non-statistical interactions of the resonances due to coupling to the neutron channel perturbs the GOE [Celardo *et al.* PRL (2011), Volya *et al.* PRL (2015), Bogomolny PRL (2017)].

$$H^{\text{eff}} = H^{\text{GOE}} + \Delta - i\pi W W^T$$

effective Hamiltonian of CN resonances

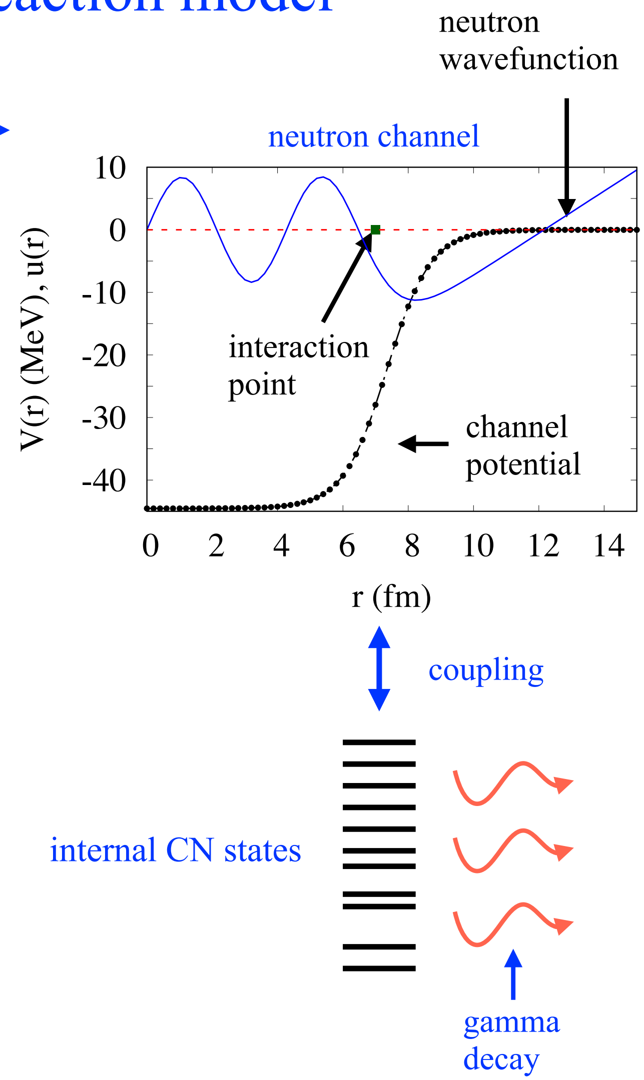
GOE Hamiltonian of bound CN states

real (imaginary) shift due to off (on)-shell couplings to channels

- Explanations have not been assessed together in a realistic model. Origin of observed PTD violation still an open question

Realistic resonance-reaction model

$$\mathbf{H} = \begin{pmatrix} \mathbf{H_n} & \mathbf{V} \\ \mathbf{V}^T & \mathbf{H_c} \end{pmatrix}$$

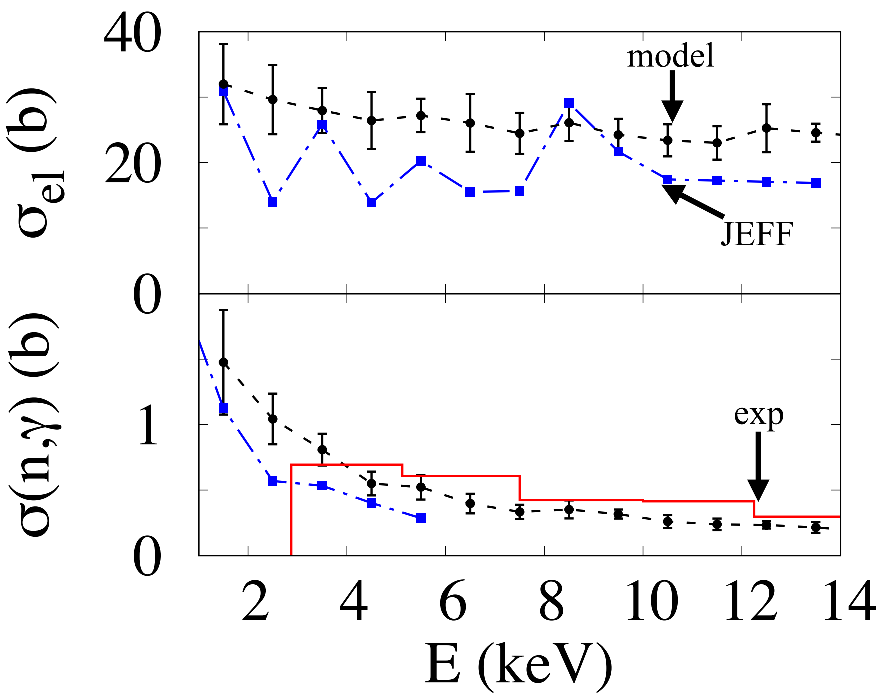


- Neutron channel represented on a spatial mesh with a Woods-Saxon potential.
- CN states have GOE spectrum with a constant imaginary shift to describe gamma decay.
- Channel coupled to CN states at one interaction point. Coupling strengths have a Gaussian distribution.
- Able to calculate resonance energies and widths, as well as cross sections, within the same framework.
- Based on Mazama code of G. F. Bertsch.

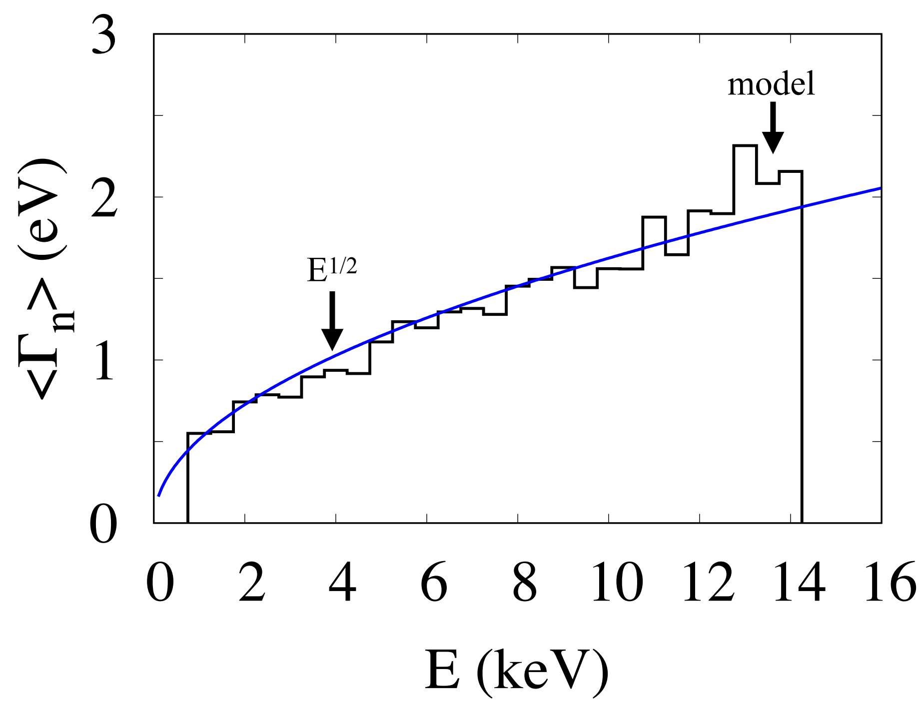
Bertsch, Brown, and Davis, arXiv:1804.00364 (2018).

n + ^{194}Pt: Baseline Model

- Woods-Saxon: V_0 = -44.54 MeV, r_0 = 1.27 fm, a_0 = 0.67 fm from Bohr and Mottelson.
- Average resonance spacing D = 82 eV, gamma width Γ_γ = 72 meV from RIPL-3.
- Average coupling strength parameter v_0 = 11 keV-fm$^{1/2}$ tuned to match RIPL-3 neutron strength function at 8 keV neutron energy.

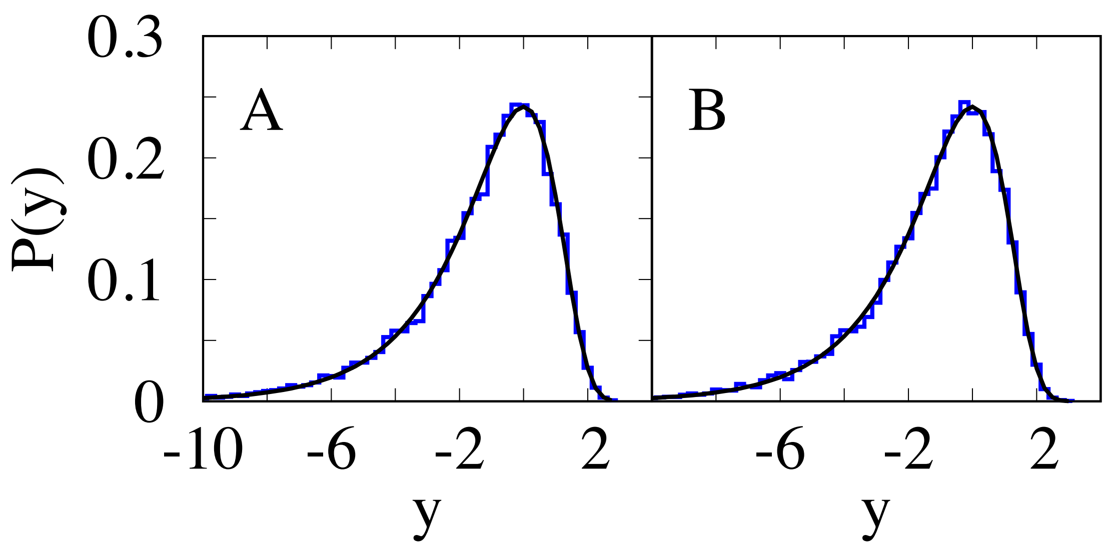


Model cross sections agree with JEFF-3.2 library and experimental cross section [Koehler and Guber, PRC (2012)].



Average neutron width matches the $E^{1/2}$ expectation.

Baseline neutron width fluctuations



histogram is model
solid line is PTD

- Reduced neutron width: $\gamma_{n,r} = \Gamma_{n,r}/\langle\Gamma_n\rangle(E_r)$
- Reduction A: average neutron width taken from model.
- Reduction B: assumes $\langle\Gamma_n\rangle(E) \propto E^{1/2}$
- Figure shows distribution of y = ln(x), $x = \gamma/\langle\gamma\rangle$

Parameter variation

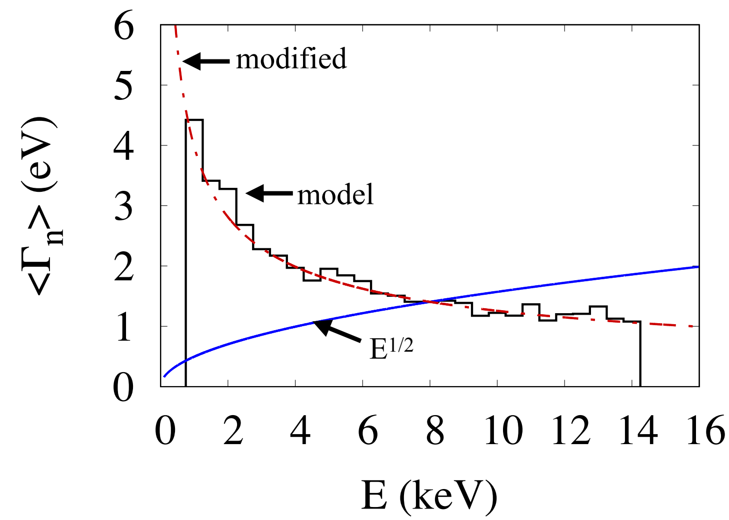
- Reducing neutron channel potential depth by a few MeV creates a near-threshold bound or virtual state of the channel.

- Modifies the $E^{1/2}$ behavior of the average width, agrees with the formula of Weidenmüller: $\langle \Gamma_n \rangle (E) \propto \frac{\sqrt{E}}{E + |E_0|}$

- The reduced width distribution extracted with the $E^{1/2}$ form is broader than PTD.

- Results insensitive to physically large changes in average channel-CN coupling.

results for $E_0 = 0$



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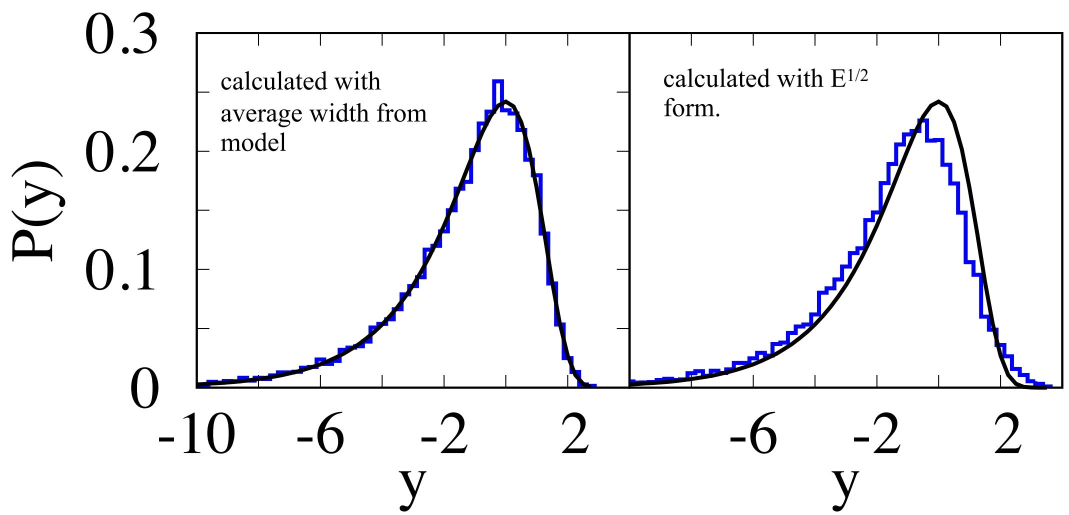

Summary and Conclusions

- In our realistic model, the PTD describes the neutron width fluctuations well if the energy dependence of the average neutron width is correctly described.
- This study excludes explanations for the observed PTD violation based on coupling to the neutron channel.
- Within a reasonable parameter range, there can be a near-threshold bound or virtual state of the neutron channel. In this case, using the $E^{1/2}$ form will produce PTD violation. Near-threshold state must be within a few keV to have a significant effect.
- A reanalysis by the experimentalists [Koehler *et al.* arXiv:1101.4533 (2011)] found that using this state did not improve their agreement with the PTD. Problem with multilevel R-matrix analysis?
- The elastic and total cross sections are quite sensitive to the existence of such a state.
- More experimental investigation into resonance width statistics would be useful to explore possible breakdowns of statistical model.

Thank you for your attention!

Extra Slides

Resonance determination

- To find the complex resonance wavenumbers k_r, solve Schrödinger equation with appropriate boundary conditions
  - neutron wavefunction is regular at origin.
  - neutron wavefunction is purely outgoing.

$$u(0) = 0$$

$$u(r) \rightarrow B(k)e^{ikr} \quad \text{for large r}$$

$$\Rightarrow u(N_n + 1) = u(N_n)e^{ik\Delta r}$$

- With discretized approach, obtain a nonlinear eigenvalue problem (NEVP).

$$\mathbf{M}(k)\vec{u} = [\mathbf{H} - E - te^{ik\Delta r}\mathbf{C}]\vec{u}$$

- Solve NEVP with an iterative method to find resonance wavenumbers k_r.

$$t = \hbar^2/2m\Delta r^2 \ , \ \mathbf{C}_{ij} = \delta_{i,N_n}\delta_{ij}$$

- Find resonance energies, total widths, and neutron widths from wavenumbers.

$$E_r - \frac{i}{2}\Gamma_r = \frac{\hbar^2 k_r^2}{2m}$$

- Can calculate elastic and capture cross sections [details in additional slides].

$$\Gamma_{n,r} = \Gamma_r - \Gamma_\gamma$$

Non-statistical interactions

$$H^{\mathrm{eff}} = H^{\mathrm{GOE}} + \Delta - i\pi W W^T$$

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