Towards a non-local microscopic description of scattering observables of nucleons on deformed nuclei

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Why study nucleon induced reactions?

- Elastic scattering
- Inelastic scattering
- Radiative capture
- Particle emission
- Fission

• Direct
• Preequilibrium
• and compound processes

Input for other reaction mechanisms’ models
Direct reactions: scattering

- Direct elastic and inelastic scattering
  - DWBA
  - Local potential
  - Non local potential
  - Coupled channels
  - Phenomonologic approach
  - Microscopic approach
I. Coupled channels and DWBA calculations

II. Non local potentials
III. Our method
IV. Applications
V. Conclusion and perspectives
Theoretical reminders : Coupled Channels / DWBA

\[ [E_{\text{incident}} - K - \langle I|V|I \rangle] u_I(r) = \sum_{j \neq I} \langle I|V|J \rangle u_j(r) \]

I = elastic channel
F = inelastic channel

\[ [(E_{\text{incident}} - \epsilon_F) - K - \langle F|V|F \rangle] u_F(r) = \sum_{j \neq F} \langle F|V|J \rangle u_j(r) \]

Coupled Channels

DWBA

\[ [E_{\text{incident}} - K - \langle I|V|I \rangle] u_I(r) \approx 0 \]

\[ [(E_{\text{incident}} - \epsilon_F) - K - \langle F|V|F \rangle] u_F(r) = \sum_{j \neq F} \langle F|V|J \rangle u_j(r) \]

hypothesis : \( \sigma_{\text{elastic}} \gg \sigma_{\text{inelastic}} \)
Reminders: nucleon scattering on deformed target

$^{208}\text{Pb}$
First excitations: surface vibrations
- DWBA
- CC

$^{238}\text{U}$: axially deformed
First excitations: rotations
- DWBA
- CC

DWBA: good approximation
CC: bad approximation

$\text{factor up to 10 between DWBA and CC}$
I. Coupled Channels and DWBA calculations

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In nuclear physics, the potential of interaction is non local:

\[-\frac{\hbar^2}{2\mu} \nabla_r^2 \Psi(r) + \int U_{\text{non local}}(r, r') \Psi(r') \, dr' = E\Psi(r)\]

Usual approximation:

\[-\frac{\hbar^2}{2\mu} \nabla_r^2 \Psi(r) + U_{\text{local}}(r) \Psi(r) = E\Psi(r)\]
First phenomenologic studies with non local potentials done in the 1960s, uneasy numerical treatment

Today, many phenomenological approaches:

- Dispersive non local potential (W. Dickhoff)
- Nucleon transfer description with non local potentials (NLAT code)
- Global dispersive non local potential of B. Morillon (CEA DAM)


Derivation of a microscopic non local optical potential and application to nucleon elastic scattering on spherical target: Nuclear structure method

Microscopic potentials derived from nuclear matter approaches applied to the description of elastic and inelastic scattering of nucleons and transfer reactions

Potentials obtained by ab initio methods

Objectives of the Ph.D. project

Describe nucleon-nucleus scattering observables with a microscopic, parameter free method.

Study direct elastic and inelastic scattering, and preequilibrium emissions.

Requirements: CC code for nonlocal potentials, another code to compute nonlocal potentials.
Possible applications of interest

Description of the rotational band of $^{238}$U basing the modelling on:

- Melbourne $G$ matrix
- HFB (+ QRPA)

$$|JM K^{\pi} \rangle = \sqrt{\frac{2J+1}{16\pi^2}} \int D_{MK}^J(\Omega) R(\Omega) |K^{\pi} \rangle d\Omega$$

$$+ (-1)^{J+K} D_{M-K}^J(\Omega) R(\Omega) |\bar{K}^{\pi} \rangle$$

$|K^{\pi} \rangle$ Target’s state in the intrinsic frame

Pre-equilibrium calculations: contributions of 1 and 2-phonon excitations (2$^{nd}$ order)

- Melbourne $G$ matrix
- RPA

$$|1 \otimes 2 \rangle = |3 \rangle$$

$$|1 \rangle = \Theta_{N1, J1}^{+} |\bar{0} \rangle$$

$$|2 \rangle = \Theta_{N2, J2}^{+} |\bar{0} \rangle$$

$|\bar{0} \rangle$
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CC calculation with non local potentials

\[ U_{FI}(\vec{r}, \vec{r}') = \sum_{24}^{24} \langle \vec{r} | 2 | V_{\text{effective}} | \vec{r}' \rangle \rho_{24}^{FI} \]

\[
\begin{align*}
(E_{\text{inc}} - K - U_{II}) \varphi_I &= \sum_{j \neq I} U_{Ij} \varphi_j \\
(E' - K - U_{FF}) \varphi_F &= \sum_{j \neq F} U_{Fj} \varphi_j
\end{align*}
\]

Potentials computation

-effective interaction

-target's structure description

Melbourne G matrix

mean field and beyond approach: HF/HFB + RPA/QRPA

target nuclei for validations: \(^{208}\text{Pb}\) and \(^{90}\text{Zr}\)

Goal of the Ph.D. project

- Write a scalable CC code for non local potentials: **ECANOL**
- Write a scalable code to compute microscopic potentials

\[
i : \text{target nucleus' ground state} \\
f : \text{target nucleus' discrete excited state} \\
f' : \text{another discrete excited state of the target nucleus}
\]

### Codes

<table>
<thead>
<tr>
<th>Potential calculation</th>
<th>Scattering problem resolution</th>
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<tr>
<td>microscopic</td>
<td>DWBA</td>
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<tr>
<td>( \langle f</td>
<td>V_{\text{eff}}</td>
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<td>( \langle f</td>
<td>V_{\text{eff}}</td>
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<tr>
<th>DWBA 98</th>
<th>ECIS-06</th>
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**target**

- **Spherical target**
  - DWBA 98
  - DWBA 98
  - ECIS-06

- **Deformed target**
  - ECIS-06
  - ECIS-06

**DWBA 98 and ECIS-06**: **non scalable**!!!

- Write a **scalable CC code for non local potentials**: **ECANOL**
- Write a **scalable code to compute microscopic potentials**
Method proposed and developed by Hugo F. Arellano, successfully applied to the description of charge exchange reactions Phys. Rev. C 76, 014616

Solve numerically

\[
\begin{pmatrix}
\phi^I \\
\phi^F
\end{pmatrix}
= \begin{pmatrix}
\frac{F}{k_i} \\
0
\end{pmatrix}
+ \begin{pmatrix}
G(k_i)U^{II} & G(k_i)U^{IF} \\
G(k_F)U^{FI} & G(k_F)U^{FF}
\end{pmatrix}
\begin{pmatrix}
\phi^I \\
\phi^F
\end{pmatrix}
\]

Example: 10 MeV (n,n') ²⁰⁸Pb  
2 excited states: N₁ = 1⁻ and N₂ = 2⁺  
\( I^\pi = \frac{3^+}{2} \)

\[
\begin{pmatrix}
\phi_{\frac{1}{2}, 0}^{N_1} \\
\phi_{\frac{3}{2}, 1}^{N_1} \\
\phi_{\frac{3}{2}, 1}^{N_2} \\
\phi_{\frac{1}{2}, 0}^{N_2} \\
\phi_{\frac{7}{2}, 4}^{N_2}
\end{pmatrix}
= 1^{-}
\begin{pmatrix}
G_2(k_i)U^{00}_{\frac{3}{2}, 2}0^+, (\frac{3}{2}, 2)0^+ & G_2(k_i)U^{0N_1}_{\frac{3}{2}, 2}0^+, (\frac{3}{2}, 1)1^- & G_2(k_i)U^{0N_1}_{\frac{3}{2}, 2}0^+, (\frac{3}{2}, 1)1^- & \cdots & G_2(k_i)U^{0N_2}_{\frac{3}{2}, 2}0^+, (\frac{3}{2}, 4)2^+ \\
G_1(k_1)U^{N_1, 1}_{\frac{1}{2}, 1}1^- (\frac{3}{2}, 2)0^+ & G_1(k_1)U^{N_1, N_1}_{\frac{3}{2}, 1}1^-, (\frac{3}{2}, 1)1^- & G_1(k_1)U^{N_1, N_1}_{\frac{3}{2}, 1}1^-, (\frac{3}{2}, 1)1^- & \cdots & G_1(k_1)U^{N_1, N_2}_{\frac{1}{2}, 1}1^-, (\frac{3}{2}, 4)2^+ \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
G_0(k_2)U^{N_2, 0}_{\frac{3}{2}, 0}2^+, (\frac{3}{2}, 2)0^+ & G_0(k_2)U^{N_2, N_1}_{\frac{3}{2}, 0}2^+, (\frac{3}{2}, 1)1^- & G_0(k_2)U^{N_2, N_1}_{\frac{3}{2}, 0}2^+, (\frac{3}{2}, 1)1^- & \cdots & G_0(k_2)U^{N_2, N_2}_{\frac{3}{2}, 0}2^+, (\frac{3}{2}, 4)2^+ \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
G_4(k_2)U^{N_2, 0}_{\frac{3}{2}, 4}2^+, (\frac{3}{2}, 2)0^+ & G_4(k_2)U^{N_2, N_1}_{\frac{3}{2}, 4}2^+, (\frac{3}{2}, 1)1^- & G_4(k_2)U^{N_2, N_1}_{\frac{3}{2}, 4}2^+, (\frac{3}{2}, 1)1^- & \cdots & G_4(k_2)U^{N_2, N_2}_{\frac{3}{2}, 4}2^+, (\frac{3}{2}, 4)2^+ \\
\end{pmatrix}^{-1}
\begin{pmatrix}
\frac{F_0}{k_i} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]
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Application to pre-equilibrium calculation: context

Contribution of all RPA 1-phonon states to pre-equilibrium emission cross section with fully microscopic approach.

Good description of energy spectrum for $E^* < \sim 30\text{MeV}$

Good description of angular distribution at forward angles

Unsatisfying description for backward angles and high excitation energy

Contributions missing!

From 1-phonon states, construct 2-phonon excitations:

\[
|N_1 \otimes N_2\rangle = |N_3\rangle
\]

\[
|N_1\rangle = \Theta_{N_1,J_1}^{+} |\tilde{0}\rangle \quad \leftrightarrow \quad |N_2\rangle = \Theta_{N_2,J_2}^{+} |\tilde{0}\rangle
\]

- Couplings in full arrows computed with MINOLOP code
- Couplings in dashed arrows neglected
- XS calculation done with ECANOL

Large contribution of 2-phonon states given their number
First results for the contribution of 2-phonon states up to $E^* = 16\text{MeV}$

2-phonon contribution to the angular distribution is strong at backward angles even for low excitation energy
V. Conclusion and perspectives
Conclusion

- ECANOL, CC code for non local potentials validated, ready to use
- MINOLOP, code for microscopic potentials computed from Melbourne G matrix + structure input ready and validated (central part only)
- First application to the contribution of 2-phonon excitations to pre-equilibrium emission shows their important contribution even at low excitation energies on the double differential cross section

To-do list:

- Add spin-orbit, tensor and Coulomb parts of the interaction
- Calculate 2-phonon contributions to (p,xp) pre-equilibrium emission with the complete interaction
- Investigate the impact of non locality on spherical targets
Extend the study to deformed targets

Description of the rotational band of $^{238}\text{U}$ basing the modelling on:

$$|JMK^\pi\rangle = \sqrt{\frac{2J+1}{16\pi^2}} \int D_{MK}^J(\Omega)R(\Omega)|K^\pi\rangle d\Omega$$

$$+ (-1)^{J+K}D_{M-K}^J(\Omega)R(\Omega)|\overline{K^\pi}\rangle$$

$|K^\pi\rangle$ Target’s state in the intrinsic frame
Thank you for your attention.
\[ V_{\text{eff}} = V_{\text{central}} + V_{\text{spin-orbit}} + V_{\text{tensor}} \]

<table>
<thead>
<tr>
<th>MINOLOP</th>
<th>Optical potential</th>
<th>Coupling potentials</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( \langle i</td>
<td>V_{\text{eff}}</td>
</tr>
<tr>
<td>local</td>
<td>ready</td>
<td>ready</td>
</tr>
<tr>
<td>non local</td>
<td>ready</td>
<td>ready</td>
</tr>
<tr>
<td>local</td>
<td>Being validated</td>
<td>Being validated</td>
</tr>
<tr>
<td>non local</td>
<td>Being validated</td>
<td>Being validated</td>
</tr>
<tr>
<td>CENTRAL TERM</td>
<td>ready</td>
<td>ready</td>
</tr>
<tr>
<td>SPIN-ORB / TENSOR</td>
<td>Being validated</td>
<td>Being validated</td>
</tr>
</tbody>
</table>
### Nucleon-nucleus scattering resolution

<table>
<thead>
<tr>
<th>DWBA</th>
<th>CC</th>
</tr>
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<tbody>
<tr>
<td>local</td>
<td>local</td>
</tr>
<tr>
<td>non local</td>
<td>non local</td>
</tr>
</tbody>
</table>

- **ready**
- **ready**
- **ready**
- **ready**

Validation done by comparison to DWBA98 et ECIS-06
Contribution of each 2-phonon up to $E^* = 14\text{MeV}$ to $\sigma_{\text{tot}}$

\begin{align*}
\sigma \text{ (mb)} & \\
\text{spin (h units)} & 0 2 4 6 8 10 12 14 16
\end{align*}

- par etat
- somme
80 MeV (n,n') on $^{90}$Zr

d$\sigma$/d$\theta$ (mb/sr/deg)

θ (°)
80 MeV (n,n') on $^{90}$Zr

$\frac{d\sigma}{d\theta}$ (mb/srd)

$\theta$ (°)

2+ sum
2+ interf
4+ sum
4+ interf