The Statistical Model of Nuclear Reactions: Open Problems

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1. Motivation

In recent years, detailed experimental tests of predictions of the statistical model of nuclear reactions: Scattering of slow neutrons (s-wave and p-wave only) , investigation of numerous isolated compound-nuclear (CN) resonances, statistical distribution of resonance parameters. Statistical model predicts that reduced partial widths have Porter-Thomas distribution (PTD) (i.e., a χ^2distribution with one degree of freedom). Baffling results:

(i) In the target nuclei $^{192}\mathrm{Pt}$ and $^{194}\mathrm{Pt}$, a total of 158 and 411 resonances, respectively, were measured. The data reject the validity of the PTD with 99.997 % statistical significance.
P. E. Koehler et al., Phys. Rev. Lett. 105 (2010) 072502.

(ii) Reanalysis of the Nuclear Data Ensemble rejects the validity of the PTD with 99.97 % statistical significance.
P. E. Koehler, Phys. Rev. C 84 (2011) 034312.

(iii) Total gamma decay widths of CN resonances in neutron scattering are sums over very many gamma channels. The distribution of these widths should be very narrow but in $^{96}\mathrm{Mo}$ is actually much wider than predicted.
P. E. Koehler et al., Phys. Rev. C 88 (2013) 041305(R).

Both s-wave and p-wave resonances contribute. Target nucleus has spin/parity 5/2+. The resonances have spin/parity 2+ and 3+ (s-waves) and 1-, 2-, 3-, 4- (p-waves). The cumulative distributions of gamma decay widths (dark lines with error bars) are much wider than predicted (red curves). The maxima of the distributions are at significantly larger values of the widths than predicted.

How to account for these discrepancies?

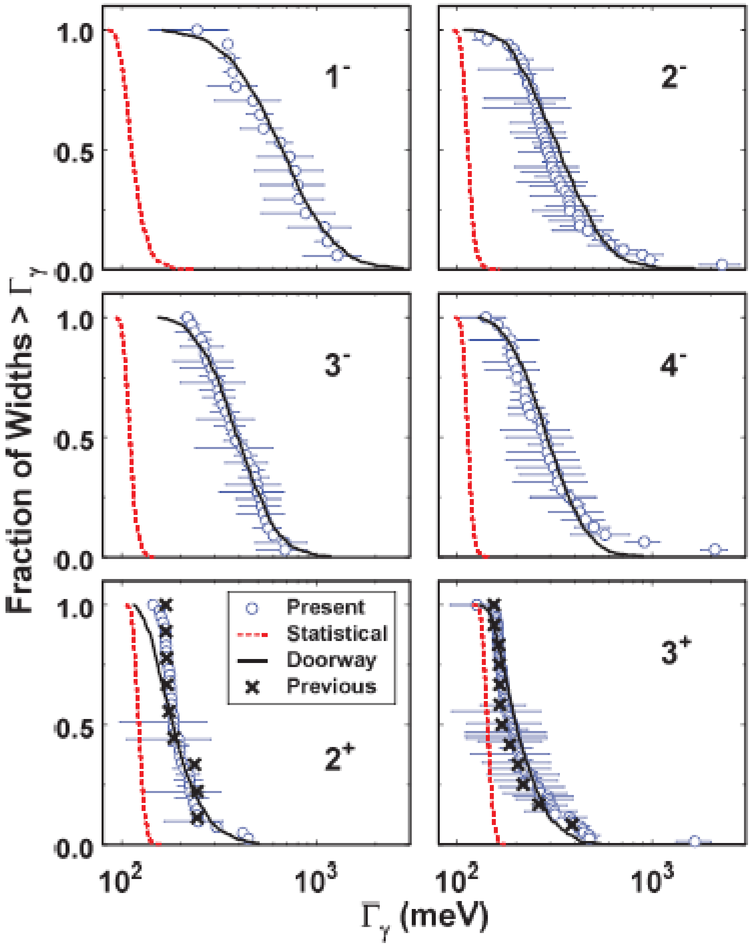


FIG. 6. (Color online) Cumulative Γ_γ distributions for ^{95}Mo + n resonances. Each panel shows the fraction of resonances having Γ_γ larger than a certain size vs the size. Data from the present work and those based on previous firm J^π values are shown as open blue circles and black x's, respectively. Error bars depict one-standard-deviation uncertainties as reported by SAMMY. Statistical-model simulations with and without an added doorway are shown as solid black and dashed red curves, respectively. Simulated Γ_γ values have been normalized by factors of 0.75, 0.9, 0.85, 1.1, 1.0, and 1.2 for 1^-, 2^-, 3^-, 4^-, 2^+, and 3^+ resonances, respectively. Not shown is the smallest 2^- Γ_γ (39.7 ± 2.9 meV).

P. E. Koehler et al., Phys. Rev. C 88 (2013) 041305(R).

2. Statistical Model

G. E. Mitchell, A. Richter, and H. A. Weidenmüller, Rev. Mod. Phys. 82 (2010) 2845

Channels c with c = 1 neutron channel (s-wave or p-wave) and c = 2, …, Λ gamma channels. No direct reactions. Scattering matrix is

$$S_{cc'}(E) = \delta_{cc'} - 2i\pi \sum_{\mu\nu} W_{c\mu}(E - H^{\rm eff})^{-1}_{\mu\nu} W_{\nu c'} \ .$$

Effective non-Hermitean Hamiltonian is

$$H^{\rm eff}_{\mu\nu} = H^{\rm GOE}_{\mu\nu} + \sum_c \mathcal{P} \int_{E_c}^{\infty} {\rm d}E' \ \frac{W_{\mu c} W_{c\nu}}{E - E'} - i\pi \sum_c W_{\mu c} W_{c\nu} \ .$$

Here $H^{\rm GOE}$ drawn from the GOE, $W_{c\mu}(E) = W_{\mu c}(E)$ couples channels and space of compound states with $\mu = 1, \ldots, N \gg 1$, $\mathcal{P}$ denotes the principal-value integral, and E is the excitation energy of the CN, with E = 0 at the CN ground state. Matrix elements $W_{c\mu}(E)$ are defined for $E \geq E_c$ (threshold energy for channel c). Neglect principal-value integrals for gamma channels. For neutron channel, $W_{1\mu} \propto (E - E_1)^{1/4}$ for s-waves and $W_{1\mu} \propto (E - E_1)^{3/4}$ for p-waves. Use that and channel orthogonality to write

$$H^{\rm eff}_{\mu\nu} = H^{\rm GOE}_{\mu\nu} + \delta_{\mu\nu} V_\mu \ , V_1 = \lambda \left(\frac{1}{\pi} \mathcal{P} \int_{E_1}^{\infty} {\rm d}E' \frac{\kappa_c}{E - E'} - i\kappa_c \right) \ , V_c = -i\lambda\kappa_c \ \text{for} \ c \geq 2 \ ,$$

but $V_c = 0$ for c > Λ, and λ = N d / π. Average S-matrix is $\langle S_{cc'} \rangle = \frac{1-\kappa_c}{1+\kappa_c}$. GOE Hamiltonian is orthogonally invariant. That implies PTD for partial widths. Coupling terms to channels V_μ violate orthogonal invariance. Are violations of statistical-model predictions due to violation of orthogonal invariance by coupling terms V_μ ?

3. Non-statistical Effects: Thomas-Ehrman shift

In the Pt isotopes, the 4s single-particle state of the shell model is close to neutron threshold, increasing the coupling matrix elements V_1 and the principal-value integral (the shift function). Can that explain the deviations from PTD in Pt? Neglect gamma channels.

A. Volya, H. A. Weidenmüller, and V. Zelevinsky, Phys.Rev. Lett. 115 (2015) 052501,
E. Bogomolny, Phys. Rev. Lett. 118 (2017) 022501,
P. Fanto, G. F. Bertsch, and Y. Alhassid, arXiv:1710.00792 (2017).

For reasonably strong V_1 , Volya et al. find numerically significant deviations from PTD. Bogomolny diagonalizes H^{eff} analytically for constant V_1 in the limit of large matrix dimension N. Deviations from PTD do arise but locally distribution is PT. Fanto et al. confirm that result by a realistic calculation.

As a result, the deviations from PTD in Pt isotopes (and in the NDE) cannot be accounted for by violations of orthogonal invariance due to the channel coupling matrix elements (as long as these have realistic values).

Disregard shift function in what follows.

4. Nonstatistical Effects: Many Gamma Channels

For medium-weight and heavy CN, the number of gamma decay channels for each neutron resonance is very large (10^6 or so). Can that account for deviations from PTD in the neutron channel, and can it account for disagreement of statistical-model prediction with measured distribution of total gamma decay widths? Model that case:

Neutron resonance μ with spin/parity J^π decays by emission of photons of multipolarity L and parity π (E1, M1, E2, M2, …, jointly written as XL) to final states f with spins/parities I^π_f . Partial decay width is $\Gamma^{J^\pi}_{\mu\gamma I^\pi_f f XL}$. Total gamma decay width is

$$\Gamma^{J^\pi}_{\mu\gamma} = \sum_{XL} \sum_{I^\pi_f f} \Gamma^{J^\pi}_{\mu\gamma I^\pi_f f XL} = \sum_{XL} \int_0^{E_1} \mathrm{d}E_\gamma \ \rho(E_\mu - E_\gamma, I^\pi_f) \ \Gamma^{J^\pi}_{\mu\gamma I^\pi_j f XL} \ .$$

The level density ρ is a sum of delta functions. Two steps: (i) Average value of partial width is given by photon strength function (which also yields coupling parameters),

$$\langle \Gamma^{J^\pi}_{\mu\gamma I^\pi_f f XL} \rangle = d_{J^\pi} E_\gamma^{2L+1} f_{XL}(E_\gamma) = d_{J^\pi} \frac{2}{\pi} \kappa_f \ .$$

For the gamma channels, $\kappa \approx 10^{-5}$ (E1) and $\kappa \approx 10^{-6}$ (M1). (For neutrons $\kappa = 2.4 \times 10^{-4}$). (ii) Using average level density, generate numerically set of final states. Fluctuations of partial widths are accounted for by putting for each final state

$$\Gamma^{J^\pi}_{\mu\gamma I^\pi_f f XL} = x_f^2 \langle \Gamma^{J^\pi}_{\mu\gamma I^\pi_f f XL} \rangle \ .$$

Here x_f^2 is a random variable with mean value unity and a PTD.

Model assumes PTD, requires average level density and strength function.

5. Gamma Decay of the $^{96}\mathrm{Mo}$ Compound Nucleus

Ground state of $^{95}\mathrm{Mo}$ has spin/parity 5/2+. CN resonances have spin/parity values 2+, 3+ (s-waves), 1-, 2-, 3-, 4- (p-waves). For level density, use back-shifted Fermi gas model or back-shifted Bethe formula with spin-cutoff. CN states with opposite parities have same level density. Consider only gamma transitions with L = 1 (E1 and M1). E1 strength function: S. A. Sheets et al., Phys. Rev. C 79 (2009) 024301.

Too many final states: Below an excitation energy of 2.79 MeV, use actual discrete levels. Above that energy, use coarse graining and define representative channels c, with c = 1, …, Λ. Group together final states f close in energy with equal quantum numbers. Average level density of final states c is taken proportional to actual average level density. Effective coupling parameters κ_c are sums over κ_f with f in the group. GOE matrix of dimension N = 1000, number of channels Λ = 401 (one neutron channel, 200 representative E1 channels, 200 representative M1 channels). Results shown below taken from middle of the spectrum to avoid edge effects.

Two-step process: (i) Use that scheme to check for deviations from PTD of partial widths, both for neutron and for gamma channels. (ii) Use the scheme of section 4 to calculate distribution of total gamma decay widths of neutron resonances.

(i) Distribution of partial widths

Write $x = \Gamma / \langle \Gamma \rangle$, $y = \ln x$. PTD is $P(y) = \sqrt{\frac{x}{2\pi}} e^{-x/2}$. Compare results for neutron channel and for most strongly coupled E1 gamma channel with PTD, for neutron resonances with spin/parity 2+.

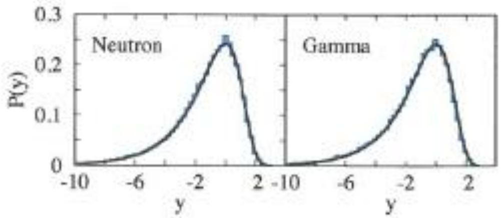
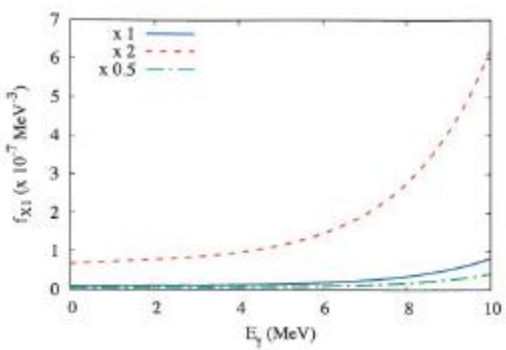


FIG. 3. Distribution of the $y = \log x$, where $x = \hat{\Gamma}/\langle\hat{\Gamma}\rangle$ is the normalized reduced partial width. The left-hand panel shows results for the neutron channel, while the right-hand panel shows results for the most strongly coupled gamma channel. Histograms are model calculations. The solid black line is the PTD.

Differences are negligible. Similar results for other spin/parity values and for less strongly coupled gamma channels. We conclude that a large number of gamma decay channels with realistic coupling strengths does not alter the PTD of partial widths in any of the channels (neutron or proton).

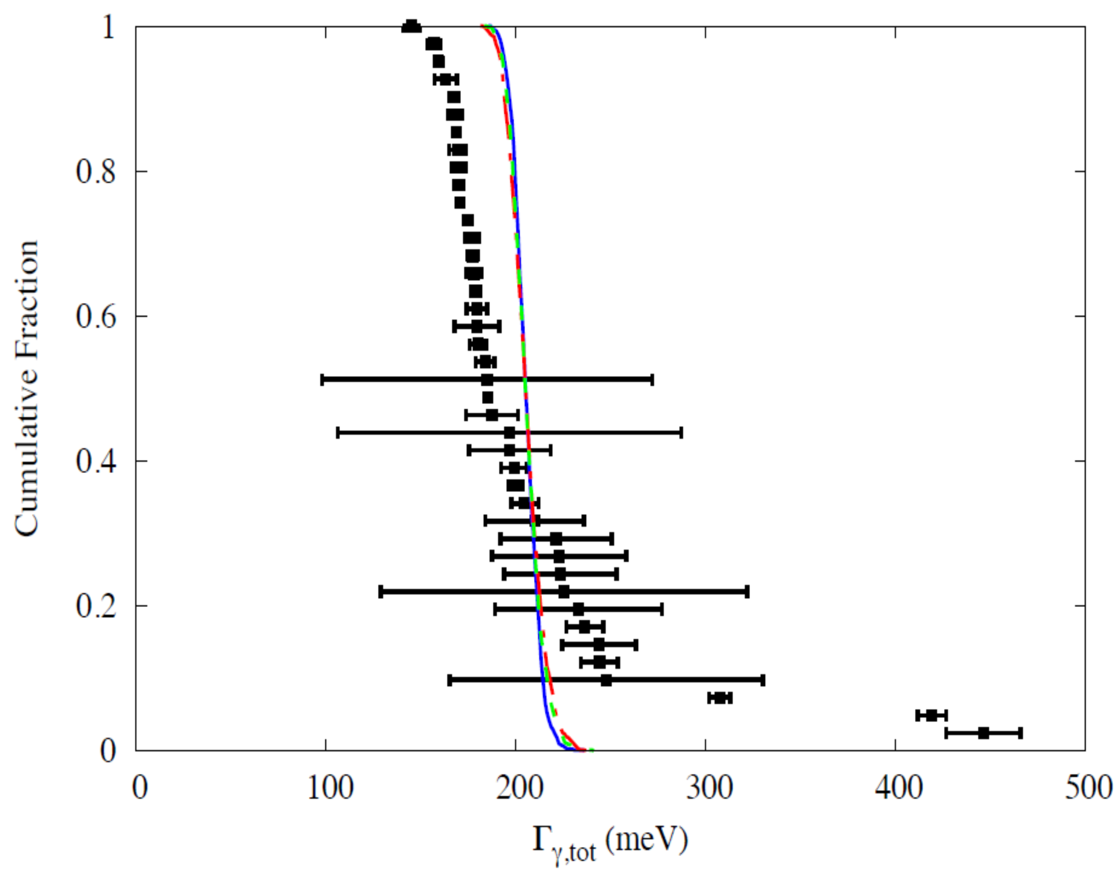
(ii) Distribution of total gamma decay widths

Use standard parameters of E1 strength function plus results of parameter variations (left figure). Compare results for 2+ resonances with (shifted) cumulative distribution of total gamma decay widths with error bars.



Three forms of E1 strength function used in simulations.

The differences persist.



Likewise, the locations of the peaks of the distributions found in the simulation disagree with the data.

J^π	2^+	3^+	1^-	2^-	3^-	4^-
$\langle\Gamma_{\gamma,\text{sim}}\rangle$ (meV)	165.5	157.5	191.2	172.8	169.2	153.8
$\langle\Gamma_{\gamma,\text{exp}}\rangle$ (meV)	206 (31)	240 (58)	670 (225)	374 (115)	404 (100)	361 (106)

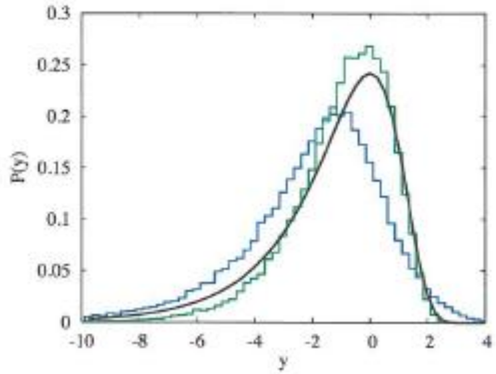
TABLE II. Comparison of simulated average total gamma widths $\langle\Gamma_{\gamma,\text{sim}}\rangle$ with the experimental results $\langle\Gamma_{\gamma,\text{exp}}\rangle$. Simulated results are calculated with baseline parameter values.

Differences are substantial and show lack of gamma transition strength.

Are these results sensitive to deviations of distribution of partial widths from the PTD? Choose in Hamiltonian a very strong coupling to generate such deviation,

$$H^{\text{eff}}_{\mu\nu} = H^{\text{GOE}}_{\mu\nu} + Z\delta_{\mu 1}\delta_{\nu 1}$$

with Z = - i λ κ, and κ = 0.8, about 3000 times bigger than for neutron channel. That changes PTD. Blue histogram for neutron, green for gamma channel.



The resulting modifications of the width of the distributions are negligible:

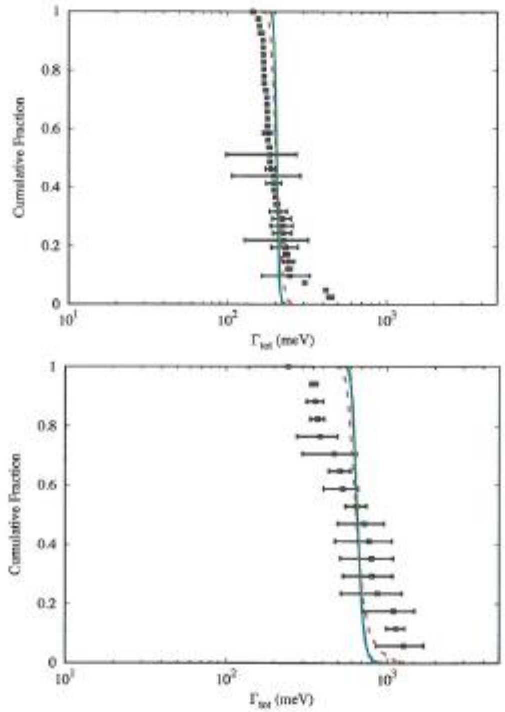


FIG. 7. Simulated total width distributions compared width data for resonances of spin-parity 2^+ (top panel) and 1^- (bottom panel). The blue solid lines are obtained using PTD fluctuations of the partial widths. The red dashed lines are obtained using the modified distribution corresponding to $i = 1$ shown in Fig. 6 for the partial width fluctuations. The green dashed-dotted lines are obtained using the modified distribution corresponding to $i = 2$ shown in Fig. 6 for the partial width fluctuations. The black dots show the experimental data. The simulated results are normalized to match the experimental average total width.

Not surprising: Summing over 200 channels effectively causes distribution to become very narrow. Large width of distribution points to few channels.

5. Results and Conclusions

Experimental results for Pt and for NDE show strong deviations of fluctuations of partial widths from PTD. Are these due to violations of orthogonal invariance?

Two possible mechanisms: Thomas-Ehrman shift and large number of gamma channels. Effect of Thomas-Ehrman shift is understood and ruled out. Simulations for many gamma channels yield perfect agreement of distribution of partial widths with PTD in all channels.

Distribution of total gamma decay widths in Mo96 disagrees with predictions of statistical model. Widths of simulated distributions are too small. Peak locations are off, with experimental values up to three times bigger than simulated ones, indicating that gamma transition strength is missing in the model. Widths of simulated distributions insensitive to changes of E1 strength function, and to modifications of PTD of partial widths. Individual simulated peak locations can be fitted by changing E1 strength function, but not all of them simultaneously.

In summary, the statistical model fails to account for data in Pt and Mo. The model contains terms that violate orthogonal invariance. Intensive studies of several theoretical groups have shown that these cannot be held responsible for the failure. Both the observed deviations from the PTD and the observed widths of the distributions of total gamma decay widths suggest that at neutron threshold, the mixing of CN states is less complete than implied in the statistical model by the use of the GOE Hamiltonian. The lack of total gamma decay strength in the model poses a perhaps unrelated problem. It raises the question whether the Brink-Axel hypothesis applies for all gamma transitions that contribute significantly to the gamma decay of the CN resonances. Both consequences are rather drastic. An independent confirmation of the experimental results (or another test of the statistical model) by another group is, therefore, highly desirable.