Statistical multi-step direct reaction models and the RPA

E. V. Chimanski, B. V. Carlson
Depto de Física, Instituto Tecnológico de Aeronáutica, SJC-SP Brazil

R. Capote Noy, A. Koning
NAPC-Nuclear Data Section, International Atomic Energy Agency, Vienna Austria

chimanski@ita.br;E.Chimanski@iaea.org

June 15, 2018
Underlying statistics in Pre-equilibrium reactions

- *Multi*-step direct formalism;
- *Leading* particle nature;
- *Response* function and target excited states;
- *Particles* in continuum and inelastic scattering.
Intro

– Pre-equilibrium –

Time scales:

▶ Compound nucleus reactions
  \[ a + A \rightarrow C^* \rightarrow b + B : \Delta t \sim 10^{-15} \text{s} \]

▶ Pre-equilibrium reactions.

▶ Direct reactions
  \[ a + A \rightarrow b + B : \Delta t \sim 10^{-21} \text{s} \]
Quantum mechanical models

Step by step

- Continuum $P$ and bound state $Q$ components:

\[
P = P_1 + P_3 + P_5 + P_7 + \ldots \\
Q = + Q_3 + Q_5 + Q_7 + \ldots
\]

Reactions occurring in the space $P$ are known as multi-step direct reactions (MSD)\(^1\) and those happening in $Q$ space are multi-step compound reactions (MSC)\(^2\).

- MSD models – leading particle type. Fast incident particle creates new particle-hole pairs on its way through the target nucleus, leaving it after a few interactions (steps).

- Incoherent sum of n-step processes:

\[
\frac{d^2 \sigma}{d \Omega dE_k} \propto \left[ \left| \langle \psi_{k_f}^{(-)} | \langle f | V | 0 \rangle | \psi_{k_i}^{(+)} \rangle \right|^2 + \left| \langle \psi_{k_f}^{(-)} | \langle f | V G V | 0 \rangle | \psi_{k_i}^{(+)} \rangle \right|^2 \right] \ldots
\]

\[
\propto \sum_{n=1}^{\infty} \frac{d^2 \sigma^n}{d \Omega dE_k}
\]


Multi-step direct model: first step
and the Random Phase approximation:

\[ \frac{d^2 \sigma^{(1)}}{d\Omega dE_f} \propto \left| \langle \psi_{k_f}^{(+)} | f | V | 0 \rangle | \psi_{k_i}^{(-)} \rangle \right|^2 ; | f \rangle = \sum_{\mu} a_{\mu}^f | \mu \rangle ; \sum_{\mu, \mu'} a_{\mu}^f a_{\mu}^f \to \delta_{\mu \mu'} \sum_{\mu} | a_{\mu}^f |^2 \]

the nucleus excited states \( \rightarrow \) Response function – often Gaussian distributions.

\[ \frac{d^2 \sigma^{(1)}}{d\Omega dE_f} = \frac{m^2}{(2\pi \hbar^2)^2} \frac{k_{f}}{k_{0}} \sum_{\mu} \rho_{\mu}(E_x) \left| \langle \psi_{k_f}^{(+)} | \langle \mu | V | 0 \rangle | \psi_{k_i}^{(-)} \rangle \right|^2 . \]

Self-consistent RPA \(^3\): given \( J^\pi \): \( Q_x^{\dagger} = \sum_{mi} X_{mi}^{x} a_{m}^{\dagger} a_{i} + \sum_{mi} Y_{mi}^{x} a_{i}^{\dagger} a_{m} \)

ph amplitudes \( a_{\mu}^{f} \equiv a_{\mu}^{x} \equiv X_{\mu}^{x} + Y_{\mu}^{x} \),

Strength function

\[ | a_{\mu}^{x} |^2 \equiv \rho_{\mu}(E_x) = \frac{1}{2\pi} \frac{\gamma}{(E_x - \bar{E})^2 + (\gamma/2)^2} \]

Distorted projectile waves in continuum $\psi^{(\pm)}$

Eikonal (Glauber) + $t\rho$ approx and the elastic cross-section

$$
\psi_{k}^{(+)} (\vec{r}) = \exp \left[ i \vec{k} \cdot \vec{r} - \frac{i}{\hbar v} \int_{-\infty}^{z} U(r) dz' \right]
$$

Proton-target optical potential

$$
U (\vec{r}) = -\frac{\hbar v^2}{2} \left[ \sigma_{pp}^T (i + \alpha_{pp}) \rho_{p} (\vec{r}) + \sigma_{pn}^T (i + \alpha_{pn}) \rho_{n} (\vec{r}) \right]
$$

Inelastic Scattering

Assuming $V(\mathbf{r} - \mathbf{r}') = V_0 \delta(\mathbf{r} - \mathbf{r}')$, an one-step individual amplitude becomes,

$$\left\langle \vec{k}_f | \langle ph | V | A \rangle | \vec{k}_i \right\rangle = \int d^3r \, \psi_{k_f}^{(-)*}(\mathbf{r}) \, \psi_p^{*}(\mathbf{r}) \, V_0 \, \psi_h(\mathbf{r}) \, \psi_{k_i}^{(+)}(\mathbf{r}),$$

where $\psi_h$ is an occupied orbital in the initial nucleus (a hole state after the collision) and $\psi_p$ is an unoccupied orbital or continuum state of the initial nucleus.

The particle-hole energy is

$$E_\mu = E_{ph} = E_p - E_h,$$

and the excitation energy

$$E_x = \varepsilon^* = E_f - E_i.$$
Inelastic Scattering

The one-step cross-section is obtained by:

\[
\frac{d^2\sigma_{f\leftarrow i}}{d\Omega dE_{k_f}} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i} \frac{1}{S} \sum_{ph} \rho(E_x) \left|\langle \vec{k}_f; ph | T^{(1)} | \vec{k}_i \rangle\right|^2 \delta(E_i - E_f - E_x),
\]

where Strength function \( \rho(E_x) = \frac{1}{2\pi} \frac{\gamma}{(E_x - E_\mu)^2 + (\gamma/2)^2}, \quad E_{ph} \equiv E_\mu. \)

![Graph showing angular distribution of inelastic scattering for Zr (p,p') at E_x = 20.0 MeV with different incident energies.](attachment:graph.png)
Inelastic Scattering

Lower excitation energies

\[ (X^X + Y^X)^2 \]

\[ \theta \text{ (deg)} \]

Uncoupled

Coupled

\[ \frac{d\sigma}{d\Omega} \text{ (mb/sr)} \]

\[ Zr(p,p') - E_i = 200 \text{ MeV}; \ 3^- E_x = 2.74 \text{ MeV} \]

\[ E_\mu = 3.136 \text{ MeV} \]

\[ E_\mu = 3.754 \text{ MeV} \]

\[ \text{data} \]
Inelastic Scattering

\[ \frac{d\sigma}{d\Omega} \text{(mb/sr)} \]

\[ 90Zr \ (p, p') \ E_i = 200 \text{ MeV} \]

\[ 92Mo \ (p, p') \ E_i = 200 \text{ MeV} \]

\[ 90Zr \ (p, p') \ E_i = 160 \text{ MeV} \]

\[ 90Zr \]

\[ (n, n')E_i = 60.0 \text{ MeV} \]
\[ (p, p')E_i = 60.0 \text{ MeV} \]
\[ (n, n')E_i = 80.0 \text{ MeV} \]
\[ (p, p')E_i = 80.0 \text{ MeV} \]
\[ (n, n')E_i = 200.0 \text{ MeV} \]
\[ (p, p')E_i = 200.0 \text{ MeV} \]
Forthcoming

1. At sufficiently high incident energy, both of the final particles can be in the continuum. The DWBA amplitude will then be

\[
\langle \vec{k}_{f_1}, \vec{k}_{f_2}; \hbar | T | \vec{k}_i \rangle = \int d^3r \, \psi_{\vec{k}_{f_1}}^{(-)*} (\vec{r}) \, \psi_{\vec{k}_{f_2}}^{(-)*} (\vec{r}) \, V_0 \, \psi_h (\vec{r}) \, \psi_{\vec{k}_i}^{(+)} (\vec{r}).
\]

2. At extremely low energy, both of the particles can occupy previously unoccupied bound states of the nucleus. In this case, we would say that the incident nucleon was absorbed. The corresponding DWBA amplitude is

\[
\langle p_1 p_2 \hbar | T | \vec{k}_i \rangle = \int d^3r \, \psi_{p_1}^* (\vec{r}) \, \psi_{p_2}^* (\vec{r}) \, V_0 \, \psi_h (\vec{r}) \, \psi_{\vec{k}_i}^{(+)} (\vec{r}).
\]

3. Two-step: One particle in continuum

\[
\langle \vec{k}_f; p_2 \hbar_2, p_1 \hbar_1 \mid T^{(2)} \mid \vec{k}_i \rangle = \int d^3r \, \psi_{\vec{k}_f}^{(-)*} (z, \vec{b}) \, \psi_{p_2}^* (z, \vec{b}) \, \psi_{h_2} (z, \vec{b}) \\
\times G_m^{(+)} (z, \vec{b}; z, \vec{b}) \, \psi_{p_1}^* (z, \vec{b}) \, \psi_{h_1} (z, \vec{b}) \, \psi_{\vec{k}_i}^{(+)} (z, \vec{b})
\]
Summary

- We have taken into account channels where the projectile creates new p-h pairs with bound or unbound (continuum) particle states;
- One of the two particles remains in the continuum while the second occupies a previously unoccupied bound state of the nucleus. This is the situation assumed in all multi-step direct models;
- One step is limited to around 40 MeV of excitation energies;
- $t\rho$ approximation results are in agreement with high energy scattering;
- Extension of the first reaction step to include 0, 1 or 2 nucleons in the continuum;
- In the second step of the reaction, 0, 1, 2 or 3 can be in the continuum;
Acknowledgments

FAPESP – 2016/07398-8,2017/13693-5 ; IAEA; ITA

To be submitted

Chimanski+, PRC 2018