Statistical multi-step direct reaction models and the RPA

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Intro

Outline

`Underlying` statistics in Pre-equilibrium reactions

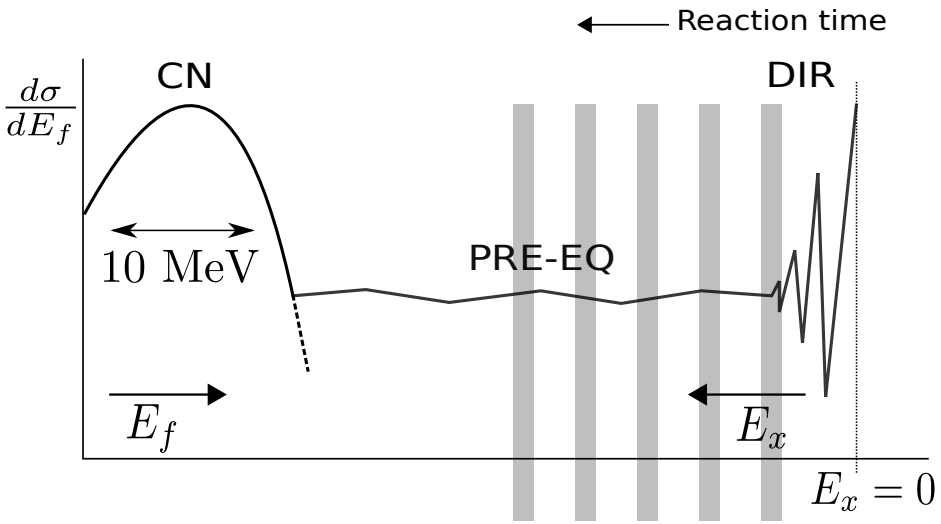
- *Multi*-step direct formalism;
- *Leading* particle nature;
- *Response* function and target excited states;
- *Particles* in continuum and inelastic scattering.

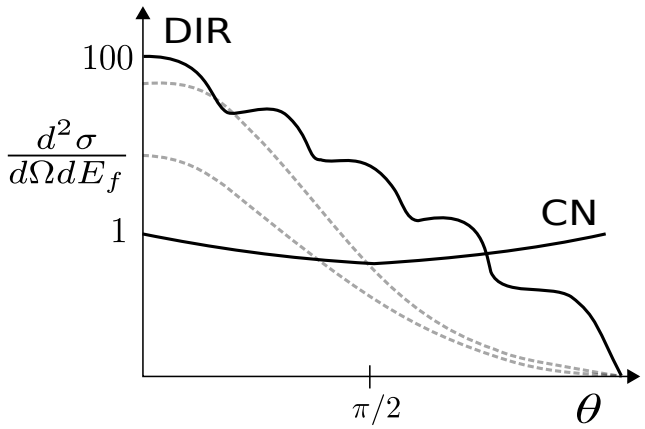
Intro

– Pre-equilibrium –

Time scales:

- Compound nucleus reactions
  a+A $\rightarrow$ C* $\rightarrow$ b+B :
  $\Delta t \sim 10^{-15} s$
- Pre-equilibrium reactions.
- Direct reactions
  a+A $\rightarrow$ b+B :
  $\Delta t \sim 10^{-21} s$.





Quantum mechanical models

Step by step

- Continuum P and bound state Q components:

$$P = P_1 + P_3 + P_5 + P_7 + \dots$$
$$Q = \quad + Q_3 + Q_5 + Q_7 + \dots$$

  Reactions occuring in the space P are known as multi-step direct reactions (MSD)[1] and those happing in Q space are multi-step compound reactions (MSC)[2]

- MSD models – leading particle type. Fast incident particle creates new particle-hole pairs on its way through the target nucleus, leaving it after a few interactions (steps).

- Incoherent sum of n-step processes :

$$\frac{d^2\sigma}{d\Omega dE_k} \propto \left[\left| \langle \psi^{(-)}_{\mathbf{k}_f} | \langle f | V | 0 \rangle | \psi^{(+)}_{\mathbf{k}_i} \rangle \right|^2 + \left| \langle \psi^{(-)}_{\mathbf{k}_f} | \langle f | VGV | 0 \rangle | \psi^{(+)}_{\mathbf{k}_i} \rangle \right|^2 \dots \right]$$
$$\propto \sum_{n=1}^{\infty} \frac{d^2\sigma^n}{d\Omega dE_k}$$

[1] H. Feshbach, et. al.,Ann. Phys., 1980; T. Tamura, et. al. PRC, 1982; H. Nishioka, et. al., Ann. Phys., 1988.

[2] D. Agassi, et. al. Phys Rep., 1975; H. Nishioka, et. al. Ann. Phys., 1986.

Multi-step direct model: first step

and the Random Phase approximation:

$$\frac{d^2\sigma^{(1)}}{d\Omega dE_f} \propto \left|\langle\psi^{(+)}_{\mathbf{k}_f}|\langle f|V|0\rangle|\psi^{(-)}_{\mathbf{k}_i}\rangle\right|^2;\ |f\rangle = \sum_\mu a^f_\mu|\mu\rangle;\ \sum_{\mu,\mu'} a^f_{\mu'}a^f_\mu \to \delta_{\mu\mu'}\sum_\mu |a^f_\mu|^2$$

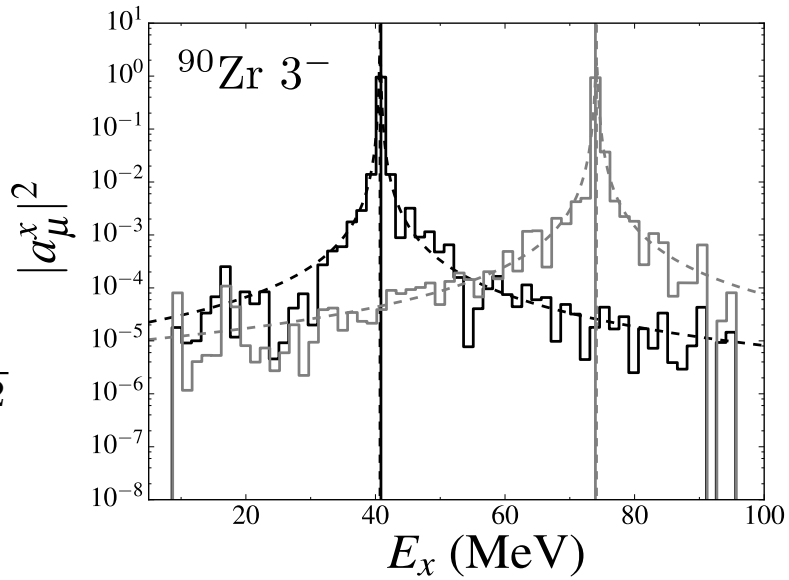
the nucleus excited states $\to$ Response function – often Gaussian distributions.

$$\frac{d^2\sigma^{(1)}}{d\Omega dE_f} = \frac{m^2}{(2\pi\hbar^2)^2}\frac{k_f}{k_0}\sum_\mu \rho_\mu(E_x)\left|\langle\psi^{(+)}_{\mathbf{k}_f}|\langle\mu|V|0\rangle|\psi^{(-)}_{\mathbf{k}_i}\rangle\right|^2.$$

Self-consistent RPA [3]: given J^π: $Q^\dagger_x = \sum_{mi} X^x_{mi}a^\dagger_m a_i - \sum_{mi} Y^x_{mi}a^\dagger_i a_m$

ph amplitudes $a^f_\mu \equiv a^x_\mu = X^x_\mu + Y^x_\mu$,
Strength function

$$|a^x_\mu|^2 \equiv \rho_\mu(E_x) = \frac{1}{2\pi}\frac{\gamma}{(E_x-\bar{E})^2+(\gamma/2)^2}$$

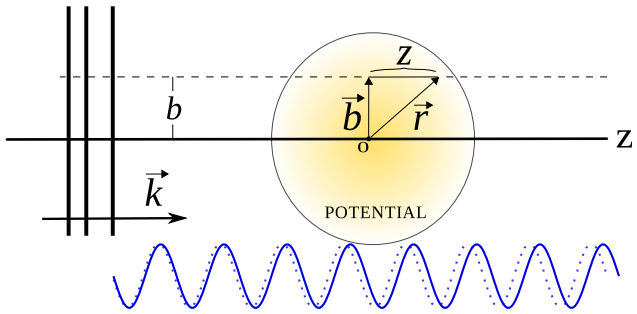


[3] Gianluca Colò, et. al. Comput Phys Commun 184 (2013) 142–161.

Distorted projectile waves in continuum $\psi^{(\pm)}$

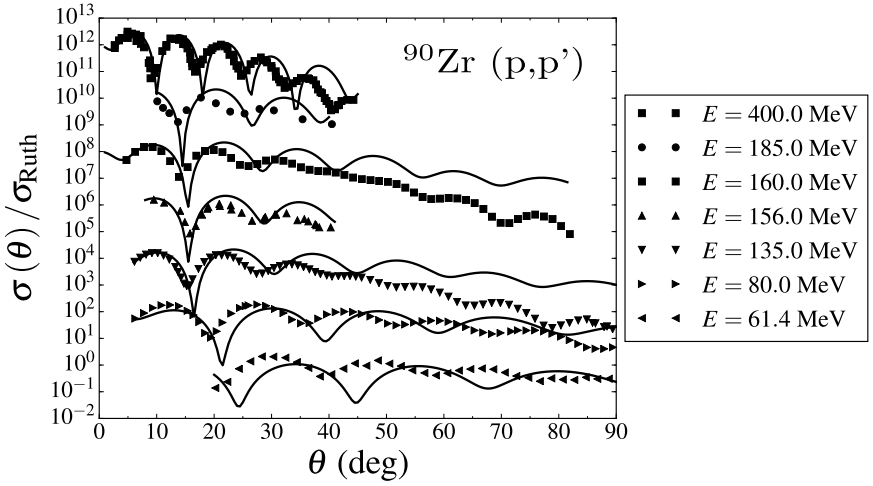
Eikonal (Glauber) + $t\rho$ approx and the elastic cross-section

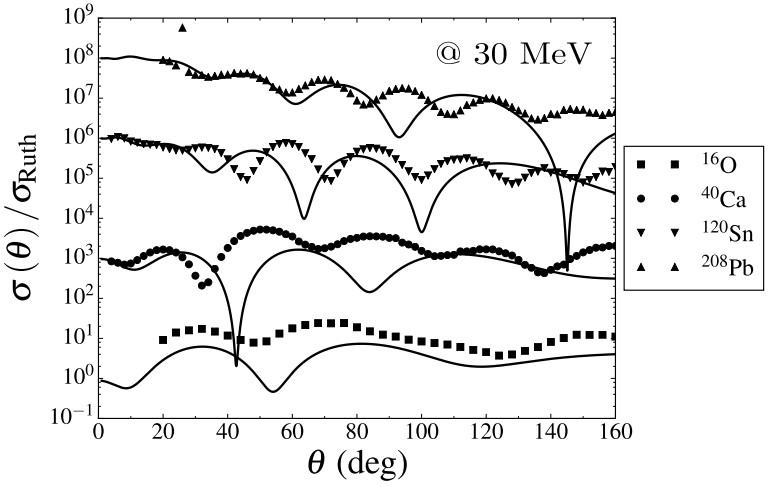
$$\psi_k^{(+)}(\vec{r}) = \exp\left[i\vec{k}\cdot\vec{r} - \frac{i}{\hbar v}\int_{-\infty}^{z} U(r)dz'\right]$$



Proton-target optical potential

$$U(\vec{r}) = -\frac{\hbar v}{2}\left[\sigma_{pp}^{T}(i+\alpha_{pp})\rho_p(\vec{r}) + \sigma_{pn}^{T}(i+\alpha_{pn})\rho_n(\vec{r})\right]$$





Lee et al. 1988,Nadasen et al. 1981, Hagberg et al.1971, Comparat et al. 1974, FULMER et al.1969, Ridley,Turner 1964.

Inelastic Scattering

Assuming $V\left(\vec{r}-\vec{r}'\right) = V_0\delta\left(\vec{r}-\vec{r}'\right)$, an one-step individual amplitude becomes,

$$\left\langle \vec{k}_f \left|\langle ph|V|A\rangle\right| \vec{k}_i \right\rangle = \int d^3r\, \psi_{k_f}^{(-)*}\left(\vec{r}\right)\, \psi_p^*\left(\vec{r}\right) V_0\, \psi_h\left(\vec{r}\right)\, \psi_{k_i}^{(+)}\left(\vec{r}\right)\ ,$$

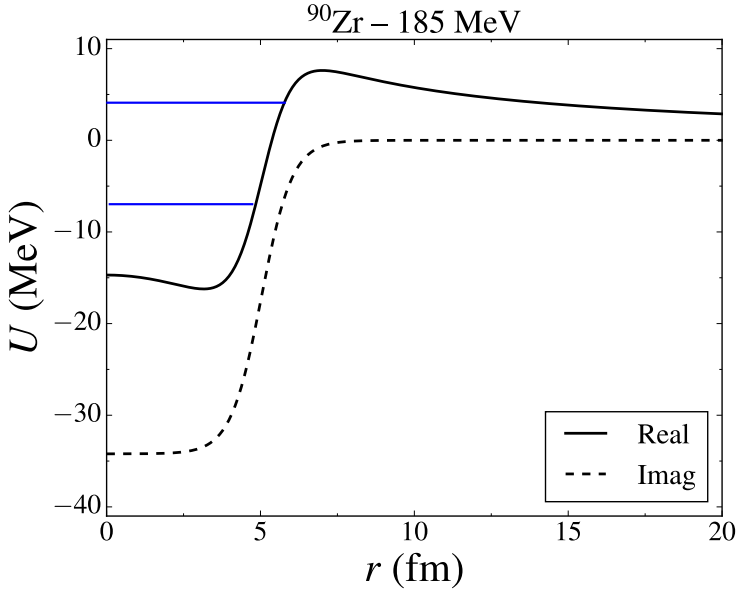
where ψ_h is an occupied orbital in the initial nucleus (a hole state after the collision) and $\psi_p$is an unoccupied orbital or continuum state of the initial nucleus.

The particle-hole energy is

$$E_\mu = E_{ph} = E_p - E_h,$$

and the excitation energy

$$E_x = \varepsilon^* = E_f - E_i.$$

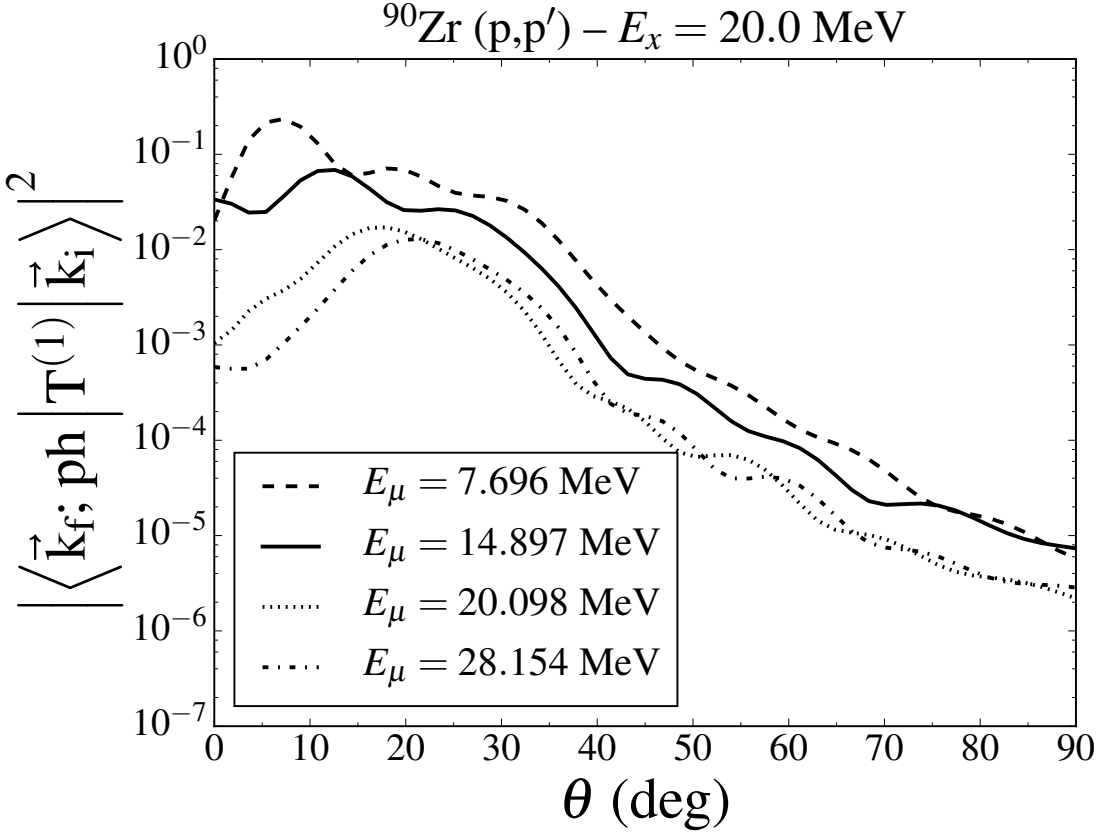


Inelastic Scattering

The one-step cross-section is obtained by:

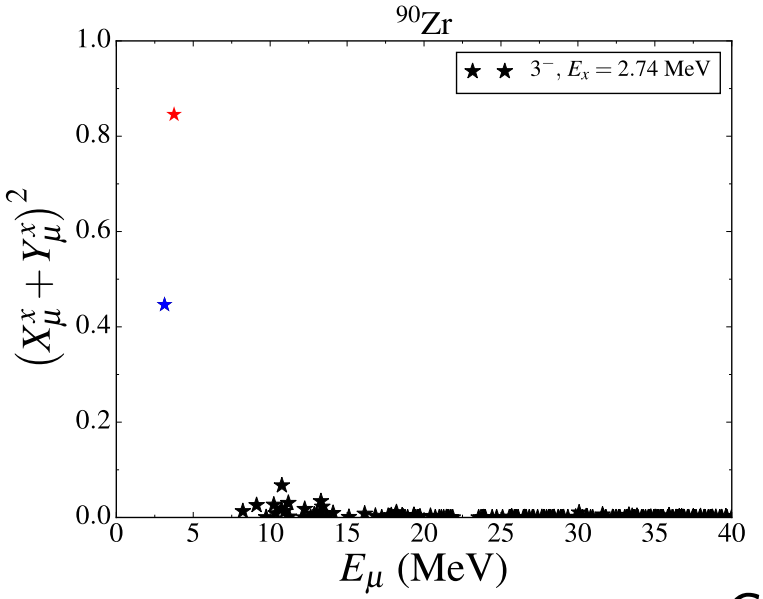
$$\frac{d^2\sigma_{f\leftarrow i}}{d\Omega dE_{k_f}} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i}\frac{1}{S}\sum_{ph}\rho(E_x)\left|\left\langle \vec{k}_f;\, ph\left|T^{(1)}\right|\vec{k}_i\right\rangle\right|^2 \delta(E_i - E_f - E_x),$$

where Strength function $\rho(E_x) = \frac{1}{2\pi}\frac{\gamma}{(E_x - E_\mu)^2 + (\gamma/2)^2}, \qquad E_{ph} \equiv E_\mu.$

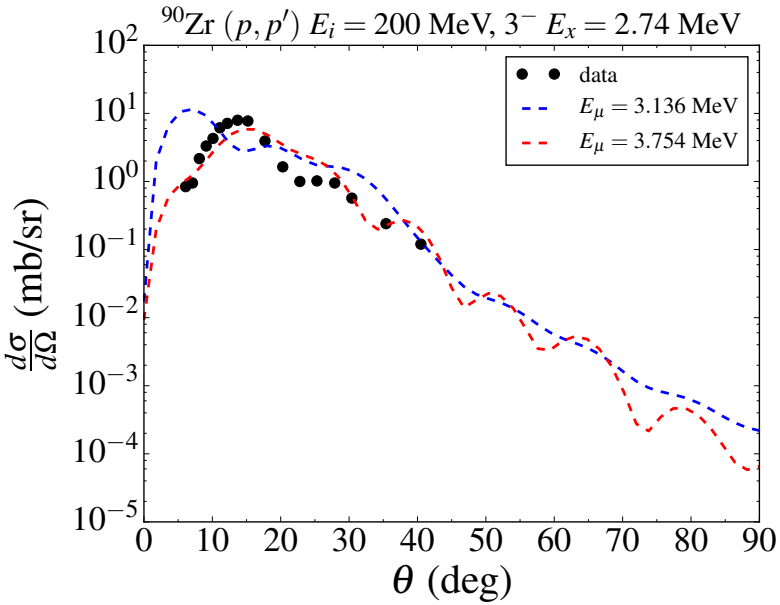


Inelastic Scattering

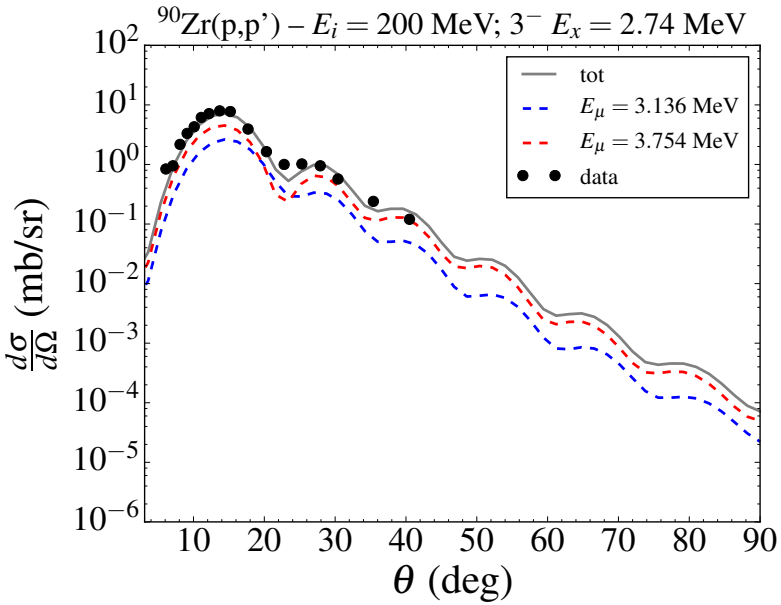
Lower excitation energies



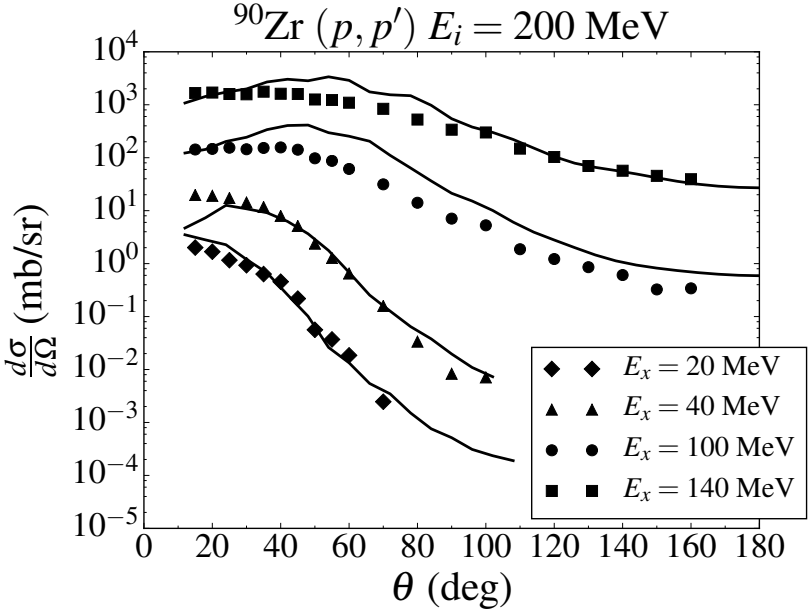
Uncoupled

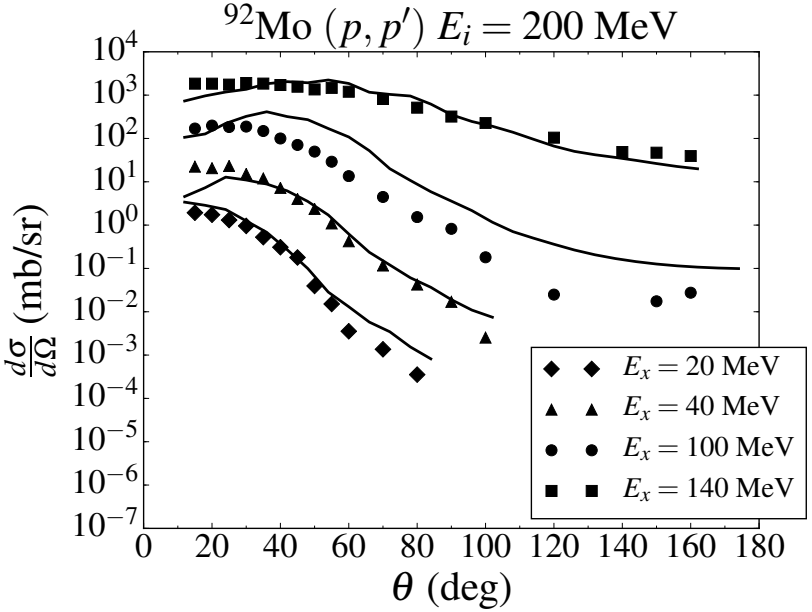


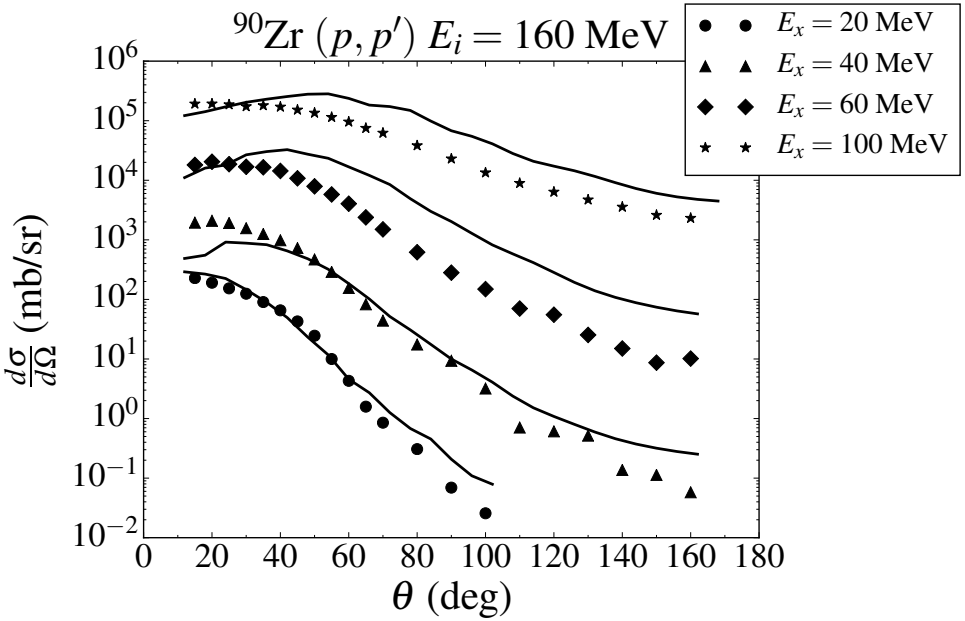
Coupled

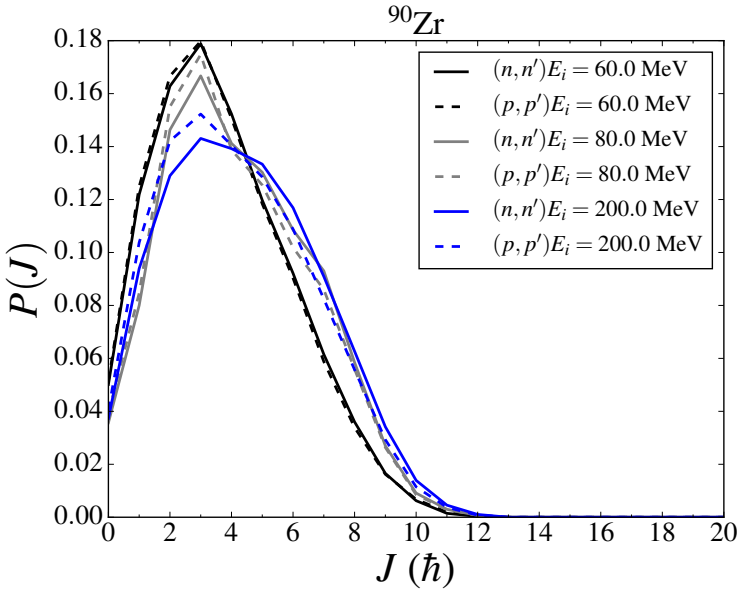


Inelastic Scattering









Forthcoming

1. At sufficiently high incident energy, both of the final particles can be in the continuum. The DWBA amplitude will then be

$$\left\langle \vec{k}_{f_1}, \vec{k}_{f_2}; h \left| T \right| \vec{k}_i \right\rangle = \int d^3r \, \psi^{(-)*}_{k_{f_1}} (\vec{r}) \, \psi^{(-)*}_{k_{f_2}} (\vec{r}) \, V_0 \, \psi_h (\vec{r}) \, \psi^{(+)}_{k_i} (\vec{r}) \, .$$

2. At extremely low energy, both of the particles can occupy previously unoccupied bound states of the nucleus. In this case, we would say that the incident nucleon was absorbed. The corresponding DWBA amplitude is

$$\left\langle p_1 p_2 h \left| T \right| \vec{k}_i \right\rangle = \int d^3r \, \psi^*_{p_1} (\vec{r}) \, \psi^*_{p_2} (\vec{r}) \, V_0 \, \psi_h (\vec{r}) \, \psi^{(+)}_{k_i} (\vec{r}) \, .$$

3. Two-step: One particle in continuum

$$\left\langle \vec{k}_f; \, p_2 h_2, \, p_1 h_1 \left| T^{(2)} \right| \vec{k}_i \right\rangle = \int d^3r \, \psi^{(-)*}_{k_f} \left(z, \, \vec{b}\right) \, \psi^*_{p_2} \left(z, \, \vec{b}\right) \psi_{h_2} \left(z, \, \vec{b}\right)$$
$$\times G^{(+)}_m \left(z, \, \vec{b}; \, z, \, \vec{b}\right) \, \psi^*_{p_1} \left(z, \, \vec{b}\right) \psi_{h_1} \left(z, \, \vec{b}\right) \, \psi^{(+)}_{k_i} \left(z, \, \vec{b}\right)$$

Summary

- We have taken into account channels where the projectile creates new p-h pairs with bound or unbound (continuum) particle states;
- One of the two particles remains in the continuum while the second occupies a previously unoccupied bound state of the nucleus. This is the situation assumed in all multi-step direct models;
- One step is limited to around 40 MeV of excitation energies;
- $t\rho$ approximation results are in agreement with high energy scattering;
- Extension of the first reaction step to include 0, 1 or 2 nucleons in the continuum;
- In the second step of the reaction, 0, 1, 2 or 3 can be in the continuum;

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To be submitted

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