Effects of Nucleon Correlations on Nuclear Structure and Reactions

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Theoretical Correlation Methods Used:

- The Coherent Density Fluctuation Model (CDFM) [Sofia, 1979-till now]
  based on the delta-function approximation for the overlap and energy kernels of the Generator Coordinate Method

- The Generator Coordinate Method

- The Jastrow Correlation Method

- The Natural Orbital and Overlap Functions Representations

- The Nuclear Density Functional Theory

...and others
Coherent Density Fluctuation Model (CDFM)


\[ \rho(r, r') = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_x(r, r') \]  
(1)

\[ \rho_x(r, r') = 3\rho_0(x) j_1(k_F(x)|r - r'|) \Theta \left( x - \frac{|r + r'|}{2} \right) \]  
(2)

\[ k_F(x) = \left( \frac{3\pi^2}{2} \rho_0(x) \right)^{1/3} \equiv \frac{\beta}{x} ; \quad \rho_0(x) = \frac{3A}{4\pi x^3} \]  
(3)

\[ \beta = \left( \frac{9\pi A}{8} \right)^{1/3} \simeq 1.52A^{1/3} \]  
(4)

\[ \rho(r) = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x) \Theta(x - |r|) \]  
(5)

\[ |\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)} \left. \frac{d\rho(r)}{dr} \right|_{r=x} ; \quad \left( \frac{d\rho}{dr} \leq 0 \right) ; \quad \int_0^\infty dx |\mathcal{F}(x)|^2 = 1 \]  
(6)

\[ n(k) = \int_0^\infty dx |\mathcal{F}(x)|^2 \frac{4}{3} \pi x^3 \Theta(k_F(x) - |k|) \]  
(7)
Nucleon momentum distribution for $^4\text{He}$: the black squares are the exp. data, the exp (S)-method (dotted line), the correlation method of Akaishi (curve 1) and the CDFM (curve 2). Normalization: $\int n(k)dk = 1$


Spectral functions for $^{40}\text{Ca}$ in CDFM

Natural Orbitals

- Löwdin (1955)

\[
\rho(r, r') = \sum_{\alpha} N_{\alpha} \psi^*_\alpha(r) \psi_\alpha(r')
\]  

(1)

\[0 \leq N_{\alpha} \leq 1, \quad \sum_{\alpha} N_{\alpha} = A \]

(2)

\[
\rho(r) = \sum_{\alpha} N_{\alpha} |\psi_\alpha(r)|^2
\]  

(3)

\[
n(k) = \sum_{\alpha} N_{\alpha} |\psi_\alpha(k)|^2
\]  

(4)

\[\{\psi_\alpha(r)\}: \text{complete orthonormal set} \]

\[
\int \rho(r, r')\psi^*_\alpha(r')dr' = N_{\alpha} \psi_\alpha(r)
\]  

(5)

\[
\int \rho(k, k')\psi^*_\alpha(k')dk' = N_{\alpha} \psi_\alpha(k)
\]  

(6)
Overlap Functions

- One-body overlap functions

\[ \phi_\alpha(r) = \langle \Psi_\alpha^{(A-1)} | a(r) | \Psi^{(A)} \rangle \]  \hspace{1cm} (1)

Spectroscopic factor:

\[ S_\alpha = \langle \phi_\alpha | \phi_\alpha \rangle \]  \hspace{1cm} (2)

\[ \tilde{\phi}_\alpha(r) = S_\alpha^{-1/2} \phi_\alpha(r) \]  \hspace{1cm} (3)

\[ \rho(r, r') = \sum_\alpha \phi^*_\alpha(r) \phi_\alpha(r') = \sum_\alpha S_\alpha \tilde{\phi}^*_\alpha(r) \tilde{\phi}_\alpha(r') \]  \hspace{1cm} (4)


\[ \phi_{n_0 l_j}(r) = \frac{\rho_{l_j}(r, a)}{C_{n_0 l_j} \exp(-k_{n_0 l_j} a)/a} \]  \hspace{1cm} (5)
FIG. 1. Overlap functions (solid line), self-consistent Hartree-Fock single-particle wave functions (dot-dashed line), and natural orbitals (dashed line) for the nucleus $^{40}$Ca.
\( \Phi_{a_0J\Sigma L \Pi L_R}(r, R) \) 

\[ \frac{\rho_{J\Sigma L \Pi L_R}^{(2)}(r, R; a, b)}{\Phi_{a_0J\Sigma L \Pi L_R}(a, b)} = \frac{\rho_{J\Sigma L \Pi L_R}^{(2)}(r, R; a, b)}{N \exp\{-k\sqrt[4]{b^2 + (1/4)a^2}\}[b^2 + (1/4)a^2]^{-5/2}}. \]  

(25) 


Removal of \( ^1S_0 \) and \( ^3P_1 \) (pp) pairs from \( ^{16}\text{O}(e,e'pp)^{14}\text{C}_{\text{g.s.}} \)

Partial waves: \( 2S+1 \ell \_L \); \( \overrightarrow{L} = \overrightarrow{l} + \overrightarrow{L}_R \)

FIG. 1. The $^3S_0$ two-proton overlap functions for the nucleus $^{16}\text{O}$ leading to the $0^+$ ground state of $^{14}\text{C}$ extracted from the JCM (left) and uncorrelated (right) two-body density matrices.

FIG. 2. The $^3P_1$ two-proton overlap functions for the nucleus $^{16}\text{O}$ leading to the $0^+$ ground state of $^{14}\text{C}$ extracted from the JCM (left) and uncorrelated (right) two-body density matrices.
Exotic Nuclei (Structure)


Nuclear Symmetry Energy and Its Volume and Surface Components

\[ E = -c_1 A + c_2 A^{2/3} + c_3' \frac{(N - Z)^2}{A} + \text{Coulomb term} + \text{pairing energy contribution} + \text{shell corrections} + \cdots \quad (1) \]

⇒ Feenberg (1947); Cameron (1957); Green (1958); Myers and Swiatecki (1966, 1969); Bethe (1971); Danielewicz; Dieperink and Van Isacker, E. Suraud, Agrawal, Viñas, De, Samaddar, Centelles; Tsang, Warda, and others

\[ c_3' \equiv s = c_3 - \frac{c_4}{A^{1/3}} = c_3 \left( 1 - \frac{\chi}{A^{1/3}} \right) \quad (2) \]

\[ \chi = \frac{c_4}{c_3} \quad (3) \]

\[ c_3 = \frac{s}{1 - \frac{\chi}{A^{1/3}}}, \quad c_4 = \chi \left( \frac{s}{1 - \frac{\chi}{A^{1/3}}} \right) \quad (4) \]
\[ E_{\text{sym}} = \frac{a_a(A)}{A} (N - Z)^2 \]  \hspace{1cm} (6)

- Agrawal et al. (2014); Myers and Swiatecki (1969); Lipparini and Stringari (1982); Jiang et al. (2012); P.-G. Reinhard et al. (2006)

\[ a_a(A) = \frac{a^V_A}{1 + A^{-1/3} a^V_A a^S_A} \simeq c_3 - \frac{c_4}{A^{1/3}}, \] \hspace{1cm} (7)

(at \( A \geq 27 \))

if \( c_3 = a^V_A \) and \( c_4 = (a^V_A)^2 / a^S_A \).
\[ s = \int_0^\infty dx |\mathcal{F}(x)|^2 S(\rho(x)) \]  \hspace{1cm} (8)


\[ \frac{a_A^V}{a_A^S} = \frac{3}{r_0 \rho_0} \int_0^\infty dx |\mathcal{F}(x)|^2 x \rho_0(x) \left\{ \frac{S(\rho_0)}{S(\rho(x))} - 1 \right\} \]  \hspace{1cm} (9)

\[ s \equiv a_\alpha(A) \]  \hspace{1cm} (10)

Let

\[ \kappa \equiv \frac{a_A^V}{a_A^S} \]  \hspace{1cm} (11)

\[ s = \frac{a_A^V}{1 + A^{-1/3} \kappa} \]  \hspace{1cm} (12)

\[ a_A^V = s(1 + A^{-1/3} \kappa) \]  \hspace{1cm} (13)

\[ a_A^S = \frac{s}{\kappa}(1 + A^{-1/3} \kappa) \]  \hspace{1cm} (14)

- Considered Ni, Sn, and Pb isotopic chains
Exotic Nuclei (processes)

Microscopic optical potential; elastic scattering; breakup reactions

\[ U_{opt}(r) = N_R V^F(r) + iN_I W^H(r) \] (1)

1. Direct and exchange parts of the real OP (ReOP)

Folding:

\[ V^F(r) = V^D(r) + V^{EX}(r) \] (2)

\( V^D_{IS}, V^D_{IV}, V^{EX}_{IS}, V^{EX}_{IV} \)

\( v^D_{(00)(01)}(\rho, E), v^{EX}_{(00)(01)}(\rho, E) \) – M3Y effective interactions

2. Imaginary part of the OP (ImOP) within the high-energy approximation

\[ W^H(r) = -\frac{1}{2\pi^2} \frac{E}{k} \bar{\sigma}_{NN} \int_0^\infty j_0(qr) \rho_p(q) \rho_t(q) f_{NN}(q) q^2 dq \] (3)

Superscaling in Electron- and Neutrino- Nuclei Scattering

PWIA; \((e, e' N)\):

\[
\left[ \frac{d\sigma}{de' d\Omega' dp_N d\Omega_N} \right]^{PWIA}_{(e, e' N)} = K \sigma^{eN}(q, \omega; p, \mathcal{E}, \phi_N)S(p, \mathcal{E})
\] (1)

\[
F(q, \omega) \simeq \frac{[d\sigma/d\epsilon' d\Omega']_{(e, e')}}{\sigma^{eN}(q, \omega; p = |y|, \mathcal{E} = 0)}
\] (2)

RFG:

\[
f_{RFG}(\psi') \simeq \frac{3}{4} \left(1 - \psi'^2\right) \theta \left(1 - \psi'^2\right)
\] (3)

\[
S(p, \mathcal{E}) = \sum_i 2(2j_i + 1)n_i(p)L_{\Gamma_i}(\mathcal{E} - \mathcal{E}_i);
\] (4)

\[
L_{\Gamma_i}(\mathcal{E} - \mathcal{E}_i) = \frac{1}{\pi} \frac{\Gamma_i/2}{(\mathcal{E} - \mathcal{E}_i)^2 + (\Gamma_i/2)^2};
\] (5)

\((\Gamma_{1p} = 6 \text{ MeV and } \Gamma_{1s} = 20 \text{ MeV})\)

\[
\rho(r, r') = \sum_{\alpha} N_{\alpha} \varphi_{\alpha}^*(r) \varphi_{\alpha}(r'); \ [0 \leq N_{\alpha} \leq 1; \sum_{\alpha} N_{\alpha} = A];
\] (6)
Information on the nucleon momentum distributions from
the scaling function

– Amado, Woloshyn (1976–77):

\[ n(k) \xrightarrow{k \to \infty} \left( \frac{\tilde{V}_{NN}(k)}{k^2} \right)^2 \]  

(unknown if \(k\) or \(k/A\) must be large)

\[ f(\psi') = 0.12 \left( \frac{1 + m}{2 + m} \right) \frac{1}{|\psi'|^{2+m}} \]

\[ n(k) \sim \frac{1}{k^{4+m}}; \quad \text{Results: } m \simeq 4.5 \]

For \(m = 4\) \(V_{NN}(r) \sim \frac{1}{r}\) (at \(r \to 0\))

For \(m = 5\) \(V_{NN}(r) \sim \frac{1}{r^{1/2}}\) (at \(r \to 0\))

FIG. 4. Momentum distribution in the dilute Fermi gas model with realistic NN forces can serve as an "effective"

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distribution Eq. (13) for different values of

\[ n \]

Therefore, we look for the proper value of \( n \). The factor \( 0 \) \( k > k_F \)

normalization (for

we use

\( x \)

Finally, from Eq. (8) one can obtain the following expres-

\( \psi \)

\( k \)

\( N \)

is obtained by the total normalization of

\( f \)

\( \frac{4 \pi k_F^n}{\mu} \) [Eq. (2)] from Ref. [44], but for

\( m \)

\( f(k/k_F) \) \[ \text{Eq. (2)} \] from Ref. [44], which, being a

\( m \)

\( \sim \)

\( 1 \)

\( k \)

\( 8 \) [25] which, being a

\( m \)

\( f(\psi') \) \[ n(k) \sim 1/k^{4+m} \], \( m = 1 \ldots 5 \); Phys. Rev. C 75, 034319 (2007)
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