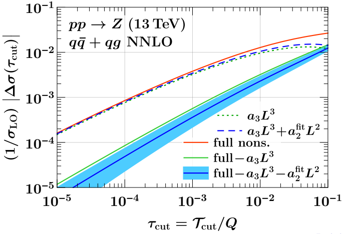
Subleading Power Corrections for *N*-Jettiness Subtractions

Ian Moult

with Lorena Rothen, Iain Stewart, Frank Tackmann, and HuaXing Zhu

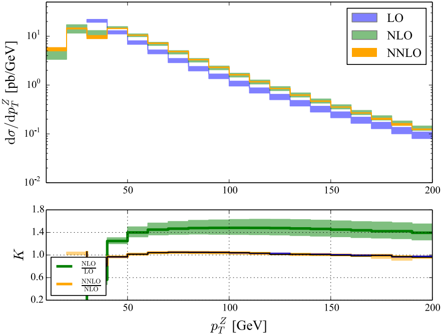
Berkeley Center For Theoretical Physics/ Lawrence Berkeley Laboratory



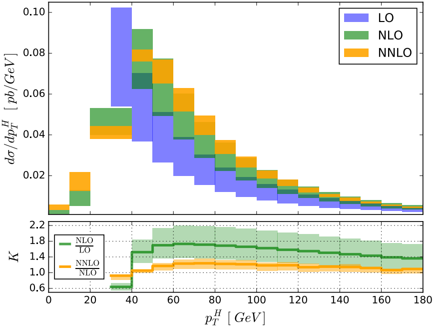
Precision Calculations for the LHC

- Precision calculations play an essential role at the LHC.
  - e.g. characterize Higgs, SM backgrounds, SM measurements,...
- State of the art is next-to-next-to-leading order (NNLO).

p_T Spectrum in $Z+$ Jet



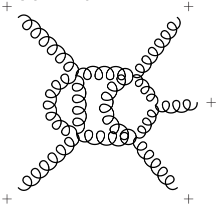
p_T Spectrum in $H+$ Jet



[Boughezal, Focke, Giele, Petriello, Liu]

NNLO Amplitudes

- Well developed set of tools for calculating amplitudes
  - Integration by Parts Chetyrkin, Tchakov
  - Differential Equations Kotikov, Remiddi, Gehrmann, Henn
  - Symbols, Polylogarithms, Special Functions
- Many two-loop amplitudes of phenomenological relevance known.
- $2 \to 2$ NNLO virtual corrections:
  - Di-jet: [Anastasiou, Glover, Oleari, Tejeda-Yeomans], [Bern, De Freitas, Dixon]
  - $\gamma+$ jet: [Anastasiou, Glover, Tejeda-Yeomans]
  - $W+$ jet: [Gehrmann, Tancredi]
  - $Z+$ jet: [Gehrmann, Tancredi, Weihs]
  - $H+$ jet: [Gehrmann, Jaquier, Glover, Koukoutsakis]
  - Diboson: [Gehrmann, von Manteuffel, Tancredi], [Caola, Henn, Melnikov, Smirnov, Smirnov]

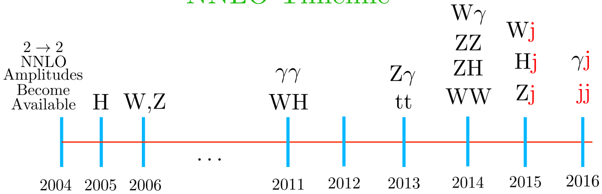


- Progress towards $2 \to 3$:
  - Integrands: [Badger, Frellesvig, Zhang], [Badger, Mogull, Ochirov, O'Connell], [Badger, Mogull, Peraro]
  - Integrals: [Gehrmann, Henn, Lo Presti], [Dunbar, Perkins]

NNLO Cross Sections

- Progress towards NNLO phenomenology slower.
- Significant complexity in translating amplitudes $\rightarrow$ cross sections.

NNLO Timeline

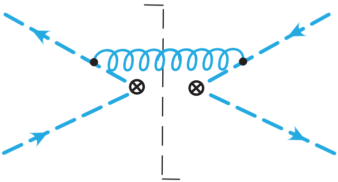


- Recent rapid progress in NNLO calculations involving jets.

NNLO Cross Sections

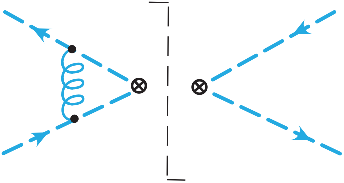
- Higher order calculations require cancellation of soft and collinear divergences between real and virtual diagrams.
- NLO:

Real



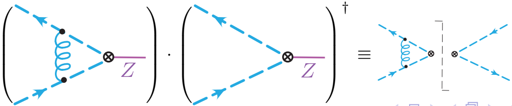
"Hidden" $\frac{1}{\epsilon^2}$ from integration over phase space.

Virtual



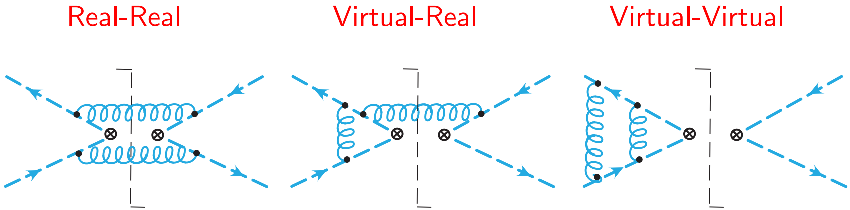
Explicit $\frac{1}{\epsilon^2}$ from loop.

- Note that for cross sections I will use a "cut" notation:



NNLO Cross Sections

- Higher order calculations require cancellation of soft and collinear divergences between real and virtual diagrams.
- NNLO:

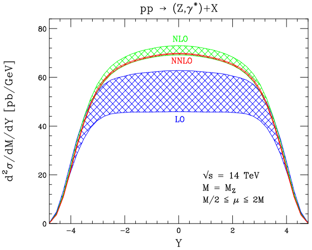


- Complicated overlapping divergences.
- IR structure of cross sections is bottleneck to NNLO phenomenology at LHC.

Inclusive Cross Sections

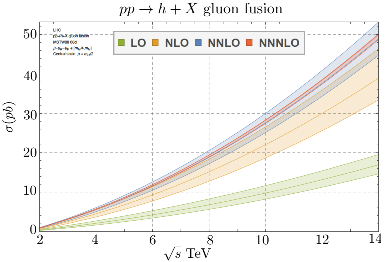
- Relatively inclusive cross sections can be computed analytically.
- Recent progress: "Higgs Differential" [Dulat, Mistlberger et al.]

Z Boson Rapidity Spectrum



[Anastasiou, Dixon, Melnikov, Petriello]

Inclusive Higgs Cross Section



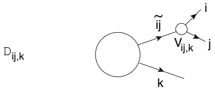
[Anastasiou, Duhr, Dulat, Herzog, Mistlberger]

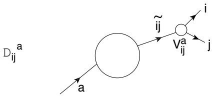
- For many cases would like to do phase space integrals numerically.
- Difficult due to the presence of IR divergences.

Local Subtractions

- Local subtractions: Subtract from real emission graphs a function which reproduces soft and collinear limits, but can be analytically integrated to extract poles.
  $\Longrightarrow$ real graphs finite, can be integrated numerically. Poles can be analytically cancelled against virtual graphs.

- Well understood at NLO:
  - FKS [Frixione, Kunszt, Signer]
  - Catani-Seymour [Catani, Seymour]





- Progress/ successful implementations at NNLO:
  - Colorful NNLO [Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, Tulipant]
  - Sector Decomposition [Anastasiou, Melnikov, Petriello]
  - Antenna Subtraction [Gehrmann-De Ridder, Gehrmann, Glover et al.]
  - Sector-improved Residue Subtraction [Czakon], [Caola, Melnikov, Rontsch]
  - Projection to Born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi]

- Typically extremely complicated, scale poorly with multiplicity.

Global Subtractions at NNLO

q_T: [Catani, Grazzini]
N-jettiness:[Boughezal, Focke, Petriello, Liu]
[Gaunt, Stahlhofen,Tackmann, Walsh]

- Use an observable to regulate phase space.

$$\boxed{\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}}$$

- Want $\mathcal{T}_N$ to isolate collinear and soft singularities around an N-jet configuration.

$$\sigma(\mathcal{T}_N^{\text{cut}}) = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

Compute using factorization in soft/collinear limits:

$$\int_{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

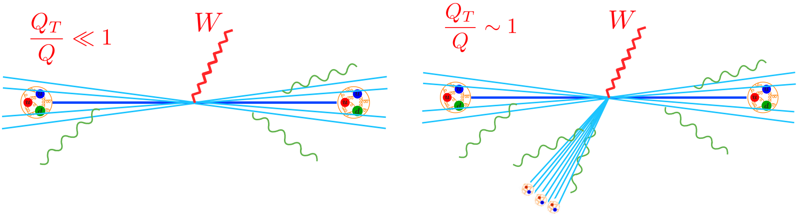
Additional jet resolved. Away from singularity.

$$\frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1}$$

Q_T Subtractions

[Catani, Grazzini]

- For color singlet production can use Q_T.



- All orders factorization theorem:

[Collins, Soper, Sterman]

$$\frac{d\sigma}{dQ_T^2} = H B_a \otimes B_b \otimes S + \mathcal{O}(Q_T^2/Q^2)$$

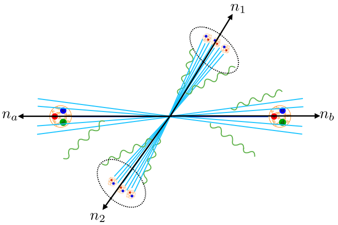
- Successfully applied to
$pp \to H, W, Z, \gamma\gamma, WH, ZH, ZZ, W^+W^-, Z\gamma, W\gamma$

[Catani, Grazzini, Kallweit, Wiesemann]
[MATRIX]

N-Jettiness

[Stewart, Tackmann, Waalewijn]

- N-jettiness: Inclusive event shape to identify N jets.



$$q_i = Q n_i$$

$$\boxed{\tau_N = \frac{2}{Q^2} \sum_k \min\left\{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \cdots, q_N \cdot p_k \right\}}$$

- $\tau_N \ll 1 \implies N$ isolated jets + beams.
- All orders factorization theorem:

$$\frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_N)$$

N-jettiness Subtractions

- Ingredients for factorization known to NNLO

$$\frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_N)$$

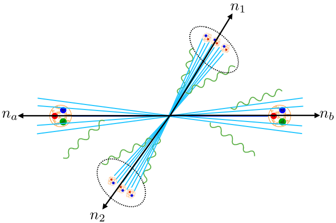
- Beam Functions: [Gaunt, Stahlhofen, Tackmann]
- Jet Functions: [Becher, Neubert], [Becher, Bell]
- Soft Functions:
  - Color singlet known analytically [Kelley, Schwartz, Schabinger, Zhu], [Monni, Gehrmann, Luisoni] [Hornig, Lee, Stewart, Walsh, Zuberi], [Kang, Labun, Lee]
  - 1-jettiness known numerically [Boughezal, Liu, Petriello] [Campbell, Ellis, Mondini, Williams]
- Hard Functions:
  - Process dependent.
  - Many $2 \to 1$ and $2 \to 2$ NNLO amplitudes known.

N-jettiness Subtractions

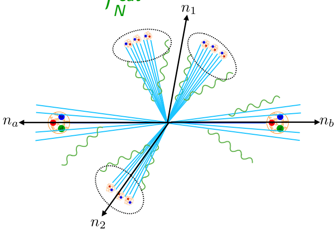
[Boughezal, Focke, Petriello, Liu]
[Gaunt, Stahlhofen, Tackmann, Walsh]

- N-jettiness subtractions: general method for NNLO subtractions allowing for jets in final state.

$$\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$



NNLO calculation in singular limit

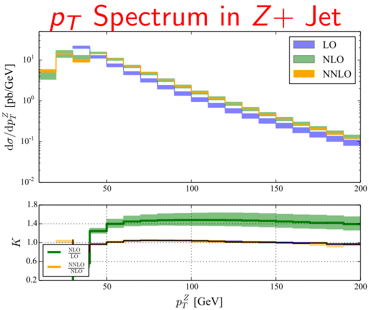


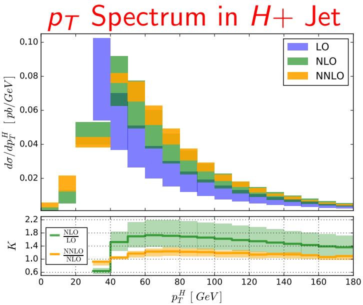
NLO calculation in resolved limit

New NNLO Results with N-jettiness

[Boughezal, Focke, Giele, Petriello, Liu]

- Impressive new results with jets in the final state: $W/Z/H$+jet at NNLO





- Implemented in MCFM for color singlet production at NNLO.

[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams]

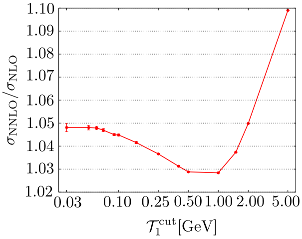
- Conceptually simple, extendable to higher orders, multiplicities.

Power Corrections

- Why is this not a perfect scheme?
- Standard factorization drops power corrections in $\mathcal{T}_N^{\mathrm{cut}}$.

$$\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\mathrm{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\mathrm{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

$\mathcal{T}^{\mathrm{cut}}$ Dependence for NNLO $Z+$ Jet



[Boughezal, Focke, Giele, Petriello, Liu]

- Very difficult numerically to go to low $\mathcal{T}^{\mathrm{cut}}$ values.

Power Corrections

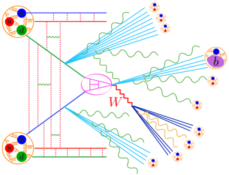
- Would like to compute and understand structure of power corrections.
- Want to systematically compute cross section as a power series in collinear and soft limits:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \underbrace{\frac{d\sigma^{(0)}}{d\tau}}_{\mathcal{O}(\tau^{-1})} + \underbrace{\frac{d\sigma^{(2)}}{d\tau}}_{\mathcal{O}(\tau^{0})} + \underbrace{\frac{d\sigma^{(4)}}{d\tau}}_{\mathcal{O}(\tau^{1})} + \cdots$$

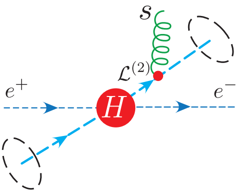
- Practical aspects:
    - Numerical effect.
    - Dependence on Born kinematics.
- More generally:
    - Understand all orders structure in α_s.
    - Identify universal structures, e.g. Y_n, $\Gamma_{\rm cusp}(\alpha_s)$.

Outline

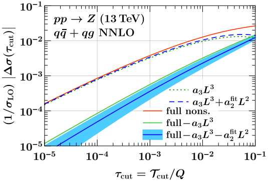
- Factorization, Power Corrections and N-Jettiness Subtractions



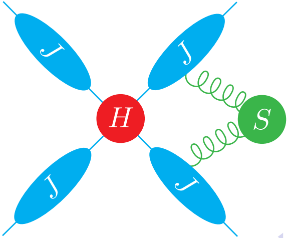
- Perturbative Event Shapes at Subleading Power



- Power Corrections for N-jettiness Subtractions



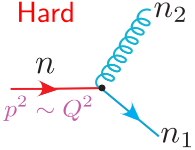
Factorization, Power Corrections and N-Jettiness Subtractions



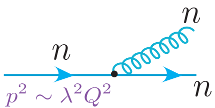
Perturbative Factorization

- Behavior in the collinear and soft limits: $\lambda \ll 1$

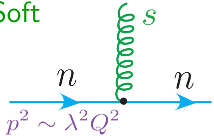
Hard



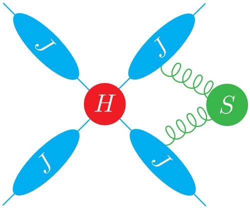
Collinear



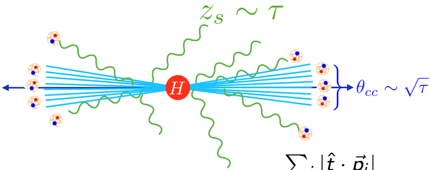
Soft



Formal Limit



Enforced by Measurement
(p_T, threshold, event shapes)

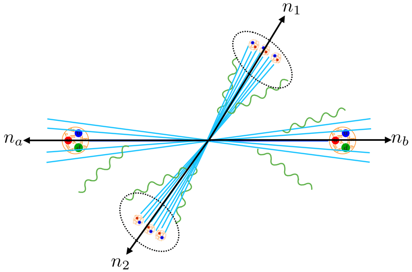


$$\tau = 1 - \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \ll 1$$

- Generic Factorization: $H \otimes \prod J_i \otimes S$

Cross Section Level

- Approach to collinear and soft limit physically realized by event shape/ jet observables. e.g. N-jettiness



- $\tau_N \ll 1 \implies N$ isolated jets + beams (collinear) + soft.
- All orders factorization theorem:

$$\frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_N)$$

- Allows simple calculation in $\tau_N \ll 1$ limit.

Power Corrections: General Structure

- These factorization theorems represent the first term in a "power" expansion in soft and collinear limits, in $\tau \ll 1$.
- General structure for an IRC safe observable τ:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ \quad \text{Leading Power (LP)}$$

$$+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau \quad \text{Next to Leading Power (NLP)}$$

$$+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)} \tau \log^m \tau$$

$$+ \cdots$$

$$= \frac{d\sigma^{(0)}}{d\tau} + \frac{d\sigma^{(2)}}{d\tau} + \frac{d\sigma^{(4)}}{d\tau} + \cdots$$

Power Corrections to N-Jettiness Subtractions

- Can use this to understand the power corrections to N-jettiness subtractions.
- Recall:

$$\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

- Object of interest:

$$\sigma(\tau_{\text{cut}}) = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

- Error made:

$$\Delta\sigma(\tau_{\text{cut}}) = \sigma(\tau_{\text{cut}})_{\text{exact}} - \sigma(\tau_{\text{cut}})_{\text{approx}}$$

Power Corrections to N-Jettiness Subtractions

- Approximation in the singular region receives power corrections

$$\frac{d\sigma}{d\tau} = \overbrace{\sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+}^{\text{LP Approximation}} + \overbrace{\sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau + \sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)}\, \tau \log^m \tau + \cdots}^{\text{Power Corrections}}$$

- Gives rise to power corrections in the integrated cross section

$$\Longrightarrow \sigma(\tau_{\rm cut}) = \overbrace{\sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n} \tilde{c}_{nm}^{(0)} \log^m(\tau_{\rm cut})}^{\text{LP Approximation}} + \overbrace{\tau_{\rm cut}\sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} \tilde{c}_{nm}^{(2)} \log^m(\tau_{\rm cut}) + \cdots}^{\text{Power Corrections}}$$

- The function $\tau_{\rm cut}\log^m(\tau_{\rm cut})$ approaches zero slowly!
  - NLO: $\tau_{\rm cut}\log(\tau_{\rm cut}) + \cdots$
  - NNLO: $\tau_{\rm cut}\log^3(\tau_{\rm cut}) + \cdots$
  - NNNLO: $\tau_{\rm cut}\log^5(\tau_{\rm cut}) + \cdots$
- Very small values of $\tau_{\rm cut}$ are required.

Power Corrections

[IM, Rothen, Stewart, Tackmann, Zhu] 1612.00450

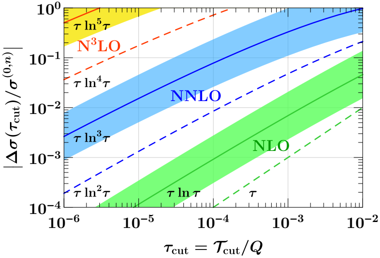
- Use functional form to estimate size of power corrections

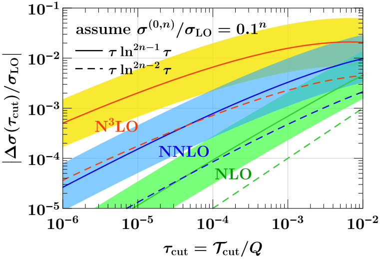
$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut})_{\sf exact} - \sigma(\tau_{\rm cut})_{\sf approx}$$

Solid=LP Dashed=remove LL NLP

$$\sigma(\tau_{\rm cut}) = \sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}_{nm}^{(0)} \log^m(\tau_{\rm cut}) + \tau_{\rm cut} \sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}_{nm}^{(2)} \log^m(\tau_{\rm cut}) + \cdots$$

Estimated Missing Correction





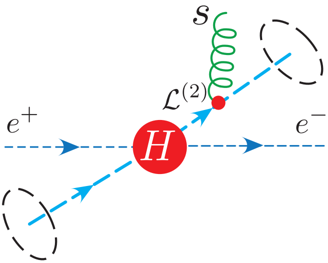
- Factor of ~ 10 improvement by calculating leading log (LL) at NLP.

[IM, Vita, Stewart] 1703.03408

[Feige, Kolodrubetz, IM, Stewart] 1703.03411

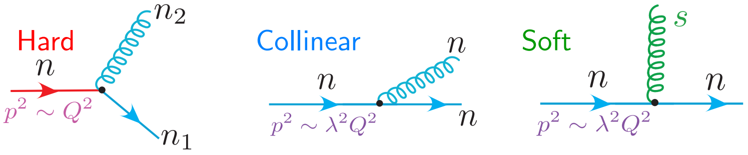
[IM, Rothen, Stewart, Tackmann, Zhu] 1612.00450, 1710.03227

Perturbative Event Shapes Beyond Leading Power



Effective Field Theory

- Use approach of Effective Field Theory:
  - Focus on relevant degrees of freedom.
  - Integrate out irrelevant degrees of freedom.



- Effective theory for long wavelength dynamics of soft and collinear radiation in the presence of a hard scattering source

  $\Longrightarrow$ Soft Collinear Effective Theory [Bauer, Fleming, Pirjol, Stewart]

Soft Collinear Effective Theory

[Bauer, Fleming, Pirjol, Stewart]

- SCET has proven to be a powerful framework for studying factorization.
- Allows a systematic expansion about soft and collinear limits in a power counting parameter λ.
- Separate fields for soft and collinear particles.
- Fields/Lagrangians have a definite power counting in λ.

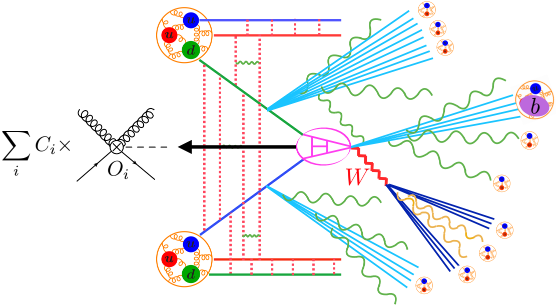
Operator	$\mathcal{B}^{\mu}_{n_i\perp}$	χ_{n_i}	$\mathcal{P}^{\mu}_{\perp}$	q_{us}	D^{μ}_{us}
Power Counting	λ	λ	λ	λ^3	λ^2

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm dyn} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\rm hard} + \sum_{i\geq 0} \mathcal{L}^{(i)}$$

Soft Collinear Effective Theory

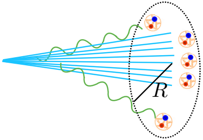
[Bauer, Fleming, Pirjol, Stewart]

- Hard scattering is described by operators in EFT



- Long wavelength dynamics of soft and collinear radiation described by Lagrangian

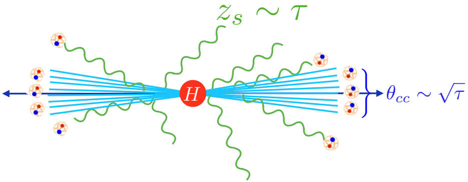
$\mathcal{L}_{\text{dyn}}$:



Fixed Order Thrust at NLP

- Simple playground is Thrust (2-jettiness) in e^+e^-

$$\tau = 1 - \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



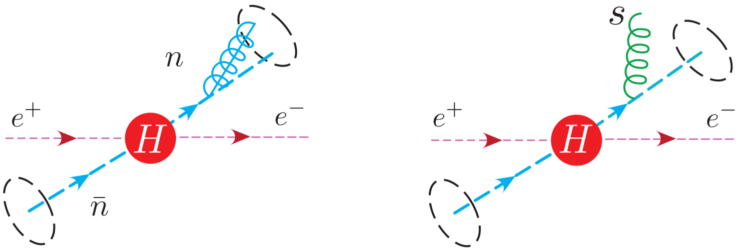
- NLO result can be used as a check.
- Can be "crossed" to color singlet production at the LHC.

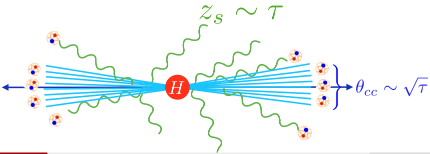
Leading Power SCET

- Leading Power SCET:
  - Leading Power Hard Scattering Operators:

$$\mathcal{O} = C(Q^2)\bar{\chi}_n \Gamma \chi_{\bar{n}}$$

  - Leading power Lagrangian (eikonal/ collinear)



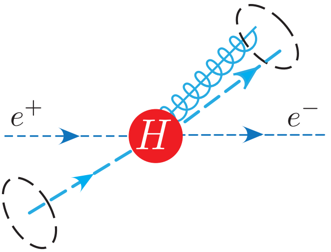


Subleading Power SCET

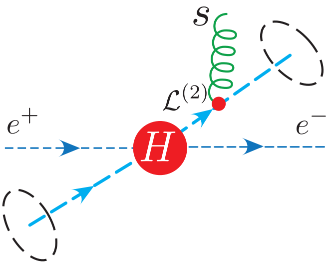
- Subleading Power in SCET:

$$\mathcal{L}_{\mathsf{SCET}} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm dyn} = \sum_{i\geq 0} \mathcal{L}_{\rm hard}^{(i)} + \sum_{i\geq 0} \mathcal{L}^{(i)}$$

Subleading Hard Scattering Operators

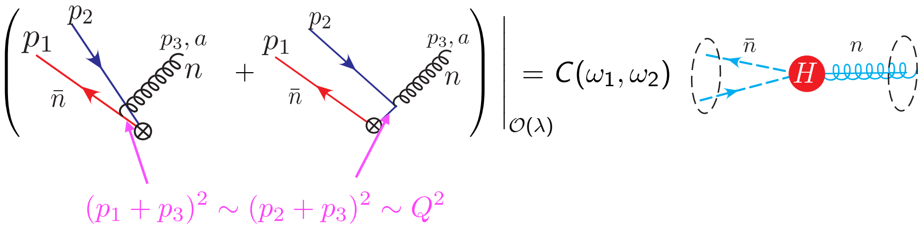


Subleading Lagrangians

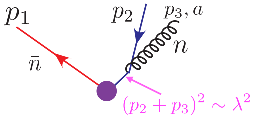


Matching

- Subleading power operators obtained by matching.
- Wilson coefficients, $C(\omega_1, \omega_2)$ depends on large momentum fraction, i.e. $z, 1-z$.



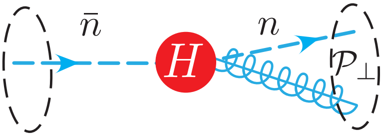
- Contrast with a collinear splitting.
  Described by the Lagrangian of the EFT



Relevant Hard Scattering Operators

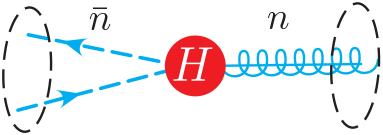
- For leading log (singularity), $\alpha_s^n \log^{2n-1}(\tau)$, two relevant hard scattering operators:

qg In Same Sector



$\bar{\chi}_n \chi_{\bar{n}} \mathcal{P}_\perp \mathcal{B}_n$

$q\bar{q}$ In Same Sector



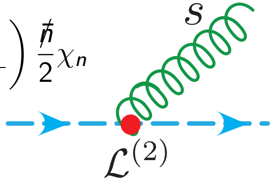
$\bar{\chi}_{\bar{n}} \chi_{\bar{n}} \mathcal{B}_n$

- $q\bar{q}$ in same sector has no LP analog.

Subleading Lagrangian

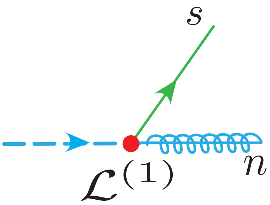
- Subleading Lagrangians are universal, and known.
- Correct the dynamics of soft and collinear particles. e.g.
    - Correction to eikonal emission:

$$\mathcal{L}^{(2)}_{\chi_n} = \bar{\chi}_n \left(i\not{D}_{us\perp} \frac{1}{\overline{\mathcal{P}}} i\not{D}_{us\perp} - i\not{\mathcal{D}}_{n\perp} \frac{i\bar{n}\cdot D_{us}}{(\overline{\mathcal{P}})^2} i\not{\mathcal{D}}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \chi_n$$



    - Emission of soft quarks:

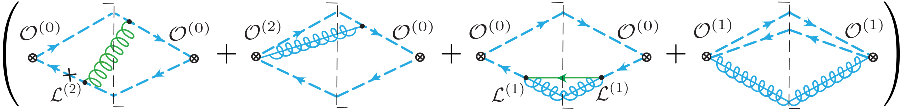
$$\mathcal{L}^{(1)}_{\chi_n q_{us}} = \bar{\chi}_n \frac{1}{\overline{\mathcal{P}}} g \not{\mathcal{B}}_{n\perp} q_{us} + \text{h.c.}$$



- Not decoupled into Wilson lines.

NLO Thrust at NLP

- Sum four graphs, reproduces the NLP piece of well known NLO thrust result:



Total

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma_1^{(2)}}{d\tau} = 4C_F\left(\frac{\alpha_s}{4\pi}\right)\log(\tau)}$$

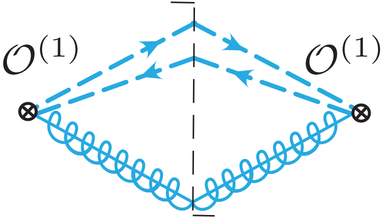
$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma^{\text{NLO}}}{\mathrm{d}\tau} = \frac{1}{\tau}\frac{C_F\alpha_s}{4\pi}[-6-8\log(\tau)] + \frac{C_F\alpha_s}{4\pi}[-4+4\log(\tau)] + \tau[\cdots] + \cdots$$

- Could have also just done textbook NLO calculation, and expanded...
- Result gives directly (no expansions) the NLP contribution.
- Sets up going to higher order, where result is not known.

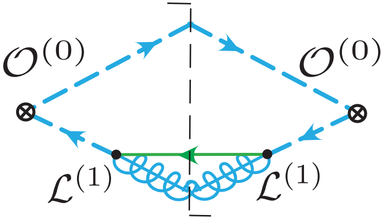
NNLO Thrust at NLP

- NNLO calculation made simple by consistency relation in EFT.
- EFT modes have well defined scaling:

Collinear: $\mu_c^2 \sim Q^2\tau$



Soft : $\mu_s^2 \sim Q^2\tau^2$



$$\frac{1}{\sigma_0}\frac{d\sigma_1^{(2)}}{d\tau} = -4C_F\left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{Q^2\tau}\right)\right] + 4C_F\left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{Q^2\tau^2}\right)\right]$$

Fixed Order Consistency Relations

- Fixed order calculations made simple by consistency relation in EFT.
- EFT modes have well defined scaling:

$$\mu_h^2 = Q^2 , \qquad \mu_c^2 = Q^2\tau , \qquad \mu_s^2 = Q^2\tau^2$$

- General form of n-loop fixed order calculation:

$$\frac{\mathrm{d}\sigma^{(2,n)}}{\mathrm{d}\tau} = \sum_{\kappa} \sum_{i=0}^{2n-1} \frac{c_{\kappa,i}}{\epsilon^i} \left(\frac{\mu^{2n}}{Q^{2n}\tau^{m(\kappa)}} \right)^{\epsilon} + \sum_{\gamma} \sum_{i=0}^{2n-2} \frac{d_{\gamma,i}}{\epsilon^i} \left(\frac{\mu^{2(n-1)}}{Q^{2(n-1)}\tau^{m(\gamma)}} \right)^{\epsilon}$$

  - 1-loop:

$$\begin{aligned} &\text{soft:} & \kappa &= s\,, & m(\kappa) &= 2\,, \\ &\text{collinear:} & \kappa &= c\,, & m(\kappa) &= 1 \end{aligned}$$

  - 2-loop:

$$\begin{aligned} &\text{hard-collinear:} & \kappa &= hc\,, & m(\kappa) &= 1\,, \\ &\text{hard-soft:} & \kappa &= hs\,, & m(\kappa) &= 2\,, \\ &\text{collinear-collinear:} & \kappa &= cc\,, & m(\kappa) &= 2\,, \\ &\text{collinear-soft:} & \kappa &= cs\,, & m(\kappa) &= 3\,, \\ &\text{soft-soft:} & \kappa &= ss\,, & m(\kappa) &= 4 \end{aligned}$$

- Pole terms must cancel $\implies$ non-trivial constraints.

Fixed Order Consistency Relations

- Solving the set of equations, one finds:
  - 1-loop:

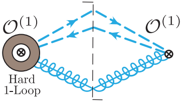
$$c_{s,1} = -c_{c,1}$$

  - 2-loop:

$$\begin{aligned}
c_{hc,3} = \frac{c_{cs,3}}{3} = -c_{ss,3} &= -\frac{1}{3}(c_{hs,3} + c_{cc,3})\,, \\
c_{cs,2} &= c_{hc,2} - 2c_{ss,2} + d_{c,2}\,, \\
c_{hs,2} + c_{cc,2} &= -2c_{hc,2} + c_{ss,2} - d_{c,2}\,, \\
c_{hs,1} + c_{cc,1} &= -(c_{cs,1} + c_{hc,1} + c_{ss,1} + d_{c,1} + d_{s,1})
\end{aligned}$$

- 2-loop NLP result can be written:

$$\begin{aligned}
\frac{d\sigma^{(2,2)}}{d\tau} = {}& c_{hc,3} \ln^3 \tau + (c_{hc,2} + c_{ss,2} + d_{c,2}) \ln^2 \tau \\
&+ \left(-c_{cs,1} + c_{hc,1} - 2c_{ss,1} + d_{c,1}\right) \ln \tau \\
&+ d_{c,2} \ln \frac{Q^2}{\mu^2} \ln \tau + \text{const}
\end{aligned}$$



- LL can be computed from only the hard-collinear contribution.
- Can easily show this extends to all orders.

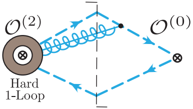
NNLO Thrust at NLP

[Ellis, Ross, Terrano]

[Garland, Gehrmann, Glover, Koukoutsakis, Remiddi]

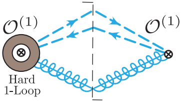
- 1 (and 2) loop results for $e^+e^- \to 3$ partons known.
- Calculation of NLP $\alpha_s^2 \log^3(\tau)$ straightforward using consistency relations: Only need hard-collinear contribution.

Quark Channel



$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,2)}_{\mathrm{Cat.1}}}{d\tau} = -32 C_F^2 \left(\frac{\alpha_s}{4\pi}\right)^2 \log^3(\tau)$$

Gluon Channel



$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,2)}_{\mathrm{Cat.2}}}{d\tau} = 8 C_F (C_F + C_A) \left(\frac{\alpha_s}{4\pi}\right)^2 \log^3(\tau)$$

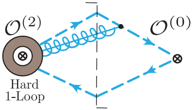
Total

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma^{(2,2)}}{d\tau} = 8 C_F (C_A - 3 C_F) \left(\frac{\alpha_s}{4\pi}\right)^2 \log^3(\tau)}$$

NNLO Thrust at NLP

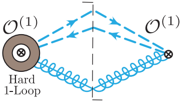
- Very different color structures of leading logs.
- Gluon channel has no leading power analog $\Longrightarrow$ More interesting
- All orders result can be derived from renormalization of (dressed) Wilson loops.

Quark Channel



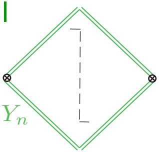
$$\frac{1}{\sigma_0}\frac{d\sigma_2^{(2)}}{d\tau} = -32C_F^2\left(\frac{\alpha_s}{4\pi}\right)^2\log^3(\tau)$$

Gluon Channel

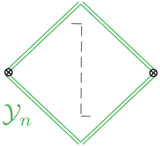


$$\frac{1}{\sigma_0}\frac{d\sigma_2^{(2)}}{d\tau} = 8C_F(C_F+C_A)\left(\frac{\alpha_s}{4\pi}\right)^2\log^3(\tau)$$

Fundamental Cusp:



Adjoint Cusp:



Extension to pp

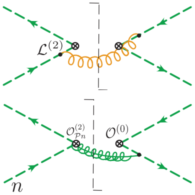
- Operators and Lagrangians also applicable to perturbative power corrections in pp.

$$\mathrm{d}\sigma = \sum_{ij} \int \mathrm{d}\xi_a \mathrm{d}\xi_b \, f_i(\xi_a) \, f_j(\xi_b) \, \mathrm{d}\hat{\sigma}_{ij}(\xi_a, \xi_b)$$

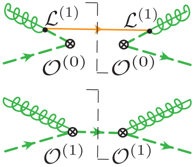
- Partonic cross section at $\mathcal{O}(\tau^0)$ written as

$$\frac{\mathrm{d}\hat{\sigma}^{(2,n)}_{ij}(\xi_a, \xi_b; X)}{\mathrm{d}Q^2 \, \mathrm{d}Y \, \mathrm{d}\tau} = \sigma_{q0}(Q, X) \left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{2n-1} C^{(2,n)}_{ij,m}(\xi_a, \xi_b) \ln^m \tau$$

$q\bar{q}$ channel



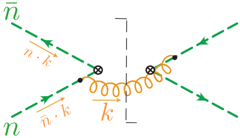
qg channel



Extension to *pp*

- Power corrections arise from residual momentum routed into pdfs.
- Must be expanded homogeneously:

$$f_i\left[\xi\left(1+\frac{k}{Q}\right)\right] = f_i(\xi) + \frac{k}{Q}\,\xi f_i'(\xi) + \cdots$$



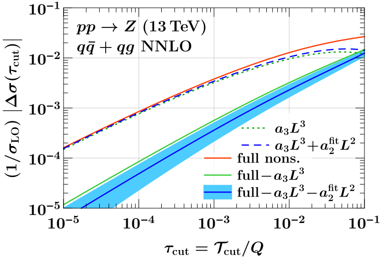
- We take $\xi f_i'(\xi) \sim f_i(\xi)$.
- Coefficients of partonic cross section at $\mathcal{O}(\tau^0)$ involve δ'.
- Use the shorthand notation

$$\delta_a' \equiv x_a\,\delta'(\xi_a - x_a)\,, \qquad \delta_b' \equiv x_b\,\delta'(\xi_b - x_b)$$

for the δ' acting on either beam direction.

[IM, Rothen, Stewart, Tackmann, Zhu] 1612.00450,1710.03227

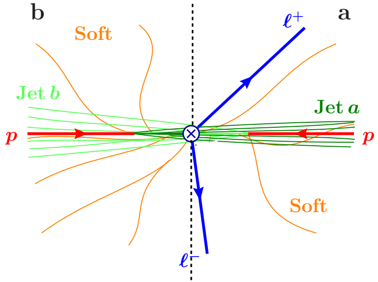
Power Corrections for *N*-Jettiness Subtractions



0-Jettiness

- Start with simplest case: 0-jettiness (beam thrust)
- Applicable for color singlet production.
- Divide event into two halves using beam.

$$\tau_0 = \frac{1}{Q}\sum_k \min\{n_a \cdot k, n_b \cdot k\}$$

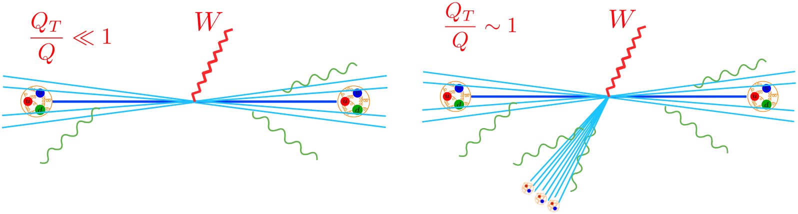


- Can be thought of as “crossing” of thrust.

Observable Dependence: Q_T Subtractions

- Highly desirable for power corrections to be independent of Born kinematics

  $\Longrightarrow$ e.g. Q_T subtractions: Q_T/Q independent of rapidity.



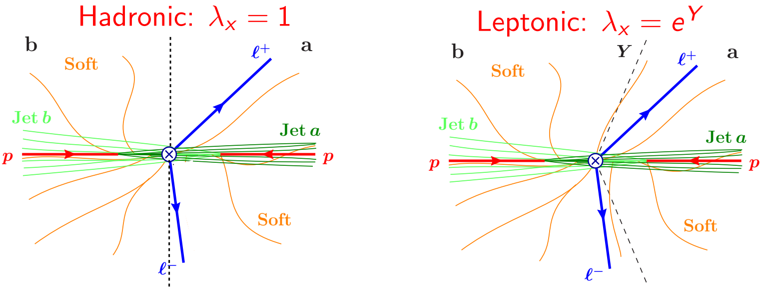
- Want to choose definition of N-jettiness such that power expansion is well behaved throughout phase space:

  $\Longrightarrow$ Completely general feature that can be understood in the 0-jettiness case.

Observable Dependence

- Two natural definitions of 0-jettiness:

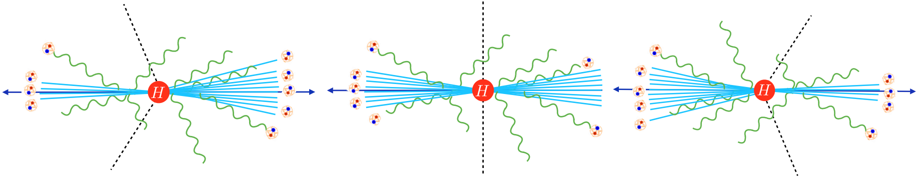
$$\mathcal{T}_0^x = \sum_k \min\Big\{\lambda_x\, n_a \cdot k,\, \lambda_x^{-1}\, n_b \cdot k\Big\}$$



- Hadronic definition used in initial studies.

Observable Dependence

- Consider boosting thrust in $e^+e^- \to$ dijets by Y.
- Power expansion is invariant ONLY IF observable changes accordingly.



$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma^{\mathrm{NLO}}}{\mathrm{d}\tau} = \underbrace{\frac{1}{\tau}\frac{C_F\alpha_s}{4\pi}[-6-8\log(\tau)]}_{\text{Leading Power}} + \underbrace{\frac{C_F\alpha_s}{4\pi}[-4+4\log(\tau)]}_{\text{Next to Leading Power}} + \tau[\cdots] + \cdots$$

- Failure to boost observable definition
  $\implies$ expansion parameter is $\lambda^2 \sim \tau e^{|Y|}$.

Results for Beam Thrust in Drell Yan: Leptonic Definition

$$\boxed{\frac{\mathrm{d}\hat{\sigma}_{ij}^{(2,n)}(\xi_a,\xi_b;X)}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}\tau} = \sigma_{q0}(Q,X)\left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{2n-1} C_{ij,m}^{(2,n)}(\xi_a,\xi_b)\ln^m\tau}$$

- NLO results for $\mathcal{O}(\tau^0)$ power correction in leptonic definition:

$$C_{q\bar{q},1}^{(2,1)}(\xi_a,\xi_b) = 8C_F\left(\delta_a\delta_b + \frac{\delta_a'\delta_b}{2} + \frac{\delta_a\delta_b'}{2}\right)$$
$$C_{qg,1}^{(2,1)}(\xi_a,\xi_b) = -2T_F\,\delta_a\delta_b$$

- NNLO results obtained from hard-collinear contribution using consistency:

$$C_{q\bar{q},3}^{(2,2)}(\xi_a,\xi_b) = -32C_F^2\left(\delta_a\delta_b + \frac{\delta_a'\delta_b}{2} + \frac{\delta_a\delta_b'}{2}\right)$$
$$C_{qg,3}^{(2,2)}(\xi_a,\xi_b) = 4T_F(C_F+C_A)\,\delta_a\delta_b$$

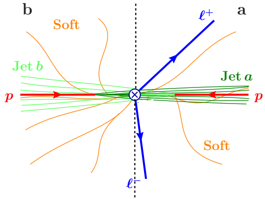
- No explicit dependence on rapidity.

Results for Beam Thrust in Drell Yan: Hadronic Definition

- Power corrections for hadronic definition are enhanced by $e^{|Y|}$!

$$\widetilde{C}^{(2,2)}_{q\bar{q},3}(\xi_a,\xi_b) = -16C_F^2\Big[e^{Y}\delta_a(\delta_b+\delta_b') + e^{-Y}(\delta_a+\delta_a')\delta_b\Big]$$
$$\widetilde{C}^{(2,2)}_{qg,3}(\xi_a,\xi_b) = 4T_F(C_F+C_A)e^{Y}\delta_a\delta_b$$
$$\widetilde{C}^{(2,2)}_{gq,3}(\xi_a,\xi_b) = 4T_F(C_F+C_A)e^{-Y}\delta_a\delta_b$$

- Constraint on radiation depends on Y:

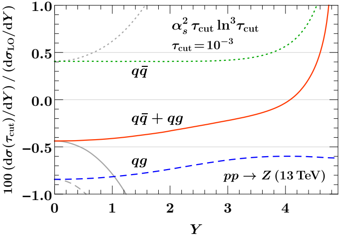


- Expansion parameter for hadronic definition is $\lambda^2 \sim \tau e^{|Y|}$.
- Breaks down away from central rapidity.

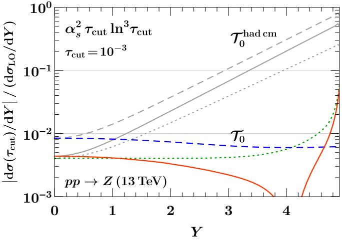
Observable Dependence

- Exponential growth of power corrections for hadronic definition.
- Power corrections for leptonic definition close to rapidity independent!
- Very important when computing differential distributions. (e.g. rapidity spectrum) $\boxed{\Longrightarrow \text{ Leptonic Definition Strongly Preferred!}}$

Power Correction (Linear)



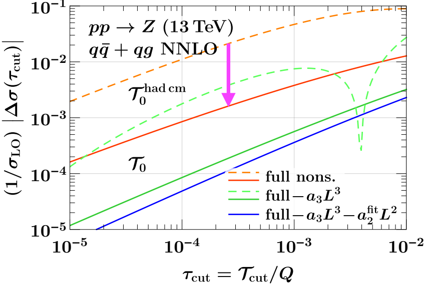
Power Correction (Log)



Observable Dependence

- Before including power corrections, improved definition
  $\Longrightarrow$ ~ 10 reduction in power correction for integrated cross section.

NNLO Power Correction



- Adapted in recent applications of N-jettiness. e.g. 1710.06294 1708.02925
- General lesson for N-jettiness slicing that applies beyond the color singlet case.

Numerical Comparisons

[Campbell, Ellis, Williams]

- Exact fixed order result can be computed numerically (MCFM).
- Subtract known leading power result to obtain power corrections:

$$\overbrace{\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}}^{\text{MCFM}} - \underbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+}_{\text{Leading Power}} = \overbrace{\sum_{n=0}^{n=\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} c_{nm}^{(2)} \log^m \tau + \cdots}^{\text{Power Corrections} \,\equiv\, \text{Nonsingular}}$$

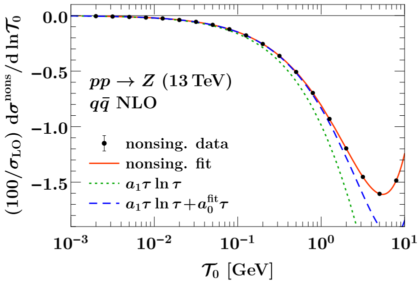
- Allows a numerical study of the size of power corrections in N-jettiness subtraction scheme.

NLO Beam Thrust at NLP

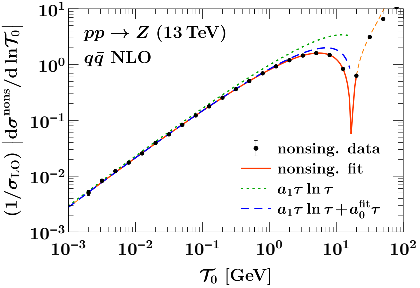
- NLO nonsingular cross section well approximated by leading logarithmic power correction

$$\frac{d\sigma^{\rm nons}}{d\tau} = \left(\frac{\alpha_s}{\pi}\right)\left(\overbrace{\tilde{c}^{(1)}\log(\tau)}^{\text{Calculated}} + \tilde{c}^{(0)} + \mathcal{O}(\tau)\right)$$

Nonsingular (Linear)



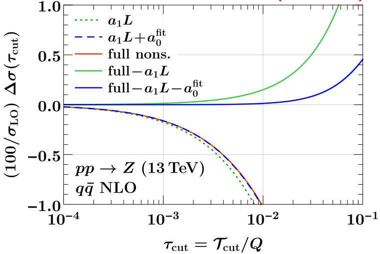
Nonsingular (Log)



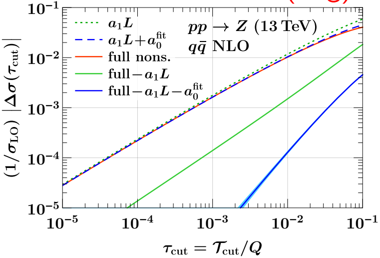
NLO Beam Thrust at NLP

- Significant reduction in power correction, $\Delta\sigma(\tau_{\rm cut})$.

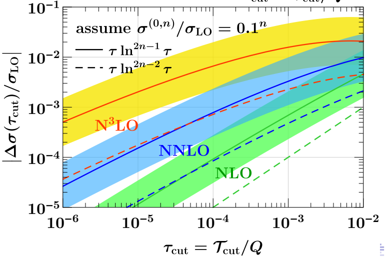
Power Correction (Linear)



Power Correction (Log)



- Agrees well with scaling estimate.

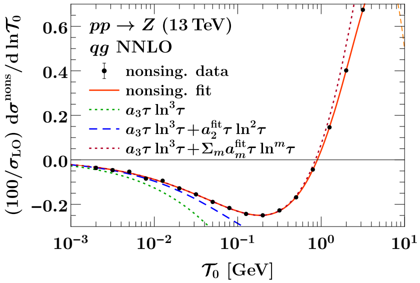


NNLO Beam Thrust at NLP

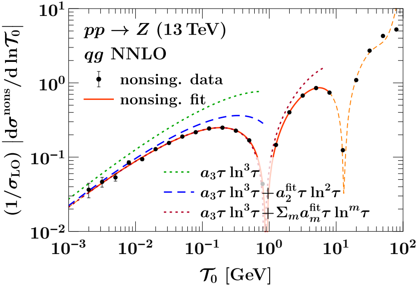
- Leading logarithm provides good approximation at NNLO.
- At NNLO there are subleading logarithms which we have not (yet) calculated.

$$\frac{d\sigma^{\text{nons}}}{d\tau} = \left(\frac{\alpha_s}{\pi}\right)^2 \left(\overbrace{\tilde{c}^{(3)} \log^3(\tau)}^{\text{Calculated}} + \tilde{c}^{(2)} \log^2(\tau) + \tilde{c}^{(1)} \log(\tau) + \tilde{c}^{(0)} + \mathcal{O}(\tau) \right)$$

Nonsingular (Linear)



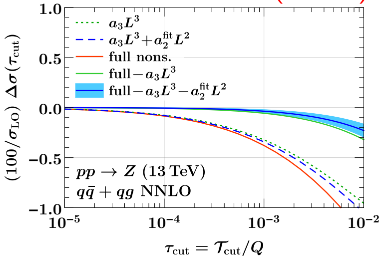
Nonsingular (Log)



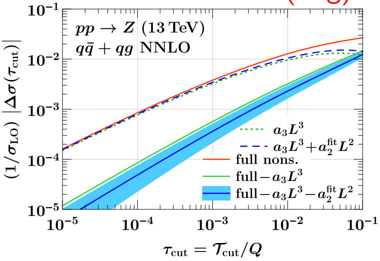
NNLO Beam Thrust at NLP

- Combined result for $\Delta\sigma(\tau_{\rm cut})$ for both channels at NNLO.

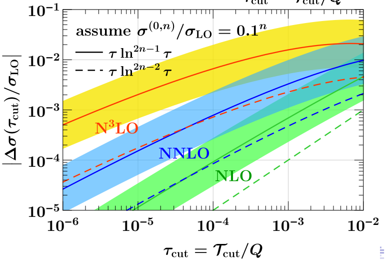
Power Correction (Linear)



Power Correction (Log)



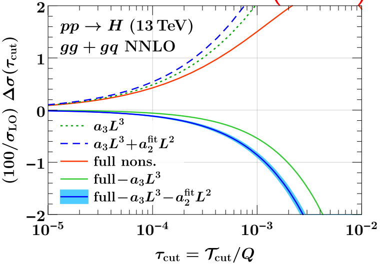
- Agrees well with scaling estimate.



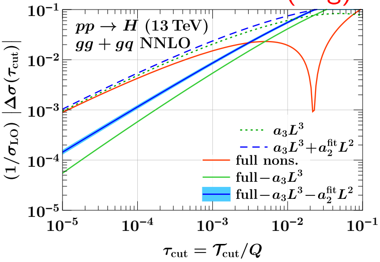
NNLO Beam Thrust at NLP: Higgs

- Similar behavior observed for the Higgs case.

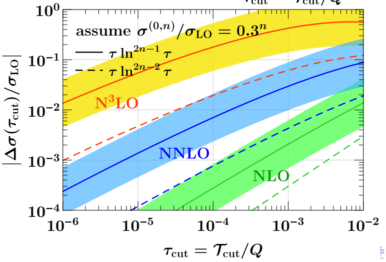
Power Correction (Linear)



Power Correction (Log)



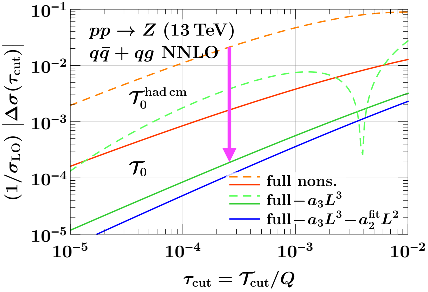
- Agrees well with scaling estimate.



Summary of Improvement

- Analytic leading power correction + improved observable definition $\implies \sim 100$ improvement.
- Power corrections under good analytic control.

NNLO Power Correction



- N-jettiness subtractions promising as an NNLO subtraction scheme.

Future Directions

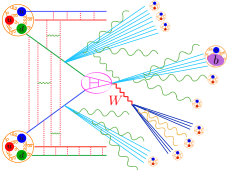
- This was a specific case, namely Drell-Yan/ Higgs production.
- Many questions/ directions remain:
  - Extension to final state jets:
    - Relation $\frac{d\sigma^{(2,2)}}{d\tau} = c_{hc,3} \ln^3 \tau + \cdots$ hints at simple structure.
  - Calculation of further subleading logs.
    - Conceptually straightforward in our framework.
  - Extension to N^{3}LO?
    - $\alpha_s^3 \log^5 \tau$ power corrections known using above. Need leading power 3-loop soft/beam functions (3-loop soft function known for p_T).
      [Li, Zhu]

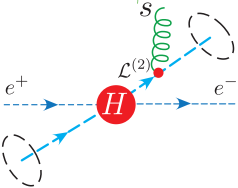
Conclusions

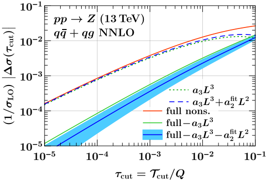
- *N*-Jettiness subtractions provides a general approach to NNLO calculations involving jets.

- Factorization and EFTs can be used to compute subleading power corrections for perturbative event shapes.

- Power corrections for *N*-jettiness subtractions are under analytic control.







Thanks!