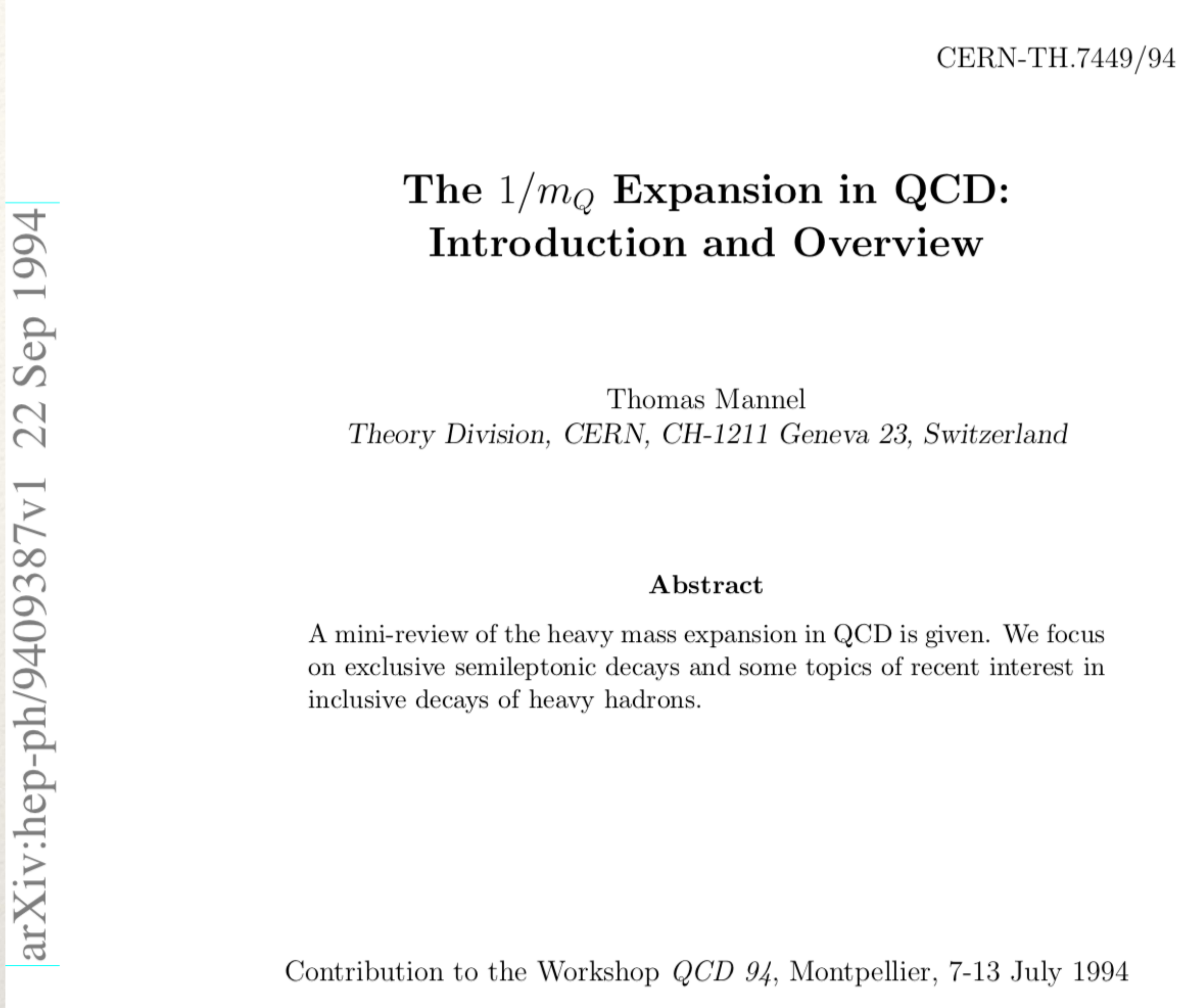


arXiv:hep-ph/9409387v1 22 Sep 1994

CERN-TH.7449/94

The $1/m_Q$ Expansion in QCD:
Introduction and Overview

Thomas Mannel
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

A mini-review of the heavy mass expansion in QCD is given. We focus on exclusive semileptonic decays and some topics of recent interest in inclusive decays of heavy hadrons.

Contribution to the Workshop *QCD 94*, Montpellier, 7-13 July 1994

Heavy Quarks through the looking glass *Siegen* *4./5.10.2018*

The 1/mQ Expansion in QCD

Alexander Lenz
IPPP, Durham University

Outline

Florian Bernlochner - Beauty is Back BELLE 2

Johannes Albrecht - LHCb

Outline

1 Introduction
2 HQET
  2.1 The Heavy Quark Limit and additional Symmetries
  2.2 **Strategy of a HQET Calculation**
  2.3 An Application: B -> D(*) l nu
3 The Heavy-Mass Limit for Inclusive Decays
  3.1 Operator Product Expansion
  3.2 Inclusive Semileptonic Decays
  3.3 The Endpoint Region
  3.4 **Inclusive Non-leptonic** and Rare Decays
4 Concluding Remarks

Gil Paz - Effective Theories to all orders

Christian Bauer - SCET

Susanne Westhoff - Tau polarimetry

Martin Jung - NP in sl b->c

Robert Harlander- OPE

Frank Tackmann- SIMBA

Paolo Gambino - Endless chase for Vcb

Mikolai Misiak - b -> s gamma

Svjetlana Fajfer - Flavour Anomalies

Andrzej Buras -eps’

Javier Virto - ?

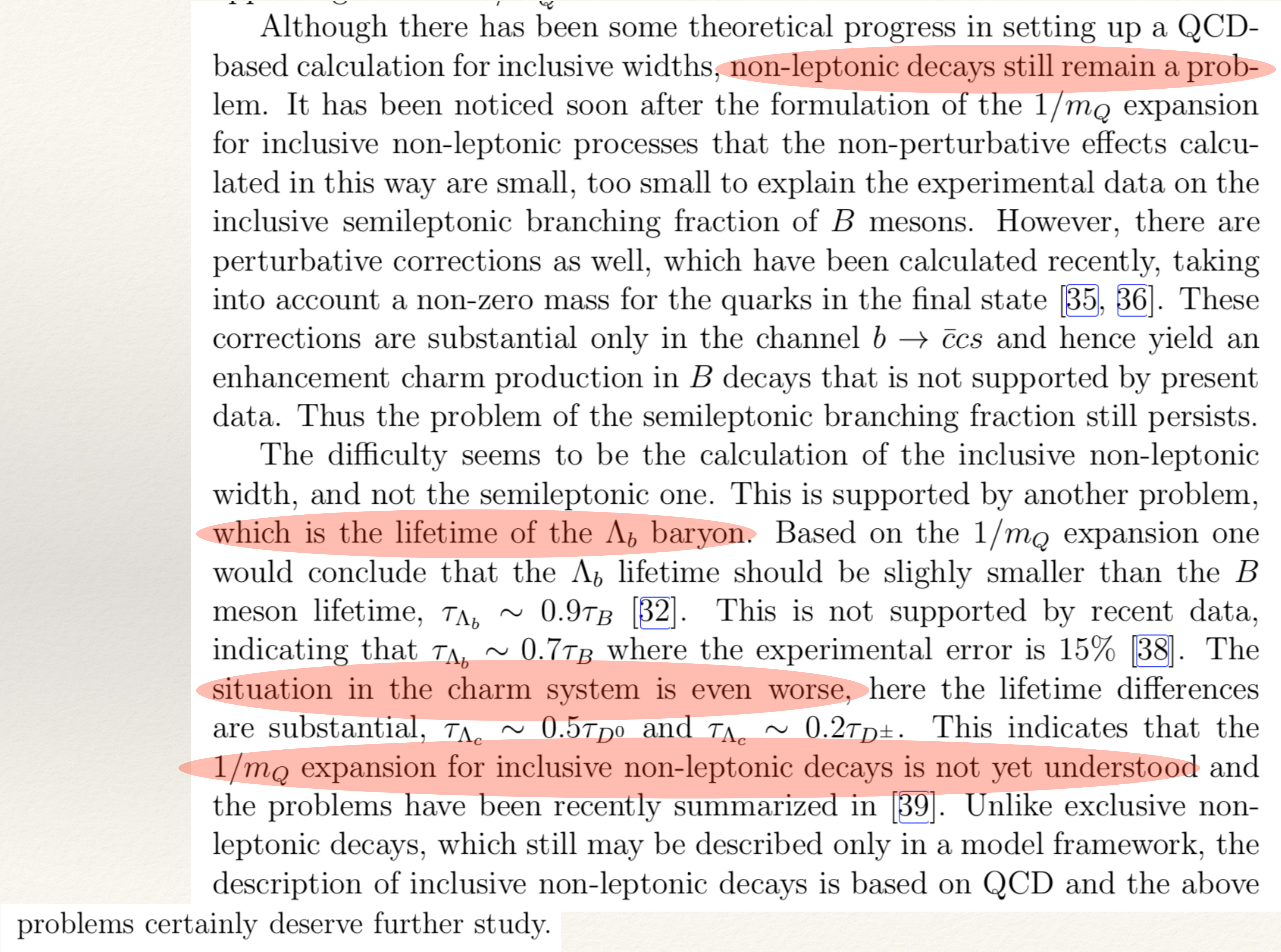
Bastian Kubis - Pion TFF & g-2

Uli Nierste - CPV in charm

Alexey Petrov - LFV in tau decays

Claus Grupen - Flavour gives variety to life

Non-leptonic Decays

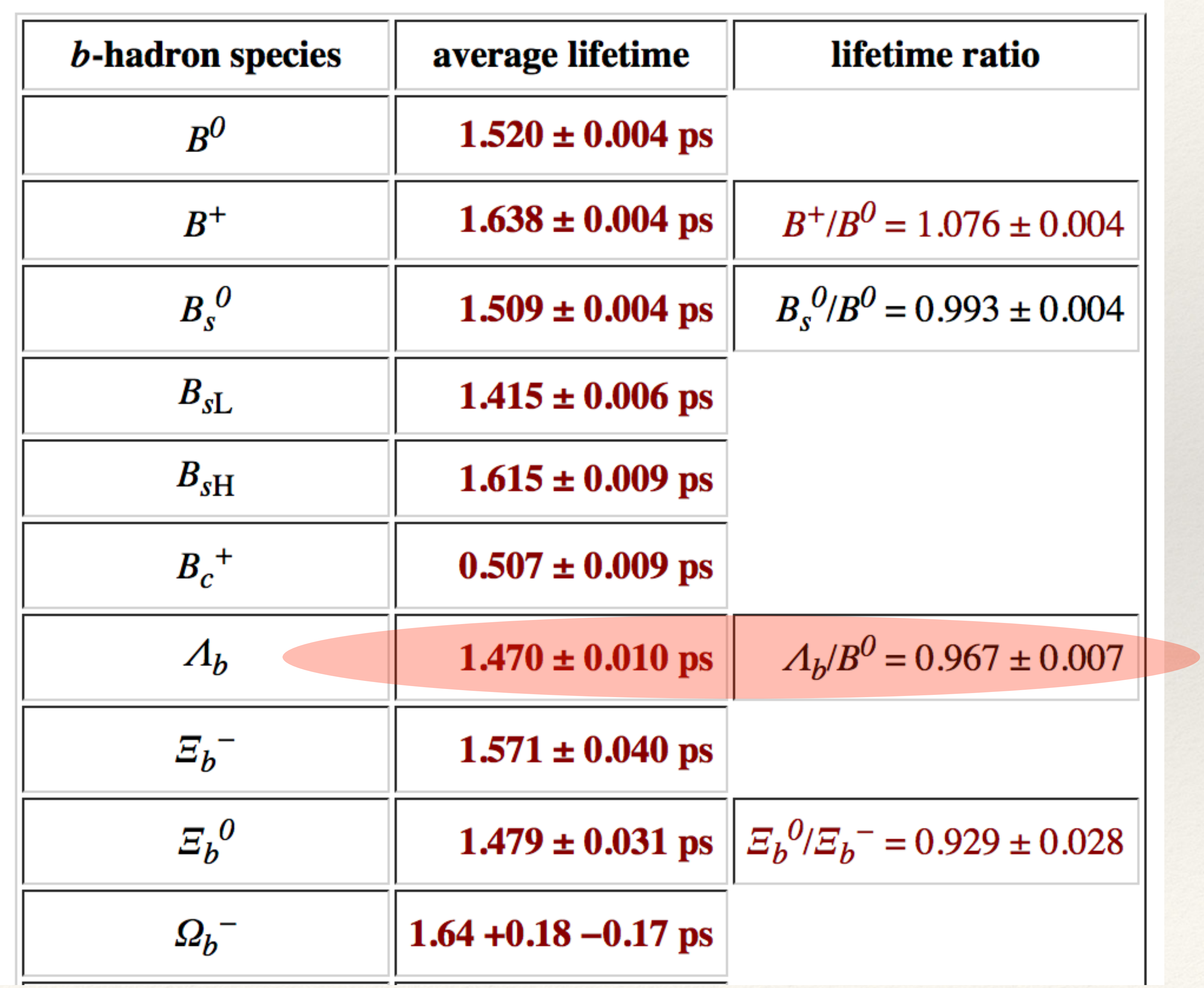


Although there has been some theoretical progress in setting up a QCD-based calculation for inclusive widths, non-leptonic decays still remain a problem. It has been noticed soon after the formulation of the $1/m_Q$ expansion for inclusive non-leptonic processes that the non-perturbative effects calculated in this way are small, too small to explain the experimental data on the inclusive semileptonic branching fraction of B mesons. However, there are perturbative corrections as well, which have been calculated recently, taking into account a non-zero mass for the quarks in the final state [35, 36]. These corrections are substantial only in the channel $b \to \bar{c}cs$ and hence yield an enhancement charm production in B decays that is not supported by present data. Thus the problem of the semileptonic branching fraction still persists.

The difficulty seems to be the calculation of the inclusive non-leptonic width, and not the semileptonic one. This is supported by another problem, which is the lifetime of the Λ_b baryon. Based on the $1/m_Q$ expansion one would conclude that the Λ_b lifetime should be slighly smaller than the B meson lifetime, $\tau_{\Lambda_b} \sim 0.9\tau_B$ [32]. This is not supported by recent data, indicating that $\tau_{\Lambda_b} \sim 0.7\tau_B$ where the experimental error is 15% [38]. The situation in the charm system is even worse, here the lifetime differences are substantial, $\tau_{\Lambda_c} \sim 0.5\tau_{D^0}$ and $\tau_{\Lambda_c} \sim 0.2\tau_{D^\pm}$. This indicates that the $1/m_Q$ expansion for inclusive non-leptonic decays is not yet understood and the problems have been recently summarized in [39]. Unlike exclusive non-leptonic decays, which still may be described only in a model framework, the description of inclusive non-leptonic decays is based on QCD and the above problems certainly deserve further study.

24 years later

Finally our experimental friends were working harder :-)



b-hadron species	average lifetime	lifetime ratio
B^0	1.520 ± 0.004 ps	
B^+	1.638 ± 0.004 ps	$B^+/B^0 = 1.076 \pm 0.004$
B_s^0	1.509 ± 0.004 ps	$B_s^0/B^0 = 0.993 \pm 0.004$
$B_{s\mathrm{L}}$	1.415 ± 0.006 ps	
$B_{s\mathrm{H}}$	1.615 ± 0.009 ps	
B_c^+	0.507 ± 0.009 ps	
Λ_b	1.470 ± 0.010 ps	$\Lambda_b/B^0 = 0.967 \pm 0.007$
Ξ_b^-	1.571 ± 0.040 ps	
Ξ_b^0	1.479 ± 0.031 ps	$\Xi_b^0/\Xi_b^- = 0.929 \pm 0.028$
Ω_b^-	1.64 +0.18 −0.17 ps	

HFLAV 2018

24 years later

Did we also work harder ?

HQE:

$$\frac{1}{\tau} = \Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots .$$

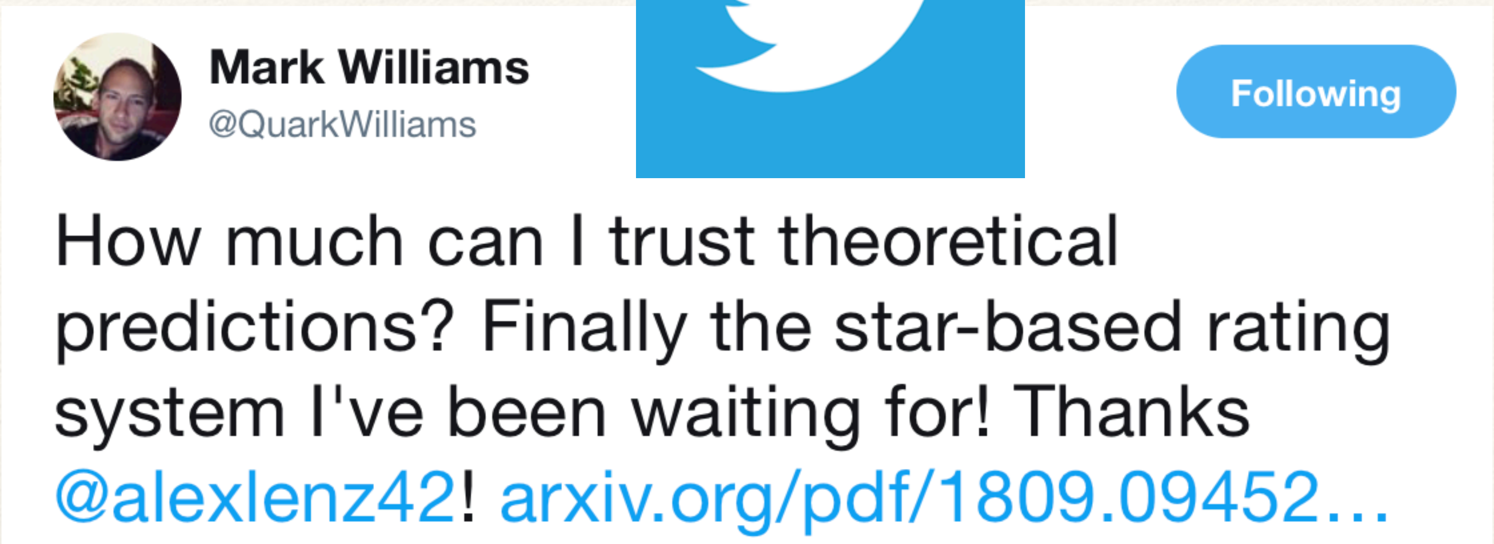
Each term can be split up into a perturbative Wilson coefficient and a non-perturbative matrix element

$$\Gamma_i = \left[\Gamma_i^{(0)} + \frac{\alpha_S}{4\pi}\Gamma_i^{(1)} + \frac{\alpha_S^2}{(4\pi)^2}\Gamma_i^{(2)} + \ldots ,\right] \langle O^{d=i+3} \rangle$$

For mixing a similar expansion holds - starting at the third order

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots .$$

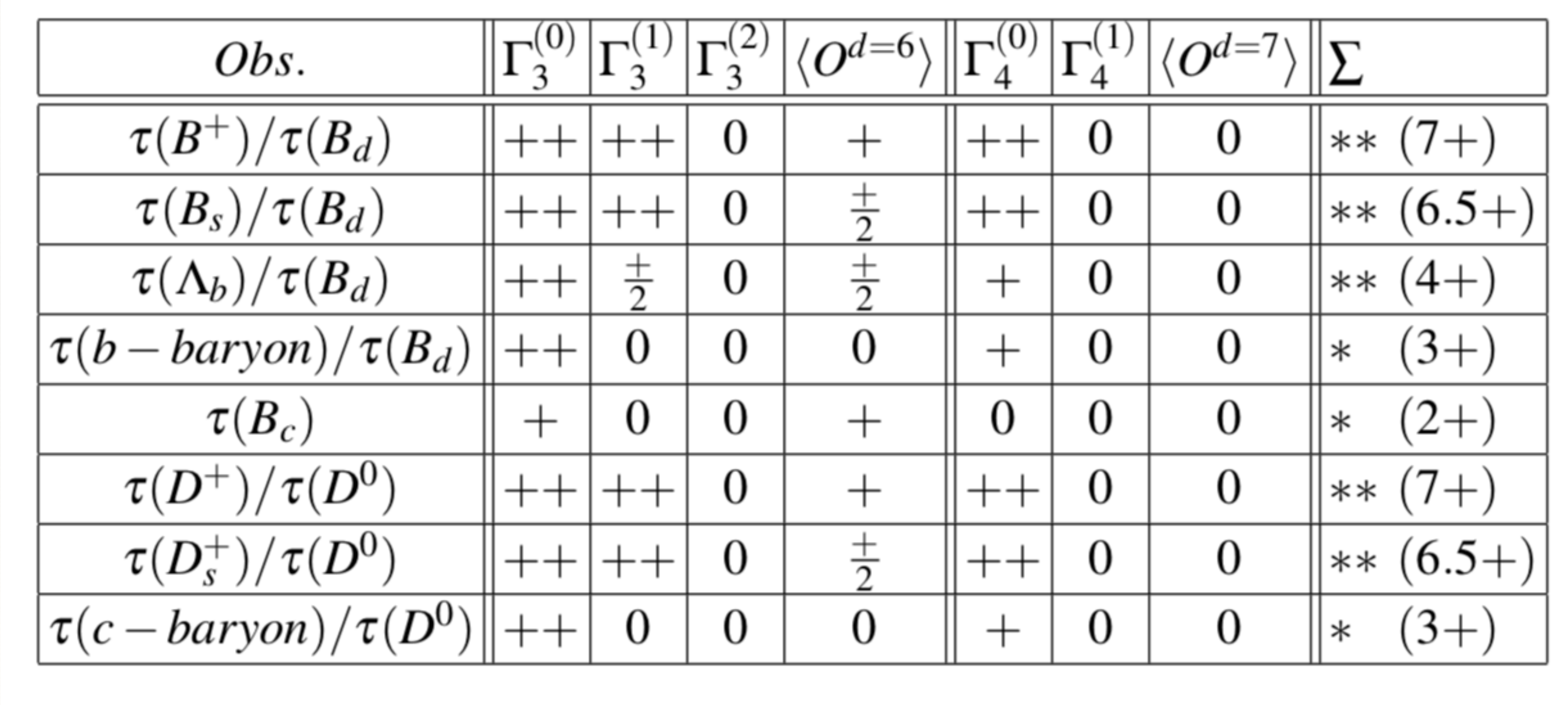
24 years later



Mark Williams
@QuarkWilliams

How much can I trust theoretical predictions? Finally the star-based rating system I've been waiting for! Thanks @alexlenz42! arxiv.org/pdf/1809.09452…

A + for each independent calculation
At most ++
At most +++ for <>: 2 lattice, 1 sum rule
Punishment: A - - for no <Q6>
A 0 for quark model et al for <Q6>

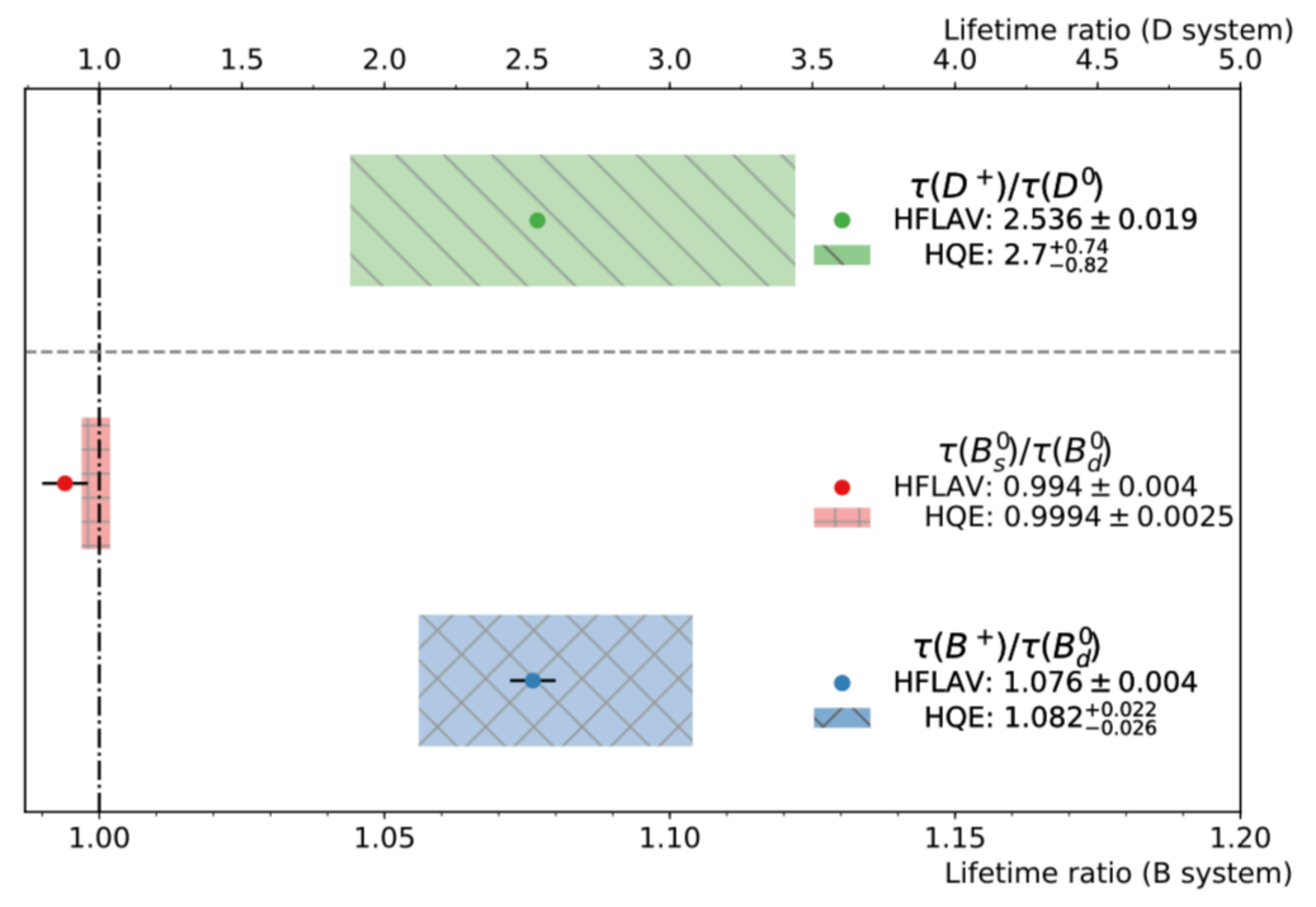


Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6}\rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7}\rangle$	Σ
$\tau(B^+)/\tau(B_d)$	++	++	0	+	++	0	0	** (7+)
$\tau(B_s)/\tau(B_d)$	++	++	0	$\frac{+}{2}$	++	0	0	** (6.5+)
$\tau(\Lambda_b)/\tau(B_d)$	++	$\frac{+}{2}$	0	$\frac{+}{2}$	+	0	0	** (4+)
$\tau(b-baryon)/\tau(B_d)$	++	0	0	0	+	0	0	* (3+)
$\tau(B_c)$	+	0	0	+	0	0	0	* (2+)
$\tau(D^+)/\tau(D^0)$	++	++	0	+	++	0	0	** (7+)
$\tau(D_s^+)/\tau(D^0)$	++	++	0	$\frac{+}{2}$	++	0	0	** (6.5+)
$\tau(c-baryon)/\tau(D^0)$	++	0	0	0	+	0	0	* (3+)

****: 12-15 *** 8 -11.5 **: 4-7.5 *: 2-3.5

24 years later

- Lifetimes agree perfectly with experiment
- Even a convergence in the charm system seems to be feasible
- Lattice confirmation of matrix elements would be very desirable
- Precision of HQET sum rule can be improved



$$\frac{\tau(D^+)}{\tau(D^0)} = 2.7 = 1 + 16\pi^2 \, (0.25)^3 \, (1 - 0.34)$$

Kirk, AL, Rauh 1711.02100
pert. NLO-QCD:
AL, Rauh 1305.3588

Expansion parameter for HQE in charm = 0.3
not a back of envelope statement, but real calculations

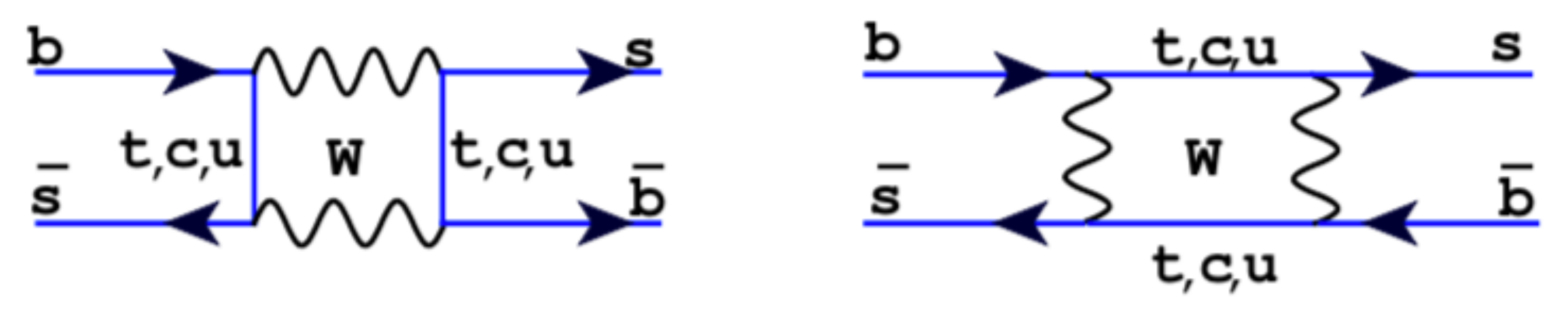
d=6 calculated with sum rules
lattice confirmation urgently needed

d=7 estimated in vacuum insertion approximation
do sum rule/lattice

Rauh, Kirk, AL
JHEP 2017

24 years later

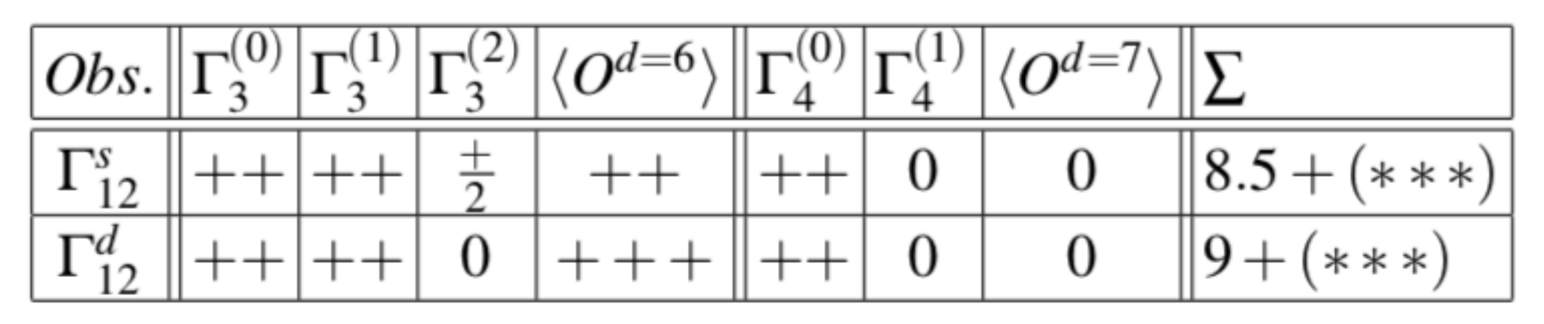
The box diagrams



Physical observables

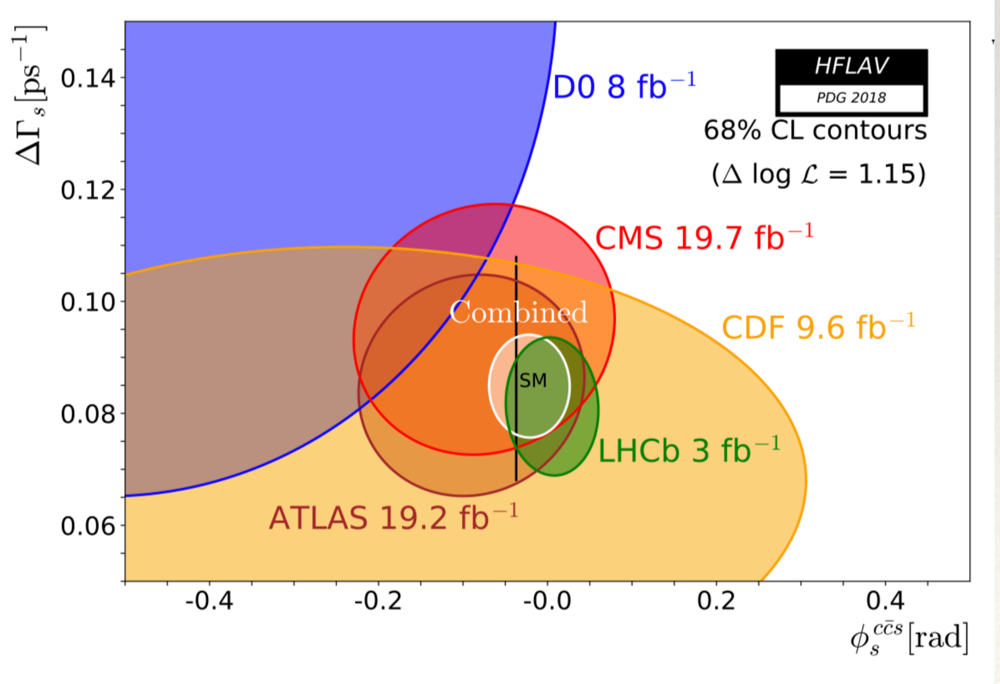
$$\Delta M_q \approx 2\left|M_{12}^q\right|, \qquad \Delta\Gamma_q \approx 2\left|\Gamma_{12}^q\right|\cos\phi_{12}^q, \qquad a_{sl}^q \approx \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right|\sin\phi_{12}^q .$$

Status quo for Gamma_12



Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6}\rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7}\rangle$	Σ
Γ_{12}^s	$++$	$++$	$\frac{+}{2}$	$++$	$++$	0	0	$8.5+(***)$
Γ_{12}^d	$++$	$++$	0	$+++$	$++$	0	0	$9+(**)$

$$\Delta\Gamma_s = (0.098 \pm 0.014)\mathrm{ps}^{-1}, \quad a_{sl}^s = (2.27 \pm 0.25)\cdot 10^{-5},$$
$$\Delta\Gamma_d = (2.99 \pm 0.52)\cdot 10^{-3}\mathrm{ps}^{-1}, \quad a_{sl}^d = -(4.90 \pm 0.54)\cdot 10^{-4}.$$



24 years later

The mass difference of neutral B mesons

$$\Delta M_s^{\mathrm{Exp}} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$$

HFLAV 2018

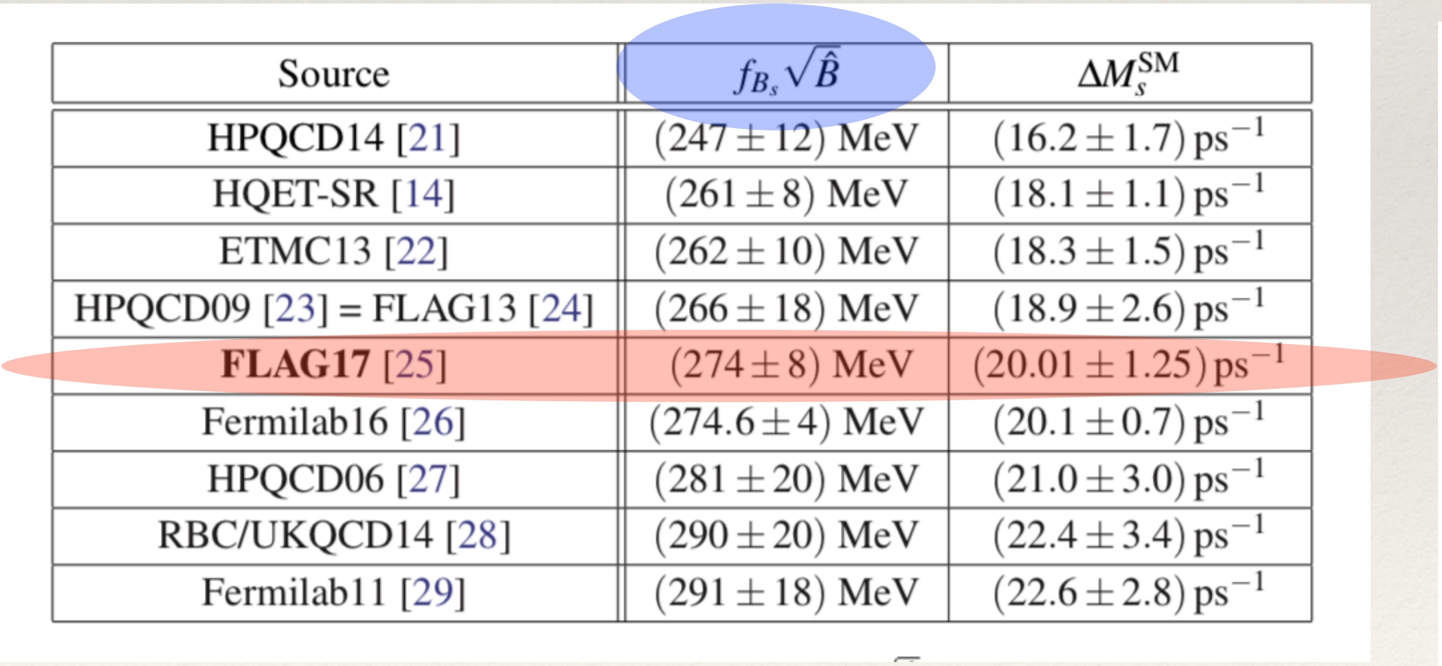
$$M_{12}^q = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B f_{B_q}^2 M_{B_q} \hat{\eta}_B \,,$$

$$Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta$$

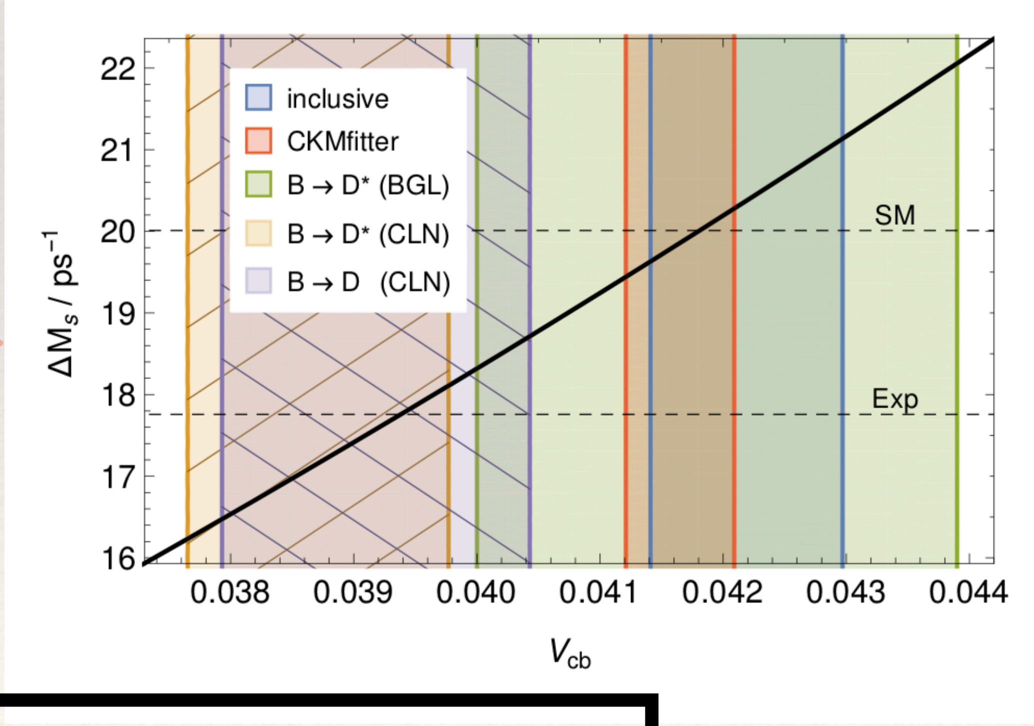
$$\langle Q \rangle \equiv \langle B_s^0 | Q | \bar{B}_s^0 \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B(\mu)$$

by far dominant uncertainty

significant CKM dependence



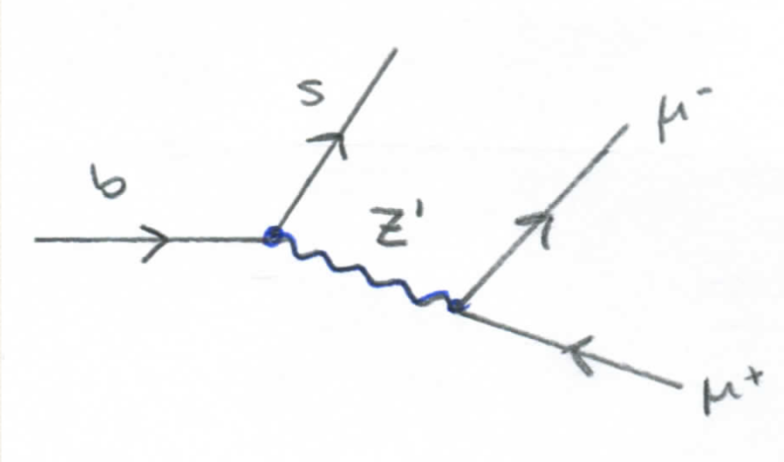
Source	$f_{B_s}\sqrt{\hat{B}}$	ΔM_s^{SM}
HPQCD14 [21]	(247 ± 12) MeV	$(16.2 \pm 1.7)\,\mathrm{ps}^{-1}$
HQET-SR [14]	(261 ± 8) MeV	$(18.1 \pm 1.1)\,\mathrm{ps}^{-1}$
ETMC13 [22]	(262 ± 10) MeV	$(18.3 \pm 1.5)\,\mathrm{ps}^{-1}$
HPQCD09 [23] = FLAG13 [24]	(266 ± 18) MeV	$(18.9 \pm 2.6)\,\mathrm{ps}^{-1}$
FLAG17 [25]	(274 ± 8) MeV	$(20.01 \pm 1.25)\,\mathrm{ps}^{-1}$
Fermilab16 [26]	(274.6 ± 4) MeV	$(20.1 \pm 0.7)\,\mathrm{ps}^{-1}$
HPQCD06 [27]	(281 ± 20) MeV	$(21.0 \pm 3.0)\,\mathrm{ps}^{-1}$
RBC/UKQCD14 [28]	(290 ± 20) MeV	$(22.4 \pm 3.4)\,\mathrm{ps}^{-1}$
Fermilab11 [29]	(291 ± 18) MeV	$(22.6 \pm 2.8)\,\mathrm{ps}^{-1}$



Taking the most recent FLAG average gives 2 sigma discrepancy

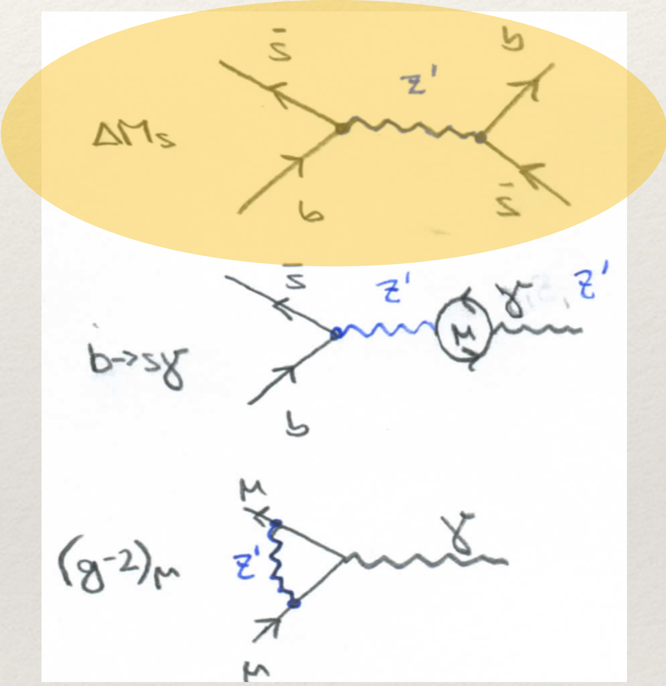
24 years later

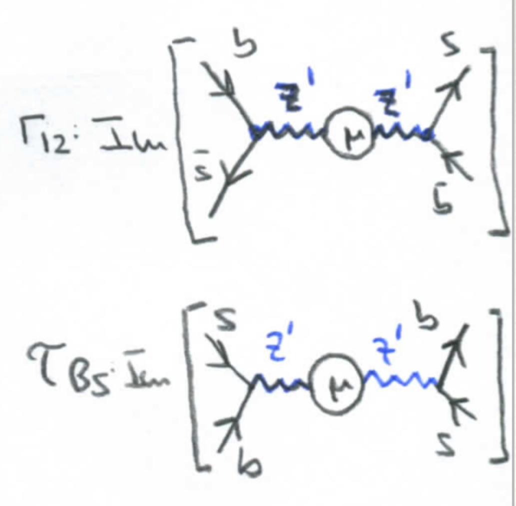
A popular BSM model for solving the anomalies related to loop-level (semi) leptonic decays are Z' models:



Such a new tree-level transition will also affect many other observables, most notably B-mixing at tree-level, but also many loop processes.

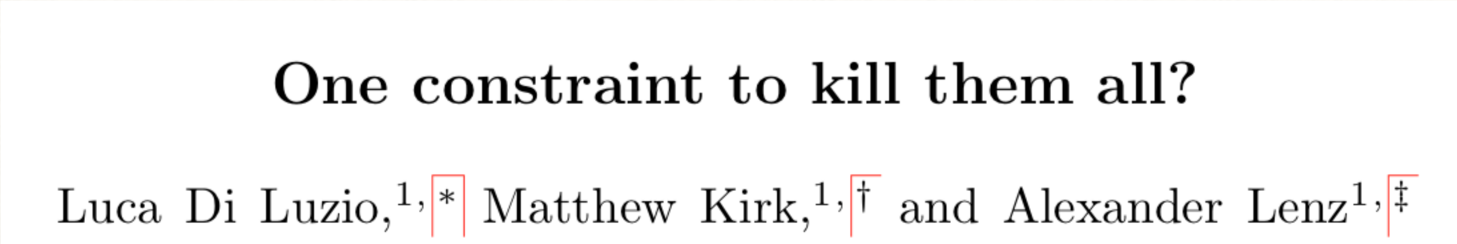
Make sure all relevant bounds are included, e.g. electro-weak precision bounds





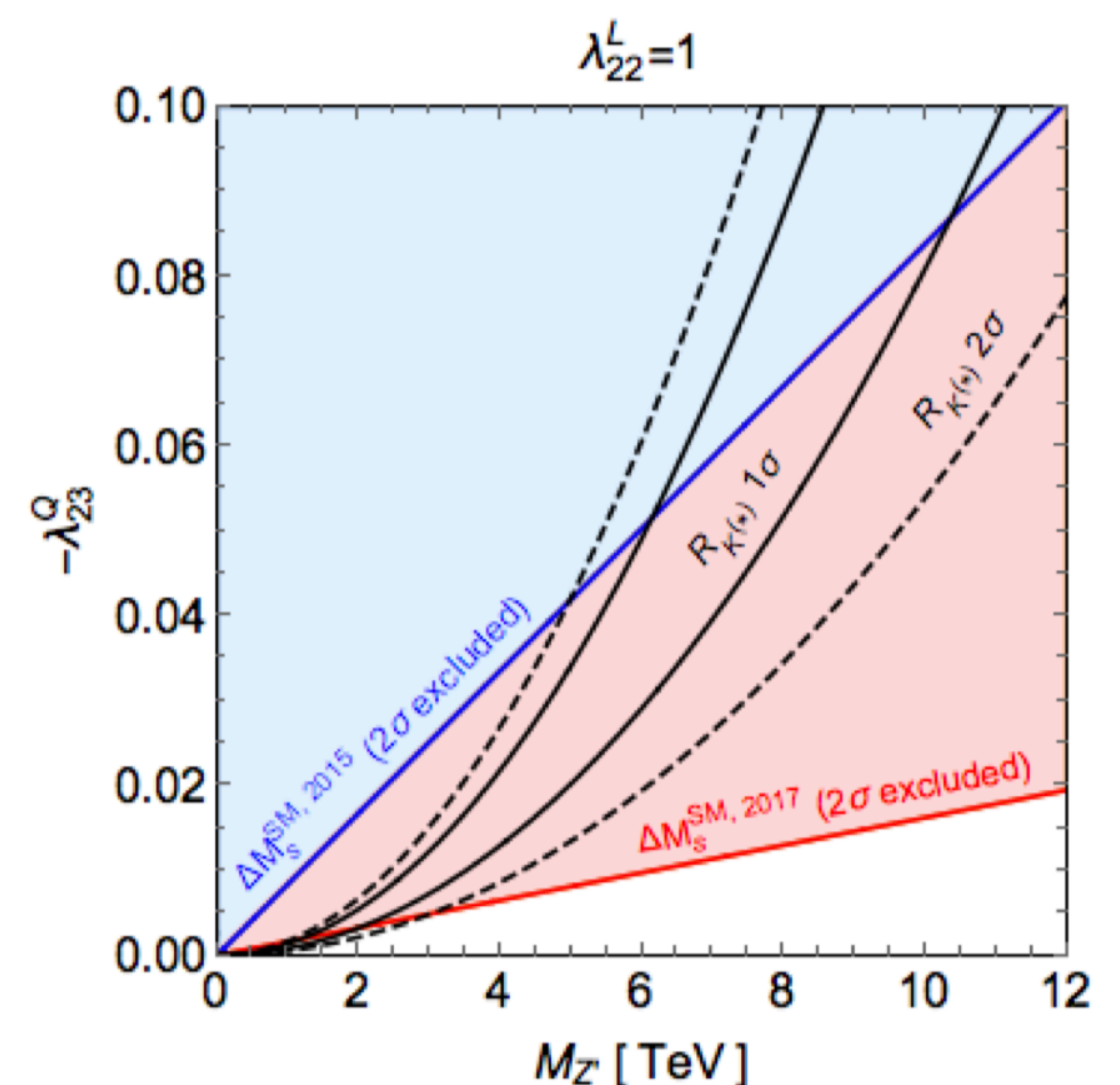
24 years later

If the new lattice values are correct
and if Vcb is close to the inclusive value
=> severe consequences for BSM physics



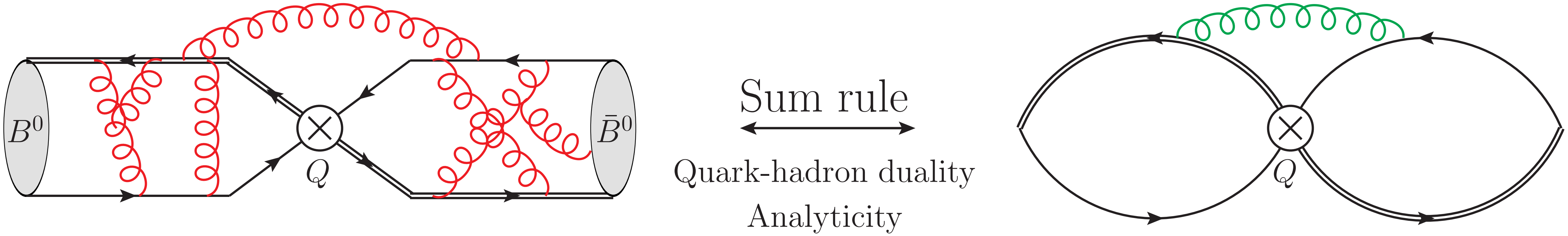
One constraint to kill them all?

Luca Di Luzio,[1,*] Matthew Kirk,[1,†] and Alexander Lenz[1,‡]



24 years later

Independent cross-check of FNAL/MILC highly desirable



Sum rule

↔

Quark-hadron duality

Analyticity

Hadronic matrix element

Characteristic scale: Λ_{QCD}

$\alpha_s\left(\Lambda_{\text{QCD}}\right) \sim \mathcal{O}(1)$

$\Rightarrow$ non-perturbative

Correlation function

Characteristic scale: 'virtuality' ω

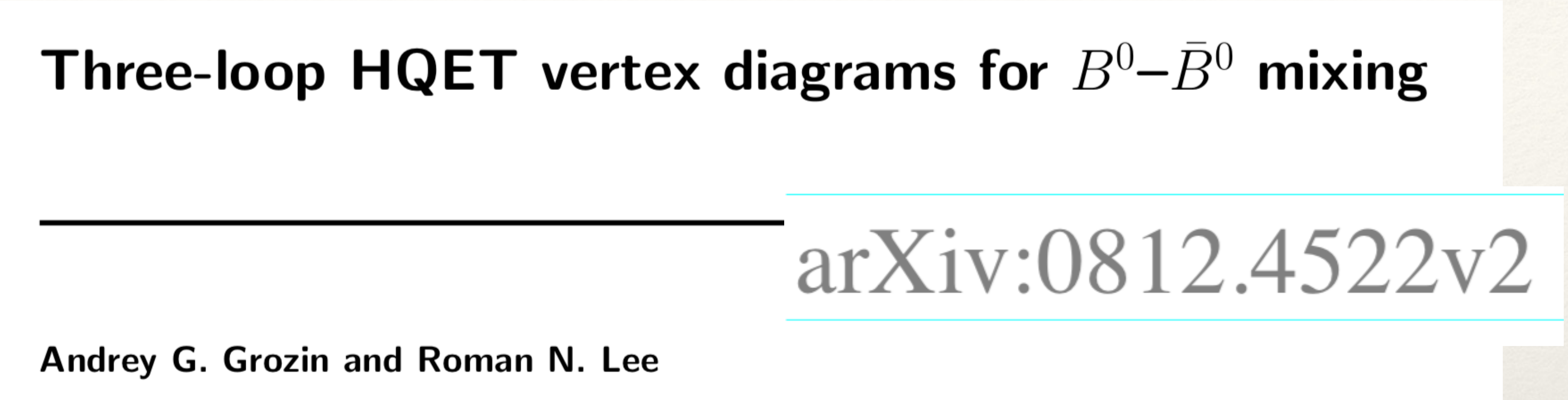
Choose ω s.t. $\alpha_s(\omega) \ll 1$

$\Rightarrow$ perturbatively calculable

1. **HQET sum rule at hadronic scale**
2. **HQET running to scale mb**
3. **HQET-QCD matching**

24 years later

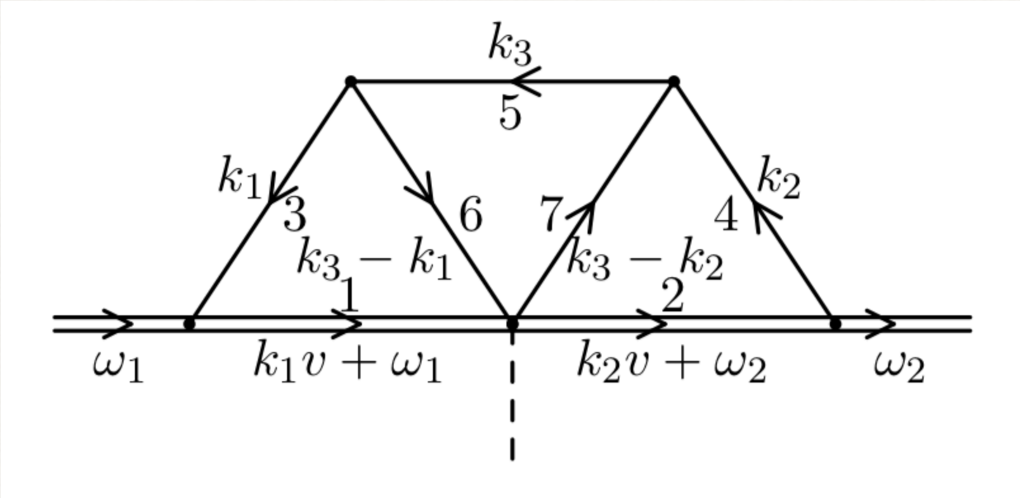
Independent cross-check of FNAL/MILC highly desirable



Three-loop HQET vertex diagrams for B^0–$\bar{B}^0$ mixing

arXiv:0812.4522v2

Andrey G. Grozin and Roman N. Lee



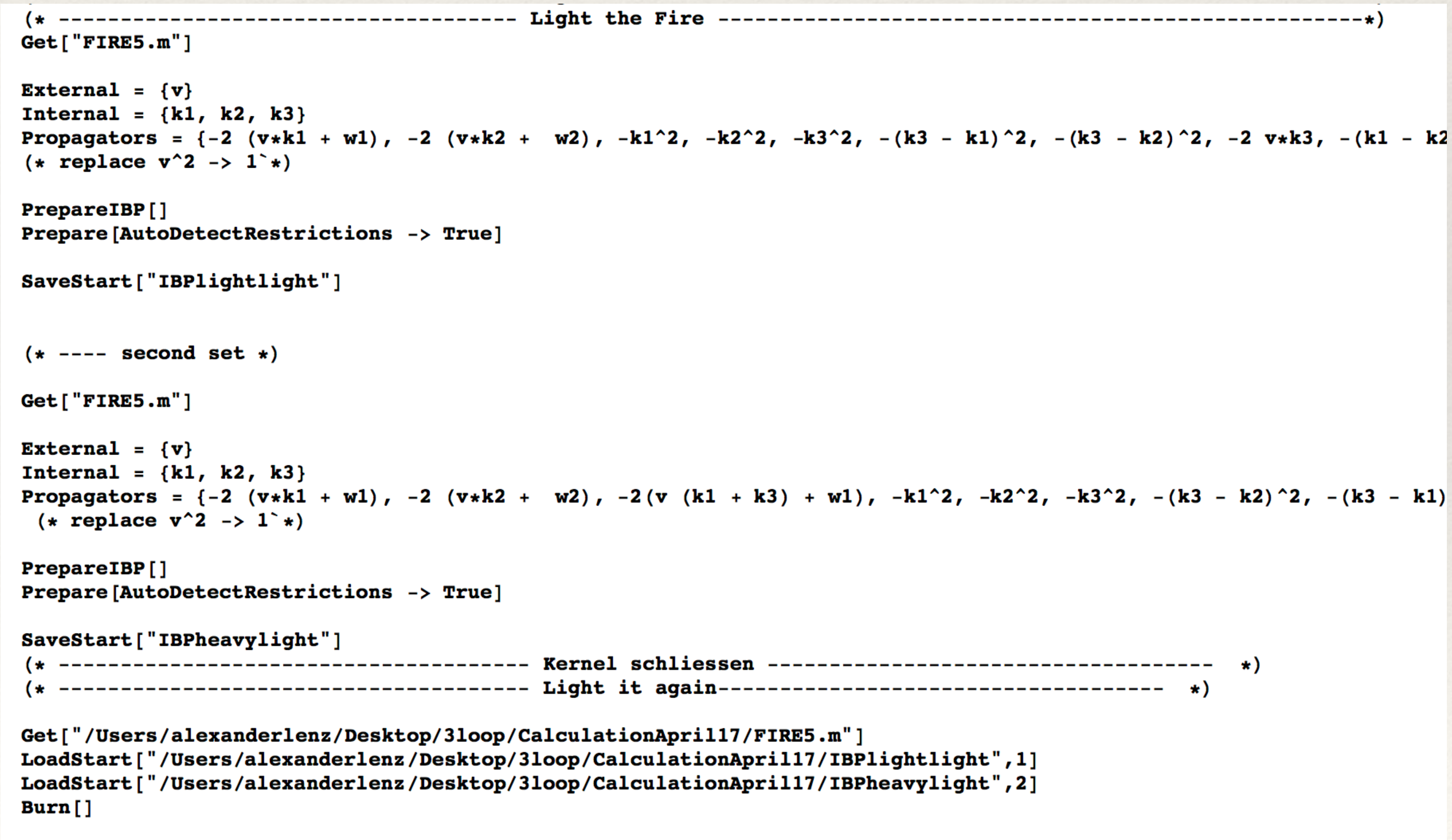
Do integration by parts with FIRE

arXiv.org > hep-ph > arXiv:1408.2372

High Energy Physics – Phenomenology

FIRE5: a C++ implementation of Feynman Integral REduction

Alexander V. Smirnov



```
(* ------------------------------------- Light the Fire ---------------------------------------------------*)
Get["FIRE5.m"]

External = {v}
Internal = {k1, k2, k3}
Propagators = {-2 (v*k1 + w1), -2 (v*k2 +  w2), -k1^2, -k2^2, -k3^2, -(k3 - k1)^2, -(k3 - k2)^2, -2 v*k3, -(k1 - k2
(* replace v^2 -> 1`*)

PrepareIBP[]
Prepare[AutoDetectRestrictions -> True]

SaveStart["IBPlightlight"]


(* ---- second set *)

Get["FIRE5.m"]

External = {v}
Internal = {k1, k2, k3}
Propagators = {-2 (v*k1 + w1), -2 (v*k2 +  w2), -2(v (k1 + k3) + w1), -k1^2, -k2^2, -k3^2, -(k3 - k2)^2, -(k3 - k1)
 (* replace v^2 -> 1`*)

PrepareIBP[]
Prepare[AutoDetectRestrictions -> True]

SaveStart["IBPheavylight"]
(* ------------------------------------- Kernel schliessen -----------------------------------   *)
(* ------------------------------------- Light it again----------------------------------   *)

Get["/Users/alexanderlenz/Desktop/3loop/CalculationApril17/FIRE5.m"]
LoadStart["/Users/alexanderlenz/Desktop/3loop/CalculationApril17/IBPlightlight",1]
LoadStart["/Users/alexanderlenz/Desktop/3loop/CalculationApril17/IBPheavylight",2]
Burn[]
```

24 years later

Complete SR calculation for the SM B-mixing operator (V-A)x(V-A) for Bd mesons

PHYSICAL REVIEW D **94,** 034024 (2016)

B^0-$\bar{B}^0$ mixing at next-to-leading order

Andrey G. Grozin

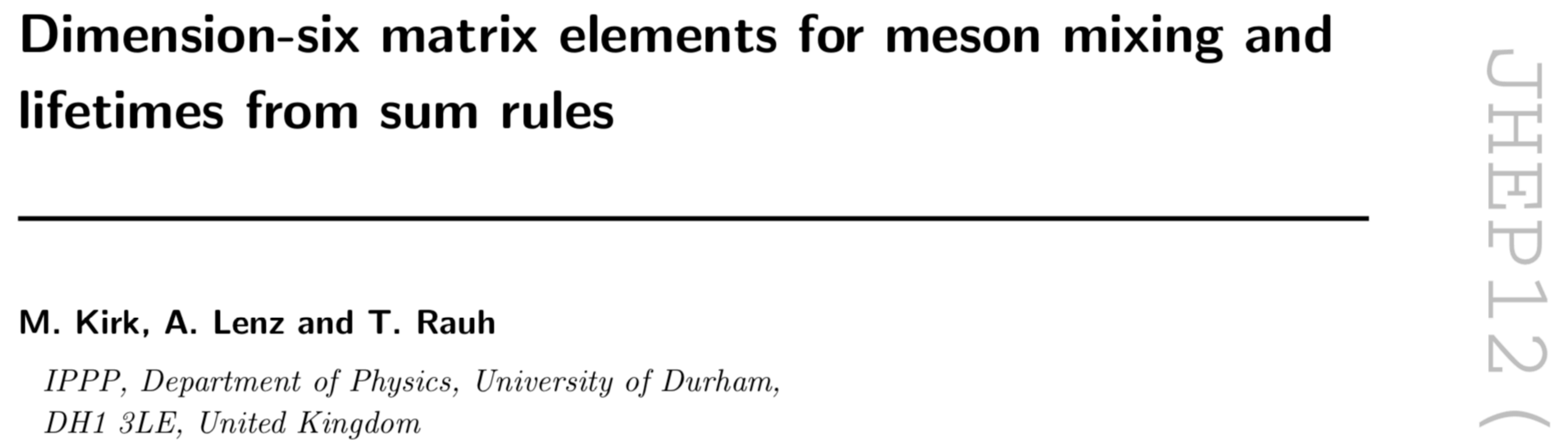
Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia
and Novosibirsk State University, Novosibirsk 630090, Russia

Rebecca Klein, Thomas Mannel, and Alexei A. Pivovarov

Theoretische Elementarteilchenphysik, Naturwiss.- techn. Fakultät, Universität Siegen,
57068 Siegen, Germany

(Received 4 July 2016; published 11 August 2016)

Complete SR calculation for all 5 B-mixing operators and all lifetime operators



Dimension-six matrix elements for meson mixing and lifetimes from sum rules

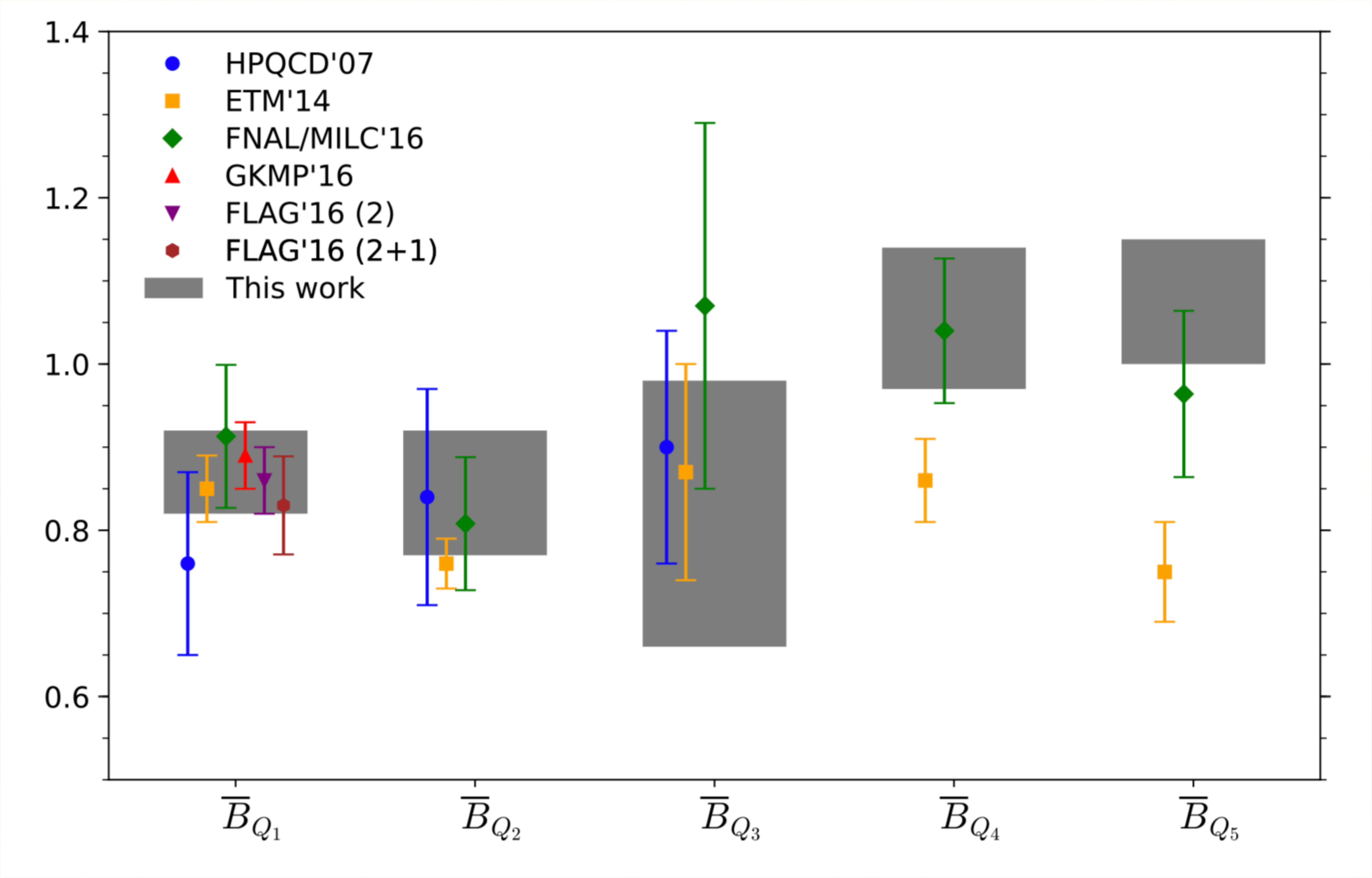
---

M. Kirk, A. Lenz and T. Rauh

IPPP, Department of Physics, University of Durham,
DH1 3LE, United Kingdom

24 years later

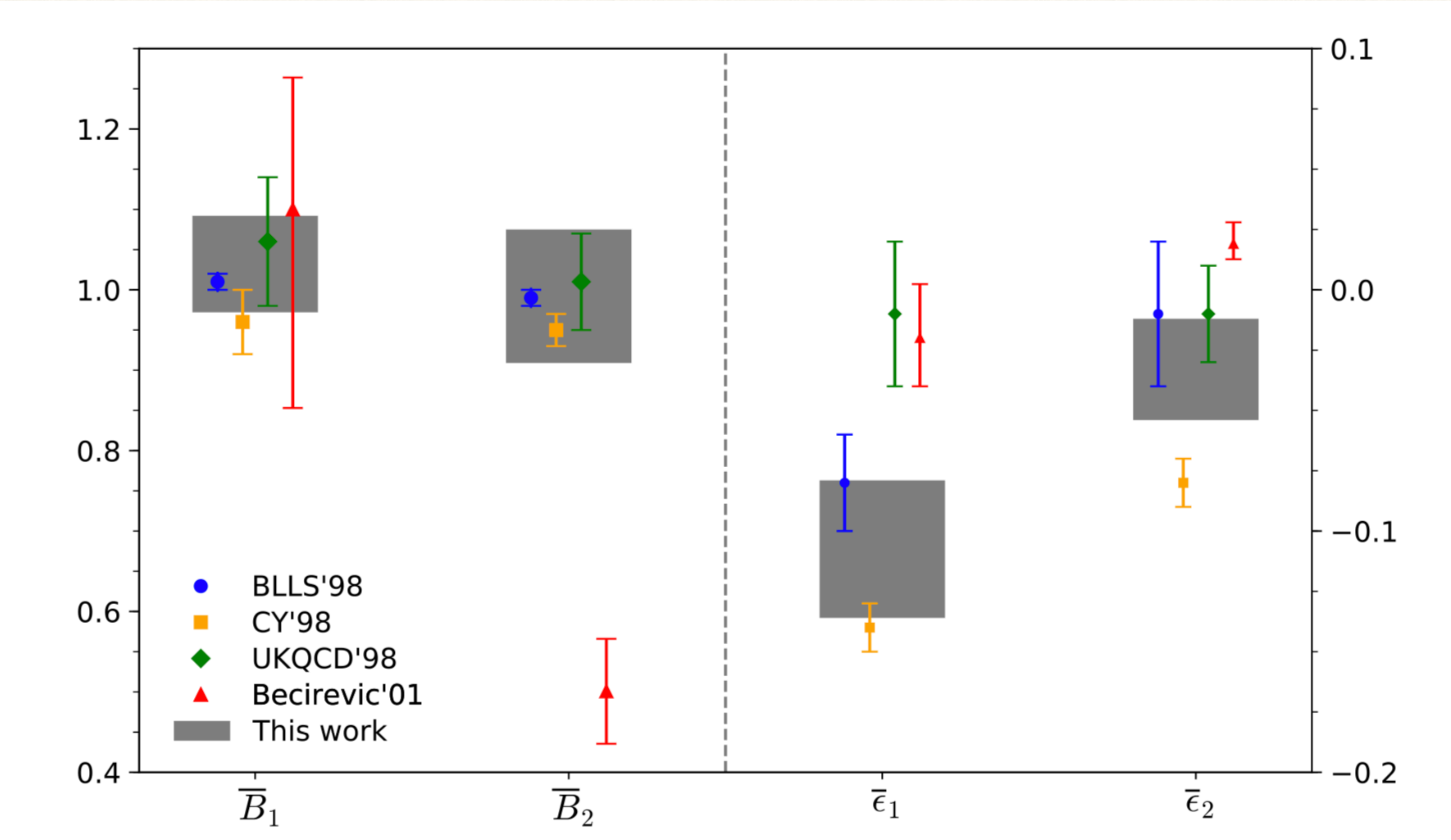
Good agreement for SM B-mixing operator (V-A)x(V-A) for Bd mesons



- SR are competitive since B-1 can be calculated
- Dominant uncertainty stems from matching!

24 years later

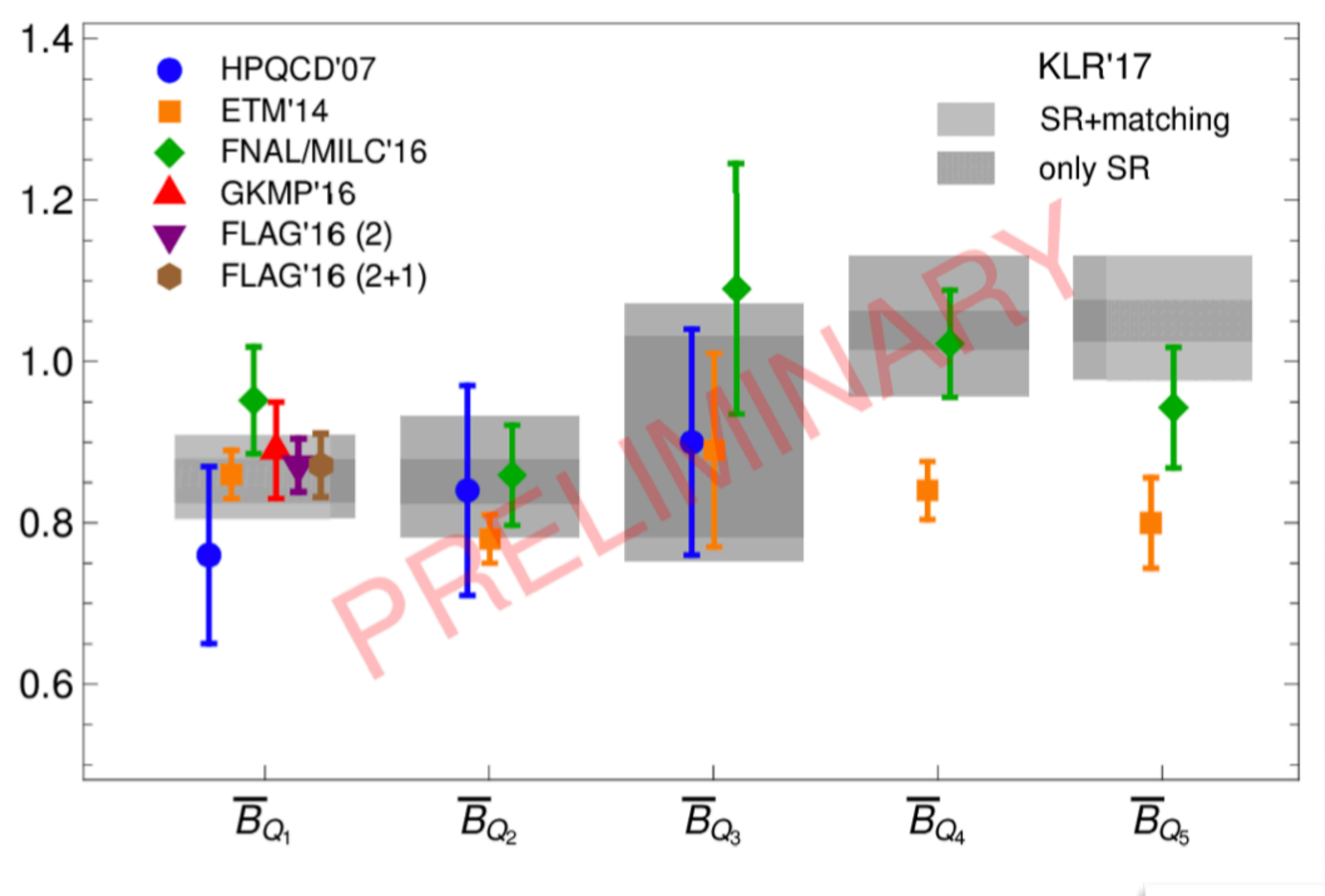
Only reliable non-perturbative determination of matrix lifetimes



Should be cross-checked by a lattice evaluation

24 years later

Preliminary determination of SM B-mixing operator (V-A)x(V-A) for Bs mesons



- **Slightly smaller then FNAL/MILC**
- **Mass difference close to experimental number**
- **Independent lattice evaluation desirable**

Rauh, King, AL
In progress

24 years later

To reduce SR uncertainty (by a factor of 2?) matching has to be done at NNLO

PHYSICAL REVIEW D **96,** 074032 (2017)

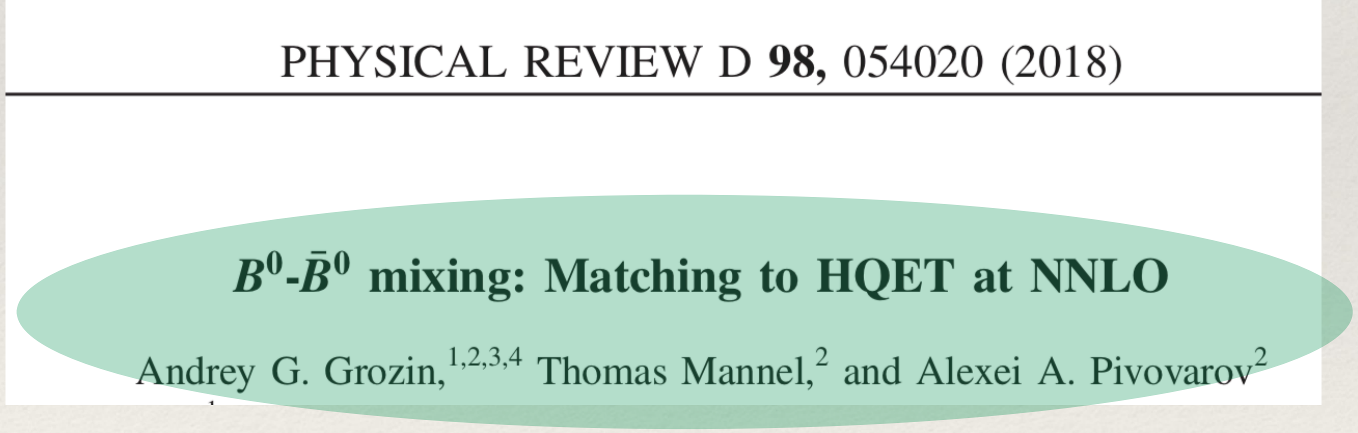
Towards a next-to-next-to-leading order analysis of matching in $B^0-\bar{B}^0$ mixing

Andrey G. Grozin

Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia and Novosibirsk State University, Novosibirsk 630090, Russia

Thomas Mannel and Alexei A. Pivovarov

Theoretische Elementarteilchenphysik, Naturwiss.- techn. Fakultät, Universität Siegen, 57068 Siegen, Germany



PHYSICAL REVIEW D **98,** 054020 (2018)

B^0-$\bar{B}^0$ mixing: Matching to HQET at NNLO

Andrey G. Grozin,[1,2,3,4] Thomas Mannel,[2] and Alexei A. Pivovarov[2]

$$\begin{aligned} C_l(m_b) &= 1 + \frac{\alpha_s(m_b)}{4\pi} C_l^{(1)} + \left(\frac{\alpha_s(m_b)}{4\pi}\right)^2 C_l^{(2)} \\ C_p(m_b) &= \frac{\alpha_s(m_b)}{4\pi} C_p^{(1)} + \left(\frac{\alpha_s(m_b)}{4\pi}\right)^2 C_p^{(2)} . \end{aligned}$$

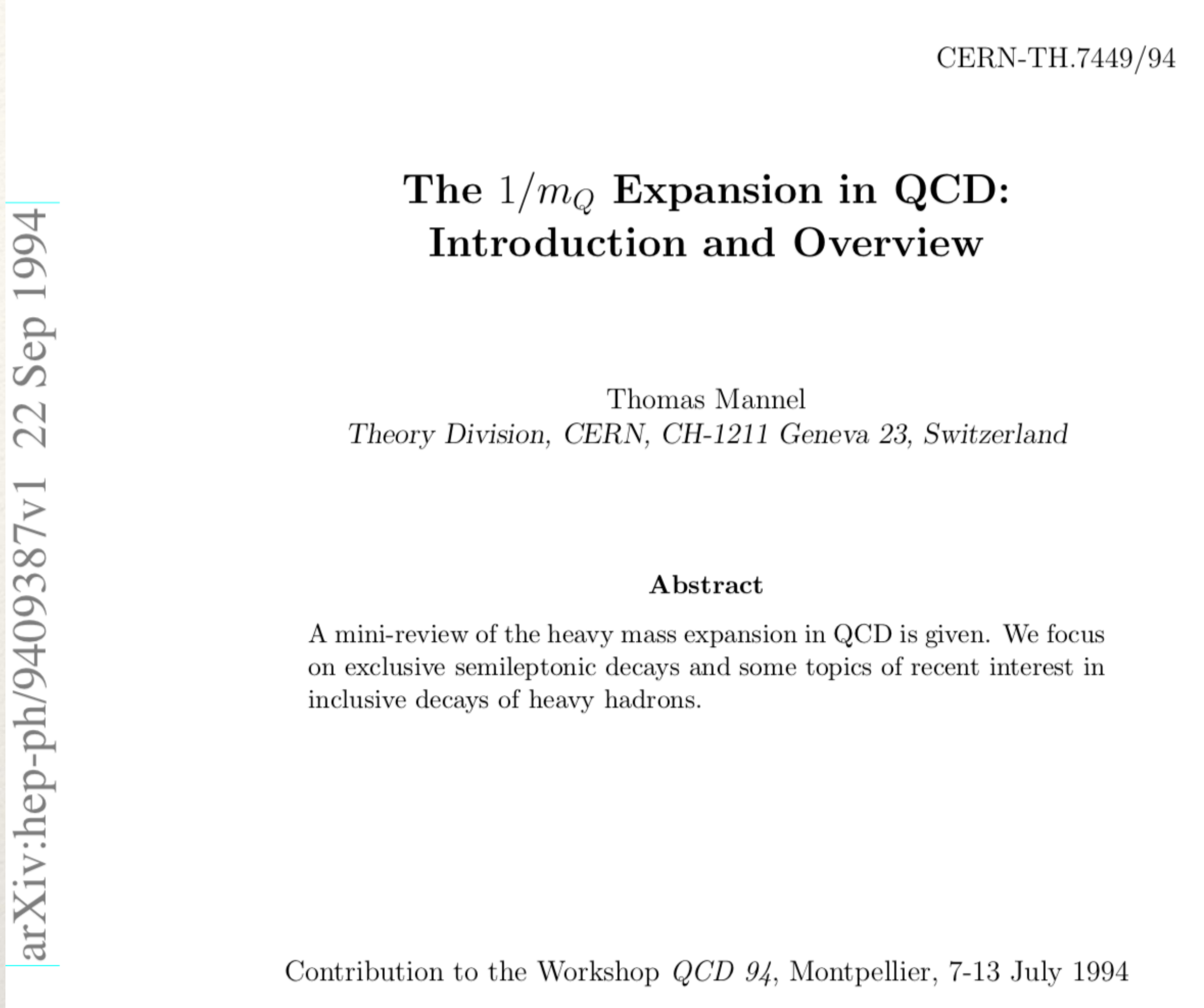
$$\begin{aligned} C_l^{(2)} = (N-1)\Bigg[&\left(\frac{N+3}{3N}\pi^2 + \frac{123N+211}{24N}\right) n_l - 2\frac{N^2+N+1}{N^2} I_0 \\ &- 2(N-1)\frac{N^2+2N+2}{N^2}\zeta(3) - \frac{43N^3+111N^2-111N-275}{48N^2}\pi^2 \\ &- \frac{13518N^3+8456N^2-7981N+35037}{576N^2}\Bigg] . \end{aligned}$$

$$\begin{aligned} C_p^{(2)} = (N+1)\Bigg[&\frac{38}{9} n_l - \frac{8}{3} I_0 - \frac{2}{9}\frac{4N^2+9N-29}{N}\pi^2 \\ &- \frac{686N^3-563N^2+1599N+18}{36N^2}\Bigg] . \end{aligned}$$

$$I_0 = \pi^2 \log(2) - \frac{3}{2}\zeta(3) = 5.038\ldots$$

$$\begin{aligned} C_l(m_b) &= 1 - 12a_s - 175.6a_s^2 , \\ C_p(m_b) &= -8a_s - 311.2a_s^2 , \end{aligned}$$

$$\Delta m = \text{const}\left(1 - 6.4a_s - (4.9 + x_l^{(2)})a_s^2\right) f_B^2 .$$



CERN-TH.7449/94

The $1/m_Q$ Expansion in QCD:
Introduction and Overview

Thomas Mannel
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

A mini-review of the heavy mass expansion in QCD is given. We focus on exclusive semileptonic decays and some topics of recent interest in inclusive decays of heavy hadrons.

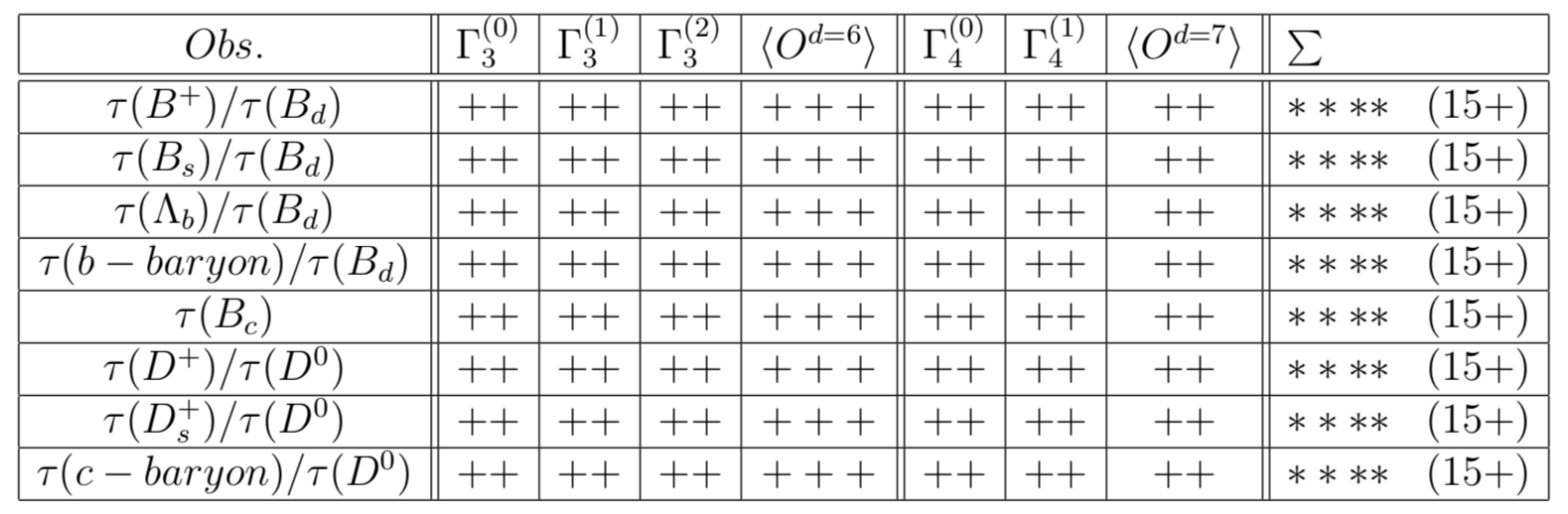
Contribution to the Workshop *QCD 94*, Montpellier, 7-13 July 1994

Heavy Quarks through the looking glass *Siegen* **4./5.10.2042**

The 1/mQ Expansion in QCD

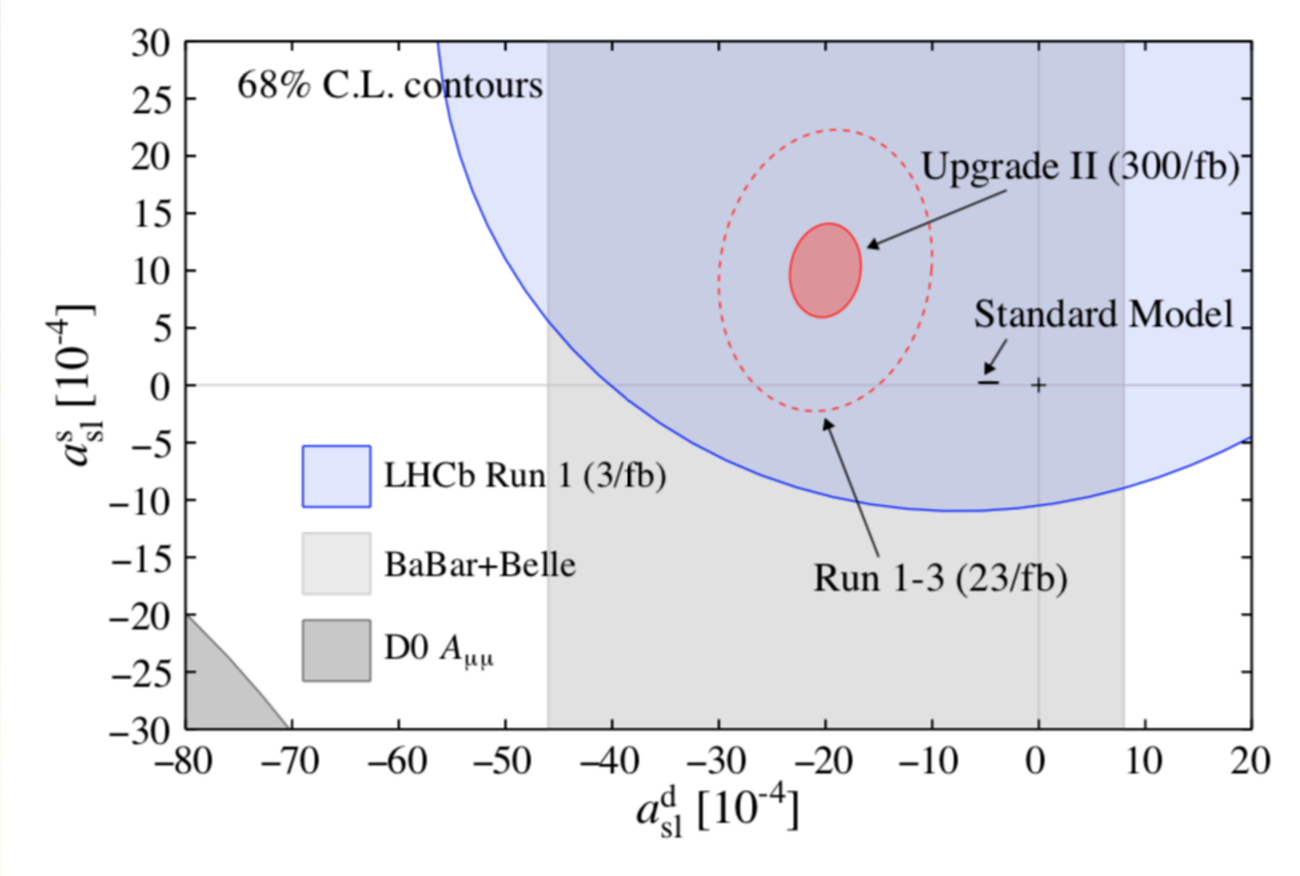
Alexander Lenz
IPPP, Durham University

48 years later

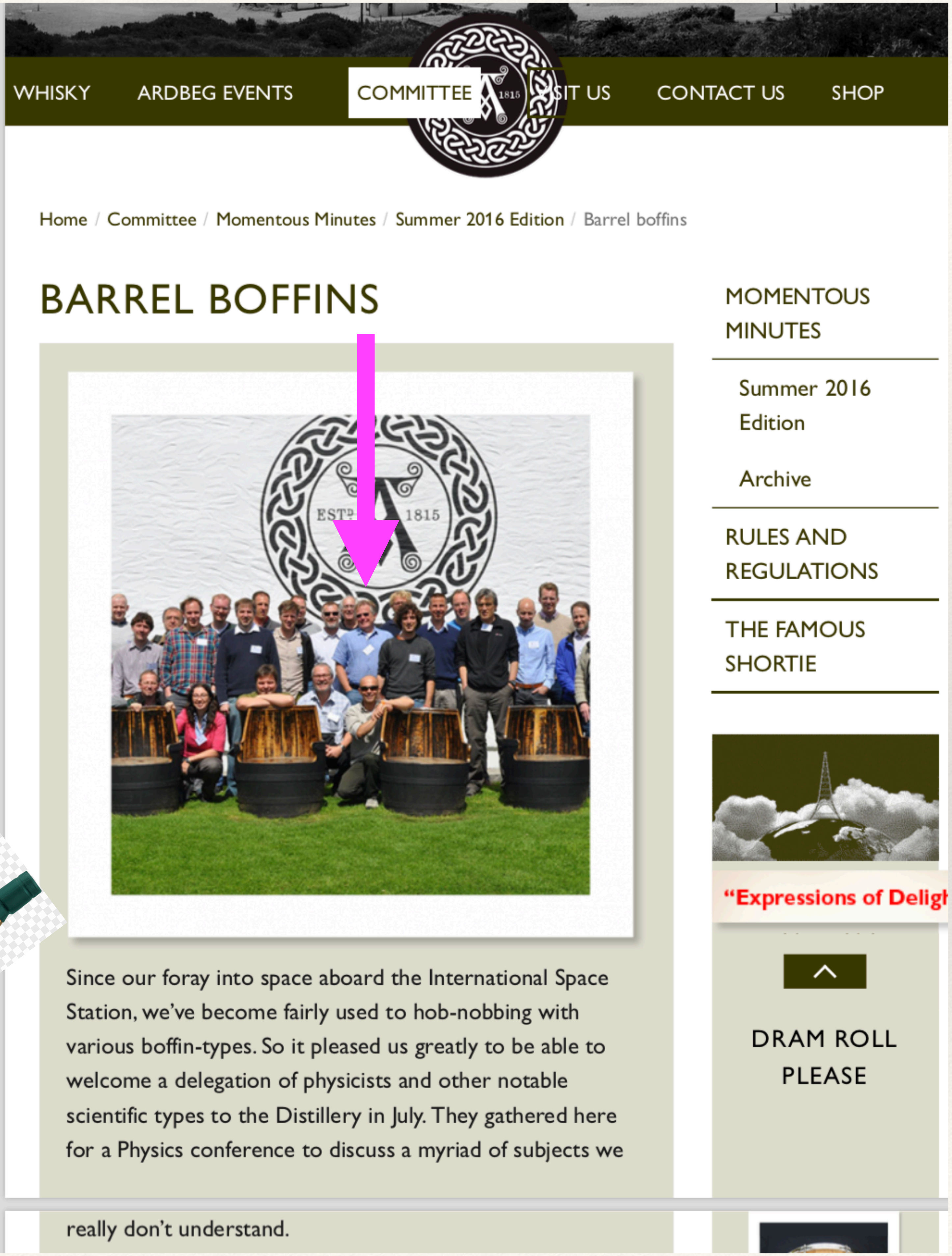


$Obs.$	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	Σ
$\tau(B^+)/\tau(B_d)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(B_s)/\tau(B_d)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(\Lambda_b)/\tau(B_d)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(b - baryon)/\tau(B_d)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(B_c)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(D^+)/\tau(D^0)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(D_s^+)/\tau(D^0)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)
$\tau(c - baryon)/\tau(D^0)$	$++$	$++$	$++$	$+++$	$++$	$++$	$++$	$* * **$ (15+)

9.4 sigma discrepancy of experimental numbers for semi-leptonic asymmetries from SM predictions confirmed



Happy birthday Thomas!



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BARREL BOFFINS

Since our foray into space aboard the International Space Station, we've become fairly used to hob-nobbing with various boffin-types. So it pleased us greatly to be able to welcome a delegation of physicists and other notable scientific types to the Distillery in July. They gathered here for a Physics conference to discuss a myriad of subjects we really don't understand.

MOMENTOUS MINUTES

Summer 2016 Edition

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