Effective Field Theories to All Orders

Gil Paz

Department of Physics and Astronomy,

Wayne State University,

Detroit, Michigan, USA

Introduction

Motivation

- Effective field theories allow to systematically describe effects suppressed by a ratio of two scales, e.g. $\Lambda_{\rm QCD}/m_b$

Motivation

- Effective field theories allow to systematically describe effects suppressed by a ratio of two scales, e.g. $\Lambda_{\rm QCD}/m_b$

- Current applications require higher dimensional operators:

- $|V_{cb}|$ extraction from inclusive B decays
  uses dimension 7 and 8 HQET operators
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

Motivation

- Effective field theories allow to systematically describe effects suppressed by a ratio of two scales, e.g. $\Lambda_{\rm QCD}/m_b$

- Current applications require higher dimensional operators:

- $|V_{cb}|$ extraction from inclusive B decays uses dimension 7 and 8 HQET operators [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

- Applications of NRQED to proton structure effects in spectroscopy require Wilson coefficients of operators of dimension 5,6, and 7 [Hill, GP, PRL **107** 160402 (2011)]

Motivation

- Effective field theories allow to systematically describe effects suppressed by a ratio of two scales, e.g. $\Lambda_{\mathrm{QCD}}/m_b$

- Current applications require higher dimensional operators:
- $|V_{cb}|$ extraction from inclusive B decays
  uses dimension 7 and 8 HQET operators
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

- Applications of NRQED to proton structure effects in spectroscopy require Wilson coefficients of operators of dimension 5,6, and 7
  [Hill, GP, PRL **107** 160402 (2011)]

- The structure of the SM EFT is simpler than expected
  [Henning, Lu, Melia, Murayama, JHEP **1708**, 016 (2017)]
  What about HQET and NRQCD (NRQED)?

Higher Dimensional Operators

- Effective field theories allow to systematically describe effects suppressed by a ratio of two scales, e.g. $\Lambda_{\mathrm{QCD}}/m_b$

- Questions:

1) What is the number of operators at each order?
2) What are the operators at each order?
3) Can we calculate the non-perturbative input (matrix elements or Wilson coefficients)?
4) What are the convergence properties of the resulting series?

Higher Dimensional Operators

- Effective field theories allow to systematically describe effects suppressed by a ratio of two scales, e.g. $\Lambda_{\rm QCD}/m_b$

- Questions:

1) What is the number of operators at each order?
2) What are the operators at each order?
3) Can we calculate the non-perturbative input (matrix elements or Wilson coefficients)?
4) What are the convergence properties of the resulting series?

- Thanks to the pioneering work of Thomas in

  Thomas Mannel "Higher order 1/m corrections at zero recoil" Phys. Rev. D **50**, 428 (1994) [hep-ph/9403249]

  we can now answer the first two questions

Outline

- Introduction
- A little bit of history
- Thomas Mannel "Higher order 1/m corrections at zero recoil" Phys. Rev. D **50**, 428 (1994) [hep-ph/9403249]
- Later developments
- Conclusions and Outlook

A little bit of history

A tale of two effective field theories

- HQET: Heavy Quark Effective Theory

$$\mathcal{L}_{HQET}^{kinetic} = \bar{h}\, iv \cdot D\, h$$

- NRQCD: Non Relativistic Quantum Chromodynamics
  (NRQED: Non Relativistic Quantum Electrodynamics)

$$\mathcal{L}_{NRQCD}^{kinetic} = \psi^\dagger iD_t \psi + \psi^\dagger \frac{\boldsymbol{D}^2}{2M}\psi$$

- Different kinetic term and power counting

- Lagrangians can be related by
  - $h \to \psi$
  - Choosing $v = (1, 0, 0, 0)$

- The relation is not as well known as it should be

Prehistory

$$D_t = \frac{\partial}{\partial t} + ieA^0, \quad \boldsymbol{D} = \boldsymbol{\nabla} - ie\boldsymbol{A}$$

- Schrödinger equation: $iD_t + \dfrac{\boldsymbol{D}^2}{2M}$ (1926)
- Hydrogen Fine Structure:

Spin-Orbit:	$\boldsymbol{\sigma} \cdot \boldsymbol{B}$	(1927)
Relativistic correction:	$\boldsymbol{D}^4$	(1905?)
Darwin term:	$\boldsymbol{\partial} \cdot \boldsymbol{E}$	(1928)

Prehistory

$$D_t = \frac{\partial}{\partial t} + ieA^0, \quad \boldsymbol{D} = \boldsymbol{\nabla} - ie\boldsymbol{A}$$

- Schrödinger equation: $iD_t + \frac{\boldsymbol{D}^2}{2M}$ (1926)
- Hydrogen Fine Structure:

Spin-Orbit:	$\boldsymbol{\sigma}\cdot\boldsymbol{B}$	(1927)
Relativistic correction:	$\boldsymbol{D}^4$	(1905?)
Darwin term:	$\boldsymbol{\partial}\cdot\boldsymbol{E}$	(1928)

- Organize operators in Lagrangian form
- The dim=5,6 were given in [Caswell, Lepage PLB **167**, 437 (1986)]

$$\mathcal{L}_{\text{NRQED}}^{\text{dim=5,6}} = \psi^\dagger\left\{iD_t + \frac{\boldsymbol{D}^2}{2M} + \frac{\boldsymbol{D}^4}{8M^3} + c_F g\frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g\frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8M^2} + ic_S g\frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})}{8M^2} + c_{W1}g\frac{\{\boldsymbol{D}^2,\boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3}\right\}\psi$$

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\text{HQET}}^{\text{dim}=5} = \bar{h}\, iv\cdot D\, h + \frac{\bar{h}\,(iD)^2\, h}{2M} + c_F g \frac{\bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h}{4M}$$

[Falk, Grinstein, Luke, NPB 357, 185 (1991)]

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\mathrm{HQET}}^{\mathrm{dim}=5} = \bar{h}\, iv \cdot D\, h + \frac{\bar{h}\,(iD)^2\, h}{2M} + c_F\, g\, \frac{\bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h}{4M}$$

[Falk, Grinstein, Luke, NPB 357, 185 (1991)]

- Compared to

$$\mathcal{L}_{\mathrm{NRQED}}^{\mathrm{dim}=5,6} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2M} + \frac{\boldsymbol{D}^4}{8M^3} + c_F\, g\, \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}}{2M} + c_D\, g\, \frac{[\partial \cdot \boldsymbol{E}]}{8M^2} + i c_S\, g\, \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D})}{8M^2} + c_{W1}\, g\, \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma} \cdot \boldsymbol{B}\}}{8M^3} \Big\} \psi$$

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\text{HQET}}^{\text{dim}=5} = \bar{h}\, iv \cdot D\, h + \frac{\bar{h}\,(iD)^2\, h}{2M} + c_F g \frac{\bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h}{4M}$$

[Falk, Grinstein, Luke, NPB 357, 185 (1991)]

- Compared to

$$\mathcal{L}_{\text{NRQED}}^{\text{dim}=5,6} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2M} + \frac{\boldsymbol{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8M^2} + i c_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2} + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3} \Big\} \psi$$

- We can see the analogy between HQET and NRQED

	NRQED (1920's-1980's)	HQET(1990's)
Dimension 5	$\boldsymbol{D}^2$	$(iD)^2$
	$\boldsymbol{\sigma}\cdot\boldsymbol{B}$	$\sigma_{\mu\nu}G^{\mu\nu}/2$

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\text{HQET}}^{\text{dim}=5} = \bar{h}\, iv\cdot D\, h + \frac{\bar{h}\,(iD)^2\, h}{2M} + c_F g \frac{\bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h}{4M}$$

[Falk, Grinstein, Luke, NPB 357, 185 (1991)]

- Compared to

$$\mathcal{L}_{\text{NRQED}}^{\text{dim}=5,6} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2M} + \frac{\boldsymbol{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\partial\cdot\boldsymbol{E}]}{8M^2} + ic_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2} + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3} \Big\}\psi$$

- We can see the analogy between HQET and NRQED

	NRQED (1920's-1980's)	HQET(1990's)
Dimension 5	$\boldsymbol{D}^2$	$(iD)^2$
	$\boldsymbol{\sigma}\cdot\boldsymbol{B}$	$\sigma_{\mu\nu}G^{\mu\nu}/2$

- Can even "guess" dimension 6 HQET operators, but this wasn't done

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\text{HQET}}^{\text{dim}=5} = \bar{h}\, iv \cdot D\, h + \frac{\bar{h}\,(iD)^2\, h}{2M} + c_F g \frac{\bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h}{4M}$$

[Falk, Grinstein, Luke, NPB 357, 185 (1991)]

- Compared to

$$\mathcal{L}_{\text{NRQED}}^{\text{dim}=5,6} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2M} + \frac{\boldsymbol{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\partial\cdot\boldsymbol{E}]}{8M^2} + ic_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})}{8M^2} + c_{W1} g \frac{\{\boldsymbol{D}^2,\boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3} \Big\}\psi$$

- We can see the analogy between HQET and NRQED

	NRQED (1920's-1980's)	HQET(1990's)
Dimension 5	$\boldsymbol{D}^2$	$(iD)^2$
	$\boldsymbol{\sigma}\cdot\boldsymbol{B}$	$\sigma_{\mu\nu}G^{\mu\nu}/2$

- Can even "guess" dimension 6 HQET operators, but this wasn't done
- What about higher dimensional operators?

Thomas Mannel

"Higher order 1/m corrections at zero recoil"

Phys. Rev. D **50**, 428 (1994) [hep-ph/9403249]

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

- Between HQET fields $\bar{h} \ldots h$ the Dirac basis reduces to $\{1, \boldsymbol{\sigma}\} = \{1, s^\lambda\}$ with $v \cdot s = 0$

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

- Between HQET fields $\bar{h} \dots h$ the Dirac basis reduces to
  $\{1, \boldsymbol{\sigma}\} = \{1, s^\lambda\}$ with $v \cdot s = 0$

- $\bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n}(s^\lambda) h$ is the general operator
  Since $iv \cdot Dh = 0$

  - $v_{\mu_1} \bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n}(s^\lambda) h = 0$
  - $v_{\mu_n} \bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n}(s^\lambda) h = 0$
  - $v_\lambda \bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n}(s^\lambda) h = 0$

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

- Between HQET fields $\bar{h}\ldots h$ the Dirac basis reduces to
  $\{1, \boldsymbol{\sigma}\} = \{1, s^\lambda\}$ with $v \cdot s = 0$

- $\bar{h}\, iD^{\mu_1} \ldots\ iD^{\mu_n}(s^\lambda)h$ is the general operator
  Since $iv \cdot Dh = 0$

- $v_{\mu_1}\bar{h}\, iD^{\mu_1} \ldots\ iD^{\mu_n}(s^\lambda)h = 0$
- $v_{\mu_n}\bar{h}\, iD^{\mu_1} \ldots\ iD^{\mu_n}(s^\lambda)h = 0$
- $v_\lambda\bar{h}\, iD^{\mu_1} \ldots\ iD^{\mu_n}(s^\lambda)h = 0$

- Dimension 3: $\bar{h}h$
- Dimension 4: $\bar{h}iD^\mu h \to 0$
- Dimension 5: Two operators $\bar{h}\, iD^{\mu_1} iD^{\mu_2} h, \quad \bar{h}iD^{\mu_1} iD^{\mu_2} s^\lambda h$
- Dimension 6: Two operators $\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} h, \quad \bar{h}iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} s^\lambda h$

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

- Dimension 5:

$$\begin{aligned}\langle B(v)|\bar{h}(iD_\alpha)(iD_\beta)h|B(v)\rangle &= 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]\frac{1}{3}\lambda_1 \\ \langle B(v)|\bar{h}(iD_\alpha)(iD_\beta)s_\lambda h|B(v)\rangle &= 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu \frac{1}{6}\lambda_2\end{aligned}$$

- Dimension 6:

$$\begin{aligned}\langle B(v)|\bar{h}(iD_\alpha)(iD_\mu)(iD_\beta)h_v|B(v)\rangle &= 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]v_\mu \frac{1}{3}\rho_1 \\ \langle B(v)|\bar{h}(iD_\alpha)(iD_\mu)(iD_\beta)s_\lambda h|B(v)\rangle &= 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu v_\mu \frac{1}{6}\rho_2.\end{aligned}$$

$d_H = 3$ pseudo scalar meson, $d_H = -1$ vector meson

Beyond dimension 6 HQET operators

- In the same paper Thomas had the **vision** to go beyond dimension 6

- For Spin Independent

- before equation (20):

  "one obtains for the forward matrix element... the general expression"

- before equation (21):

  "It is a simple combinatorical exercise to show that the number... of independent scalar parameters is"

- For spin dependent

- before equation (22):

  "The general form of these matrix elements ... is given by"

A difficult problem

- Counting and listing the number of operators at each order is a *difficult* problem

A difficult problem

- Counting and listing the number of operators at each order is a *difficult* problem

- Results had to be revised, e.g.

- Manohar: hep-ph/9701294 v1 $\rightarrow$ v2

  "*Two terms added to Lagrangian*"

A difficult problem

- Counting and listing the number of operators at each order is a *difficult* problem

- Results had to be revised, e.g.

- Manohar: hep-ph/9701294 v1 $\rightarrow$ v2

  "*Two terms added to Lagrangian*"

- Paz et al. arXiv:1702.0890 v1 $\rightarrow$ v2

  "*discussion of operators with multiple color structures was added*"

A difficult problem

- Counting and listing the number of operators at each order is a *difficult* problem

- Results had to be revised, e.g.

- Manohar: hep-ph/9701294 v1 $\rightarrow$ v2

  "*Two terms added to Lagrangian*"

- Paz et al. arXiv:1702.0890 v1 $\rightarrow$ v2

  "*discussion of operators with multiple color structures was added*"

- Problem only solved in 2017,

  23(!) years after Thomas's seminal paper

- [Gunawardna, GP JHEP **1707** 137 (2017)]

- [Kobach, Pal PLB **772** 225 (2017)]

Later developments

Dimension 7 NRQCD operators

- The dimension 7 operators listed in [Manohar PRD **56**, 230 (1997)]

$$
\begin{aligned}
\mathcal{L}_{\text{NRQCD}}^{\text{dim}=7} = \psi^\dagger \Bigg\{ & \frac{\boldsymbol{D}^4}{8M^3} + i c_M g \frac{\{\boldsymbol{D}^i, [\boldsymbol{\partial} \times \boldsymbol{B}]^i\}}{8m_p^3} \\
& + c_{A1} g^2 \, \frac{\left(\boldsymbol{B}_a^i \boldsymbol{B}_b^i - \boldsymbol{E}_a^i \boldsymbol{E}_b^i\right) \; T^a T^b}{8M^3} - c_{A2} g^2 \, \frac{\boldsymbol{E}_a^i \boldsymbol{E}_b^i \; T^a T^b}{16M^3} \\
& + c_{A3} g^2 \, \frac{\left(\boldsymbol{B}_a^i \boldsymbol{B}_b^i - \boldsymbol{E}_a^i \boldsymbol{E}_b^i\right) \delta^{ab}}{8M^3} - c_{A4} g^2 \, \frac{\boldsymbol{E}_a^i \boldsymbol{E}_b^i \, \delta^{ab}}{16M^3} \\
& + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma} \cdot \boldsymbol{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma} \cdot \boldsymbol{B} D^i}{4m_p^3} + c_{p'p} g \frac{\boldsymbol{\sigma} \cdot \boldsymbol{D} \boldsymbol{B} \cdot \boldsymbol{D} + \boldsymbol{D} \cdot \boldsymbol{B} \boldsymbol{\sigma} \cdot \boldsymbol{D}}{8m_p^3} \\
& - c_{B1} g^2 \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{B}_a \times \boldsymbol{B}_b - \boldsymbol{E}_a \times \boldsymbol{E}_b) f^{abc} T^c}{16M^3} + c_{B2} g^2 \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{E}_a \times \boldsymbol{E}_b) f^{abc} T^c}{16M^3} \Bigg\} \psi
\end{aligned}
$$

Dimension 7 NRQCD operators

- The dimension 7 operators listed in [Manohar PRD **56**, 230 (1997)]

$$\mathcal{L}^{\text{dim}=7}_{\text{NRQCD}} = \psi^\dagger \Bigg\{ \frac{\boldsymbol{D}^4}{8M^3} + i c_M g \frac{\{\boldsymbol{D}^i, [\boldsymbol{\partial}\times \boldsymbol{B}]^i\}}{8m_p^3}$$
$$+ c_{A1} g^2 \, \frac{\left(\boldsymbol{B}^i_a \boldsymbol{B}^i_b - \boldsymbol{E}^i_a \boldsymbol{E}^i_b\right) \; T^a T^b}{8M^3} - c_{A2} g^2 \, \frac{\boldsymbol{E}^i_a \boldsymbol{E}^i_b \; T^a T^b}{16M^3}$$
$$+ c_{A3} g^2 \, \frac{\left(\boldsymbol{B}^i_a \boldsymbol{B}^i_b - \boldsymbol{E}^i_a \boldsymbol{E}^i_b\right) \delta^{ab}}{8M^3} - c_{A4} g^2 \, \frac{\boldsymbol{E}^i_a \boldsymbol{E}^i_b \, \delta^{ab}}{16M^3}$$
$$+ c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot \boldsymbol{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma}\cdot \boldsymbol{B} D^i}{4m_p^3} + c_{p'p} g \frac{\boldsymbol{\sigma}\cdot \boldsymbol{D}\boldsymbol{B}\cdot \boldsymbol{D} + \boldsymbol{D}\cdot \boldsymbol{B}\boldsymbol{\sigma}\cdot \boldsymbol{D}}{8m_p^3}$$
$$- c_{B1} g^2 \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{B}_a \times \boldsymbol{B}_b - \boldsymbol{E}_a \times \boldsymbol{E}_b) f^{abc} T^c}{16M^3} + c_{B2} g^2 \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{E}_a \times \boldsymbol{E}_b) f^{abc} T^c}{16M^3} \Bigg\} \psi$$

- Comments:
  - Explicit color structures are taken from [Gunawardna, GP JHEP **1707** 137 (2017)]
  - Last line vanishes for NRQED but not for NRQCD

Dimension 7 HQET operators

- In 2010 Mannel, Turczyk, and Uraltsev calculated the contribution of dimension 7 & 8 HQET operators to inclusive semileptonic B decays [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Dimension 7 HQET operators

- In 2010 Mannel, Turczyk, and Uraltsev calculated the contribution of dimension 7 & 8 HQET operators to inclusive semileptonic B decays
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- Dimension 7 inclusive semileptonic B decays need

- 4 Spin Independent (SI) operators
- 5 Spin Dependent (SD) operators

  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Dimension 7 HQET operators

- In 2010 Mannel, Turczyk, and Uraltsev calculated the contribution of dimension 7 & 8 HQET operators to inclusive semileptonic B decays
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- Dimension 7 inclusive semileptonic B decays need

- 4 Spin Independent (SI) operators
- 5 Spin Dependent (SD) operators
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- Dimension 7 NRQCD Lagrangian

- 6 Spin Independent (SI) operators
- 5 Spin Dependent (SD) operators
  [Manohar PRD **56**, 230 (1997)]

Dimension 7 HQET operators

- In 2010 Mannel, Turczyk, and Uraltsev calculated the contribution of dimension 7 & 8 HQET operators to inclusive semileptonic B decays
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- Dimension 7 inclusive semileptonic B decays need
  - 4 Spin Independent (SI) operators
  - 5 Spin Dependent (SD) operators

  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- Dimension 7 NRQCD Lagrangian
  - 6 Spin Independent (SI) operators
  - 5 Spin Dependent (SD) operators

  [Manohar PRD **56**, 230 (1997)]

- Why the difference?

Dimension 8 HQET/NRQED operators

- Dimension 8 inclusive semileptonic B decays need 7 SI operators and 11 SD operators
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Dimension 8 HQET/NRQED operators

- Dimension 8 inclusive semileptonic B decays need 7 SI operators and 11 SD operators
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

$$
\begin{aligned}
\mathcal{L}^{\text{dim}=8}_{\text{NRQED}} = \psi^\dagger \Big\{ & c_{X1} g \frac{[\boldsymbol{D}^2, \boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{D}]}{M^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\}}{M^4} \\
& + c_{X3} g \frac{[\boldsymbol{\partial}^2 \boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + i c_{X4} g^2 \frac{\{\boldsymbol{D}^i, [\boldsymbol{E}\times\boldsymbol{B}]^i\}}{M^4} \\
& + i c_{X5} g \frac{D^i \boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D}) D^i}{M^4} + i c_{X6} g \frac{\epsilon^{ijk}\sigma^i D^j [\boldsymbol{\partial}\cdot\boldsymbol{E}] D^k}{M^4} \\
& + c_{X7} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + c_{X8} g^2 \frac{[\boldsymbol{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{B}]}{M^4} + c_{X9} g^2 \frac{[\boldsymbol{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{E}]}{M^4} \\
& + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{E}\times[\partial_t \boldsymbol{E} - \boldsymbol{\partial}\times\boldsymbol{B}]}{M^4} \Big\} \psi
\end{aligned}
$$

Dimension 8 HQET/NRQED operators

- Dimension 8 inclusive semileptonic B decays need
  7 SI operators and 11 SD operators
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

- The dim=8 NRQED Lagrangian was given in
  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

$$
\begin{aligned}
\mathcal{L}^{\text{dim}=8}_{\text{NRQED}} = \psi^\dagger \Big\{ & c_{X1} g \frac{[\boldsymbol{D}^2, \boldsymbol{D}\cdot\boldsymbol{E}+\boldsymbol{E}\cdot\boldsymbol{D}]}{M^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\}}{M^4} \\
& + c_{X3} g \frac{[\boldsymbol{\partial}^2 \boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + i c_{X4} g^2 \frac{\{\boldsymbol{D}^i, [\boldsymbol{E}\times\boldsymbol{B}]^i\}}{M^4} \\
& + i c_{X5} g \frac{D^i \boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D}) D^i}{M^4} + i c_{X6} g \frac{\epsilon^{ijk}\sigma^i D^j [\boldsymbol{\partial}\cdot\boldsymbol{E}] D^k}{M^4} \\
& + c_{X7} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + c_{X8} g^2 \frac{[\boldsymbol{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{B}]}{M^4} + c_{X9} g^2 \frac{[\boldsymbol{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{E}]}{M^4} \\
& + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{E}\times[\partial_t \boldsymbol{E} - \boldsymbol{\partial}\times\boldsymbol{B}]}{M^4} \Big\} \psi
\end{aligned}
$$

- 4 SI operators and 8 SD operators
- Missing operators are presumably NRQCD operators

Dimension 8 HQET/NRQED operators

- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- Lagrangian can be constructed by considering all possible combinations of iD_t, $i\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{\sigma}$ that are
  - Rotationally invariant
  - P and T even
  - Hermitian

Dimension 8 HQET/NRQED operators

- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- Lagrangian can be constructed by considering all possible combinations of iD_t, $i\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{\sigma}$ that are

- Rotationally invariant
- P and T even
- Hermitian

- Such construction of $\mathcal{L}_{\text{NRQED}}^{\text{dim}\leq 6}$ was demonstrated in [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

Dimension 8 HQET/NRQED operators

- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- Lagrangian can be constructed by considering all possible combinations of iD_t, $i\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{\sigma}$ that are
- Rotationally invariant
- P and T even
- Hermitian

- Such construction of $\mathcal{L}_{\text{NRQED}}^{\text{dim}\leq 6}$ was demonstrated in [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

- For higher dimensional operators this becomes tedious:
- Different choices for operators
- Are operators linearly independent?
- How many linearly independent operators?

Dimension 8 HQET/NRQED operators

- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- Lagrangian can be constructed by considering all possible combinations of iD_t, $i\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{\sigma}$ that are
- Rotationally invariant
- P and T even
- Hermitian

- Such construction of $\mathcal{L}_{\text{NRQED}}^{\text{dim}\leq 6}$ was demonstrated in [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

- For higher dimensional operators this becomes tedious:
- Different choices for operators
- Are operators linearly independent?
- How many linearly independent operators?

- Is there an easier way?

General method

- My graduate student Ayesh Gunawardana and I looked at this problem in [Gunawardna, GP JHEP **1707** 137 (2017)]
- Following [Mannel, PRD 50, 428 (1994)] we considered matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

General method

- My graduate student Ayesh Gunawardana and I looked at this problem in [Gunawardna, GP JHEP **1707** 137 (2017)]

- Following [Mannel, PRD 50, 428 (1994)] we considered matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- We used the constraints

- Orthogonality of v to μ_1, μ_n, λ [Mannel, PRD 50, 428 (1994)]
- Parity and Time reversal symmetry
- Hermitian conjugation
- Four dimensions
- Possible multiple color structures [Kobach, Pal PLB **772** 225 (2017)]

General method

- My graduate student Ayesh Gunawardana and I looked at this problem in [Gunawardna, GP JHEP **1707** 137 (2017)]

- Following [Mannel, PRD 50, 428 (1994)]
  we considered matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- We used the constraints
- Orthogonality of v to μ_1, μ_n, λ [Mannel, PRD 50, 428 (1994)]
- Parity and Time reversal symmetry
- Hermitian conjugation
- Four dimensions
- Possible multiple color structures [Kobach, Pal PLB **772** 225 (2017)]

- To decompose them in terms of the tensors
- v^{μ_i}, $\Pi^{\mu\nu} = g^{\mu\nu} - v^{\mu}v^{\nu}$, $\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$

General method

- We considered matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- We expressed them in terms of $v^{\mu_i}, \Pi^{\mu_i\mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}v_\rho$ using
- Orthogonality: $v_{\mu_1} = v_{\mu_n} = v_\lambda = 0$
- P, T, and Hermitian conjugation:
  SI (SD) matrix elements are sym. (anti-sym.) under index inversion
- Four dimensions:
  not all tensors are linearly independent
- Checking possible multiple color structures

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i \mu_j}$,
- We can have $\Pi\,\Pi$:

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
- We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
- We can have $\Pi\,\Pi$: $\quad \Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} \qquad \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
- We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
- or $\Pi\, vv$:

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
- We can have $\Pi\,\Pi$: $\quad \Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} \quad \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} \quad \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
- or $\Pi\, vv$: $\quad \Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
  - We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
  - or $\Pi\, vv$: $\Pi^{\mu_1\mu_4}v^{\mu_2}v^{\mu_3}$

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + $$
$$+ a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4}v^{\mu_2}v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
  - We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
  - or $\Pi\, vv$: $\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + $$
$$+ a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\, h$:

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
  - We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
  - or $\Pi\, vv$: $\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4}\, h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + \\ + a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$$

  Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\, h$:
  $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_2}], [iD^{\mu_3}, iD^{\mu_4}]\}\, h$, $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_3}], [iD^{\mu_2}, iD^{\mu_4}]\}\, h$, and
  $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_4}], [iD^{\mu_2}, iD^{\mu_3}]\}\, h$

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
  - We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
  - or $\Pi\, vv$: $\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + \\ + a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\, h$:
  $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_2}], [iD^{\mu_3}, iD^{\mu_4}]\}\, h$, $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_3}], [iD^{\mu_2}, iD^{\mu_4}]\}\, h$, and $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_4}], [iD^{\mu_2}, iD^{\mu_3}]\}\, h$
  Contracting with tensors above: $a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$
  $\Rightarrow$ 2 op. with 2 color structures: 6 in total but only 4 at tree level

Example: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$,
- We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
- or $\Pi\, vv$: $\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + \\ + a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

- Multiple color structure arise from $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\, h$:
  $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_2}], [iD^{\mu_3}, iD^{\mu_4}]\}\, h$, $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_3}], [iD^{\mu_2}, iD^{\mu_4}]\}\, h$, and $\bar{h}\,\{[iD^{\mu_1}, iD^{\mu_4}], [iD^{\mu_2}, iD^{\mu_3}]\}\, h$
  Contracting with tensors above: $a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$
  $\Rightarrow$ 2 op. with 2 color structures: 6 in total but only 4 at tree level
- Explains 4 HQET SI op. in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)] and 6 NRQCD SI in [Manohar PRD **56**, 230 (1997)]

New Result: Dimension 8 NRQCD Lagrangian

[Gunawardna, GP JHEP **1707** 137 (2017), Kobach, Pal PLB **772** 225 (2017)]

New Result: Dimension 8 NRQCD Lagrangian

[Gunawardna, GP JHEP **1707** 137 (2017), Kobach, Pal PLB **772** 225 (2017)]

$$\mathcal{L}_{\text{NRQCD}}^{\text{dim}=8} = \psi^\dagger \Big\{ \ldots c_{X1} g \frac{[\boldsymbol{D}^2, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{D}^i, \boldsymbol{E}^i]\}}{m_p^4} + c_{X3} g \frac{[\boldsymbol{D}^i, [\boldsymbol{D}^i, [\boldsymbol{D}^j, \boldsymbol{E}^j]]]}{m_p^4}$$

$$+ic_{X4a}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}_a^j \boldsymbol{B}_b^k \{T^a, T^b\}\}}{2M^4} + ic_{X4b}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}_a^j \boldsymbol{B}_b^k\, \delta^{ab}\}}{m_p^4} + ic_{X5} g \frac{\boldsymbol{D}^i \boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D}) \boldsymbol{D}^i}{m_p^4}$$

$$+ic_{X6} g \frac{\epsilon^{ijk} \sigma^i \boldsymbol{D}^j [\boldsymbol{D}^l, \boldsymbol{E}^l] \boldsymbol{D}^k}{m_p^4} + c_{X7a}\, g^2 \frac{\{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a T^a, [\boldsymbol{D}^i, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X7b}\, g^2 \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a [\boldsymbol{D}^i, \boldsymbol{E}^i]_a}{m_p^4}$$

$$+c_{X8a}\, g^2 \frac{\{\boldsymbol{E}_a^i T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_b T^b\}}{2M^4} + c_{X8b}\, g^2 \frac{\boldsymbol{E}_a^i [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_a}{m_p^4} + c_{X9a}\, g^2 \frac{\{\boldsymbol{B}_a^i T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_b T^b\}}{2M^4}$$

$$+c_{X9b}\, g^2 \frac{\boldsymbol{B}_a^i [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_a}{m_p^4} + c_{X10a}\, g^2 \frac{\{\boldsymbol{E}_a^i T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_b T^b\}}{2M^4} + c_{X10b}\, g^2 \frac{\boldsymbol{E}_a^i [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_a}{m_p^4}$$

$$+c_{X11a}\, g^2 \frac{\{\boldsymbol{B}_a^i T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X11b}\, g^2 \frac{\boldsymbol{B}_a^i [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_a}{m_p^4} + \tilde{c}_{X12a}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}_a^j [D_t, \boldsymbol{E}^k]_b \{T^a, T^b\}}{2M^4}$$

$$+\tilde{c}_{X12b}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}_a^j [D_t, \boldsymbol{E}^k]_a}{m_p^4} + ic_{X13} g^2 \frac{[\boldsymbol{E}^i, [D_t, \boldsymbol{E}^i]]}{m_p^4} + ic_{X14} g^2 \frac{[\boldsymbol{B}^i, (\boldsymbol{D} \times \boldsymbol{E} + \boldsymbol{E} \times \boldsymbol{D})^i]}{m_p^4}$$

$$+ic_{X15} g^2 \frac{[\boldsymbol{E}^i, (\boldsymbol{D} \times \boldsymbol{B} + \boldsymbol{B} \times \boldsymbol{D})^i]}{m_p^4} + c_{X16} g^2 \frac{[\boldsymbol{\sigma} \cdot \boldsymbol{B}, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X17} g^2 \frac{[\boldsymbol{B}^i, \{\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}\}]}{m_p^4} + c_{X18} g^2 \frac{[\boldsymbol{E}^i, \{\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i\}]}{m_p^4} \Big\} \psi$$

- 25 operators
- c_{Xib} start at $\mathcal{O}(\alpha_s)$
- Tree level operators agree with

  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators with dim≤ 8

  [Kobach, Pal PLB **772** 225 (2017)]

Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators with dim≤ 8

  [Kobach, Pal PLB **772** 225 (2017)]

- "While the Hilbert series can count the number of operators that are invariant under the given symmetries, it does not say how the indices within each operator are contracted. In general, this needs to be done by hand." [Kobach, Pal PLB **772** 225 (2017)]

Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators with dim≤ 8
  [Kobach, Pal PLB **772** 225 (2017)]

- "While the Hilbert series can count the number of operators that are invariant under the given symmetries, it does not say how the indices within each operator are contracted. In general, this needs to be done by hand." [Kobach, Pal PLB **772** 225 (2017)]

- Imposing T invariance is an issue: "Unlike NRQED, we have not found an automated way to implement invariance under time reversal in NRQCD/HQET" [Kobach, Pal PLB **772** 225 (2017)]

Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators with dim≤ 8
  [Kobach, Pal PLB **772** 225 (2017)]

- "While the Hilbert series can count the number of operators that are invariant under the given symmetries, it does not say how the indices within each operator are contracted. In general, this needs to be done by hand." [Kobach, Pal PLB **772** 225 (2017)]

- Imposing T invariance is an issue: "Unlike NRQED, we have not found an automated way to implement invariance under time reversal in NRQCD/HQET" [Kobach, Pal PLB **772** 225 (2017)]

- Still, a very useful check

Conclusions and Outlook

Conclusions

- Effective field theories are a very useful tool in contemporary physics

Conclusions

- Effective field theories are a very useful tool in contemporary physics
- The question of higher dimensional NRQED/HQET/NRQCD operators has roots in the early days of quantum mechanics

Conclusions

- Effective field theories are a very useful tool in contemporary physics
- The question of higher dimensional NRQED/HQET/NRQCD operators has roots in the early days of quantum mechanics
- The first discussion of number and identity of such operators was in
  Thomas Mannel "Higher order 1/m corrections at zero recoil"
  Phys. Rev. D **50**, 428 (1994) [hep-ph/9403249]

Conclusions

- Effective field theories are a very useful tool in contemporary physics
- The question of higher dimensional NRQED/HQET/NRQCD operators has roots in the early days of quantum mechanics
- The first discussion of number and identity of such operators was in
  Thomas Mannel "Higher order 1/m corrections at zero recoil"
  Phys. Rev. D **50**, 428 (1994) [hep-ph/9403249]
- In 2017 the problem was solved in

- [Gunawardna, GP JHEP **1707** 137 (2017)]
- [Kobach, Pal PLB **772** 225 (2017)]

Outlook: open questions

3) Non-perturbative input is needed for physical applications, e.g.

Outlook: open questions

3) Non-perturbative input is needed for physical applications, e.g.

- $\mathcal{L}^{\text{dim}=5}_{\text{HQET}} \ni c_F g \bar{h}\sigma_{\mu\nu}G^{\mu\nu}\, h/4M, \quad \mathcal{L}^{\text{dim}=5}_{\text{NRQED}} \ni c_F g \psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\psi/2M$

Outlook: open questions

3) Non-perturbative input is needed for physical applications, e.g.

- $\mathcal{L}^{\text{dim}=5}_{\text{HQET}} \ni c_F g \bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h/4M, \quad \mathcal{L}^{\text{dim}=5}_{\text{NRQED}} \ni c_F g \psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\psi/2M$
- For inclusive B decays

  Perturbative Wilson coefficients, Non-perturbative matrix elements:
  $\langle \bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h\rangle$ related to $B - B^*$ mass difference

Outlook: open questions

3) Non-perturbative input is needed for physical applications, e.g.

- $\mathcal{L}_{\text{HQET}}^{\text{dim}=5} \ni c_F g \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h/4M, \quad \mathcal{L}_{\text{NRQED}}^{\text{dim}=5} \ni c_F g \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{B} \psi/2M$
- For inclusive B decays

  Perturbative Wilson coefficients, Non-perturbative matrix elements:

  $\langle \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h \rangle$ related to $B - B^*$ mass difference
- For proton structure effects in spectroscopy

  Non-perturbative Wilson coefficients, Perturbative matrix elements:

  c_F related to proton magnetic moment

Outlook: open questions

3) Non-perturbative input is needed for physical applications, e.g.

- $\mathcal{L}_{\text{HQET}}^{\text{dim=5}} \ni c_F g \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h/4M, \quad \mathcal{L}_{\text{NRQED}}^{\text{dim=5}} \ni c_F g \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{B} \psi/2M$
- For inclusive B decays
  Perturbative Wilson coefficients, Non-perturbative matrix elements:
  $\langle \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h \rangle$ related to $B - B^*$ mass difference
- For proton structure effects in spectroscopy
  Non-perturbative Wilson coefficients, Perturbative matrix elements:
  c_F related to proton magnetic moment

- Some can be extracted from data, other require Lattice QCD
  What can we calculate in the next 10, 20, or even 100 years?

Outlook: open questions

3) Non-perturbative input is needed for physical applications, e.g.

- $\mathcal{L}^{\text{dim}=5}_{\text{HQET}} \ni c_F g \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h/4M, \quad \mathcal{L}^{\text{dim}=5}_{\text{NRQED}} \ni c_F g \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{B} \psi/2M$
- For inclusive B decays
  Perturbative Wilson coefficients, Non-perturbative matrix elements:
  $\langle \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h \rangle$ related to $B - B^*$ mass difference
- For proton structure effects in spectroscopy
  Non-perturbative Wilson coefficients, Perturbative matrix elements:
  c_F related to proton magnetic moment

- Some can be extracted from data, other require Lattice QCD
  What can we calculate in the next 10, 20, or even 100 years?

4) Convergence:
   "..it has been argued that the OPE results in an asymptotic series with limitations paralleling those for the perturbative series."
   [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Personal note

- Thomas belongs to "Late Boomers" generation (born 1956-1964)
  I belong to "Generation X" (born 1965-1980)
  For definitions see, e.g., http://www.pewresearch.org/

Personal note

- Thomas belongs to "Late Boomers" generation (born 1956-1964)
  I belong to "Generation X" (born 1965-1980)
  For definitions see, e.g., http://www.pewresearch.org/
- Growing up, Thomas's generation was a role model for my generation

Personal note

- Thomas belongs to "Late Boomers" generation (born 1956-1964)
  I belong to "Generation X" (born 1965-1980)
  For definitions see, e.g., http://www.pewresearch.org/

- Growing up, Thomas's generation was a role model for my generation

- Scientifically, they pioneered the use of effective field theories

Personal note

- Thomas belongs to "Late Boomers" generation (born 1956-1964)
  I belong to "Generation X" (born 1965-1980)
  For definitions see, e.g., http://www.pewresearch.org/
- Growing up, Thomas's generation was a role model for my generation
- Scientifically, they pioneered the use of effective field theories
- Thomas exemplifies these qualities

Personal note

- Thomas belongs to "Late Boomers" generation (born 1956-1964)
  I belong to "Generation X" (born 1965-1980)
  For definitions see, e.g., http://www.pewresearch.org/
- Growing up, Thomas's generation was a role model for my generation
- Scientifically, they pioneered the use of effective field theories
- Thomas exemplifies these qualities
- Thomas, I wish you many more years of productive work!

Backup

HQET Motivation

- Inclusive semileptonic B decays and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\gamma$ can be described by local OPE

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

HQET Motivation

- Inclusive semileptonic B decays and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\gamma$ can be described by local OPE

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

- $c_{k,n}$ are Wilson coefficients: perturbative

HQET Motivation

- Inclusive semileptonic B decays and
  $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\gamma$
  can be described by local OPE

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

- $c_{k,n}$ are Wilson coefficients: perturbative

- $\langle O_{k,n} \rangle$ are matrix elements of HQET operators: non-perturbative
  Often called HQET parameters
  $O_{k,n} \sim \bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n}(s^\lambda)h \cdot T_{\mu_1 \ldots \mu_n \lambda}$, $\quad$ T is a Lorentz tensor

HQET Motivation

- Inclusive semileptonic B decays and
  $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\gamma$
  can be described by local OPE

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

- $c_{k,n}$ are Wilson coefficients: perturbative

- $\langle O_{k,n} \rangle$ are matrix elements of HQET operators: non-perturbative
  Often called HQET parameters
  $O_{k,n} \sim \bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} (s^\lambda) h \cdot T_{\mu_1 \ldots \mu_n \lambda}$, T is a Lorentz tensor

- $|V_{cb}|$ extraction from inclusive B decays
  uses dimension 7 and 8 HQET operators
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

NRQED Motivation

- The proton radius puzzle
- Muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- Regular hydrogen [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm

NRQED Motivation

- The proton radius puzzle
  - Muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p = 0.84184(67)$ fm
  - Regular hydrogen [Mohr et al. RMP **80**, 633 (2008)]
    $r_E^p = 0.87680(690)$ fm

- Use NRQED for proton structure effects in spectroscopy
  [Hill, GP, PRL **107** 160402 (2011)]

$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \ldots$$

  - Dimension 5 operator: $a_p = 1.793$
  - Dimension 6 operator: $r_E^H = 0.8751(61)$ fm or $r_E^{\mu H} = 0.84087(26)(29)$ fm
  - Dimension 7 operators: $r_M = 0.776(34)(17)$ fm, $\bar{\beta} = 2.5(4) \times 10^{-4}$ fm^3

General Motivation

- What can we learn about the structure of effective field theories?
- For SM EFT the structure is simpler than expected
  [Henning, Lu, Melia, Murayama, JHEP **1708**, 016 (2017)]
- What about HQET and NRQCD (NRQED)?

General method: Orthogonality

- Consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

General method: Orthogonality

- Consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}\, h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}\, s^\lambda h|H\rangle$

- Since $iv\cdot Dh = 0$
- $v_{\mu_1}\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h = 0$
- $v_{\mu_n}\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h = 0$
- $v_\lambda\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h = 0$
  [Mannel, PRD 50, 428 (1994)]

- More accurately, the $1/M$ corrections to $iv\cdot Dh = 0$ give rise to higher dimensional operators. One can impose this order by order.

- Similarly for NRQCD (NRQED):
  $\psi^\dagger\,(iD_t O + OiD_t)\,\psi/M^n$ can be eliminated by $\psi\to\psi - O\psi/M^n$
  [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

General method: PT symmetry

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

General method: PT symmetry

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \dots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Parity and Time reversal are symmetries of HQET
  In particular under PT:

- $p = (p^0, \vec{p}\,) \stackrel{PT}{\to} (p^0, \vec{p}\,) = p \Rightarrow v = p/m \stackrel{PT}{\to} v$
- $iD^{\mu} \stackrel{PT}{\to} iD^{\mu}$
- $\bar{h}h \stackrel{PT}{\to} \bar{h}h$
- $\bar{h}s^{\lambda}h \stackrel{PT}{\to} -\bar{h}s^{\lambda}h$

General method: PT symmetry

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Parity and Time reversal are symmetries of HQET
  In particular under PT:

- $p = (p^0, \vec{p}) \stackrel{PT}{\to} (p^0, \vec{p}) = p \Rightarrow v = p/m \stackrel{PT}{\to} v$
- $iD^{\mu} \stackrel{PT}{\to} iD^{\mu}$
- $\bar{h}h \stackrel{PT}{\to} \bar{h}h$
- $\bar{h}s^{\lambda}h \stackrel{PT}{\to} -\bar{h}s^{\lambda}h$

- Since T is anti-linear

$$\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle \stackrel{PT}{=} \langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle^*$$
$$\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle \stackrel{PT}{=} -\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle^*$$

- SI matrix elements are real, SD matrix elements are imaginary

General method: Hermitian conjugation

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

General method: Hermitian conjugation

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h|H\rangle$

- $\bar{h}h$, $\bar{h}s^\lambda h$, iD^μ are hermitian
  using Hermitian conjugation

$$\begin{aligned}\langle H|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h|H\rangle &= \langle H|\left(\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h\right)^\dagger|H\rangle^* \\ &= \langle H|\bar{h}\, iD^{\mu_n} \dots iD^{\mu_1}(s^\lambda)h|H\rangle^*\end{aligned}$$

General method: Hermitian conjugation

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h|H\rangle$

- $\bar{h}h$, $\bar{h}s^\lambda h$, iD^μ are hermitian
  using Hermitian conjugation

$$\begin{aligned}\langle H|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h|H\rangle &= \langle H|\left(\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h\right)^\dagger|H\rangle^* \\ &= \langle H|\bar{h}\, iD^{\mu_n} \dots iD^{\mu_1}(s^\lambda)h|H\rangle^*\end{aligned}$$

- Combining with the PT constraints
  Under inversion of the indices:
- SI matrix elements are symmetric
- SD matrix elements are anti-symmetric

General method: Tensor decomposition

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

General method: Tensor decomposition

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- B is a pseudo-scalar $\Rightarrow$ matrix element can only depend on $v^{\mu_i}, g^{\mu_i\mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}$

General method: Tensor decomposition

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

- B is a pseudo-scalar $\Rightarrow$ matrix element can only depend on $v^{\mu_i}, g^{\mu_i \mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}$

- Alternatively following
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  Define $\Pi^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$
  For the standard choice of $v = (1, 0, 0, 0)$: $\Pi^{00} = 0$ and $\Pi^{ij} = -\delta^{ij}$

General method: Tensor decomposition

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

- B is a pseudo-scalar $\Rightarrow$ matrix element can only depend on $v^{\mu_i}, g^{\mu_i \mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}$

- Alternatively following
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  Define $\Pi^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$
  For the standard choice of $v = (1,0,0,0)$: $\Pi^{00} = 0$ and $\Pi^{ij} = -\delta^{ij}$

- Since the indices in $\epsilon^{\rho\sigma\alpha\beta}$ cannot be linearly independent of v^μ replace $\epsilon^{\rho\sigma\alpha\beta}$ by $\epsilon^{\rho\sigma\alpha\beta} v_\rho$

General method: Tensor decomposition

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

- B is a pseudo-scalar $\Rightarrow$ matrix element can only depend on $v^{\mu_i}, g^{\mu_i\mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}$

- Alternatively following
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  Define $\Pi^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$
  For the standard choice of $v = (1,0,0,0)$: $\Pi^{00} = 0$ and $\Pi^{ij} = -\delta^{ij}$

- Since the indices in $\epsilon^{\rho\sigma\alpha\beta}$ cannot be linearly independent of v^μ replace $\epsilon^{\rho\sigma\alpha\beta}$ by $\epsilon^{\rho\sigma\alpha\beta} v_\rho$

- Matrix element depend on
  $v^{\mu_i}, \Pi^{\mu_i\mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta} v_\rho$

General method: Four dimensions

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

General method: Four dimensions

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Four dimensions $\Rightarrow$ only four independent directions

General method: Four dimensions

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Four dimensions $\Rightarrow$ only four independent directions

- Not all tensors with more than four indices are independent

General method: Four dimensions

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Four dimensions $\Rightarrow$ only four independent directions

- Not all tensors with more than four indices are independent

- Example: for dimension 7 SD HQET operators
  need $\Pi^{\mu\nu}\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$: three indices are the same
  Tensors obtained by permuting indices are not linearly independent

General method: Four dimensions

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Four dimensions $\Rightarrow$ only four independent directions

- Not all tensors with more than four indices are independent

- Example: for dimension 7 SD HQET operators
  need $\Pi^{\mu\nu}\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$: three indices are the same
  Tensors obtained by permuting indices are not linearly independent

- Example: for dimension 11 SI HQET operators
  need $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6}\Pi^{\mu_7\mu_8}$: four indices are the same

General method: Color factors

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

General method: Color factors

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

- Starting at dimension 7 we can have multiple color factors
  E.g. consider $\psi^\dagger E_a^i T^a E_b^i T^b \psi$ [Kobach, Pal PLB **772** 225 (2017)]
  $\left\{T^a, T^b\right\} = \frac{1}{3}\delta^{ab} + d^{abc}T^c \Rightarrow$ two color structures

General method: Color factors

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n} s^\lambda h|H\rangle$

- Starting at dimension 7 we can have multiple color factors
  E.g. consider $\psi^\dagger E_a^i T^a E_b^i T^b \psi$ [Kobach, Pal PLB **772** 225 (2017)]
  $\{T^a, T^b\} = \frac{1}{3}\delta^{ab} + d^{abc} T^c \Rightarrow$ two color structures

- Use basis $\{T^a, T^b\}$ and δ^{ab}:
- $\psi^\dagger E_a^i E_b^i \{T^a, T^b\}\psi$: generated by commutators and anti-commutators

General method: Color factors

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

- Starting at dimension 7 we can have multiple color factors
  E.g. consider $\psi^\dagger E^i_a T^a E^i_b T^b \psi$ [Kobach, Pal PLB **772** 225 (2017)]
  $\left\{T^a, T^b\right\} = \frac{1}{3}\delta^{ab} + d^{abc}T^c \Rightarrow$ two color structures

- Use basis $\left\{T^a, T^b\right\}$ and δ^{ab}:

- $\psi^\dagger E^i_a E^i_b \left\{T^a, T^b\right\}\psi$: generated by commutators and anti-commutators
- $\psi^\dagger E^i_a E^i_b \delta^{ab}\psi$: generated by one-gluon exchange between $\psi^\dagger$ and ψ $\Rightarrow$ extra α_s suppression $\Rightarrow$ not needed at $\mathcal{O}(\alpha_s^0)$

General method: Color factors

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

- Starting at dimension 7 we can have multiple color factors
  E.g. consider $\psi^\dagger E_a^i T^a E_b^i T^b \psi$ [Kobach, Pal PLB **772** 225 (2017)]
  $\left\{T^a, T^b\right\} = \frac{1}{3}\delta^{ab} + d^{abc}T^c \Rightarrow$ two color structures

- Use basis $\left\{T^a, T^b\right\}$ and δ^{ab}:
  - $\psi^\dagger E_a^i E_b^i \left\{T^a, T^b\right\}\psi$: generated by commutators and anti-commutators
  - $\psi^\dagger E_a^i E_b^i \delta^{ab}\psi$: generated by one-gluon exchange between $\psi^\dagger$ and ψ $\Rightarrow$ extra α_s suppression $\Rightarrow$ not needed at $\mathcal{O}(\alpha_s^0)$

- Decomposition of $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} (s^\lambda) h|H\rangle$ does not distinguish $\left\{T^a, T^b\right\}$ from δ^{ab}. Need to be put "by hand".

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda}v_\rho$
  The 2 other indices can be

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^{\lambda} h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^{\lambda} h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_{\rho}$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

$$
\begin{aligned}
\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle &= i\tilde{a}^{(7)}_{12}\left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) \\
&+ \quad i\tilde{a}^{(7)}_{13}\left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) + \\
&+ \quad i\tilde{a}^{(7)}_{14}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda}v_\rho + i\tilde{a}^{(7)}_{23}\Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho + i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho
\end{aligned}
$$

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

$$
\begin{aligned}
\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle &= i\tilde{a}_{12}^{(7)} \left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda} v_\rho\right) \\
&+ \quad i\tilde{a}_{13}^{(7)} \left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda} v_\rho\right) + \\
&+ \quad i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + i\tilde{b}^{(7)} v^{\mu_2} v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda} v_\rho
\end{aligned}
$$

- Multiple color structure arise from $\bar{h}\left\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\right\} h$
  Contractions with tensors above give *no* contribution

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

$$\begin{aligned}\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle &= i\tilde{a}^{(7)}_{12}\left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right)\\ &+ \quad i\tilde{a}^{(7)}_{13}\left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +\\ &+ \quad i\tilde{a}^{(7)}_{14}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda}v_\rho + i\tilde{a}^{(7)}_{23}\Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho + i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho\end{aligned}$$

- Multiple color structure arise from $\bar{h}\left\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\right\} h$
  Contractions with tensors above give *no* contribution

- Explains 5 HQET SD op. in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)] and 5 NRQCD SD in [Manohar PRD **56**, 230 (1997)]

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

- Multiple color structures arise from
  - $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]]\}\, h$: 20 possibilities
  - $\bar{h}\,\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}\, h$: 15 possibilities

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

- Multiple color structures arise from
  - $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]]\}\, h$: 20 possibilities
  - $\bar{h}\,\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}\, h$: 15 possibilities

  Contractions with tensors above give 1 contribution

  $\Rightarrow$ 1 op. with 2 color structures: 8 in total but only 7 at tree level

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4}+\Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right)+$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4}+\Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right)+a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right)+$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3}+b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3}+b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3}+$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

- Multiple color structures arise from
  - $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]]\}\, h$: 20 possibilities
  - $\bar{h}\,\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}\, h$: 15 possibilities

  Contractions with tensors above give 1 contribution

  $\Rightarrow$ 1 op. with 2 color structures: 8 in total but only 7 at tree level

- Explains 7 HQET dimesion 8 SI operators in
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  The new operator will be listed below

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^{\lambda}\, h|H\rangle =$$

$$i\tilde{a}^{(8)}_{12}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}^{(8)}_{14}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{a}^{(8)}_{15}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}^{(8)}_{24}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+i\tilde{b}^{(8)}_{13}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}^{(8)}_{14}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}^{(8)}_{15}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}^{(8)}_{34}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}^{(8)}_{35}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}^{(8)}_{45}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho .$$

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^{\lambda}\, h|H\rangle =$$

$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{a}_{15}^{(8)}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+ i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+ i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho.$$

- Checking for multiple color structures as before
  Contractions with tensors above give 6 contributions
  $\Rightarrow$ 6 op. with 2 color structures: 17 in total but only 11 at tree level

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^{\lambda}\, h|H\rangle =$$

$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{a}_{15}^{(8)}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho .$$

- Checking for multiple color structures as before
  Contractions with tensors above give 6 contributions
  $\Rightarrow$ 6 op. with 2 color structures: 17 in total but only 11 at tree level

- Explains 11 HQET dimesion 8 SD operators in
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

$$\frac{1}{2M_H}\langle H|\bar h\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} iD^{\mu_6} h|H\rangle = a^{(9)}_{12,34}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} +$$
$$+a^{(9)}_{12,35}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6}+\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + a^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) +$$
$$+a^{(9)}_{13,25}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + a^{(9)}_{14,25}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} +$$
$$+a^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}) + a^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + a^{(9)}_{16,23}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} +$$
$$+a^{(9)}_{16,24}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + a^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + b^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_3}) +$$
$$+b^{(9)}_{12,46}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{12,56}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6}v^{\mu_3}v^{\mu_4} +$$
$$+b^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{13,46}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_5} +$$
$$+b^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{14,36}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_5} + b^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_4} +$$
$$+b^{(9)}_{16,23}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{16,24}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5}v^{\mu_2}v^{\mu_4}) +$$
$$+b^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}v^{\mu_3}v^{\mu_4} + b^{(9)}_{16,34}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4}v^{\mu_2}v^{\mu_5} + c^{(9)}\,\Pi^{\mu_1\mu_6}v^{\mu_2}v^{\mu_3}v^{\mu_4}v^{\mu_5}$$

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, iD^{\mu_6}\, h|H\rangle = a^{(9)}_{12,34}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} +$$
$$+a^{(9)}_{12,35}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6}+\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + a^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) +$$
$$+a^{(9)}_{13,25}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + a^{(9)}_{14,25}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} +$$
$$+a^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}) + a^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + a^{(9)}_{16,23}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} +$$
$$+a^{(9)}_{16,24}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + a^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + b^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_3}) +$$
$$+b^{(9)}_{12,46}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{12,56}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6}v^{\mu_3}v^{\mu_4} +$$
$$+b^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{13,46}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_5} +$$
$$+b^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{14,36}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_5} + b^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_4} +$$
$$+b^{(9)}_{16,23}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{16,24}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5}v^{\mu_2}v^{\mu_4}) +$$
$$+b^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}v^{\mu_3}v^{\mu_4} + b^{(9)}_{16,34}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4}v^{\mu_2}v^{\mu_5} + c^{(9)}\,\Pi^{\mu_1\mu_6}v^{\mu_2}v^{\mu_3}v^{\mu_4}v^{\mu_5}$$

- There are also multiple color structures
  Arise from combining pure color octets:
  $[iD^{\mu_i}, iD^{\mu_j}]$, $[iD^{\mu_i}, [iD^{\mu_j}, iD^{\mu_k}]]$, $[iD^{\mu_i}, [iD^{\mu_j}, [iD^{\mu_k}, iD^{\mu_l}]]]$
  For phenomenological applications at the current level of precision only T^aT^b is needed

New Result: Moments of the leading power shape function

- The shape function is important for charmless inclusive B decays
  Its moments are related to HQET parameters
  The matrix elements decomposition makes their calculation easy

$$2M_B \int d\omega \, \omega^k \, S(\omega) = n_{\mu_1} ... n_{\mu_k} \langle \bar{B}(v) | \bar{h} \, iD^{\mu_1} ... iD^{\mu_k} \, h | \bar{B}(v) \rangle$$

New Result: Moments of the leading power shape function

- The shape function is important for charmless inclusive B decays
  Its moments are related to HQET parameters
  The matrix elements decomposition makes their calculation easy

$$2M_B \int d\omega\, \omega^k\, S(\omega) = n_{\mu_1} ... n_{\mu_k} \langle \bar{B}(v)| \bar{h}\, iD^{\mu_1} ... iD^{\mu_k}\, h|\bar{B}(v)\rangle$$

$$\int d\omega\, S(\omega) = 1, \qquad \int d\omega\, \omega\, S(\omega) = 0, \qquad \int d\omega\, \omega^2\, S(\omega) = -a^{(5)} = -\lambda_1/3,$$

$$\int d\omega\, \omega^3\, S(\omega) = -a^{(6)} = -\rho_1/3,$$

$$\int d\omega\, \omega^4\, S(\omega) = a^{(7)}_{12} + a^{(7)}_{13} + a^{(7)}_{14} - b^{(7)} = m_1/5 - m_2/3,$$

$$\begin{aligned}\int d\omega\, \omega^5\, S(\omega) &= 2a^{(8)}_{12} + 2a^{(8)}_{13} + 2a^{(8)}_{15} + b^{(8)}_{12} + b^{(8)}_{14} + b^{(8)}_{15} - c^{(8)} = \\ &= \left(-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7\right)/15,\end{aligned}$$

$$\begin{aligned}\int d\omega\, \omega^6\, S(\omega) = &-a^{(9)}_{12,34} - 2a^{(9)}_{12,35} - 2a^{(9)}_{12,36} - a^{(9)}_{13,25} - 2a^{(9)}_{13,26} - a^{(9)}_{14,25} - 2a^{(9)}_{14,26} - a^{(9)}_{15,26} \\ &-a^{(9)}_{16,23} - a^{(9)}_{16,24} - a^{(9)}_{16,25} + 2b^{(9)}_{12,36} + 2b^{(9)}_{12,46} + b^{(9)}_{12,56} + 2b^{(9)}_{13,26} + b^{(9)}_{13,46} \\ &+2b^{(9)}_{14,26} + b^{(9)}_{14,36} + b^{(9)}_{15,26} + 2b^{(9)}_{16,23} + 2b^{(9)}_{16,24} + b^{(9)}_{16,25} + b^{(9)}_{16,34} - c^{(9)}\end{aligned}$$