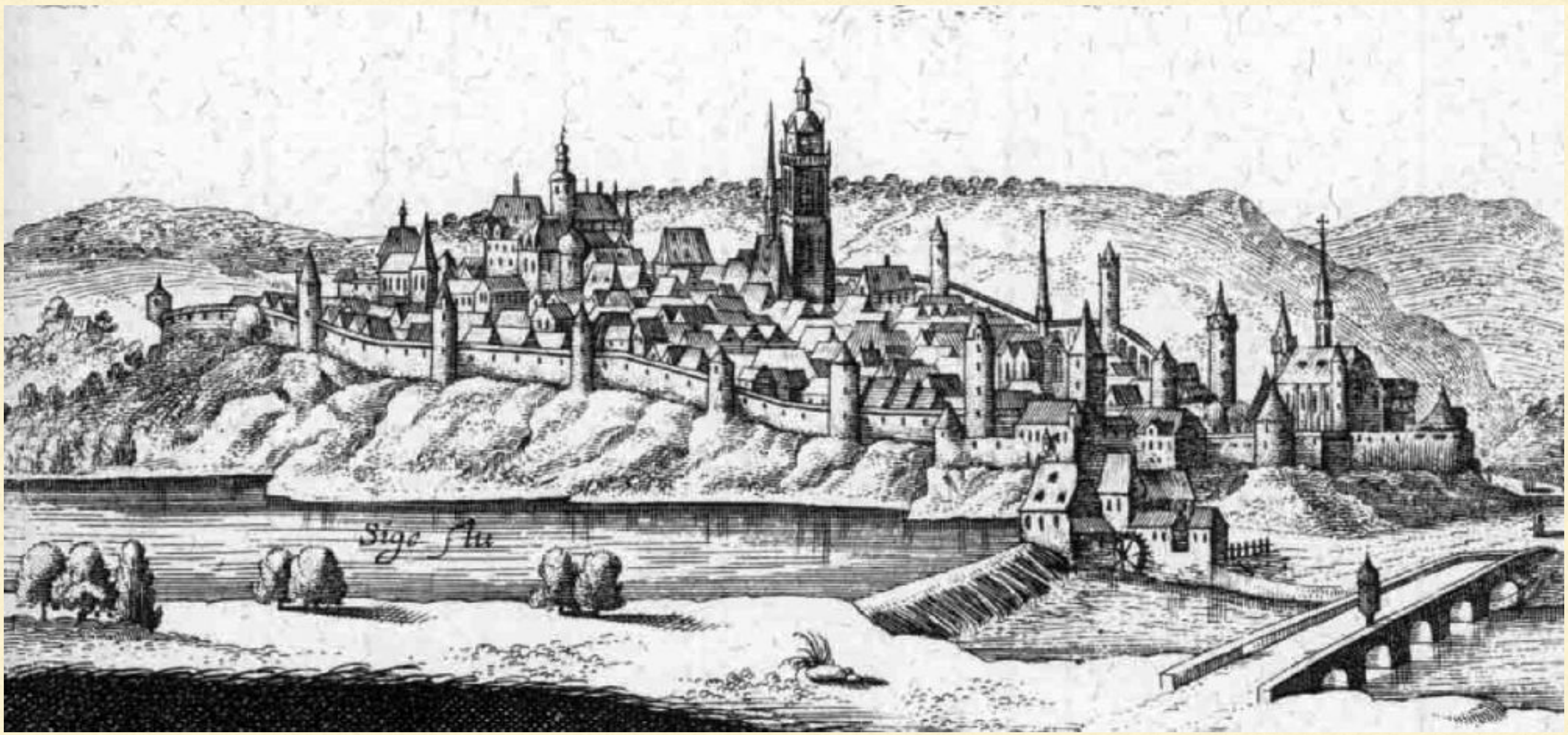
THE (ENDLESS) CHASE FOR V_{cb}

Paolo Gambino Università di Torino & INFN, Torino



Heavy quarks through the looking glass, Siegen, 4-5 october 2018

30 YEARS OF RELEVANT CONTRIBUTIONS

Imprecated, yet impeccable: on the theoretical evaluation of $\Gamma(B \to X_c \ell \nu)$

D. Benson [a,b], I.I. Bigi [a,b], Th. Mannel [b], N. Uraltsev [c,1]

Higher order $1/m$ corrections at zero r

Thomas Mannel

Phys. Rev. D **50**, 428 – Published 1

hep-ph/9308262

Operator Product Expansion
for Inclusive Semileptonic Decays
in Heavy Quark Effective Field Theory

Thomas Mannel

The European Physical Jou

April 2009, Volume 60, Is

$B \to D^{(*)}$ form factors from QCD light-cone sum rules

Physics Letters B

Volume 254, Issues 1–2, 17 January 1991, Pages 274-278

ELSEVIER

open access

Testing the heavy quark effective theory in $B^0 \to D^{*+} e^- \nu$ ☆

Thomas Mannel [a, b, 1], Winston Roberts [a, 2], Zbigniew Ryzak [a, 3]

Inclusive semileptonic B decays from QCD with NLO accuracy for power-suppressed terms

Thomas Mannel [a], Alexei A. Pivovarov [a,b], Denis Rosenthal [a,*]

[a] Theoretische Physik 1, Universität Siegen, D-57068 Siegen, Germany

Journal of High Energy Physics

November 2010, 2010:109 | Cite as

Higher order power corrections in inclusive B decays

Authors | Authors and affiliations

Th. Mannel, S. Turczyk, N. Uraltsev

$B \to D^*$ zero-recoil formfactor and the heavy quark expansion in QCD: a systematic study

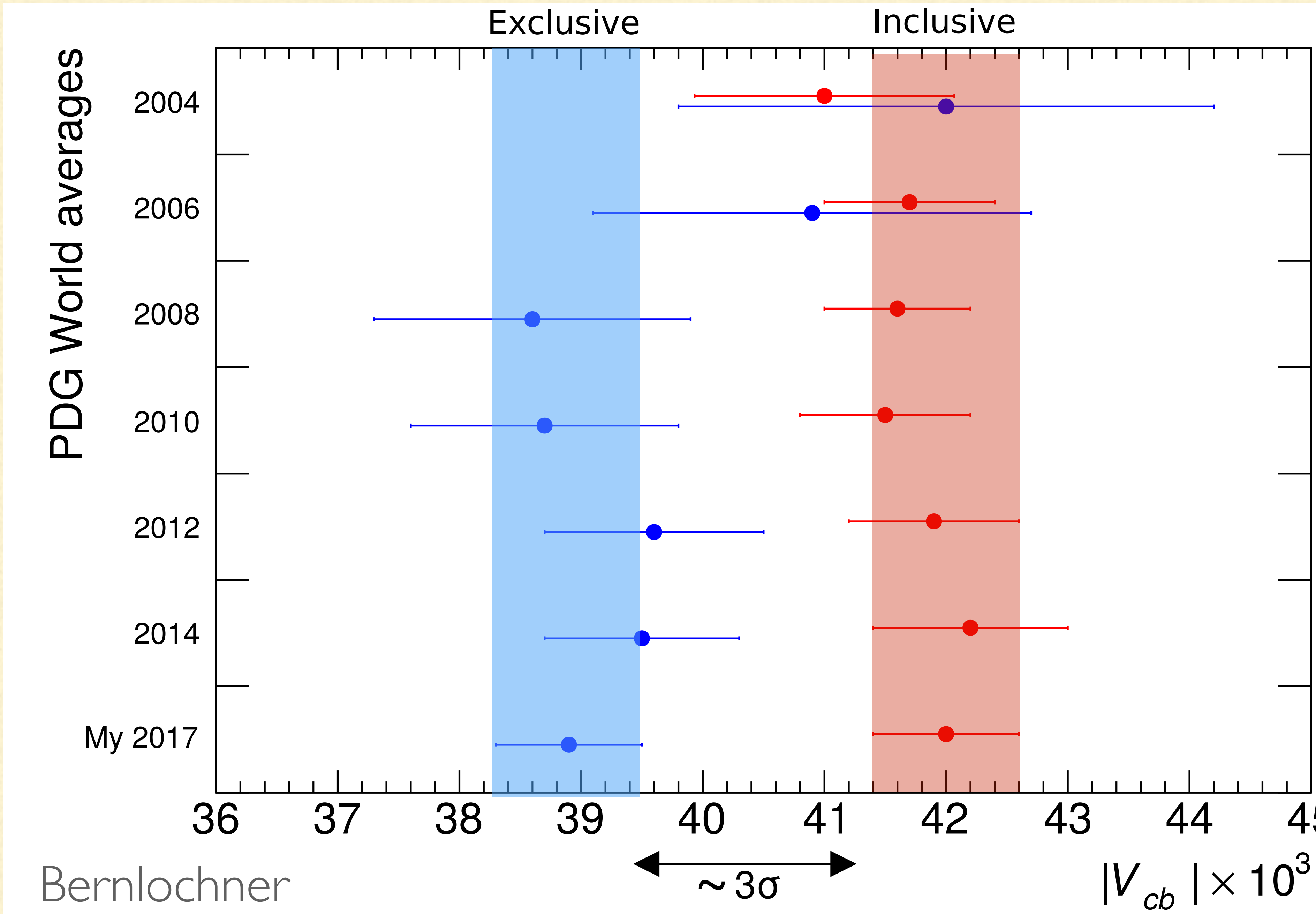
Parton–Hadron Duality in B Meson Decays

Ikaros I. Bigi[1] and Thomas Mannel

PDG particle data group 1957 2017 Reviews, Tables and Plots

89. Semileptonic b-Hadron Decays, Determination of V_{cb}, V_{ub}

Updated October 2017 by R. Kowalewski (Univ. of Victoria, Canada) and T. Mannel (Univ. of Siegen, Germany)



Babar physics book 1999
"if the two results agree"

$$|V_{cb}| = \ldots \pm 0.0004 \pm 0.0012 \; (\pm 1\% \pm 3\%)$$

ex th

The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)

V_{cb} plays an important role in UT

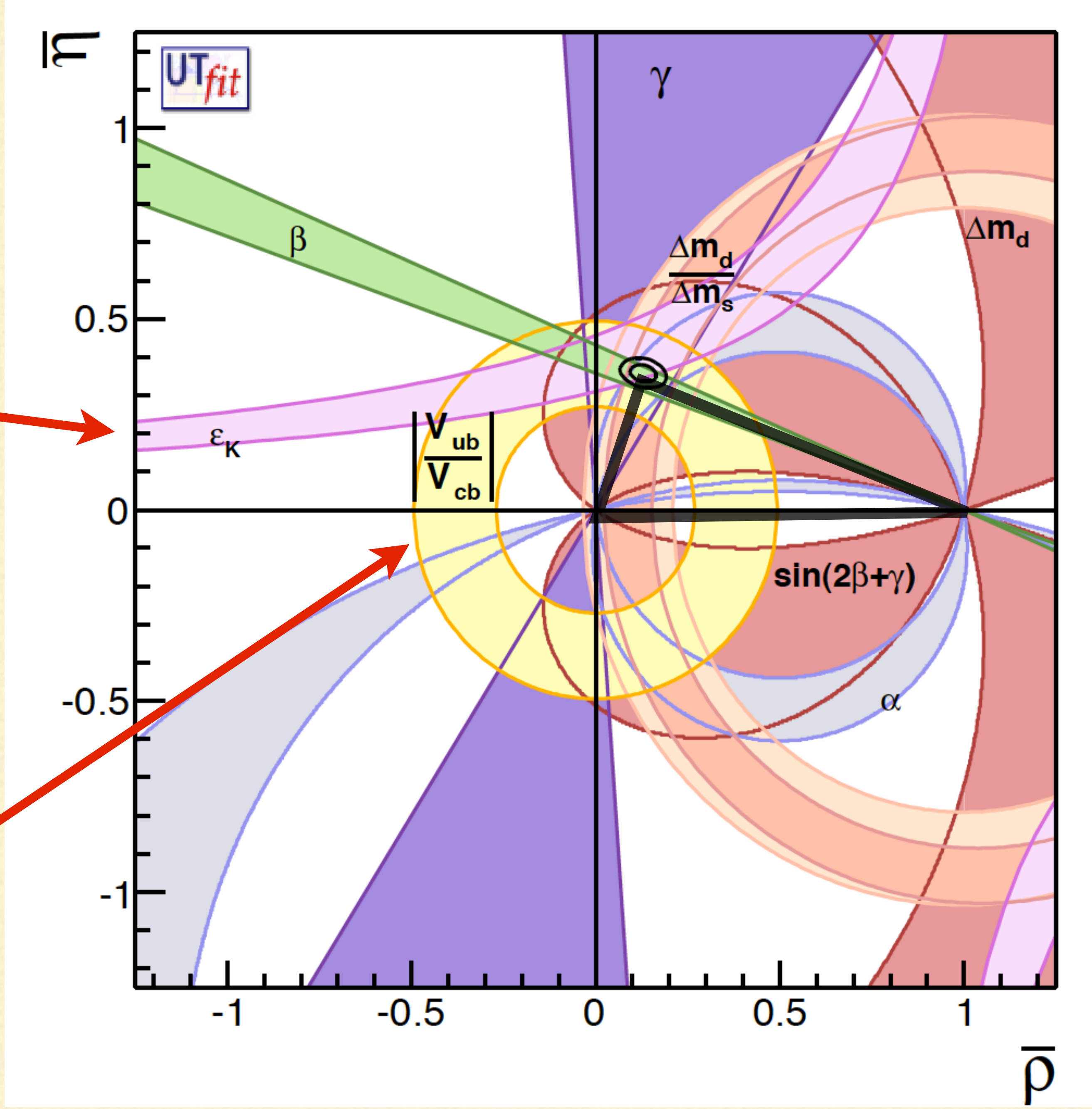
$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

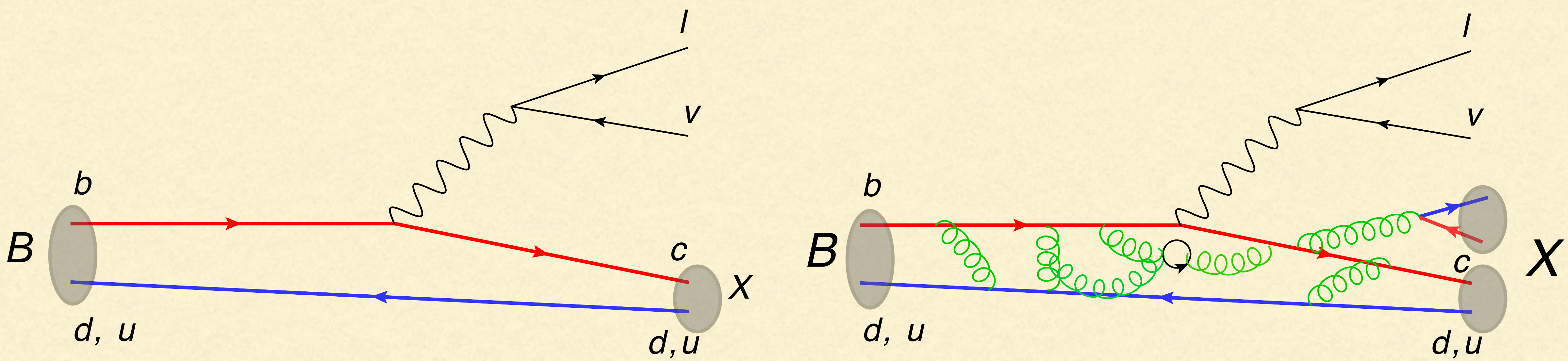
$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

where it often dominates the theoretical uncertainty.

V_{ub}/V_{cb} constrains directly the UT



INCLUSIVE DECAYS: BASICS



- Simple idea: inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: double series in α_s, Λ/m_b

- Lowest order: decay of a free b, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$
\begin{aligned}
M_i = & M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2} \\
& + \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \ldots
\end{aligned}
$$

$$
\mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \middle| \bar{b} \left(i\vec{D}\right)^2 b \middle| B \right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B} \left\langle B \middle| \bar{b} \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b \middle| B \right\rangle_\mu
$$

OPE valid for inclusive enough measurements, away from perturbative singularities: semileptonic width, moments

Most fits include 6 non-pert parameters

m_b m_c $\mu^2_{\pi,G}$ $\rho^3_{D,LS}$

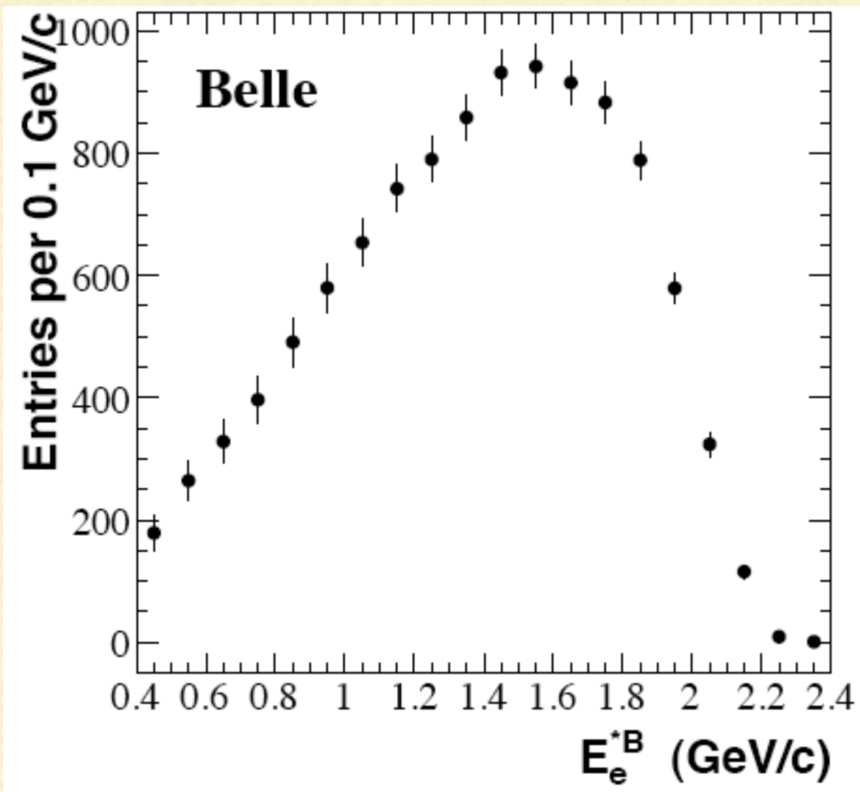
and all known corrections up to $O(\Lambda^3/m_b^3)$

TAU final states as well!

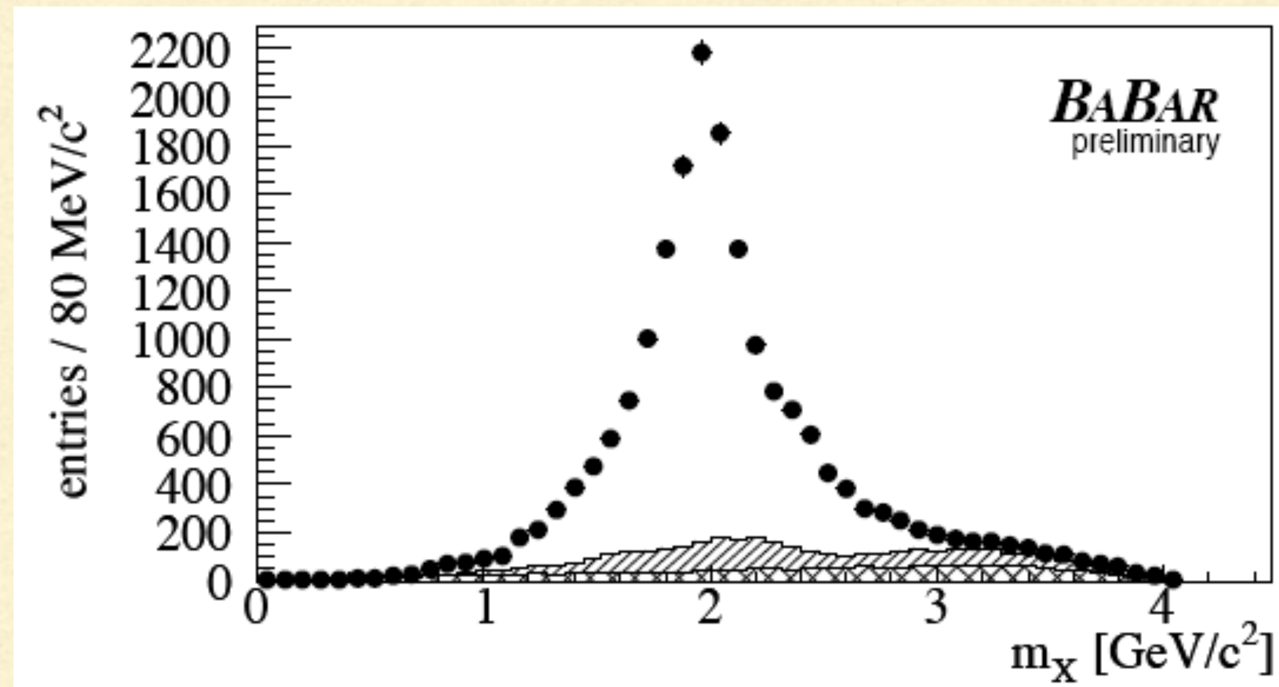
Ligeti, Tackmann, **Mannel**, Rusov, Shaharian

EXTRACTION OF THE OPE PARAMETERS

lepton energy spectrum



hadronic mass spectrum



Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

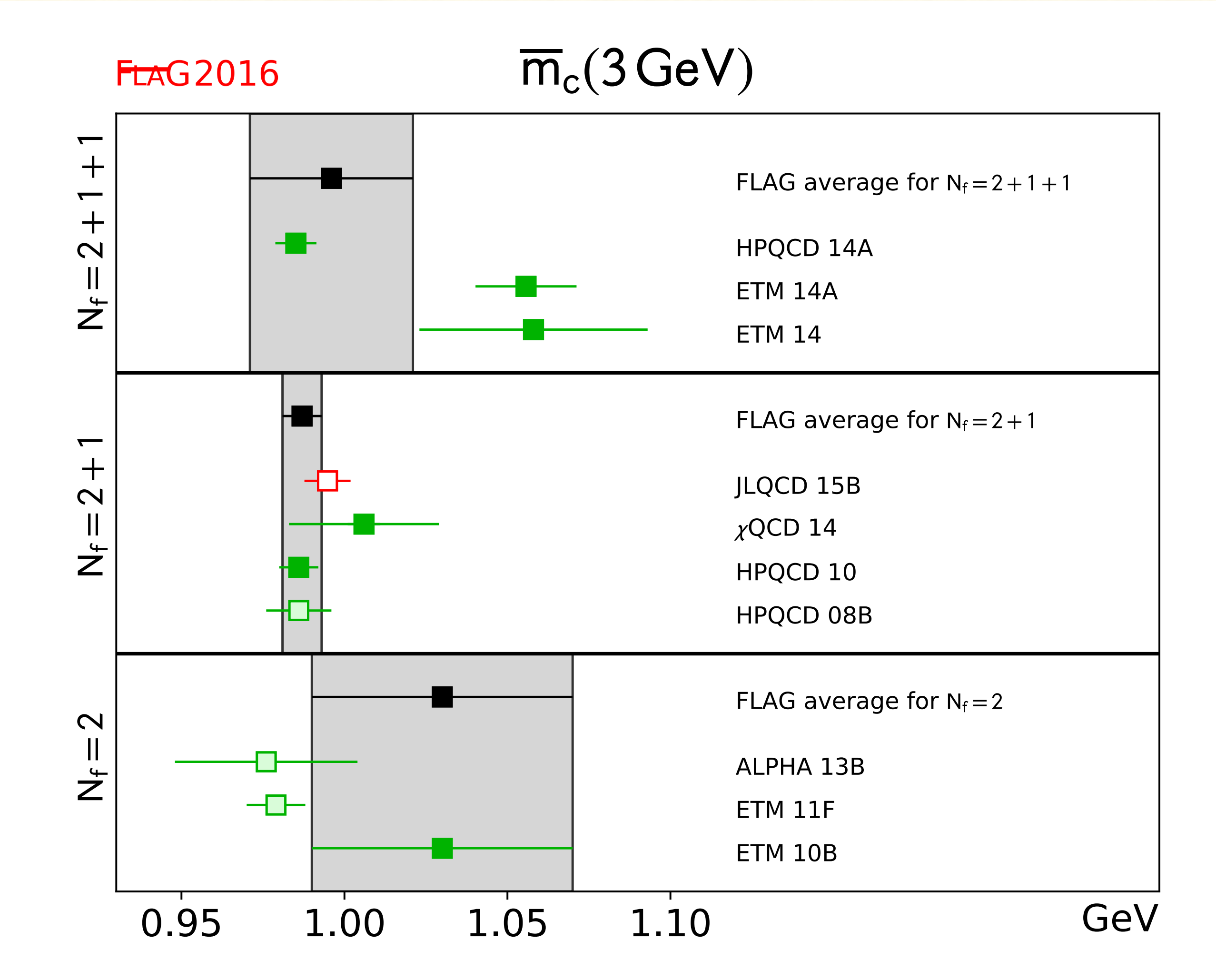
OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

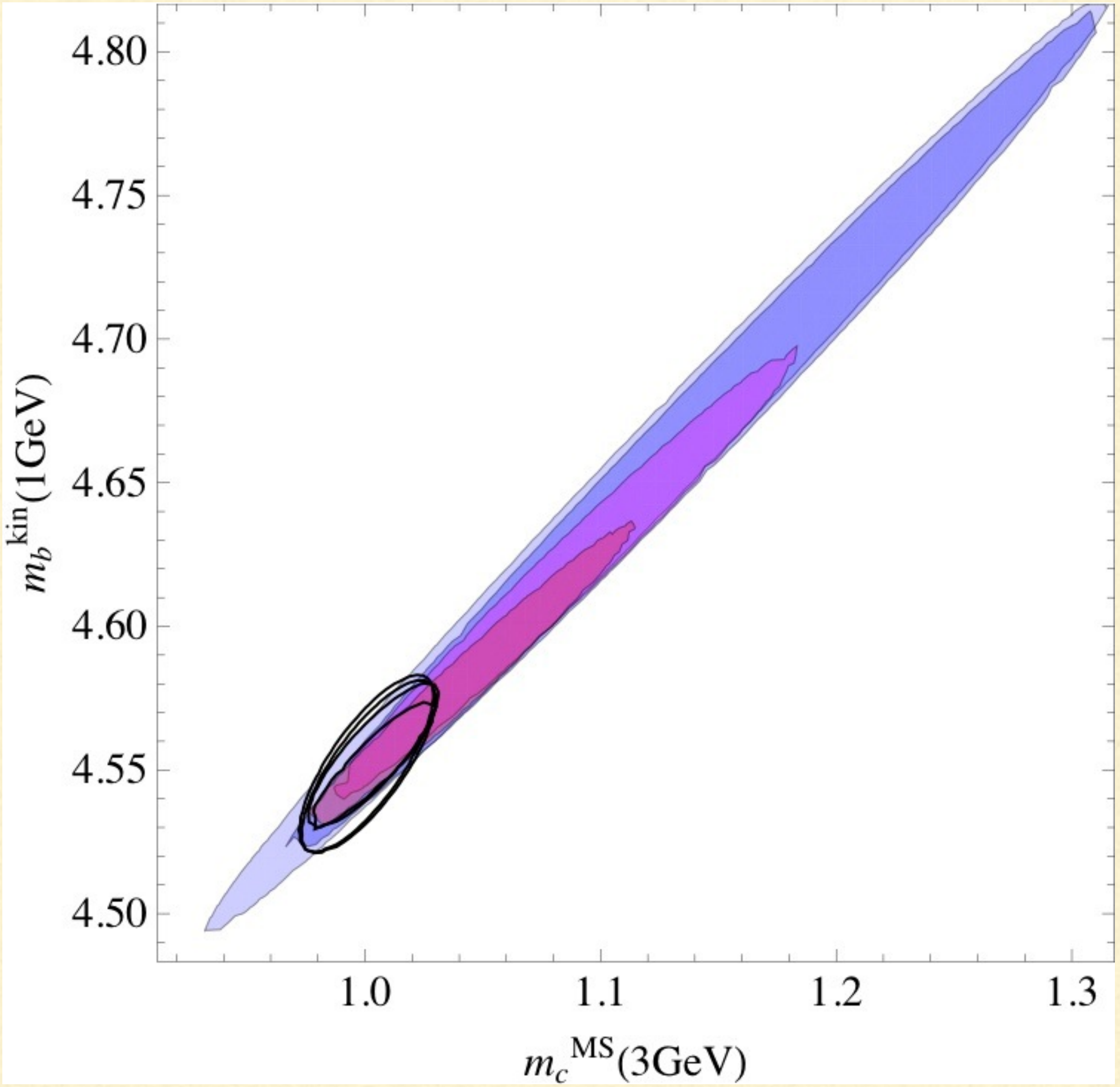
THE CURRENT HFLAV S.L. FIT

- kinetic scheme calculation based on 1411.6560,1107.3100; hep-ph/0401063
- includes all $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$ corrections Melnikov, Biswas, Czarnecki, Pak, Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG, **Mannel**, Pivovarov, Rosenthal
- Reliability of the method depends on our control of higher order effects. **Quark-hadron duality violation** would manifest as inconsistency in the fit.
- reassessment of theoretical errors, realistic theoretical correlations following Schwanda, PG, 1307.4551
- external constraints: precise heavy quark mass determinations, mild constraints on μ^2_G from hyperfine splitting and ρ^3_{LS} from sum rules

Older fits, like Buchmuller & Flaecher (2005), Bauer et al (2004) (1S scheme) are not up-to-date

CHARM MASS





Remarkable improvement in the last decade. We used m_c as input to fix m_b , but higher precision on m_b should be exploited as well

FIT RESULTS

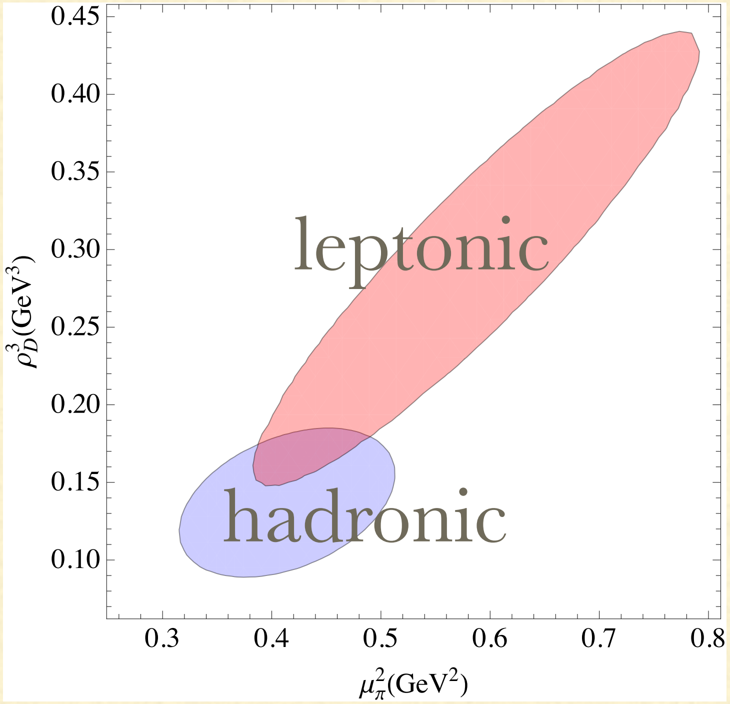
m_b^{kin}	$\overline{m}_c(3\,\text{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

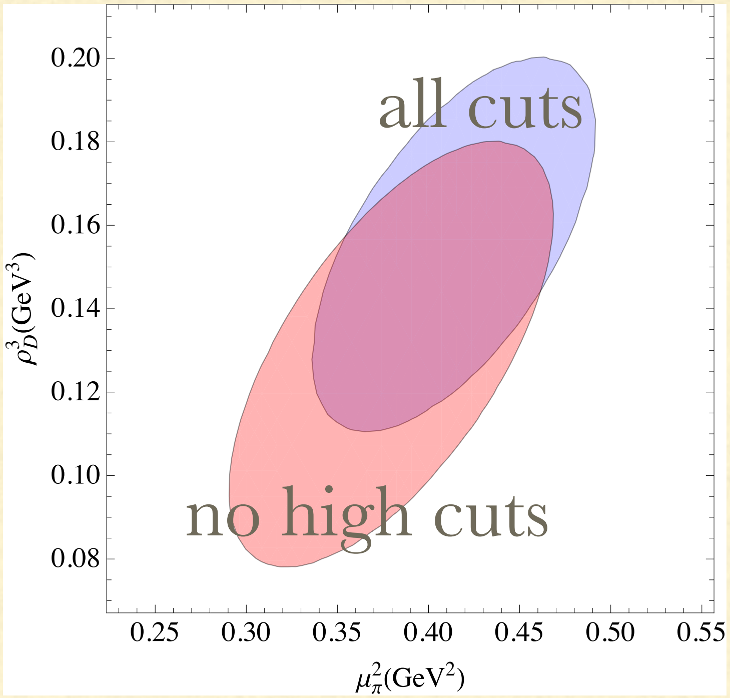
Alberti, Healey, Nandi, PG, 1411.6560

WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1\text{GeV}) - 0.85\,\overline{m}_c(3\text{GeV}) = 3.714 \pm 0.018\,\text{GeV}$$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results





HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

Mannel, Turczyk, Uraltsev 1009.4622

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$

Lowest Lying State Saturation Approx (LLSA) truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen, **Mannel** 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\, \mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\, \mu_G^2 \qquad \epsilon \sim 0.4\mathrm{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

INCLUDING HIGHER ORDERS

Healey, Turczyk, PG 1606.06174

- Using LLSA one observes a good convergence of the HQE in width and moments
- We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers
- The rest of the fit is unchanged, with slightly smaller theoretical errors

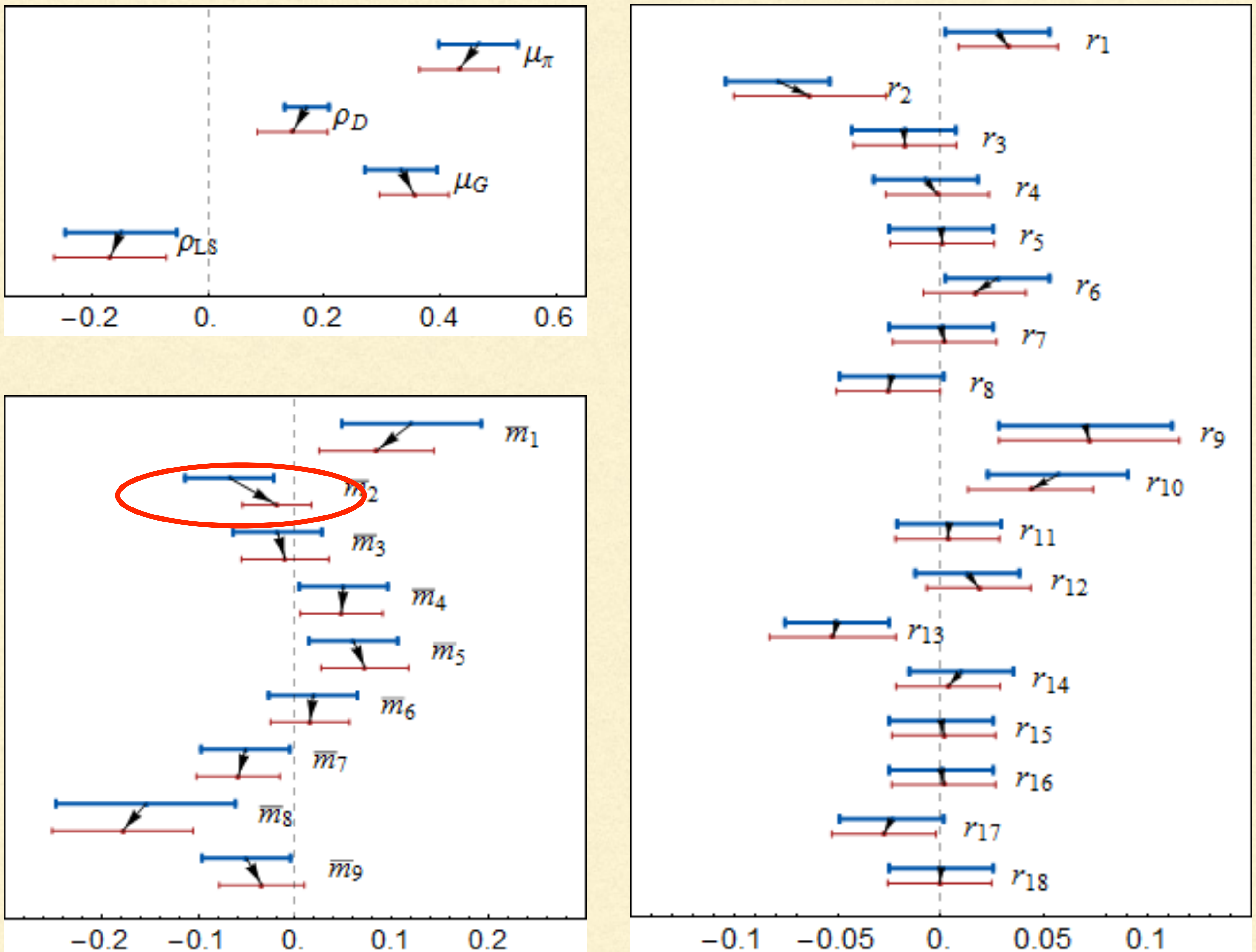
$$|V_{cb}| = 42.11(74) \times 10^{-3}$$ (0.25% reduction)

Using $m_c(2\text{GeV})$ and including the PDG average for m_b, 1.5% uncertainty on V_{cb}

$$\boxed{|V_{cb}| = 42.00(64) \times 10^{-3}}$$

SENSITIVITY TO HIGHER POWER CORRECTIONS

Moderate changes to the leading HQE m.e.

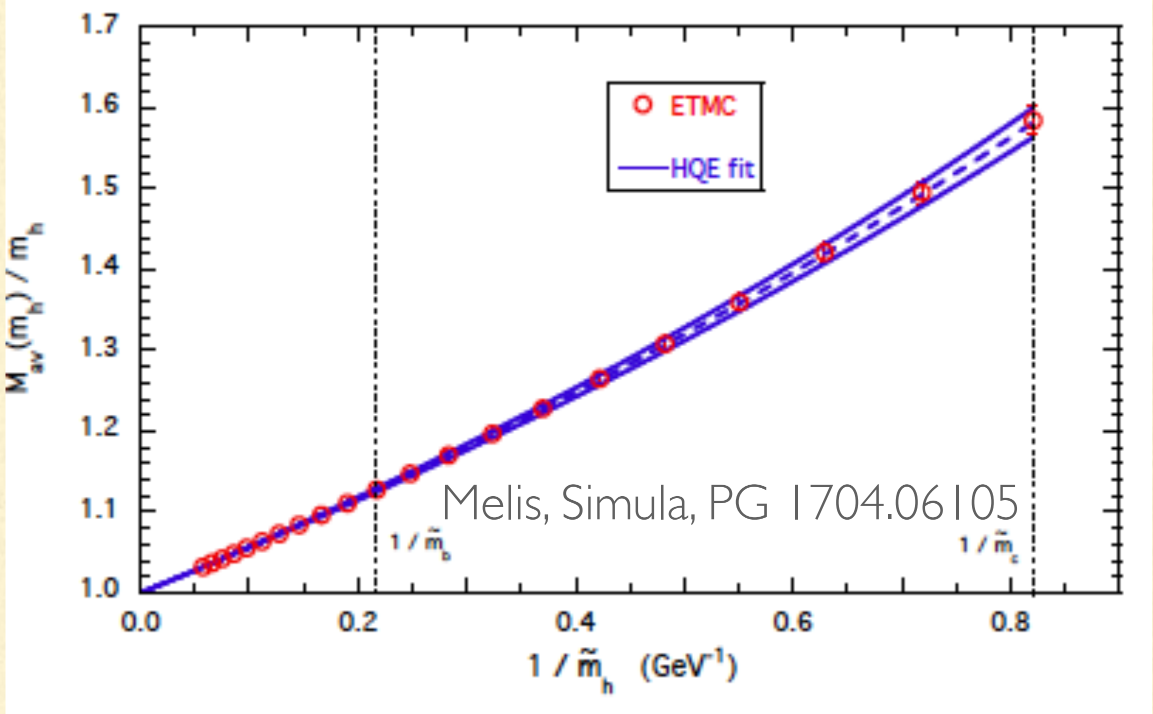


PROSPECTS for INCLUSIVE V_{cb}

- Theoretical uncertainties already dominant
- $O(\alpha_s/m_b^3)$ calculation under way
- $O(1/m_Q^{4,5})$ effects need further investigation but small effect on V_{cb}
- NNNLO corrections to total width feasible, needed for 1% uncertainty?
- Electroweak (QED) corrections require attention
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now
- **Lattice QCD** information on local matrix elements is the next frontier (eg Hashimoto method and meson masses)

MESON MASSES FROM ETMC

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \dots$$



Melis, Simula, PG 1704.06105

- on the lattice one can compute mesons for arbitrary quark masses
  see also Kronfeld & Simone hep-ph/0006345, 1802.04248
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m_π=210-450 MeV, heavy masses from m_c to $3m_c$, ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to $1/m^3$ corrections

RESULTS AND IMPLICATIONS

$$m_b^{\rm kin}(1\ \text{GeV}) = 4.61(20)\ \text{GeV}$$

$$\overline{\Lambda} = 0.552\ (26)\ \text{GeV}\ ,$$

$$\mu_\pi^2 = 0.321\ (32)\ \text{GeV}^2\ ,$$

$$\mu_G^2(m_b) = 0.253\ (25)\ \text{GeV}^2\ .$$

$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.153\ (34)\ \text{GeV}^3\ ,$$

$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.158\ (84)\ \text{GeV}^3\ .$$

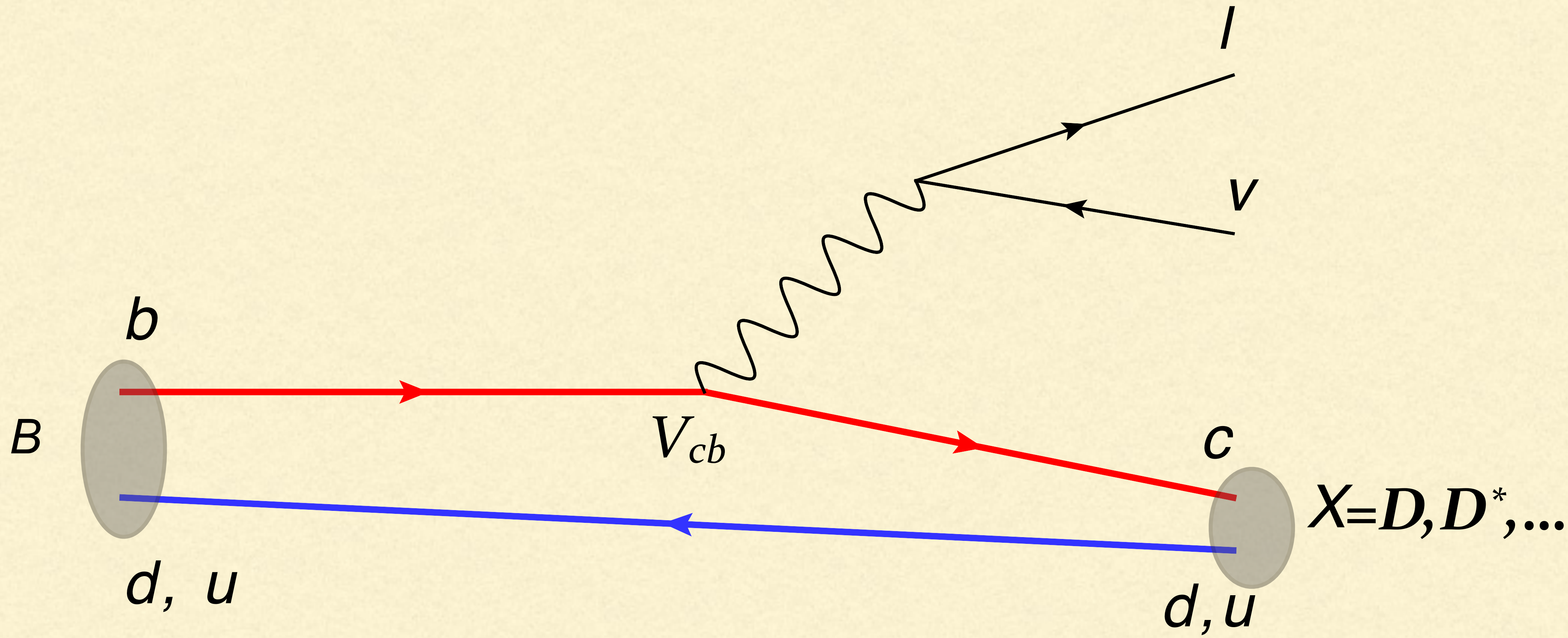
10% (30%) precision on dim 5 and 6 m.e. Competitive with the moments fits?

$$\mu_\pi^2|_B = \mu_\pi^2|_\infty - \frac{\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2)\ ,$$

$$\mu_G^2(m_b)|_B = \mu_G^2(m_b)|_\infty + \frac{\rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2)$$

- comparison with fits shows that $\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3 \approx -0.51(35)\,\text{GeV}^3$
- since $\rho_{\pi\pi}^3 + \rho_{\pi G}^3 + \rho_A^3 + \rho_S^3 \geq 0$
- it follows that $\mu_G^2(m_b) \geq 0.36(8)\text{GeV}^2$
- good consistency with fits, but only 4 combinations are determined

EXCLUSIVE DECAYS



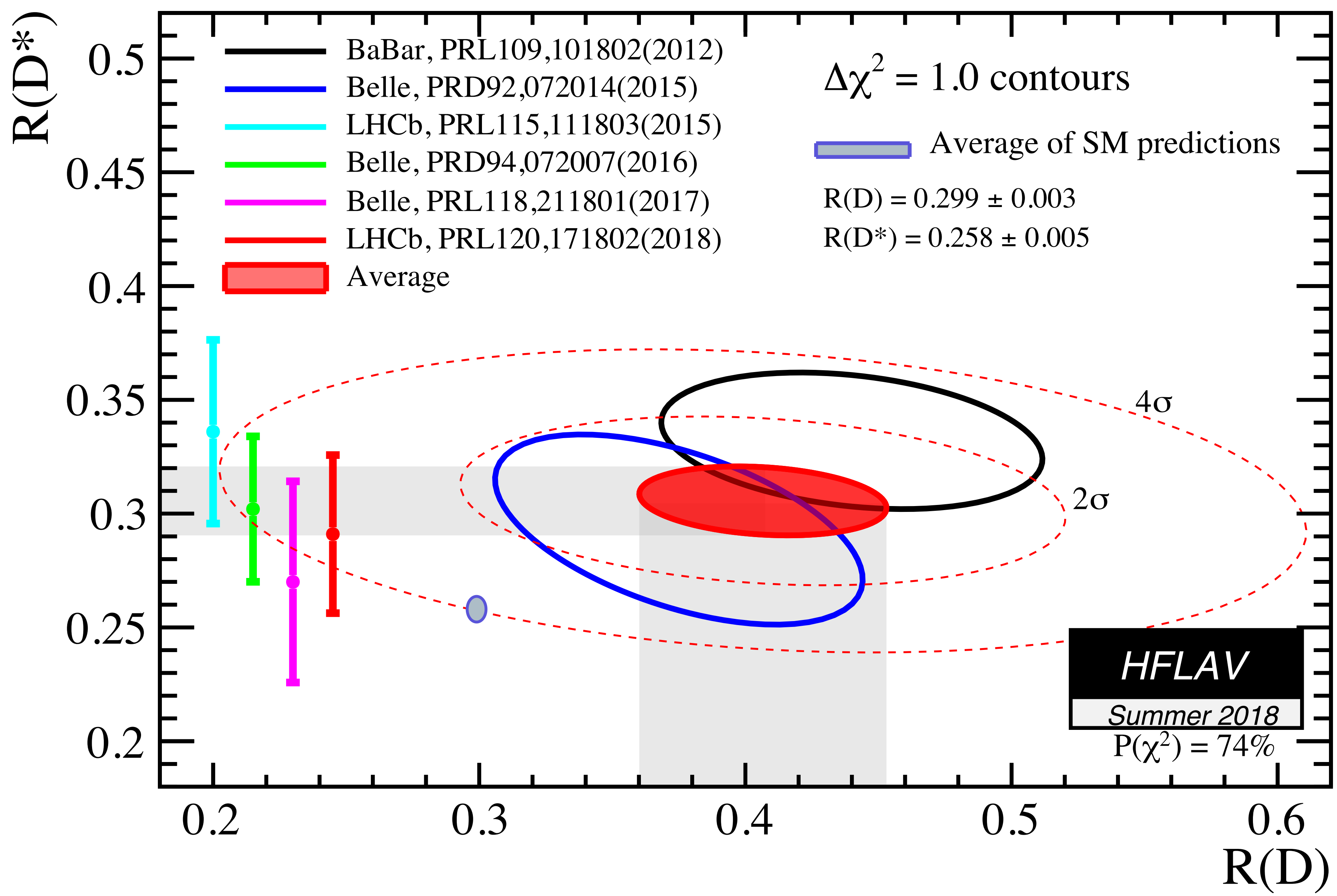
There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

$$\langle D|\bar{c}\gamma^{\mu}b|B\rangle \propto f_{+,0}(q^2)$$

Information on FFs comes from LQCD (at high q^2), LCSR (at low q^2), exp…

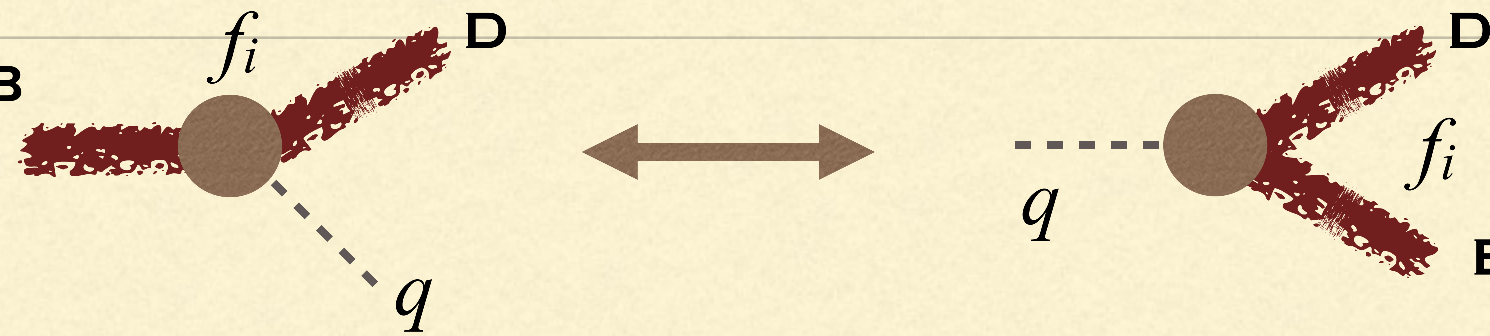
VIOLATION OF LFU WITH TAUS

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



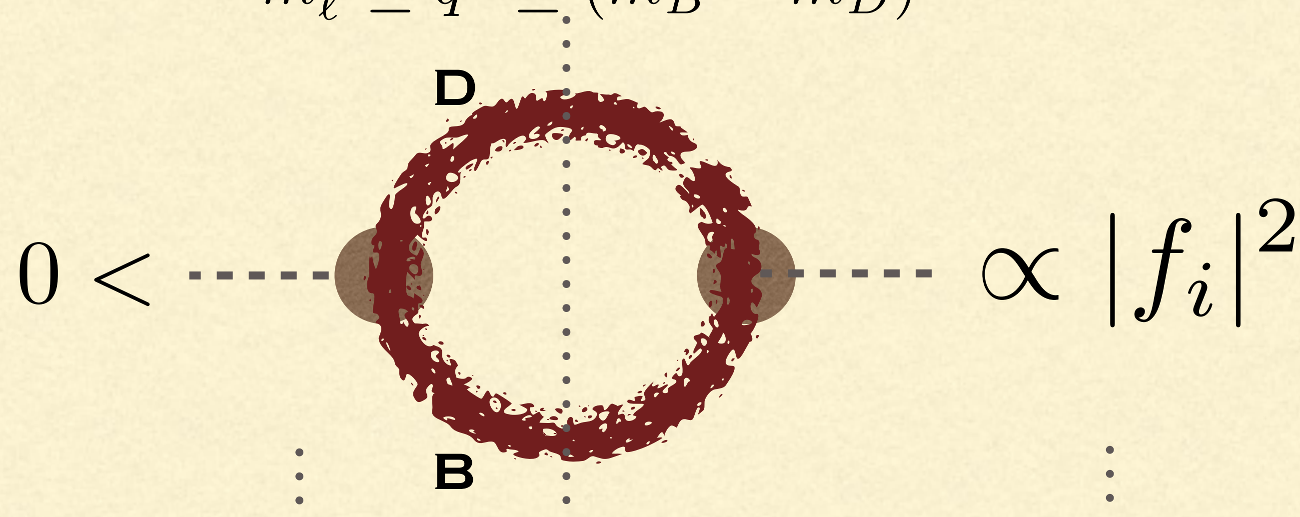
MODEL INDEPENDENT FF PARAMETRIZATION

CROSSING +
ANALITYCITY

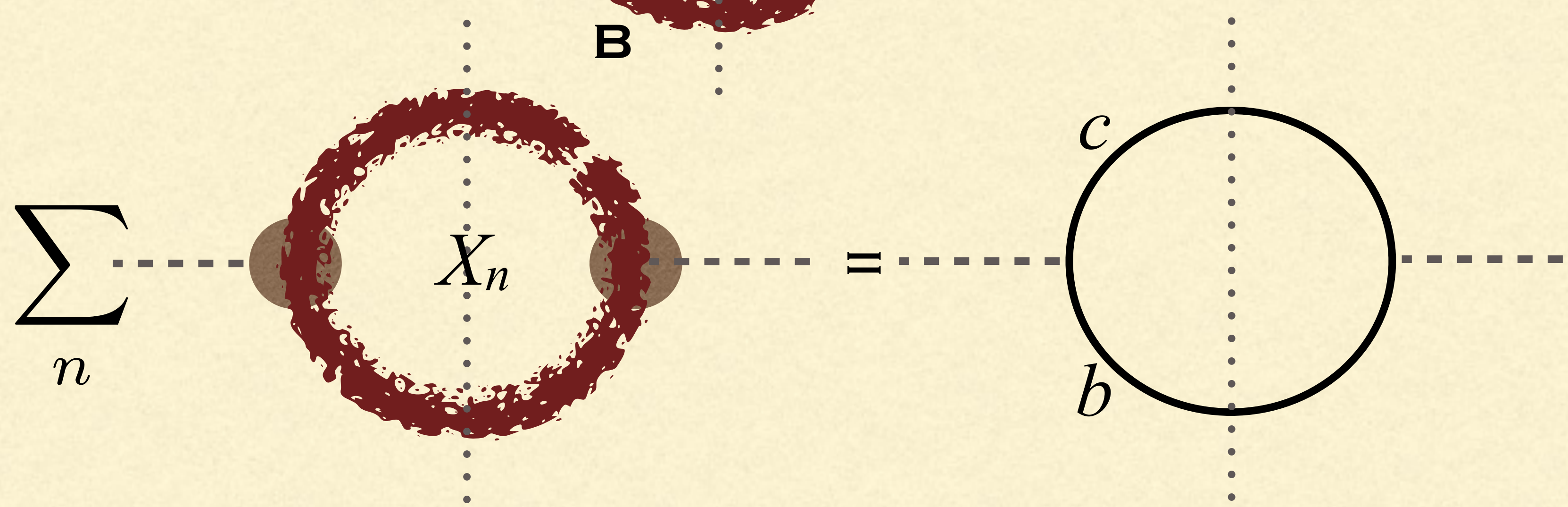


PHYSICAL SEMILEPTONIC REGION
$m_\ell^2 \leq q^2 \leq (m_B - m_D)^2$

2-POINT CORRELATOR CUTS
$q^2 \geq (m_B + m_D)^2$



POLES AT $q^2 = m^2_{Bc}$ ETC



+PERT CORR
(+CONDENSATES)

USING QUARK-HADRON DUALITY

UNITARITY CONSTRAINTS

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \qquad 0 < z < 0.056$$

$$f_i(z) = \frac{\sqrt{\chi_i}}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n$$

BGL Boyd Grinstein Lebed 1997

BLASCHKE FACTORS REMOVE POLES BELOW THRESHOLD

PHASE SPACE FACTORS

TRUNCATED AT ORDER N

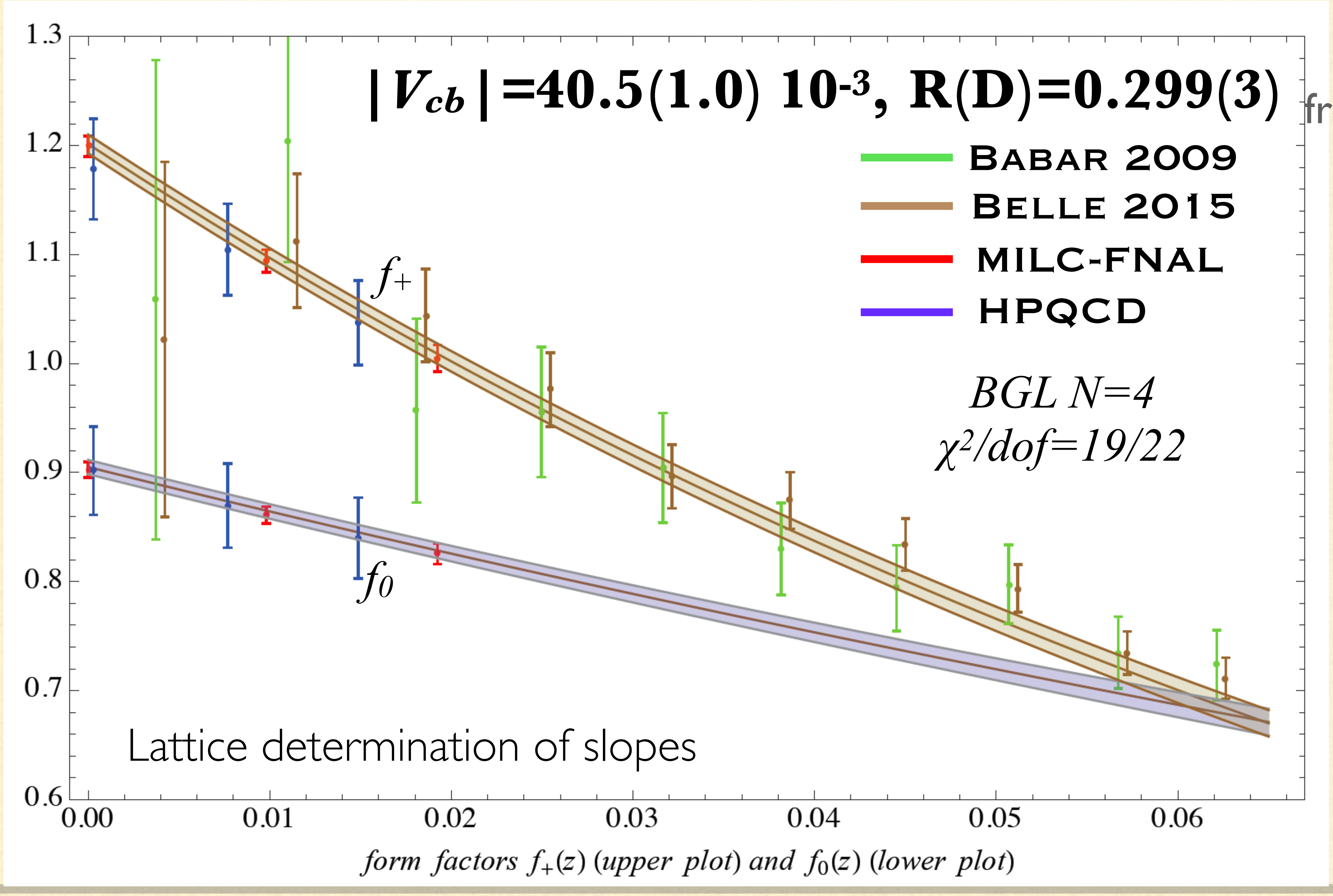
$$\sum_{n=0}^{N} (a_n^i)^2 < 1$$

WEAK UNITARITY CONSTRAINTS

assuming saturation
by single hadron channel

LATTICE + EXP FIT for $B \to Dl\nu$

Bigi, PG 1606.08030



2.4σ from exp

Strong Unitarity Bounds

Information on other channels makes the bounds tighter.
HQS implies that all $B^{(*)} \to D^{(*)}$ ff either vanish or are prop to the Isgur-Wise function: any ff F_j can be expressed as

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\mathrm{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the a_i space for S, P, V, A currents

Caprini Lellouch Neubert (CLN, 1998) exploit NLO HQET relations between form factors + QCD sum rules to **reduce parameters** for ff… up to < 2% uncertainty, never included in exp analysis.

$$h_{A1}(z) = h_{A1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2,$$

nice: only 2+2 parameters! but theoretical uncertainty?

$|V_{cb}|$ from $B\to D^*l\nu$ (traditional way)

So far LQCD gives only light lepton FF at zero recoil, w=1, where rate vanishes. Experimental results must therefore be **extrapolated to zero-recoil**

Exp error only ~1.3%: $\mathcal{F}(1)\eta_{ew}|V_{cb}|$ =35.61(45) $\times 10^{-3}$
(HFLAV extrapolation with CLN parameterization)

Two unquenched lattice calculations

$\mathcal{F}(1)$ =0.906(13) **$\mathcal{F}(1)$ =0.895(26)**

Bailey et al 1403.0635 (FNAL/MILC) Harrison et al 1711.11013 (HPQCD)

Using their average 0.904(12): ***$|V_{cb}|$= 39.13(75) 10^{-3}***

~ 2.9σ or ~ 7% from inclusive determination 42.00(65) 10^{-3}

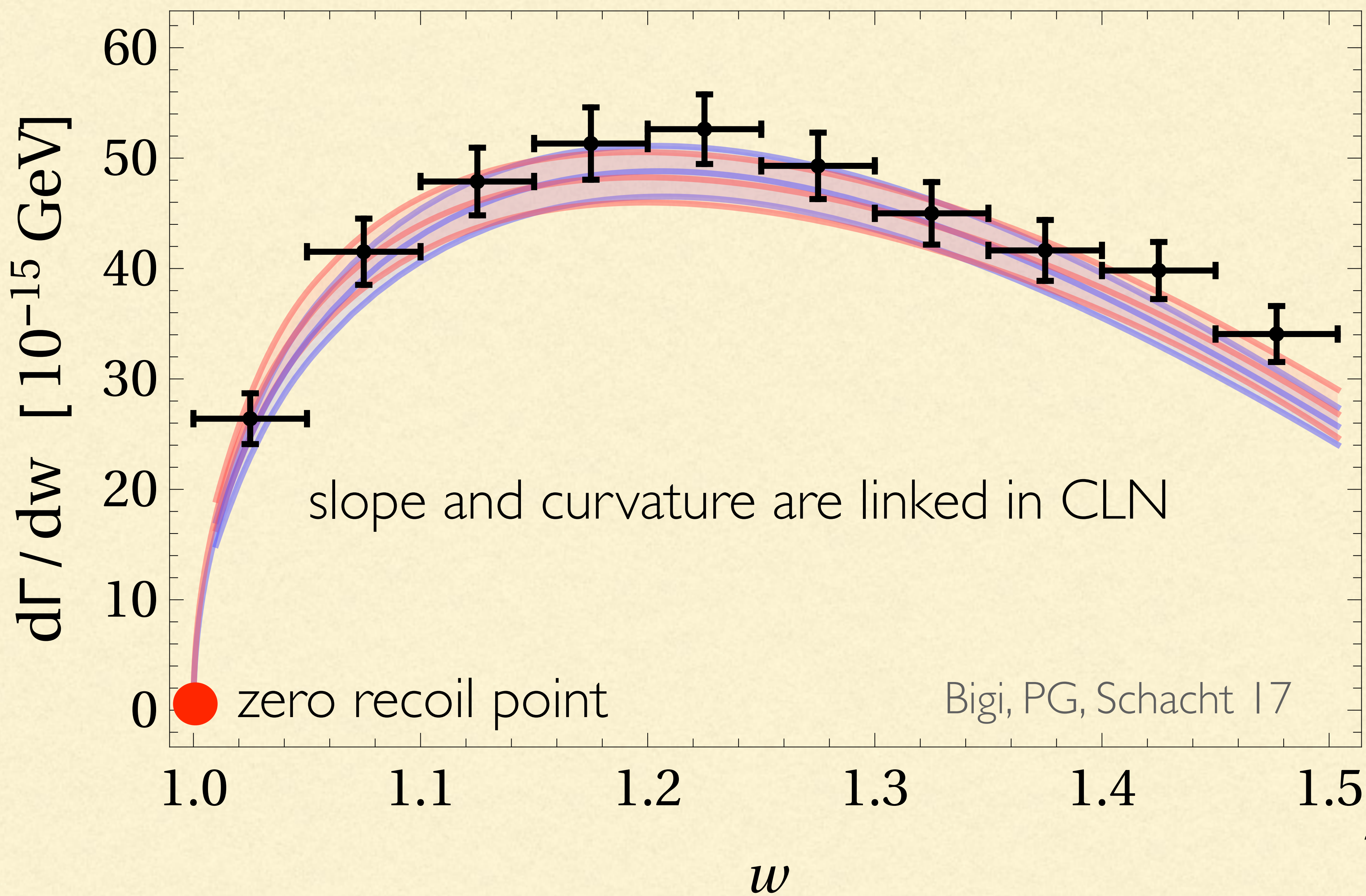
PG,Healey,Turczyk 2016

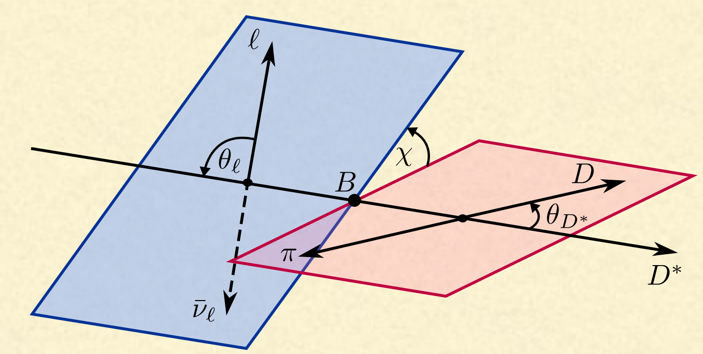
Heavy quark sum rules F(1)< 0.925 and estimate of inelastic contribution F(1)≈0.86

Mannel, Uraltsev, PG, 2012

2017 preliminary Belle analysis **1702.01521**

w and angular deconvoluted distributions (independent of parameterization).
All previous analyses are CLN based.





see also
Kobach, Grinstein

$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

Bands show two parametrizations both fitting data well, with 6% different V_{cb}

HQS breaking in FF relations

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \dots\right] \quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$c_{b,c}$ can be computed using subleading IW functions from QCD sumrules

Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

RATIOS $\frac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \dots\right] \quad w_1 = w - 1$

Roughly $\epsilon_c \sim 0.25, \quad \epsilon_c^2 \sim 0.06$ but coefficients??

In a few cases we can compare these ratios with recent lattice results: there are 5-13% differences, always > NLO correction. For ex.:

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\mathrm{LQCD}} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\mathrm{HQET}} = 0.966(28)$$

Looking at NLO HQET corrections, NNLO can be sizeable, naturally O(10-20)%

Updating Strong Unitarity Bounds

Fit to new Belle's data + total branching ratio (world average) in 1707.09509
with UPDATED strong unit. bounds (including uncertainties & LQCD inputs)

for reference CLN fit: $|V_{cb}|$=0.0392(12)

BGL Fit:	Data + lattice	Data + lattice + LCSR	Data + lattice	Data + lattice + LCSR
unitarity	weak	weak	strong	strong
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36
$\lvert V_{cb}\rvert$	0.0424 (18)	0.0413 (14)	0.0415 (13)	0.0406 $\left(^{+12}_{-13}\right)$

LCSR: Light Cone Sum Rule results from Faller, Khodjamirian, Klein, **Mannel**, 0809.0222

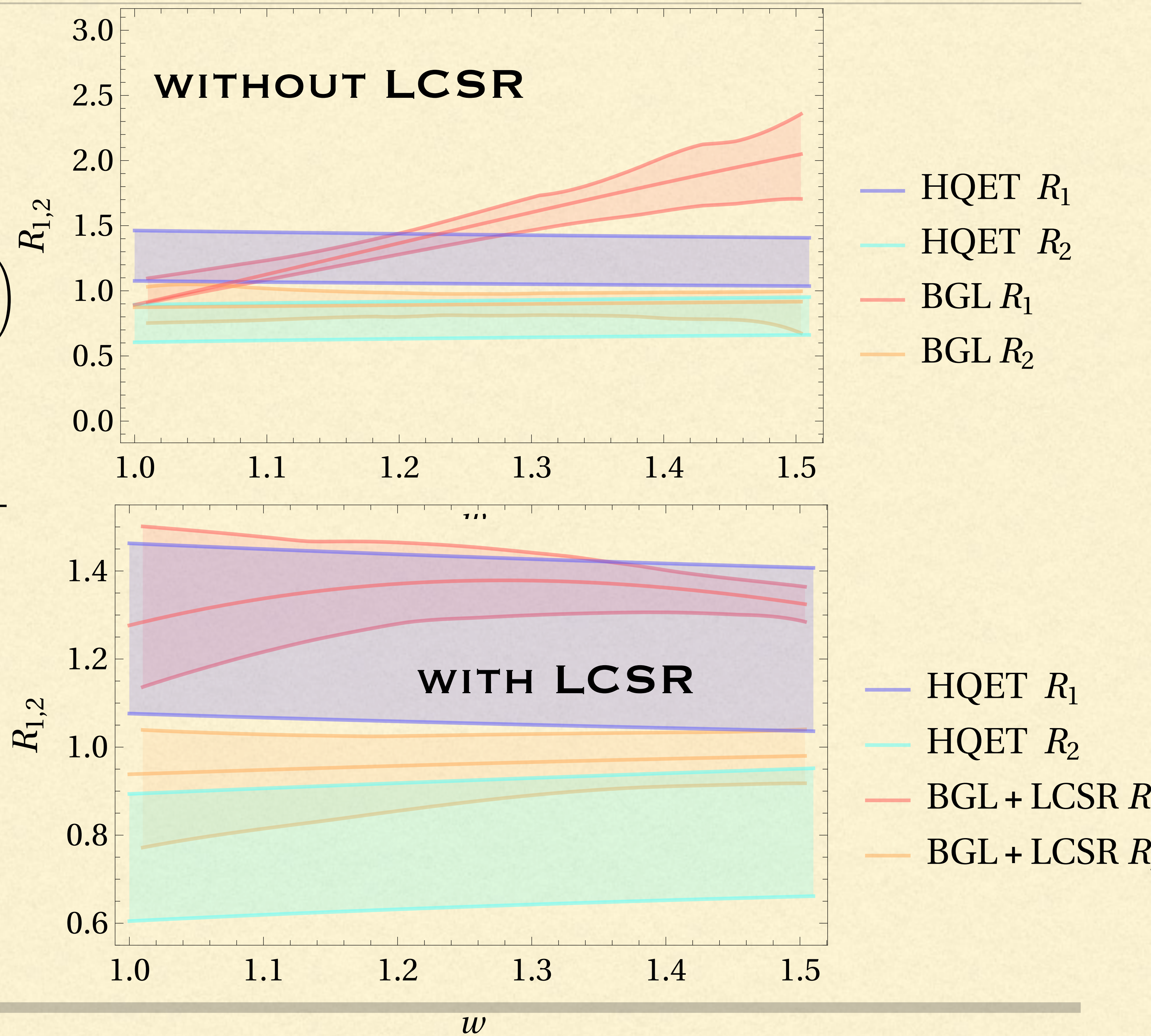
Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but 3.5-5% difference persists

CONSISTENCY WITH HQET

$$R_1(w) = \frac{V_4(w)}{A_1(w)}$$

$$R_2(w) = \frac{w-r}{w-1}\left(1 - \frac{1-r}{w-r}\frac{A_5(w)}{A_1(w)}\right)$$

Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)



NEW BELLE UNTAGGED RESULT

Results and Outlook @Belle

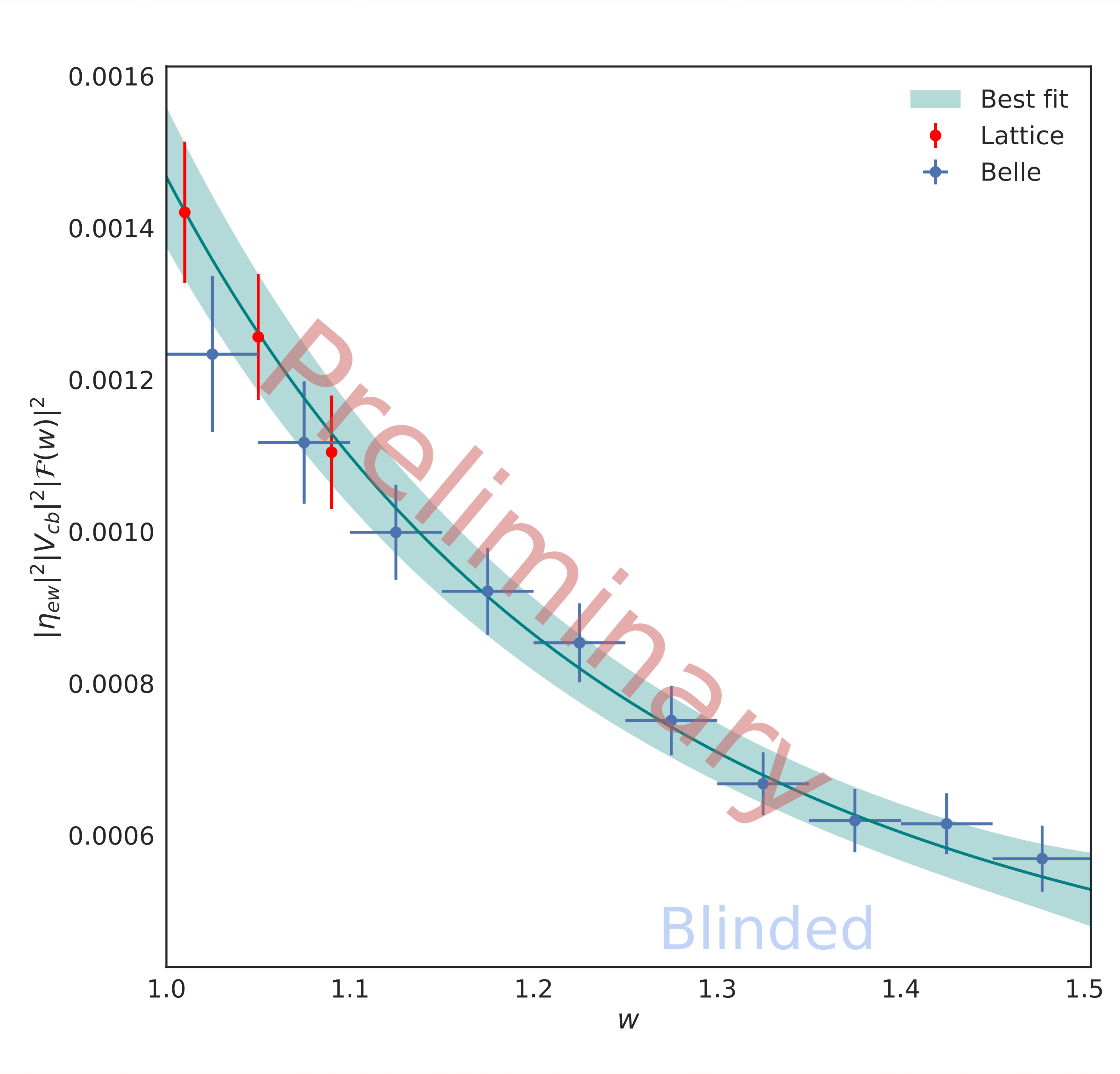
Link	Channel	Tag	$\lvert V_{cb}\rvert \times 10^3$ (CLN)	$\lvert V_{cb}\rvert \times 10^3$ (BGL)	Unfold	Notes
Phys.Rev. D82 112007	$D^* \ell^- \bar{\nu}_\ell$	No	35.5 ± 1.5			
1809.03290	$D^* \ell^- \bar{\nu}_\ell$	No	38.4 ± 0.9	42.5 ± 1.0	Soon	
1702.01521	$D^* \ell^- \bar{\nu}_\ell$	Had.	37.4 ± 1.3		Yes	Soon: Separate results $\ell = e$ and $\ell = \mu$
Phys.Rev. D93 no.3, 032006	$D \ell^- \bar{\nu}_\ell$	Had.	39.9 ± 1.3	40.8 ± 1.1		

cf. current PDG: $V_{cb,\text{incl.}} = (42.2 \pm 0.8) \times 10^{-3}$

Full Belle data, most precise result to date.
Parametrization dependence is not linked to
a particular dataset and has nothing to do with unfolding

BLINDED FNAL-MILC RESULTS

First unquenched calculation of B→D* form factors at non zero recoil

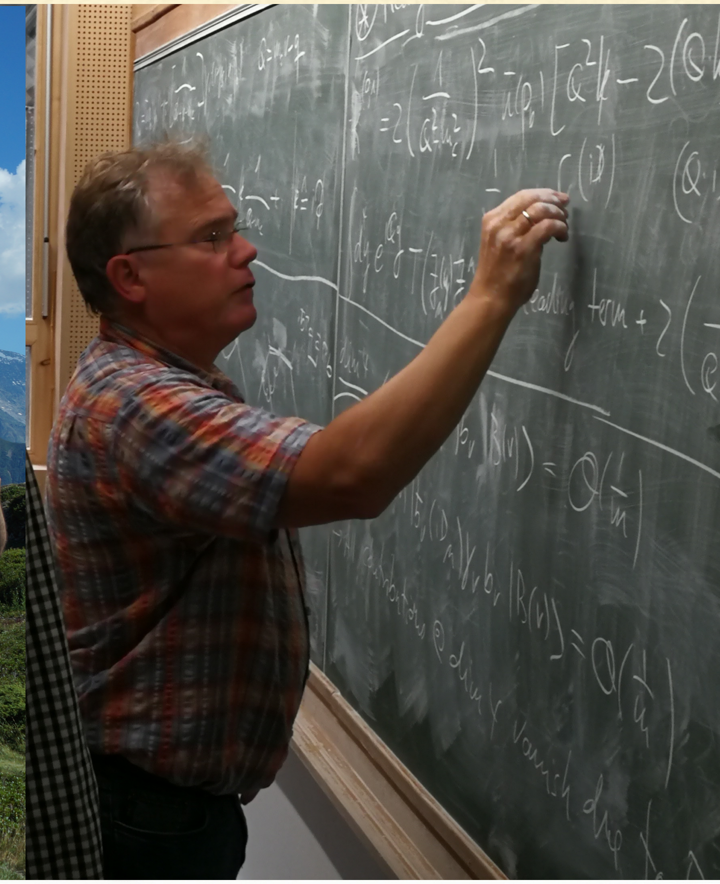


Vaquero, talk at CKM 2018

combined fit with Belle spectrum

CONCLUSIONS

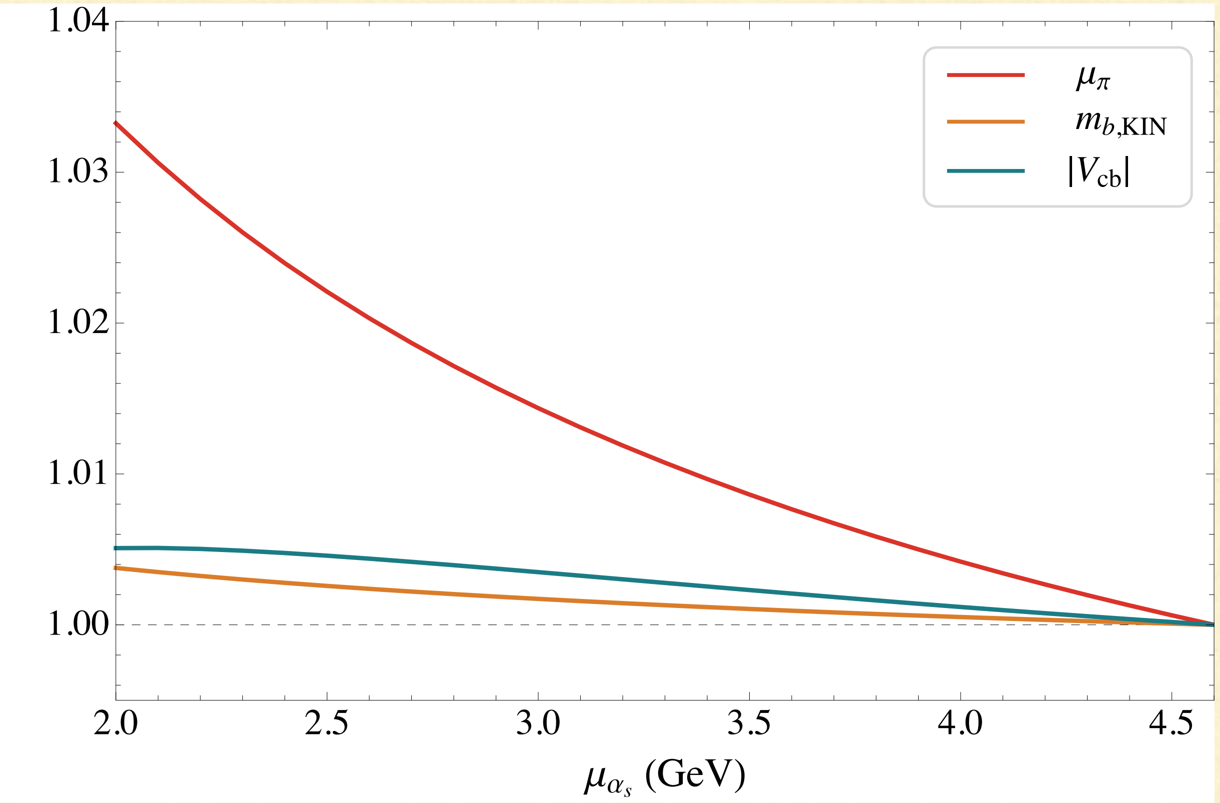
- The efforts to improve the theory of inclusive semileptonic B decays continue. No sign of inconsistency so far, several ways to improve and decrease uncertainty.
- We understand better higher power contributions and have checked their impact on the fit. They seem to have been tamed. LQCD determination of HQE parameters from masses is promising
- The choice of parametrization has a surprisingly large effect on the extraction of V_{cb} from B->D*lv.
- The V_{cb} puzzle is not yet over. The exclusive determination was biased and uncertainties underestimated, but we need (unblinded) LQCD results to understand the level of agreement with the inclusive one.

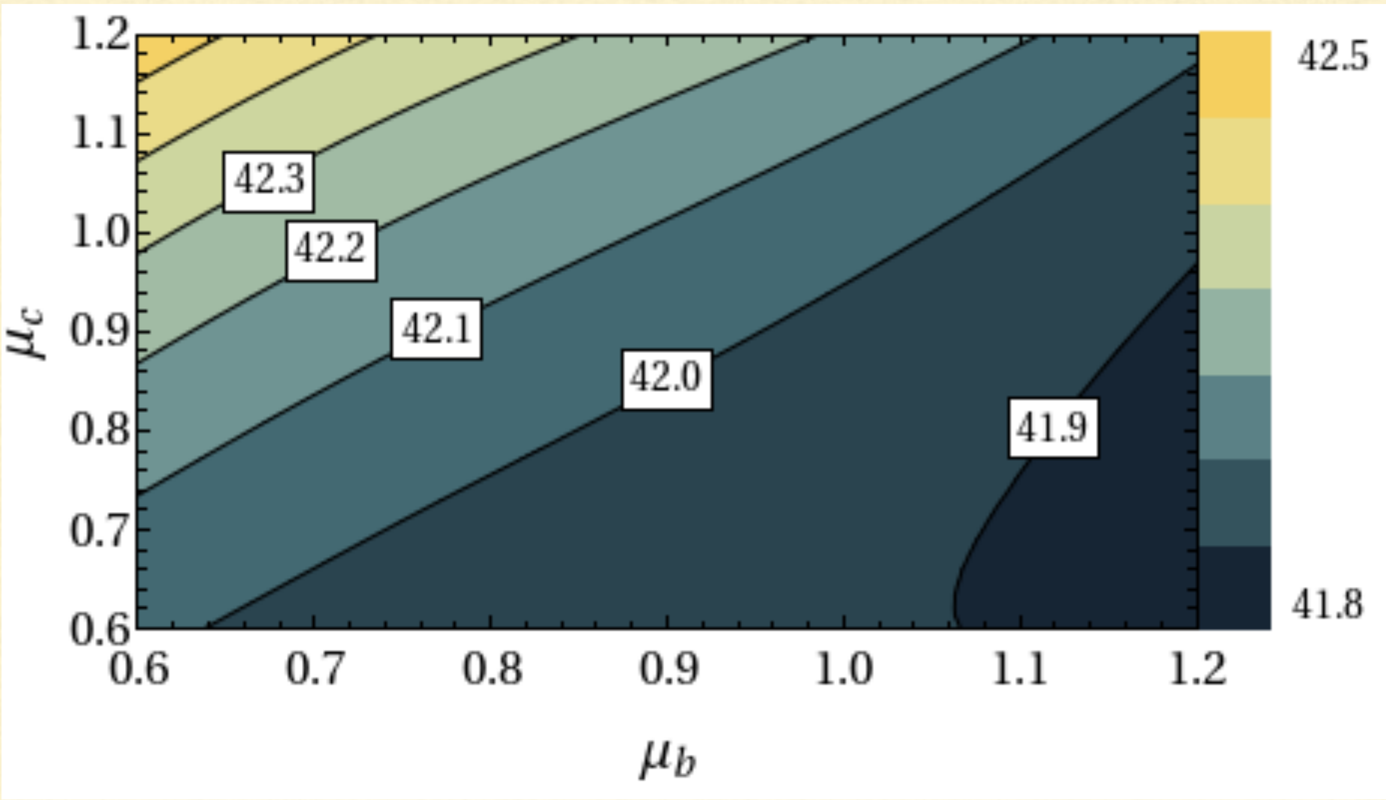


Les Houches 2017 summer school
EFT in particle physics and cosmology

BACKUP

SCALE DEPENDENCE

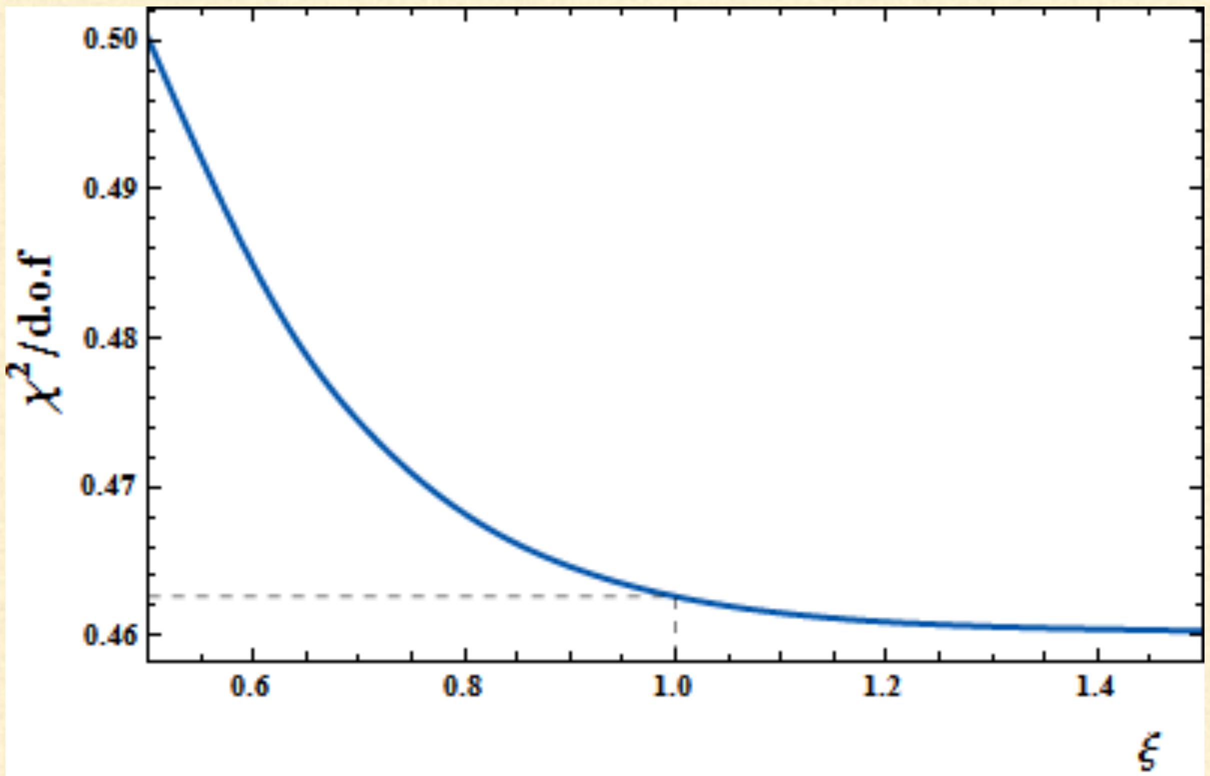


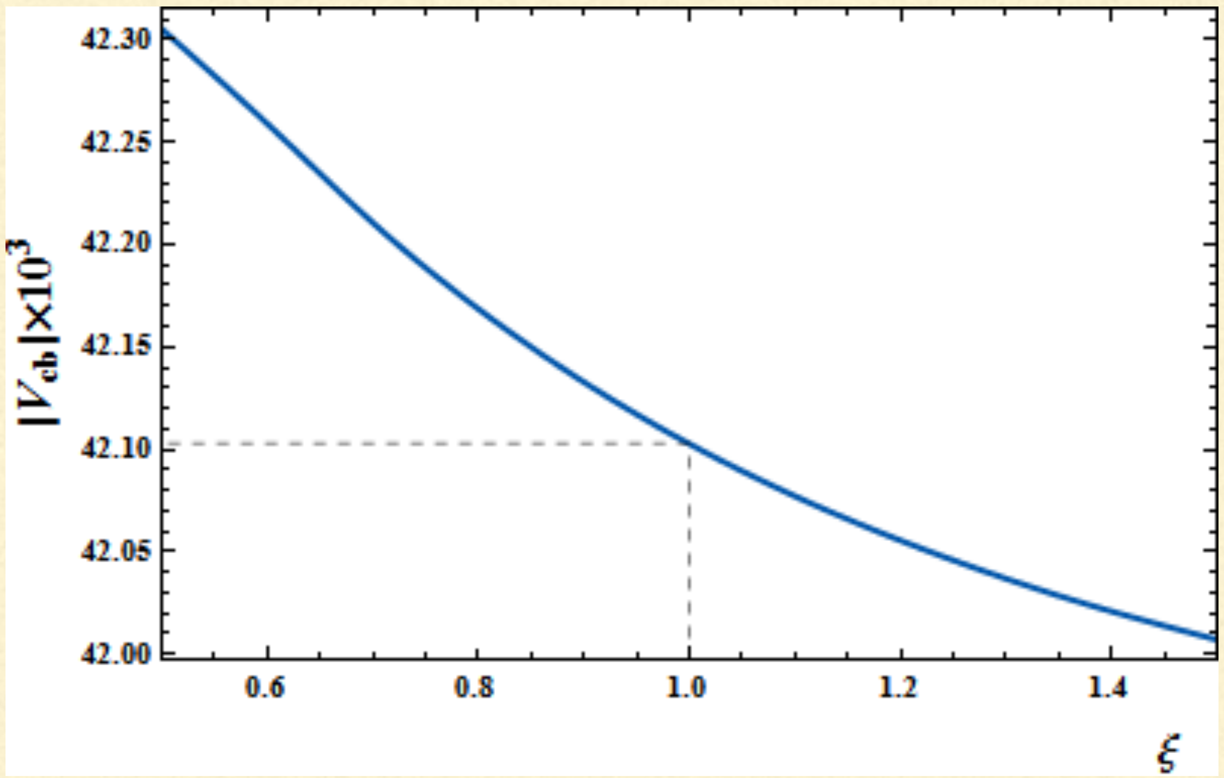


dependence on the scale of α_s

dependence on the kin scale of m_c and m_b

DEPENDENCE ON LLSA UNCERTAINTY



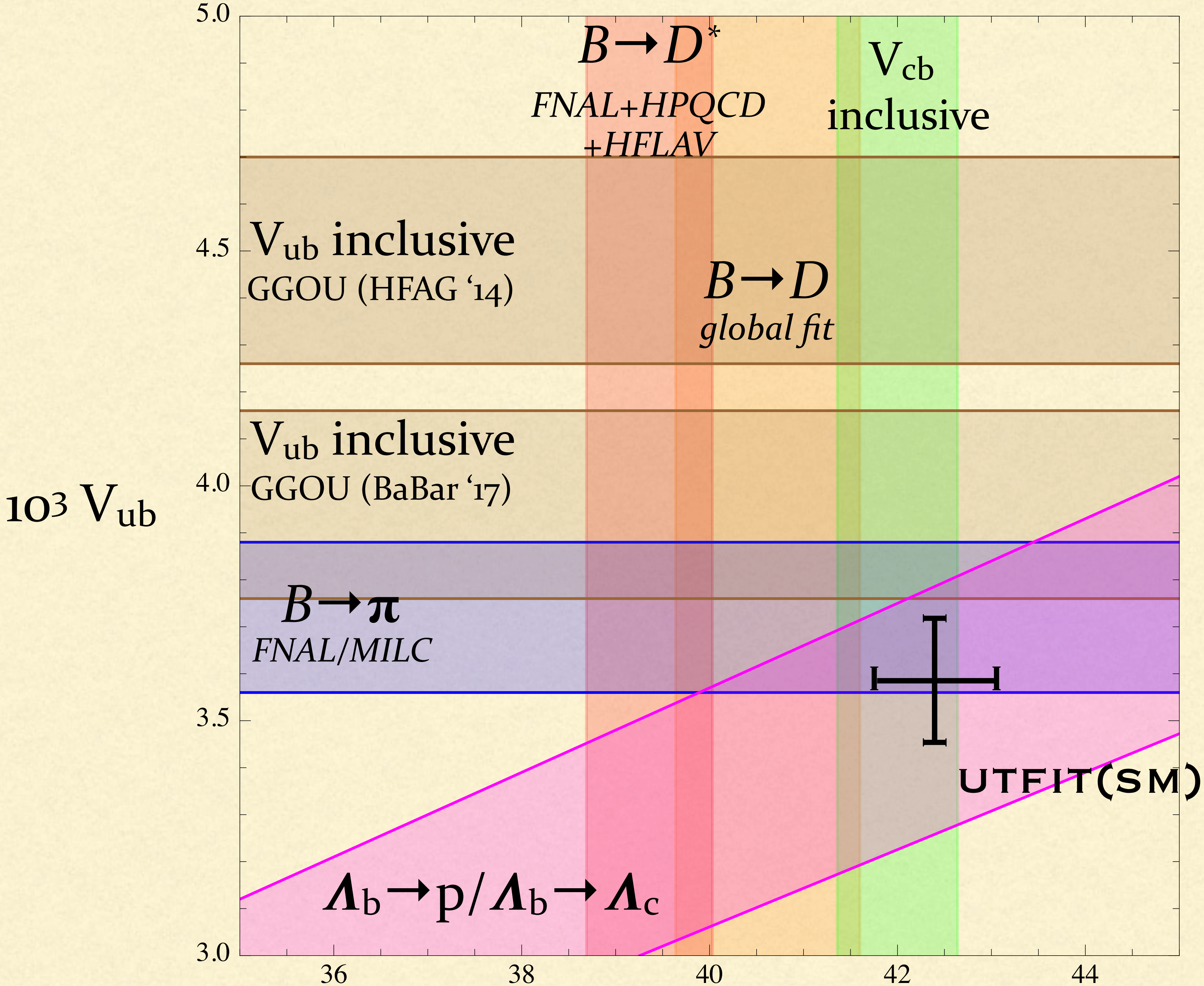


if we rescale all LLSA uncertainties by a factor ξ the results change very little.
Similar tiny deviations if we vary ε between 0.3 and 0.5 GeV

RECENT DEVELOPMENTS

- New Belle $B \to D^* l\nu$ tagged analysis (deconvoluted spectra)
- Strong dependence of V_{cb} on FF parametrisation (CLN vs BGL)
- BGL alone may be equally misleading: role of HQS constraints
- New Belle $B \to D^* l\nu$ tagged analysis with both CLN and BGL
- Upcoming FNAL/MILC results on all $B \to D^* l\nu$ FFs at non-zero recoil

STATUS of V_{cb} and V_{ub}



The size of NLO corrections varies strongly. Some ff are protected by Luke's theorem (no 1/m corrections at zero recoil), others are linked by kinematic relations at max recoil to those protected

NNLO corrections can be sizeable and are naturally O(10-20)%

$$\frac{F_j(w)}{V_1(w)} = A_j \left[1 + B_j\, w_1 + C_j\, w_1^2 + D_j\, w_1^3 + \ldots\right]$$

F_j	A_j	B_j	C_j	D_j
S_1	1.0208	−0.0436	0.0201	−0.0105
S_2	1.0208	−0.0749	−0.0846	0.0418
S_3	1.0208	0.0710	−0.1903	0.0947
P_1	1.2089	−0.2164	0.0026	−0.0007
P_2	0.8938	−0.0949	0.0034	−0.0009
P_3	1.0544	−0.2490	0.0030	−0.0008
V_1	1	0	0	0
V_2	1.0894	−0.2251	0.0000	0.0000
V_3	1.1777	−0.2651	0.0000	0.0000
V_4	1.2351	−0.1492	−0.0012	0.0003
V_5	1.0399	−0.0440	−0.0014	0.0004
V_6	1.5808	−0.1835	−0.0009	0.0003
V_7	1.3856	−0.1821	−0.0011	0.0003
A_1	0.9656	−0.0704	−0.0580	0.0276
A_2	0.9656	−0.0280	−0.0074	0.0023
A_3	0.9656	−0.0629	−0.0969	0.0470
A_4	0.9656	−0.0009	−0.1475	0.0723
A_5	0.9656	0.3488	−0.2944	0.1456
A_6	0.9656	−0.2548	0.0978	−0.0504
A_7	0.9656	−0.0528	−0.0942	0.0455

HQS breaking in FF relations

HQET: $F_i(w) = \xi(w)\left[1 + c^i_{\alpha_s}\frac{\alpha_s}{\pi} + c^i_b\,\epsilon_b + c^i_c\,\epsilon_c + \ldots\right]$ $\quad \epsilon_{b,c} = \overline{\Lambda}/2m_{b,c}$

$c_{b,c}$ can be computed using subleading IW functions from QCD sumrules
Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

RATIOS $\quad \dfrac{F_j(w)}{V_1(w)} = A_j\left[1 + B_j\,w_1 + C_j\,w_1^2 + D_j\,w_1^3 + \ldots\right]$ $\quad w_1 = w - 1$

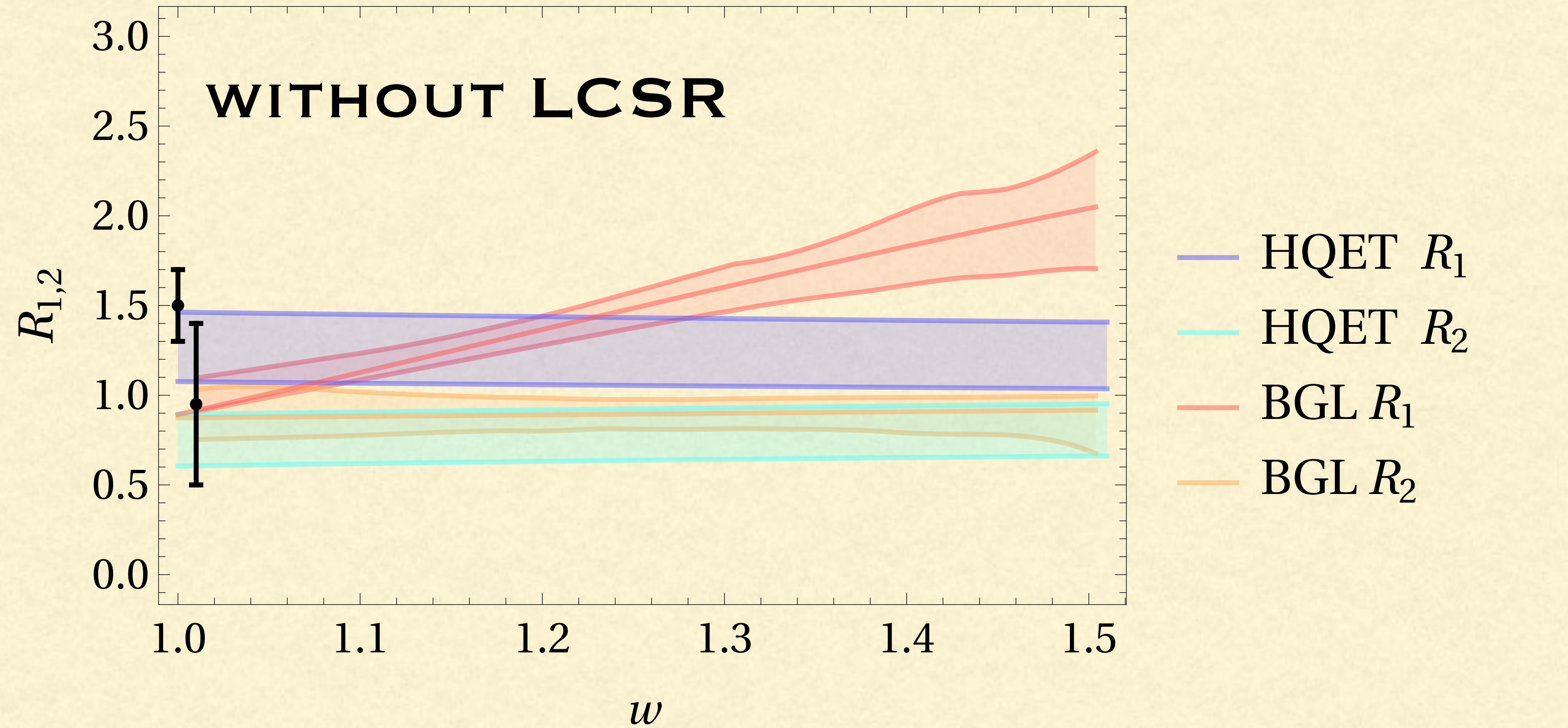
Roughly $\quad \epsilon_c \sim 0.25, \qquad \epsilon_c^2 \sim 0.06 \qquad$ but coefficients??

In a few cases we can compare these ratios with recent lattice results:
there are 5-13% differences, always > NLO correction. For ex.:

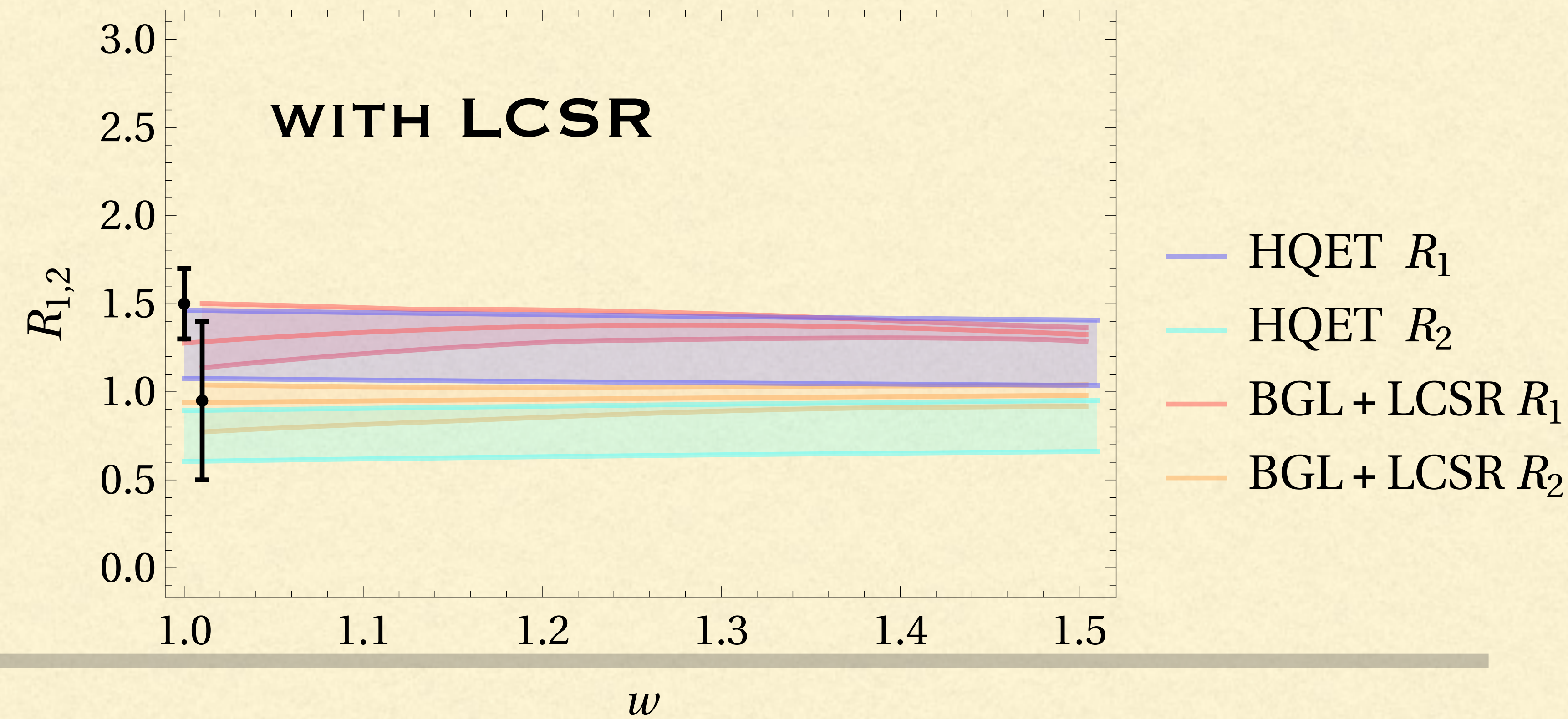
$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET} = 0.966(28)$$

CONSISTENCY WITH HQET

Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)



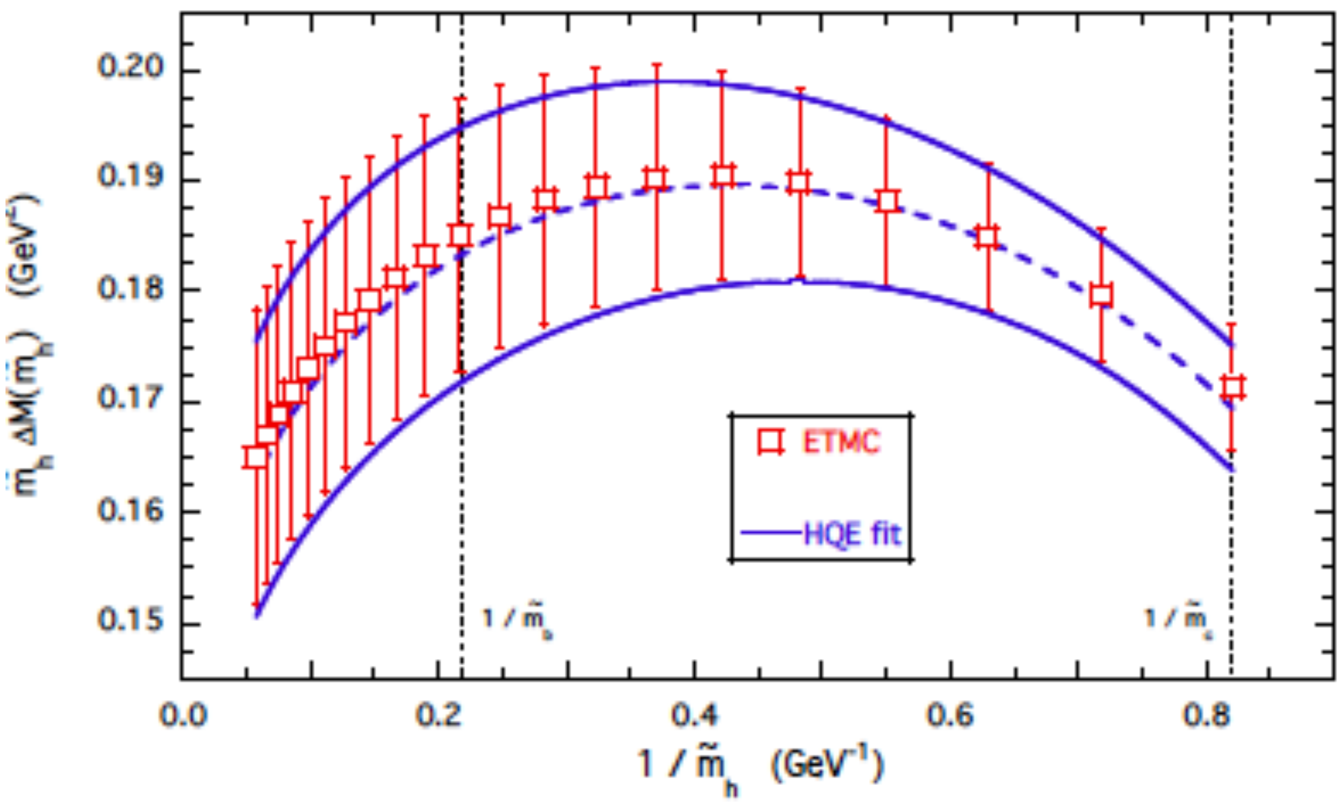
black points from preliminary FNAL-MILC calculation according to Bernlochner et al 1708.07134 (before continuum and chiral extrapolations…)



HQE expansion parameters from $\Delta M(\widetilde{m}_h)$

The chain equation can be extended beyond the b quark point, we choose $n \sim 20$ ($\widetilde{m}_h \simeq 4\widetilde{m}_b$),

$$\widetilde{m}_h^{(n)} \frac{\Delta M(\widetilde{m}_h^{(n)})}{c_G(\widetilde{m}_h^{(n)}, \widetilde{m}_b)} = \widetilde{m}_c \frac{\Delta M(\widetilde{m}_c)}{c_G(\widetilde{m}_c, \widetilde{m}_b)} \prod_{i=2}^{n} \overline{y}_{\Delta M}(\widetilde{m}_h^{(i)}, \lambda) \ .$$



We apply a correlated fit based on the HQE expansion

$$\widetilde{m}_h \Delta M(\widetilde{m}_h) = \frac{2}{3} c_G(\widetilde{m}_h, \widetilde{m}_b) \mu_G^2(\widetilde{m}_b) + \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3}{3\widetilde{m}_h} + \frac{\Delta\sigma^4}{\widetilde{m}_h^4} \ .$$

Results of the dim-6 fit $\widetilde{m}_h > \widetilde{m}_c$

$$\mu_G^2(\widetilde{m}_b) = 0.250\ (18)_{\rm stat}\ (8)_{\rm syst}\ \mathrm{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.143\ (57)_{\rm stat}\ (21)_{\rm syst}\ \mathrm{GeV}^3$$

Results of the dim-6 fit $\widetilde{m}_h > 2\widetilde{m}_c$

$$\mu_G^2(\widetilde{m}_b) = 0.254\ (22)\ \mathrm{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.158\ (70)\ \mathrm{GeV}^3$$

Results of the dim-7 fit

$$\mu_G^2(\widetilde{m}_b) = 0.254\ (20)_{\rm stat}\ (9)_{\rm syst}\ \mathrm{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.173\ (74)_{\rm stat}\ (25)_{\rm syst}\ \mathrm{GeV}^3$$
$$\Delta\sigma^4 = 0.0092\ (58)_{\rm stat}\ (14)_{\rm syst}\ \mathrm{GeV}^4$$

Courtesy of A. Melis