Signals in Micro Pattern Gaseous Detectors, including resistive elements

RD51 Open Lectures, Dec. 11th 2017

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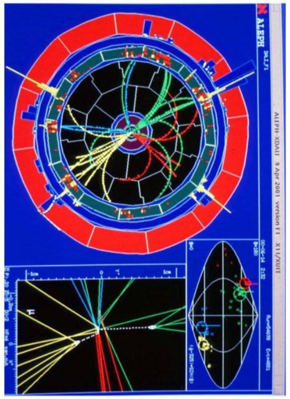
Creation of the Signal

Charged particles leave a trail of ions (and excited atoms) along their path: Electron-Ion pairs in gases and liquids, electron hole pairs in solids.

Photons from de-excitation are usually converted to electrons for detection.

The produced charges can be registered → Position measurement → Time measurement → Tracking Detectors ....



Cloud Chamber: Charges create drops → photography.
Bubble Chamber: Charges create bubbles → photography.
Emulsion: Charges 'blacked' the film.
Spark Chamber: Charges produce a conductive channel that create a discharge → photography

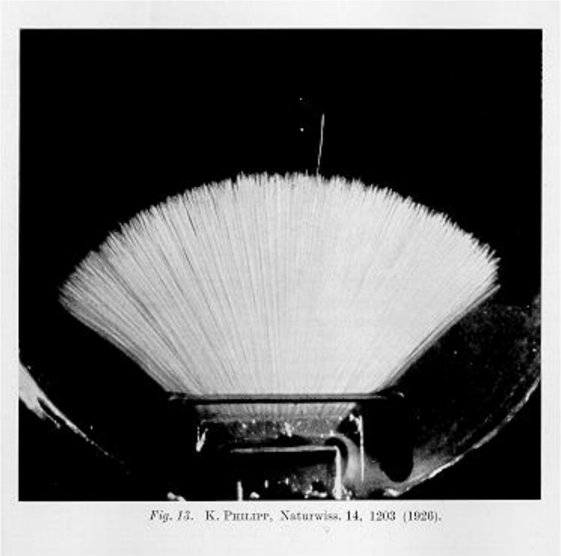
Gas and Solid State Detectors: Moving Charges (electric fields) induce electronic signals on metallic electrons that can be read by dedicated electronics.

→In some solid state detectors, the charge created by the incoming particle is sufficient (silicon strip, pad, pixel sensors)

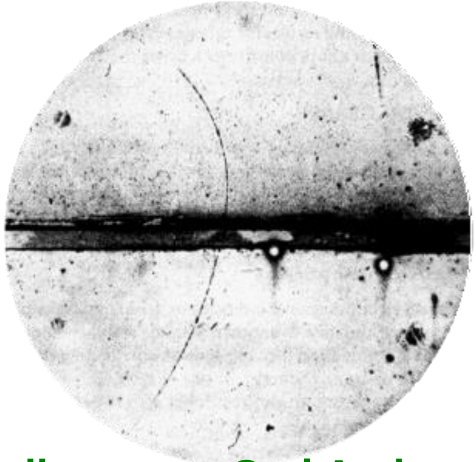
→In gas detectors (wire chambers, GEMs, Micromegas …) and some solid state detectors (APDs, LGADs, SiPMs) the charges are internally multiplied in order to provide a measurable signal.

Cloud Chamber, C.T.R. Wilson 1910

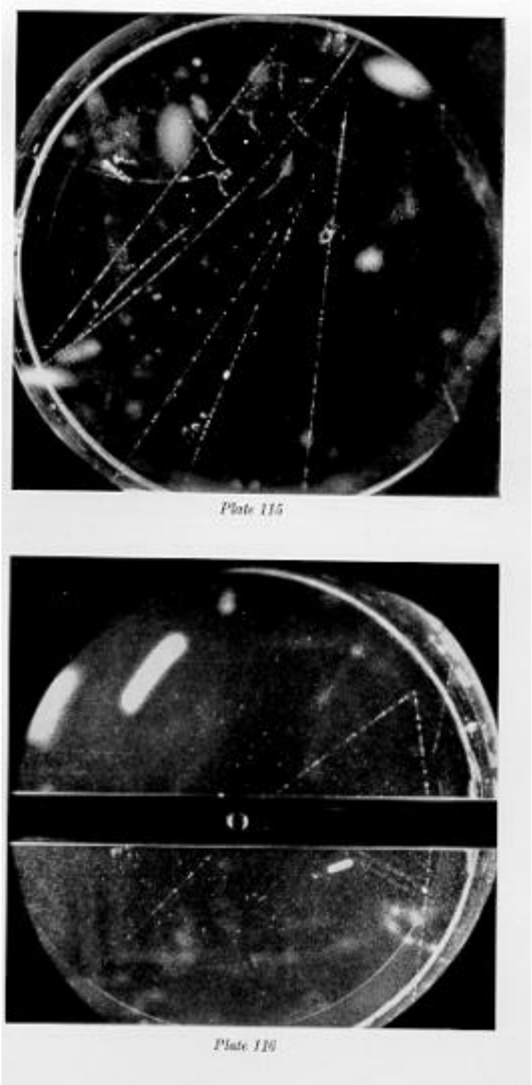
Charges act as condensation nuclei in supersaturated water vapor



Alphas, Philipp 1926



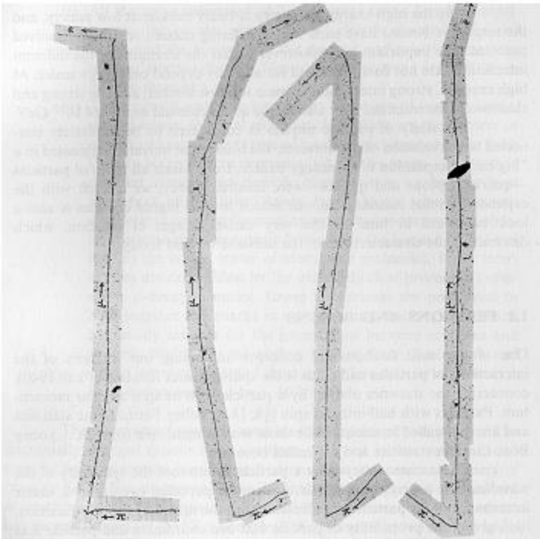
Positron discovery, Carl Andersen 1933



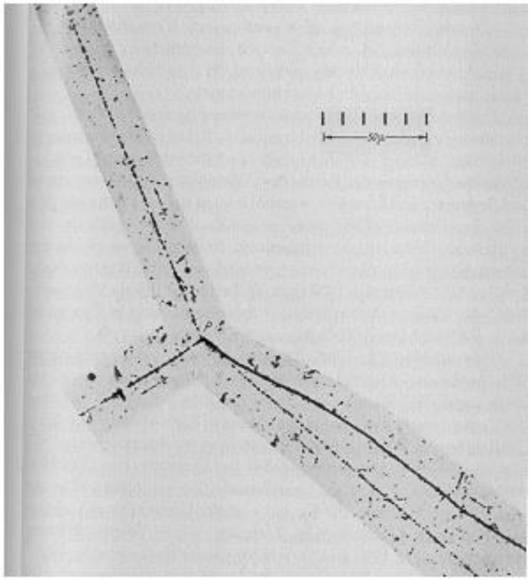
V- particles, Rochester and Wilson, 1940ies

Nuclear Emulsion, M. Blau 1930ies

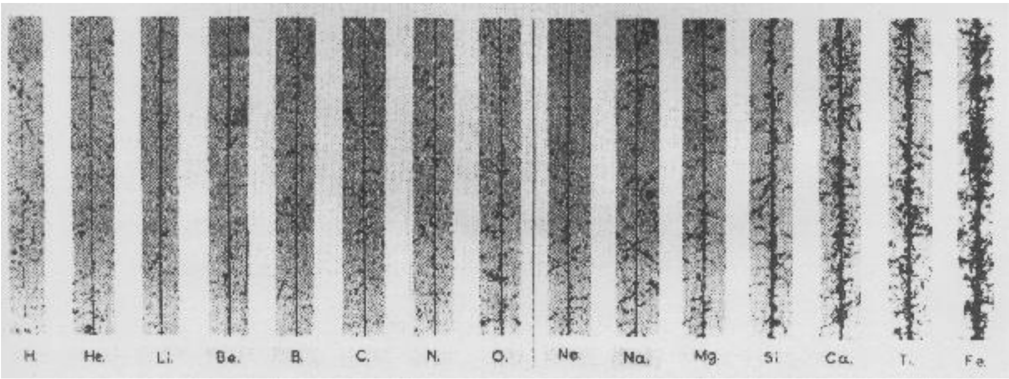
Charges initiate a chemical reaction that blackens the emulsion (film)



C. Powell, Discovery of muon and pion, 1947



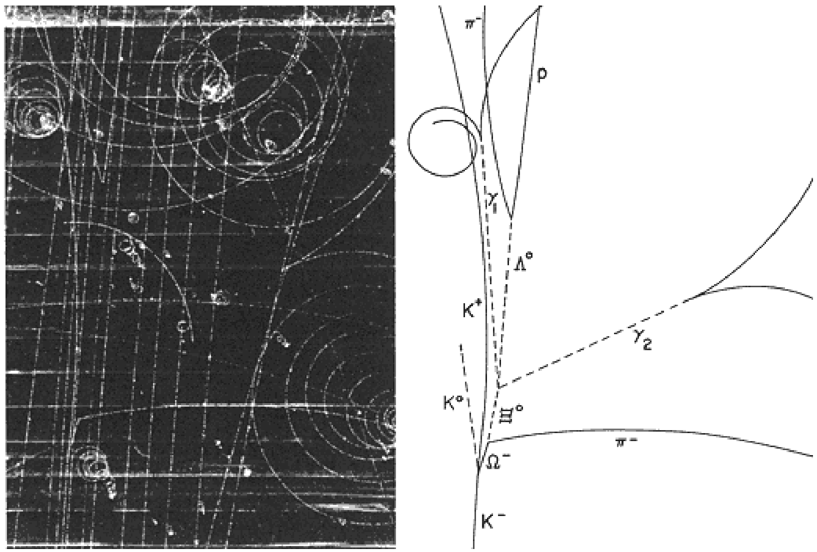
Kaon Decay into 3 pions, 1949

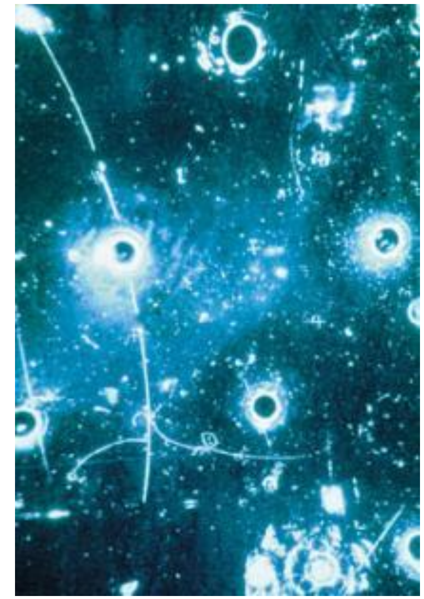


Cosmic Ray Composition

Bubble Chamber, D. Glaser 1952

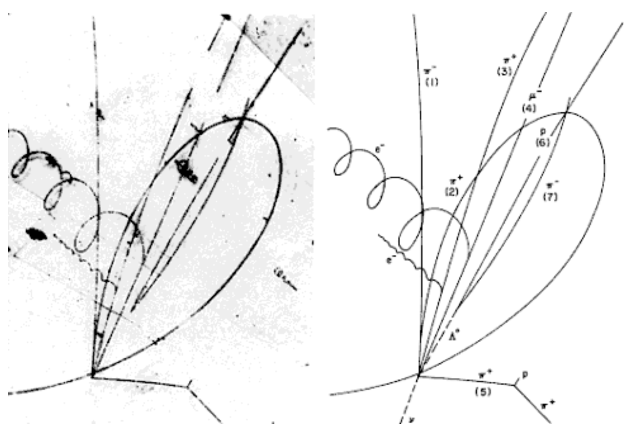
Charges create bubbles in superheated liquid, e.g. propane or Hydrogen (Alvarez)





Discovery of the Ω^- in 1964

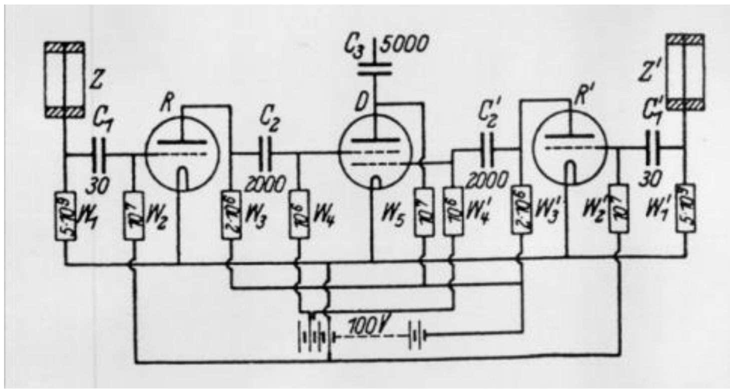
Neutral Currents 1973



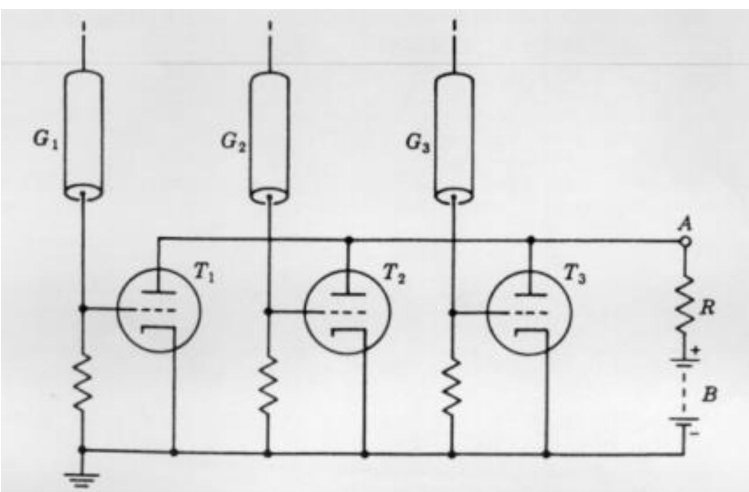
Charmed Baryon, 1975

Electric Registration of Geiger Müller Tube Signals

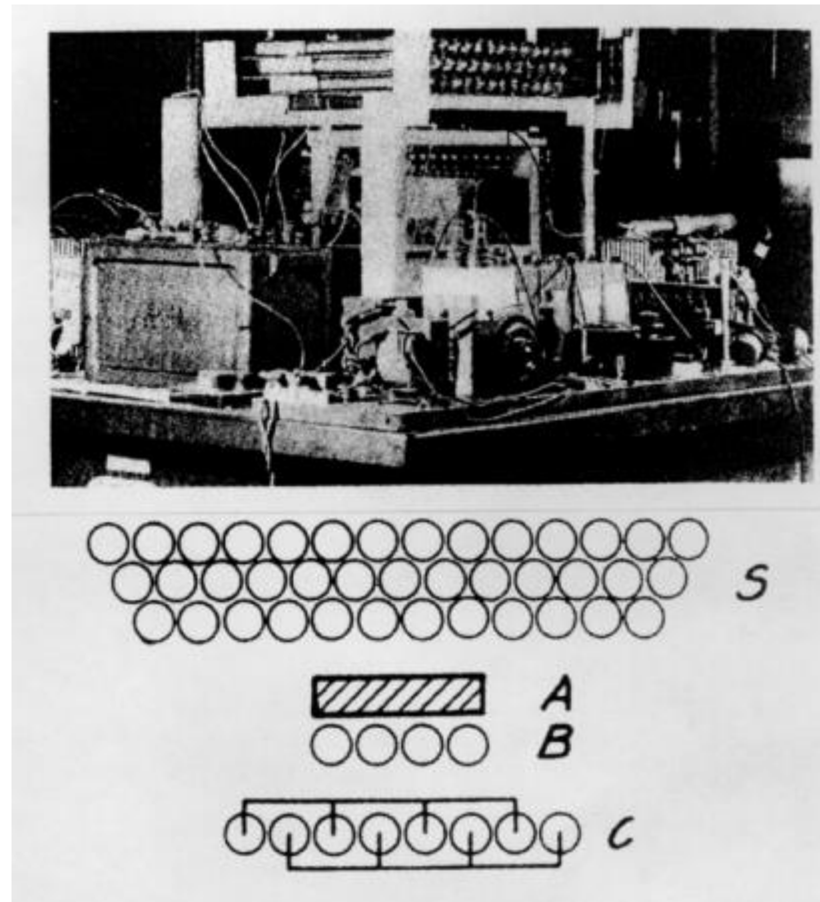
Charges create a discharge in a cylinder with a thin wire set to HV. The charge is measured with a electronics circuit consisting of tubes → electronic signal.



W. Bothe, 1928



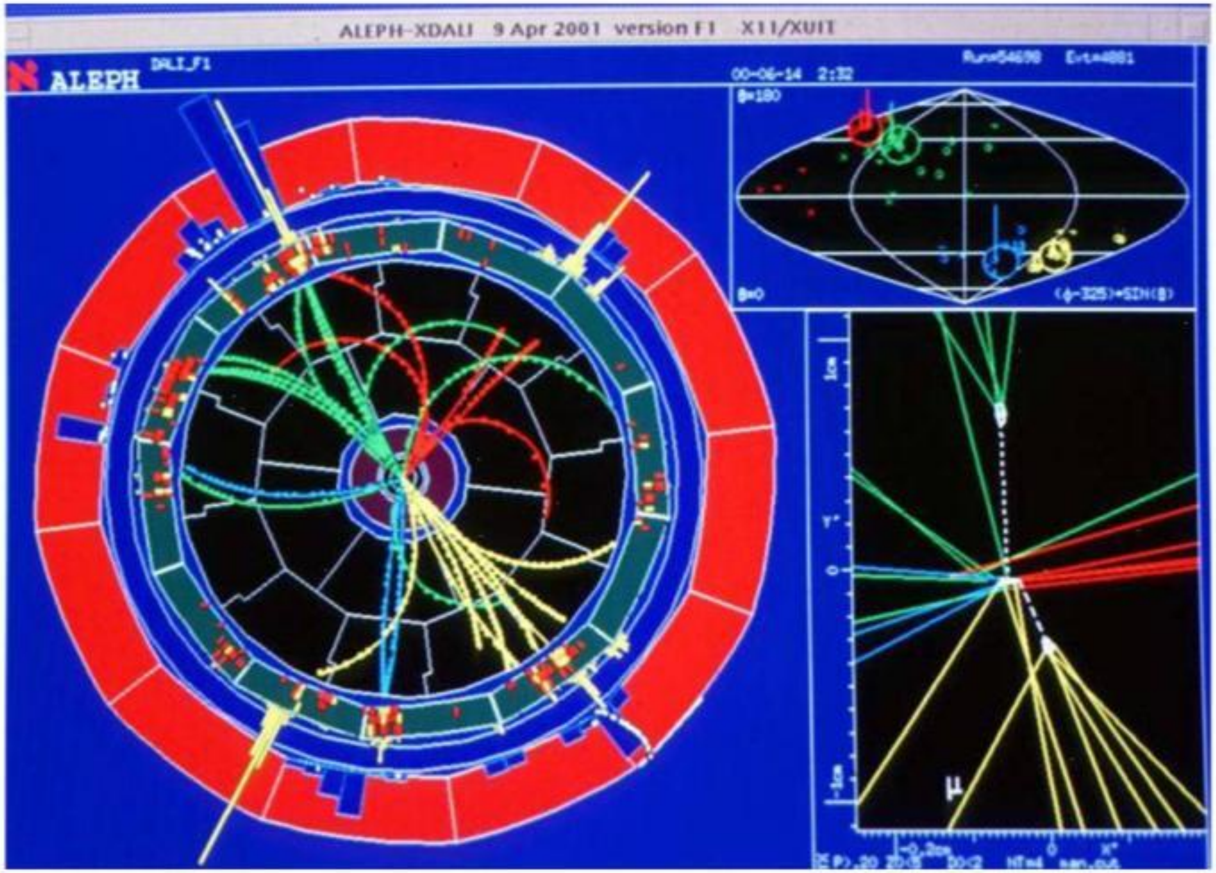
B. Rossi, 1932



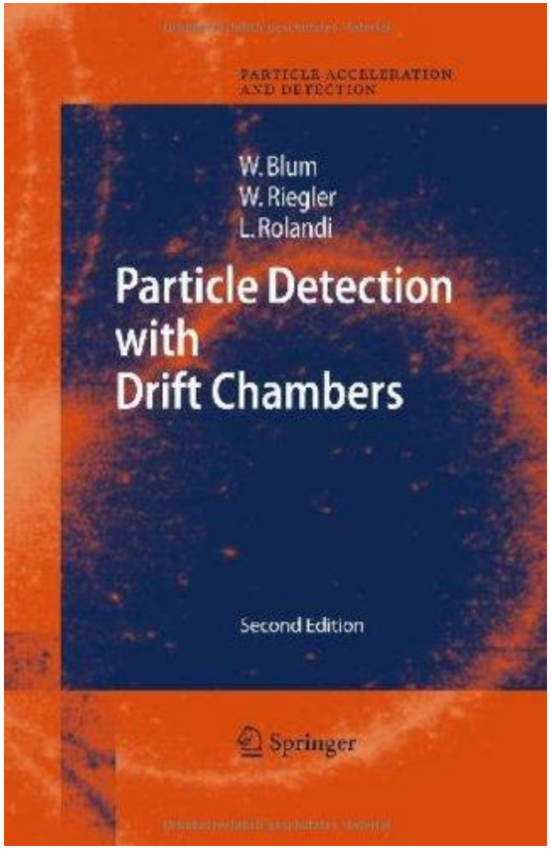
Cosmic Ray Telescope 1930ies

Ionization Chambers, Wire Chambers, Solid State Detectors

! The movement of charges in electric fields induces signal on readout electrodes (No discharge, there is no charge flowing from cathode to Anode) !



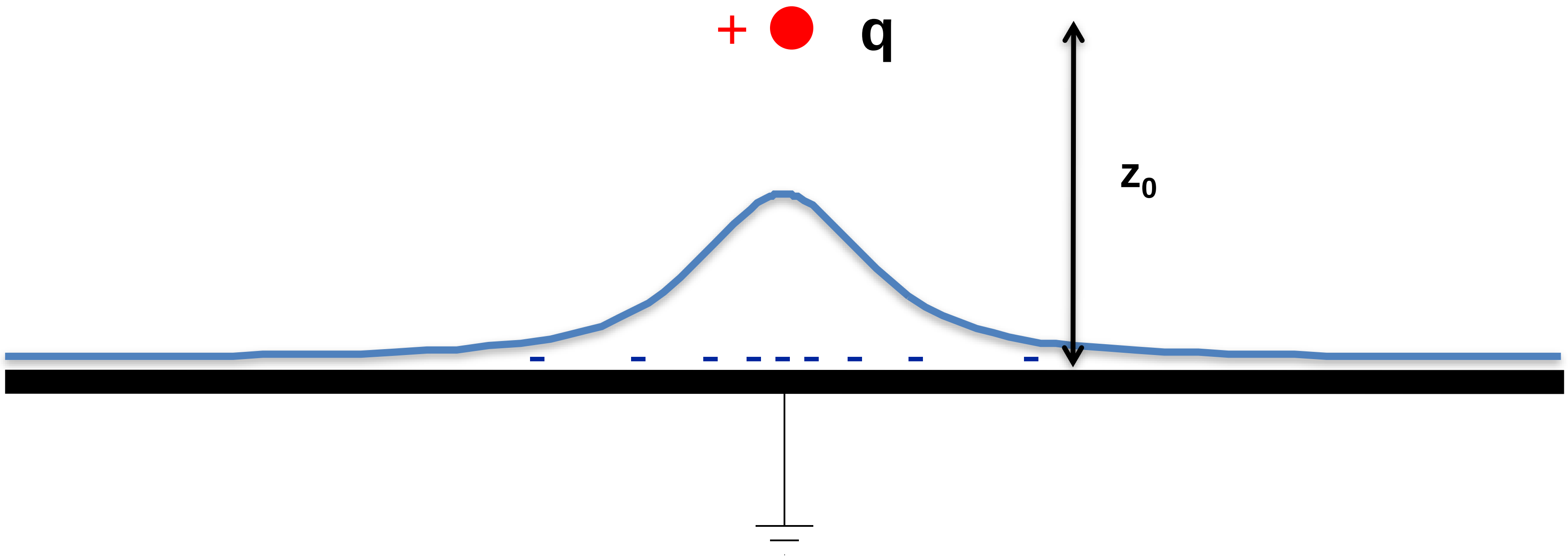
The Principle of Signal Induction on Metal Electrodes by Moving Charges



Many details on signal induction principles, electronics noise and signal processing can be found in this book.

Induced Charges

A point charge q at a distance z_0 above a grounded metal plate 'induces' a surface charge.



Electrostatics, Things we Know

Poisson Equation:

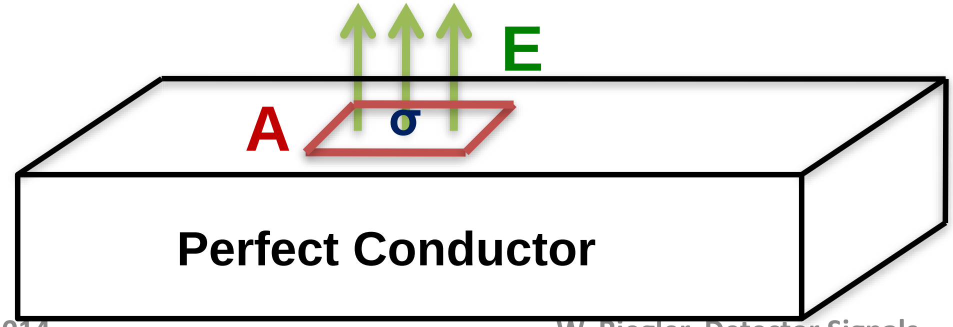
$$\Delta\varphi = -\frac{\rho}{\varepsilon_0} \qquad \vec{E} = -\vec{\nabla}\varphi$$

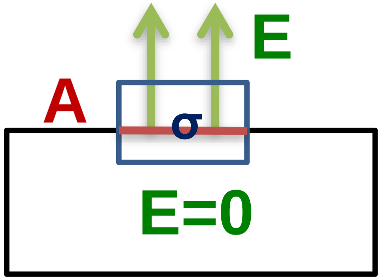
Gauss Law:

$$\oint \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0}\oint \rho\, dV$$

→ Metal Surface: Electric Field is perpendicular to the surface. Charges are only on the surface. Surface Charge Density σ and electric E field on the surface are related by

$$E\,A = \frac{1}{\varepsilon_0}\sigma\,A \qquad \rightarrow \qquad \sigma = \varepsilon_0 E$$





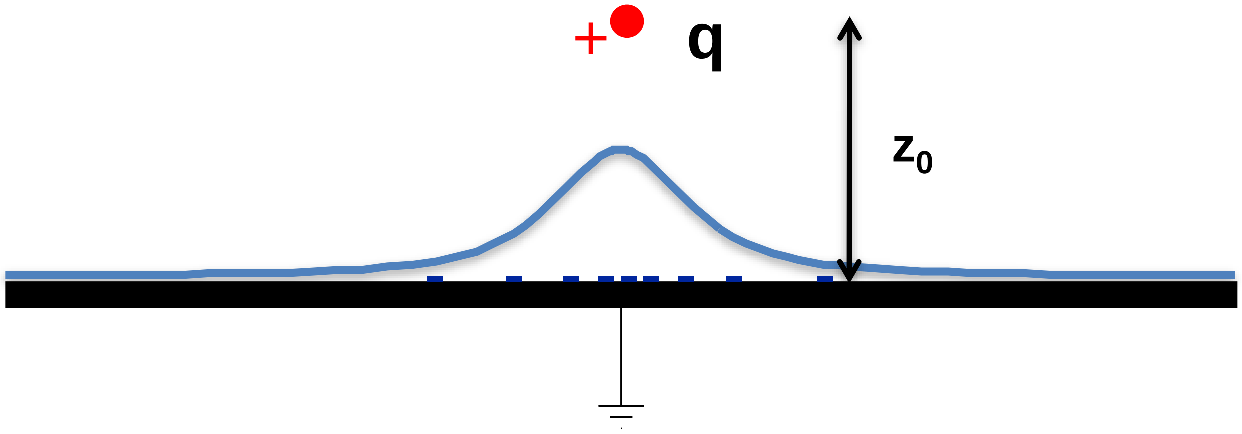
Induced Charges

In order to find the charge induced on an electrode we therefore have to

a)Solve the Poisson equation with boundary condition that φ=0 on the conductor surface.

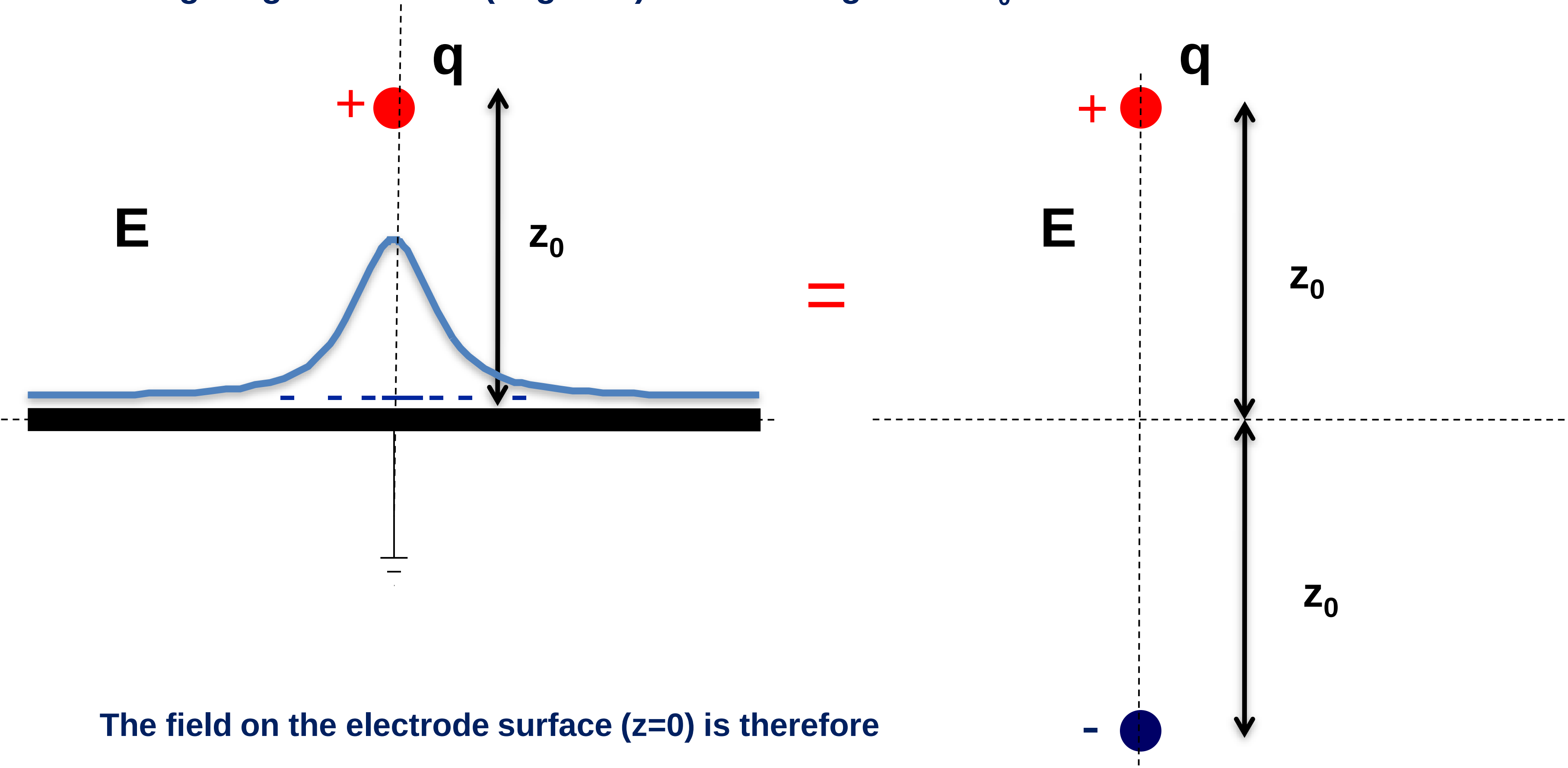
b)Calculate the electric field E on the surface of the conductor

c)Integrate e_0E over the electrode surface.



Induced Charges

The solution for the field of a point charge in front of a metal plate is equal to the solution of the charge together with a (negative) mirror charge at $z=-z_0$.



The field on the electrode surface (z=0) is therefore

$$E_z(x,y) = -\frac{q z_0}{2\pi\varepsilon_0 (x^2+y^2+z_0^2)^{\frac{3}{2}}} \qquad E_x = E_y = 0$$

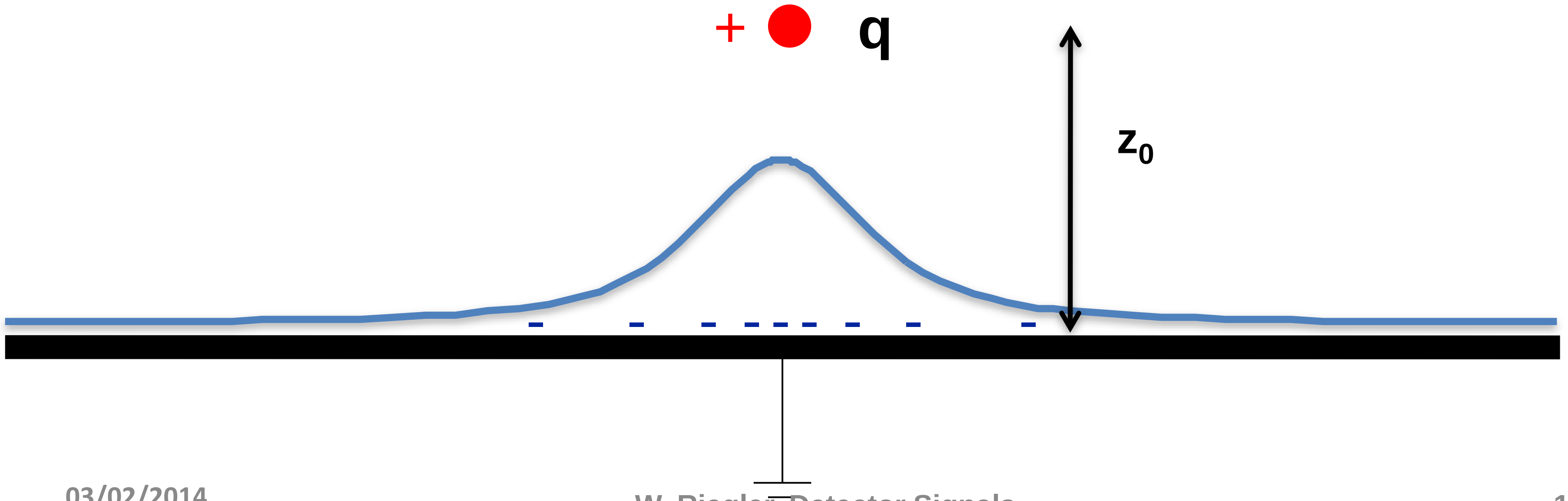
Induced Charges

We therefore find a surface charge density of

$$\sigma(x,y) = \varepsilon_0 E_z(x,y) = -\frac{q z_0}{2\pi(x^2+y^2+z_0^2)^{\frac{3}{2}}}$$

And therefore a total induced charge of

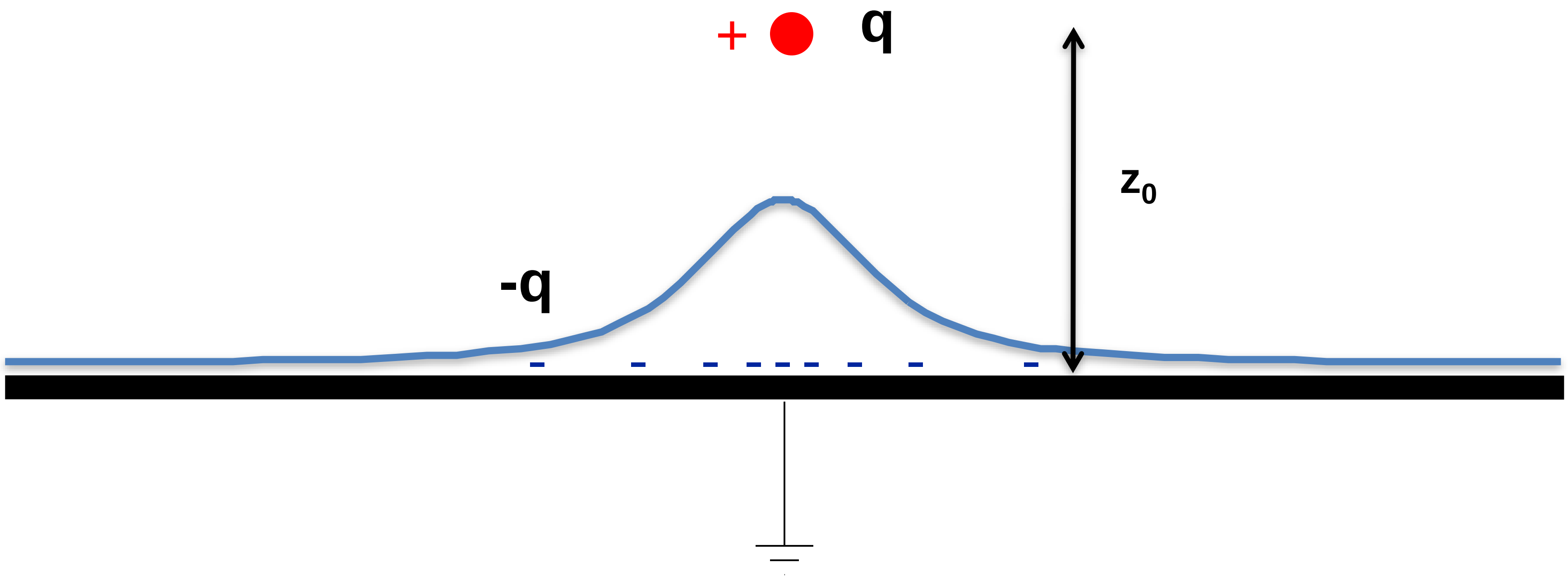
$$Q = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sigma(x,y)\,dx\,dy = -q$$



Induced Charges

The total charge induced by a point charge q on an infinitely large grounded metal plate is equal to –q, independent of the distance of the charge from the plate.

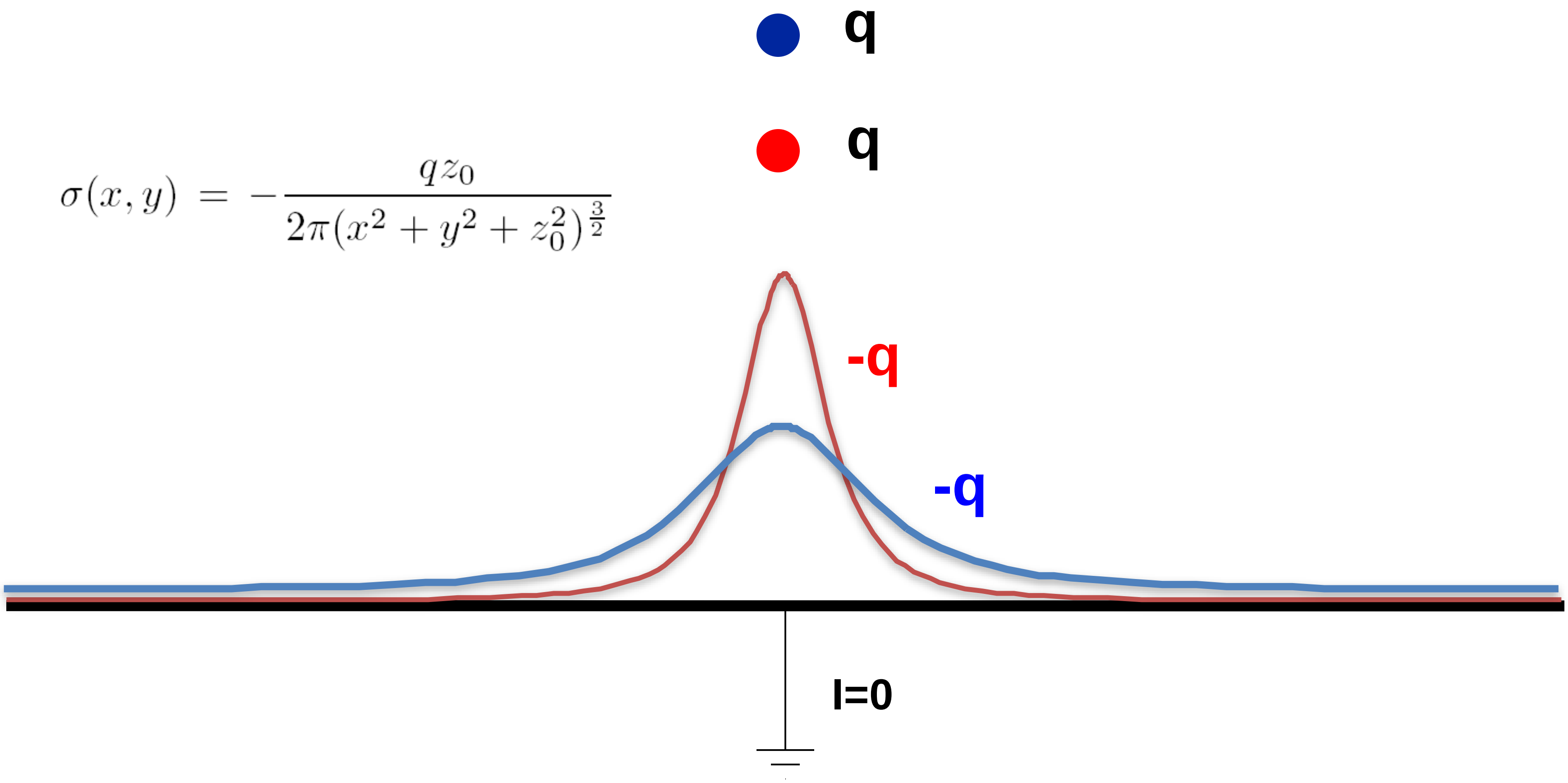
The surface charge distribution is however depending on the distance z_0 of the charge q.



Induced Charges

Moving the point charge closer to the metal plate, the surface charge distribution becomes more peaked, the total induced charge is however always equal to –q.

$$\sigma(x,y) = -\frac{qz_0}{2\pi(x^2+y^2+z_0^2)^{\frac{3}{2}}}$$



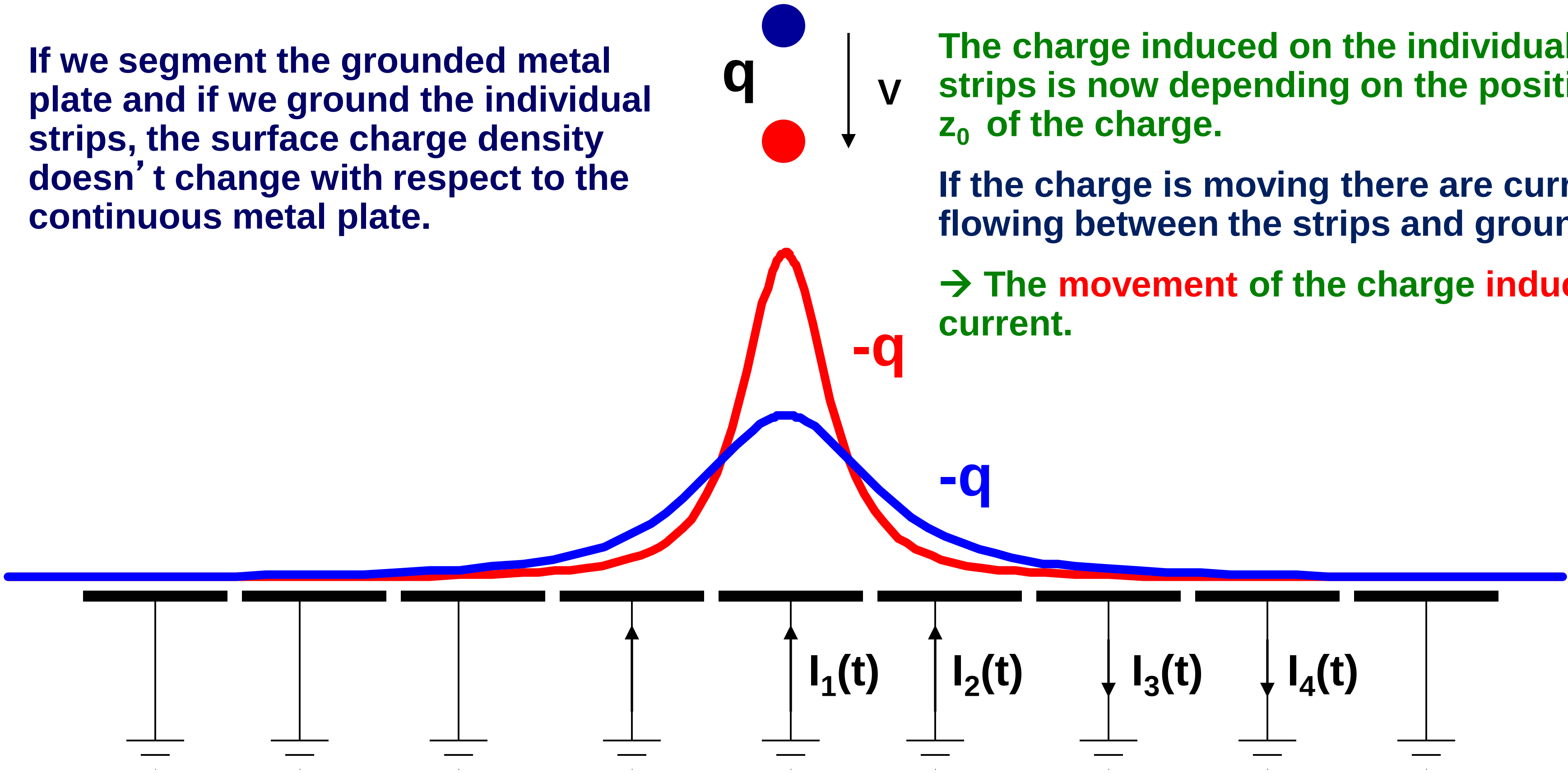
Signal Induction by Moving Charges

If we segment the grounded metal plate and if we ground the individual strips, the surface charge density doesn't change with respect to the continuous metal plate.

The charge induced on the individual strips is now depending on the position z_0 of the charge.

If the charge is moving there are currents flowing between the strips and ground.

→ The movement of the charge induces a current.



$$Q_1(z_0) = \int_{-\infty}^{\infty}\int_{-w/2}^{w/2} \sigma(x,y)\,dx\,dy = -\frac{2q}{\pi}\arctan\left(\frac{w}{2z_0}\right) \qquad z_0(t) = z_0 - vt$$

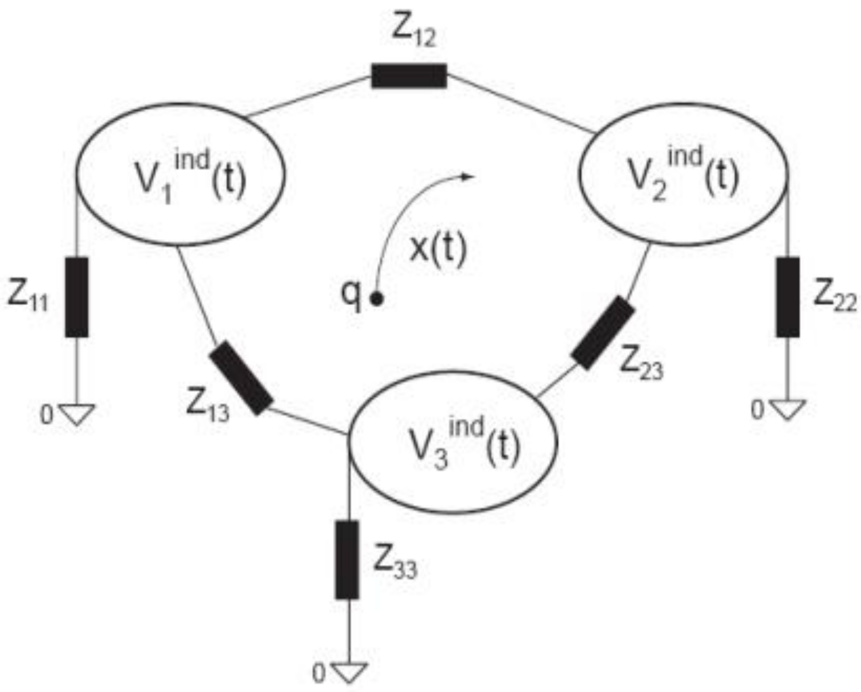
$$I_1^{ind}(t) = -\frac{d}{dt}Q_1[z_0(t)] = -\frac{\partial Q_1[z_0(t)]}{\partial z_0}\frac{dz_0(t)}{dt} = \frac{4qw}{\pi[4z_0(t)^2 + w^2]}v$$

Formulation of the Problem

In a real particle detector, the electrodes (wires, cathode strips, silicon strips, plate electrodes …) are not grounded but they are connected to readout electronics and interconnected by other discrete elements.

We want to answer the question:

What are the voltages induced on metal electrodes by a charge q moving along a trajectory x(t), in case these metal electrodes are connected by arbitrary linear impedance components ?

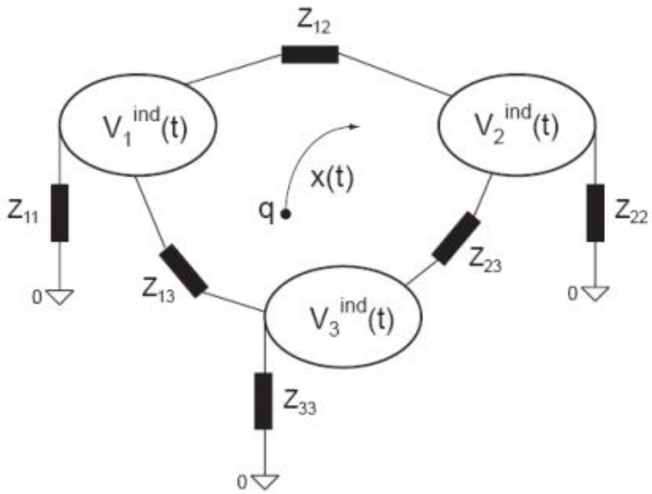


Formulation of the Problem

We will divide the problem into two parts:

We first calculate the currents induced on grounded electrodes.

We then have to place these currents as ideal current sources on a circuit containing the discrete components and the mutual electrode capacitances



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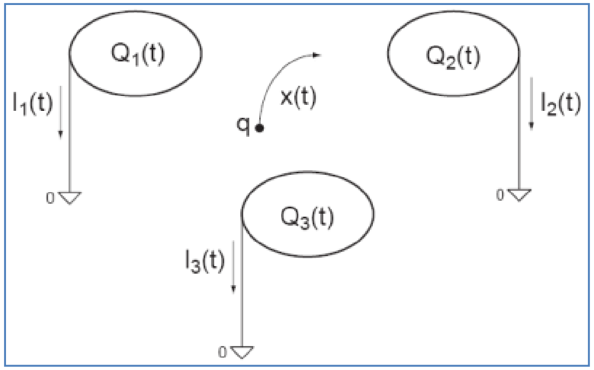

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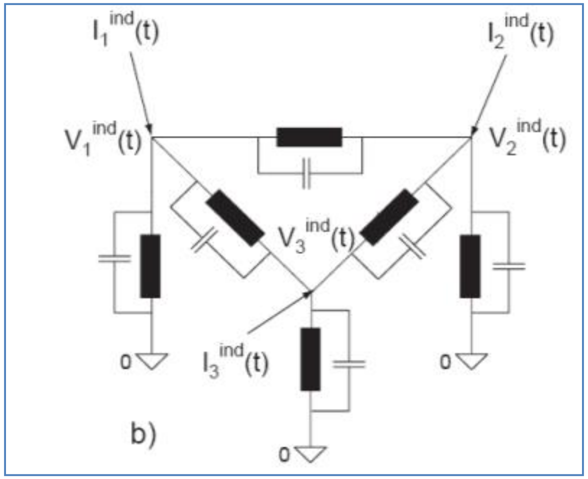
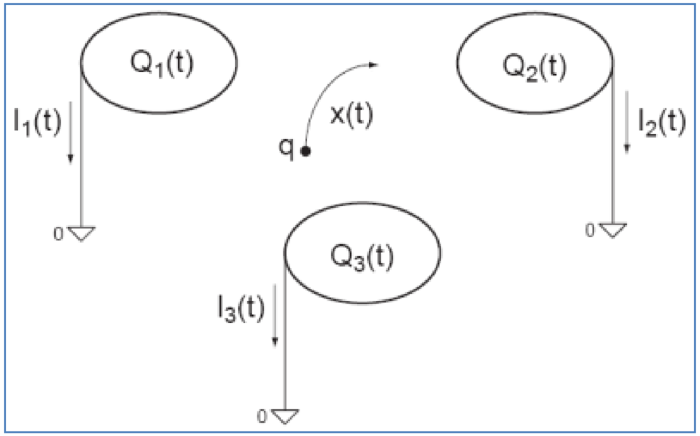


The second step is typically performed by using an analog circuit simulation program. We will first focus on the induced currents.

Currents on Grounded Electrodes

We can imagine this case by reading the signal with an ideal current amplifier of zero input impedance



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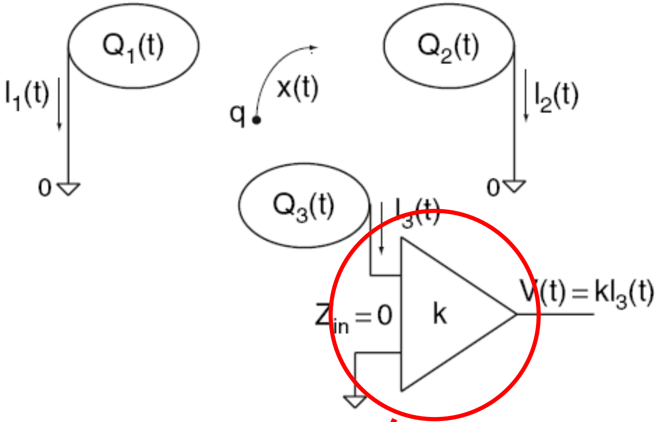
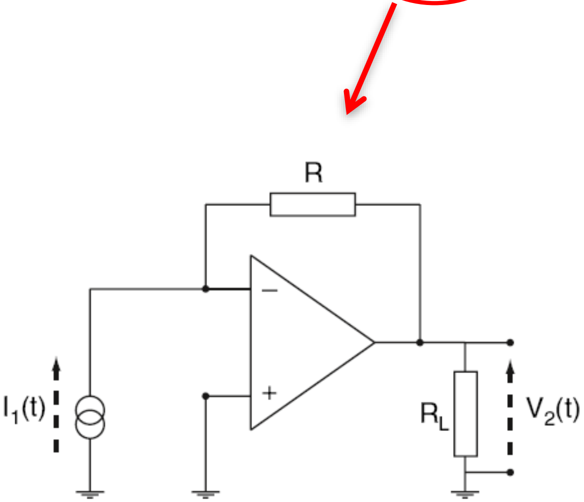
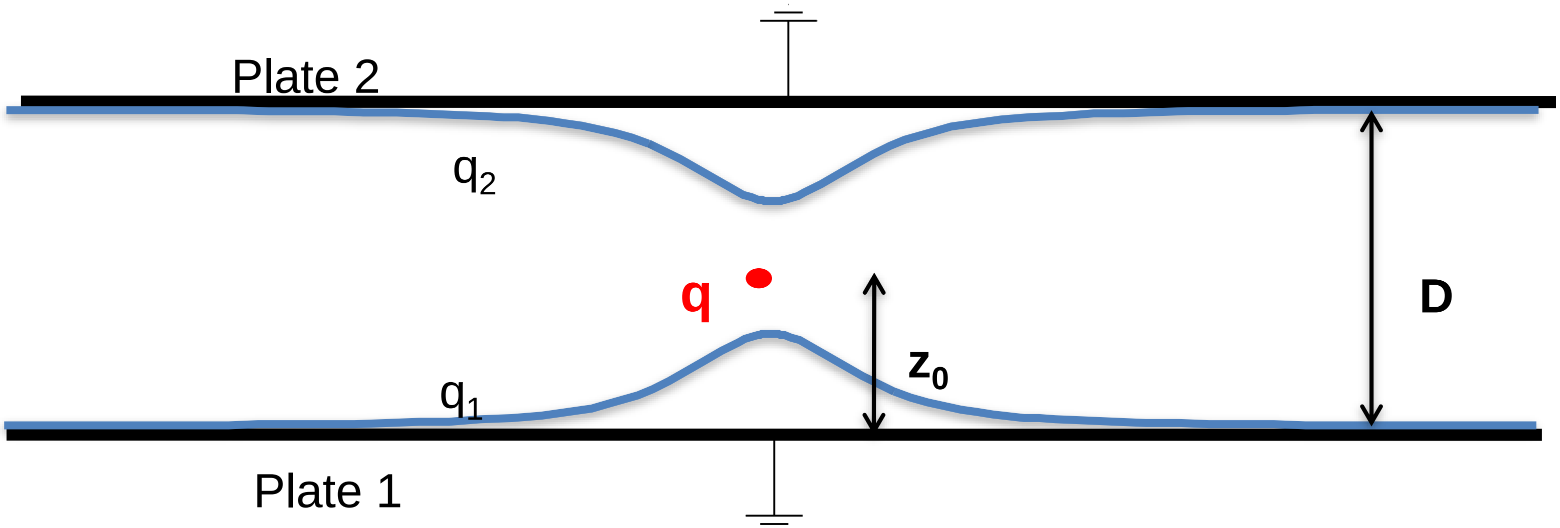




$V_2(t) = -R\ I_1(t)$

Parallel Plate Chamber



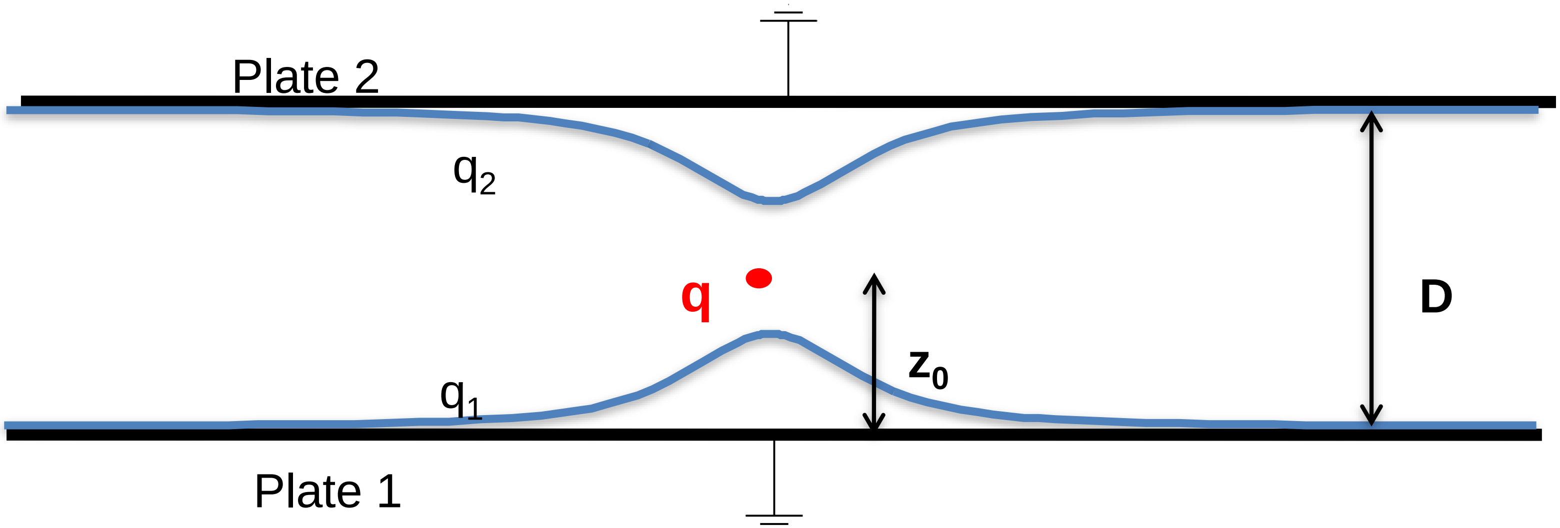
$$\phi(r,z) = \frac{q}{\varepsilon_0 \pi D} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{D} z\right) \sin\left(\frac{n\pi}{D} z_0\right) K_0\left(\frac{n\pi}{D} r\right)$$ [5]

$$E(r,z) = \frac{q}{\varepsilon_0 \pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D} \cos\left(\frac{n\pi}{D} z\right) \sin\left(\frac{n\pi}{D} z_0\right) K_0\left(\frac{n\pi}{D} r\right)$$

$$\sigma_1(r) = \varepsilon_0 E(r, z=0) = \frac{q}{\pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D} \sin\left(\frac{n\pi}{D} z_0\right) K_0\left(\frac{n\pi}{D} r\right)$$

[5] C. Y. Fong, C. Kittel, Induced Charge on Capacitor Plates, Am. J . Phys. 35(1967)1091.

Parallel Plate Chamber



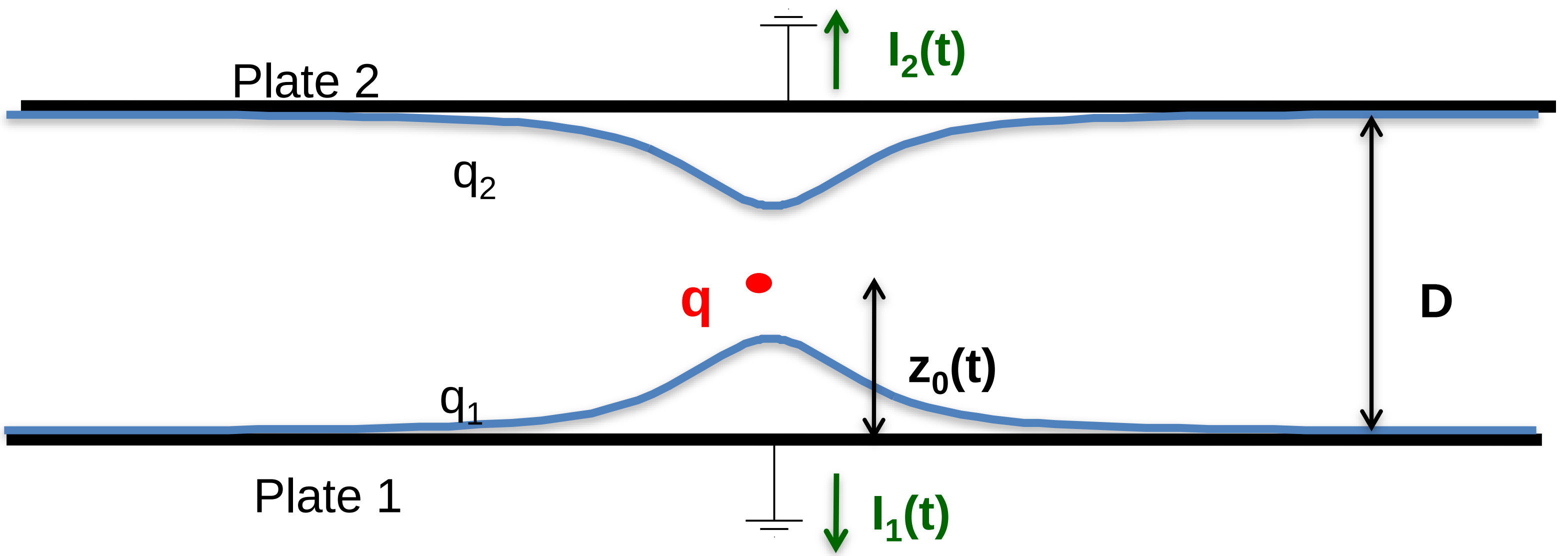
$$q_1 = \int_0^\infty 2r\pi\sigma(r)dr = \frac{q}{\pi D}\sum_{n=1}^{\infty}\frac{n\pi}{D}\sin\left(\frac{n\pi}{D}z_0\right)\int_0^\infty 2r\pi K_0\left(\frac{n\pi}{D}r\right)dr = \frac{2q}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{D}z_0\right) =$$

$$= -q\left(1-\frac{z_0}{D}\right)$$

$$q_2 = \ldots = -q\,\frac{z_0}{D}$$

$$q_1 + q_2 = -q$$

Parallel Plate Chamber

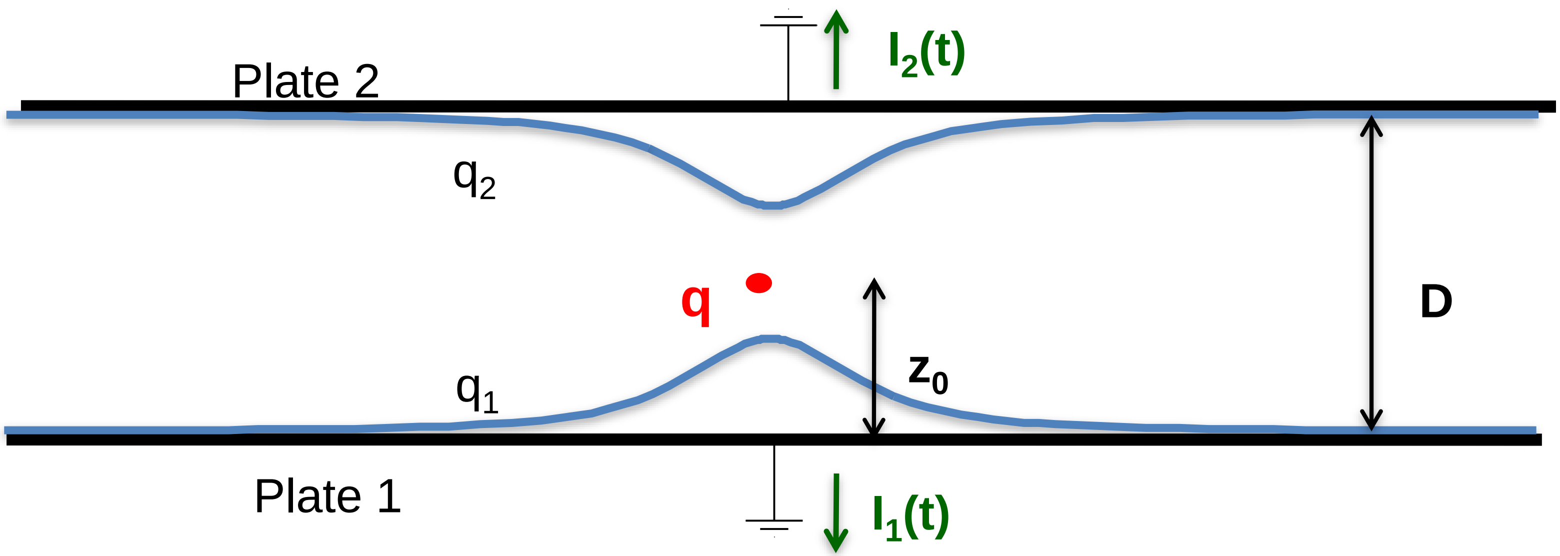


$$q_1 = -q\left(1 - \frac{z_0}{D}\right) \qquad q_2 = -q\frac{z_0}{D} \qquad z_0(t) = vt$$

$$q_1(t) = -q\left(1 - \frac{vt}{D}\right) \qquad q_2(t) = -q\frac{vt}{D} \qquad q_1(t) + q_2(t) = q$$

$$I_1(t) = -\frac{dq_1(t)}{dt} = -\frac{qv}{D} \qquad I_2(t) = -\frac{dq_2(t)}{dt} = +\frac{qv}{D} \qquad I_1(t) + I_2(t) = 0$$

Parallel Plate Chamber



$$q_1(t) = -q\left(1 - \frac{vt}{D}\right) \qquad q_2(t) = -q\frac{vt}{D} \qquad I_1(t) = -\frac{dq_1(t)}{dt} = -\frac{qv}{D} \qquad I_2(t) = -\frac{dq_2(t)}{dt} = +\frac{qv}{D}$$

The sum of all induced charges is equal to the moving charge at any time.

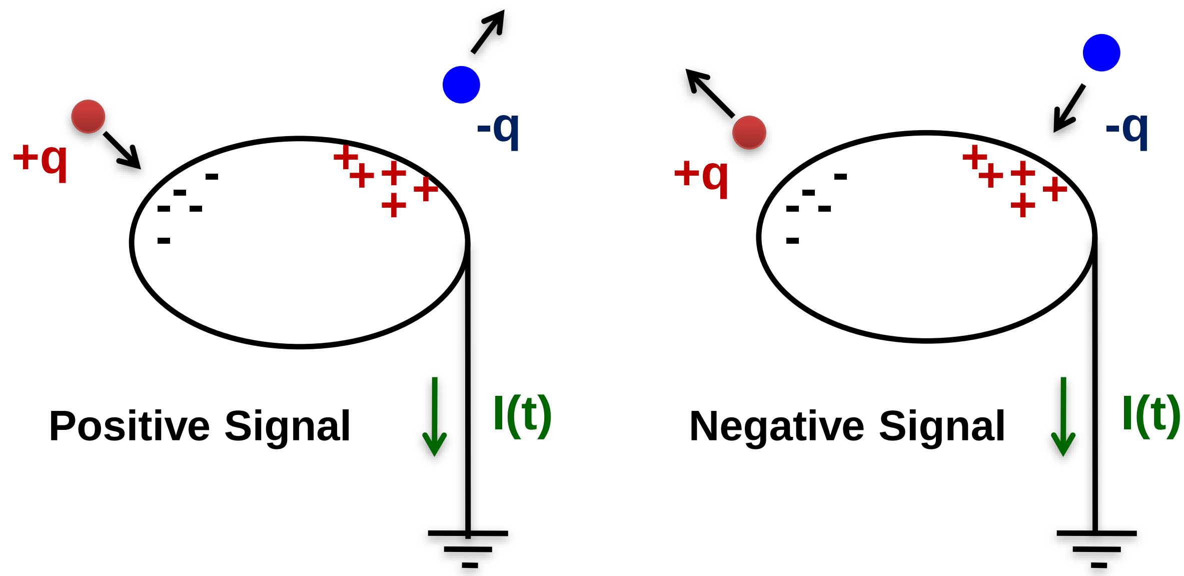
The sum of the induced currents is zero at any time.

The field calculation is complicated, the formula for the induced signal is however very simple – there might be an easier way to calculate the signals ?

→ Ramo-Shockley theorem !

Signal Polarity Definition

$$I(t) = -\frac{dQ(t)}{dt}$$



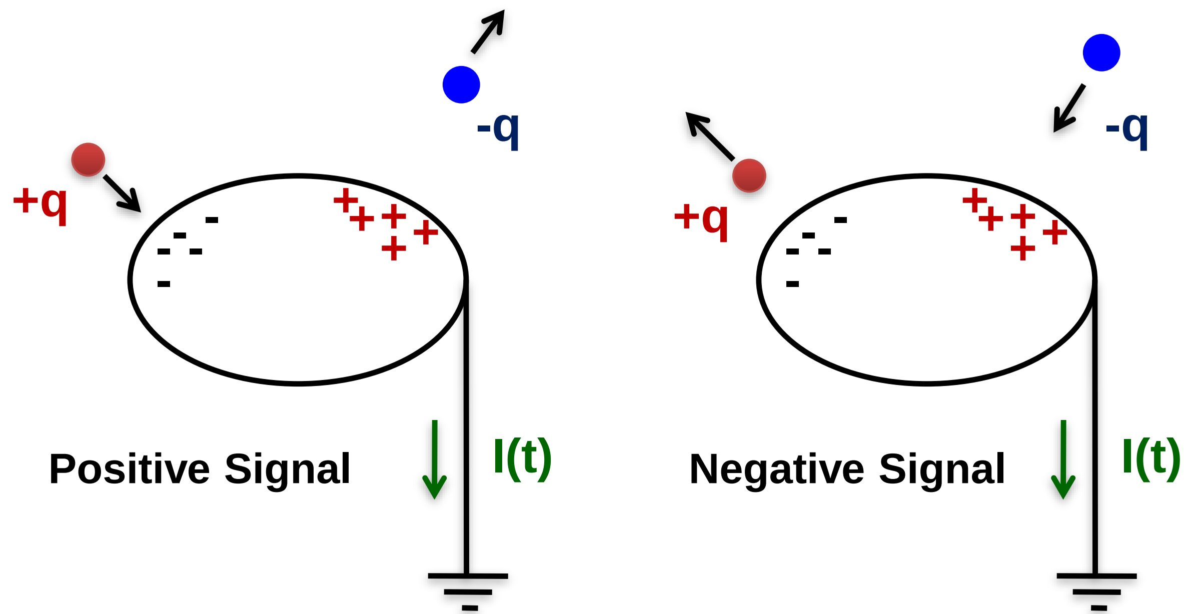
The definition of I=-dQ/dt states that the positive current is pointing away from the electrode.

The signal is positive if:
Positive charge is moving from electrode to ground or
Negative charge is moving from ground to the electrode

The signal is negative if:
Negative charge is moving from electrode to ground or
Positive charge is moving from ground to the electrode

Signal Polarity Definition

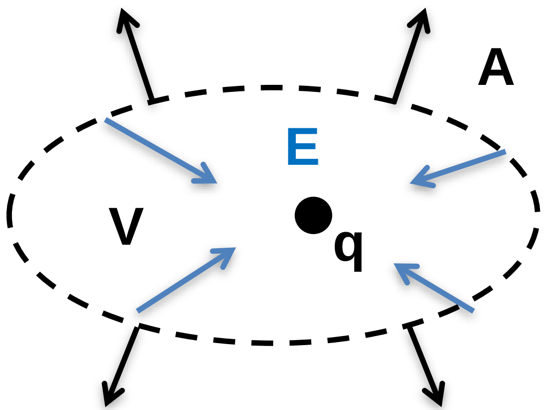
$$I(t) = -\frac{dQ(t)}{dt}$$



By this we can guess the signal polarities:

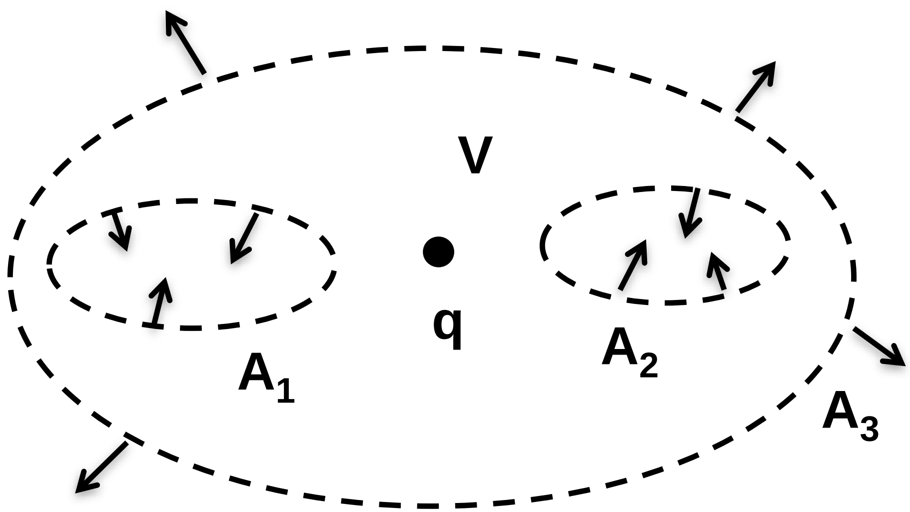
In a wire chamber, the electrons are moving towards the wire, which means that they attract positive charges that are moving from ground to the electrode. The signal of a wire that collects electrons is therefore negative.

Sum of Induced Charges and Currents



$$\oint_{\vec{A}} \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0} \oint_V \rho\, dV = \frac{q}{\varepsilon_0}$$

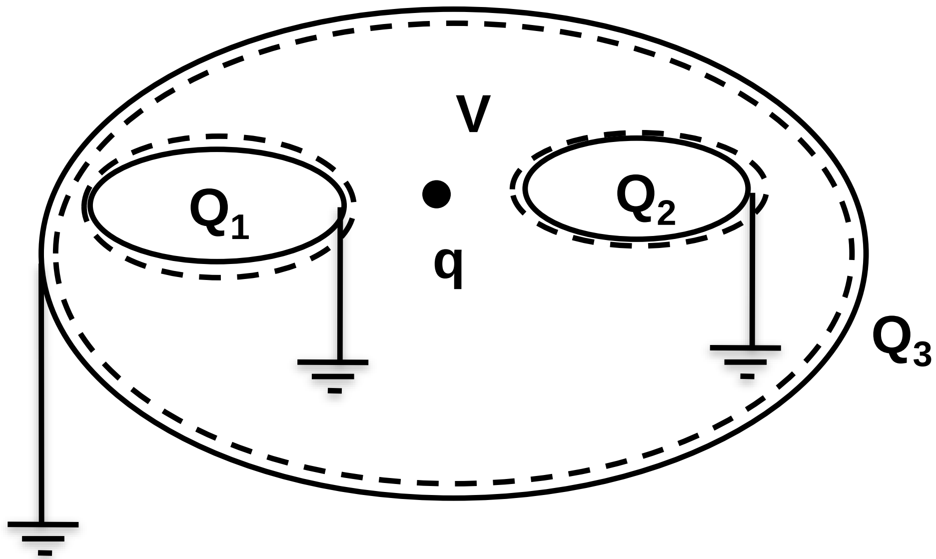
The surface A must be oriented towards the outside of the volume V.



$A = A_1 + A_2 + A_3$

$$\oint_{\vec{A}} \vec{E}\, d\vec{A} = \oint_{\vec{A}_1} \vec{E}\, d\vec{A} + \oint_{\vec{A}_2} \vec{E}\, d\vec{A} + \oint_{\vec{A}_3} \vec{E}\, d\vec{A} = \frac{q}{\varepsilon_0}$$

Sum of Induced Charges and Currents



$$\oint_{\vec{A}} \vec{E}\, d\vec{A} = \oint_{\vec{A}_1} \vec{E}\, d\vec{A} + \oint_{\vec{A}_2} \vec{E}\, d\vec{A} + \oint_{\vec{A}_3} \vec{E}\, d\vec{A} = \frac{q}{\varepsilon_0}$$

In case the surfaces are metal electrodes we know that

$$Q_1 = -\oint_{\vec{A}_1} \varepsilon_0 \vec{E}\, d\vec{A} \qquad Q_2 = -\oint_{\vec{A}_2} \varepsilon_0 \vec{E}\, d\vec{A} \qquad Q_3 = -\oint_{\vec{A}_3} \varepsilon_0 \vec{E}\, d\vec{A}$$

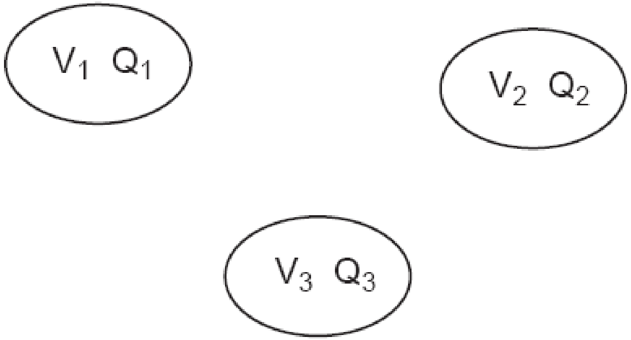
And we therefore have

$$Q_1 + Q_2 + Q_3 = -q$$

In case there is one electrode enclosing all the others, the sum of all induced charges is always equal to the point charge.

The sum of all induced currents is therefore zero at any time !

Charged Electrodes



Setting the three electrodes to potentials V_1, V_2, V_3 results in charges Q_1, Q_2, Q_3. In order to find them we have to solve the Laplace equation

$$\Delta\varphi = 0$$

with boundary condition

$$\varphi|_{\vec{A}_1} = V_1 \qquad \varphi|_{\vec{A}_2} = V_2 \qquad \varphi|_{\vec{A}_3} = V_3$$

And the calculate

$$Q_1 = \oint_{\vec{A}_1} -\vec{\nabla}\varphi \, d\vec{A} \qquad Q_2 = \oint_{\vec{A}_2} -\vec{\nabla}\varphi \, d\vec{A} \qquad Q_3 = \oint_{\vec{A}_3} -\vec{\nabla}\varphi \, d\vec{A}$$

Green's Second Theorem

Gauss Law which is valid for Vector Field and Volume V surrounded by the Surface A:

$$\oint_{\vec{A}} \vec{E}\, d\vec{A} = \oint_{V} \vec{\nabla}\vec{E}\, dV$$

By setting

$$\vec{E} = \varphi\vec{\nabla}\psi \qquad \oint_{\vec{A}} \varphi\vec{\nabla}\psi\, d\vec{A} = \oint_{V} \vec{\nabla}\varphi\vec{\nabla}\psi\, dV + \oint_{V} \varphi\Delta\psi\, dV$$

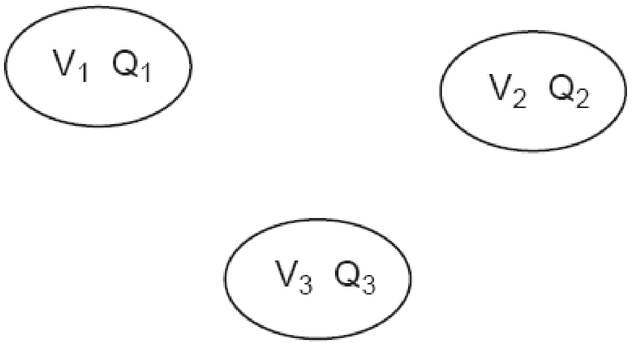
and setting

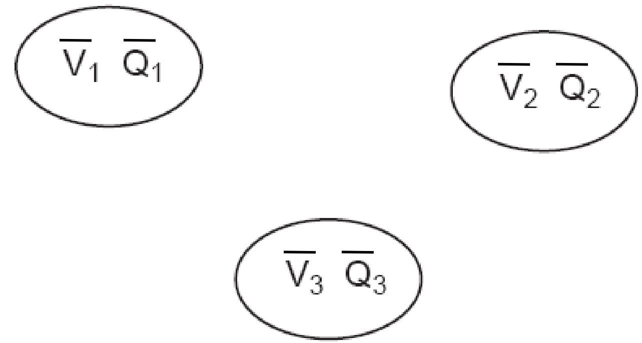
$$\vec{E} = \psi\vec{\nabla}\varphi \qquad \oint_{\vec{A}} \psi\vec{\nabla}\varphi\, d\vec{A} = \oint_{V} \vec{\nabla}\psi\vec{\nabla}\varphi\, dV + \oint_{V} \psi\Delta\varphi\, dV$$

and subtracting the two expressions we get Green's second theorem:

$$\oint_{\vec{A}} \left(\varphi\vec{\nabla}\psi - \psi\vec{\nabla}\varphi\right) d\vec{A} = \oint_{V} (\varphi\Delta\psi - \psi\Delta\varphi)\, dV$$

Green's Theorem, Reciprocity





$$\Delta\varphi = 0 \qquad\qquad \Delta\psi = 0$$

$$\varphi|_{\vec{A}_1} = V_1 \quad \varphi|_{\vec{A}_2} = V_2 \quad \varphi|_{\vec{A}_3} = V_3 \qquad\qquad \psi|_{\vec{A}_1} = \overline{V}_1 \quad \psi|_{\vec{A}_2} = \overline{V}_2 \quad \psi|_{\vec{A}_3} = \overline{V}_3$$

$$Q_1 = \oint_{\vec{A}_1} -\vec{\nabla}\varphi\, d\vec{A} \quad Q_2 = \oint_{\vec{A}_2} -\vec{\nabla}\varphi\, d\vec{A} \quad Q_3 = \oint_{\vec{A}_3} -\vec{\nabla}\varphi\, d\vec{A} \qquad \overline{Q}_1 = \oint_{\vec{A}_1} -\vec{\nabla}\psi\, d\vec{A} \quad \overline{Q}_2 = \oint_{\vec{A}_2} -\vec{\nabla}\psi\, d\vec{A} \quad \overline{Q}_3 = \oint_{\vec{A}_3} -\vec{\nabla}\psi\, d\vec{A}$$

$$\oint_{\vec{A}} \left(\varphi\vec{\nabla}\psi - \psi\vec{\nabla}\varphi\right) d\vec{A} = \oint_V (\varphi\Delta\psi - \psi\Delta\varphi)\, dV$$

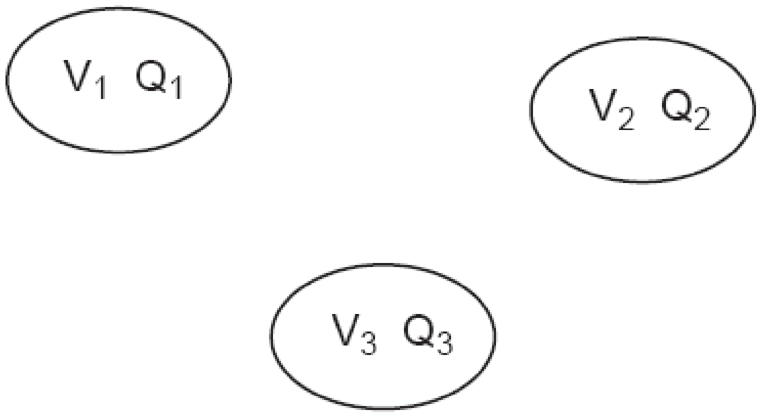
$$\sum_{n=1}^{N} Q_n \overline{V}_n = \sum_{n=1}^{N} \overline{Q}_n V_n$$

Reciprocity Theorem

It relates two electrostatic states, i.e. two sets of voltages and charges

Electrostatics, Capacitance Matrix

From the reciprocity theorem it follows that the voltages of the electrodes and the charges on the electrodes are related by a matrix



$$Q_n = \sum_{m=1}^{N} c_{nm} V_m$$

The matrix c_{nm} is called the capacitance matrix with the important properties

$$c_{nm} = c_{mn} \qquad c_{nm} < 0 \qquad \sum_{m=1}^{N} c_{nm} > 0$$

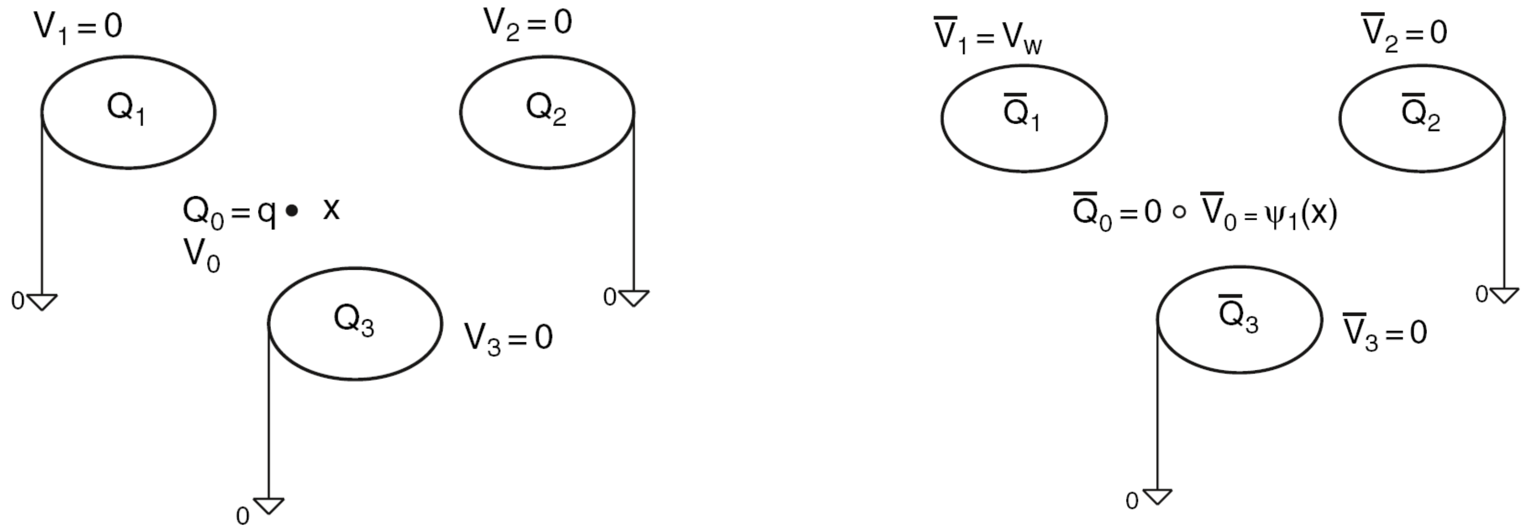
The capacitance matrix elements are not to be confused with the electrode capacitances of the equivalent circuit. They are related by

$$C_{nm} = -c_{nm} \quad n \neq m \qquad C_{nn} = \sum_{m=1}^{N} c_{nm}$$

Induced Charge

We assume three grounded electrodes and a point charge in between. We want to know the charges induced on the grounded electrodes. We assume the point charge to be an very small metal electrode with charge q, so we have a system of 4 electrodes with $V_1=0$, $V_2=0$, $V_3=0$, $Q_0=q$.

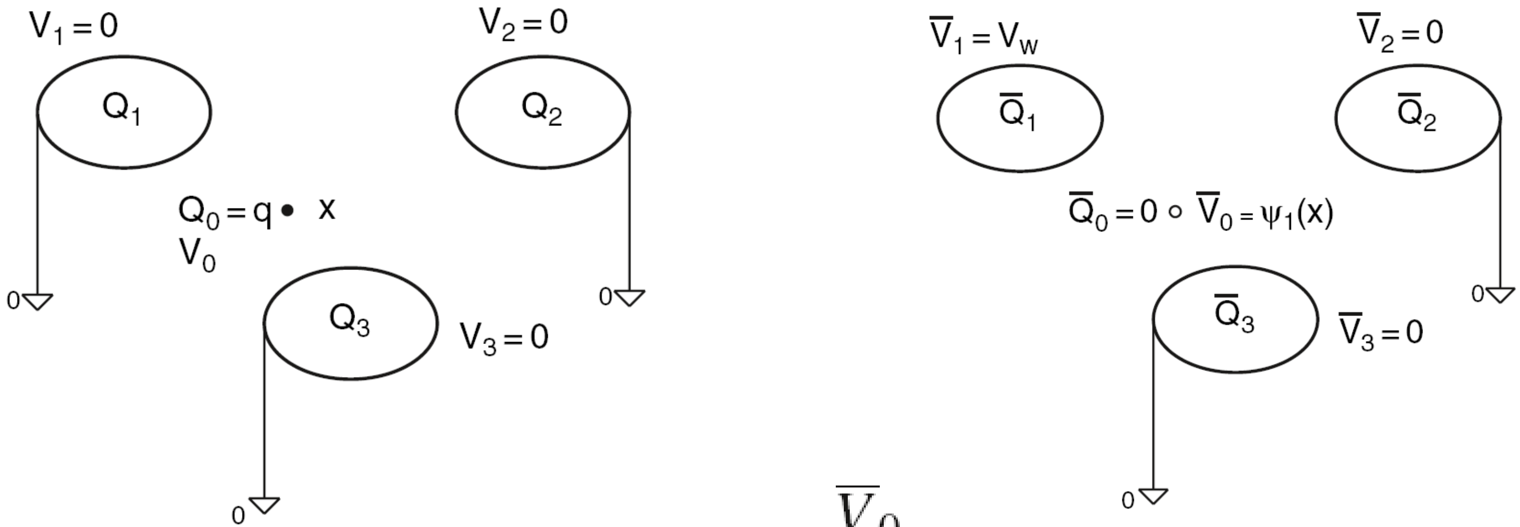
We can now assume another set of voltages and charges where we remove the charge from electrode zero, we put electrode 1 to voltage V_w and keep electrodes 2 and 3 grounded.



Using the reciprocity theorem $\sum_{n=1}^{N} Q_n \overline{V}_n = \sum_{n=1}^{N} \overline{Q}_n V_n$ we get

$$q\overline{V}_0 + Q_1 V_w = 0 \qquad \rightarrow \qquad Q_1 = -q\frac{\overline{V}_0}{V_w}\,.$$

Induced Charge

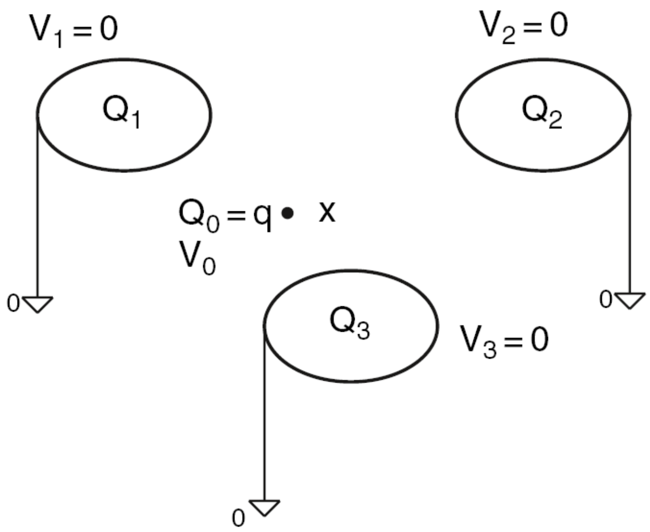


$$Q_1 = -q\frac{\overline{V}_0}{V_w}$$

The voltage $\overline{V}_0$ is the voltage of the small uncharged electrode for the second electrostatic state, and because a small uncharged electrode is equal to having no electrode, $\overline{V}_0$ is the voltage at the place x of the point charge in case the charge is removed, electrode 1 is put to voltage V_w and the other electrodes are grounded.

We call the potential ψ(x) the weighting potential of electrode 1.

Induced Charge

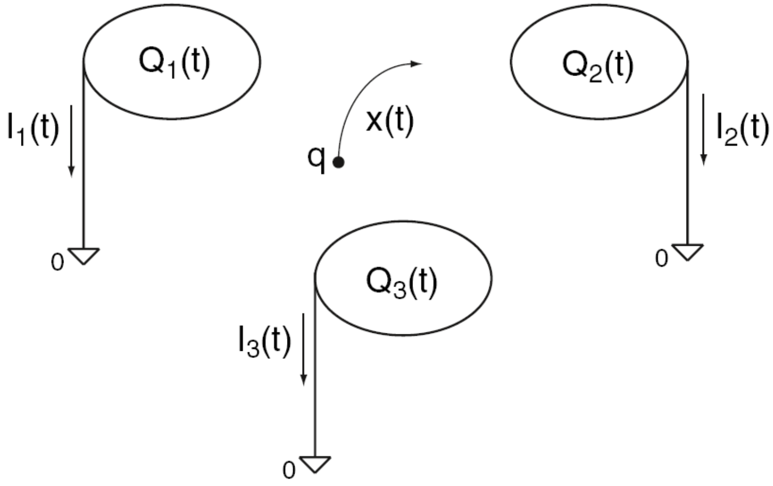


$$Q = -\frac{q}{V_w}\,\psi(\vec{x})$$

The charge induced by a point charge q at position x on a grounded electrode can be calculated the following way: One removes the point charge, puts the electrode in question to potential V_w while keeping the other electrodes grounded.

This defines the potential 'weighting potential' ψ(x) from which the induced charge can be calculated by the above formula.

Induced Current, Ramo Schottky Theorem



$$Q = -\frac{q}{V_w}\,\psi(\vec{x})$$

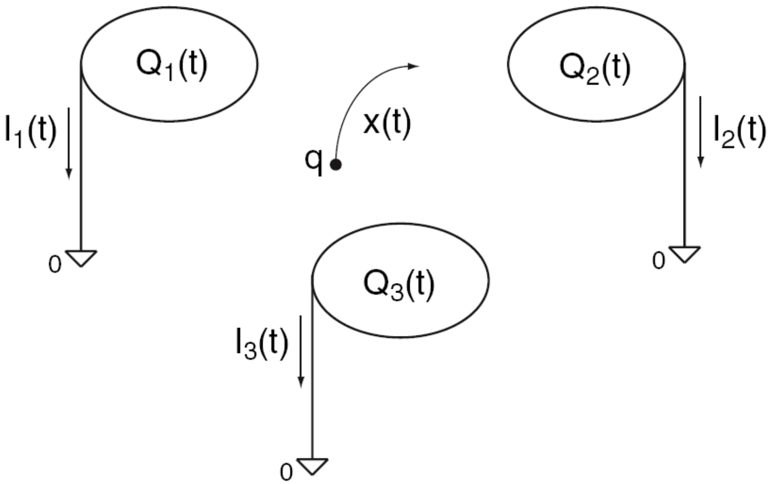
In case the charge is moving along a trajectory x(t), the time dependent induced charge is

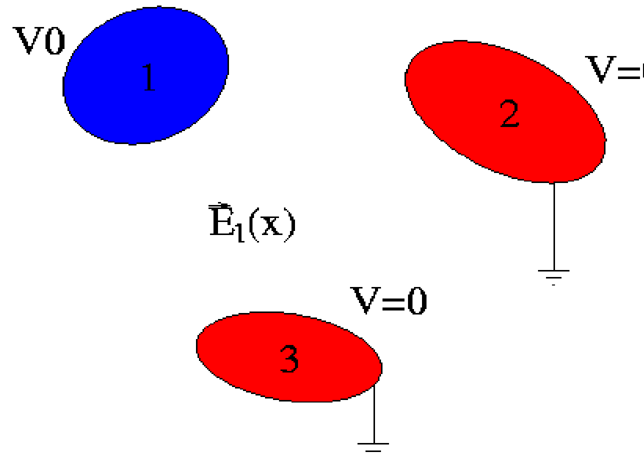
$$Q(t) = -\frac{q}{V_w}\,\psi\left(\vec{x}(t)\right)$$

And the induced current is

$$I(t) = -\frac{dQ}{dt} = \frac{q}{V_w}\,\vec{\nabla}\psi\left(\vec{x}(t)\right)\,\frac{d\vec{x}(t)}{dt} = -\frac{q}{V_w}\,\vec{E}\left(\vec{x}(t)\right)\,\vec{v}(t)$$

Induced Current





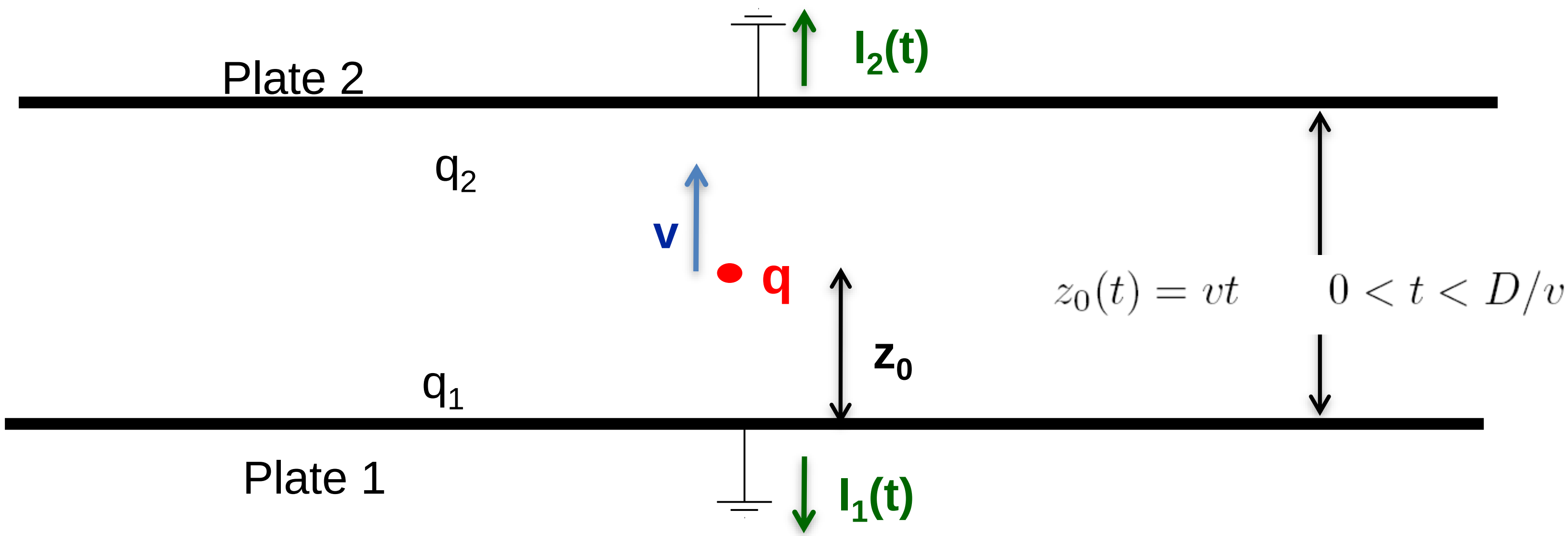
The current induced on a grounded electrode n by a moving point charge q is given by

$$I_n(t) == -\frac{q}{V_w}\vec{E_n}(\vec{x}(t))\,\vec{v}(t)$$

Where the weighting field E_n is defined by removing the point charge, setting the electrode in question to potential V_w and keeping the other electrodes grounded.

Removing the charge means that we just have to solve the Laplace equation and not the Poisson equation !

Parallel Plate Chamber



Weighting field E_1 of plate 1: Remove charge, set plate1 to V_w and keep plate 2 grounded

$$E_1 = \frac{V_w}{D}$$

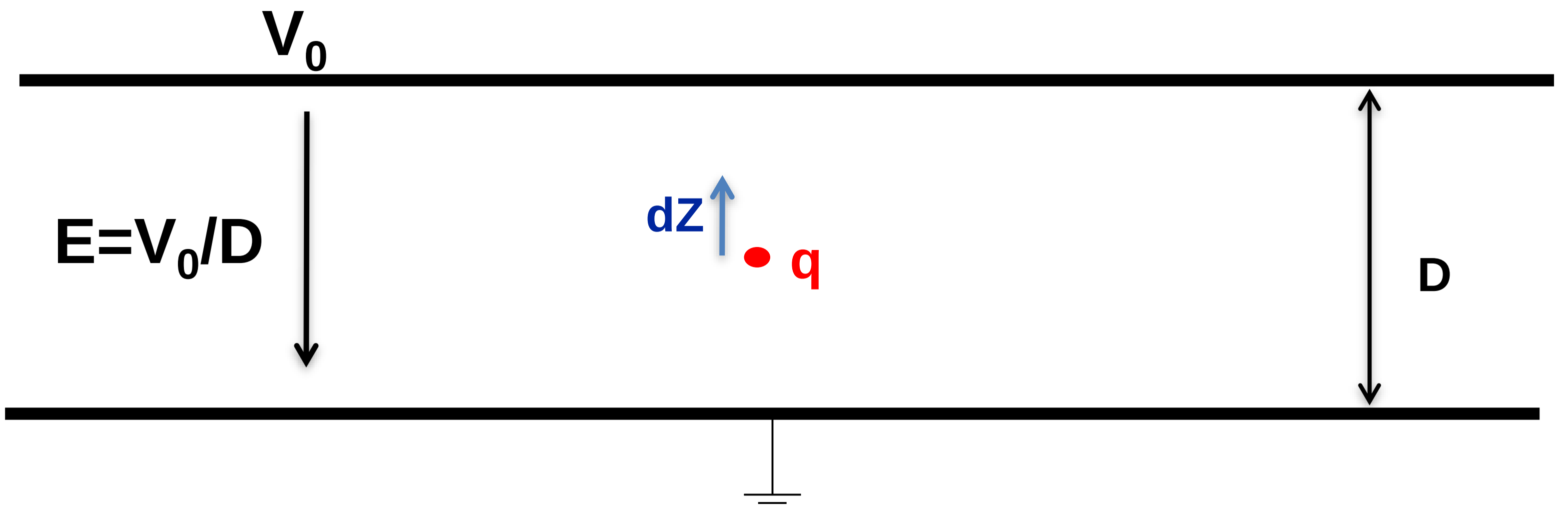
Weighting field E_2 of plate 2: Remove charge, set plate2 to V_w and keep plate1 grounded

$$E_2 = -\frac{V_w}{D}$$

So we have the induced currents

$$I_1 = -\frac{q}{V_w}\frac{V_w}{D}E_1\,v = -\frac{qv}{D} \qquad I_2 = -\frac{q}{V_w}\frac{V_w}{D}E_2\,v = \frac{qv}{D}$$

Arguing with Energy ? Not a good Idea !



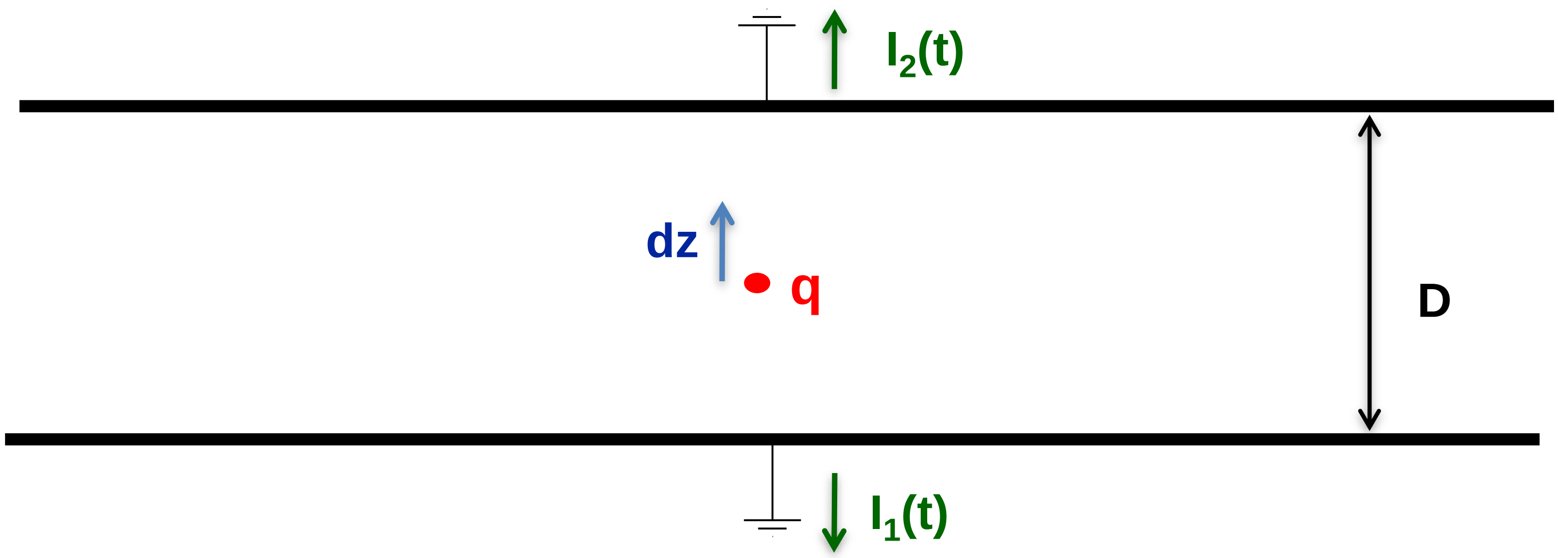
$$Energy = \frac{1}{2} C V_0^2 = \frac{1}{2} \frac{Q^2}{C} \qquad dW = qE dz = \frac{qV_0}{D} dz$$

$$d(Energy) = dW \qquad d\frac{1}{2} \frac{Q^2}{C} = \frac{qV_0}{D} dz$$

$$\frac{1}{2} \frac{1}{C} Q dQ = \frac{qV_0}{D} dz \qquad dQ = \frac{q}{D} dz$$

This argument gives the correct result, it is however only correct for a 2 electrode system because there the weighting field and the real field are equal. In addition the argument is very misleading.

Arguing with Energy ? Not a good Idea !



An induced current signal has nothing to do with Energy. In a gas detector the electrons are moving at constant speed in a constant electric field, so the energy gained by the electron in the electric field is lost into collisions with the gas, i.e. heating of the gas.

In absence of an electric field, the charge can be moved across the gap without using any force and currents are flowing.

The electric signals are due to induction !

Total Induced Charge

If a charge is moving from point x_0 to point x_1, the induced charge is

$$Q_n^{ind} = \int_{t_0}^{t_1} I_n^{ind}(t)dt = -\frac{q}{V_w}\int_{t_0}^{t_1} \boldsymbol{E}_n[\boldsymbol{x}(t)]\,\dot{\boldsymbol{x}}(t)dt = \frac{q}{V_w}[\psi_n(\boldsymbol{x}_1) - \psi_n(\boldsymbol{x}_0)]$$

If a pair of charges +q and -q is produced at point x_0 and q moves to x_1 while –q moves to x_2 , the charge induced on electrode n is given by

$$Q_n^{ind} = \int_{t_0}^{t_1} I_n^{ind}(t)dt = \frac{q}{V_w}[\psi_n(\boldsymbol{x}_1) - \psi_n(\boldsymbol{x}_2)]$$

If the charge q moves to electrode n while the charge –q moves to another electrode, the total induced charge on electrode n is q, because the ψ_n is equal to V_w on electrode n and equal to zero on all other electrodes.

In case both charges go to different electrodes the total induced charge is zero.

After ALL charges have arrived at the electrodes, the total induced charge on a given electrode is equal to the charge that has ARRIVED at this electrode.

Current signals on electrodes that don't receive a charge are therefore strictly bipolar.

Induced Charge, ‘Collected’ Charge

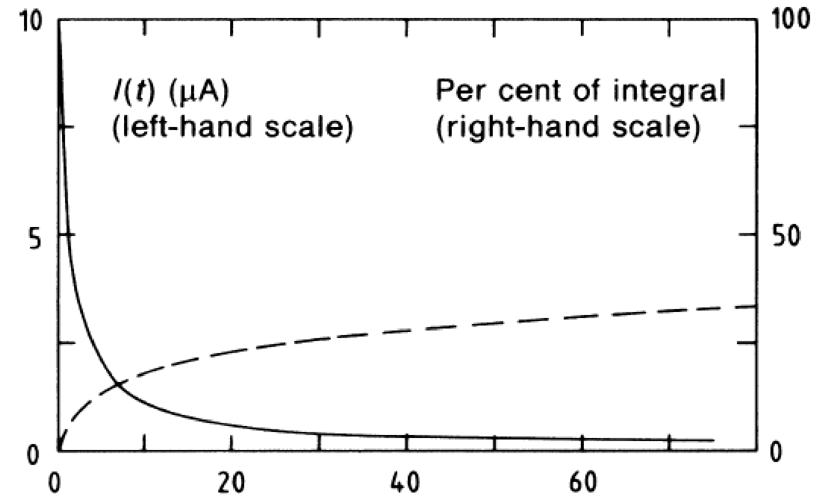
The fact that the total induced charge on an electrode, once ALL charges have arrived at the electrodes, is equal to the actual charge that has ARRIVED at the electrode, leads to very different ‘vocabulary for detectors in different detectors.

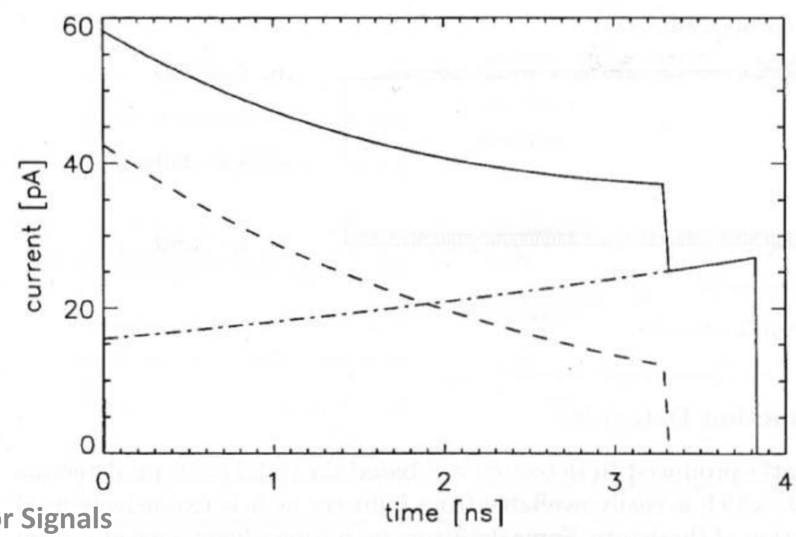
In wire chambers the ions take hundreds of microseconds to arrive at the cathodes. Because the electronics ‘integration time’ is typically much shorter than this time, the reality that the signal is ‘induced’ is very well known for wire chambers, and the signal shape is dominated by the movement of the ions.

The longer the amplifier integration time, the more charge is integrated, which is sometimes called ‘collected’, but it has nothing to do with collecting charge from the detector volume …

In Silicon Detectors, the electrons and holes take only a few ns to arrive at their electrodes, so e.g. for typical ‘integration times’ of amplifiers of 25ns, the shape is dominated by the amplifier response. The peak of the amplifier output is the proportional to the primary charge, and all the charge is ‘collected’

Still, the signal is not due to charges entering the amplifier from the detector, it is due to induction by the moving charge. Once the charge has actually arrived at the electrode, the signal is over !



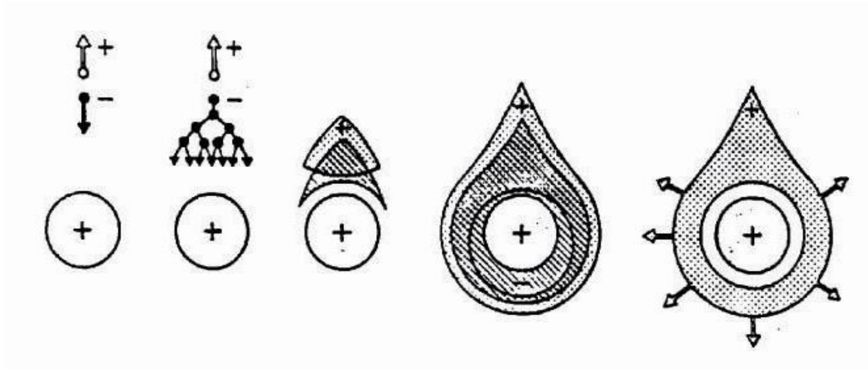


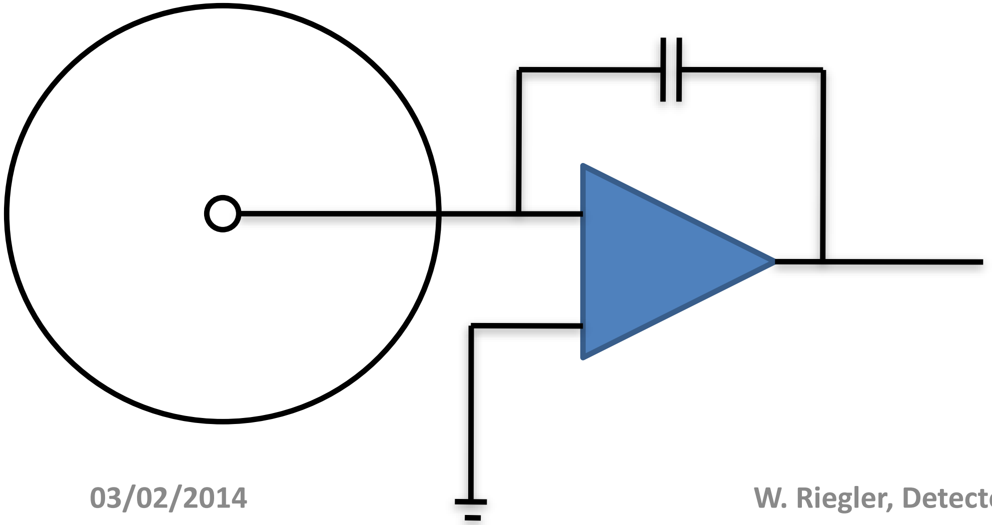
Total Induced Charge

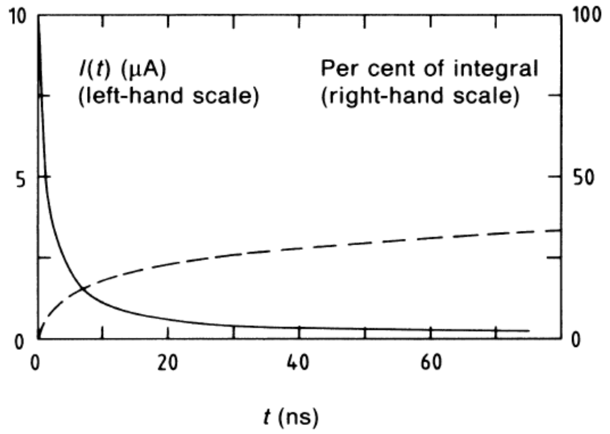
Imagine avalanche in a drift tube, caused by a single electron.
Let's assume that the gas gain is 10^4.
We read out the wire signal with an ideal integrator

The 10^4 electrons arrive at the wire within <1ns, so the integrator should instantly see the full charge of $-10^4\ e_0$ electrons ?

No ! The ions close to the wire induce the opposite charge on the wire, so in the very beginning there is zero charge on the integrator and only once the Ions have moved away from the wire the integrator measures the full $-10^4\ e_0$







Signal Calculation in 3 Steps

What are the signals induced by a moving charge on electrodes that are connected with arbitrary linear impedance elements ?

1) Calculate the particle trajectory in the ‘real’ electric field.

2) Remove all the impedance elements, connect the electrodes to ground and calculate the currents induced by the moving charge on the grounded electrodes.

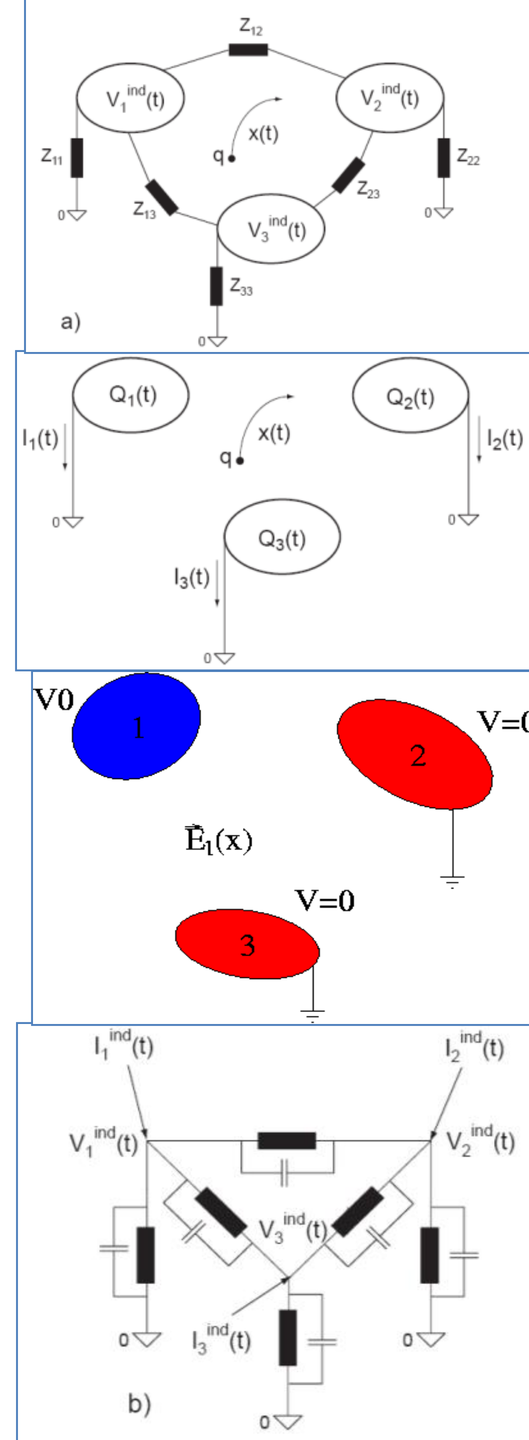
The current induced on a grounded electrode by a charge q moving along a trajectory x(t) is calculated the following way (Ramo Theorem):

One removes the charge q from the setup, puts the electrode to voltage V_0 while keeping all other electrodes grounded. This results in an electric field $E_n(x)$, the Weighting Field, in the volume between the electrodes, from which the current is calculated by

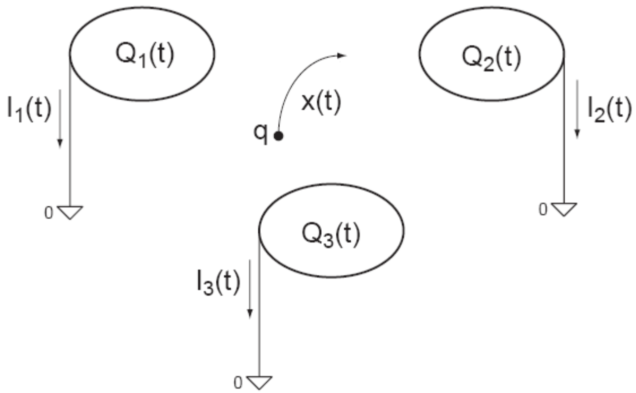
$$I_n(t) = -\frac{q}{V_0}\vec{E}_n[\vec{x}(t)]\frac{d\vec{x}(t)}{dt} = -\frac{q}{V_0}\vec{E}_n[\vec{x}(t)]\vec{v}(t)$$

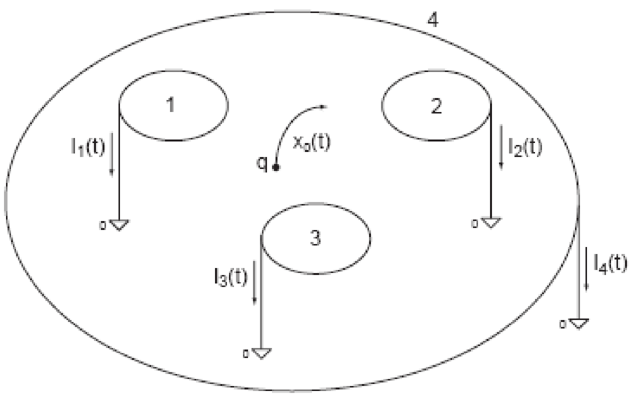
3) These currents are then placed as ideal current sources on a circuit where the electrodes are ‘shrunk’ to simple nodes and the mutual electrode capacitances are added between the nodes. These capacitances are calculated from the weighting fields by

$$c_{nm} = \frac{\varepsilon_0}{V_w}\oint_{\boldsymbol{A}_n} \boldsymbol{E}_m(\boldsymbol{x})d\boldsymbol{A} \qquad C_{nn} = \sum_m c_{nm} \qquad C_{nm} = -c_{nm} \quad n \neq m$$



General Signal Theorems





The following relations hold for the induced currents:

1) The charge induced on an electrode in case a charge in between the electrode has moved from a point x_0 to a point x_1 is

$$Q_n^{ind} = \int_{t_0}^{t_1} I_n^{ind}(t)dt = -\frac{q}{V_w}\int_{t_0}^{t_1} \boldsymbol{E}_n[\boldsymbol{x}(t)]\,\dot{\boldsymbol{x}}(t)dt = \frac{q}{V_w}[\psi_n(\boldsymbol{x}_1) - \psi_n(\boldsymbol{x}_0)]$$

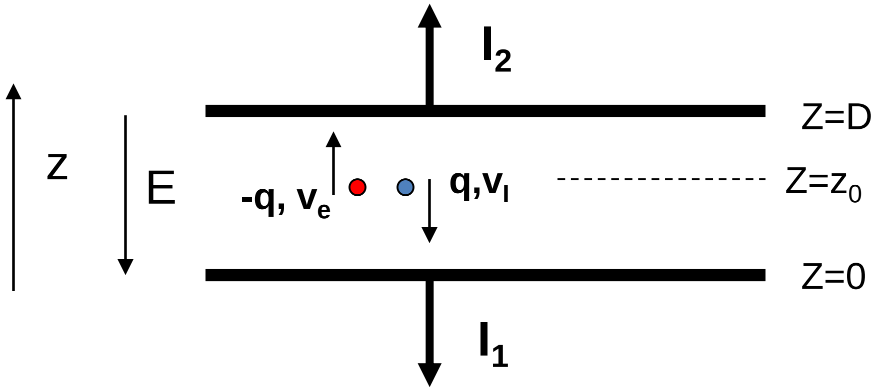
and is independent on the actual path.

2) Once ALL charges have arrived at the electrodes, the total induced charge in the electrodes is equal to the charge that has ARRIVED at this electrode.

3) In case there is one electrode enclosing all the others, the sum of all induced currents is zero at any time.

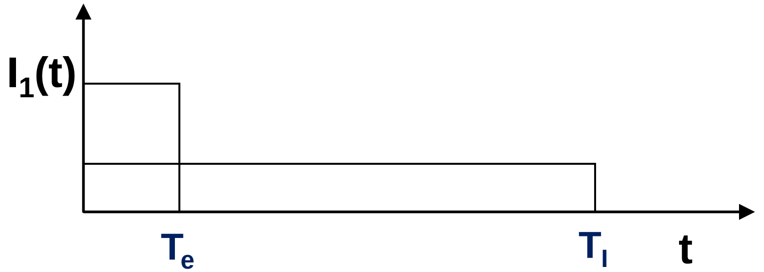
Signals in a Parallel Plate Geometry

E.g.: Electron-ion pair in gas
or Electron-ion pair in a liquid
or Electron-hole pair in a solid

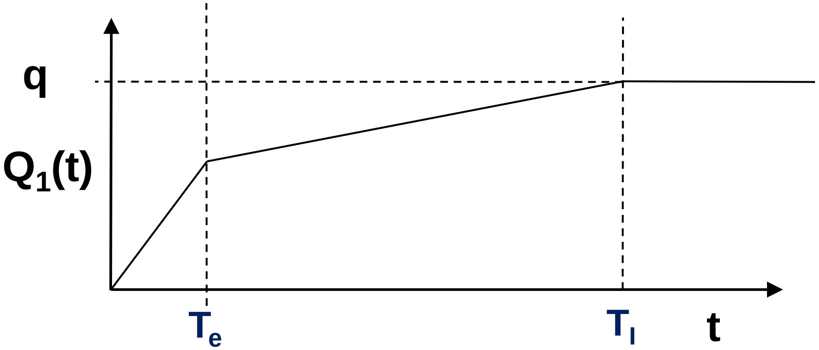


$E_1 = V_0/D$
$E_2 = -V_0/D$

$I_1 = -(-q)/V_0*(V_0/D)*v_e - q/V_0\ (V_0/D)\ (-v_I)$
$= q/D*v_e + q/D*v_I$
$I_2 = -I_1$



$Q^{tot}{}_1 = \int I_1 dt = q/D*v_e\ T_e + q/D*v_I*T_I$
$= q/D*v_e*(D-z_0)/v_e + q/D*v_I*z_0/v_I$
$= q(D-z_0)/D + qz_0/D =$
$q_e + q_I = q$



The total induced charge on a specific electrode, once all the charges have arrived at the electrodes, is equal to the charge that has arrived at this specific electrode.

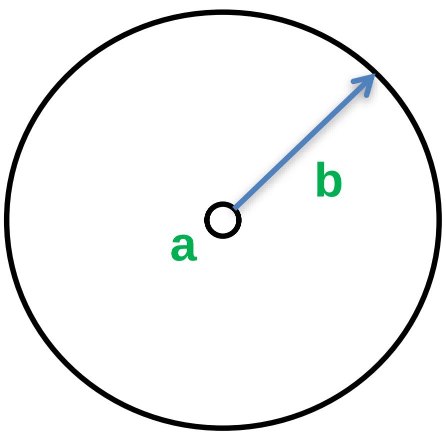
Wire Chamber Signals

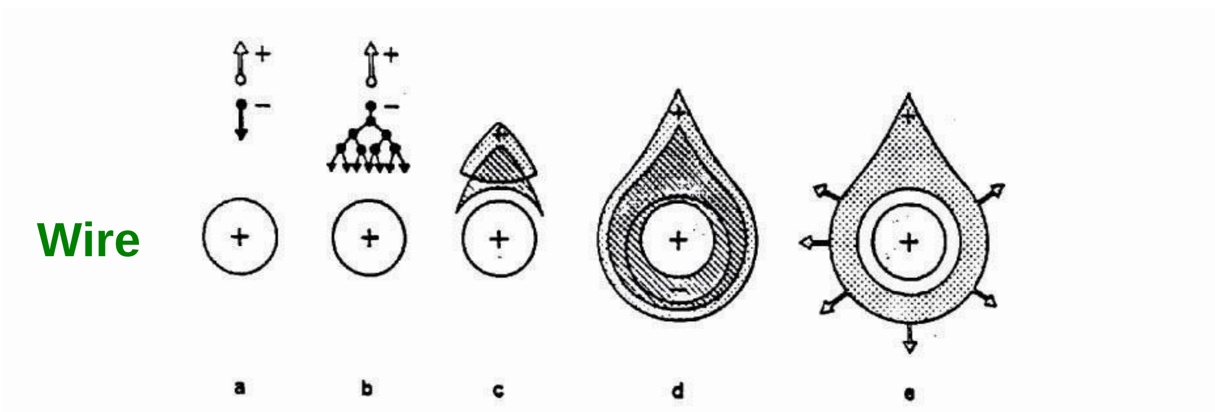
Wire with radius (10-25μm) in a tube of radius b (1-3cm):

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r} = \frac{V_0}{\ln\frac{b}{a}}\frac{1}{r}, \qquad V(r) = \frac{V_0}{\ln\frac{b}{a}}\ln\frac{r}{a},$$

Electric field close to a thin wire (100-300kV/cm). E.g. V_0=1000V, a=10μm, b=10mm, E(a)=150kV/cm

Electric field is sufficient to accelerate electrons to energies which are sufficient to produce secondary ionization → electron avalanche → signal.





Wire Chamber Signals

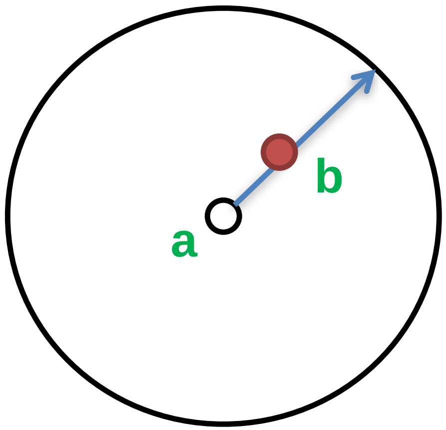
The electrons are produced very close to the wire, so for now we assume that N_{tot} ions are moving from the wire surface to the tube wall

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r} = \frac{U}{\ln\frac{b}{a}\,r}, \qquad V(r) = \frac{U}{\ln\frac{b}{a}}\ln\frac{r}{a},$$

Ions move with a velocity proportional to the electric field.

$$v = \mu E$$

$$\frac{dr(t)}{dt} = \mu\frac{U}{r(t)\ln(b/a)} \quad \rightarrow \quad r(t) = a\sqrt{1+\frac{t}{t_0}} \qquad t_0 = \frac{a^2\ln(b/a)}{2\mu U}$$



Weighting Field of the wire: Remove charge and set wire to V_w while grounding the tube wall.

$$E_1(r) = \frac{V_w}{r\ln(b/a)}$$

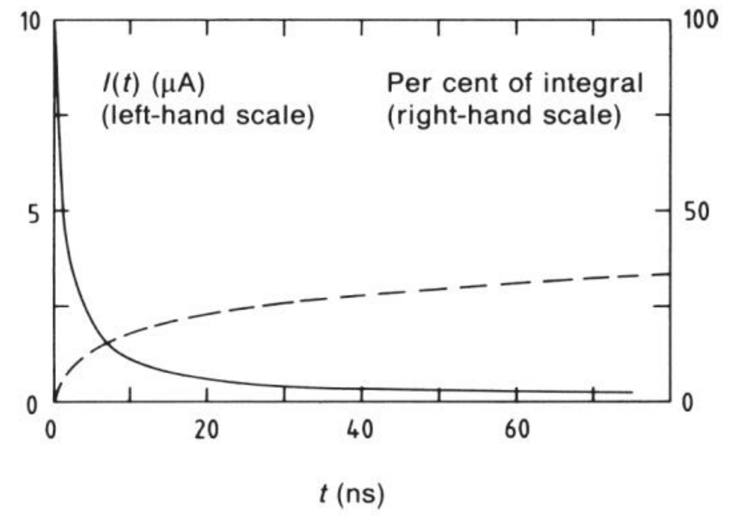
The induced current is therefore

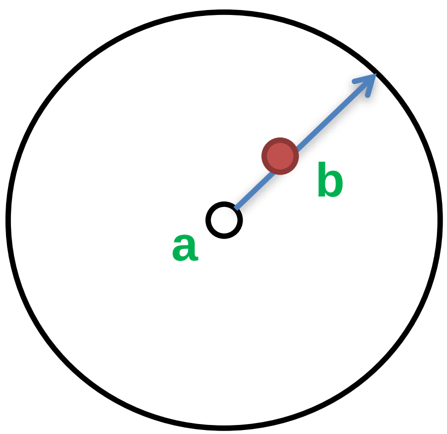
$$I_1^{ind}(t) = -\frac{N_{tot}e_0}{V_w}E_1[r(t)]\,\dot{r}(t) = -\frac{N_{tot}e_0}{2\ln(b/a)}\frac{1}{t+t_0}.$$

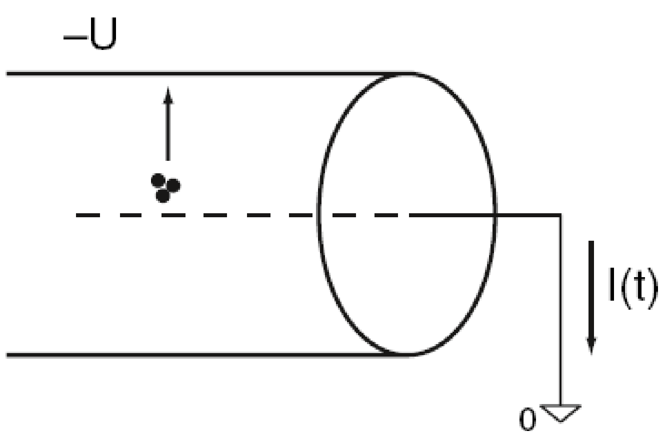
Wire Chamber Signals

$$I_1^{ind}(t) = -\frac{N_{tot} e_0}{V_w} E_1[r(t)]\, \dot{r}(t) = -\frac{N_{tot} e_0}{2\ln(b/a)} \frac{1}{t+t_0}.$$

$$Q_1^{ind}(t) = \int_0^t I_1^{ind}(t')dt' = -\frac{N_{tot} e_0}{2\ln(b/a)} \ln\left(1 + \frac{t}{t_0}\right)$$







Conclusion

This principle of signal generation is identical for Solid State Detectors, Gas Detectors and Liquid Detectors.

The signals are due to charges (currents) induced on metal electrodes by moving charges.

The easiest way to calculate signals induced by moving charges on metal electrodes is the use of Weighting fields (Ramo – Shockley Theorem) for calculation of currents induced on grounded electrodes.

These currents can then be placed as ideal current sources on an equivalent circuit diagram representing the detector.

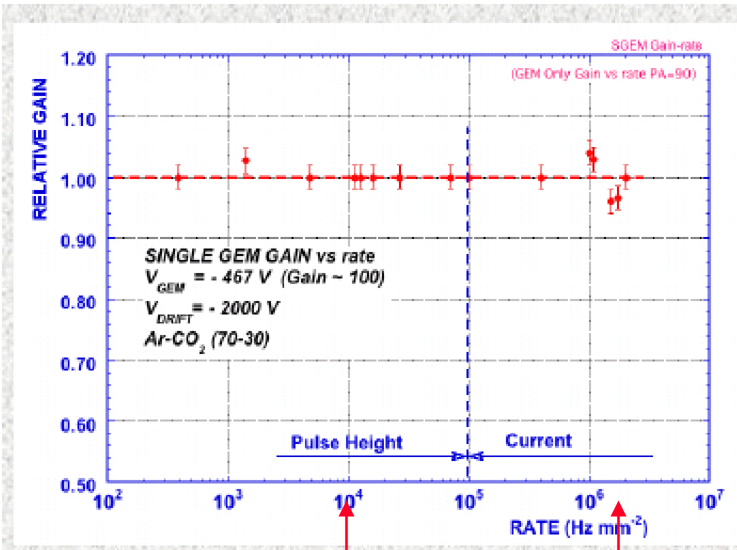
Micro Pattern Gas Detector

Signal Characteristics,

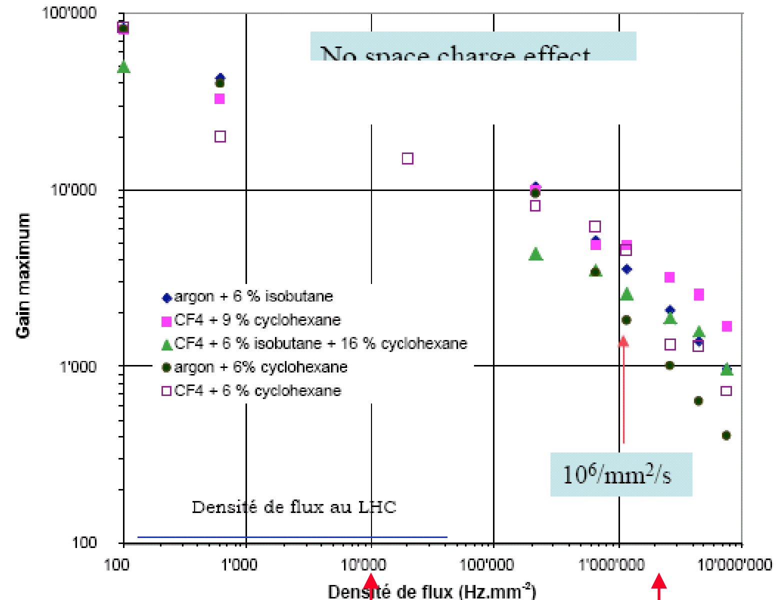
Frontend Electronics Specification

GEM/Micromega Rate Limits

GEM

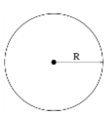


MICROMEGA

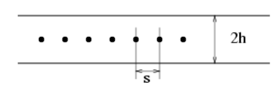


1MHz/cm² **200MHz/cm²**

1MHz/cm² **200MHz/cm²**



TRT straw tubes (R=2mm) and LHCb MWPCs (s=1.5mm, h=2.5mm) are limited at about 1MHz/cm² due to spacecharge.



In terms of rate capability due to space charge effects, MICROPATTERN detectors win over wire chambers by several orders of magnitude → BUT

Rate Limits

When talking about rate capability one also has to talk about pulse-width and pad response function and dead-time and occupancy

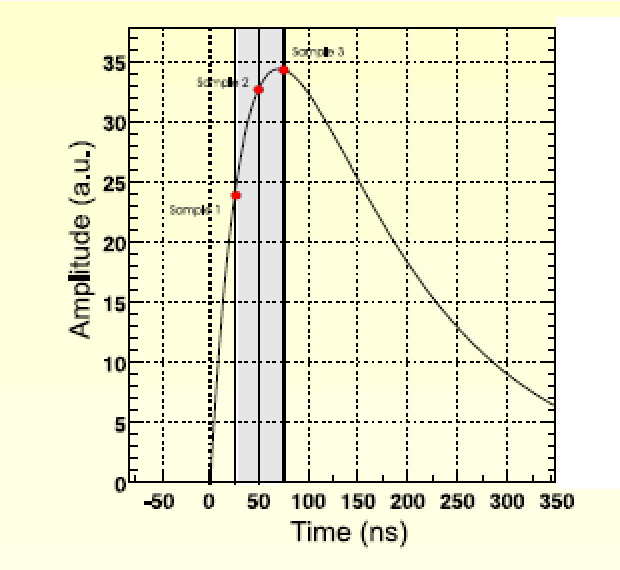
Compass Gems:

Rates of up to 2.5MHz/cm^2
Amplifier peaking time approx 50ns
occupancy up to 25% (readout strips).

Compass MicroMegas:

Up to 0.5 MHz/cm2
Rates of 200kHz/strip
Amplifier peaking time approx 85ns

E.g. 1MHz on a readout channel & 100ns pulse-width → 10% inefficiency



Clearly in order to profit from 200MHz rate capability one has to

Increase the electronics bandwidth.

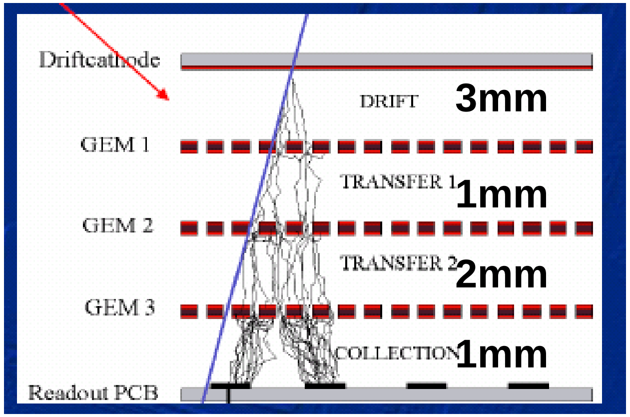
AND/OR Pixelize the readout electrodes

→ Pad response function, ballistic deficit, double track separation

GEM

V$_1$~50um/ns

V$_2$~ 100um/ns



'Only' fast electron signal, but

Single electron moving in the induction gap takes about $1mm/v_1$=10ns.

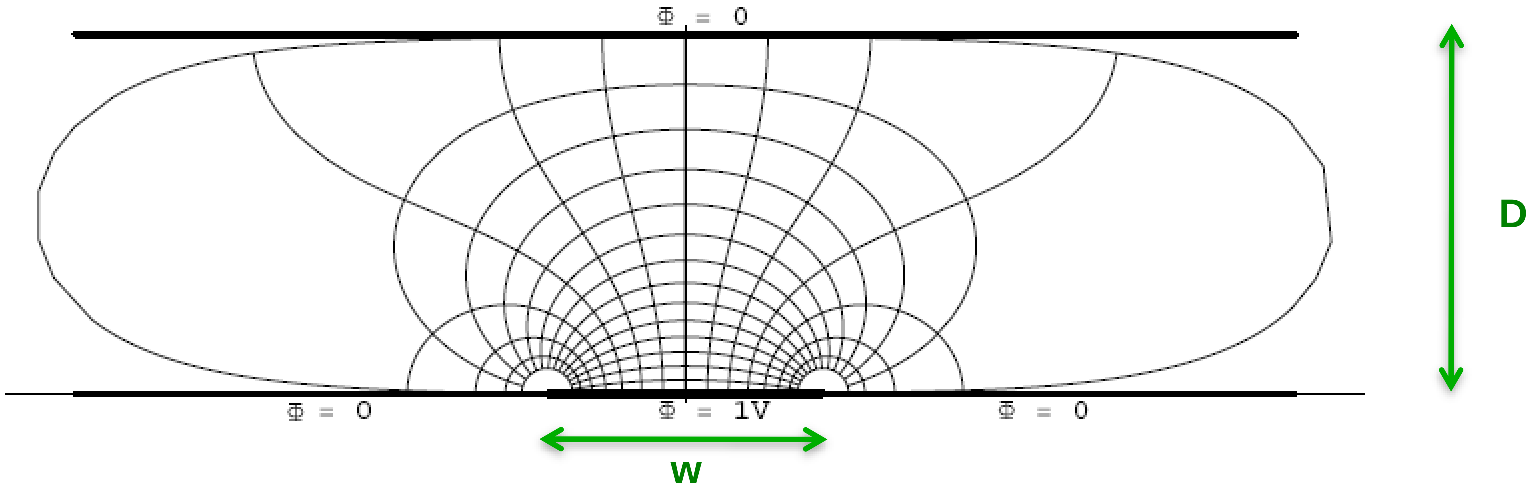
Collecting all electrons from the drift gap takes a maximum of $3mm/v_1$=60ns.

The GEM signal has a length of about 50 ns.

Use fast electronics for GEM readout.

→ Increase of Pad Response Function !

Pad response function of parallel plate geometry



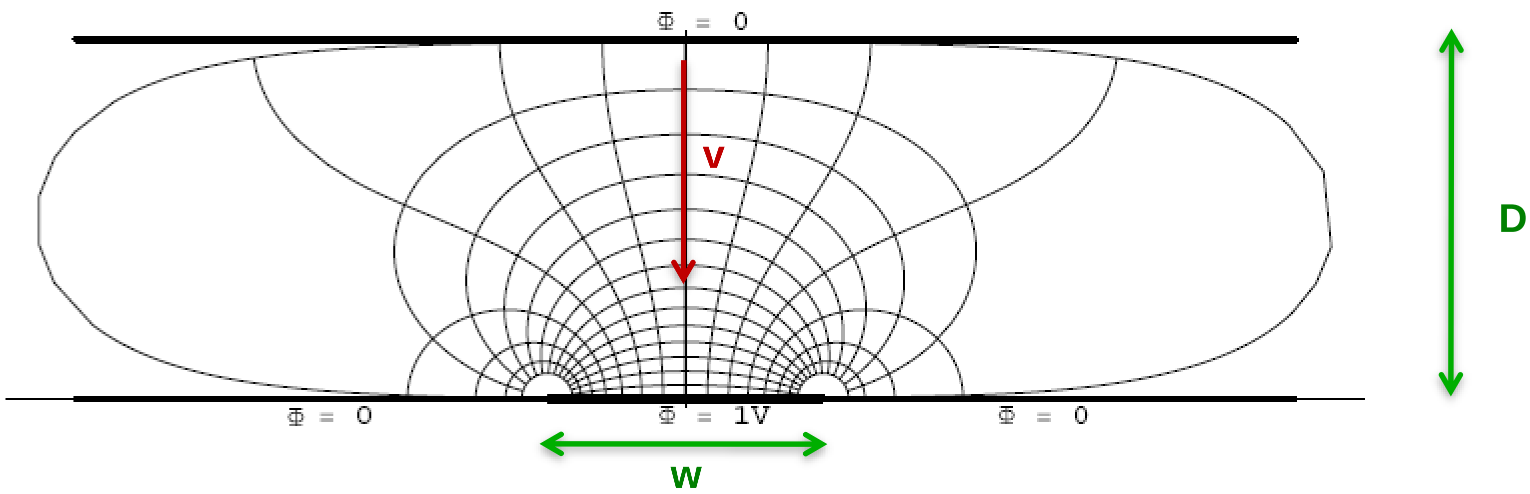
$$\Phi_1(x,z) = \frac{V_1}{\pi}\Big[\ \arctan\Big(\cot\big(\frac{z\pi}{2D}\big)\ \tanh\big(\pi\,\frac{x+w/2}{2D}\big)\Big) - \arctan\Big(\cot\big(\frac{z\pi}{2D}\big)\ \tanh\big(\pi\,\frac{x-w/2}{2D}\big)\Big)\Big]$$

$$E_{1x} = V_1\frac{1}{2D}\left[\frac{\sin\left(\frac{z\pi}{D}\right)}{\cosh\left(\pi\,\frac{x-w/2}{D}\right)-\cos\left(\frac{z\pi}{D}\right)} - \frac{\sin\left(\frac{z\pi}{D}\right)}{\cosh\left(\pi\,\frac{x+w/2}{D}\right)-\cos\left(\frac{z\pi}{D}\right)}\right]$$

Weighting Field of a strip in an GEM or Micromega

$$\boxed{E_{1z}} = -V_1\frac{1}{2D}\left[\frac{\sinh\left(\pi\,\frac{x-w/2}{D}\right)}{\cosh\left(\pi\,\frac{x-w/2}{D}\right)-\cos\left(\frac{z\pi}{D}\right)} - \frac{\sinh\left(\pi\,\frac{x+w/2}{D}\right)}{\cosh\left(\pi\,\frac{x+w/2}{D}\right)-\cos\left(\frac{z\pi}{D}\right)}\right]$$

Pad response function of parallel plate geometry



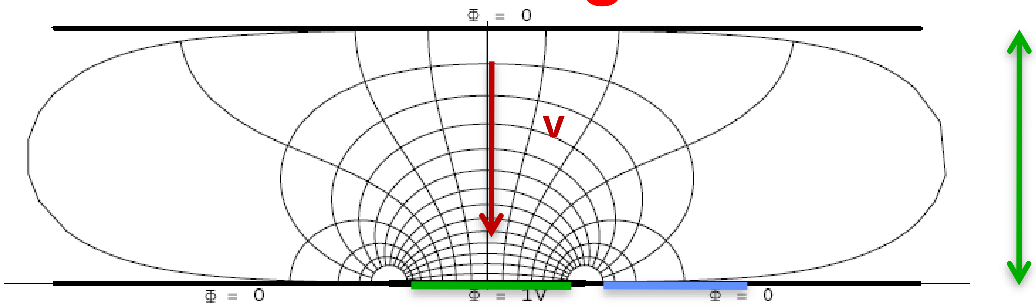
I(t,x) = $-e_0$*Ez[x,D-v*t]*v

When all charges have arrived at the electrodes the induced charge is equal to the total charge that has arrived at the electrode.

If the electronics 'integration time' ~ 'peaking time' ~ 'shaping time' is larger than the time it takes the electron to pass the induction gap, the readout strips that don't receive any charge show zero signal.

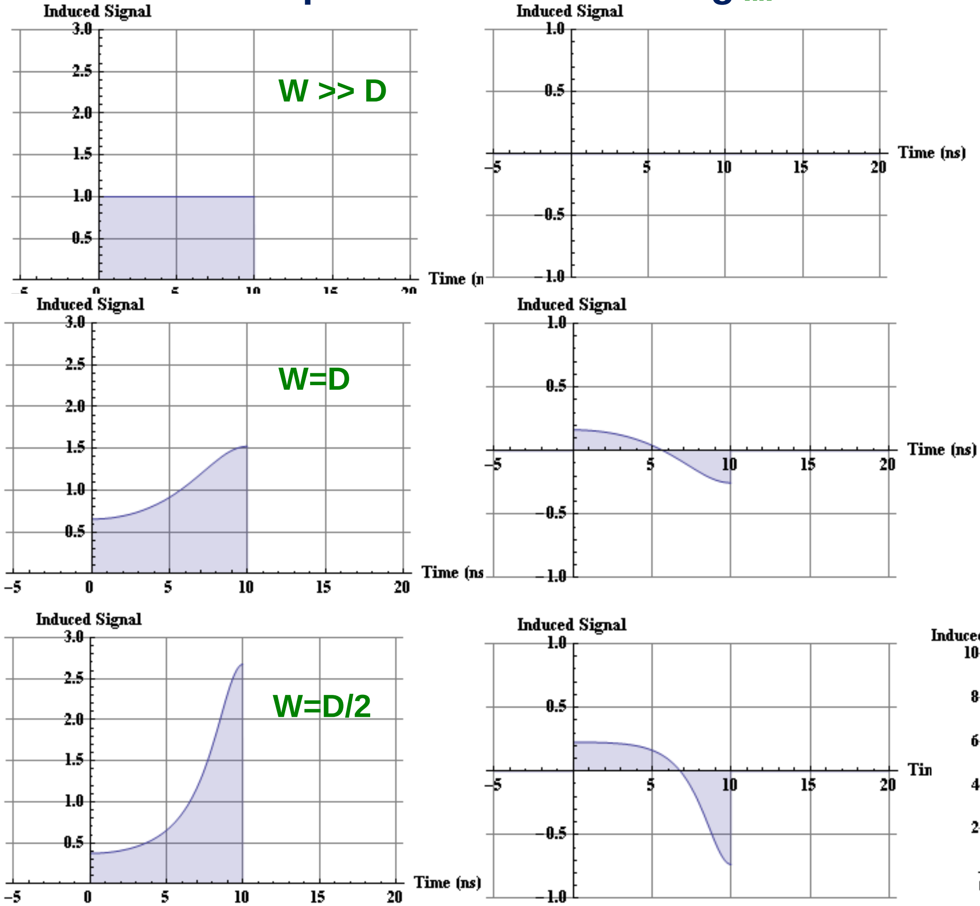
If the electronics 'integration time' ~ 'peaking time' ~ 'shaping time' is smaller than than the time it takes the electron to pass the induction gap, the readout strips that don't receive any charge show a signal different from zero that is strictly bipolar.

GEM Signals



D = 1mm

Central Strip / First Neighbour



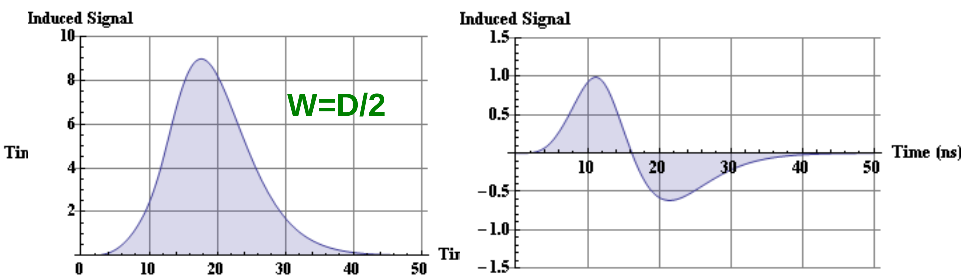
Amplifier Peaking Time = 10ns

W=D/2 (D=1mm, w=0.5mm)

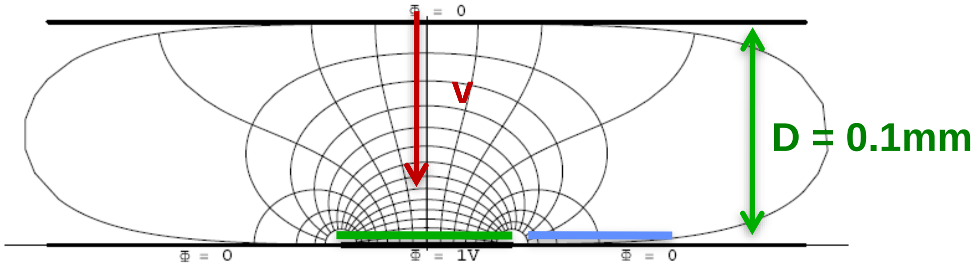
→ 10% crosstalk !

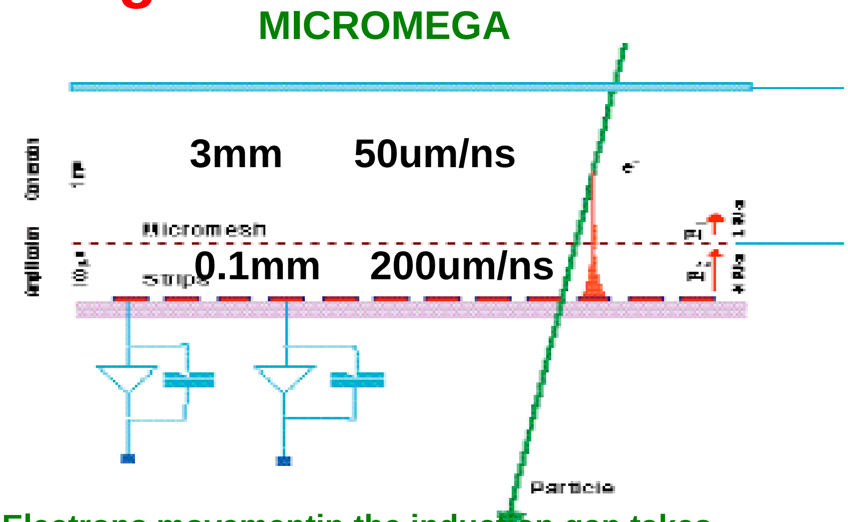
In case the electronics peaking time is smaller than the time the electron takes to pass the electron transit time the induction gap the 'cluster size' or 'pad response function' becomes larger.

Central Strip / First Neighbor



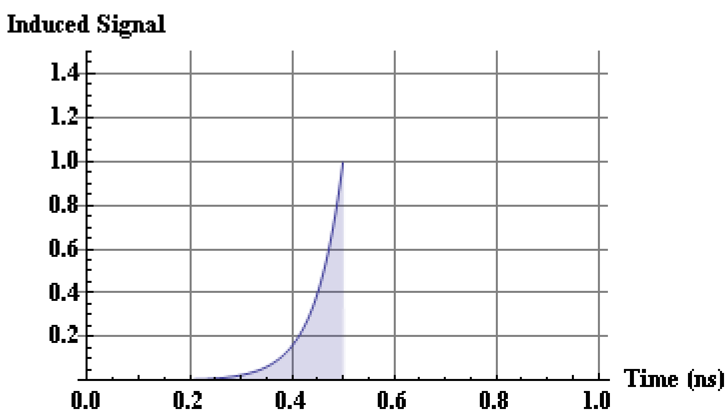
Micromega Signal





$I(t,x) = -e_0 * \mathrm{Exp}[\alpha\ v\ t] * Ez[x, D-v*t]*v$ → Electrons

Electrons movementin the induction gap takes about $0.1mm/v_1=0.5ns$.



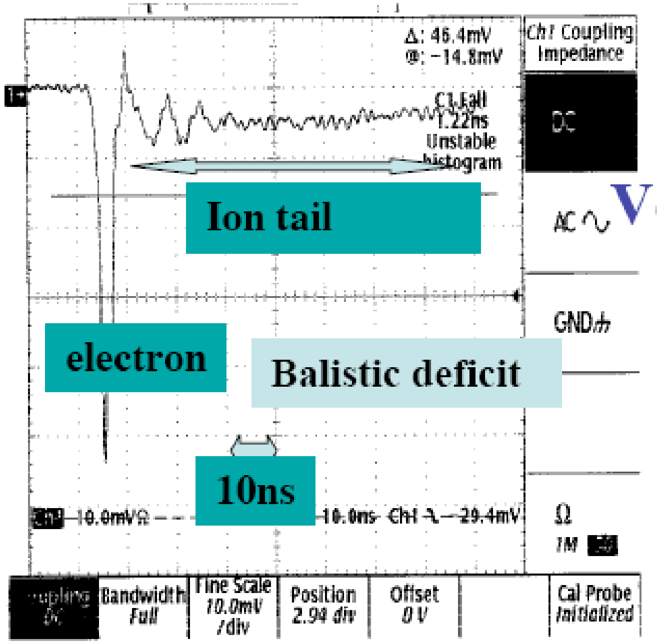
Collecting all electrons from the drift gap takes a maximum of $3mm/v_1=60ns$.

The MICROMEGA electron signal has a length of about 50 ns.

Typically w>>D – cluster size from electron component is dominated by diffusion and not by direct induction.

However, ion component has a length of about 100ns → Ballistic Deficit for fast electronis (e.g. 10ns peaking time).

Small gap → fast signals



Very-fast current signal

1 ns rise time

12 μm Micromegas is under development for NA48-3

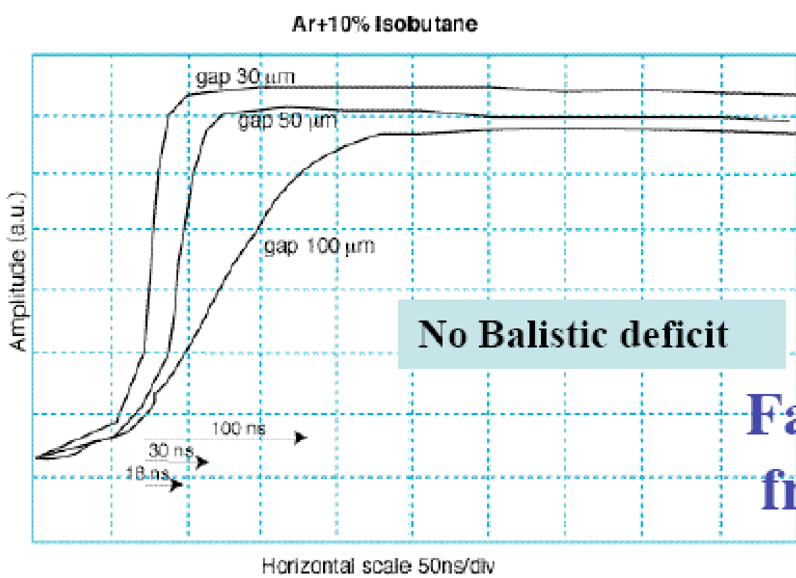
Ion collection time will be :

6 ns in Argon

2.5 ns in Neon

1 ns in Helium !!!!

Much faster than Si (x40)



Fast signal 20-100 ns from charge preamplifier

Conclusions

As long as the electronics integration time is significantly larger than the time of the charge movement in the induction gap (GEM and MICROMEGA) the signal just depends on the amount of charge that has arrived at the readout pad (‘collection’).

For the GEM detector, where only electrons move inside the induction gap, the transit time is about 10ns, so the above statement is true for electronics integration time of >>10ns.

For the Micromega, electrons are moving from the mesh to the readout plane in <0.5ns, but the ions take up to 100ns to move back to the mesh, depending on the gas, geometry and operation point.

The signal shape and crosstalk will therefore strongly depends on the electronics integration time (ballistic deficit).

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Electric fields, weighting fields, signals and charge diffusion in detectors including resistive materials

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ABSTRACT: In this report we discuss static and time dependent electric fields in detector geometries with an arbitrary number of parallel layers of a given permittivity and weak conductivity. We derive the Green's functions i.e. the field of a point charge, as well as the weighting fields for readout pads and readout strips in these geometries. The effect of 'bulk' resistivity on electric fields and signals is investigated. The spreading of charge on thin resistive layers is also discussed in detail, and the conditions for allowing the effect to be described by the diffusion equation is discussed. We apply the results to derive fields and induced signals in Resistive Plate Chambers, MICROMEGAS detectors including resistive layers for charge spreading and discharge protection as well as detectors using resistive charge division readout like the MicroCAT detector. We also discuss in detail how resistive layers affect signal shapes and increase crosstalk between readout electrodes.

KEYWORDS: Charge induction; Detector modelling and simulations II (electric fields, charge transport, multiplication and induction, pulse formation, electron emission, etc); Micropattern gaseous detectors (MSGC, GEM, THGEM, RETHGEM, MHSP, MICROPIC, MICROMEGAS, InGrid, etc); Resistive-plate chambers

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2. Quasi-static approximation of Maxwell's equations

To include the frequency dependence of ε and σ we work in the Laplace domain, i.e. we write

$$\mathscr{L}[\vec{E}(\vec{x},t)] = \bar{\vec{E}}(\vec{x},s),$$
$$\mathscr{L}\left[\frac{\partial \vec{E}(\vec{x},t)}{\partial t}\right] = s\bar{\vec{E}}(\vec{x},s), \quad \text{etc.} \tag{2}$$

where we have assumed that at $t=0$ all fields and charges are zero. Maxwell's equations for a linear isotropic medium with permittivity $\varepsilon(\vec{x},s)$ and conductivity $\sigma(\vec{x},s)$ then read as

$$\vec{\nabla}\bar{\vec{D}} = \bar{\rho} \quad \bar{\vec{D}} = \varepsilon\bar{\vec{E}}, \qquad \vec{\nabla}\bar{\vec{B}} = 0 \quad \bar{\vec{B}} = \mu\bar{\vec{H}} \tag{3}$$

$$\vec{\nabla}\times\bar{\vec{E}} = -s\bar{\vec{B}}, \qquad \vec{\nabla}\times\bar{\vec{H}} = \bar{\vec{j}}_e + \sigma\bar{\vec{E}} + s\bar{\vec{D}} \tag{4}$$

where $\bar{\vec{j}}_e$ is an 'externally impressed' current that is connected with an 'external' charge density by $\vec{\nabla}\bar{\vec{j}}_e = -s\bar{\rho}_e$. Assuming weak conductivity σ we can set

$$\vec{\nabla}\times\bar{\vec{E}} = -s\bar{\vec{B}} = 0 \quad \Rightarrow \bar{\vec{E}} = -\vec{\nabla}\bar{\Phi} \tag{5}$$

and by taking the divergence of the second equation in Eq. (4) we find

$$\vec{\nabla}[\sigma(\vec{x},s)\vec{\nabla}]\bar{\Phi}(\vec{x},s) + \vec{\nabla}[\varepsilon(\vec{x},s)\vec{\nabla}]s\bar{\Phi}(\vec{x},s)$$
$$= -s\bar{\rho}_e(\vec{x},s) \tag{6}$$

which we can write as

$$\vec{\nabla}[\epsilon(\vec{x},s)\vec{\nabla}]\bar{\Phi}(\vec{x},s) = -\bar{\rho}_e(\vec{x},s) \quad \text{with}$$
$$\epsilon(\vec{x},s) = \varepsilon(\vec{x},s) + \frac{1}{s}\sigma(\vec{x},s). \tag{7}$$

Quasistatic Approximation

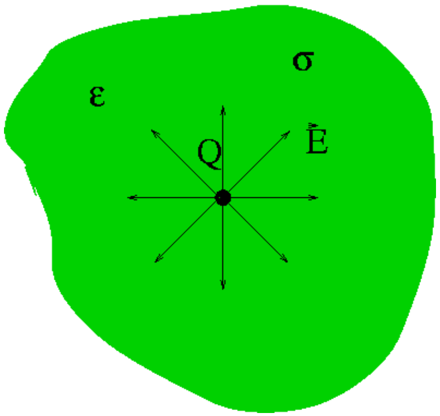
equations: Knowing the solution of the Poisson equation for a charge distribution $\rho(\vec{x})$ embedded in a geometry of a given permittivity $\varepsilon(\vec{x})$, we find the time dependent solution (in the Laplace domain with parameter s) for an 'externally impressed' charge density $\rho_e(\vec{x}, s)$ and a geometry that in addition includes a finite (weak) conductivity $\sigma(\vec{x})$ by replacing $\varepsilon(\vec{x})$ with $\varepsilon(\vec{x}) + \sigma(\vec{x})/s$ and $\rho(\vec{x})$ with $\rho_e(\vec{x}, s)$. For detector applications the volume resistivity $\rho(\vec{x}) = 1/\sigma(\vec{x})$ is traditionally used.

As an example we look at the potential of a point charge Q in a medium of constant permittivity ε, which is given by

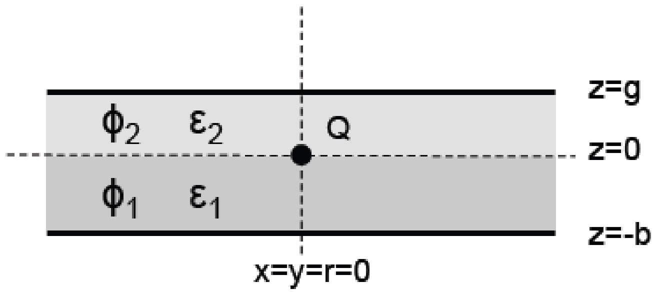
$$\phi(r) = \frac{Q}{4\varepsilon\pi\, r} \tag{1}$$

In case the medium has a conductivity σ and we place the 'external' charge Q at $t = 0$, i.e. $Q(t) = Q\Theta(t)$ and therefore $Q(s) = Q_0/s$, we replace ε by $\varepsilon + \sigma/s$ and Q by Q/s and perform the inverse Laplace transform, which gives

$$\phi(r, s) = \frac{Q}{4\pi(s\varepsilon + \sigma)r} \quad \rightarrow \quad \phi(r, t) = \frac{Q}{4\pi\varepsilon\, r}\, e^{\frac{-t}{\tau}} \qquad \tau = \varepsilon/\sigma = \rho\varepsilon \tag{2}$$



Point charge in a double layer



$$\phi_1(r,z) = \frac{1}{2\pi}\int_0^\infty J_0(kr)\left[A_1(k)e^{kz} + B_1(k)e^{-kz}\right] dk \qquad -b < z < 0$$

$$\phi_2(r,z) = \frac{1}{2\pi}\int_0^\infty J_0(kr)\left[A_2(k)e^{kz} + B_2(k)e^{-kz}\right] dk \qquad 0 < z < g$$

$z = -b$ and $z = g$ define the conditions $\phi_1(-b, r) = 0$ and $\phi_2(g, r) = 0$, which gives

$$\begin{aligned} A_1 e^{-kb} + B_1 e^{kb} &= 0 \\ A_2 e^{kg} + B_2 e^{-kg} &= 0 \end{aligned}$$

$\phi_1(r, 0) = \phi_2(r, 0)$ which gives

$$A_1 + B_1 = A_2 + B_2$$

and the εE component perpendicular to the sheet 'jumps' by $q(r)$

$$\varepsilon_1 \frac{\partial \phi_1(r,z)}{\partial z}\Big|_{z=0} - \varepsilon_2 \frac{\partial \phi_2(r,z)}{\partial z}\Big|_{z=0} = q(r) \tag{7}$$

The surface charge density corresponding to the point charge Q at $r = 0$ is $q(r) = Q\delta(r)/2\pi r$, so this last equation reads as

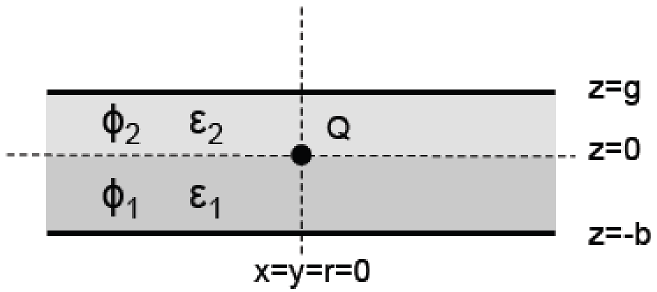
$$\frac{1}{2\pi}\int_0^\infty J_0(kr)k\left[\varepsilon_1(A_1 - B_1) - \varepsilon_2(A_2 - B_2)\right] dk = \frac{Q}{2\pi r}\delta(r)$$

Multiplying both sides of the equation with $rJ_0(k'r)$, integrating them over r from 0 to ∞ and using the relation $\int_0^\infty rJ_0(kr)J_0(k'r)dr = \delta(k-k')/k$ [18] we have

$$\varepsilon_1(A_1 - B_1) - \varepsilon_2(A_2 - B_2) = Q \tag{8}$$

→ 4 equations that define A_1, B_1, A_2, B_2

Point charge in a double layer



$$D(k) = 4[\varepsilon_1 \cosh(bk)\ \sinh(gk) + \varepsilon_2 \sinh(bk)\ \cosh(gk)] \qquad (10)$$

The solutions then read as

$$\phi_1(r,z) = \frac{Q}{2\pi}\int_0^\infty J_0(kr)\frac{4\sinh(gk)\sinh(k(b+z))}{D(k)}dk \qquad -b < z < 0 \qquad (11)$$

$$\phi_2(r,z) = \frac{Q}{2\pi}\int_0^\infty J_0(kr)\frac{4\sinh(bk)\sinh(k(g-z))}{D(k)}dk \qquad 0 < z < g \qquad (12)$$

Expressing the solution as a point charge with a correction term:

$$\frac{e^{kz}}{\varepsilon_1+\varepsilon_2} + \frac{4\sinh(gk)\sinh(k(b+z))}{D(k)} - \frac{e^{kz}}{\varepsilon_1+\varepsilon_2} = \frac{e^{kz}}{\varepsilon_1+\varepsilon_2} + f_1(k,z)$$

and arrive with Eq. 13 at

$$\phi_1(r,z) = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)}\frac{1}{\sqrt{r^2+z^2}} + \frac{Q}{2\pi}\int_0^\infty J_0(kr)f_1(k,z)dk$$

Point charge potential and weighting field of a pixel or pad in a plane condenser

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A B S T R A C T

We derive expressions for the potential of a point charge as well as the weighting potential and weighting field of a rectangular pad for a plane condenser, which are well suited for numerical evaluation. We relate the expressions to solutions employing the method of image charges, which allows discussion of convergence properties and estimation of errors, providing also an illuminating example of a problem with an infinite number of image charges.

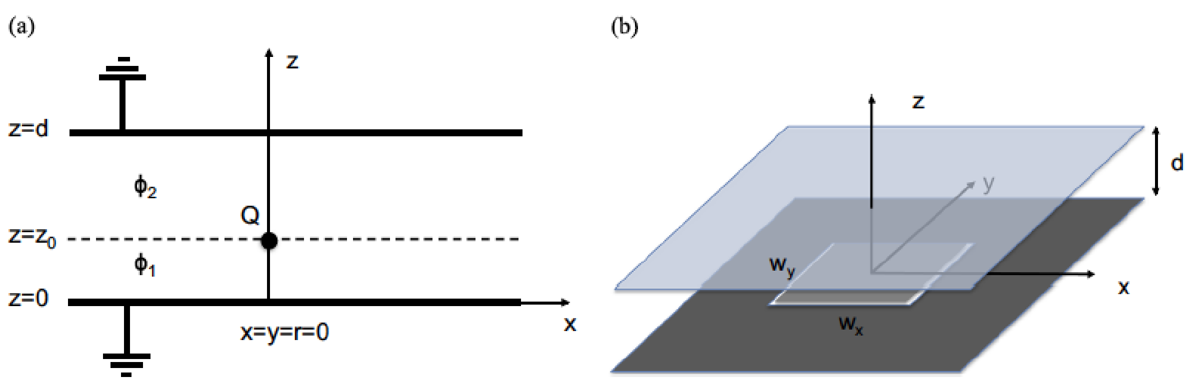


Fig. 1. (a) Point charge Q between two grounded metal planes. (b) Readout pad or pixel of dimension w_x and w_y centred at the origin.

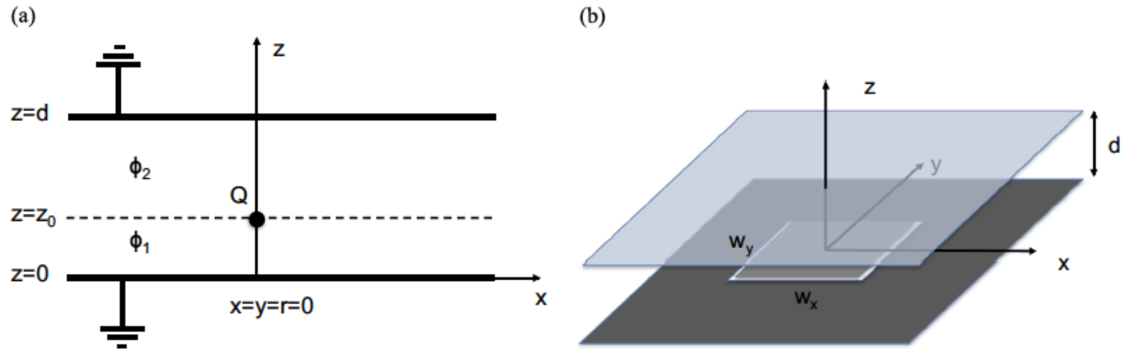


Fig. 1. (a) Point charge Q between two grounded metal planes. (b) Readout pad or pixel of dimension w_x and w_y centred at the origin.

$$\frac{4\pi\varepsilon_0}{Q}\phi(r,z)=\frac{1}{\sqrt{r^2+(z-z_0)^2}}-\frac{1}{\sqrt{r^2+(z+z_0)^2}}$$
$$+\sum_{n=1}^{N}\left[\frac{1}{\sqrt{r^2+(z+2nd-z_0)^2}}+\frac{1}{\sqrt{r^2+(z-2nd-z_0)^2}}\right.$$
$$\left.-\frac{1}{\sqrt{r^2+(z-2nd+z_0)^2}}-\frac{1}{\sqrt{r^2+(z+2nd+z_0)^2}}\right]$$
$$-\int_0^\infty 2J_0(kr)e^{-k(2N+1)d}\frac{\sin h(kz)\,\sin h(kz_0)}{\sin h(kd)}dk$$

$$\frac{\phi_w(x,y,z)}{V_w}=\frac{1}{2\pi}f(x,y,z)-\frac{1}{2\pi}\sum_{n=1}^{N}[f(x,y,2nd-z)-f(x,y,2nd+z)]$$
$$-\frac{4}{\pi^2}\int_0^\infty\int_0^\infty \cos(k_x x)\,\sin\left(k_x\frac{w_x}{2}\right)\,\cos(k_y y)$$
$$\times\sin\left(k_y\frac{w_y}{2}\right)\frac{e^{-k(2N+1)d}}{k_x k_y}\frac{\sin h(kz)}{\sin h(kd)}dk_x dk_y$$

with

$$f(x,y,u)=\int_{x-w_x/2}^{x+w_x/2}\int_{y-w_y/2}^{y+w_y/2}\frac{u}{(x'^2+y'^2+u^2)^{3/2}}dx'\,dy'$$
$$=\arctan\left(\frac{x_1y_1}{u\sqrt{x_1^2+y_1^2+u^2}}\right)+\arctan\left(\frac{x_2y_2}{u\sqrt{x_2^2+y_2^2+u^2}}\right)$$

Point charges in a geometry with N dielectric layers

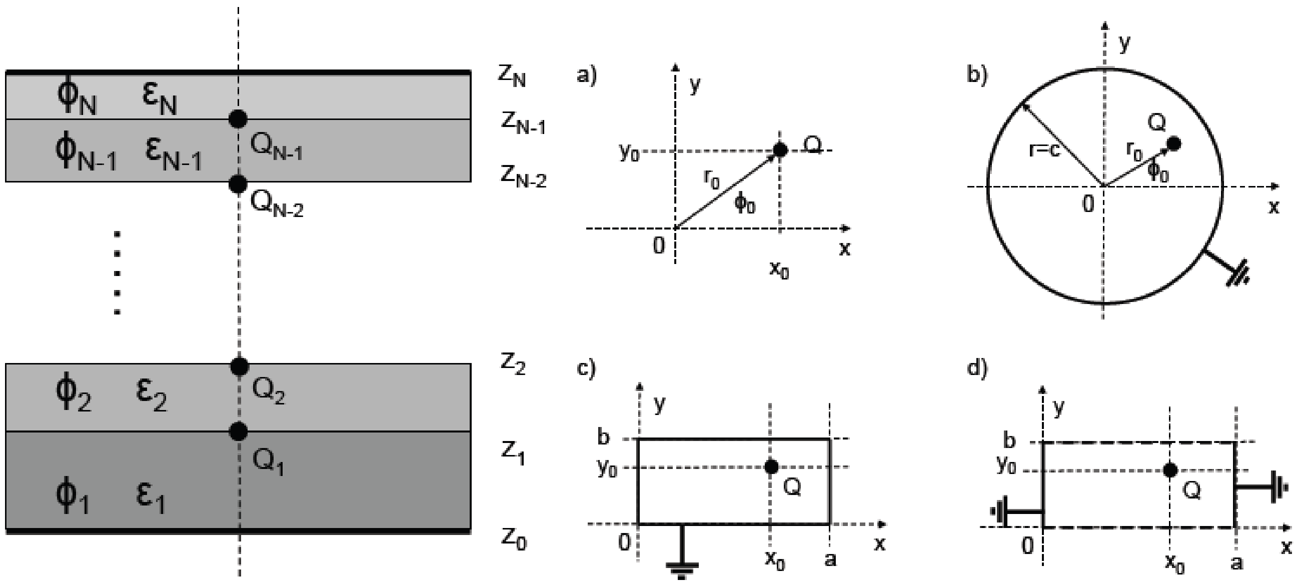


Figure 3: Left: A geometry of N dielectric layers enclosed by grounded metal plates. On the boundary between two layers at $r = 0$ there are point charges Q_n. Right: Different boundary conditions in the x-y plane.

a) $$\phi_n(r,\varphi,z) = \frac{1}{2\pi}\int_0^\infty \sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi_0)} J_m(kr)J_m(kr_0)f_n(k,z)dk$$

a) $$\phi_n(x,y,z) \;=\; \frac{1}{\pi^2}\int_0^\infty\int_0^\infty \cos[k_x(x-x_0)]\cos[k_y(y-y_0)]\frac{f_n(k,z)}{k}dk_x dk_y$$

$$f_n(k,z) = A_n e^{kz} + B_n e^{-kz} \qquad n = 1 \ldots N$$

b) $$\phi_n(r,z) = \frac{1}{c\pi}\sum_{l=1}^{\infty}\sum_{m=-\infty}^{\infty} e^{im(\varphi-\varphi_0)}\frac{J_m(k_{ml}r)J_m(k_{ml}r_0)}{j_{ml}[J_{m+1}(j_{ml})]^2} f_n(k_{ml},z)$$

c) $$\phi_n(x,y,z) = \frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(l\pi\frac{x}{a}\right)\sin\left(l\pi\frac{x_0}{a}\right)\sin\left(m\pi\frac{y}{b}\right)\sin\left(m\pi\frac{y_0}{b}\right)\frac{f_n(k_{lm},z)}{k_{lm}}$$

d) $$\phi_n(x,y,z) = \frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=0}^{\infty} \sin\left(l\pi\frac{x}{a}\right)\sin\left(l\pi\frac{x_0}{a}\right)\cos\left(m\pi\frac{y}{b}\right)\cos\left(m\pi\frac{y_0}{b}\right)\left(1-\frac{\delta_{0m}}{2}\right)\frac{f_n(k_{lm},z)}{k_{lm}}$$

Point charges in a geometry with N dielectric layers

$$f_n(k,z) = A_n e^{kz} + B_n e^{-kz} \qquad n = 1 \ldots N$$

The $2N$ coefficients $A_n(k)$ and $B_n(k)$ are defined by the two conditions at the grounded plates and at the $2(N-1)$ conditions at the $N-1$ dielectric interfaces

$$A_1 e^{kz_0} + B_1 e^{-kz_0} = 0 \qquad A_N e^{kz_N} + B_N e^{-kz_N} = 0 \tag{49}$$

$$A_n e^{kz_n} + B_n e^{-kz_n} = A_{n+1} e^{kz_n} + B_{n+1} e^{-kz_n}$$

$$\varepsilon_n A_n e^{kz_n} - \varepsilon_n B_n e^{-kz_n} = \varepsilon_{n+1} A_{n+1} e^{kz_n} - \varepsilon_{n+1} B_{n+1} e^{-kz_n} + Q_n$$

Inclusion of resistivity:

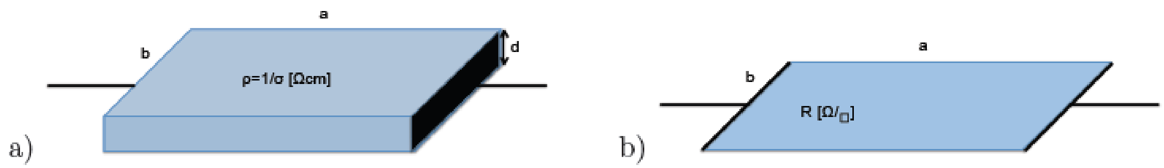


Figure 5: a) A block of material with volume resistivity $\rho\,[\Omega cm]$. b) A thin sheet of material with surface resisitivity $R\,[\Omega/\text{square}]$.

The resistance represented by the resistive sheet in Fig. 5b is given by $R\,a/b$. We can therefore conclude that for layers that have finite conductivity $\sigma_n = 1/\rho_n$, where ρ_n represents the volume resistivity of the layer, we find the fields in the Laplace domain by replacing ε_n by $\varepsilon_n + 1/(\rho_n s)$ in all expressions. In case we want a specific layer m i.e. $z_{m-1} < z < z_m$ to represent a thin sheet of a given surface resistivity $R\,[\Omega/\text{square}]$, we have to replace ε_m of this layer by

$$\varepsilon_m \quad \rightarrow \quad \varepsilon_m + \frac{1}{(z_m - z_{m-1})Rs} \tag{66}$$

In case we want to make this layer infinitely thin we have to perform the limit $\lim_{z_m \to z_{m-1}} \phi_n$ for all expressions.

Weighting fields in a geometry with N dielectric layers

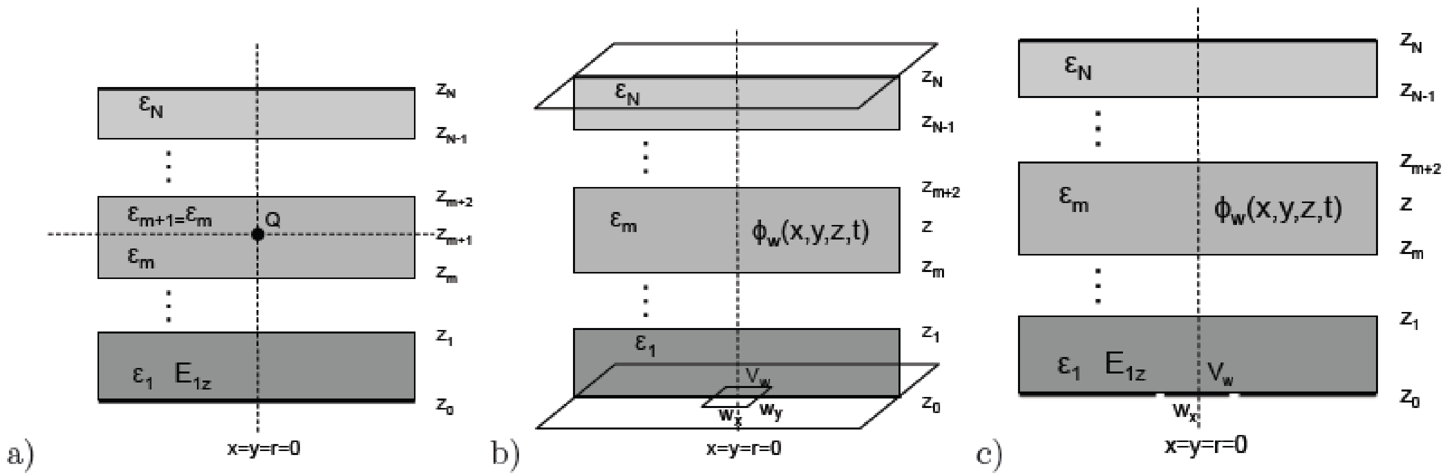


Figure 6: a) Point charge in a N layer geometry. b) Potential ϕ_w due to a rectangular pad at potential of V_w. c) Potential ϕ_w due to an infinitely extended strip at potential V_w.

Pixel:

$$\phi_n^w(x,y,z) = \varepsilon_1 \frac{V_w}{Q}\frac{4}{\pi^2}\int_0^\infty\int_0^\infty \frac{\cos(k_x x)\sin(k_x w_x/2)\cos(k_y y)\sin(k_y w_y/2)}{k_x k_y} \times \left[A_1(k, z_{n+1}=z)e^{kz_0} - B_1(k, z_{n+1}=z)e^{-kz_0}\right] dk_x dk_y \qquad (70)$$

For the case of an infinitely long strip, i.e. $w_y \to \infty$ we change variables to $s_y = k_y w_y/2$, let $w_y \to \infty$ and use $\int_0^\infty \sin(s_y)/s_y ds_y = \pi/2$ which gives

Strip:

$$\phi_n^w(x,z) = \varepsilon_1 \frac{V_w}{Q}\frac{2}{\pi}\int_0^\infty \frac{\cos(kx)\sin(kw_x/2)}{k} \times \left[A_1(k, z_{n+1}=z)e^{kz_0} - B_1(k, z_{n+1}=z)e^{-kz_0}\right] dk \qquad (71)$$

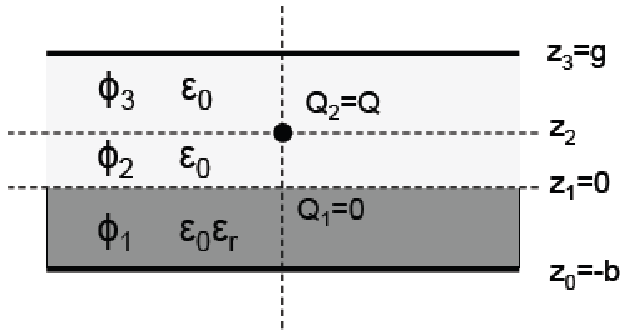
In case also w_x goes to infinity we have the weighting potential of the entire electrode which becomes

Plane:

$$\phi_n^w(z) = \varepsilon_1 \frac{V_w}{Q}\left[A_1(k=0, z_{n+1}=z) - B_1(k=0, z_{n+1}=z)\right] \qquad (72)$$

Examples

Single Gap RPC



$$\phi_2(r,z) = \frac{Q}{4\pi\varepsilon_0\sqrt{r^2+(z_2-z)^2}} + \frac{1}{2\pi}\int_0^\infty J_0(kr)\left[f_2(k,z) - \frac{Q}{2\varepsilon_0}\, e^{-k(z_2-z)}\right] dk$$

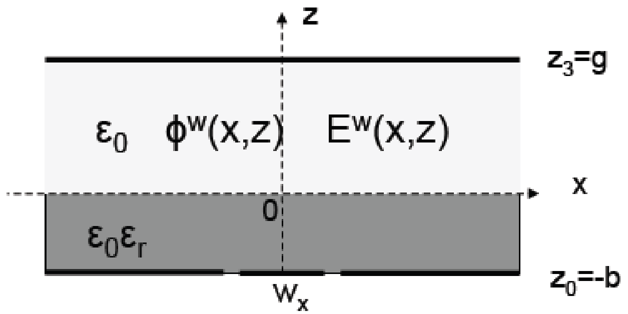
$$\phi_3(r,z) = \frac{Q}{4\pi\varepsilon_0\sqrt{r^2+(z-z_2)^2}} + \frac{1}{2\pi}\int_0^\infty J_0(kr)\left[f_3(k,z) - \frac{Q}{2\varepsilon_0}\, e^{-k(z-z_2)}\right] dk$$

$$\begin{aligned}
f_1(k,z) &= Q\sinh(k(b+z))\sinh(k(g-z_2))/(\varepsilon_0 D(k)) \\
f_2(k,z) &= Q\sinh(k(g-z_2))[\sinh(bk)\cosh(kz) + \varepsilon_r\cosh(bk)\sinh(kz)]/(\varepsilon_0 D(k)) \\
f_3(k,z) &= Q\sinh(k(g-z))[\sinh(bk)\cosh(kz_2) + \varepsilon_r\cosh(bk)\sinh(kz_2)]/(\varepsilon_0 D(k))
\end{aligned}$$

with

$$D(k) = \sinh(bk)\cosh(gk) + \varepsilon_r\cosh(bk)\sinh(gk)$$

Single Gap RPC



$$\phi^w(x,y,z) = \frac{4\varepsilon_r V_w}{\pi^2}\int_0^\infty\int_0^\infty \frac{\cos(k_x x)\sin(k_x w_x/2)\cos(k_y y)\sin(k_y w_y/2)\sinh(k(g-z))}{k_x k_y D(k)}dk_x dk_y$$

$$\phi^w(x,z) = \frac{2\varepsilon_r V_w}{\pi}\int_0^\infty \frac{\cos(kx)\sin(kw_x/2)\sinh(k(g-z))}{kD(k)}dk$$

$$\phi^w(z) = \frac{\varepsilon_r V_w(g-z)}{b+\varepsilon_r g} \qquad E_z^w = \frac{\varepsilon_r V_w}{b+\varepsilon_r g}$$

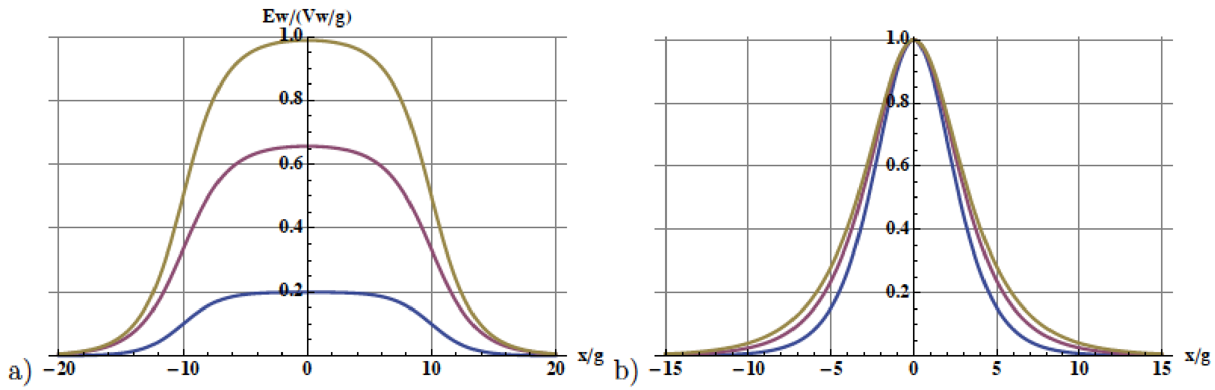
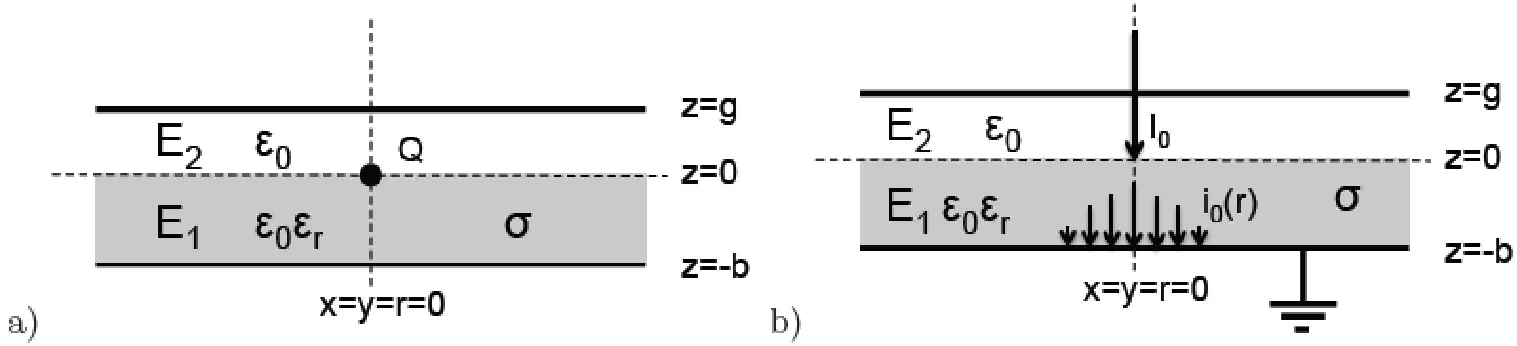


Figure 8: a) Weighting field E_z at position $z = g/2$ for $b = 4g$ and $w_x = 20g$. The three curves represent $\varepsilon_r = 1$ (bottom), $\varepsilon_r = 8$ (middle) and $\varepsilon_r = \infty$ (top). b) Normalized weighting field for the same geometry with $w_x = g$ for $\varepsilon_r = 1$(inner), $\varepsilon_r = 8$ (middle) and $\varepsilon_r = \infty$ (outer).

Single Gap RPC



$\varepsilon_1 = \varepsilon_0\varepsilon_r + \sigma/s \quad \varepsilon_2 = \varepsilon_0$ $\quad Q(t) = I_0\, t$ i.e. $Q(s) = I_0/s^2$ $\quad \lim_{t\to\infty} E(r,z,t) = \lim_{s\to 0} sE(r,z,s)$

$$i_0(r) = -\sigma E_1(r, z=-b) = \frac{I_0}{b^2\pi}\int_0^\infty \frac{1}{2} J_0\left(y\frac{r}{b}\right)\frac{y}{\cosh(y)}dy \qquad r_{50\%} \approx b \quad r_{90\%} \approx 2.3b \quad r_{99\%} \approx 3.9b$$

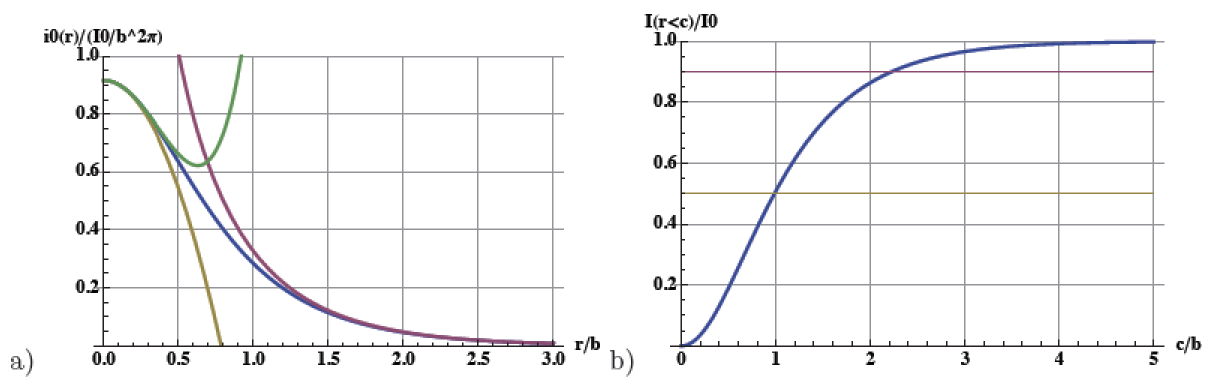
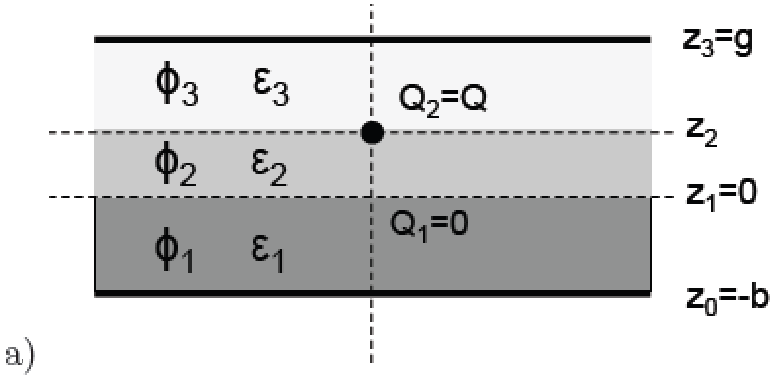
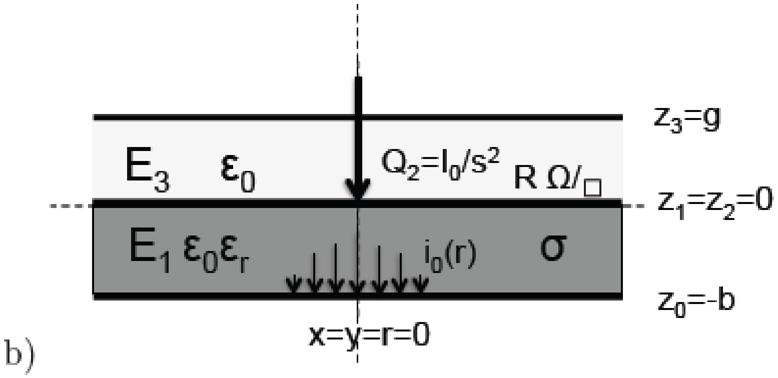


Figure 11: a) Current density $i_0(r)$ at $z=-b$. The exact curve together with the 2^{nd} order and 4^{th} order approximation from Eq. 94 and the exponential approximation from Eq. 96. b) Total current at $z=-b$ flowing inside a radius r from Eq. 97.

Single Gap RPC, increasing rate capability by a surface R



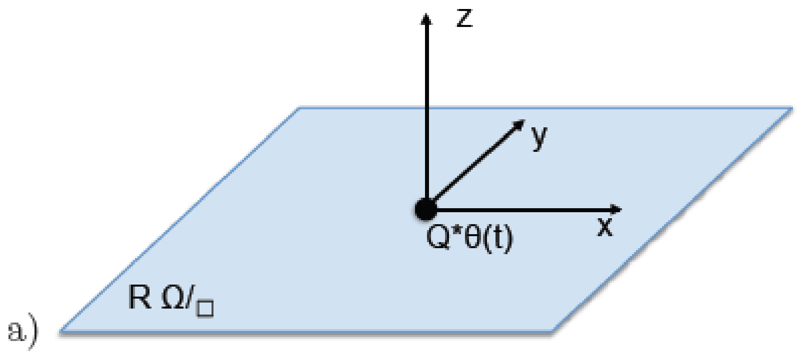


$$i_0(r) = -\sigma E_1(r, z=-b) = \frac{I_0}{b^2\pi}\int_0^\infty \frac{1}{2} J_0\left(y\frac{r}{b}\right)\frac{y}{\cosh(y)+\frac{y}{\beta^2}\sinh(y)}dy \qquad \beta^2 = R\sigma b$$

$$R < 1/(\sigma b) \rightarrow \beta^2 \ll 1$$

$$r_{50\%} \approx 1.26\sqrt{\frac{b}{R\sigma}} \quad r_{90\%} \approx 3.21\sqrt{\frac{b}{R\sigma}} \quad r_{99\%} \approx 5.77\sqrt{\frac{b}{R\sigma}}$$

Infinitely extended thin resistive layer



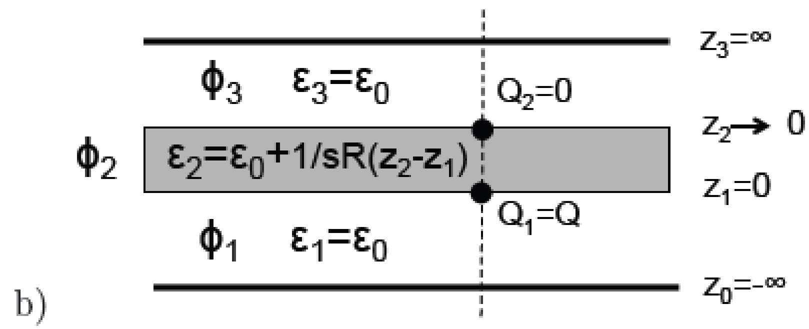


Figure 15: a) A resistive layer with surface resistance R [Ω/square]. b) The fields for this single layer can be calculated from the indicated 3-layer geometry by performing the indicated limits of the expressions for z_0, z_2, z_3.

Infinitely extended resistive layer

First we investigate an infinitely extended layer as shown in Fig. 12a. The charge Q will cause

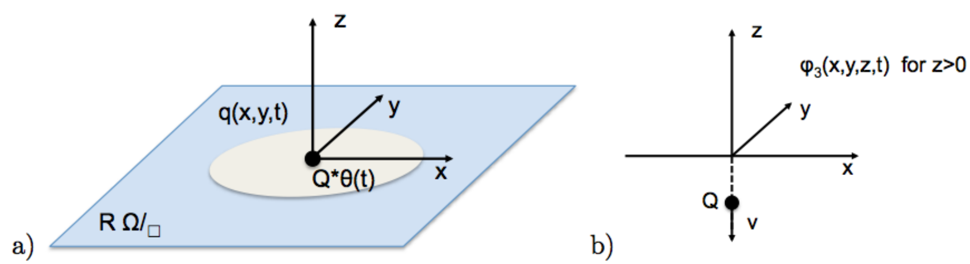


Figure 12: a) A point charge placed at an infinitely extended resistive layer at $t = 0$. b) The solution for the time dependent potential is equal to a point charge moving with velocity v along the z-axis.

$$\phi_1(r,z,t) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\sqrt{r^2 + (-z+vt)^2}} \qquad \phi_3(r,z,t) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\sqrt{r^2 + (z+vt)^2}} \tag{111}$$

We therefore conclude that the field due to a point charge placed on an infinite resistive layer at $t = 0$ is equal to the field of a charge Q that is moving with a velocity $\boxed{v = 1/2\varepsilon_0 R}$ away from the layer along the $z-$axis. As an example for a surface resistivity of $R = 1\,\mathrm{M}\Omega/\text{square}$ the velocity is $5.6\,\mathrm{cm}/\mu\mathrm{s}$.

The time dependent surface charge density on the resistive surface is given by

$$q(r,t) = \varepsilon_0 \frac{\partial \phi_1}{\partial z}|_{z=0} - \varepsilon_0 \frac{\partial \phi_3}{\partial z}|_{z=0} \tag{112}$$

which evaluates to

$$\boxed{q(r,t) = \frac{Q}{2\pi} \frac{vt}{\sqrt{(r^2 + v^2t^2)^3}}} \tag{113}$$

The total charge on the resistive surface $Q_{tot} = \int_0^\infty 2r\pi q(r,t)dr$ is equal to Q at any time. The peak and the FWHM of the charge density are given by

$$q_{max} = \frac{Q}{2\pi} \frac{1}{v^2t^2} \qquad FWHM = 2(4^{1/3} - 1)^{1/2} \approx 1.53vt \tag{114}$$

The charge is therefore 'diffusing' with a velocity v, and does not assume a gaussian shape as expected from a diffusion effect but has $1/r^3$ tails for large values of r. The radial current $I(r)$ at distance r are given by

$$I(r) = \frac{2r\pi}{R} E(r) = -\frac{2r\pi}{R} \frac{\partial \phi_1}{\partial r}|_{z=0} = \frac{Qvr^2}{(r^2 + v^2t^2)^{3/2}} \tag{115}$$

It is easily verified that the rate of change of the total charge inside a radius r i.e. $dQ_r(t)/dt = d/dt \int_0^r 2r'\pi q(r',t), dr'$ is equal the the current $I(r)$.

A point charge Q is placed on an infinitely extended resistive layer with surface resistivity of R Ohms/square at t=0.

What is the charge distribution at time t>0 ?

Note that this is not governed by any diffusion equation.

The solution is far from a Gaussian.

The timescale is giverned by the velocity $v=1/(2\varepsilon_0 R)$

Resistive layer grounded on a circle

If we now assume the geometry to be grounded at a radius $r = c$ as shown in Fig, 13a, we use Eq. 41 with $r_0 = 0$ and have the solution

$$\phi_1(r,z,t) = \frac{Q}{2\pi\varepsilon_0 c}\sum_{l=1}^{\infty}\frac{J_0(j_{0l}\frac{r}{c})}{j_{0l}J_1^2(j_{0l})}e^{-j_{0l}(t/T - z/c)} \qquad T = c/v \tag{116}$$

and $\phi_3(r,z,t) = \phi_1(r,-z,t)$. The charge inside the radius c is not a constant but it will disappear with a characteristic time constant $T = c/v$ by currents flowing into the 'grounded' ring at $r = c$. As before we can calculate the surface charge density and charge inside the radius r, which evaluate to

$$q(r,t) = \frac{Q}{c^2\pi}\sum_{l=1}^{\infty}\frac{J_0(j_{0l}r/c)}{J_1^2(j_{0l})}e^{-j_{0l}t/T} \quad Q_{tot}(t) = 2Q\sum_{l=1}^{\infty}\frac{1}{j_{0l}J_1(j_{0l})}e^{-j_{0l}t/T} \tag{117}$$

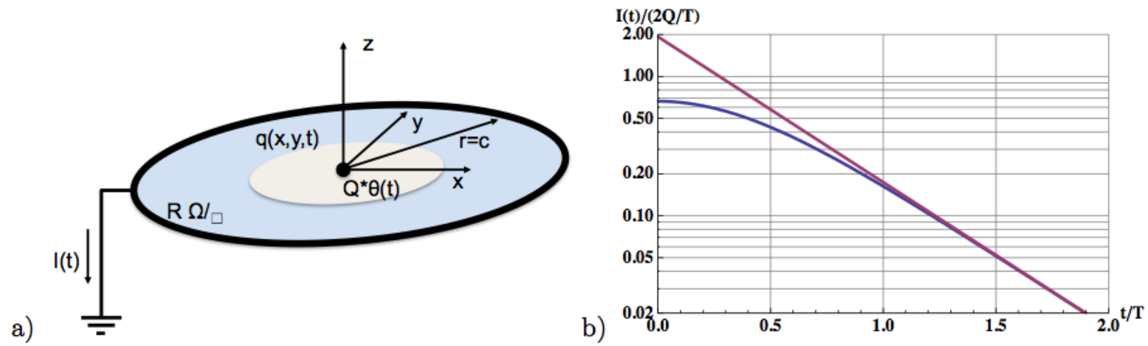


Figure 13: a) A point charge placed in the center of a resistive layer that is grounded at $r = c$. b) Current flowing to ground, where the straight line corresponds to the approximation from Eq. 119.

The current flowing into the 'grounded' ring is then again

$$I(t) = -\frac{dQ_{tot}}{dt} = \frac{2r\pi}{R}E_r(r,t) = \frac{2Q}{T}\sum_{l=1}^{\infty}\frac{1}{J_1(j_{0l})}e^{-j_{0l}t/T} \tag{118}$$

One can verify that the total amount of charge flowing to ground $\int_0^\infty I(t)dt$ is again Q. The current can be pictured to decay with an infinite number of time constants $\tau_l = T/j_{0l}$, so for large times the longest one i.e. $T/j_{01} \approx 0.42T$ will dominate and the current decays as $I(t)$. The current is plotted in Fig. 13b.

$$I(t) \approx \frac{2Q}{TJ_1(j_1)}e^{-j_{01}t/T} \qquad t \gg T \tag{119}$$

A point charge Q is placed on a resistive layer with surface resistivity of R Ohms/square that is grounded on a circle

What is the charge distribution at time t>0 ?

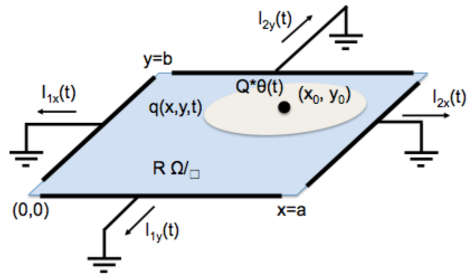
Note that this is not governed by any diffusion equation.

The solution is far from a Gaussian.

The charge disappears 'exponentially' with a time constant of T=c/v (c is the radius of the ring)

Resistive layer grounded on a rectangle

Next we assume a rectangular grounded boundar
Q at position x_0, y_0 at $t = 0$ as indicated in Fig. 14ɛ



a)

Figure 14: a) A point charge placed on a resistive layer tha
resistive layer that is grounded on at $x = 0$ and $x = a$ but in:

expression Eq. 42. Assuming the currents pointing to the outside of the boundary, the currents flowing through the 4 boundaries are

$$I_{1x} = -\frac{1}{R}\int_0^b -\frac{\partial\phi_1}{\partial x}|_{x=0}dy \qquad I_{2x} = \frac{1}{R}\int_0^b -\frac{\partial\phi_1}{\partial x}|_{x=a}dy \tag{120}$$

$$I_{1y} = -\frac{1}{R}\int_0^a -\frac{\partial\phi_1}{\partial x}|_{y=0}dx \qquad I_{2y} = \frac{1}{R}\int_0^a -\frac{\partial\phi_1}{\partial x}|_{y=b}dx \tag{121}$$

which evaluates to

$$I_{1x}(t) = \frac{4Qv}{a^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{l}{m}\frac{1}{k_{lm}}\left[1-(-1)^m\right]\sin\frac{l\pi x_0}{a}\sin\frac{l\pi y_0}{b}e^{-k_{lm}vt} \tag{122}$$

$$I_{2x}(t) = \frac{4Qv}{a^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{l}{m}\frac{1}{k_{lm}}(-1)^l\left[(-1)^m-1\right]\sin\frac{l\pi x_0}{a}\sin\frac{l\pi y_0}{b}e^{-k_{lm}vt} \tag{123}$$

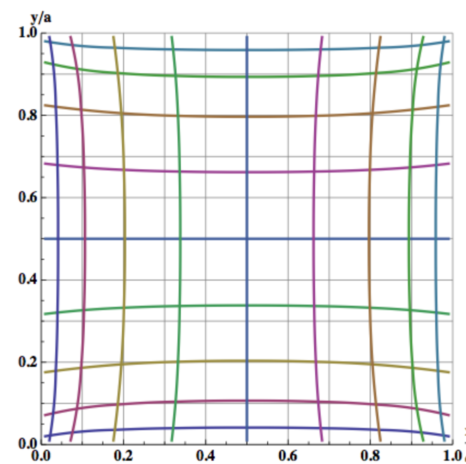
$$I_{1y}(t) = \frac{4Qv}{b^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{m}{l}\frac{1}{k_{lm}}\left[1-(-1)^l\right]\sin\frac{l\pi x_0}{a}\sin\frac{l\pi y_0}{b}e^{-k_{lm}vt} \tag{124}$$

$$I_{2y}(t) = \frac{4Qv}{b^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{m}{l}\frac{1}{k_{lm}}(-1)^m\left[(-1)^l-1\right]\sin\frac{l\pi x_0}{a}\sin\frac{l\pi y_0}{b}e^{-k_{lm}vt} \tag{125}$$

In case we want to know the total charge flowing through the grounded sides we have to integrate the above expressions from $t = 0$ to ∞ which results in the same expressions and just $e^{-k_{lm}vt}$ replaced by $1/(k_{lm}v)$. These measured currents can be used to find the position of the charge, a principle that is applied in the MicroCat detector. As an example, Fig. 15 shows the correction map that has to be applied in case one just uses linear interpolation of the measured charges.

A point charge Q is placed on a resistive layer with surface resistivity of R Ohms/square that is grounded on 4 edges

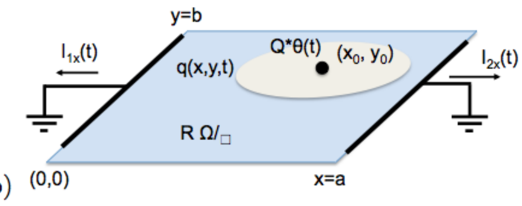
What are the currents induced on these grounded edges for time t>0 ?



› for the case where the position of the charge is determined by linear i
boundaries of the geometry in Fig. 14a.

Resistive layer grounded on two sides and i

b)



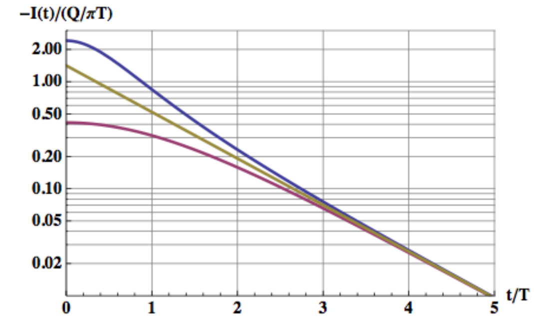


Figure 16: Currents for the geometry of Fig. 14b for $x_0 = a/4$.

5.4. Resistive layer grounded at $\pm a$ and insulated at $\pm b$.

In case the resistive layer is grounded at $x = 0, x = a$ and insulated at $y = 0, y = b$, as shown in Fig. 14, the currents are only flowing into the grounded elements at $x = 0$ and $x = a$. We use Eq. 43 and with some effort the summation can be achieved and evaluates to

$$I_{1x}(t) = -\frac{1}{R}\int_0^b -\frac{\partial \phi_1}{\partial x}|_{x=0} dy = -\frac{Q}{\pi T}\frac{\sin(\pi\frac{x_0}{a})}{\cosh(\frac{t}{T}) - \cos(\pi\frac{x_0}{a})} \tag{126}$$

$$I_{2x}(t) = \frac{1}{R}\int_0^b -\frac{\partial \phi_1}{\partial x}|_{x=a} dy = -\frac{Q}{\pi T}\frac{\sin(\pi\frac{x_0}{a})}{\cosh(\frac{t}{T}) + \cos(\pi\frac{x_0}{a})} \tag{127}$$

with $T = \frac{2a\varepsilon_0 R}{\pi} = \frac{a}{\pi v}$. For large times both expressions tend to

$$I_{1x}(t) = I_{2x}(t) \approx -\frac{2Q}{\pi T}\cos\left(\pi\frac{x_0}{a}\right) e^{-t/T} \tag{128}$$

Fig. 16 shows the two currents for a charge deposit at position $x_0 = a/4$ together with the asymptotic expression from Eq. 128. The total charge that is flowing through the grounded ends is given by

$$q_1 = \int_0^\infty I_{1x}(t)dt = Q\frac{a - x_0}{a} \qquad q_2 = \int_0^\infty I_{2x}(t)dt = Q\frac{x_0}{a} \tag{129}$$

so we learn that the charges are just shared in proportion to the distance from the grounded boundary, equal to the resistive charge division.

Possibility of position measurement in RPC and Micromegas

A point charge Q is placed on a resistive layer with surface resistivity of R Ohms/square that is grounded on 2 edges and insulated on the other two.

What are the currents induced on these grounded edges for time t>0 ?

The currents are monotonic.

Both of the currents approach exponential shape with a time constant T.

The measured total charges satisfy the simple resistive charge division formulas.

Uniform currents on resistive layers

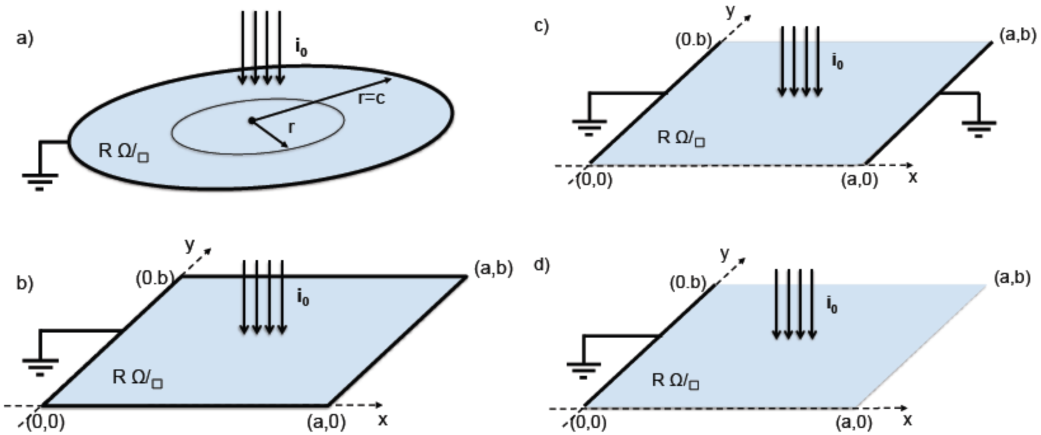


Figure 25: A uniform current 'impressed' on the resistive layer will result in a potential distribution that depends strongly on the boundary conditions. The 4 geometries shown in this figure are discussed.

Uniform illumination of the resistive layers results in 'chargeup' and related potentials.

In this section we want to discuss the potentials that are created on thin resistive layers for uniform charge deposition. In detectors like RPCs and Resistive Micromegas such resistive layers are used for application of the high voltage and for spark protection. The resistivity must be chosen small enough to ensure that potentials that are established on these layers due to charge-up are not influencing the applied electric fields responsible for the proper detector operation. If such detectors are in an environment of uniform particle irradiation the situation can be formulated by placing a uniform 'externally impressed' current per unit area i_0 [A/cm^2] on the resistive layer. For illustration we use the example of a resistive layer an absence of any grounded planes from Section 5. First we want to investigate the geometry shown in Fig. 25a) where the layer is grounded on a circle at $r = c$. The charge dq placed on an infinitesimal area at position r_0, ϕ_0 after time t is given by $dq(t) = i_0 r_0 dr_0 d\phi_0 t$, or in the Laplace domain $dq(s) = i_0 r_0 dr_0 d\phi_0 / s^2$. We therefore have to replace Q/s in Eq. 119 by $q(s)$, which results in

$$f_1(k,z,s) = \frac{i_0}{s}\frac{R r_0 dr_0 d\phi_0}{k + 2\varepsilon_0 R s}\, e^{kz} \qquad f_2(k,z,s) = \frac{i_0}{s}\frac{R r_0 dr_0 d\phi_0}{k + 2\varepsilon_0 R s}\, e^{-kz} \tag{160}$$

Since we want to know the steady situation for long times i.e. for $t \to \infty$ we $f(k,z,t \to \infty) = \lim_{s \to 0} s f(k,z,s)$ and have

$$f_1(k,z) = \frac{R i_0 r_0 dr_0 d\phi_0}{k}\, e^{kz} \qquad f_2(k,z) = \frac{R i_0 r_0 dr_0 d\phi_0}{k}\, e^{-kz} \tag{161}$$

$$\phi_1(r,z) = \phi_3(r,-z) = 2c^2 R i_0 \sum_{l=1}^{\infty} \frac{J_0(j_{0l} r/c)}{j_{0l}^3 J_1(j_{0l})} e^{j_{0l} z/c} \tag{162}$$

For $z = 0$ i.e. on the surface of the resistive layer, the expression can be summed and we have

$$\phi_1(r, z=0) = \phi_3(r, z=0) = \frac{1}{4} R i_0 (c^2 - r^2) \tag{163}$$

This expression can also be derived in an elementary way: the total current on a disc of radius r i.e. $r^2 \pi i_0$, is equal to the total radial current flowing at radius r i.e. $2r\pi E_r/R$. This defines the radial field inside the layer to $E_r = R i_0 r/2$. With the boundary condition $\phi(c) = \int_0^c E_r(r) dr = 0$ we find back the above expression. The maximum potential is therefore in the centre of the disc and is equal to

$$\phi(r=0) = \frac{c^2 \pi R i_0}{4\pi} = \frac{1}{4\pi} R I_{tot} \approx 0.08\, R I_{tot} \tag{164}$$

To find the potentials in the rectangular geometry of Fig. 25b we again have f_1, f_2 from Eq. 161 we just have to replace $r_0 dr_0 d\phi_0$ by $dx_0 dy_0$ and perform the integration $\int_0^a dx_0 \int_0^b dy_0$ of Eq. 47, which results in

$$\phi_1(x,y,z) = \phi_3(x,y,-z) = abRi_0 \frac{4}{\pi^4} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1-(-1)^l][1-(-1)^m] \sin(l\pi x/a) \sin(m\pi y/b)}{l^3 mb/a + m^3 la/b} e^{k_{lm} z} \tag{165}$$

The expression cannot be written in closed form but converges quickly, so numerical evaluation is straight forward. The peak of the potential can be found by setting $d\phi_1/dx = 0, d\phi_1/dy = 0$ and is found at $x = a/2, y = b/2$, which is also evident by the symmetry of the geometry. The maximum potential on the resistive layer is then

$$\phi_{max} = \phi(a/2, b/2, z=0) = \frac{1}{8} R i_0 a^2 b^2 \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \frac{128}{\pi^4} \frac{(-1)^{l+m}}{b^2(2l-1)^3(2m-1) + a^2(2m-1)^3(2l-1)} \tag{166}$$

For a square geometry ($b = a$) the sum evaluates to ≈ 0.59 so the peak voltage in the center is

$$\phi_{max} \approx 0.074 R i_0 a^2 = 0.074\, R I_{tot} \tag{167}$$

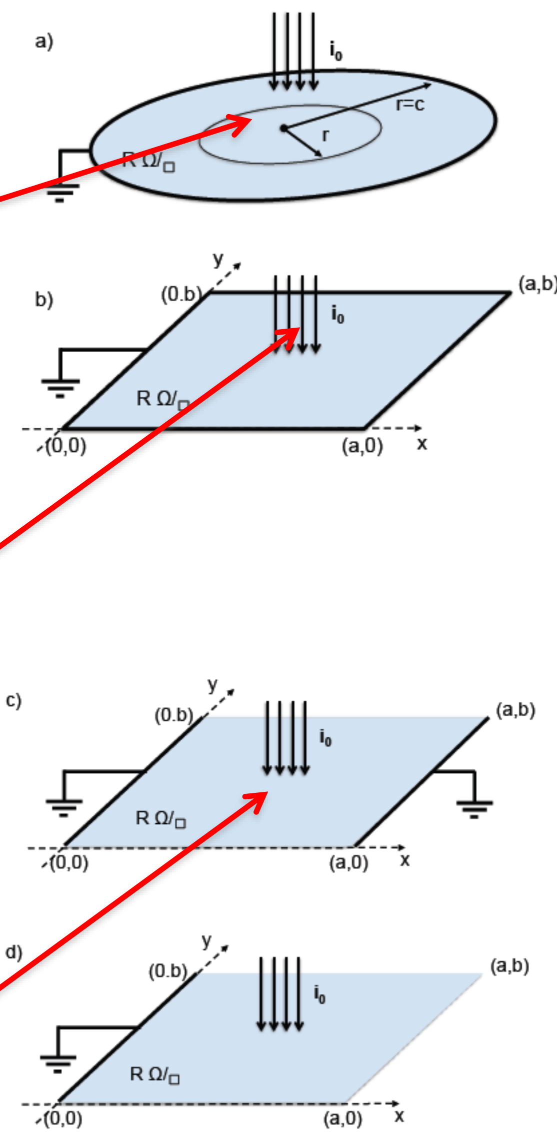
We see that the value is only less than 10 % different from the peak voltage for the circular boundary in Eq. 164.

For uniform illumination of the geometry Fig. 25c that is grounded at $x = 0, a$ and insulated at $y = 0, b$ we use expression Eq. 48 and proceed as before and find

$$\phi_1(x,z) = \phi_3(x,-z) = 2Ri_0 a^2 \sum_{l=1}^{\infty} \frac{(1-(-1)^l) \sin(l\pi x/a)}{l^3 \pi^3} e^{l\pi z/a} \tag{168}$$

The potential is is independent of y and for $z = 0$ the sum can be written inclosed form

$$\phi_1(x, z=0) = \frac{1}{2} R i_0 (ax - x^2) \qquad \phi_{max} = \frac{1}{8} a^2 R i_0 \tag{169}$$



Infinitely extended resistive layer with parallel ground plane

Assuming an infinitely extended geometry, the time dependent charge density evaluates to

$$q(r,t) = \frac{Q}{b^2\pi}\,\frac{1}{2}\int_0^\infty \kappa J_0(\kappa \frac{r}{b})\,\exp\left[-\kappa(1-e^{-2\kappa})\frac{t}{T}\right]d\kappa \qquad T = \frac{b}{v} = 2b\varepsilon_0 R \tag{134}$$

It can be verified that $\int_0^\infty 2r\pi q(r,t)dr = Q$ at any time. For long times i.e. large values of t/T we can approximate the exponent of the above expression by

$$-\kappa(1-e^{-2\kappa})\frac{t}{T} \approx -2\kappa^2\frac{t}{T} \tag{135}$$

and the integral evaluates to

$$q(r,t) = \frac{Q}{b^2\pi}\,\frac{1}{8t/T}\,e^{-\frac{r^2}{8b^2 t/T}} \tag{136}$$

In analogy to the one dimensional transmission line, the discussed geometry is often assumed to be defined by the two dimensional diffusion equation

$$\frac{\partial q}{\partial t} = h\left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2}\right) \qquad h = 1/RC \quad C = \frac{\varepsilon_0}{b} \tag{137}$$

where C is the capacitance per unit area between the resistive layer and the grounded plate. The solution of this equation for a point charge Q put at $r = 0, t = 0$ evaluates exactly to the above Gaussian expression In Fig. 19 the charge distribution from Eq. 134 is compared to the above Gaussian as well as Eq. 113 for the geometry without a ground plane. Although the order of magnitude is similar, the solution of the diffusion equation does not work very well. The reason for the discrepancy can be understood when investigating how Eq. 135 is derived: the current $\vec{j}(x,y,t)$ flowing inside the resistive layer is related to the electric field $\vec{E}(x,y,t)$ in the resistive layer by $\vec{j} = \vec{E}/R$. The relation between the current and the charge density $q(x,y,t)$ is $\vec{\nabla}\vec{j} = -\partial q/\partial t$. With $\vec{E} = -\vec{\nabla}\phi$ we then get

$$\frac{\partial q}{\partial t} = \frac{1}{R}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) \tag{138}$$

If we set $q = C\phi$ we have the diffusion equation Eq. 135. This relation between voltage and charge($Q = CU$) is however only a good approximation if the charge distribution does not have a significant gradient over distances of the order of b. For small times when the charge distribution is very peaked around zero this is certainly not a good approximation. It means that for long times when the distribution if very broad when compared to the distance b the two solutions should approach each other. Indeed this can be seen if we calculate the current that is induced on the grounded plate, which we do next. The presence of the charge on the resistive layer induces a charge on the grounded metal plane. If we assume that the metal plane is segmented into strips, as shown in Fig. 20b, we can calculate the induced charge through the electric field on the surface of the plane. Assuming a strip centred at $x = x_p$ with a width of w and infinite extension in y direction, we find the induced charge to

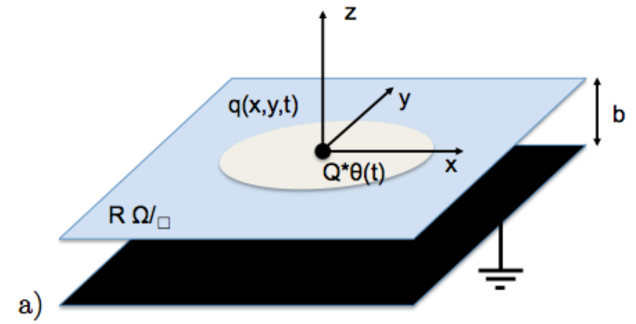


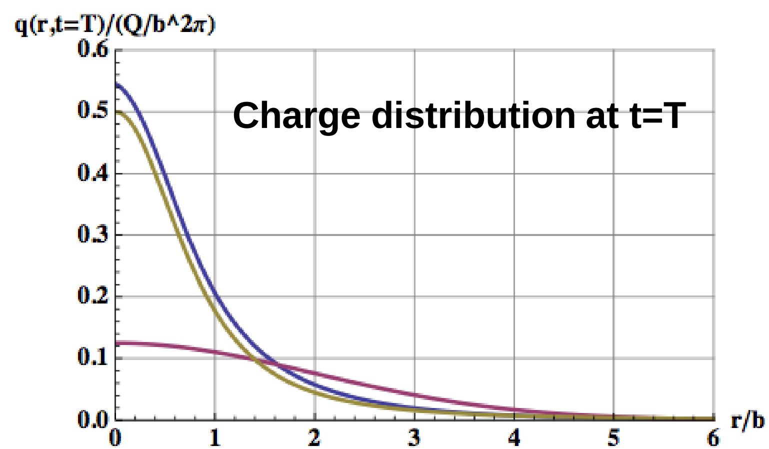
Figure 18: a) An infinitely extended resistive layer in presenc radius $r = c$.

A point charge Q is placed on an infinitely extended resistive layer with surface resistivity of R Ohms/square and a parallel ground plane at t=0.

What is the charge distribution at time t>0 ?

This process is in principle NOT governed by the diffusion equation.

In practice is is governed by the diffusion equation for long times.



Infinitely extended resistive layer with parallel ground plane

What are the charges induced metallic readout electrodes by this charge distribution?

$$Q_{ind}(t) = \int_{x_p-w/2}^{x_p+w/2}\int_{\infty}^{\infty} -\varepsilon_0 \frac{\partial \phi_1}{\partial z}|_{z=-b} dy dx \qquad (139)$$

which evaluates to

$$Q_{ind}(t) = \frac{2Q}{\pi}\int_0^{\infty} \frac{1}{\kappa}\cos(\kappa\frac{x_p}{b})\sin(\kappa\frac{w}{2b})\exp\left[-\kappa-\kappa(1-e^{-2\kappa})\frac{t}{T}\right]d\kappa \qquad (140)$$

The solution of the diffusion equation assumes the relation of a capacitor where the ground plate should just carry the charge density $-q(x,y,t)$, so the total charge on the strip is

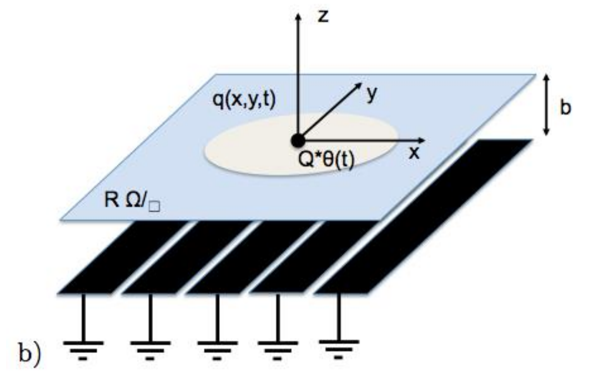
$$Q^{g}_{ind}(t) = \int_{x_p-w/2}^{x_p+w/2}\int_{\infty}^{\infty} q_g(x,y)dxdy = \frac{Q}{2}\left[\mathrm{erf}\left(\frac{2x_p+w}{4b\sqrt{2t/T}}\right)-\mathrm{erf}\left(\frac{2x_p-w}{4b\sqrt{2t/T}}\right)\right] \qquad (141)$$

Both expression are shown in Fig. 19b. Although there are significant differences at small times the curves approach each other for longer times when the charge distribution becomes broad. Indeed, if take Eq. 139 we see that for large values of t/T only small values of κ contribute to the integral, so if we expand the exponent as

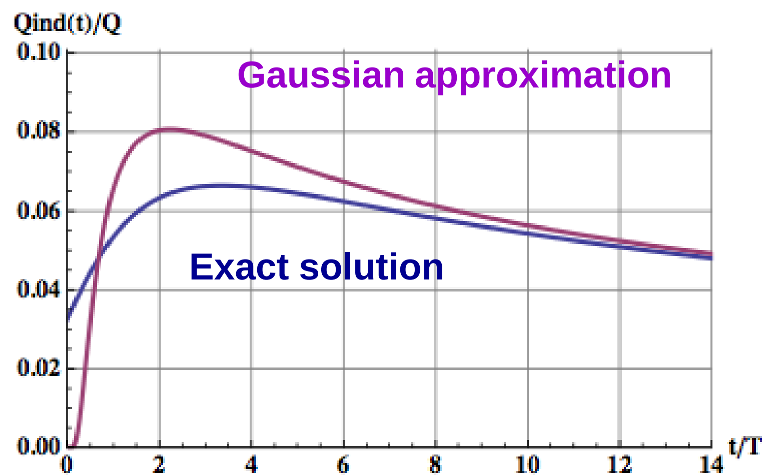
$$-\kappa-\kappa(1-e^{-2\kappa})\frac{t}{T} \approx -2\kappa^2\frac{t}{T} \qquad (142)$$

the integral evaluates precisely to expression Eq. 140.

broad. The solutions still do not represent a detector signal due to the unphysical assumption that the charge is created 'out of nowhere' at $t=0$. The correct signal on a strip due to a pair of charges $\pm Q$ moving in a detector will be discussed in Section 8.



ıce of a grounded layer. b) The same geometry grounded at a



Charge spread in e.g. a Micromega with bulk or surface resistivity

Micromega Mesh

Avalanche region

Bulk

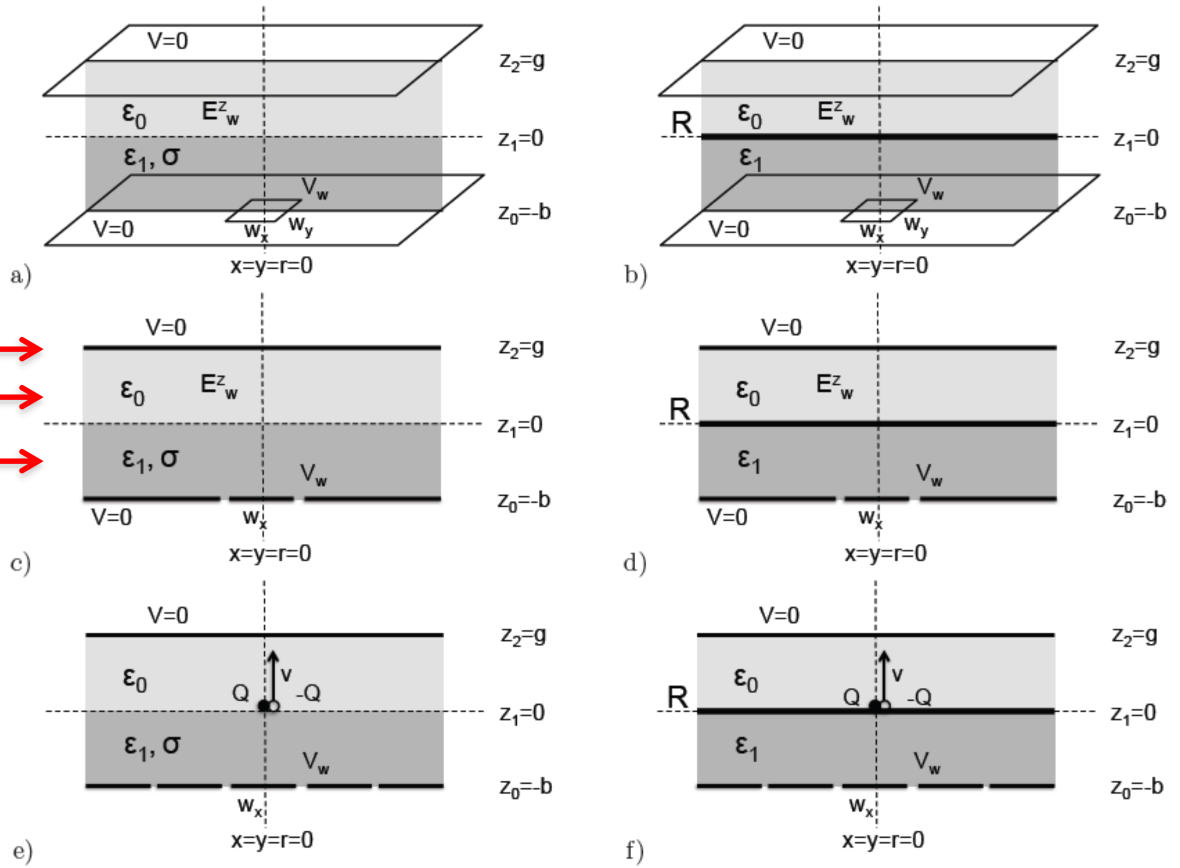
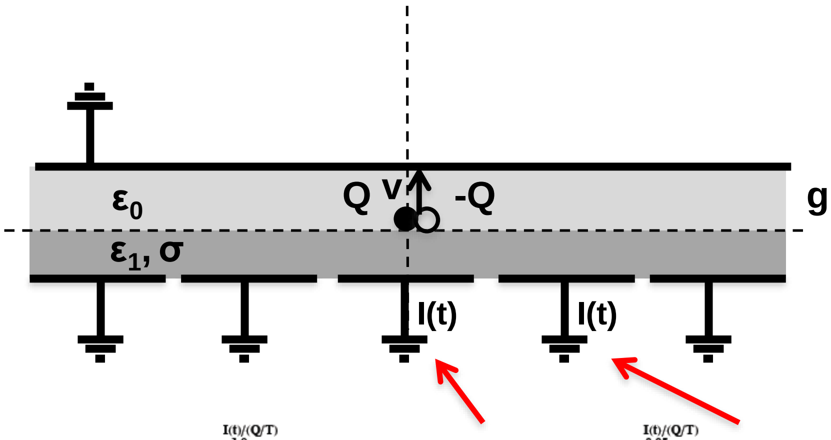


Figure 27: Weighting field for a geometry with a resistive layer having a bulk resistivity of $\rho = 1/\sigma[\Omega\text{cm}]$ (left) and a geometry with a thin resistive layer of value $R\,[\Omega/\text{square}]$ (right).

$$I(t) = -\frac{q}{V_w}\int_0^t \vec{E}_w(\vec{x}_1(t'), t-t')\dot{\vec{x}}_1(t')dt' + \frac{q}{V_w}\int_0^t \vec{E}_w(\vec{x}_2(t'), t-t')\dot{\vec{x}}_2(t')dt'$$

Charge spread in e.g. a Micromega with bulk resistivity



$$\tau_0 = \varepsilon_0/\sigma = \varepsilon_0\rho.$$

$$T = g/v$$

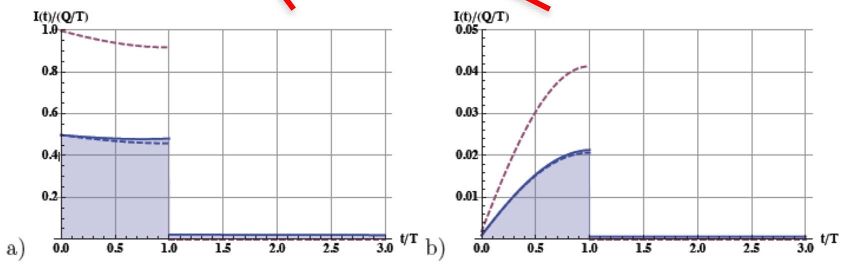


Figure 28: Uniform charge movement from $z = 0$ to $z = g$, with $\varepsilon_r = 1, w_x = 4g, b = g, \tau_0 = 10T$ for a)$x = 0$ and b) $x = 4g$.

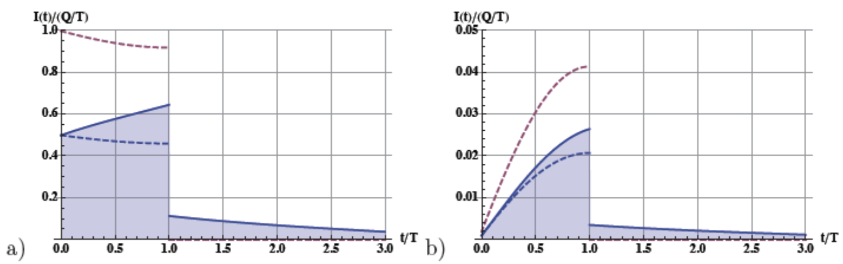


Figure 29: Uniform charge movement from $z = 0$ to $z = g$, with $\varepsilon_r = 1, w_x = 4g, b = g, \tau_0 = T$ for a)$x = 0$ and b) $x = 4g$.

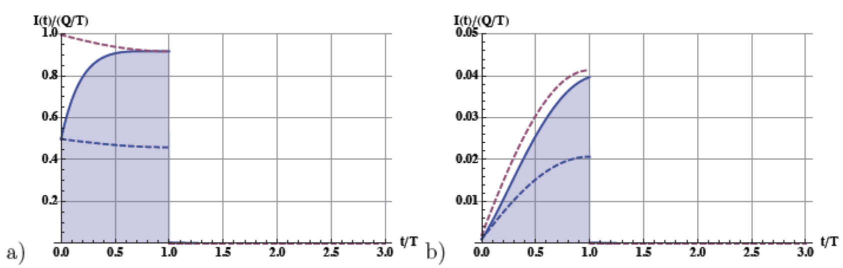


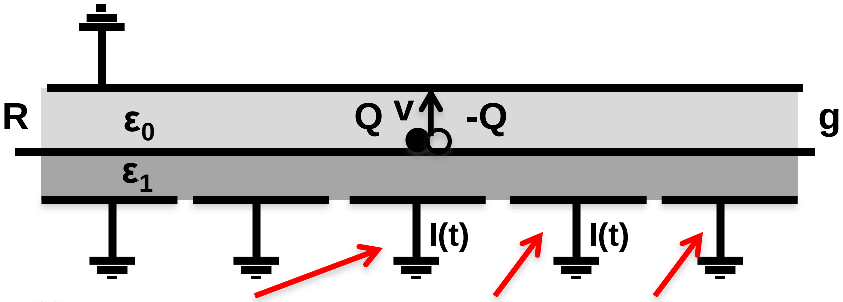
Figure 30: Uniform charge movement from $z = 0$ to $z = g$, with $\varepsilon_r = 1, w_x = 4g, b = g, \tau_0 = 0.1T$ for a)$x = 0$ and b) $x = 4g$.

Zero Resistivity

Infinite Resistivity (insulator)

All signals are unipolar since the charge that compensates Q sitting on the surface is flowing from all the strips.

Charge spread in e.g. a Micromega with surface resistivity



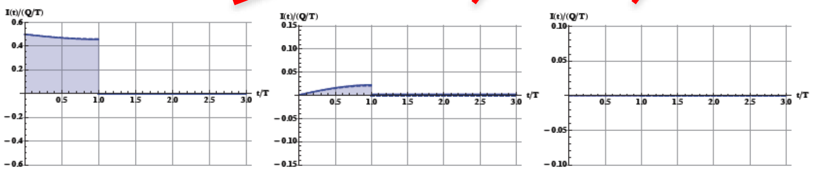


Figure 31: $\varepsilon_r = 1, w_x = 4g, b = g, T_0 = 10T$ for $x = 0, x = 4g, x = 8g$

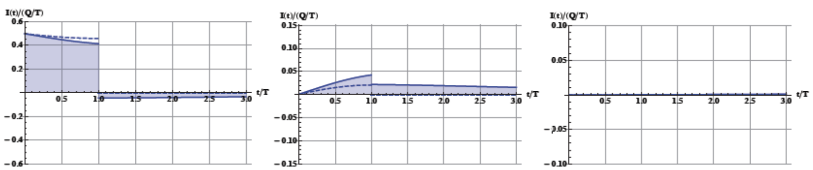


Figure 32: $\varepsilon_r = 1, w_x = 4g, b = g, T_0 = T$ for $x = 0, x = 4g, x = 8g$

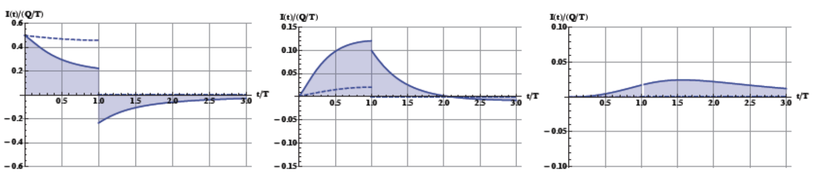


Figure 33: $\varepsilon_r = 1, w_x = 4g, b = g, T_0 = 0.1T$ for $x = 0, x = 4g, x = 8g$

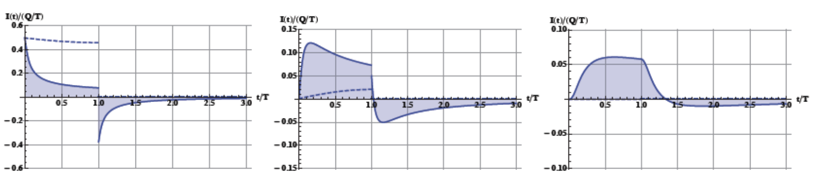


Figure 34: $\varepsilon_r = 1, w_x = 4g, b = g, T_0 = 0.01T$ for $x = 0, x = 4g, x = 8g$

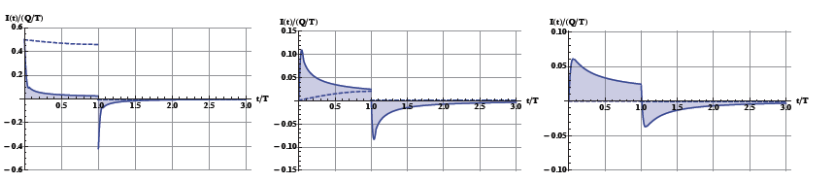


Figure 35: $\varepsilon_r = 1, w_x = 4g, b = g, T_0 = 0.001T$ for $x = 0, x = 4g, x = 8g$

$$T_0 = \varepsilon_0 R g$$

$$T = g/v$$

Zero Resistivity

Infinite Resistivity (insulator)

All signals are bipolar since the charge that compensates Q sitting on the surface is not flowing from the strips.

Summary

Fields and signals for detectors with a multilayer geometry and containing weakly conducting materials can be calculated with the presented formalism.

Charge spread, the path of currents, charge-up, signals, crosstalk can be studied in detail.

The examples can also be used a accurate benchmarks for simulation programs that calculate these geometries numerically.