Examples of analyzing MPGD data

with emphasis on precise timing

PART I

- Prelude
- Introduction
- Evaluate the number of pes per muon

Prelude

Paradigm:

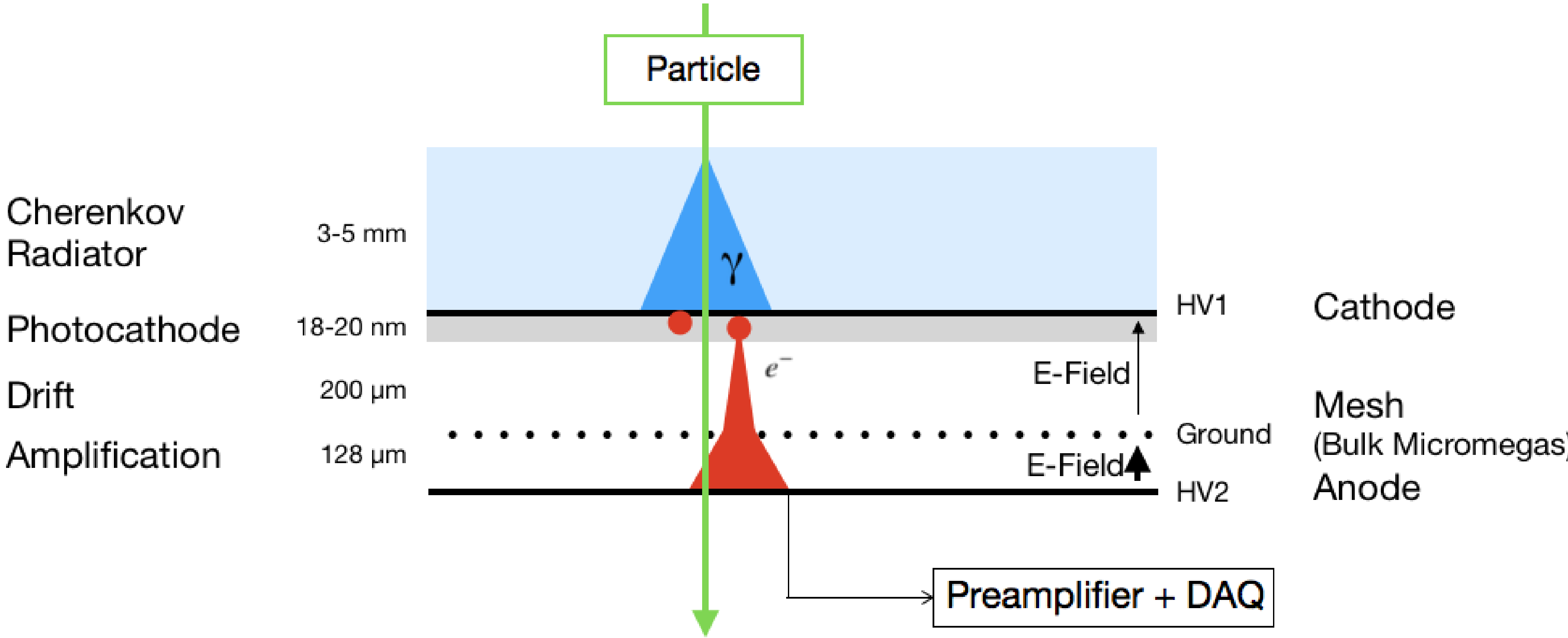
The historian of science Thomas Kuhn gave it its contemporary meaning when he adopted the word to refer to the set of concepts and practices that define a scientific discipline at any particular period of time. In his book The Structure of Scientific Revolutions (first published in 1962), Kuhn defines a scientific paradigm as: "universally recognized scientific achievements that, for a time, provide model problems and solutions for a community of practitioners, i.e.,

- *what is to be observed and scrutinized*
- *the kind of questions that are supposed to be asked and probed for answers in relation to this subject*
- *how these questions are to be structured*
- *what predictions made by the primary theory within the discipline*
- *how the results of scientific investigations should be interpreted*
- *how an experiment is to be conducted, and what equipment is available to conduct the experiment.*

Paradigm comes from Greek παράδειγμα (paradeigma), "pattern, example, sample" from the verb παραδείκνυμι (paradeiknumi), "exhibit, represent, expose" and that from παρά (para), "beside, beyond" and δείκνυμι (deiknumi), **"to show, to point out"**

Examples of analyzing MPGD data
with emphasis on precise timing

Introduction



"Compass gas" ($Ne+10\%C_2H_6+10\%CF_4$) at 1 bar.

$CF_4+20\%C_2H_6$ at 0.5 bar

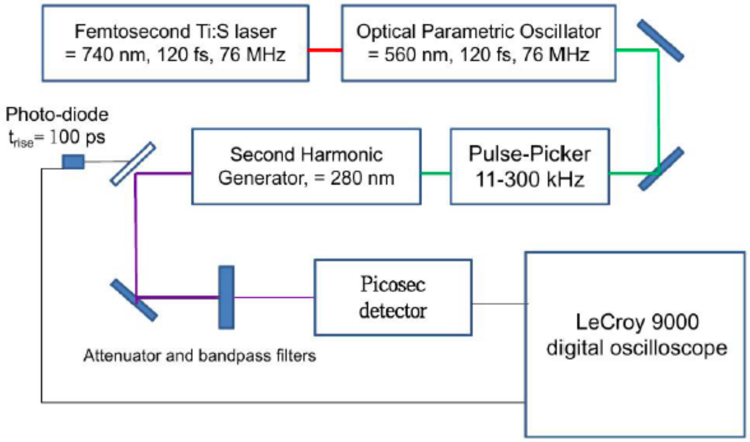
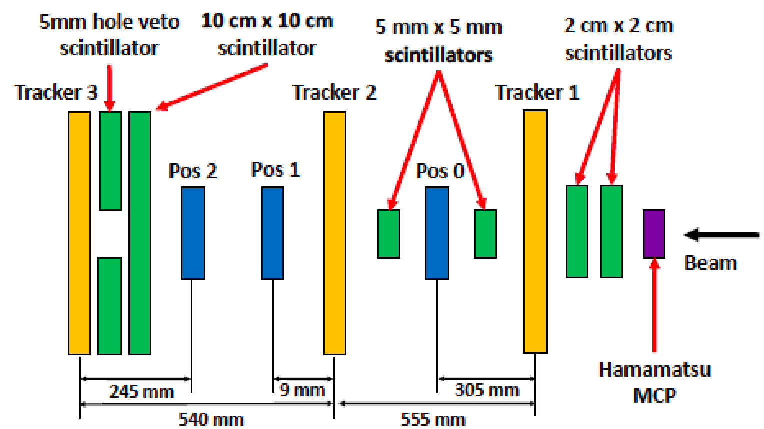


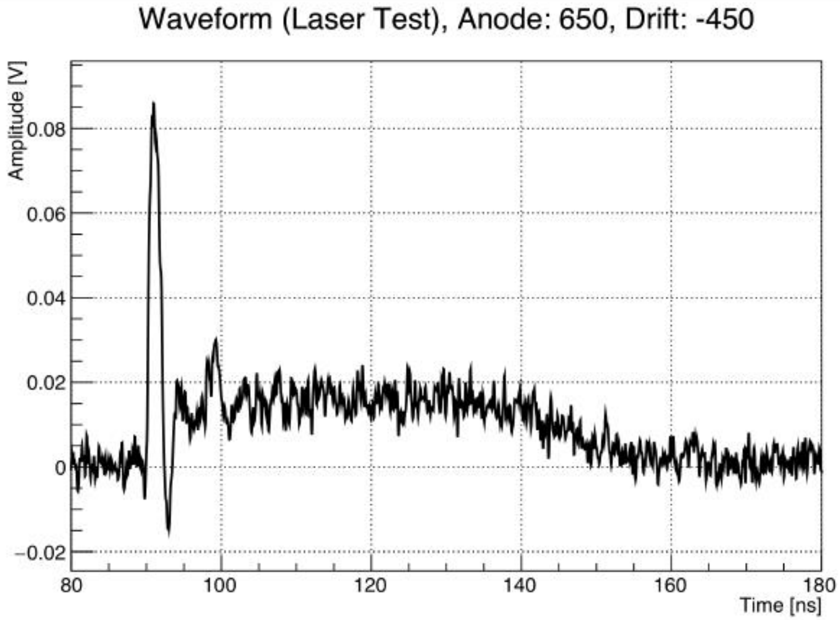
Figure 4: Schematic of the experimental setup during the laser tests, described in detail in the text.

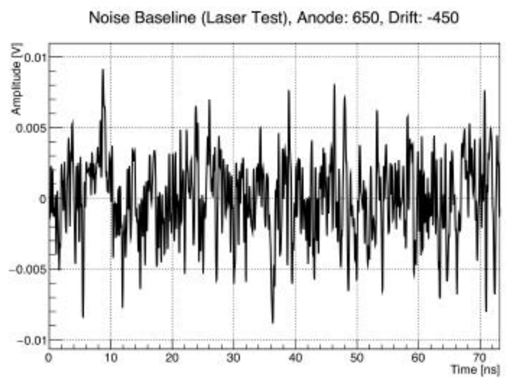
Laser calibration

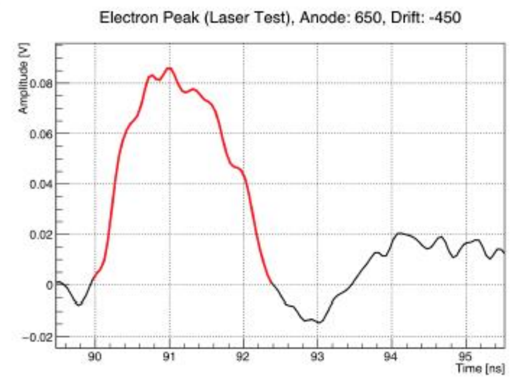
Muon test beam calibration

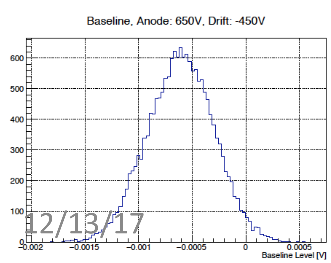


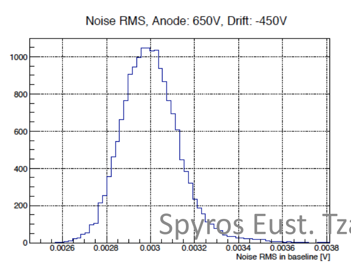
Timing Reference (T_0) Accuracy: ~5ps


Waveform (Laser Test), Anode: 650, Drift: -450
Amplitude [V]
Time [ns]

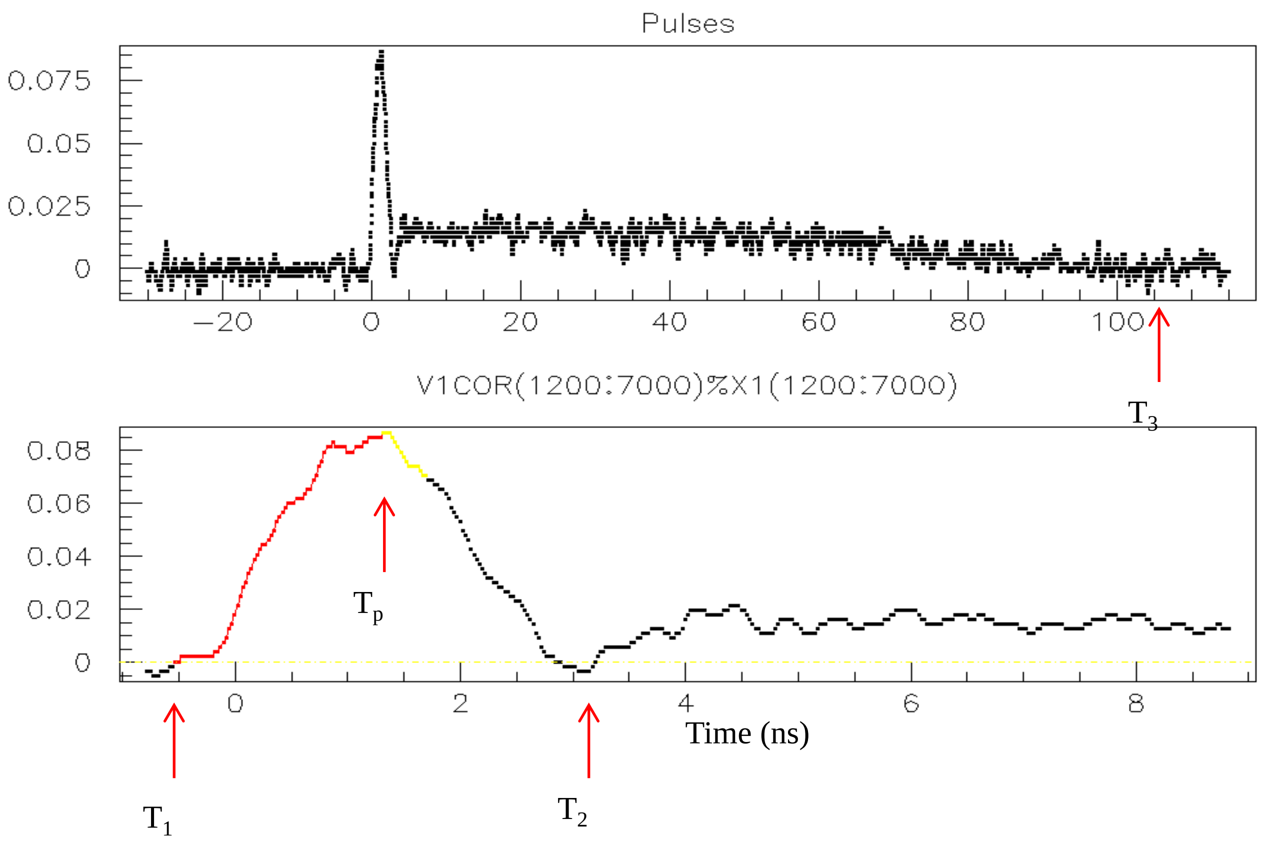

Noise Baseline (Laser Test), Anode: 650, Drift: -450
Amplitude [V]
Time [ns]


Electron Peak (Laser Test), Anode: 650, Drift: -450
Amplitude [V]
Time [ns]


Baseline, Anode: 650V, Drift: -450V
Baseline Level [V]


Noise RMS, Anode: 650V, Drift: -450V
Noise RMS in baseline [V]

Recognize the “start” and “end” of the e-peak, as well as the “end” of the ion tail

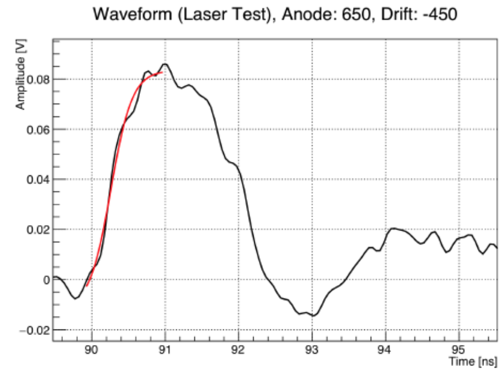


Evaluate charge by integrating the relevant part οφ the waveform

Definition of the e-peak Arrival Time
Fit the e-peak Leading Edge

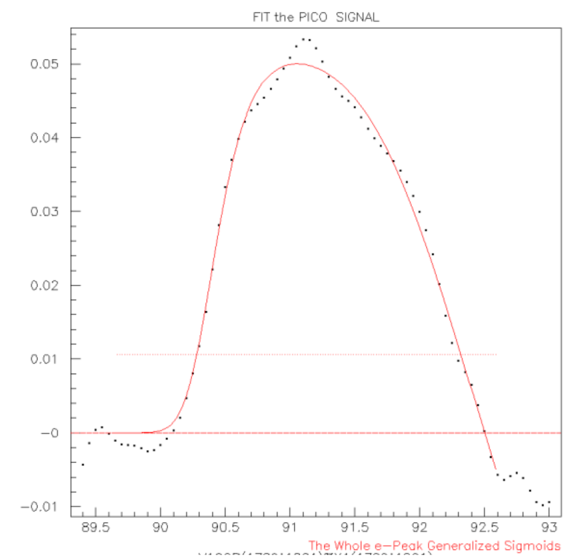
Fit the e-peak leading edge in order to neutralize noise effects.
Several Functions have been used in the fits, including quadratic and cubic polynomials as well as logistic and generalized logistic functions

$$\frac{P_0}{1+\exp\left[-P_2\left(x-P_1\right)\right]}+P_3 \quad , \quad \frac{P_0}{\left(1+\exp\left[-P_2\left(x-P_1\right)\right]\right)^{P_3}}$$



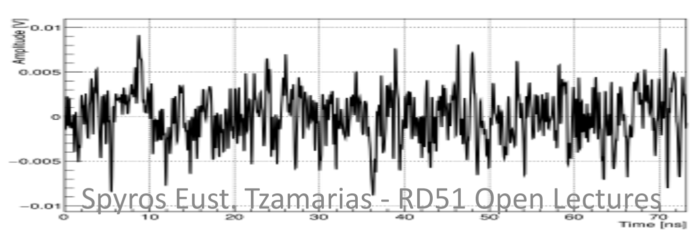
A fit of the whole e-peak was also tried using the difference of two logistic functions

$$f(t;p_0,p_1,p_2,p_3,p_4,p_5,p_6)=\frac{p_0}{\left(1+e^{-(t-p_1)p_2}\right)^{p_3}}-\frac{p_0}{\left(1+e^{-(t-p_4)p_5}\right)^{p_6}}$$



The results of these fit is also used to define the “start” and ”end” points of the e-peak waveform, to estimate charge and it is also used for timing

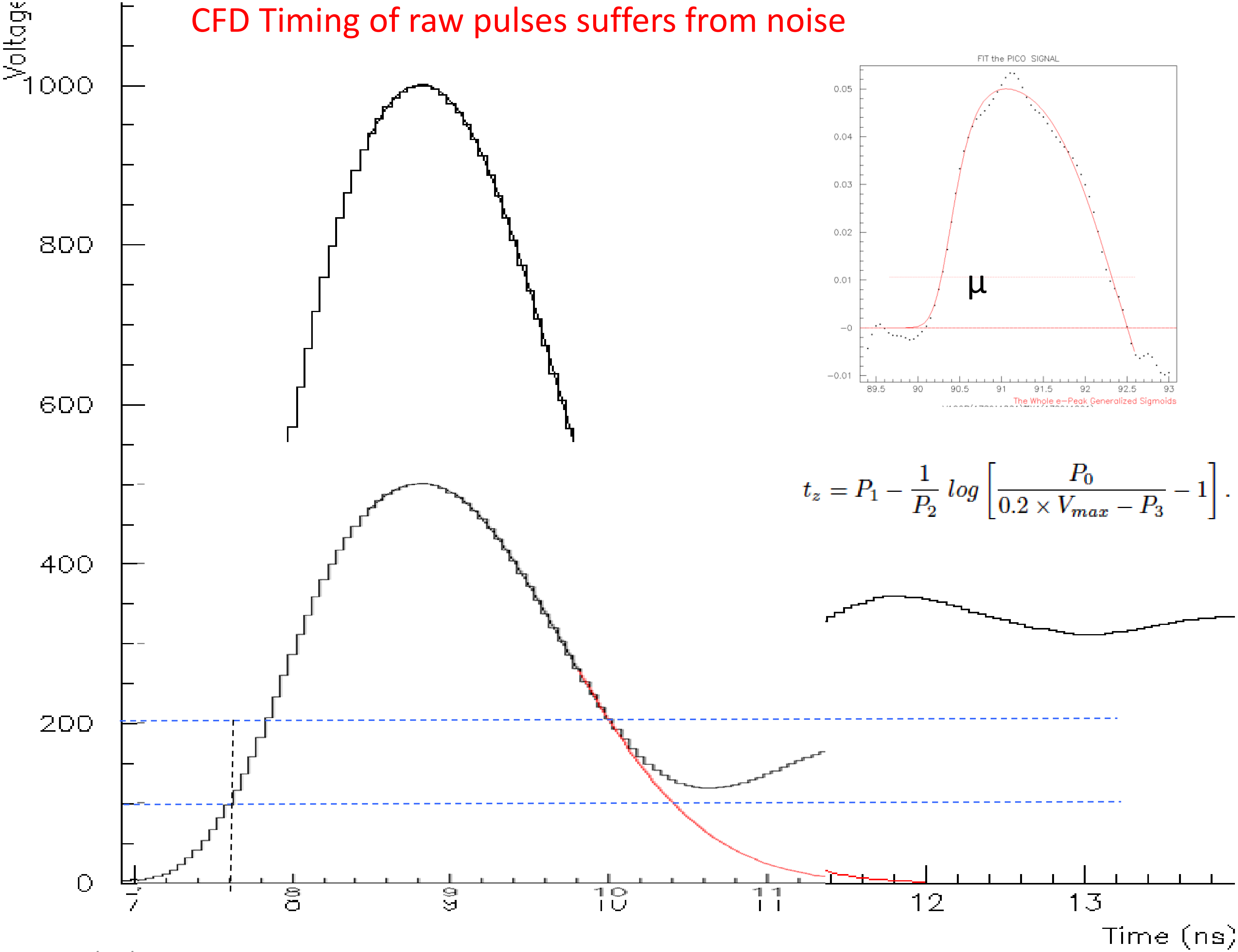
** The autocorrelation function, evaluated for a time interval Δt ~ 5 ns is almost zero*



Define the e-peak arrival time at a Constant Fraction of the peak maximum

CFD Timing minimizes “slewing effects”

CFD Timing of raw pulses suffers from noise



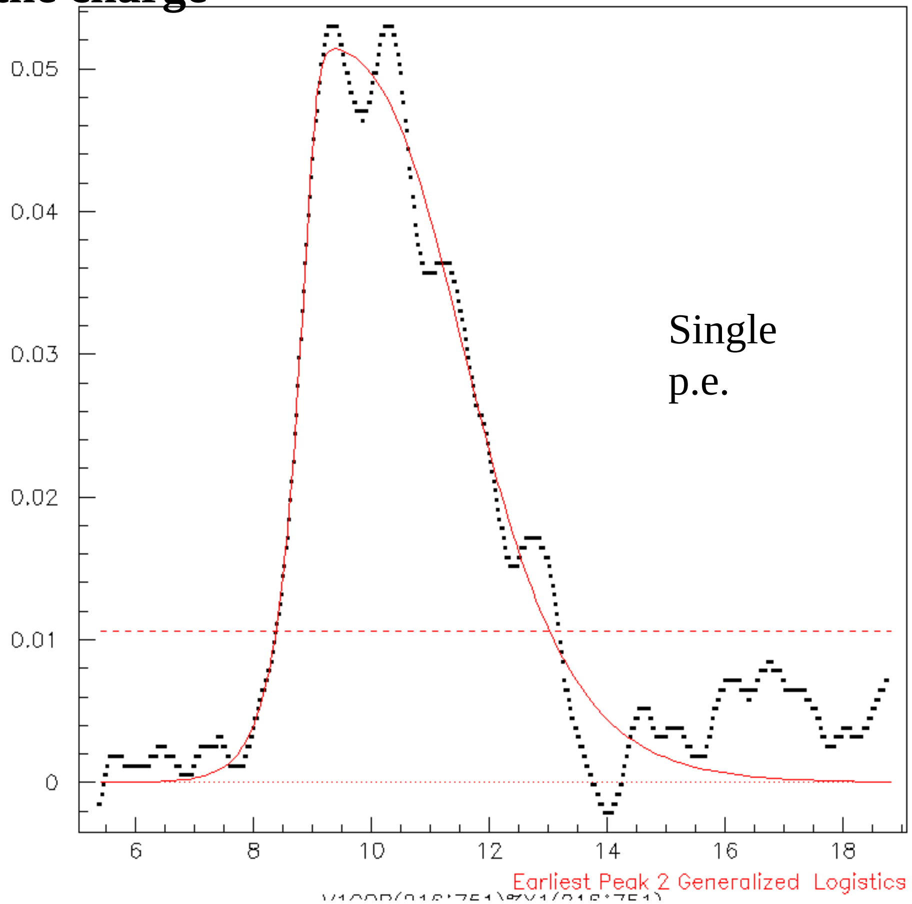
$$t_z = P_1 - \frac{1}{P_2}\, log\left[\frac{P_0}{0.2 \times V_{max} - P_3} - 1\right].$$

Fitting the e-peak waveform helps to estimate the charge in “impossible” cases

Example: LED calibration – small pulses

Define the start and the end of the e-peak

Estimate the charge

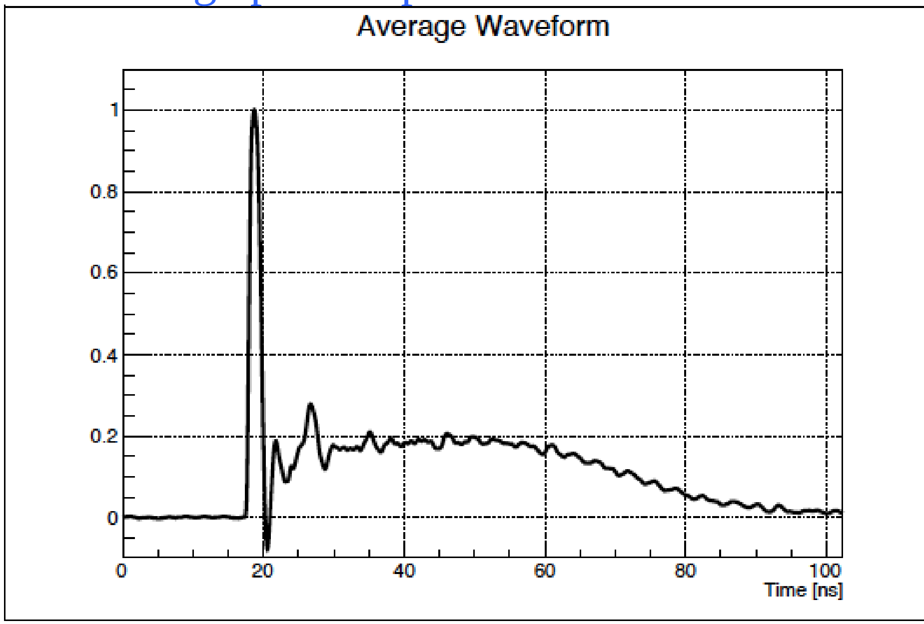


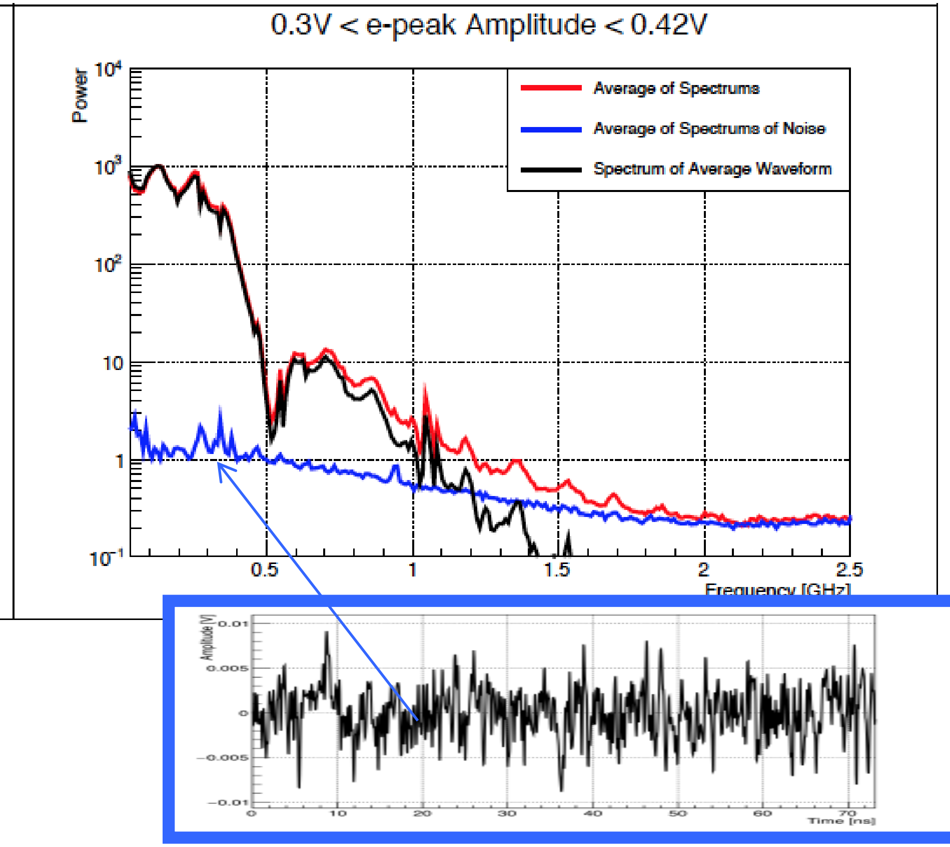
Is it possible to filter-out the noise ?

The power spectrum of the “average waveform” is free of random (uncorrelated) noise

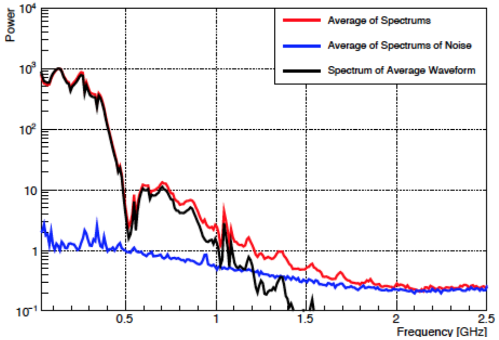
The average power spectrum of individual waveforms includes the random noise

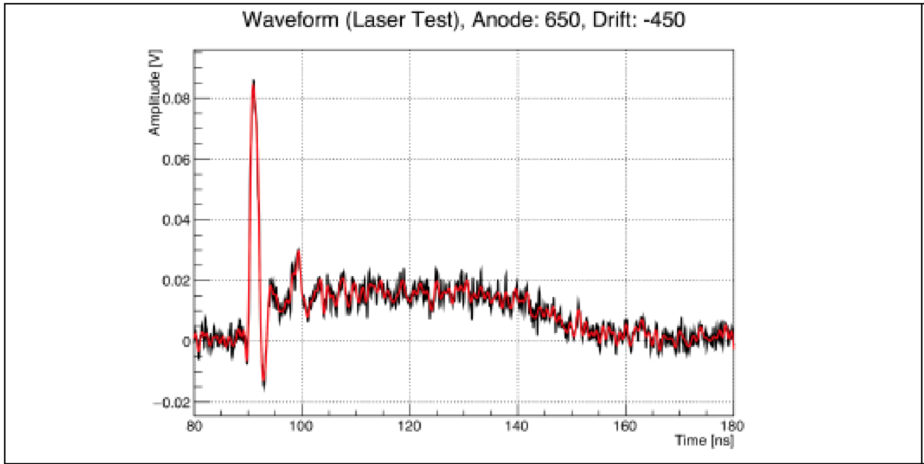
The average power spectrum of individual “noise waveforms” is the random noise

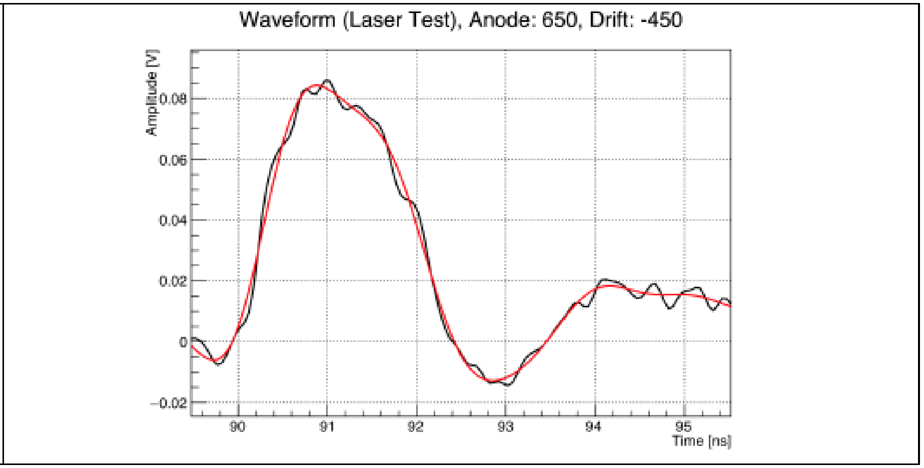




An example of filtering out the noise
(cut at 1.5 GHz)

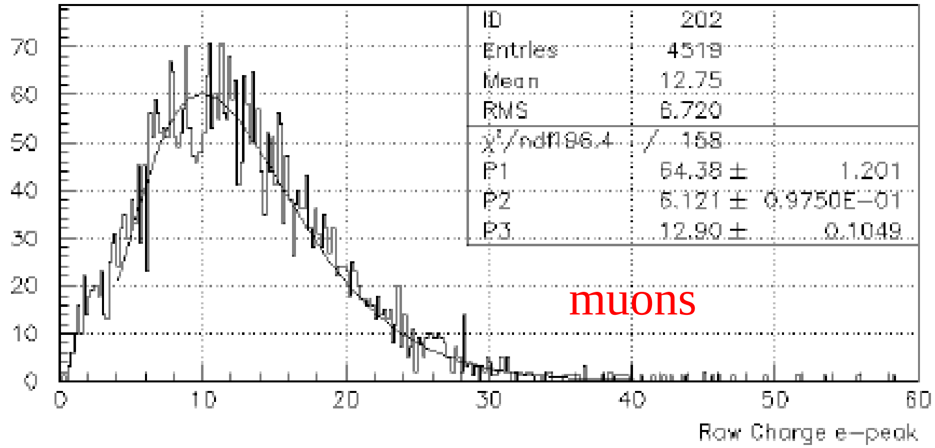


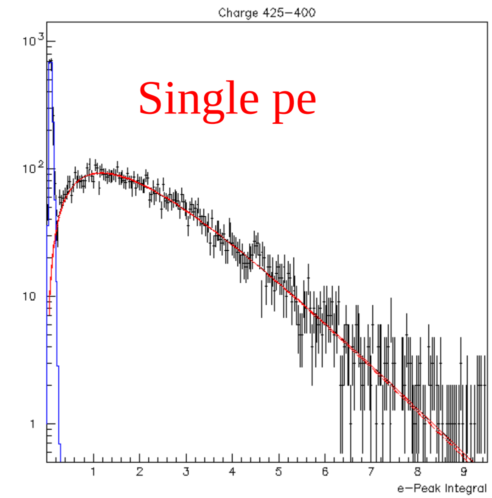




In these examples (PICOSEC-MicroMegas), the use of filtering before fitting the leading edge of the pulse DOES NOT improve the timing resolution, i.e. a conservative frequency cut does not improve the timing resolution and a strong frequency cut deforms the rising edge of the pulse worsening the time resolution

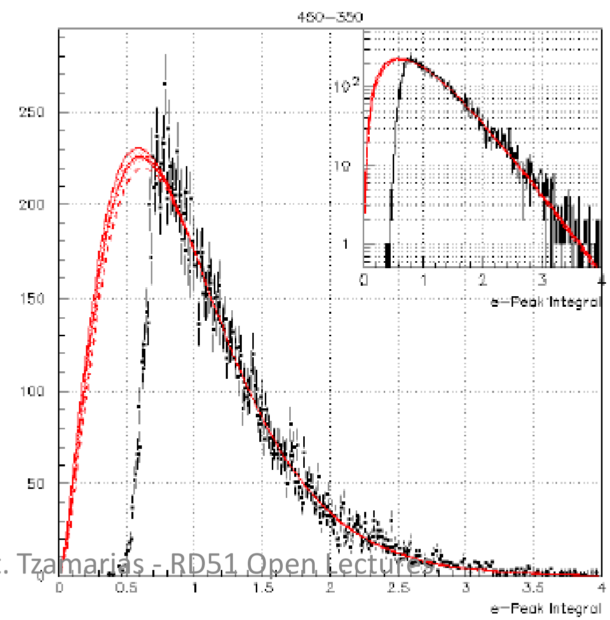
The Charge Distribution





Can we express the e-peak charge distribution of a single p.e. by a Polya function?

Usually, we accumulate signals using an event trigger that rejects noise.
Can we estimate the mean and RMS of the distribution ?



The Polya distribution is defined by the following (normalized to unity) function:

$$P_{spe}\left(Q; a=b=\theta+1, \bar{Q}_e\right)dQ = \frac{1}{Q_e}\frac{(\theta+1)^{(\theta+1)}\left(Q/\bar{Q}_e\right)^{\theta}}{\Gamma(\theta+1)}e^{-(\theta+1)Q/\bar{Q}_e}dQ$$

$$E\left[Q_{spe}\right] = \bar{Q}_e = \langle Q_e \rangle$$

$$V\left[Q_{spe}\right] = \frac{1}{\theta+1}\langle Q_e \rangle^2 = RMS^2$$

When fitting experimental distributions, we have to normalize the Polya function to the number of observed events, i.e. we fit with the function

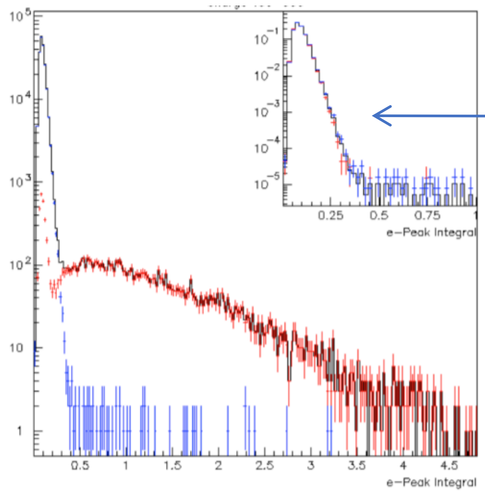
$$\left.\begin{aligned} &F(Q; C, RMS, <Q_e>) = \frac{C}{\bar{Q}_e}\frac{(\theta+1)^{(\theta+1)}\left(Q/\bar{Q}_e\right)^{\theta}}{\Gamma(\theta+1)}e^{-(\theta+1)Q/\bar{Q}_e} \\ &\bar{Q}_e = <Q_e> \\ &\theta = \frac{\langle Q_e \rangle^2}{RMS^2} - 1 \end{aligned}\right\}$$

The normalization factor, the mean and the RMS of the distribution as the free parameters to be estimated by the fit

Does the “Polya” shape describes the e-peak charge and amplitude distributions?

Laser beam tests

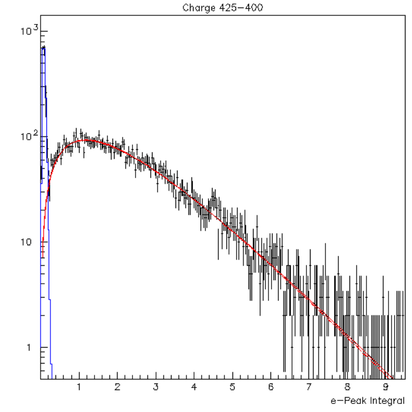
Use data sets collected without any threshold on the picosec-MM signal to test the “Polya Hypothesis”



Use out-of-time events to model the noise

$$F\left(Q;C,\theta,\bar{Q}_e\right)=\frac{C}{Q_e}\frac{(\theta+1)^{(\theta+1)}\left(Q/\bar{Q}_e\right)^{\theta}}{\Gamma(\theta+1)}e^{-(\theta+1)Q/\bar{Q}_e}$$

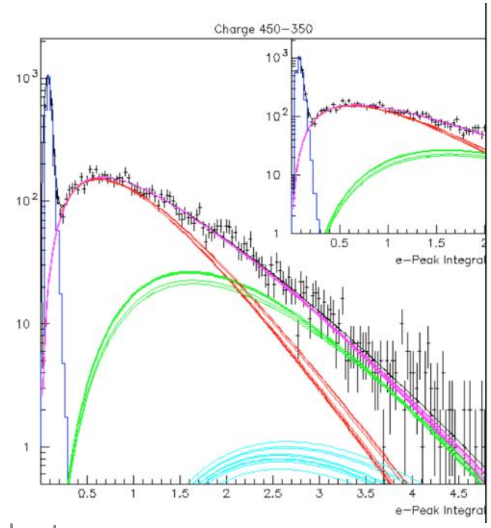
Fit the charge spectrum produced by a single photoelectron



$$G\left(Q;C_1,C_2,\cdots C_k,RMS_e,<Q_e>\right)=\sum_{N=1}^{k}\frac{C_N}{\bar{Q}_e}\frac{(\theta+1)^{N(\theta+1)}\left(Q/\bar{Q}_e\right)^{N(\theta+1)-1}}{\Gamma\left(N(\theta+1)\right)}e^{-(\theta+1)Q/\bar{Q}_e}$$

$$number\ of\ events=\sum_{N=1}^{k}C_N$$

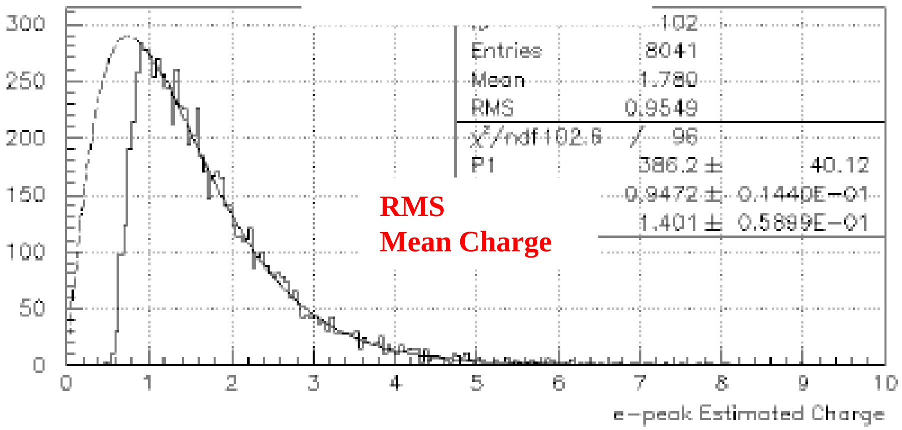
Fit the charge spectrum produced by severalphotoelectrons



Evaluate the mean number of pes per muon

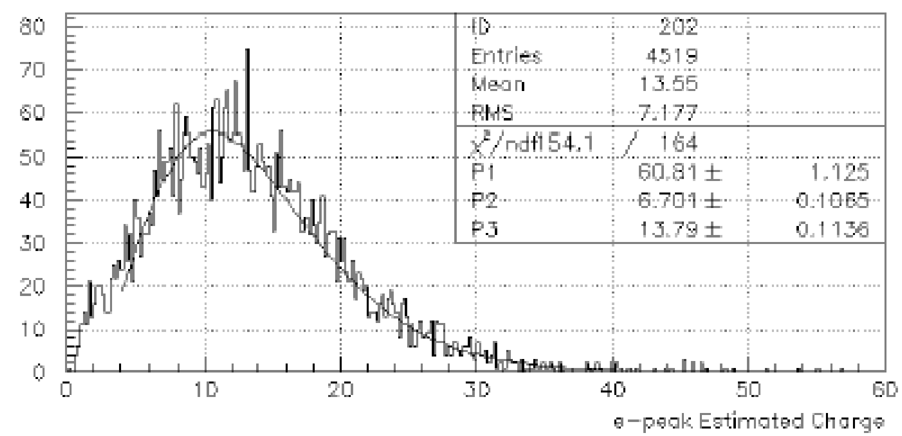
The last Christmas ghost

Rule of thumb: Sir Francis Buller ruling that a man may legally beat his wife provided that he used a stick no thicker than his **thumb**.



Laser Calibration

Fit by Polya the (truncated) single-pe charge spectrum to estimate the mean charge and the RMS. Evaluate the systematical errors.



Muon Test Beam

The beam trigger hodoscope was align to select muons passing through the active area (?) of the PICOSEC

Then… N_{pe}= 13.55/1.4 ~ 9.7pe/muon
Can I also use the RMS values of the charge distributions in order to improve the estimation of N_{pe} ?

Let us assume that the single pe's charge distribution is given by:

$$P_e\left(Q;\bar{Q}_e,V_e\right)=\begin{cases} a\ positive\ function\ for\ Q\geq 0 \\ 0\ for\ Q<0 \end{cases} \qquad (1)$$

where Q is the accumulated charge and

$$\bar{Q}_e=\langle Q\rangle=\int_{-\infty}^{\infty} Q P_e\left(Q;\bar{Q}_e,V_e\right)dQ$$

$$V_e=\langle Q^2\rangle-\bar{Q}_e^2=\int_{-\infty}^{\infty}\left(Q-\bar{Q}_e\right)^2 P_e\left(Q;\bar{Q}_e,V_e\right)dQ \qquad (2)$$

The pdf for the charge which is produced by n pe's (i.e. produced by a single track) will be:

$$P_{ne}\left(Q_n;\bar{Q}_e,V_e\right)=\underbrace{P_e\left(Q\ ;\bar{Q}_e,V_e\right)\otimes P_e\left(Q\ ;\bar{Q}_e,V_e\right)\cdots\otimes P_e\left(Q\ ;\bar{Q}_e,V_e\right)}_{n\ times}$$

$$\int_{-\infty}^{\infty} P_{ne}\left(Q_n;\bar{Q}_e,V_e\right)dQ_n=1 \qquad (3)$$

The expected value and variance of Q_n can be calculated from (3), but it is easier to consider that $Q_n=x_1+x_2+\ldots x_n$ (where x_i is the charge produced by the ith photon and it is distributed according to (1)) and that $x_i\ (i=1,2,3\ldots,n)$ are mutually independent. Then

$$\bar{Q}_{ne}=\langle Q_n\rangle=\int_{-\infty}^{\infty} Q_n P_{ne}\left(Q_n;\bar{Q}_e,V_e\right)dQ_n=n\cdot\bar{Q}_e$$

$$V_{ne}=\int_{-\infty}^{\infty}\left(Q_n-\bar{Q}_{ne}\right)^2 P_{ne}\left(Q_n;\bar{Q}_e,V_e\right)dQ_n=n\cdot V_e \qquad (4)$$

Let us also assume that there is a noise contribution $P_{0e}\left(Q;\bar{Q}_e,V_e\right) == f(Q)$ for zero pe charge, with mean 0 and variance s² (this is for completeness)

Consider now muons producing n pes, where the variable n is following some pdf $g\left(n;\mu,\vec{\mathrm{M}}\right)$ (could be Poissonian), with

$$\mu = \langle n \rangle = \sum_{n=0}^{\infty} n \cdot g\left(n;\mu,\vec{\mathrm{M}}\right)$$

$$V_g = \langle n^2 \rangle - \langle n \rangle^2 = \sum_{n=0}^{\infty} (\mu - n)^2 \cdot g\left(n;\mu,\vec{\mathrm{M}}\right) \qquad (5)$$

and $\vec{\mathrm{M}}$ is a vector of several other parameters which are relevant for the above pdf.

Then the total accumulated charge (that is proportional to the integral of the corresponding waveform) is distributed according to:

$$G\left(Q\,;\mu,\bar{Q}_e,V_e,\vec{\mathrm{M}}\right) = \sum_{n=0}^{\infty} g\left(n;\mu,\vec{\mathrm{M}}\right) \cdot P_{ne}\left(Q\,;\bar{Q}_e,V_e\right)$$

$$\int_{-\infty}^{\infty} G\left(Q\,;\mu,\bar{Q}_e,V_e,\vec{\mathrm{M}}\right) dQ = 1 \qquad (6)$$

The total charge has an expectation value which is given by

$$E[Q] = \int_{-\infty}^{\infty} Q \cdot G\left(Q\,;\mu,\bar{Q}_e,V_e,\vec{\mathrm{M}}\right) dQ = \sum_{n=0}^{\infty} g\left(n;\mu,\vec{\mathrm{M}}\right) \cdot \int_{-\infty}^{\infty} Q \cdot P_{ne}\left(Q\,;\bar{Q}_e,V_e\right) dQ$$

$$= \sum_{n=0}^{\infty} g\left(n;\mu,\vec{\mathrm{M}}\right) \cdot n \cdot \bar{Q}_e = \mu \cdot \bar{Q}_e \qquad (7)$$

The variance of the total charge should be

$$V[Q]=\int_{-\infty}^{\infty}\left(Q-\mu\cdot\bar{Q}_e\right)^2\cdot G\left(Q\,;\mu,\bar{Q}_e,V_e,\vec{\mathrm{M}}\right)dQ=\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)\cdot\int_{-\infty}^{\infty}\left(Q-\mu\cdot\bar{Q}_e\right)^2\cdot P_{ne}\left(Q\,;\bar{Q}_e,V_e\right)dQ$$

$$=\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)\cdot\int_{-\infty}^{\infty}\left\{Q^2+\left(\mu\cdot\bar{Q}_e\right)^2-2Q\mu\cdot\bar{Q}\right\}\cdot P_{ne}\left(Q\,;\bar{Q}_e,V_e\right)dQ$$

$$=\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)\left\{\underbrace{\int_{-\infty}^{\infty}Q^2\cdot P_{ne}\left(Q\,;\bar{Q}_e,V_e\right)dQ}_{V_{ne}+\bar{Q}_{ne}^2}+\left(\mu\cdot\bar{Q}_e\right)^2\underbrace{\int_{-\infty}^{\infty}P_{ne}\left(Q\,;\bar{Q}_e,V_e\right)dQ}_{1}-2\mu\cdot\bar{Q}_e\underbrace{\int_{-\infty}^{\infty}QP_{ne}\left(Q\,;\bar{Q}_e,V_e\right)dQ}_{\bar{Q}_{ne}}\right\} \quad (8)$$

$$=\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)\left\{nV_e+n^2\bar{Q}_e^2+\left(\mu\cdot\bar{Q}_e\right)^2-2\mu\cdot n\cdot\bar{Q}_e^2\right\}=\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)\left\{nV_e+\bar{Q}_e^2\left(\mu-n\right)^2\right\}$$

$$=V_e\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)n+\bar{Q}_e^2\sum_{n=0}^{\infty}g\left(n;\mu,\vec{\mathrm{M}}\right)\left(\mu-n\right)^2$$

$$=\mu V_e+\bar{Q}_e^2V_g$$

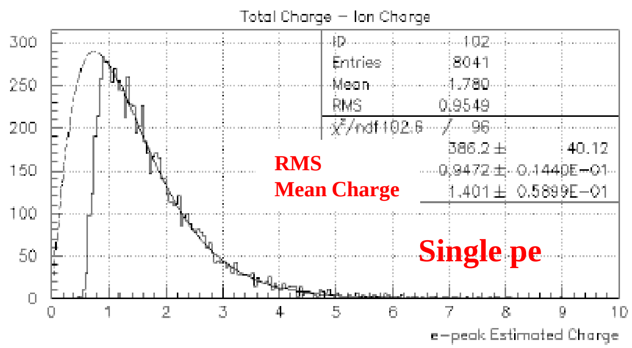
The variance of muon charge distribution is equal to (mean number of pes) x (variance of single pe charge) + (the square of the single pe mean charge) x (the variance of the number of pes)

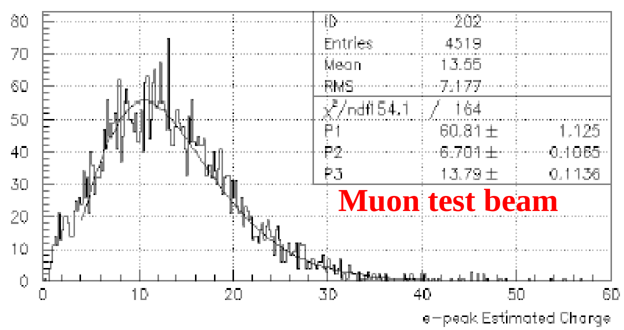
If we assume that the number of pe's is following Poissonian statistics then $V_g = \mu$ and (8) is getting the simple form:

$$V[Q] = \mu V_e + \bar{Q}_e^2 V_g = \mu V_e + \mu \bar{Q}_e^2 = \mu\left(V_e + \bar{Q}_e^2\right) \quad (9)$$

We have also found $E[Q] = \mu \cdot \bar{Q}_e \quad (7)$

Let us apply (7) and (9) to the experimental distributions

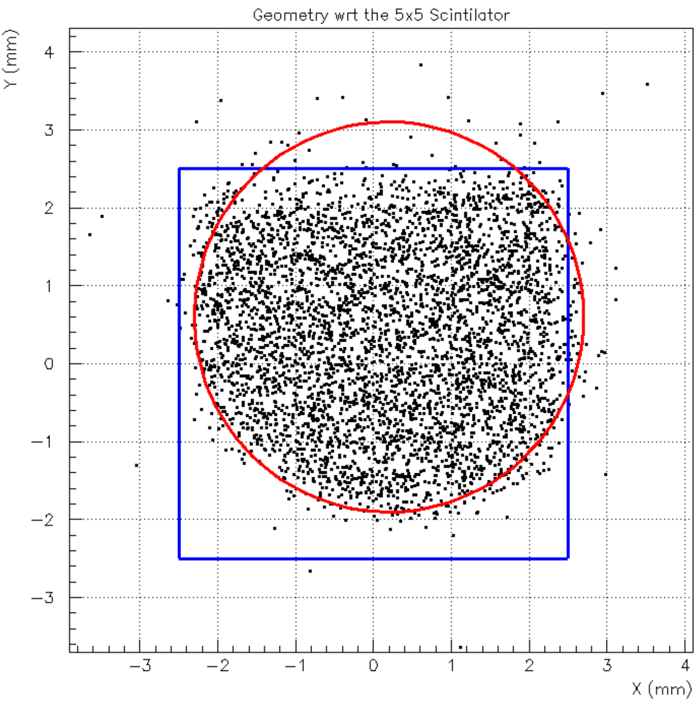


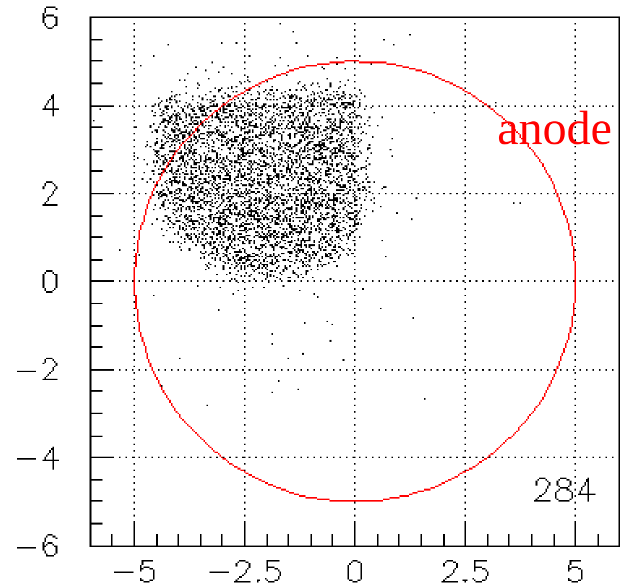


With E[Q]=13.55 and Q_e=1.4 we got μ=9.7
Then, eq. (9) with $V_e=(0.95)^2$ predicts that V[Q] should be 27.8 or that the RMS of the muon charge distribution should be 5.3.
BUT THE RMS OF THE MUON CHARGE DISTRIBUTION IS 7.2

The discrepancy could not be explained as statistical or even systematical error in estimating the relevant mean and RMS values.

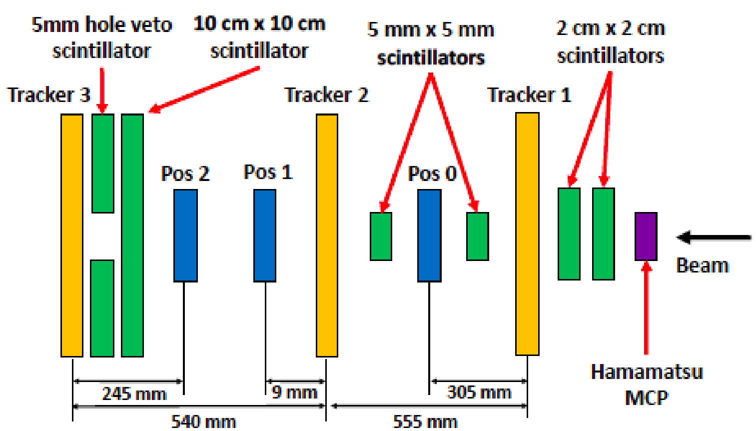
The discrepancy was an artifact of the detector misalignment with respect to the beam profile !!!





The accepted beam profile

The accepted beam profile relative to the anode



We fit to align the detector and to estimate the mean number of pes per muon

The PDF describing the PICOSEC e-peak charge distribution related to a muon is:

$$F(Q,x,y;\mu,\delta x,\delta y) = \sum_{N=1,\infty} \frac{(\mu\varepsilon(r))^{N} e^{-\mu\varepsilon(r)}}{N!} P_{Npe}(Q;RMS,\overline{Q_e})$$

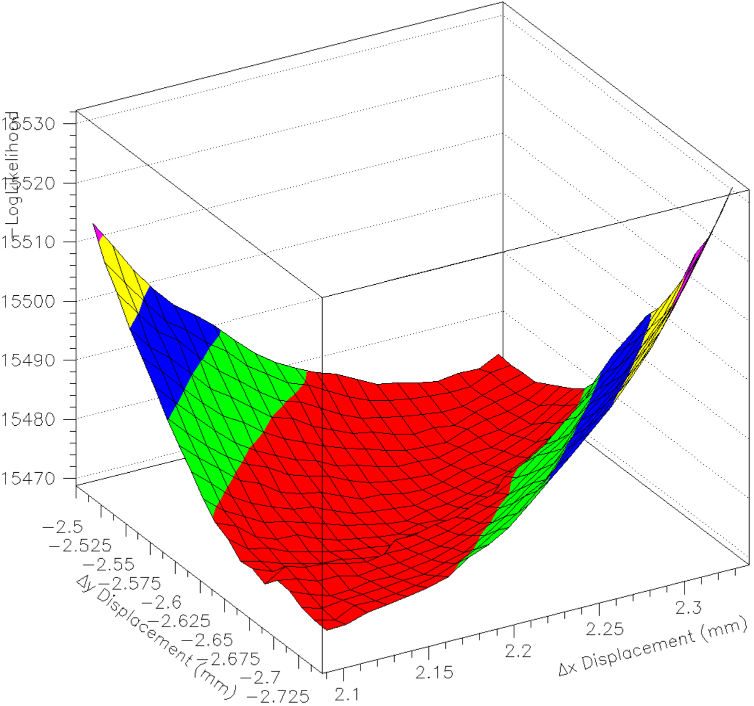
$$r = \sqrt{(x-\delta x)^2 + (y-\delta y)^2}$$

where Q is the charge of the observed PICOSEC pulse, produced by a muon with impact parameter r (wrt the centre of the anode), whilst δx and δy are the coordinates of the anode' center. The mean number of pe's per muon track is μ, P_{Npe} is the Gamma distribution for N pes whilst RMS and $\langle Q_e \rangle$ are the RMS and mean charge of the Polya distribution related to a single photoelectron.

$$P_{Npe}(Q;N,\theta,\bar{Q}_e) = \underbrace{P_{spe} \otimes P_{spe} \cdots \otimes P_{spe}}_{N\ times} = \frac{1}{\bar{Q}_e} \frac{(\theta+1)^{N(\theta+1)} (Q/\bar{Q}_e)^{N(\theta+1)-1}}{\Gamma(N(\theta+1))} e^{-(\theta+1)Q/\bar{Q}_e}$$

The function ε(r) is the geometrical acceptance, i.e. the percentage of pes, produced by a track with impact parameter r, which are produced inside the fiducial limits of the detector.

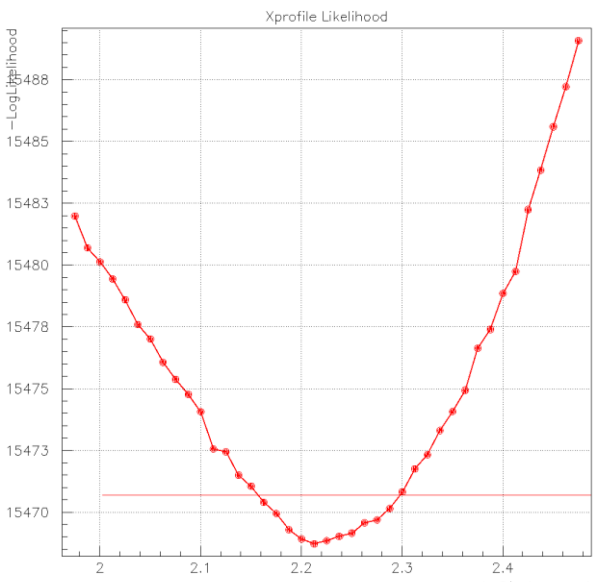
Then the likelihood function for a set of M events is
$L(\mu,\delta x,\delta y)=\prod_{i=1}^{M} F(Q_i,x_i,y_i;\mu,\delta x,\delta y)$

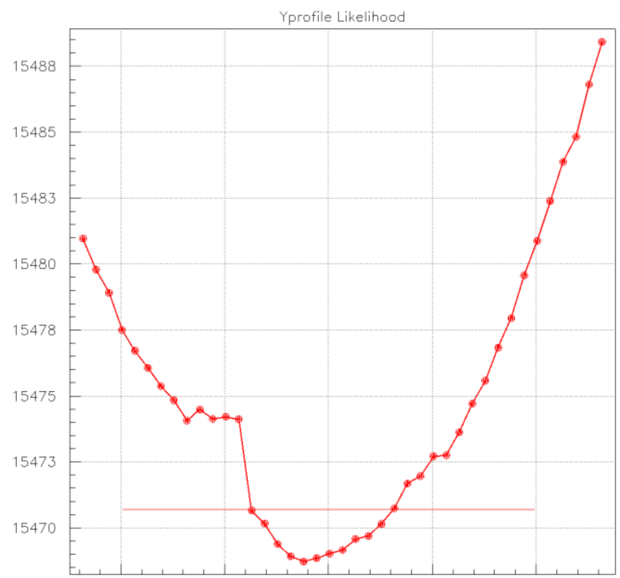


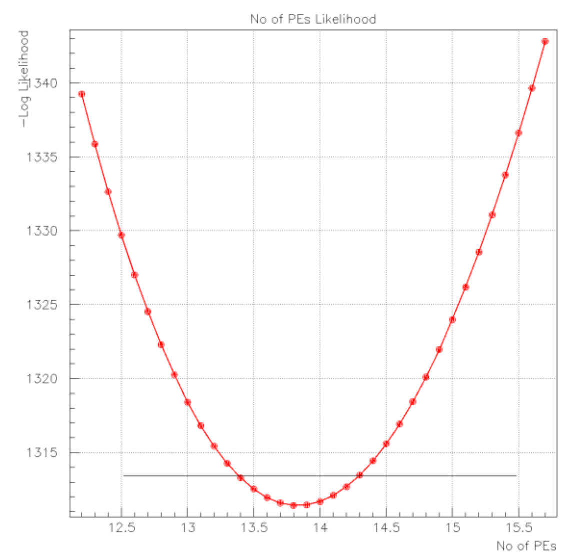
Fix the μ το the previously estimated value and perform an unbinned likelihood fit to estimate δx and δy

Fix δx and δy to the previous estimated values and perform a likelihood fit to estimate μ

Stop when the procedure has converged



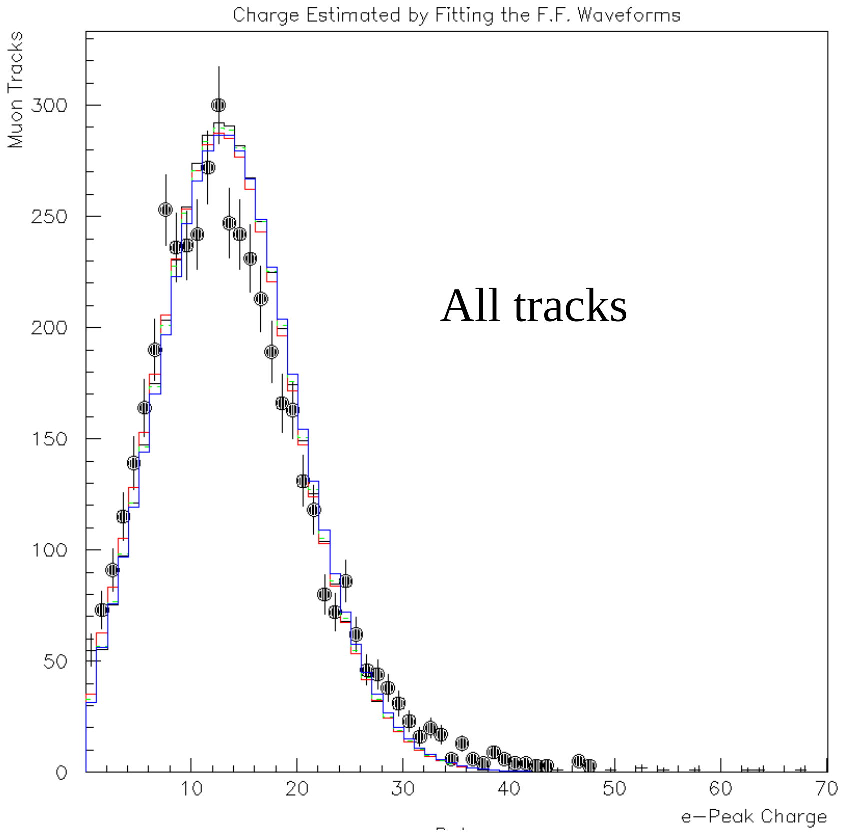


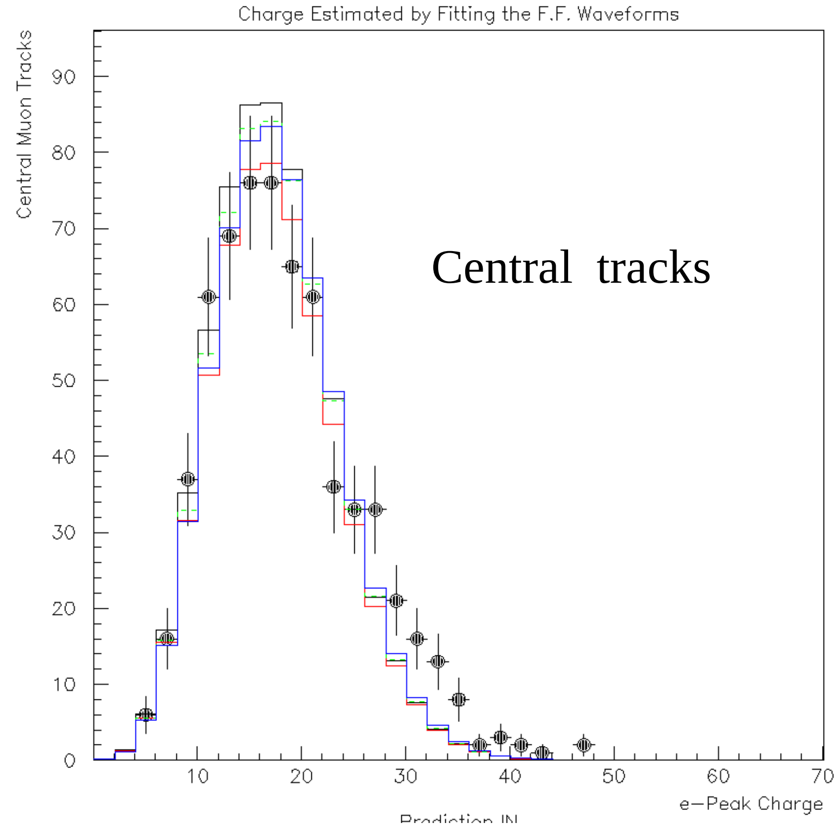


The MINOS statistical errors (at 95.5% CL) are of the order 0.5 for the μ and 0.1 mm for the displacements

SPE charge mean	SPE Charge RMS	μ_0	$\delta x\ (mm)$	$\delta y\ (mm)$
1.34	0.9137	13.024	2.11	-2.52
1.39	0.9087	12.63	2.16	2.51
1.322	0.9164	13.31	2.16	-2.51
1.365	0.9109	13.02	2.15	-2.52
1.408	0.9070	12.73	2.082	-2.507
1.383	0.9095	13.0	2.095	-2.47
1.368	0.9113	13.03	2.10	-2.457
1.347	0.9150	13.3	2.01	-2.52

Comparison of the experimental distributions with the "fit results" (different colors correspond to different Polya parameters)



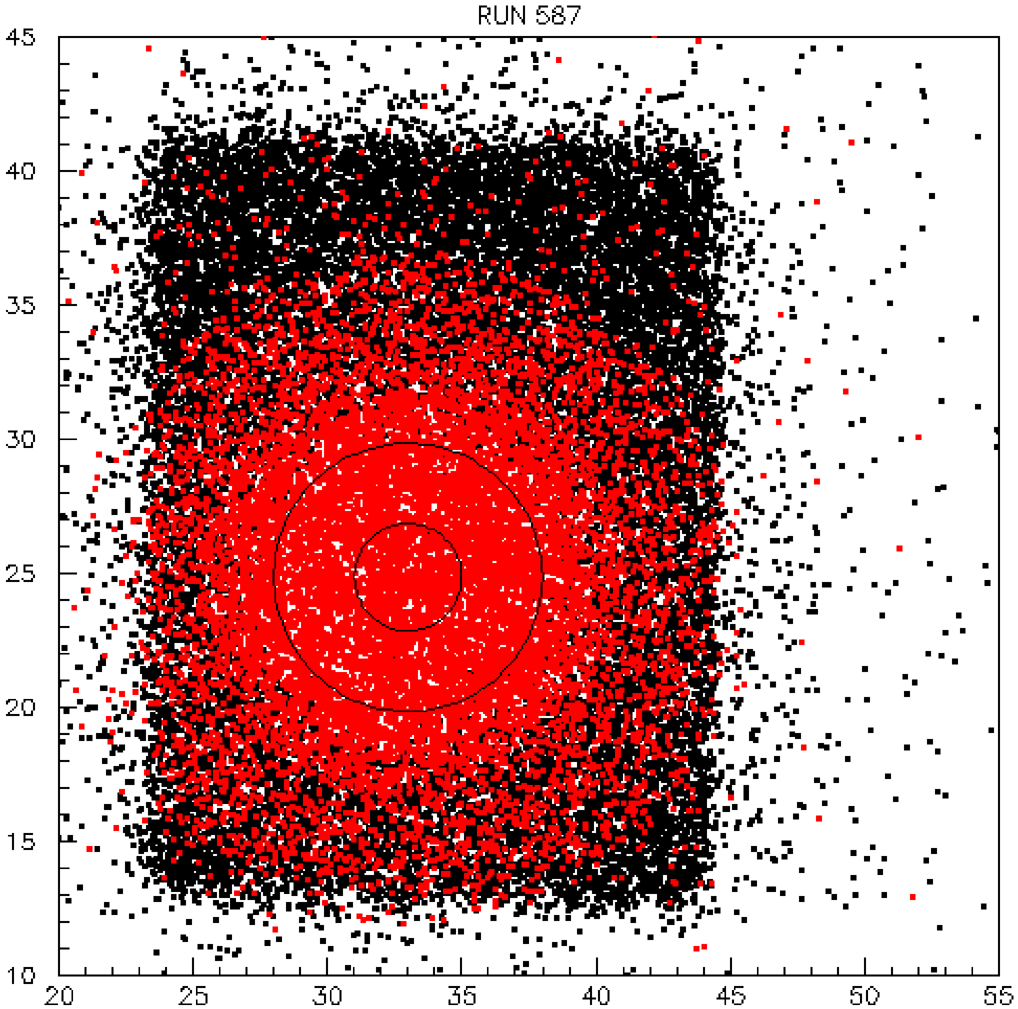


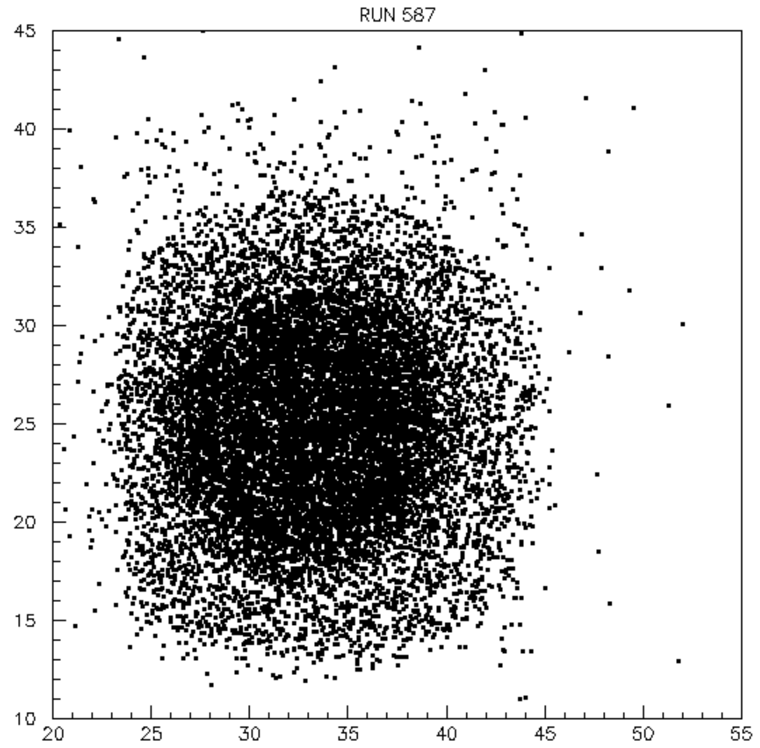
Each track, contributes to all the charge bins the histogram with weights given by:

$$F(Q,x,y;\mu,\delta x,\delta y) = \sum_{N=1,\infty} \frac{(\mu\varepsilon(r))^{n} e^{-\mu\varepsilon(r)}}{N!} P_{Npe}(Q;RMS,\overline{Q_e})$$

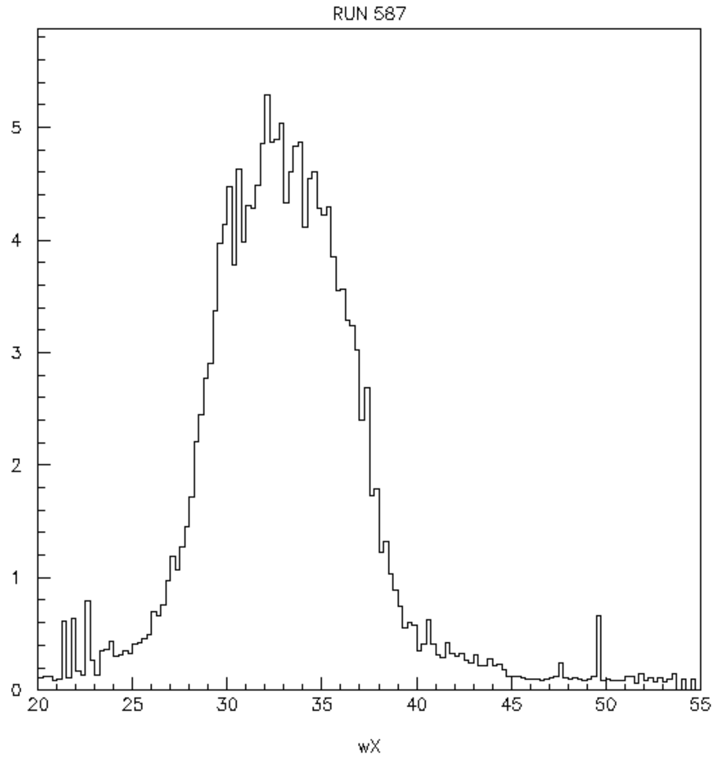
$$r = \sqrt{(x-\delta x)^2 + (y-\delta y)^2}$$

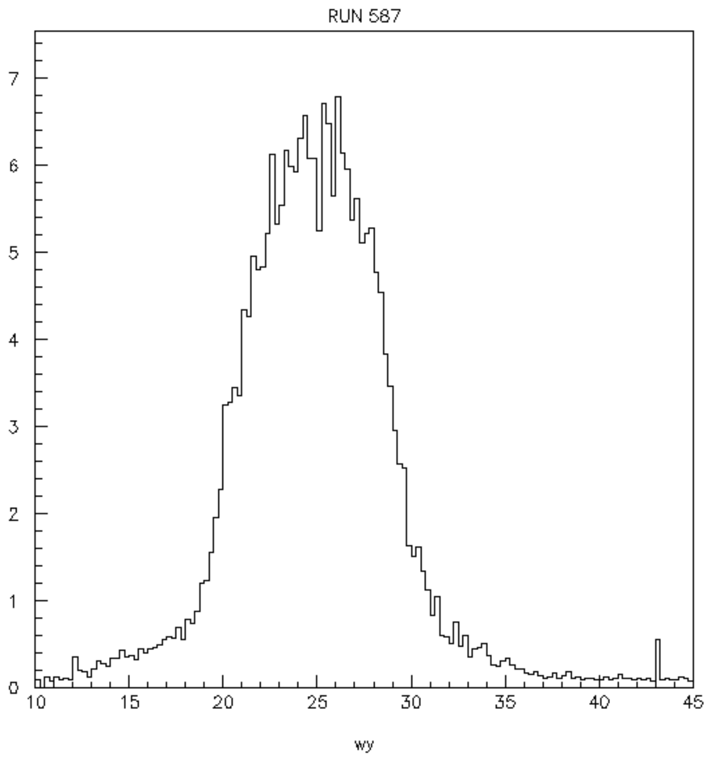
The importance to include all the effects in the estimators

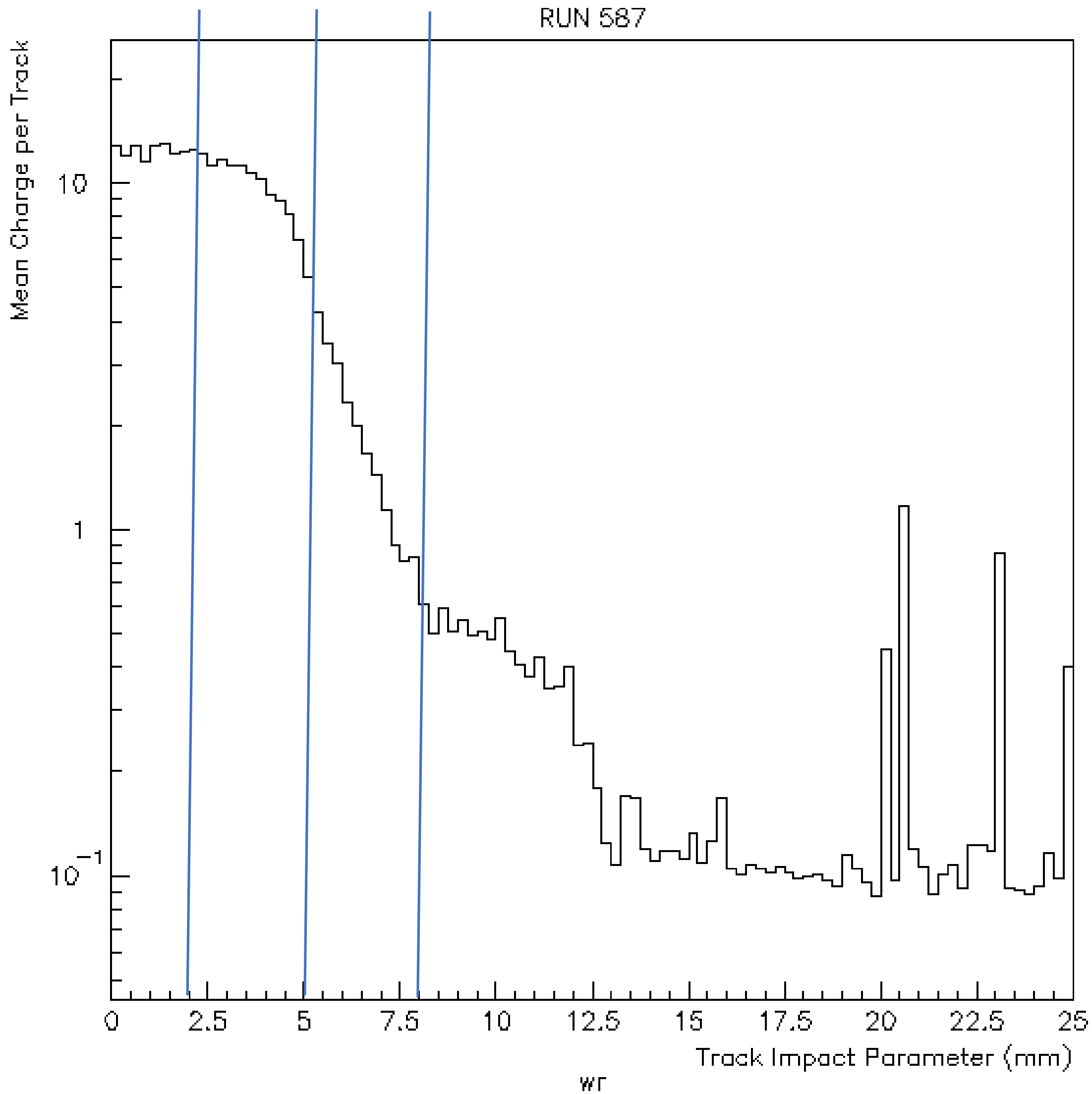


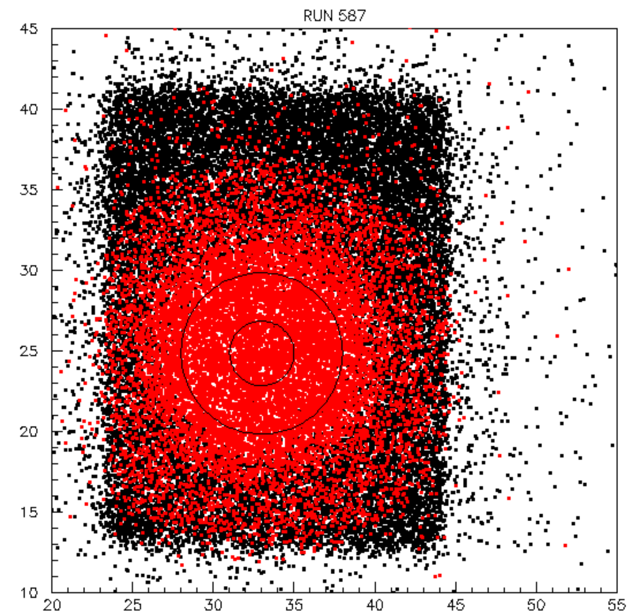


Mean Charge per Track vs X (Y) Track Impact Coordinate

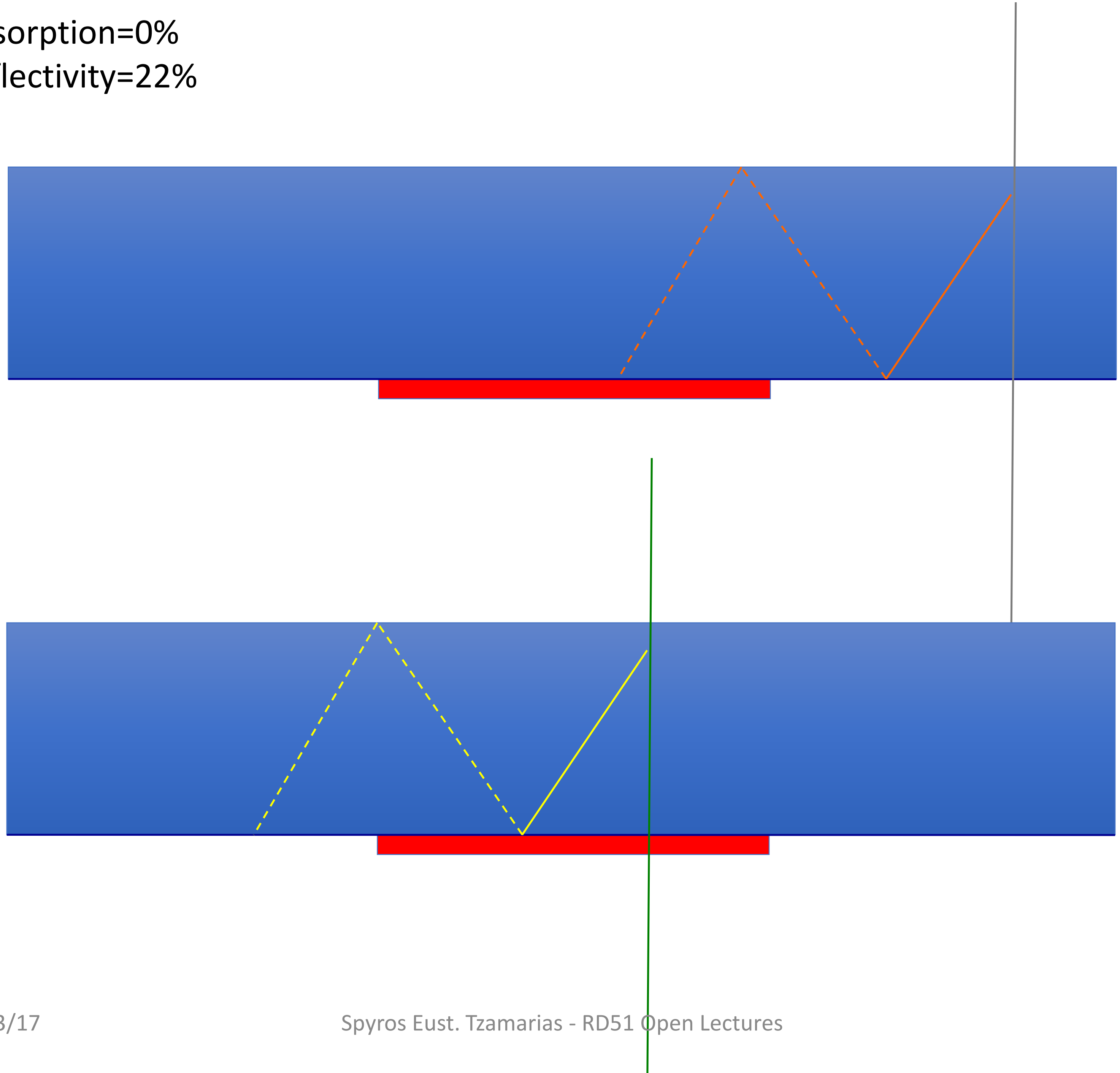


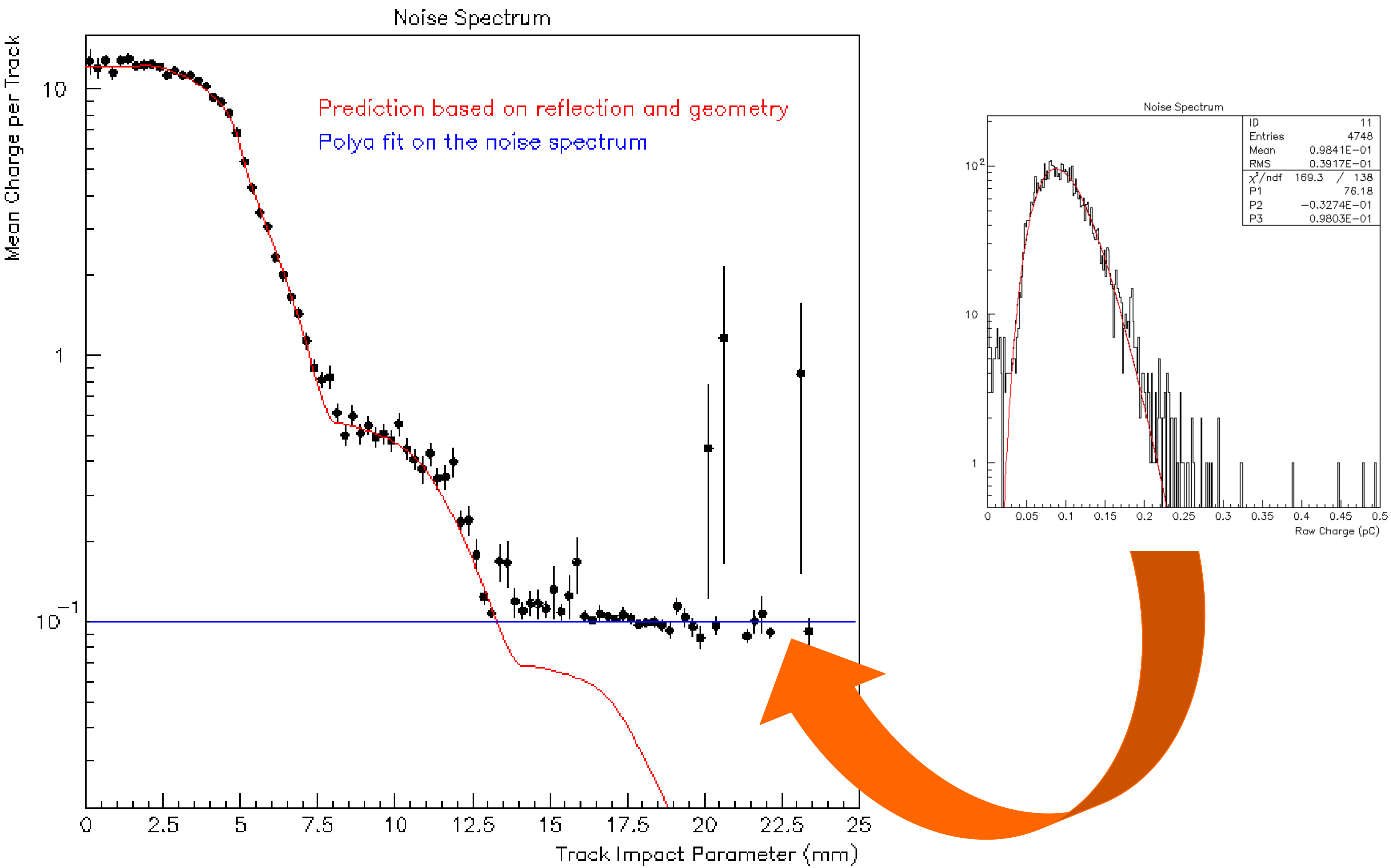



RUN 587
Mean Charge per Track
10
1
10^{-1}
0
2.5
5
7.5
10
12.5
15
17.5
20
22.5
25
Track Impact Parameter (mm)
wr


RUN 587
45
40
35
30
25
20
15
10
20
25
30
35
40
45
50
55

Absorption=0%
Reflectivity=22%





Noise Spectrum
Mean Charge per Track
Prediction based on reflection and geometry
Polya fit on the noise spectrum
Track Impact Parameter (mm)
Noise Spectrum
ID 11
Entries 4748
Mean 0.9841E−01
RMS 0.3917E−01
χ²/ndf 169.3 / 138
P1 76.18
P2 −0.3274E−01
P3 0.9803E−01
Raw Charge (pC)