

# From the Weak Gravity Conjecture to the Photon Mass

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Johns Hopkins Workshop at the GGI

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based on: several papers with Ben Heidenreich and Tom Rudelius (2015-2018);  
MR, 2018

# Does the photon have a mass?

Do any of you believe that the photon mass is nonzero?

If so, why do you think so?

If not, what's your best counterargument?

# Does the photon have a mass?

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If so, why do you think so?

If not, what's your best counterargument?

If not:

“Gauge invariance” is not a convincing answer.

“Embedding in  $SU(5)$ ” is better, but assumes a lot about the UV of our universe.

# Photon masses

Recall: gauge invariance is needed for a theory with a massless photon; the *redundancy*  $\epsilon_\mu \rightarrow \epsilon_\mu + \alpha p_\mu$  is needed to leave just the two physical helicity states.

The Proca Lagrangian which just adds a mass  $m_\gamma^2 A_\mu A^\mu / 2$  is perfectly healthy, and describes a massive gauge field with 3 propagating degrees of freedom.

We can even introduce a “gauge invariance” with the Stückelberg trick,  $f_\theta^2 (\partial_\mu \theta - e A_\mu)^2 / 2$  where under a gauge transformation  $A_\mu \mapsto A_\mu + (1/e) \partial_\mu \alpha$ ,  $\theta \mapsto \theta + \alpha$ .

In EFT, photon masses are perfectly innocent. (Unlike massive nonabelian gauge fields or gravitons—scattering amplitudes of longitudinal modes blow up.)



# Does the photon have a mass?

From Sidney Coleman's Lectures on QFT:

No matter how small  $\mu$  is, this is a system with three degrees of freedom. Everyone says the photon is massless. But suppose the photon had a mass of  $10^{-23}$  of the electron's. This would be a very hard thing to determine experimentally. Some people say, "No, absolutely not! It would be trivial to detect experimentally because we know the real massless photon has only two degrees of freedom; polarized light and so on. If we took a hot oven and let things come to thermal equilibrium, because the walls are emitting and absorbing photons, we wouldn't get the Planck Law, but instead  $\frac{3}{2}$  times the Planck Law." This is *garbage*. The amplitude for the oven walls to radiate a helicity zero photon, according to this current, goes to *zero* in the limit as  $\mu/|\mathbf{k}| \rightarrow 0$ . At every stage in the limiting process there are indeed three degrees of freedom just as you'd expect from a theory of massive vector mesons. But as  $\mu/|\mathbf{k}| \rightarrow 0$ , the amplitude for emitting the third photon goes to zero. If the photon mass is small enough, it will require twenty trillion years for that oven to reach thermal equilibrium!<sup>13</sup>

Discussed by Bass & Schrödinger, 1955

Variety of experimental bounds. Conceptually simplest: purely kinematic bound from Fast Radio Bursts.

$$m_\gamma \lesssim 10^{-14} \text{ eV}$$

(Wu et al. 1602.07835, Bonetti et al. 1602.09135, 1701.03097)

# Conclusion?

Just like neutrino masses turned out to be nonzero, and the cosmological constant turned out to be nonzero, and most of us believe the QCD theta angle will turn out to be nonzero...

The photon mass could also be nonzero. We should keep trying to measure it.

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Just like neutrino masses turned out to be nonzero, and the cosmological constant turned out to be nonzero, and most of us believe the QCD theta angle will turn out to be nonzero...

The photon mass could also be nonzero. We should keep trying to measure it.

## Not Quite.

While I certainly do believe we should keep subjecting it to experimental tests, I think the photon mass is exactly zero—and that *quantum gravity* (& experimental input) can help us see what EFT does not.



# The Swampland



Piero di Cosimo, "Perseus Freeing Andromeda," 1510-15

Even if you work with ingredients that you know exist, some novel combinations of them might not.

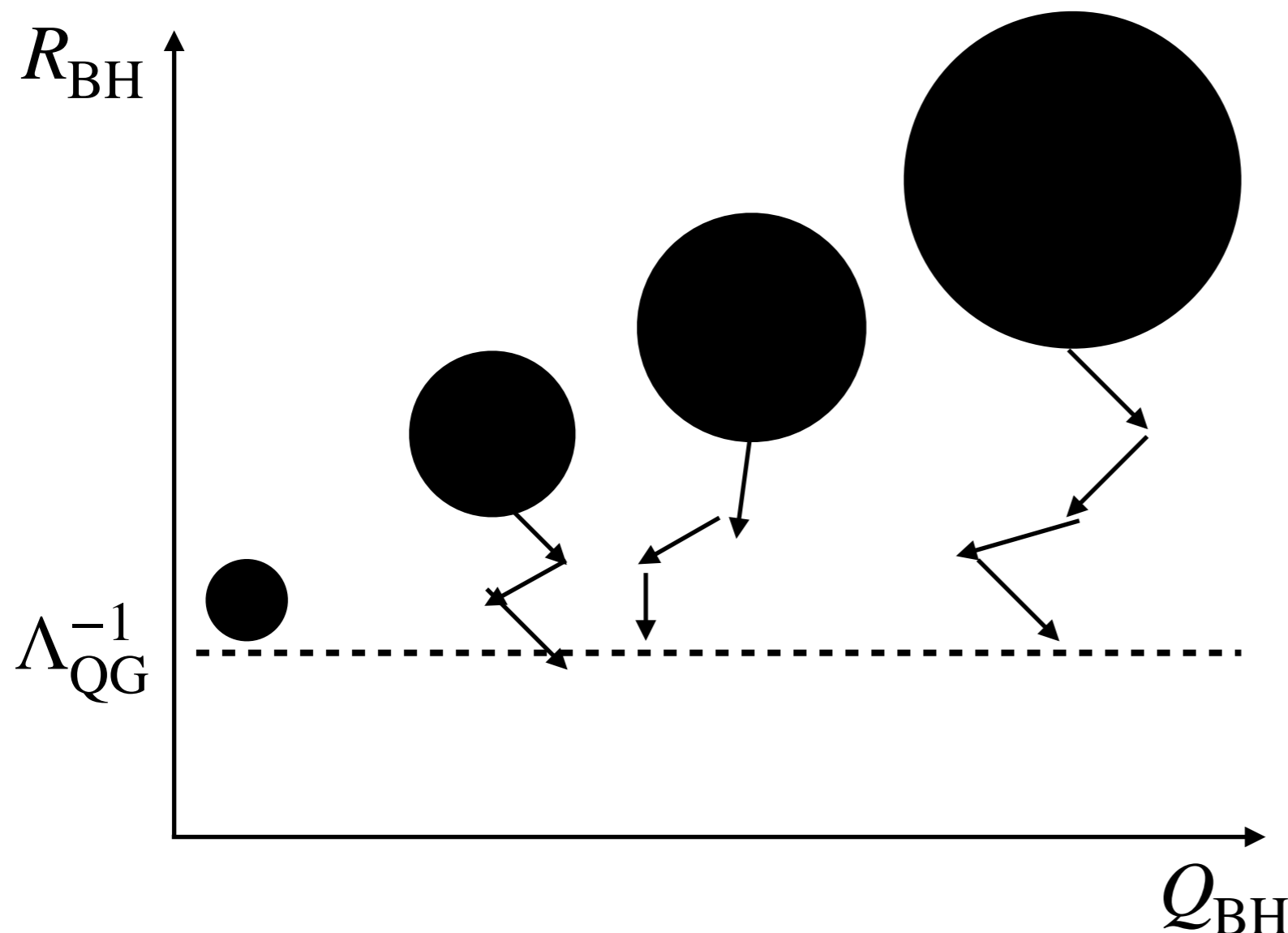


# What is the Swampland?

Certain EFTs cannot be UV-completed in quantum gravity.

Systematic studies began with: Vafa '05; Arkani-Hamed, Motl, Nicolis, Vafa '06; Ooguri, Vafa '06.

Example: no continuous *global* symmetries in QG.

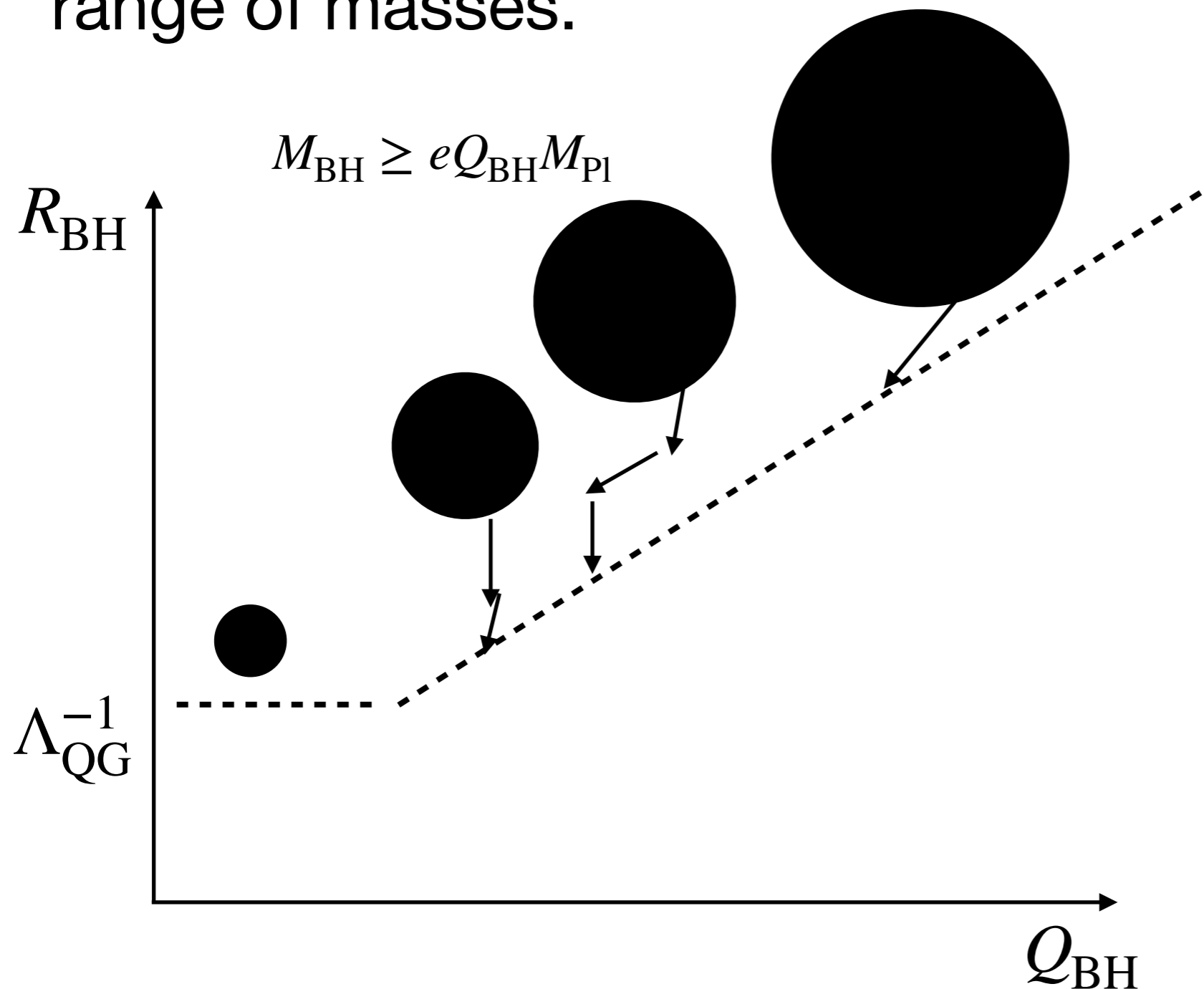


Black hole Hawking evaporation would lead to infinite entropy in finite mass range.

Banks, Seiberg '10; earlier work includes Banks, Dixon '88; Kamionkowski, March-Russell '92; ...

# Small gauge couplings

*Gauge* symmetries are perfectly allowed in QG: the extremality bound spreads the charged BHs over a range of masses.



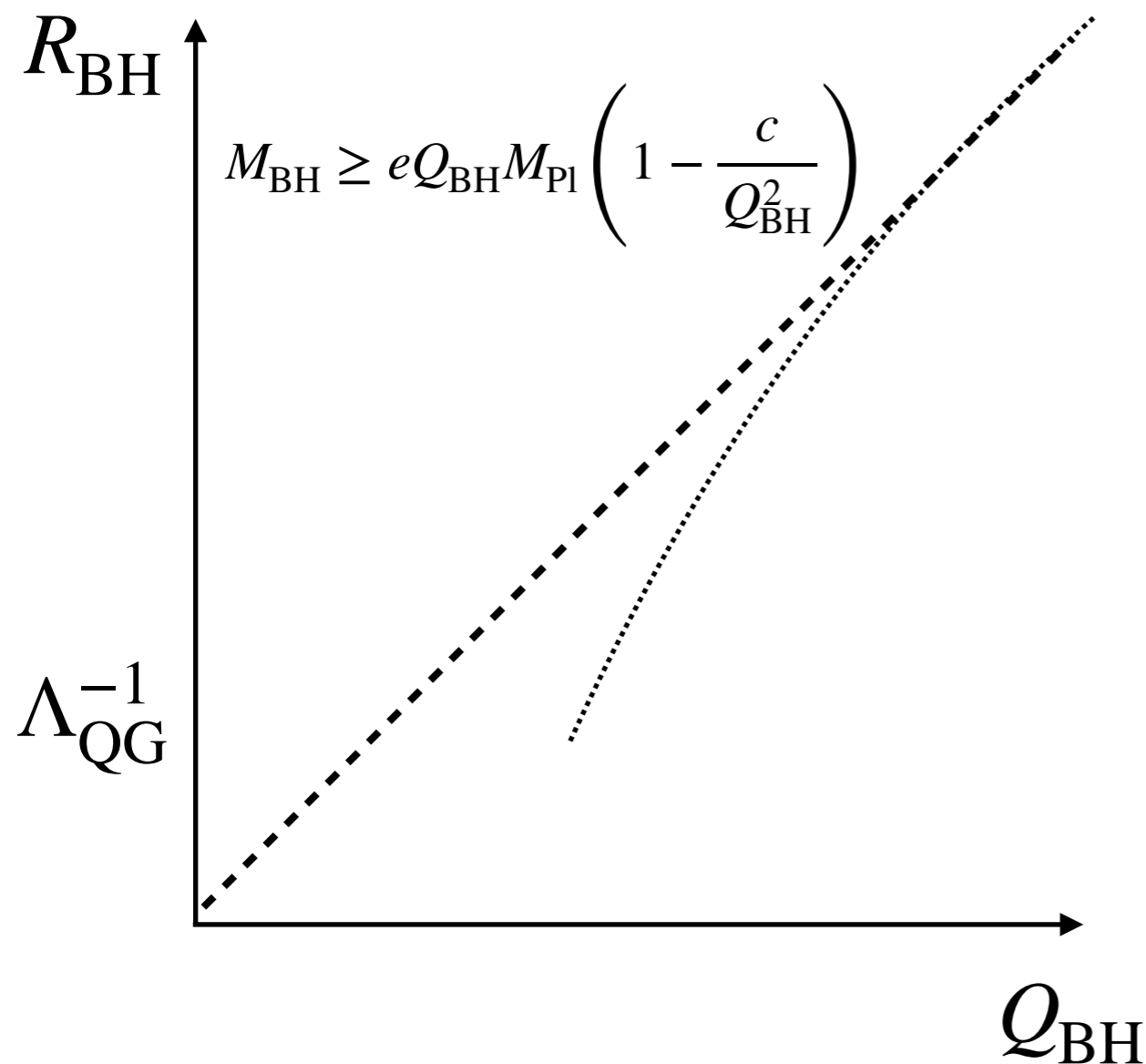
Still, the  $e \rightarrow 0$  limit of a gauge symmetry looks like a global symmetry.

Problems at tiny  $e$ ?

# Weak Gravity Conjectures

Arkani-Hamed, Motl, Nicolis, Vafa '06

**Electric:** there is at least one charged object with  $m \leq eQM_{\text{Pl}}$ . (Saturated only for BPS objects in SUSY.)



Very weak statement; can have tiny corrections to extremal black holes if we allow for huge  $Q$ .

(cf. Kats, Motl, Padi '06; Cheung, Remmen '18)

**Magnetic:** there is a *UV cutoff*  $\Lambda \lesssim eM_{\text{Pl}}$ .  
More interesting! But what sort of cutoff?

(cf. de la Fuente, Saraswat, Sundrum '14)

# Scalar Field Distances in QG

We now turn to a less famous, but possibly more useful, conjecture: the Ooguri-Vafa '06 “Distance Conjecture.”

Background: in string theory (as well as many examples in Kaluza-Klein theory), couplings are not *fixed numbers*, but rather **VEVs of scalar fields**, such as the dilaton, a radion, or more general **moduli**, e.g.:

$$\frac{1}{g^2} \sim \frac{\langle \phi \rangle}{M_{\text{Pl}}}$$

Small couplings  $\iff$  large volumes  
 large volumes  $\iff$  large VEVs

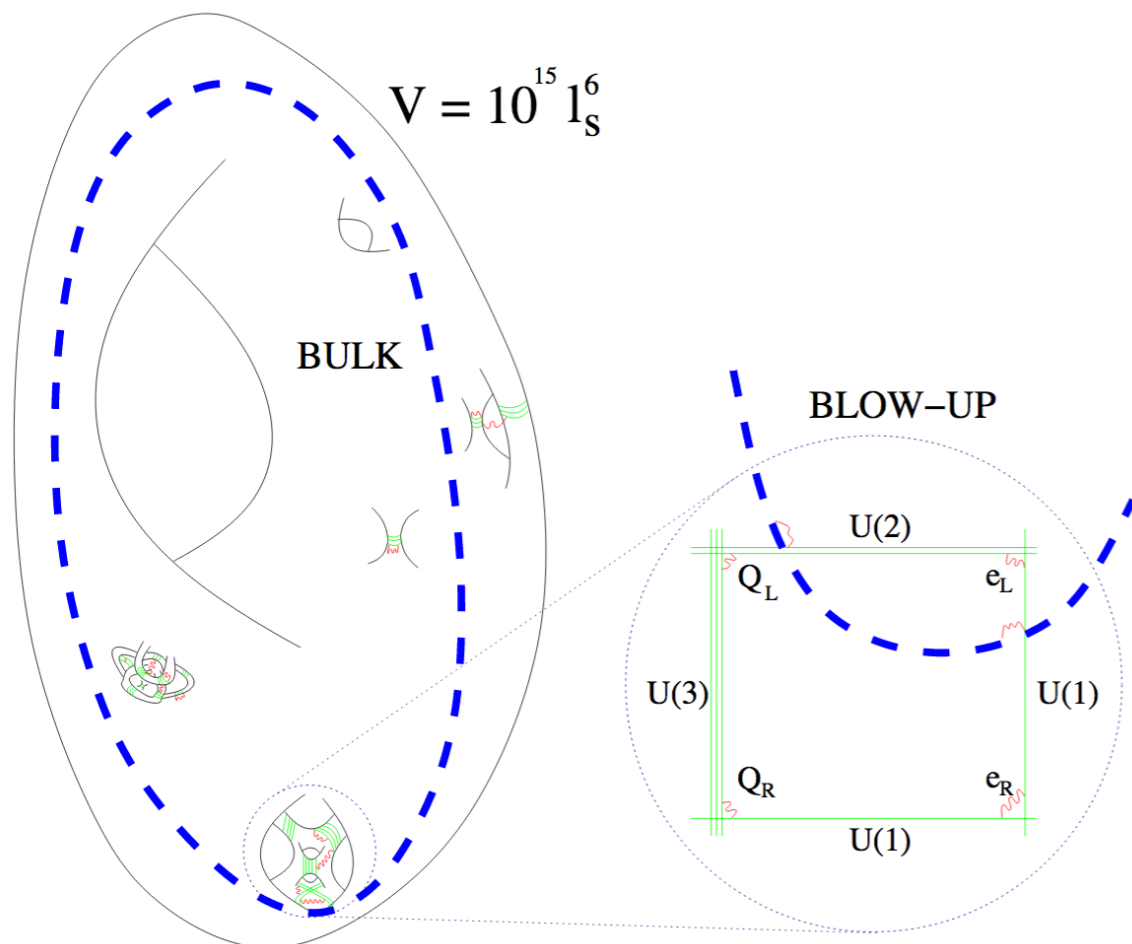


Fig. from Burgess, Conlon, Hung, Kom, Maharana, Quevedo '08



# Swampland Distance Conjecture

Ooguri and Vafa observed some features common to moduli spaces (scalar fields) in known string theories:

- Points at **infinite distance**  $d(\phi) \rightarrow \infty$  exist
- Moving a large distance  $d(\phi)$  from a fixed point in moduli space, an **infinite tower of modes** becomes light with masses trending as  $\exp(-\alpha d(\phi)/M_{\text{Pl}})$
- The constant  $\alpha$  is  $O(1)$  in known examples

(See their paper for slightly more precise statements.)

# Example: Kaluza-Klein theory

For intuition, keep in mind the classic KK theory, with an extra dimension of radius  $R$ .

$$\sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} \mathcal{R}_4 - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4e_{\text{KK}}^2} e^{\alpha\phi} F_{\mu\nu}^2 \right]$$

gravity
radion
U(1) gauge field

$$e_{\text{KK}}^2 = \frac{2}{R^2 M_{\text{Pl}}^2} \quad \text{large radius} \iff \text{small gauge coupling}$$

$$m_n = \frac{n}{R} = \frac{e_{\text{KK}}}{\sqrt{2}} M_{\text{Pl}} \quad \text{infinite tower of KK mode masses proportional to gauge coupling}$$

$$R \propto e^{\sqrt{3}\phi/(2M_{\text{Pl}})} \quad \text{radius exponential in canonically normalized radion (field space distance)}$$

# Tower Weak Gravity Conjecture

The WGC tells us to expect a low UV cutoff at small  $e$ .

In known QG theories, the way this works is that the  $e \rightarrow 0$  limit is precisely an infinite-distance limit in moduli space, just like the Kaluza-Klein example.

This motivates a stronger **Tower WGC**:

There are infinitely many charged particles of *different* charges  $q_i$ , *each of which* obeys the bound

$$m_i \leq e q_i M_{\text{Pl}}.$$

Stronger **Sublattice WGC (sLWGC)**: take the charges to lie in a sublattice (of the same dimension as full charge lattice).

## But what do we really *know*?

These conjectures don't have rigorous proofs, but strong supporting evidence.

The Sublattice WGC was proposed on the basis of *internal consistency arguments* (Heidenreich, MR, Rudelius '15/'16) that the ordinary WGC fails.

It has been generally proved in *perturbative string theory and  $AdS_3/CFT_2$*  using modular invariance (Heidenreich, MR, Rudelius '16; Montero, Shiu, Soler '16).

Recently it was proven in 6d compactifications of F-theory (*strongly coupled Type IIB string theory*) (Lee, Lerche, Weigand '18).

# Status of the conjectures

Many tests of these statements in string theory do *not* rely on supersymmetry or worldsheet perturbation theory.

Conjectures have led to mathematical/formal progress with *highly nontrivial results*, e.g. the recent F-theory evidence or the study of BPS states in Calabi-Yau complex structure moduli space (Grimm, Palti, Valenzuela '18).

The evidence (e.g. from modular invariance) directly applies to gauge fields in the IR EFT, *not* the UV.

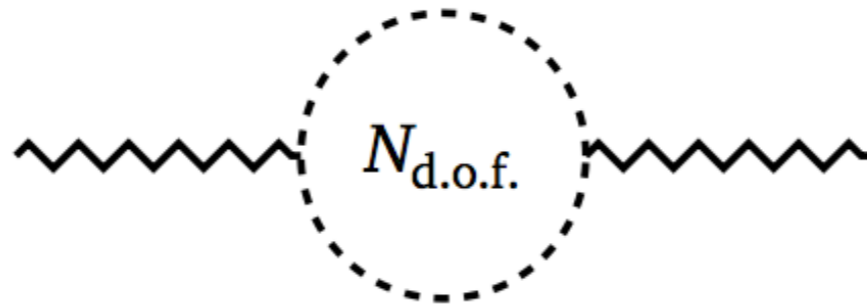
Experimental evidence for violation of these conjectures would mean QG in our universe is very different from any known theory.

# The Species Bound: Low Cutoffs from Many Weakly-Coupled Particles

In a theory with many light, weakly-coupled degrees of freedom, the UV cutoff at which gravity becomes strong is:

$$\Lambda_{\text{QG}} \lesssim \frac{M_{\text{Pl}}}{(N_{\text{d.o.f.}}(\Lambda_{\text{QG}}))^{1/(D-2)}}$$

The simplest argument for this is perturbative. Loops



renormalize the graviton kinetic term:

$$\delta M_{\text{Pl}}^{D-2} \sim N_{\text{d.o.f.}} \Lambda_{\text{QG}}^{D-2}$$

e.g. G. Dvali,  
0706.2050 and  
G. Dvali & M. Redi,  
0710.4344

# Stückelberg in the Swampland

Back to our starting point: a *Stückelberg* photon mass, introducing a Goldstone boson that shifts:

$$\frac{1}{2} f^2 (\partial_\mu \theta - e \hat{A}_\mu)^2$$

In string theory, such masses are ubiquitous. SUSY implies that a radial mode exists. Distinguishing feature is the kinetic term:

$$K(\Phi, \Phi^\dagger, V) = -M^2 \log(\Phi + \Phi^\dagger - cV)$$

The point of zero photon mass lies at infinite distance,

$$\text{Re } \Phi \rightarrow \infty, \quad m_V \sim \frac{M^2}{(\Phi + \Phi^\dagger)^2}$$

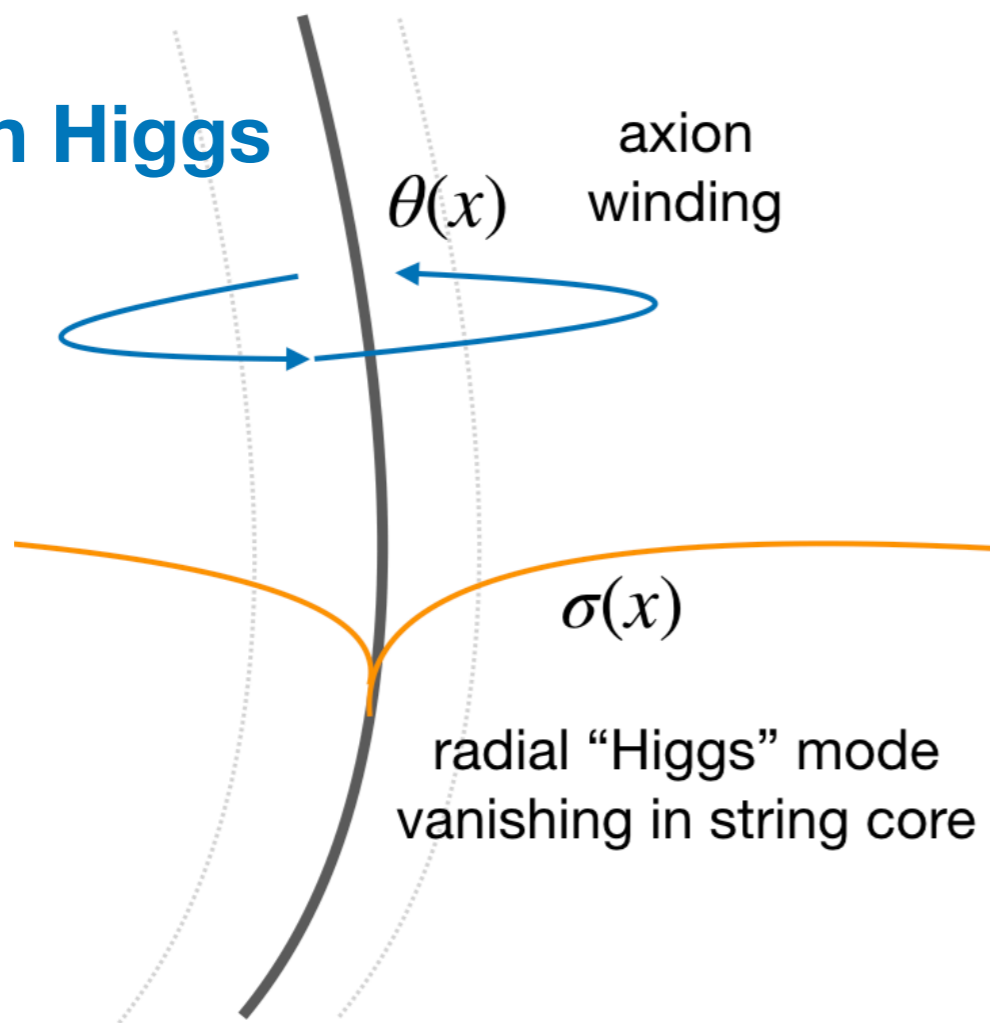
# Stückelberg in the Swampland

Dualize the eaten Goldstone boson to a 2-form gauge field  $B$ :

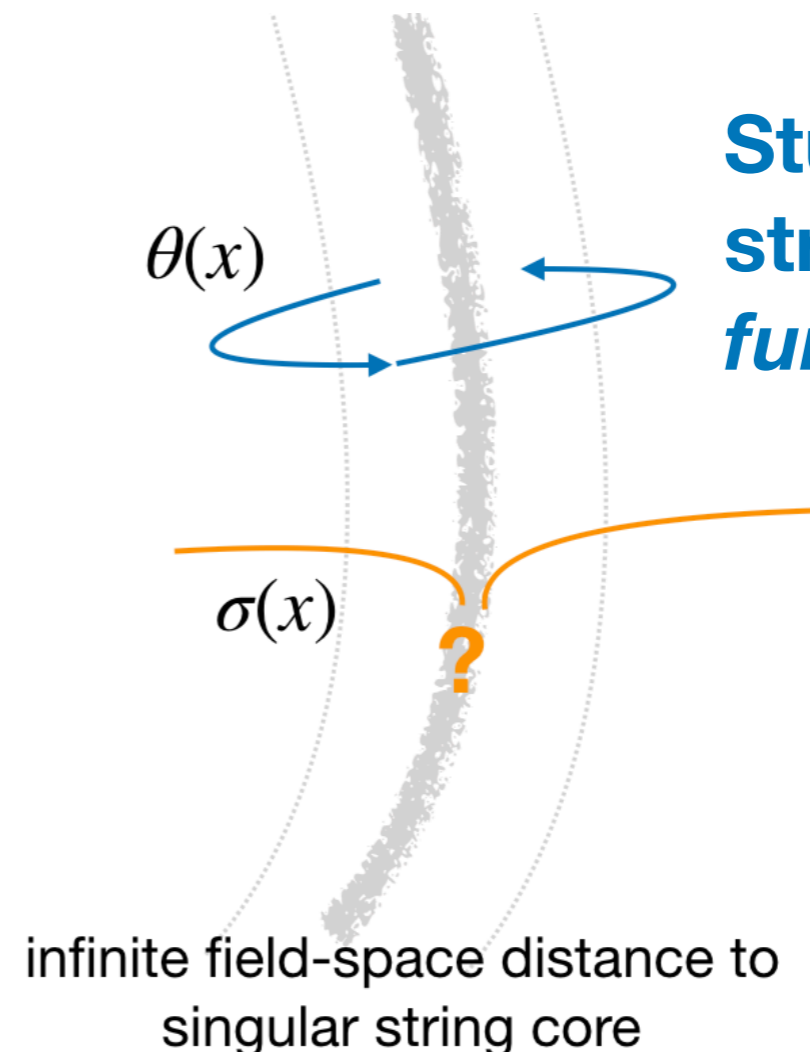
$$\epsilon^{\mu\nu\rho\lambda} \partial_{[\mu} B_{\nu\rho]} = f^2 \partial^\lambda \theta$$

Now apply the **WGC** to the  $B$ -field: charged strings exist with tension  $T \lesssim f M_{\text{Pl}}$ . (see Hebecker, Soler '17)

**Abelian Higgs string:**



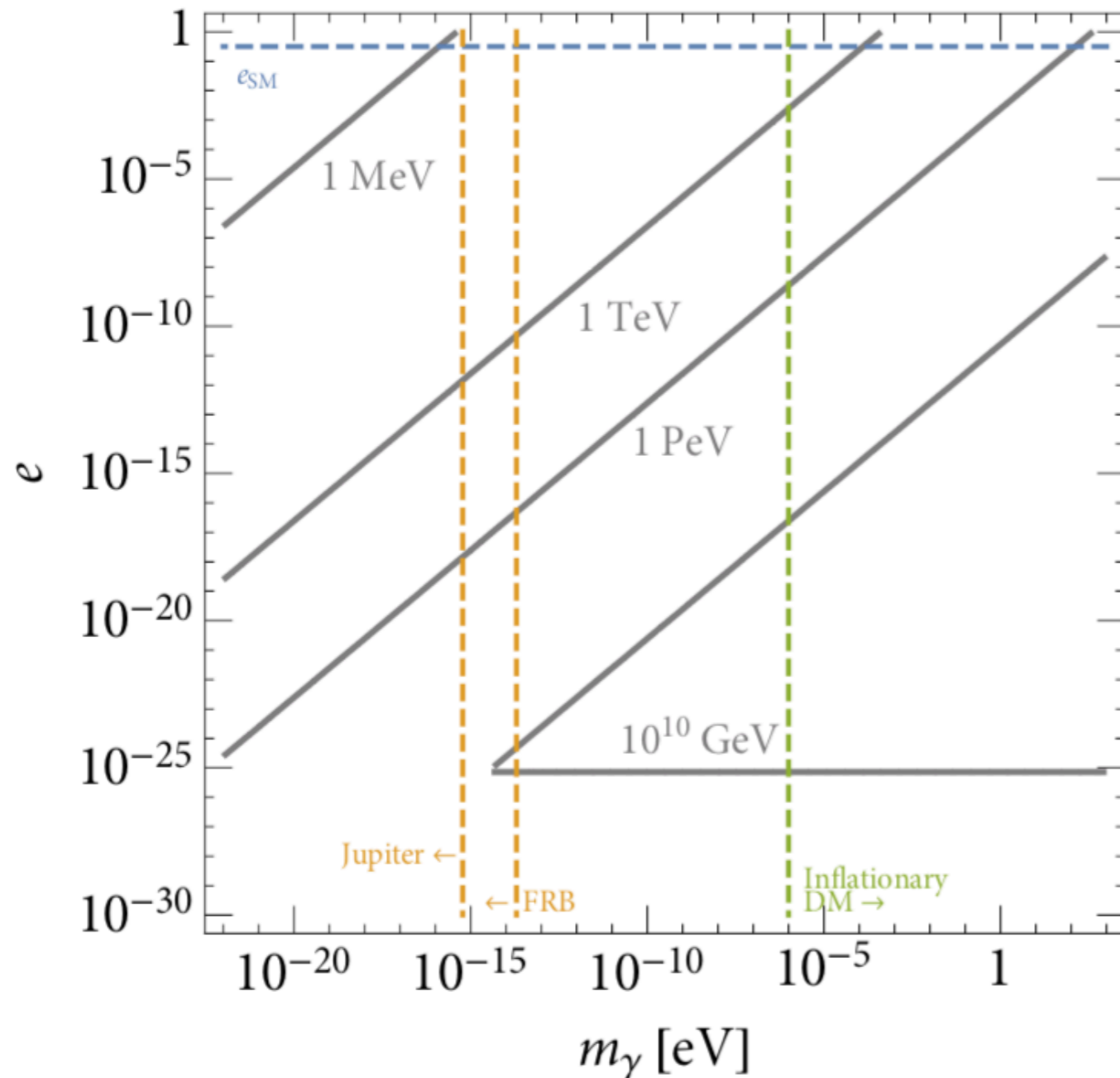
**Stückelberg strings are fundamental**





# Ultraviolet cutoffs on Stückelberg photons

Max. UV Cutoff for Stückelberg Theory



$$m_\gamma = ef$$

$e \rightarrow 0 : A_\mu$  weakly coupled

$f \rightarrow 0 : B_{\mu\nu}$  weakly coupled

$$\Lambda_{\text{QG}} \lesssim \min(e^{1/3} M_{\text{Pl}}, \sqrt{m_\gamma M_{\text{Pl}}/e})$$

species bound  
for sLWGC  
( $e \rightarrow 0$ )

tension of string  
( $f \rightarrow 0$ )

the “inflationary DM” line is dark photon dark matter produced by inflationary fluctuations: Graham, Mardon, Rajendran 2015

# Can the photon have a mass?

For the SM photon, very simple kinematic bounds (from fast radio bursts) tell us

$$m_\gamma \lesssim 10^{-14} \text{ eV}$$

A mass at this scale leads to local EFT breaking down at low energies:

$$\Lambda_{\text{QG}} \lesssim \sqrt{m_\gamma M_{\text{Pl}}/e} \lesssim 10 \text{ MeV}$$

So the SM photon can't have a Stückelberg mass.

Loophole is the unit of charge: suppose the electron charge is  $N$ , i.e. what we know as  $e$  is really  $e_0 N$  for  $N \gg 1$ .

We can push the UV cutoff above a TeV if  $N \sim 10^{14}$ .

(Or Higgs mechanism: Higgs is millicharged, similarly huge  $N$ .)

Not very *plausible*, but not logically inconsistent?

# Outlook

The original (electric) Weak Gravity Conjecture may be too weak to be very useful.

The Swampland Distance Conjecture could have a strong role to play in phenomenology.

The Tower/Sublattice WGC is a compelling combination of the two, and can be a powerful tool for theory and phenomenology.

The photon is probably massless—but all the more important to have more stringent experimental tests!



**older slides**

# Introduction

## Swampland c. 2006:

Ooguri/Vafa: large  $d(\phi) \Rightarrow$  breakdown of EFT.

Large field-space distance  $d(\phi)$ : tower of light modes

$$m \propto e^{-cd(\phi)/M_{\text{Pl}}}$$

Weak Gravity Conjecture: gauge coupling  $g \rightarrow 0$   
forbidden by QG, (magnetic) cutoff  $\Lambda \lesssim gM_{\text{Pl}}$ .

## More recently:

These are one and the same phenomenon, via  
*sublattice/tower* forms of WGC.

# Introduction

## Sublattice/Tower WGC

Beginning at the magnetic cutoff  $\Lambda \approx gM_{\text{Pl}}$ , find an infinite tower of particles in different representations (different charges, for U(1)), *each* obeying WGC.

Many particles imply **low UV cutoffs**, as all of these particles can run in loops.

Recent progress: a new assumption, of a ***universal strong coupling scale***, together with towers of particles, can unify some of the Swampland ideas.

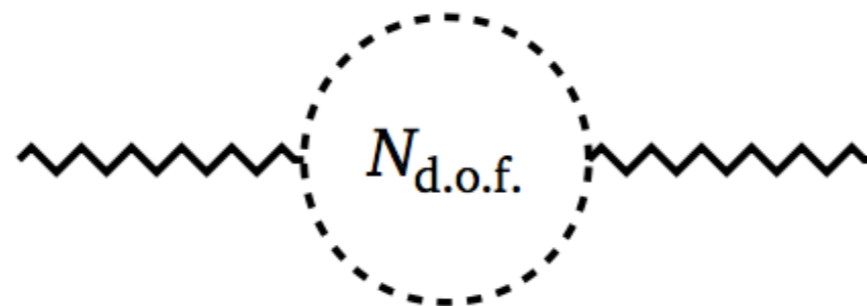
Furthermore, **quantitative predictions of UV cutoffs** help make clearer connections to phenomenology.

# The Species Bound: Low Cutoffs from Many Weakly-Coupled Particles

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# Towers and Cutoffs

In theories with a field-dependent tower of states,

$$N(\phi) \sim N_0 e^{\alpha d(\phi)/M_{\text{Pl}}}$$

the UV cutoff (in Einstein frame) decreases exponentially with field distance. Another perspective on Dine-Seiberg:

$$V(\phi) < \Lambda_{\text{QG}}^2 M_{\text{Pl}}^2 \lesssim e^{-\alpha d(\phi)/M_{\text{Pl}}} M_{\text{Pl}}^4$$

In natural theories with broken SUSY, we expect that

$$V(\phi) \sim m_{\text{SUSY}}^2 \Lambda_{\text{QG}}^2 \sim e^{-\beta d(\phi)/M_{\text{Pl}}}$$

Don't need the quasi-dS assumption invoked by Vafa and Palti yesterday or any discussion of entropy, but do need some assumption about scaling with cutoff. (Still, examples work even with tuning, e.g. compactify Standard Model.)

# Tower of States and Cutoffs

(in 4d for concreteness)

**Single U(1):**

Lattice versions of the WGC imply the existence of a tower of charged particles.

$$QeM_{\text{Pl}} \frac{\quad}{q = Q}$$

·  
·

$$3eM_{\text{Pl}} \frac{\quad}{q = 3}$$

$$2eM_{\text{Pl}} \frac{\quad}{q = 2}$$

$$eM_{\text{Pl}} \frac{\quad}{q = 1}$$

$$N_{\text{d.o.f}}(\Lambda) \gtrsim \frac{\Lambda}{eM_{\text{Pl}}}$$

(A sparse sublattice could change this, but we know no examples.)

$$\Lambda_{\text{QG}}^2 \lesssim \frac{1}{N_{\text{d.o.f}}(\Lambda_{\text{QG}})} M_{\text{Pl}}^2$$

$$\Rightarrow \Lambda_{\text{QG}} \lesssim e^{1/3} M_{\text{Pl}}$$

# Phenomenological Consequences?

We don't know any very weakly coupled U(1) gauge theory in nature. But  $B-L$  could be gauged. Current bounds (Wagner et al. 1207.2442, Heeck 1408.6845) tell us that a massless  $B-L$  force, *if it exists*, must have

$$e_{B-L} \lesssim 10^{-24}$$

so the lattice WGC species bound argument tells us that *if* we ever discover such a force, we would conclude the fundamental cutoff scale of gravity in our universe is

$$\Lambda_{\text{QG}} \lesssim 10^{-8} M_{\text{Pl}} \sim 10^{10} \text{ GeV}$$

this in turn would strongly constrain inflation, neutrino mass generation, SUSY breaking, .....

# Emergent Gauge Fields?

(in 4d for concreteness)

Single U(1):

$$N_{\text{d.o.f.}}(\Lambda) \gtrsim \frac{\Lambda}{eM_{\text{Pl}}} \Rightarrow \Lambda_{\text{QG}} \lesssim e^{1/3} M_{\text{Pl}}$$

$$QeM_{\text{Pl}} \frac{\cdot}{q = Q}$$

·  
·

$$3eM_{\text{Pl}} \frac{\cdot}{q = 3}$$

$$2eM_{\text{Pl}} \frac{\cdot}{q = 2}$$

$$eM_{\text{Pl}} \frac{\cdot}{q = 1}$$

This suggests the tower hits strong coupling at level

$$Q \sim e^{-2/3}$$

Integrating out charged degrees of freedom:

$$\frac{1}{e^2} = \frac{1}{e_{\text{UV}}^2} + \sum_{q=1}^Q \frac{q^2}{12\pi^2} \log \frac{\Lambda}{eqM_{\text{Pl}}}$$

Ignoring logs and constants, the sum is:

$$Q^3 \sim \frac{1}{e^2}$$

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$$2eM_{\text{Pl}} \quad \frac{\quad}{q = 2}$$

$$eM_{\text{Pl}} \quad \frac{\quad}{q = 1}$$

Put differently: a tower of states of different charges leads to a Landau pole for the U(1) coupling in the UV.

It also renormalizes the Planck scale, leading to a low gravitational cutoff.

A Lattice WGC tower is one for which these are (at least up to constant factors) the *same* scale!

Extends to general gauge groups!

# Nonabelian tower and UV cutoffs

**SU(2)**: states charged under Cartan U(1)

$$QgM_{\text{Pl}} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \cdots \quad \cdots$$

$$q = Q, Q-1, \dots, -(Q-1), -Q$$

$$3gM_{\text{Pl}} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad}$$

$$q = 3, 2, 1, 0, -1, -2, -3$$

$$2gM_{\text{Pl}} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad}$$

$$q = 2, 1, 0, -1, -2$$

$$gM_{\text{Pl}} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\quad}$$

$$q = 1, 0, -1$$

**States come in multiplets: linear tower leads to quadratic growth in density of states**

$$N_{\text{d.o.f.}}(\Lambda) \gtrsim \left( \frac{\Lambda}{gM_{\text{Pl}}} \right)^2$$

$$\Rightarrow \Lambda_{\text{QG}} \lesssim g^{1/2} M_{\text{Pl}}$$



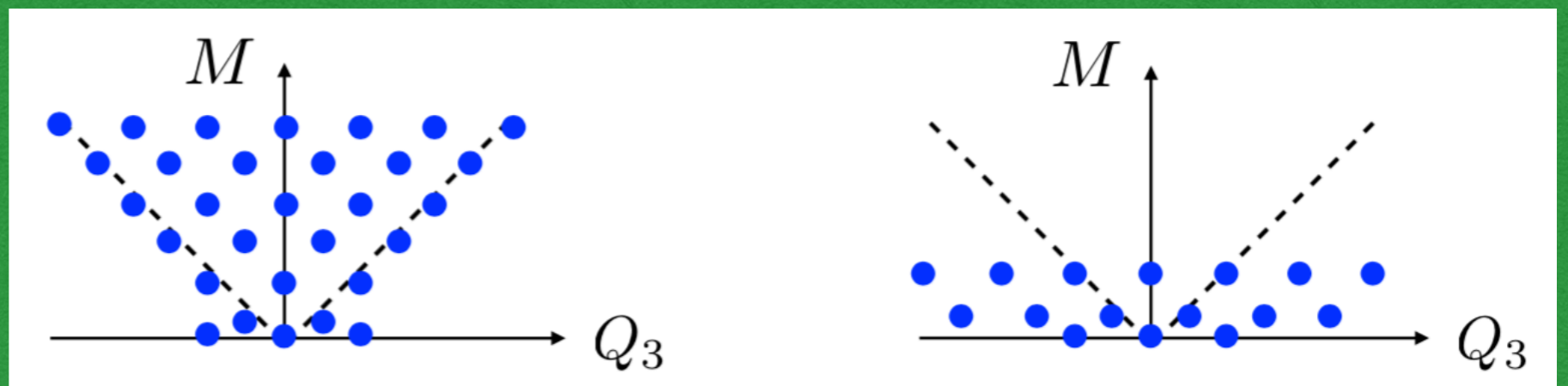
# Nonabelian tower and UV cutoffs

Need to be careful about correct form of nonabelian WGC. *Not* just the abelian WGC applied to the Cartan, in which case we can “skip rungs” in the ladder by putting bigger reps at lower mass. Rather, it’s about *representations* of the nonabelian group.

$J(1)$

**States come in multiplets: linear tower leads to quadratic growth in density of states**

$$gM_{\text{Pl}} \frac{1}{q} = \frac{1}{0},$$



# SU(2) and Emergence

As in the U(1) case, we can integrate out the tower to find corrections to the low-energy gauge coupling:

$$\frac{1}{g^2} = \frac{1}{g_{\text{UV}}^2} + \sum_{j=1/2}^J \frac{I(j)}{16\pi^2} \log \frac{\Lambda}{jgM_{\text{Pl}}}$$

where  $I(j) = \frac{1}{3}j(j+1)(2j+1) \sim j^3$  is the Dynkin index of the spin- $j$  representation. The sum scales like:

$$\frac{1}{g^2} \sim \sum_j j^3 \sim J^4 \sim \left( \frac{\Lambda}{gM_{\text{Pl}}} \right)^4$$

So again with  $\Lambda \sim g^{1/2}M_{\text{Pl}}$  we have the gauge Landau pole and the gravitational cutoff at the **same parametric scale**.

Generalizes to **arbitrary gauge groups**.



# The Converse

Some interesting converse statements are true. Requiring that a gauge theory becomes strongly coupled *at or below* the quantum gravity scale implies at least one parametrically WGC-obeying charged particle exists. Even stronger:

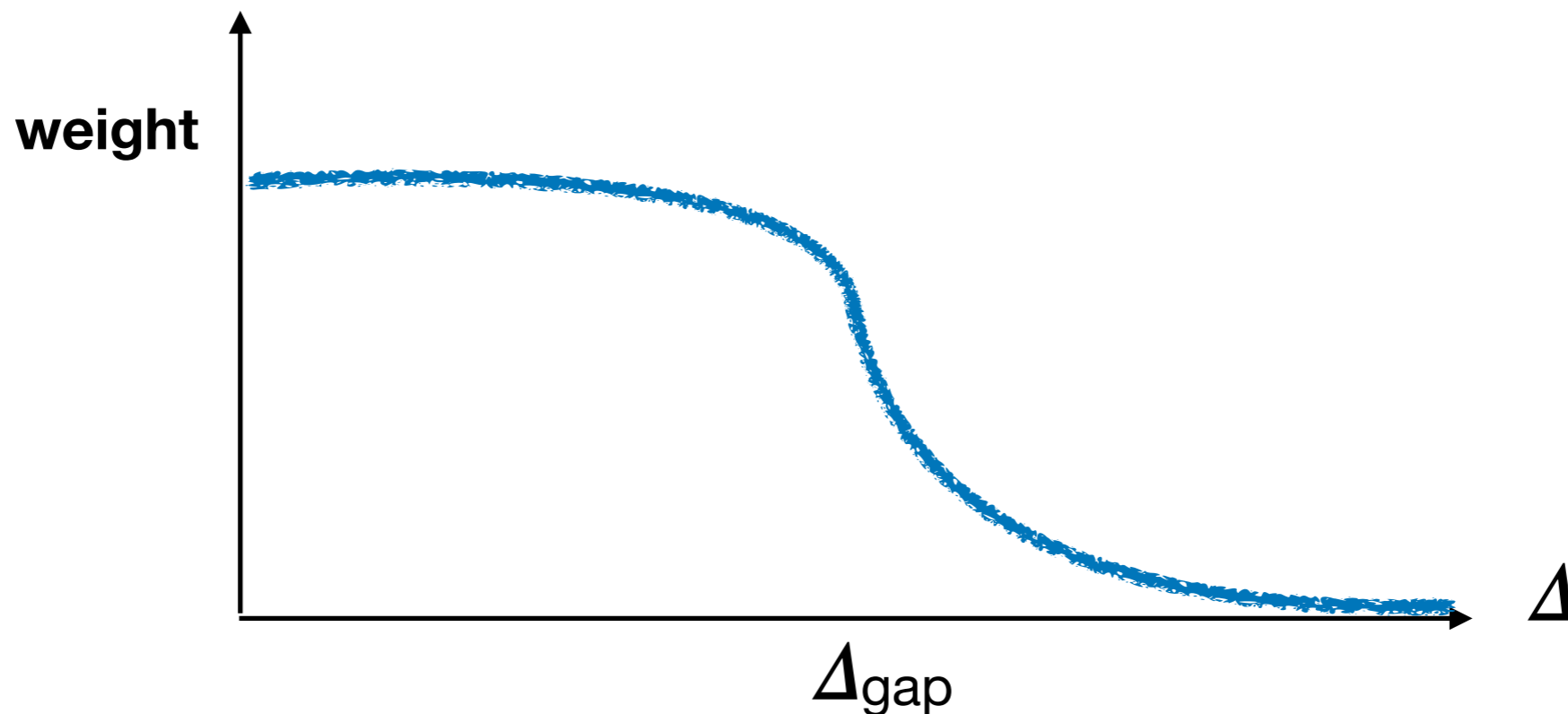
$$\Lambda_{\text{gauge}}^2 \lesssim e^2 \langle q^2 \rangle_{\Lambda_{\text{gauge}}} M_{\text{Pl}}^{D-2}$$

That is, particles below the UV cutoff will, *on average*, obey the WGC bound (parametrically).

This suggests a tight link between the WGC and UV cutoffs. A weak point arises for theories where the charged states are extended objects, and our one-loop estimates are not accurate.

# Wild Speculation / Wishful Thinking

Suppose we had a **sum rule** in CFTs that told us the central charge counts all single-trace operators below  $\Delta_{\text{gap}}$ , the analogue of the string scale or  $\Lambda_{\text{QG}}$ , but counts heavier states in some exponentially penalized way. And suppose that the coefficient  $b$  in the current-current 2-point function similarly counted charged operators proportional to  $q^2$ . This could make our  $\sim$  arguments rigorous, for AdS QG.

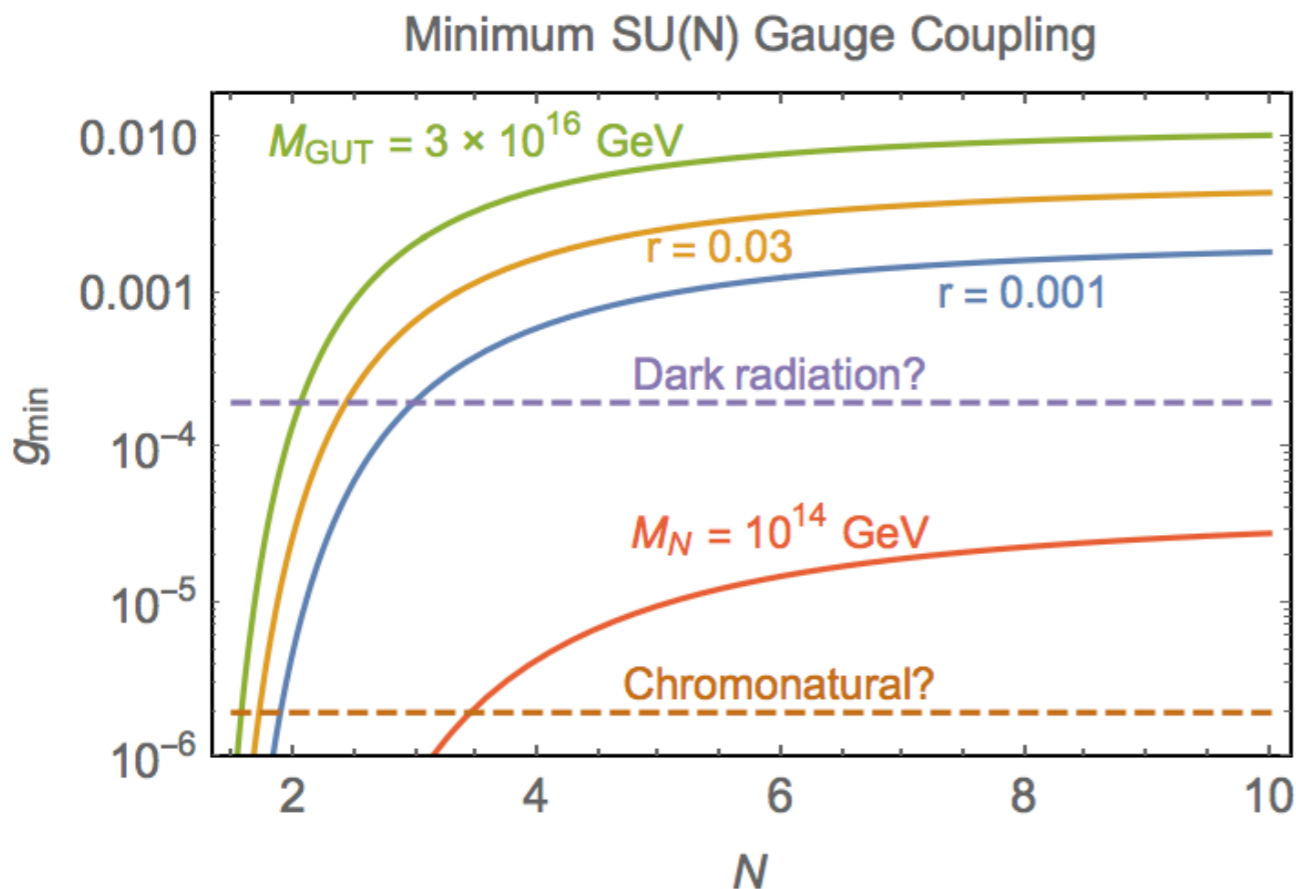


# Applications

Dark matter coupled to **dark radiation** has a detectable imprint on large-scale structure: Buen-Abad, Marques Tavares, Schmaltz 1505.03542; Lesgourgues, ... 1507.04351

$g \sim 2 \times 10^{-4}$  improves fit by  $3\sigma$

We expect small  $r$ ?  
Not *quite* constraining for SU(2).



## Chromo-natural inflation?

(Adshead, Wyman 1202.2366, Martinec, ...)

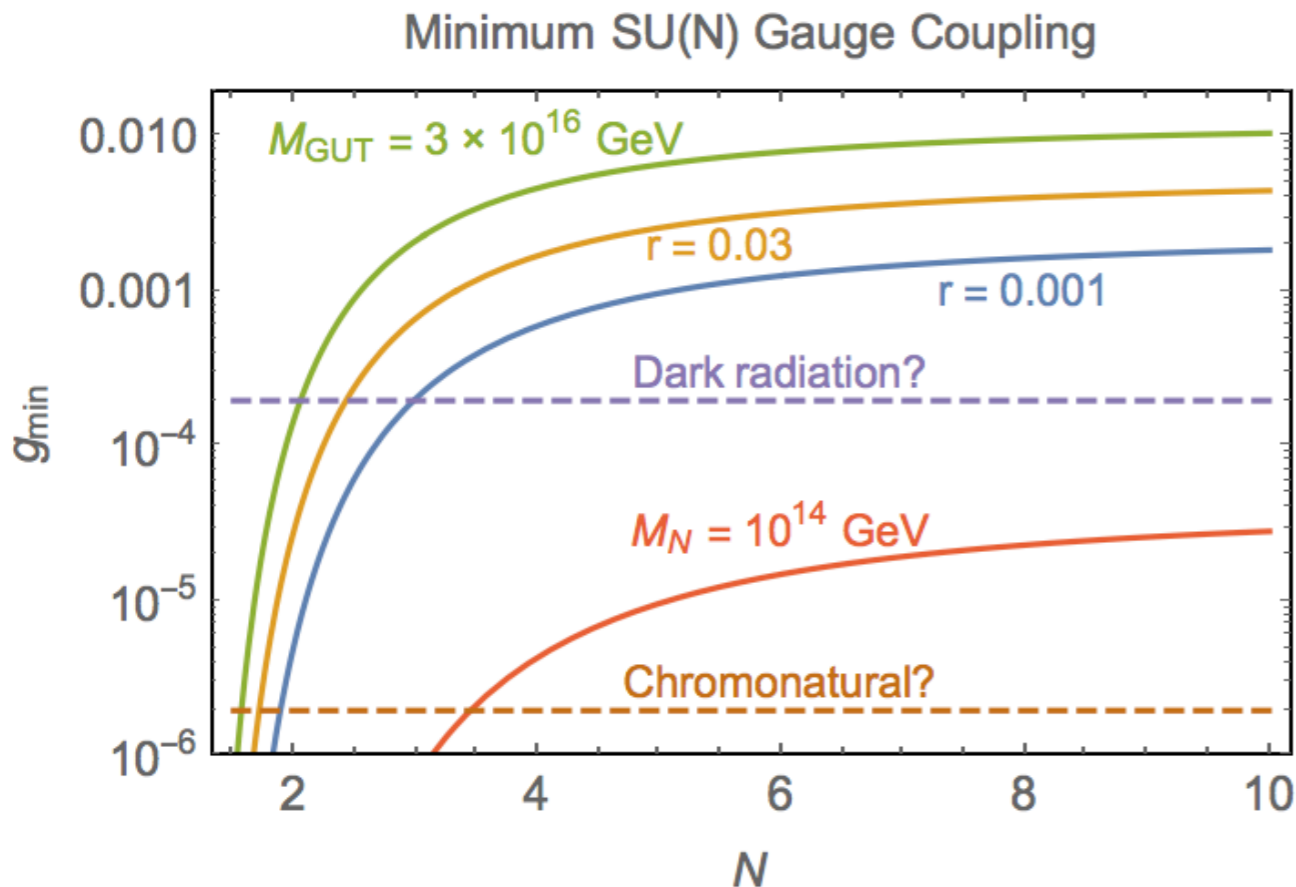
Benchmark  $g \sim 2 \times 10^{-6}$

Marginal with WGC. It has other significant problems.

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Tavares, Schmaltz 1505.03542;

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Suggests our WGC-inspired UV cutoff conjectures could be falsifiable with data.

Other applications?

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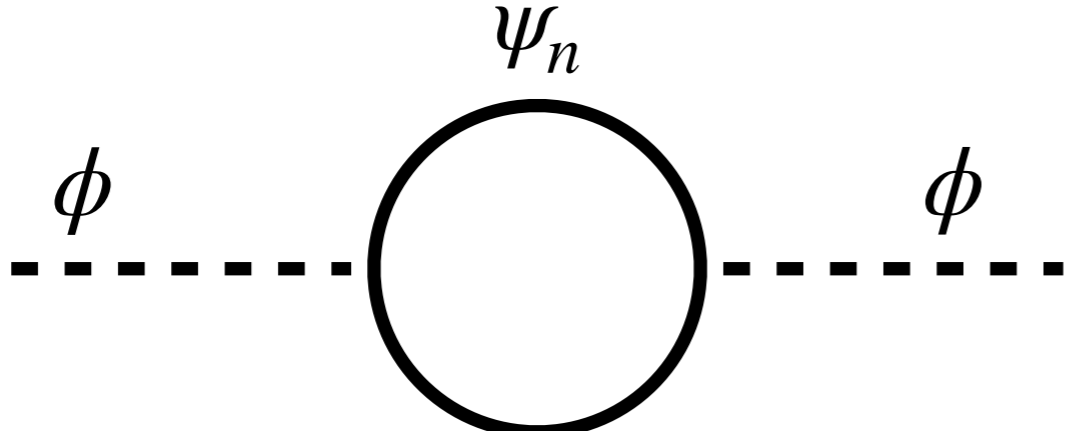
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# Moduli and the Quantum Gravity Scale

Assume fields becoming light at a special point  $\phi = 0$ .

$$\mathcal{L} = \frac{1}{2}K(\phi)(\partial\phi)^2 + \sum_n \bar{\psi}_n(i\partial - m_n(\phi))\psi_n$$

Loops:



$$\sim \frac{1}{K(\phi_0)} \left( \frac{\partial m_n}{\partial \phi} \right)^2$$

Strong coupling at same scale as species bound:

$$K(\phi_0) \sim \sum_{m_n < \Lambda_{\text{QG}}} \left( \frac{\partial m_n}{\partial \phi} \right)^2 \sim \frac{1}{\phi_0^2} \sum_{m_n < \Lambda_{\text{QG}}} m_n^2 \sim \frac{1}{\phi_0^2} N \Lambda_{\text{QG}}^2 \sim \frac{M_{\text{Pl}}^2}{\phi_0^2}$$

# Moduli and the Quantum Gravity Scale

Ooguri/Vafa 2006 *conjectured* towers become light at a rate exponential in field space distance.

Here we see it is an *output* of assuming a universal strong-coupling scale, implying a kinetic term:

$$\mathcal{L} \sim \frac{M_{\text{Pl}}^2}{\phi^2} \partial_\mu \phi \partial^\mu \phi$$

Applying a similar argument to *axion* fields:

$$\langle (\Delta m)^2 \rangle \sim \Lambda_{\text{QG}}^2 \frac{d(\phi)^2}{M_{\text{Pl}}^2}$$

**Super-Planckian field traversals require O(1) fraction of modes to pass through QG cutoff!**



# Moduli and the Quantum Gravity Scale

This is not a sharp no-go theorem, but it does suggest that one should be very careful trusting the validity of EFTs over super-Planckian field ranges in quantum gravity.

One loophole: this refers to the *field-space distance* (geodesic distance), while the *potential* might steer fields along non-geodesic paths.

(See: Hebecker, Henkenjohann, Witkowski '17; Landete, Shiu '18)

**Super-Planckian field traversals require  $O(1)$  fraction of modes to pass through QG cutoff!**



# Stückelberg in the Swampland

In effective field theory we can add masses to abelian gauge bosons and they're harmless. At small enough mass, the longitudinal mode is very weakly coupled.

We can view a photon mass as a *Stückelberg* mass, introducing a Goldstone boson that shifts:

$$\frac{1}{2} f^2 (\partial_\mu \theta - e \hat{A}_\mu)^2$$

In string theory, such masses are ubiquitous. SUSY implies that a radial mode exists. Distinguishing feature is the kinetic term:

$$K(\Phi, \Phi^\dagger, V) = -M^2 \log(\Phi + \Phi^\dagger - cV)$$

# Stückelberg in the Swampland

The point of zero photon mass lies at infinite distance,

$$\text{Re } \Phi \rightarrow \infty, \quad m_V \sim \frac{M^2}{(\Phi + \Phi^\dagger)^2}$$

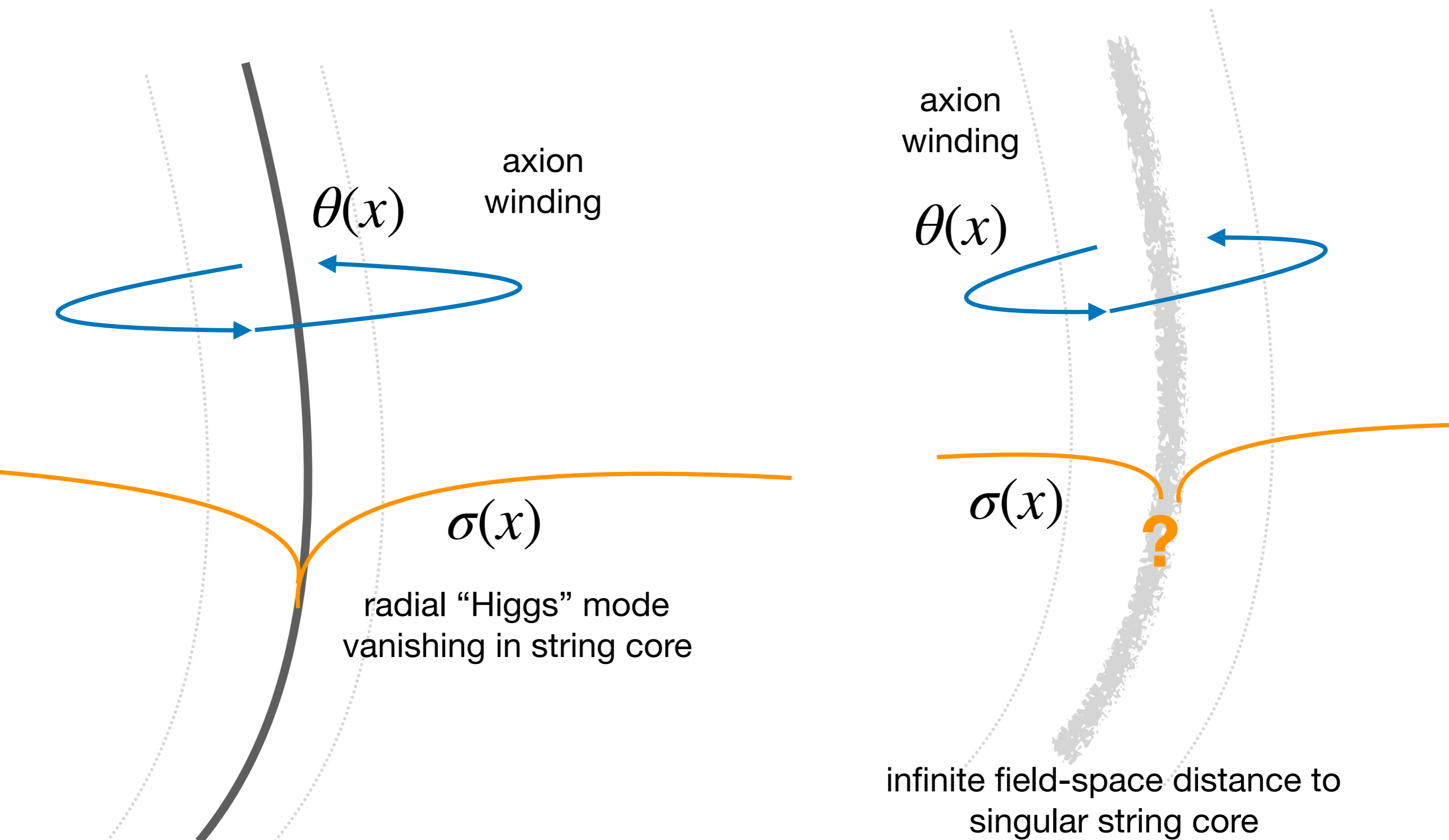
Dualize the eaten Goldstone boson to a 2-form gauge field  $B$ :

$$\epsilon^{\mu\nu\rho\lambda} \partial_{[\mu} B_{\nu\rho]} = f^2 \partial^\lambda \theta$$

Now apply the **WGC** to the  $B$ -field: charged strings exist with tension  $T \lesssim f M_{\text{Pl}}$ . (see Hebecker, Soler '17)

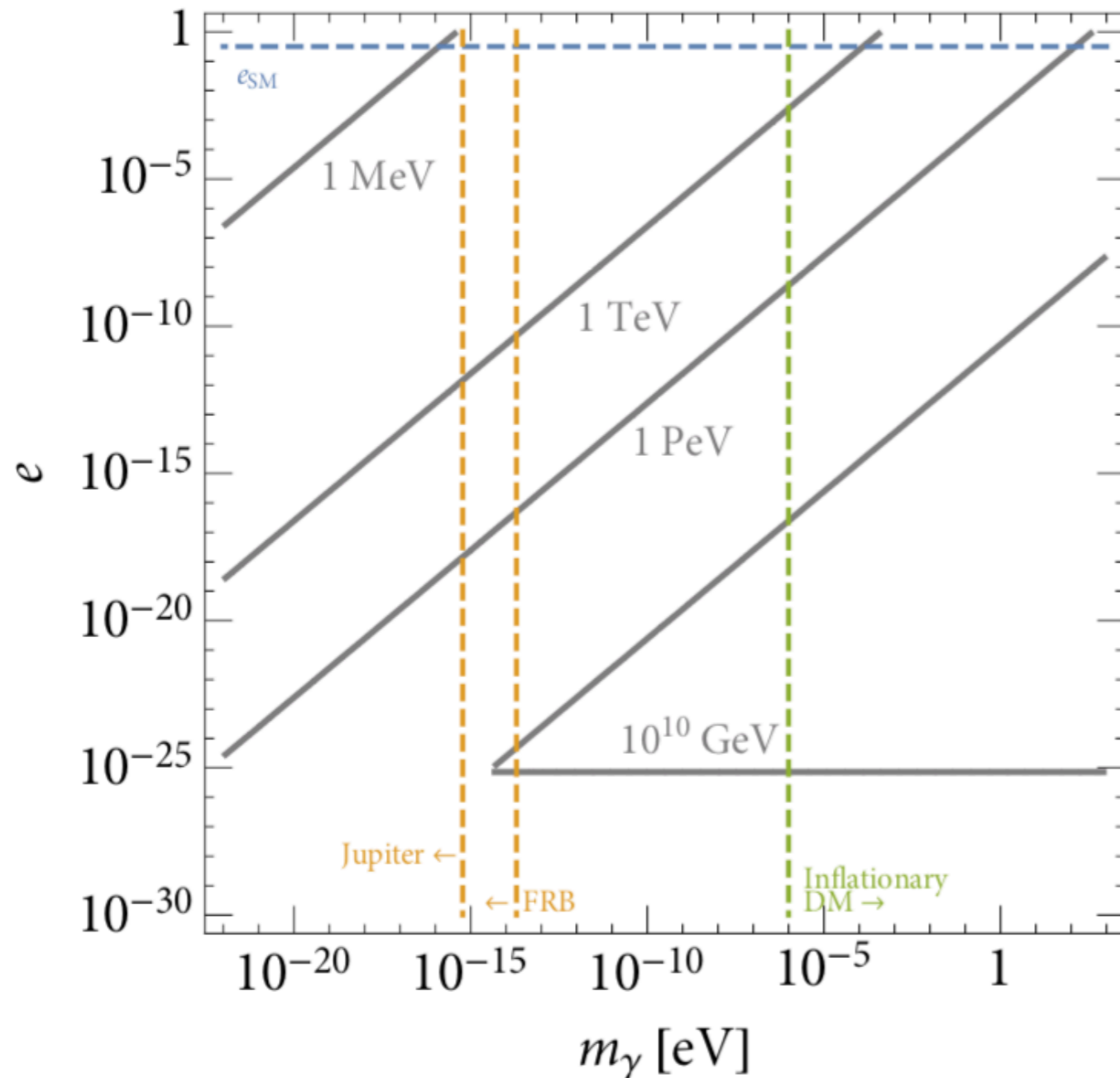
For Stückelberg masses—unlike the Higgs mechanism—these are *fundamental* strings.

# Abelian Higgs strings versus fundamental (Stückelberg) strings



# Ultraviolet cutoffs on Stückelberg photons

Max. UV Cutoff for Stückelberg Theory



$$m_\gamma = ef$$

$e \rightarrow 0 : A_\mu$  weakly coupled

$f \rightarrow 0 : B_{\mu\nu}$  weakly coupled

$$\Lambda_{\text{QGG}} \lesssim \min(e^{1/3} M_{\text{Pl}}, \sqrt{m_\gamma M_{\text{Pl}}/e})$$

the “inflationary DM” line is dark photon dark matter produced by inflationary fluctuations: Graham, Mardon, Rajendran 2015

# Can the photon have a mass?

For the SM photon, very simple kinematic bounds (from fast radio bursts) tell us

$$m_\gamma \lesssim 10^{-14} \text{ eV}$$

A mass at this scale leads to local EFT breaking down at low energies:

$$\Lambda_{\text{QG}} \lesssim \sqrt{m_\gamma M_{\text{Pl}}/e} \lesssim 10 \text{ MeV}$$

So the SM photon can't have a Stückelberg mass.

Loophole is the unit of charge: suppose the electron charge is  $N$ , i.e. what we know as  $e$  is really  $e_0 N$  for  $N \gg 1$ .

We can push the UV cutoff above a TeV if  $N \sim 10^{14}$ .

(Or Higgs mechanism: Higgs is millicharged, similarly huge  $N$ .)

Not very *plausible*, but not logically inconsistent?