Tutorial I

A: The Concept of Phase space

1. Sketch the **emittance ellipse** of a particle beam in horizontal x-x’ phase space (I) at the position of a transverse waist, (II) when the beam is divergent and (III) when the beam is convergent.

2. Phase Space Representation of a Particle Source

Consider a source at s_0 with radius w emitting particles. Make a drawing of this setup in configuration space and in phase space. Which part of phase space can be occupied by the emitted particles?

Any real beam emerging from a source like the one above will be clipped by aperture limitations of the vacuum chamber. This can be modelled by assuming that a distance d away from the source there is an iris with an opening with radius $R = w$. Make a drawing of this setup in configuration and phase space. Which part of phase space is occupied by the beam at a location after the iris?

B: Beam Transport

3. To warm up…: Solve the differential equation of second order

$$y'' - k(s)y = 0$$

Using the Ansatz .

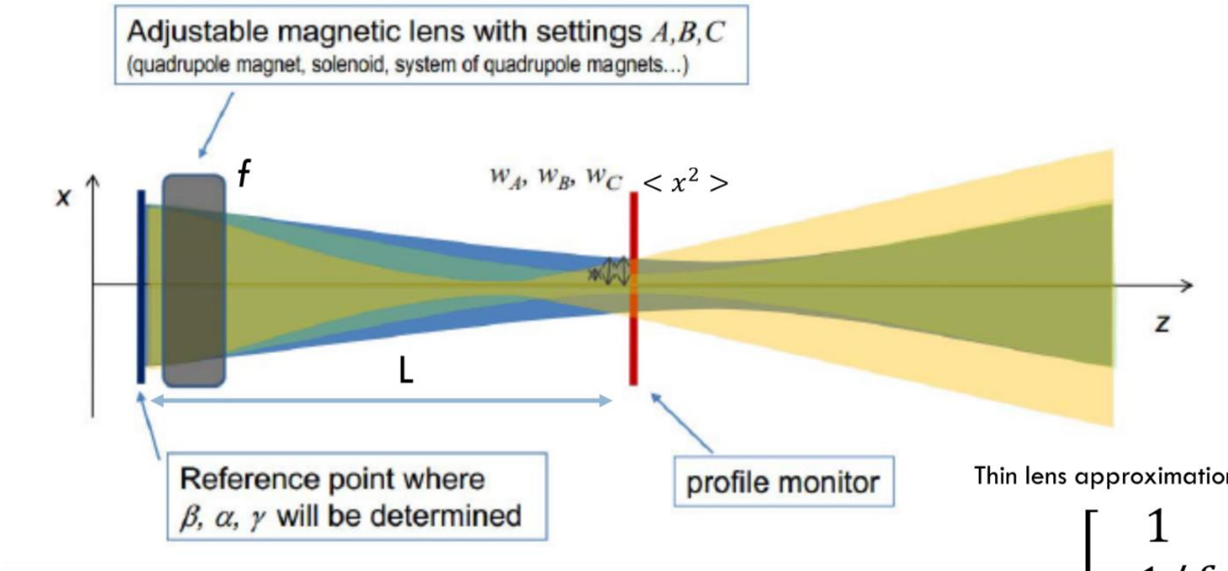
$$y(s) = A_1 e^{w(s)\cdot s} + A_2 e^{-w(s)\cdot s}$$

4. Thin lens approximation

Derive the transport matrix for a quadrupole in the thin lens approximation.

Show that for the drift matrix $R_D\,(L_1+L_2) = R_D\,(L_1) \cdot R_D\,(L_2)$.

5. Quadriple scan



Thin lens approximation

$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

Can you derive the rms emittance?

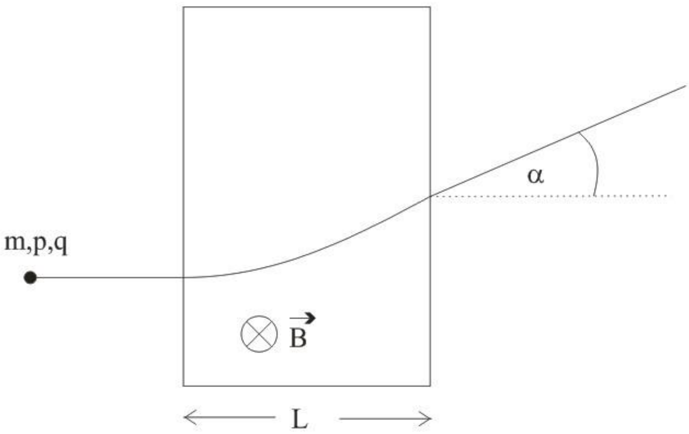
Given $\varepsilon = \sqrt{< x0^2 >< x0'^2 > - < x0x0' >^2}$

You will measure a series of $<x^2>$ in the location of screen.

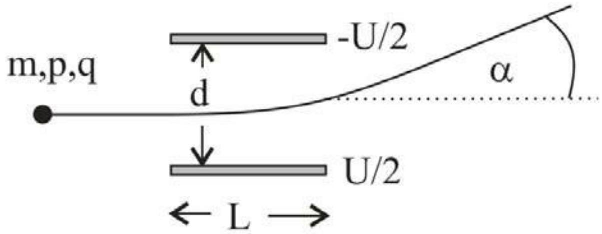
Hint: you need to derive the $<x^2>$ in term of $<x0^2>$, $<x0'^2>$ and $<x0x0'>$.

C: Particle Deflection

6. A low energy particle, $v \ll c$, with mass m, momentum p and charge q enters straight into a perpendicular magnetic field with strength B and length L. Calculate the deflection angle □□.



The same particle now enters the electric field of a parallel plate capacitor with length L.



Calculate the angle of deflection α.

Assume a voltage of *U=30 kV* is applied between the plates which have a length *L=20 cm* and are separated by a distance of *d=8 cm*. Assume furthermore a proton with a kinetic energy of *95 keV*. Show that relativistic effects can be neglected? Calculate the deflection angle α.

D: The real world – some numerical examples

7. Once fully commissioned, the **Large Hadron Collider** (LHC) will accelerate 2,808 bunches with up to 1.15×10^{11} protons per bunch to 7 TeV. The layout of the LHC is approximately circular, with a circumference of 27 km.

i. What is the total energy stored in the beam?

ii. At what speed would a car of mass m=1,500 kg drive if it had a

similar (kinetic) energy?

iii. Calculate the strength of the dipole magnets in LHC, assuming the machine is a perfect circle.

8. The **Large Electron Positron** (LEP) **collider**

The main dipole magnets of the Large Electron Positron (LEP) collider had a bending radius of 3096 m.

What was their magnetic field when LEP accelerated electrons to an energy of 105 GeV?

This field is relatively small; why was the field not increased to increase the energy ?

The LEP tunnel was about 27 km long. What fraction of it was used for bending the beam ?

LEP produced about 20 MW of synchrotron radiation when it stored electrons at an energy of 100 GeV. How much would the same number of electrons have radiated at 200 GeV ?

F: Beam Diagnostics

9. **Describe, with the aid of diagrams where appropriate, the operation principle of the ionization chamber.**

10. **Phosphor screens are routinely used to measure particle displacements x. It is more difficult to measure a particle trajectory's angle** x_0**'.**

Assume a drift section with length L. Set up the transport matrix for this drift and show how the initial angle x_0' can be derived from the displacements x_0 and x.

Now assume there is no way to measure the initial displacement, but a quadrupole doublet (focal length f and distance L between the quadrupoles) with properly tuned focusing can be used. Set up the transport matrix for this doublet in the thin lens approximation. Can we use this to measure the initial angle x_0'?

Assume electrons with a kinetic energy of *E=650 MeV* and a distance L= *931 mm* between the quadrupoles of the doublet. What focusing gradient is now required to measure the initial angle x_0'?