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# Higgsplosion

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In collaboration with Michael Spannowsky

- Before the Higgs discovery, massive Yang-Mills theory violated perturbative unitarity — problem with high-energy growth of 2 -> 2 processes

- Discovery of the (elementary) Higgs made the SM theory self-consistent

- The Higgs brings in the Hierarchy problem: radiative corrections push the Higgs mass to the new physics (high) scale: $m_h^2 \simeq m_0^2 + \delta m_{\rm new}^2$

- In this talk: consider n~100s of Higgs bosons produced in the final state n lambda >> 1. Investigate scattering processes at ~ 100 TeV energies.

- HIGGSPLOSION: n-particle rates computed in a weakly-coupled theory can become unsuppressed above critical values of n and E. Perturbative and non-perturbative semi-classical calculations. n! ~ exponential growth with n or E. (Scale n~E/m).

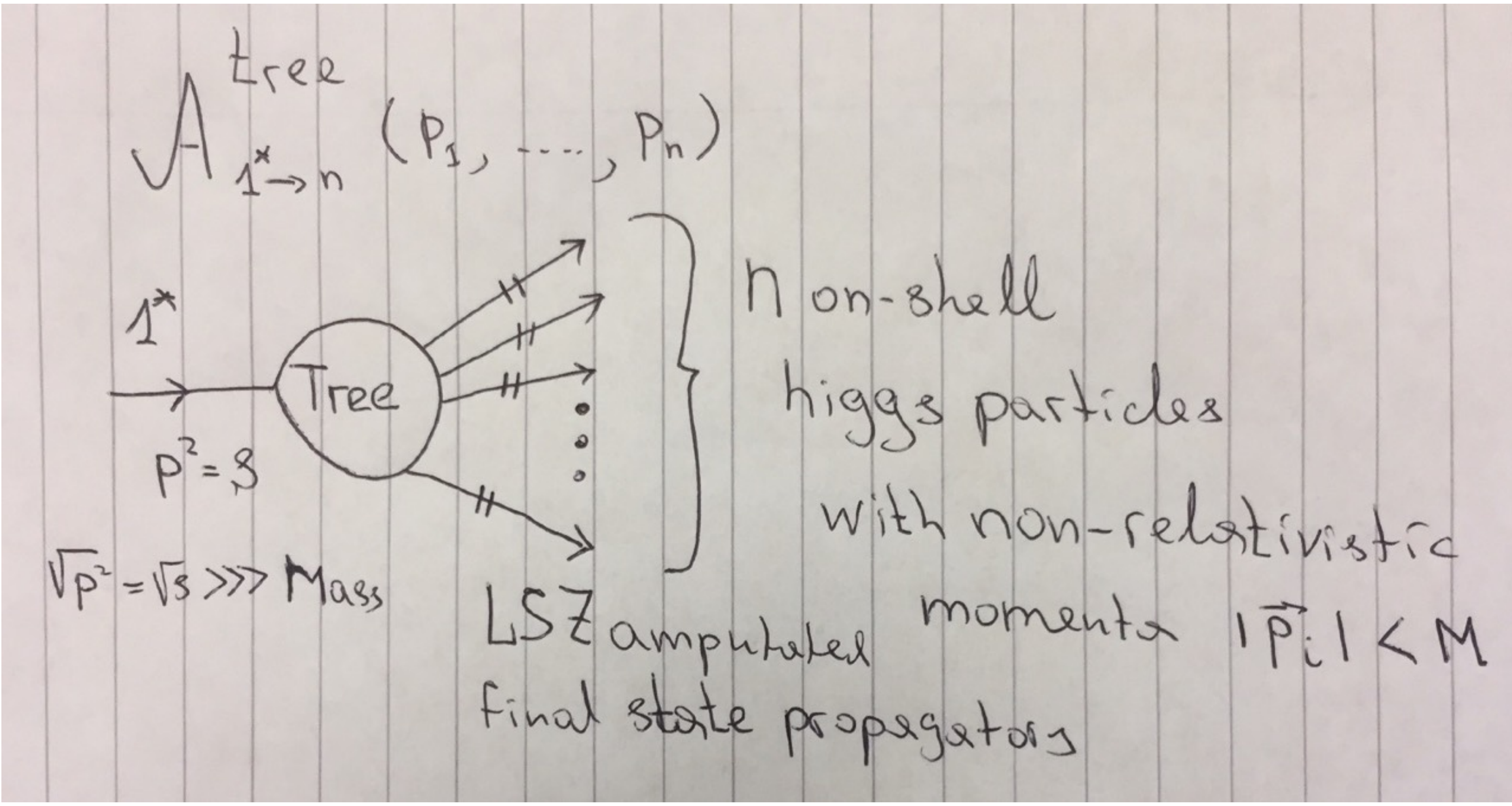
- A new unitarity problem — caused by the elementary Higgs bosons — appears to occur (?) for processes with large final state multiplicities n >> 1

- HIGGSPLOSION offers a solution to both problems: it restores the unitarity of high-multiplicity processes and dynamically cuts off the values of the loop momenta contributing to the radiative corrections to the Higgs mass.

# Organisation of the talk:

- Main idea and simple expressions for n-point amplitudes and rates
- Interpretation of tree-level results;
  quantum effects: loops and semiclassical methods.
- Higgsplosion at at e-e- and photon photon colliders
- => Summary

# Compute 1 -> n amplitudes @LO with non-relativistic final-state momenta:




see classic 1992-1994 papers:
Brown; Voloshin;
Argyres, Kleiss, Papodopoulos
Libanov, Rubakov, Son, Troitski

more recently: VVK 1411.2925

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2$$

prototype of the SM Higgs
in the unitary gauge

Tree-level $1^* \to n$ amplitudes in the limit $\varepsilon \to 0$ for any $n$ are given by

$$\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \left(1 - \frac{7}{6} n\varepsilon - \frac{1}{6}\frac{n}{n-1}\varepsilon + \mathcal{O}(\varepsilon^2)\right)$$

factorial growth

amplitude on the n-particle threshold

$$\varepsilon = \frac{1}{n\, M_h} E_n^{\rm kin} = \frac{1}{n}\frac{1}{2M_h^2}\sum_{i=1}^{n} \vec{p}_i^{\,2}$$

kinetic energy per particle per mass

In the large-$n$-non-relativistic limit the result is

$$\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \exp\left[-\frac{7}{6}n\varepsilon\right], \quad n \to \infty,\ \varepsilon \to 0,\ n\varepsilon = \text{fixed}$$

## Can now integrate over the n-particle phase-space

The cross-section and/or the $n$-particle partial decay $\Gamma_n$

$$\Gamma_n(s) \;=\; \int d\Phi_n \, \frac{1}{n!} \, \left|\mathcal{A}_{h^* \to n\times h}\right|^2$$

The $n$-particle Lorentz-invariant phase space volume element

$$\int d\Phi_n \;=\; (2\pi)^4 \delta^{(4)}(P_{\rm in} - \sum_{j=1}^{n} p_j) \, \prod_{j=1}^{n} \int \frac{d^3 p_j}{(2\pi)^3 \, 2p_j^0} \,,$$

in the large-$n$ non-relativistic limit with $n\varepsilon_h$ fixed becomes,

$$\Phi_n \;\simeq\; \frac{1}{\sqrt{n}} \left(\frac{M_h^2}{2}\right)^n \, \exp\left[\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi}+1\right) + \frac{n\varepsilon_h}{4} + \mathcal{O}(n\varepsilon_h^2)\right]$$

We find:

$$\Gamma_n^{\rm tree}(s) \;\sim\; \exp\left[n\left(\log\frac{\lambda n}{4}-1\right) + \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) - \frac{25}{12}\, n\varepsilon + \mathcal{O}(n\varepsilon^2)\right]$$

$n \gg 1, \; \varepsilon \ll 1$

Son 1995;
Libanov, Rubakov, Troitskii 1997; more recently: VVK 1411.2925

# Now **include *loop corrections*** in the exponent of Higgsplosion

The 1-loop corrected threshold amplitude for the pure $n$ Higgs production:

$$\phi^4 \text{ with SSB}: \quad \mathcal{A}_{1\to n}^{\text{tree+1loop}} = n!\,(2v)^{1-n}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

*Libanov, Rubakov, Son, Troitsky 1994*

$$\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\text{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]$$

in the limit $\lambda \to 0$, $n \to \infty$ with $\lambda n^2$ fixed. Here $B$ is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*):

$$B = +\lambda n\,\frac{\sqrt{3}}{4\pi}$$

# **Semi-classical approach** for computing Higgsplosion rate

- DT Son1995

Multi-particle decay rates $\Gamma_n$ can also be computed using an alternative semi-classical method. This is an intrinsically non-perturbative approach, with no reference in its outset made to perturbation theory.

The path integral is computed in the steepest descent method, controlled by two large parameters, $1/\lambda \to \infty$ and $n \to \infty$.

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.$$

The semi-classical computation in the regime where,

$$\lambda n = \text{fixed} \ll 1\,, \quad \varepsilon = \text{fixed} \ll 1\,,$$

reproduces the tree-level perturbative results for non-relativistic final states.

Remarkably, this semi-classical calculation also reproduces the leading-order quantum corrections arising from resumming one-loop effects.

# **Semi-classical approach** for computing Higgsplosion rate

$$\Gamma_n(s) \;\propto \mathcal{R}(\lambda; n, \varepsilon)$$

The semiclassical approach is equally applicable and more relevant to the realisation of the non-perturbative Higgsplosion case where,

$$\lambda n = \text{fixed} \gg 1\,, \quad \varepsilon = \text{fixed} \ll 1\,.$$

This calculation was carried out for the spontaneously broken theory with the result given by,

- VVK 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) \;=\; \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2}\log\frac{\varepsilon}{3\pi} - \frac{25}{12}\,\varepsilon\right)\right],$$

Higher order corrections are suppressed by $\mathcal{O}(1/\sqrt{\lambda n})$ and powers of $\varepsilon$.

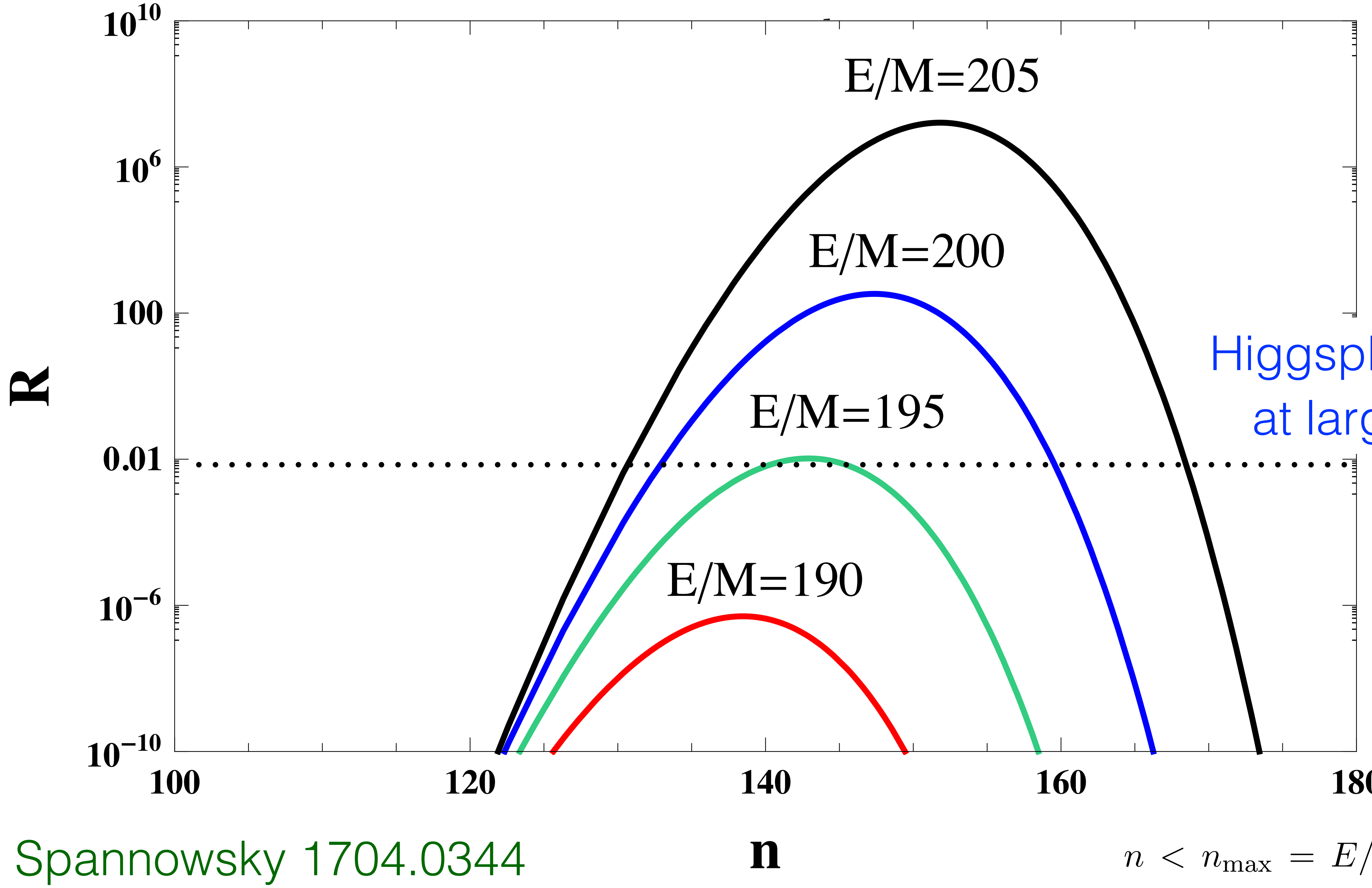
# Thus we have computed the rate R in the large lambda n limit:

using the semi-classical approach and the thin-wall approximation

VVK 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) = \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2}\log\frac{\varepsilon}{3\pi} - \frac{25}{12}\varepsilon\right)\right]$$

$\lambda n \gg 1 \quad$ small $\varepsilon$




Higgsplosion realised at large lambda n

VVK & Spannowsky 1704.0344

$n < n_{\max} = E/M_h$

*Intuitive reason for Higgsplosion:*

**Higgsplosion** ⟷ constructive interference between n! diagrams for h*-> nh elementary scalars in a spontaneously broken QFT

Why not observed somewhere else before?

Higgs lacks symmetry to prevent Higgsplosion:

- Gauge fields (gauge symmetry)

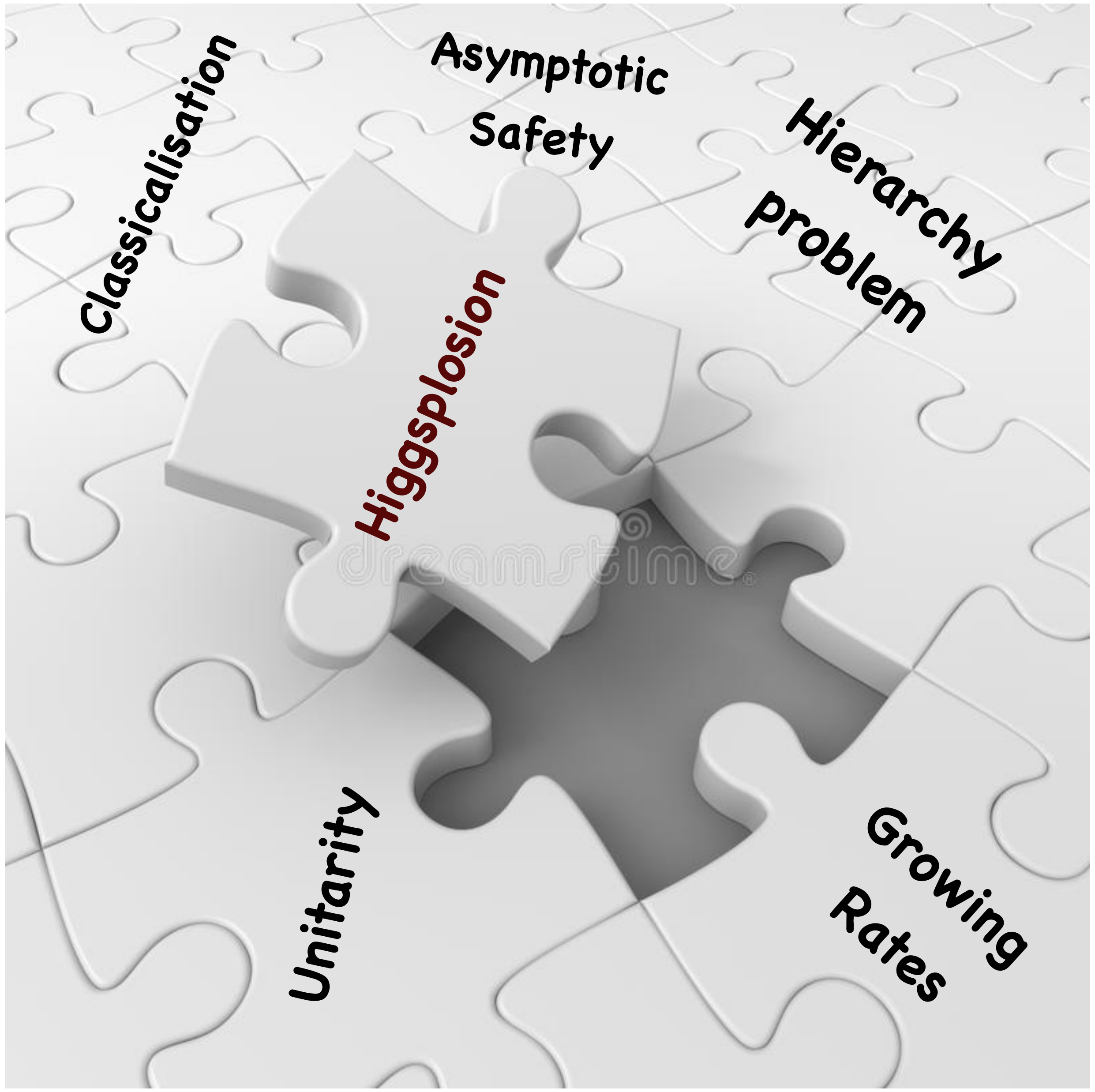
  e.g. simple Parke-Taylor '86 amplitudes

- Fermions (Pauli principle)

  zero at threshold and phase space scales 1/N!

Integrable systems in 1+1 dim or even 2+1 dimensional scalar QFTs:
The cases in <4 dimensions are special - different IR dynamics of quantum loops

Quantum Mechanics: too simple 0+1 dims

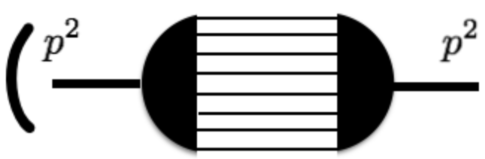



Our current understanding: **Need spontaneously broken scalar QFT in at least 3+1**

# HIGGSPLOSION and HIGGSPERSION

The optical theorem now relates the 1* -> nh amplitudes with the imaginary part of the self-energy (valid to all orders)

$$-\,\mathrm{Im}\,\Sigma_R(p^2) \;=\; m\,\Gamma(p^2)$$

- Im ( 

)

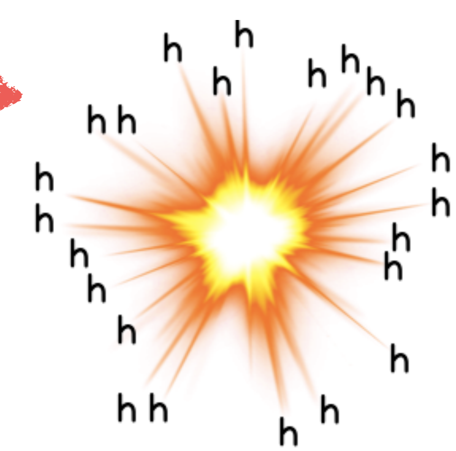
where $\Gamma(s) \;=\; \sum_{n=2}^{\infty} \Gamma_n(s)$ and $\Gamma_n(s) \;=\; \frac{1}{2m}\int \frac{d\Phi_n}{n!}\,|\mathcal{M}(1\to n)|^2$

and thus

$$\Delta_R(p) \;=\; \frac{i}{p^2 - m^2 \;-\; \mathrm{Re}\,\Sigma_R(p^2) \;+\; im\Gamma(p^2) \;+ i\epsilon}$$

No information as perturbation theory breaks down for many loops, but not possible to cancel imaginary part

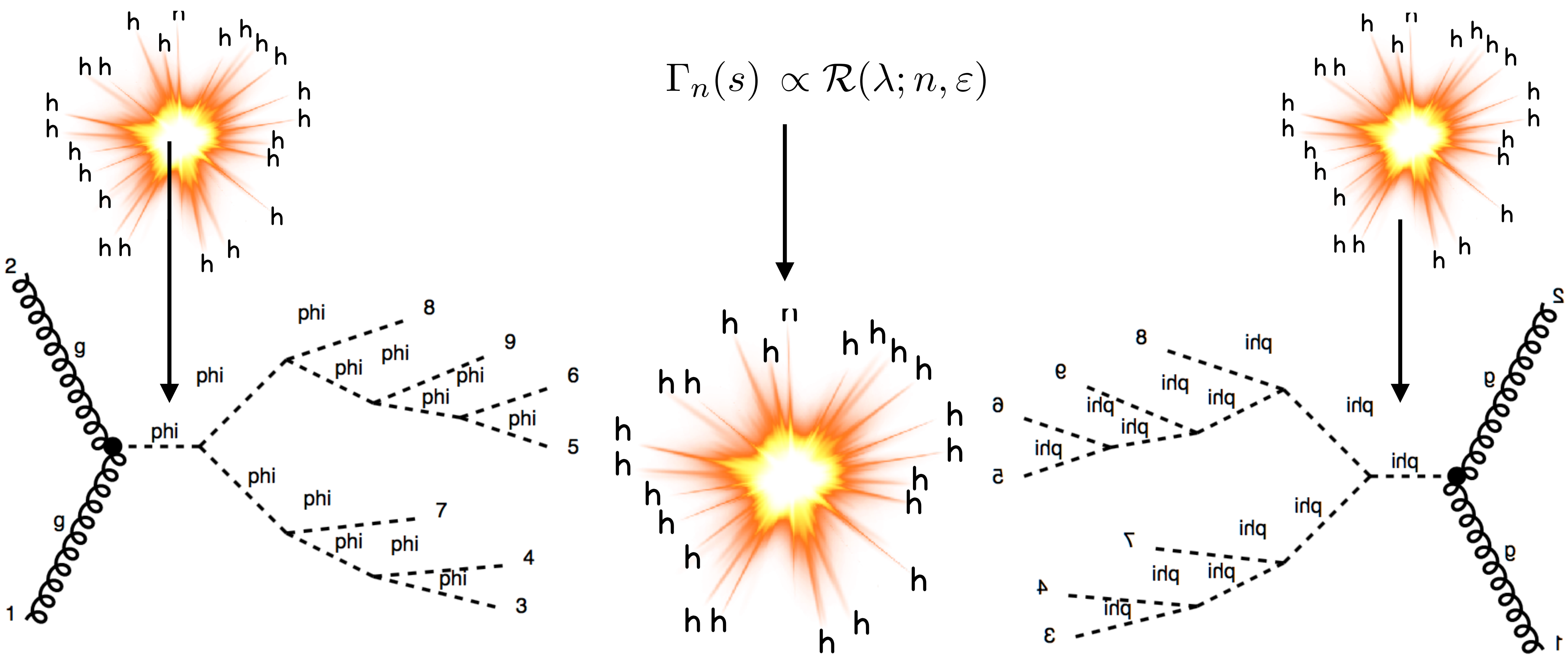
Higgsplodes



# HIGGSPLOSION and HIGGSPERSION

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - m_h^2 - Re\tilde{\Sigma}(p^2) + i m_h \Gamma(p^2)} \times \mathcal{M}_{h^*\to n\times h}$$

Include self-energy




$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 \frac{m_t^2}{m_h} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{(s - Re\Sigma(s))^2 + m_h^2\Gamma^2(s)} \times \Gamma_n(s)$$

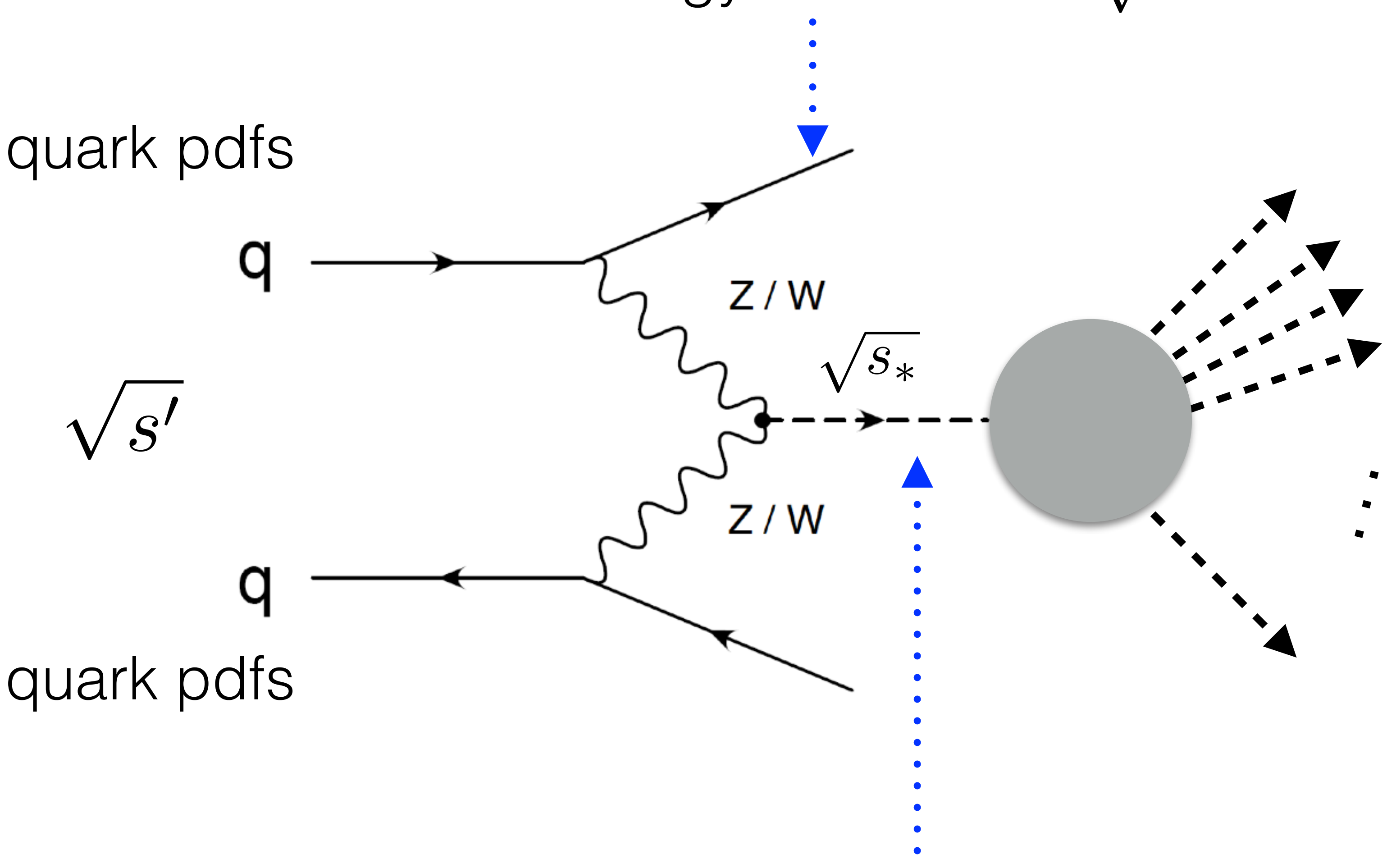
VVK & Spannowsky 1704.0344

- Unitarity restored!

# Prospects of *direct* observation of Higgslposion

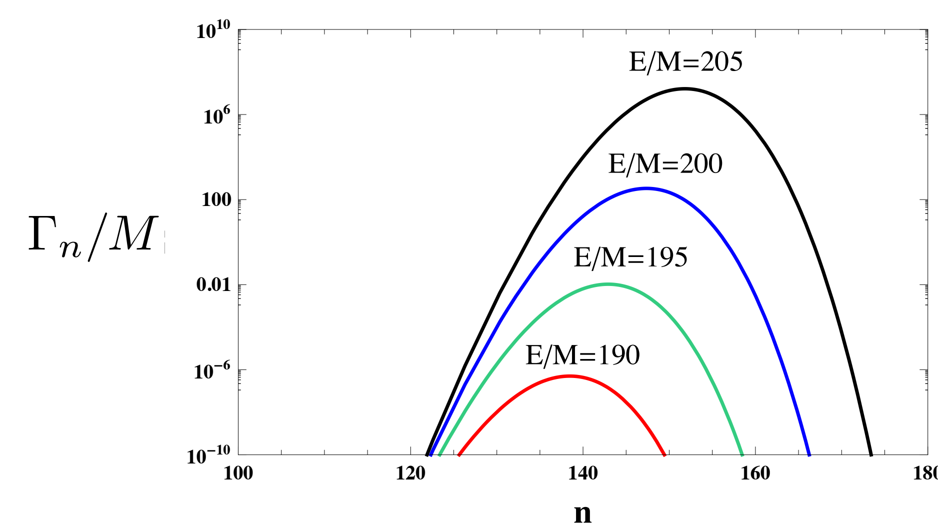
## Vector boson fusion at high-energy pp colliders (FCC)

energy excess over $\sqrt{s_*}$ carried away by jets



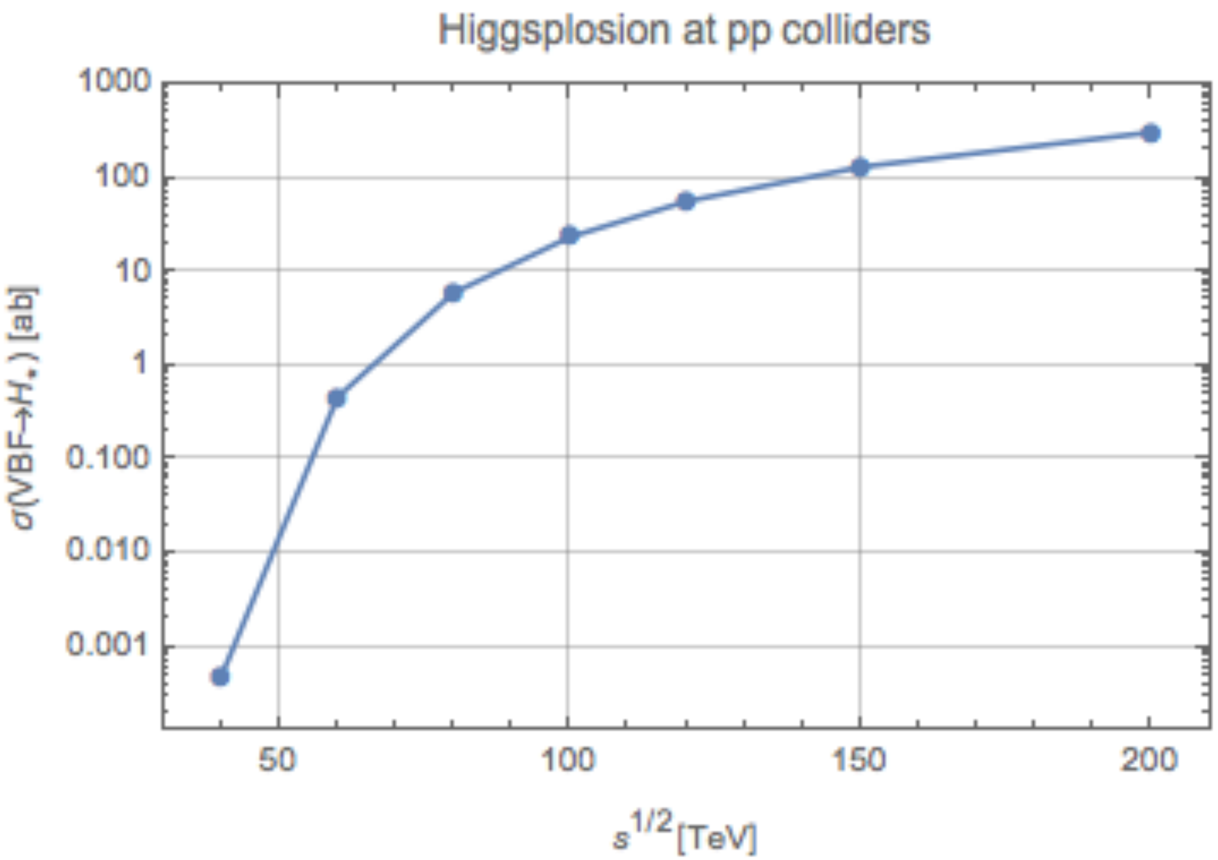

n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$

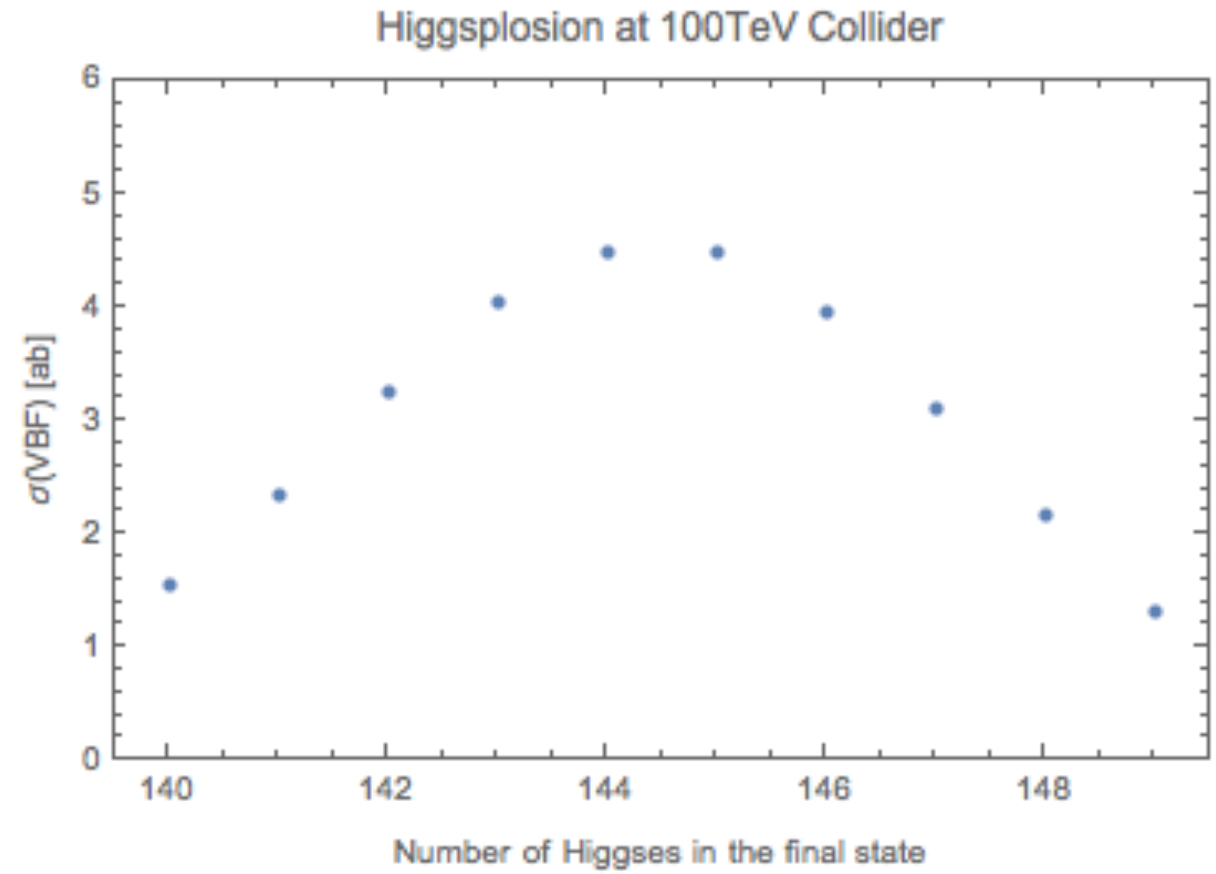
$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$




Propagator with Higgspersion at $\sqrt{s_*}$

# Vector boson fusion at high-energy pp colliders (FCC)

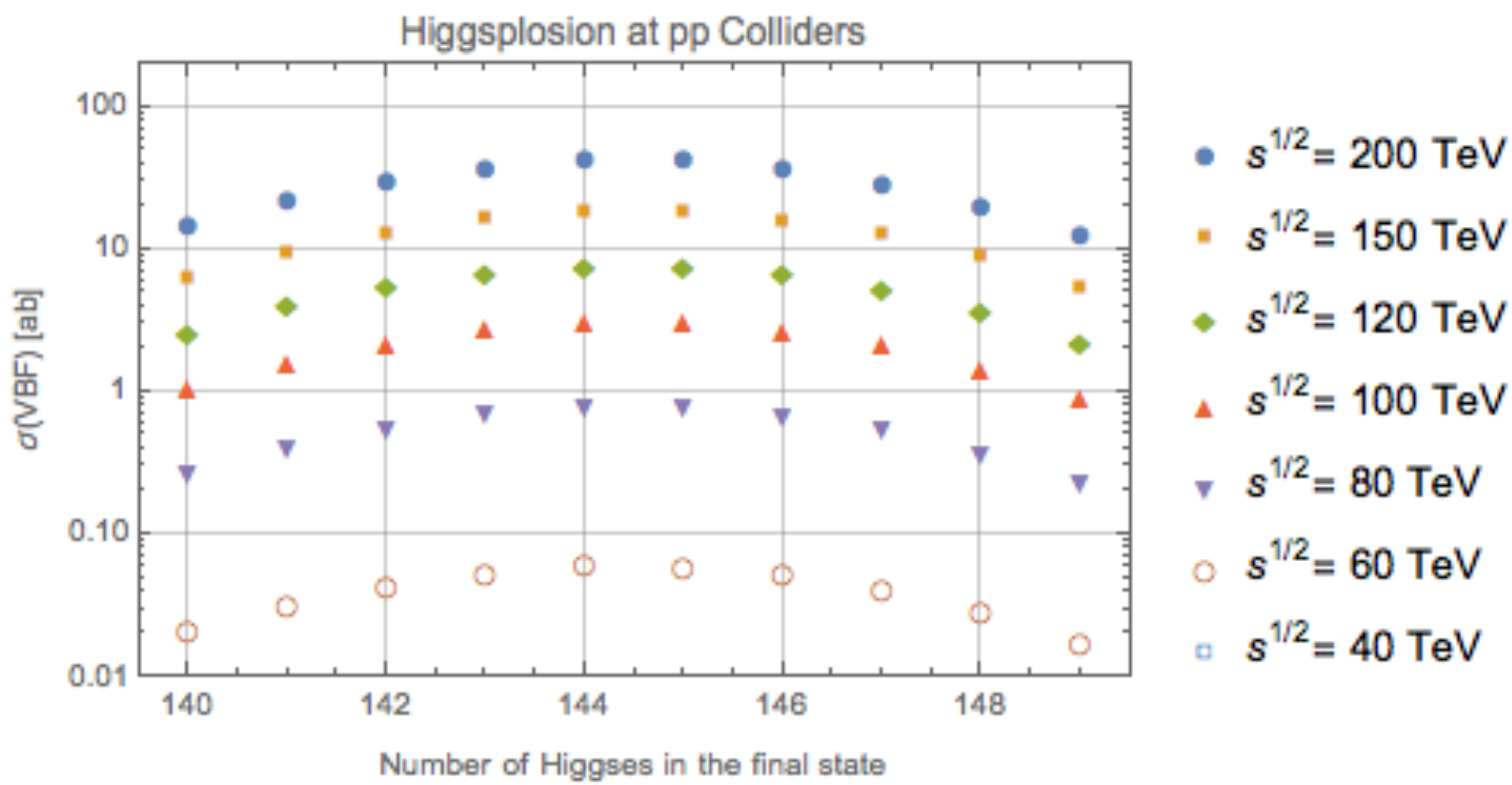



using $p_{t\,\mathrm{jet}} > 40$ GeV

VVK, J Scholtz, M Spannowsky

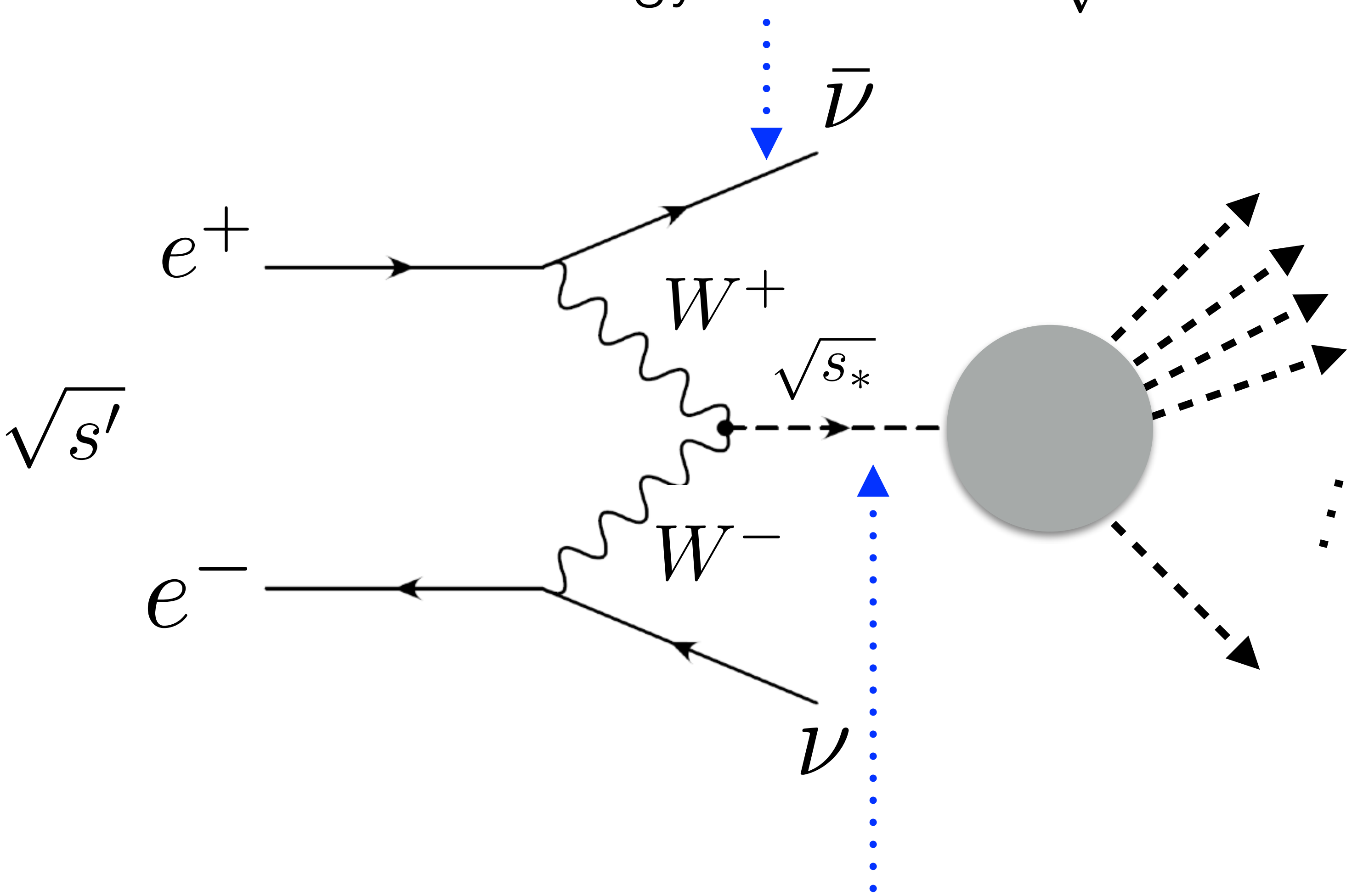
preliminary: no Higgs decays into SM d.o.f included;
& no vector bosons in final states



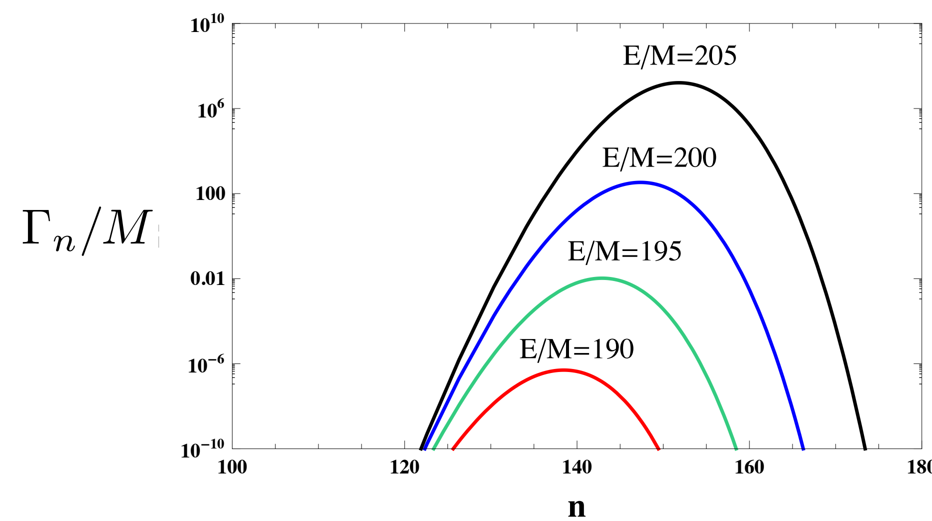
# Prospects of *direct* observation of Higgslposion

## Vector boson fusion at high-energy **e+e-** colliders: ALEGRO

energy excess over $\sqrt{s_*}$ carried away by jets




n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$




$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$

Propagator with Higgspersion at $\sqrt{s_*}$

# Vector boson fusion at high-energy **e+e-** colliders: ALEGRO

More generally should include multiple longitudinal vector boson production into Higgsplosion processes:

$$e^+e^-{}_{\rm VBF} \to h + \bar{\nu}\nu \to n \times h + \bar{\nu}\nu$$

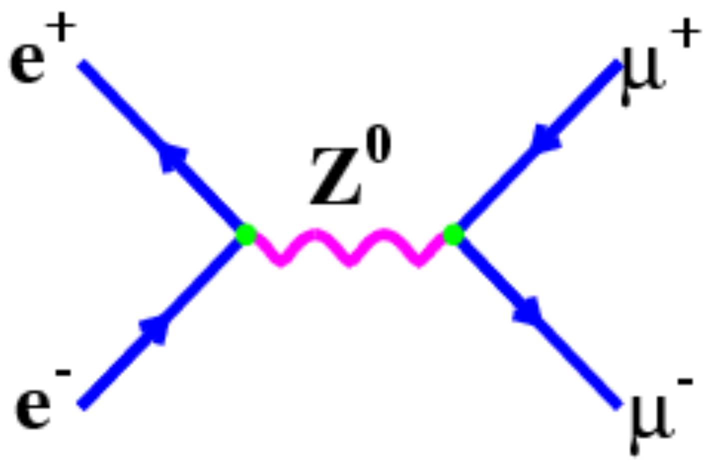
$$e^+e^-{}_{\rm VBF} \to Z^0 + \bar{\nu}\nu \to Z^0 + n \times h\ \bar{\nu}\nu$$

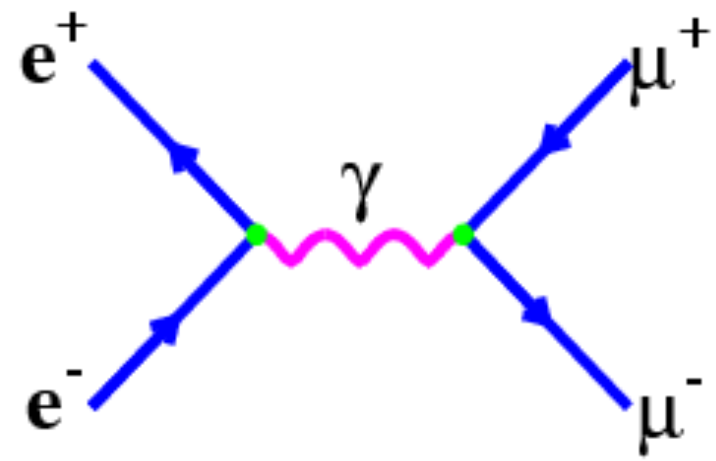
$$e^+e^-{}_{\rm VBF} \to Z^0 + \bar{\nu}\nu \to Z^0 + n_h \times h + n' \times \bar{V}_L V_L + \bar{\nu}\nu$$

Where any of the Vector boson pairs are $\bar{V}_L V_L = W^+ W-$ or $\bar{V}_L V_L = Z^0 Z^0$. This should be the dominant process.

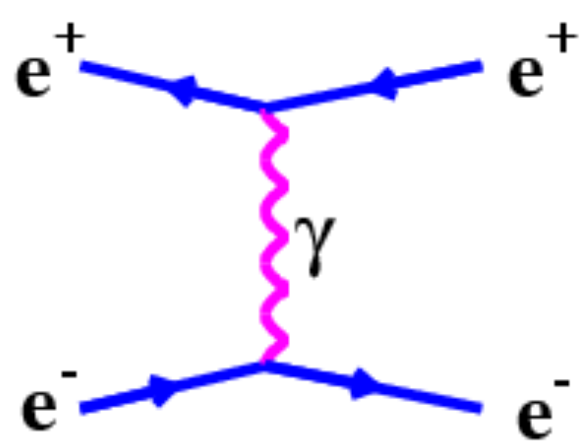
## One can also look for the disappearance of the **e+e- -> mu+ mu-** rates above E*

Higgspersion for **exclusive** 2 to 2 s-chanel processes




But not for:



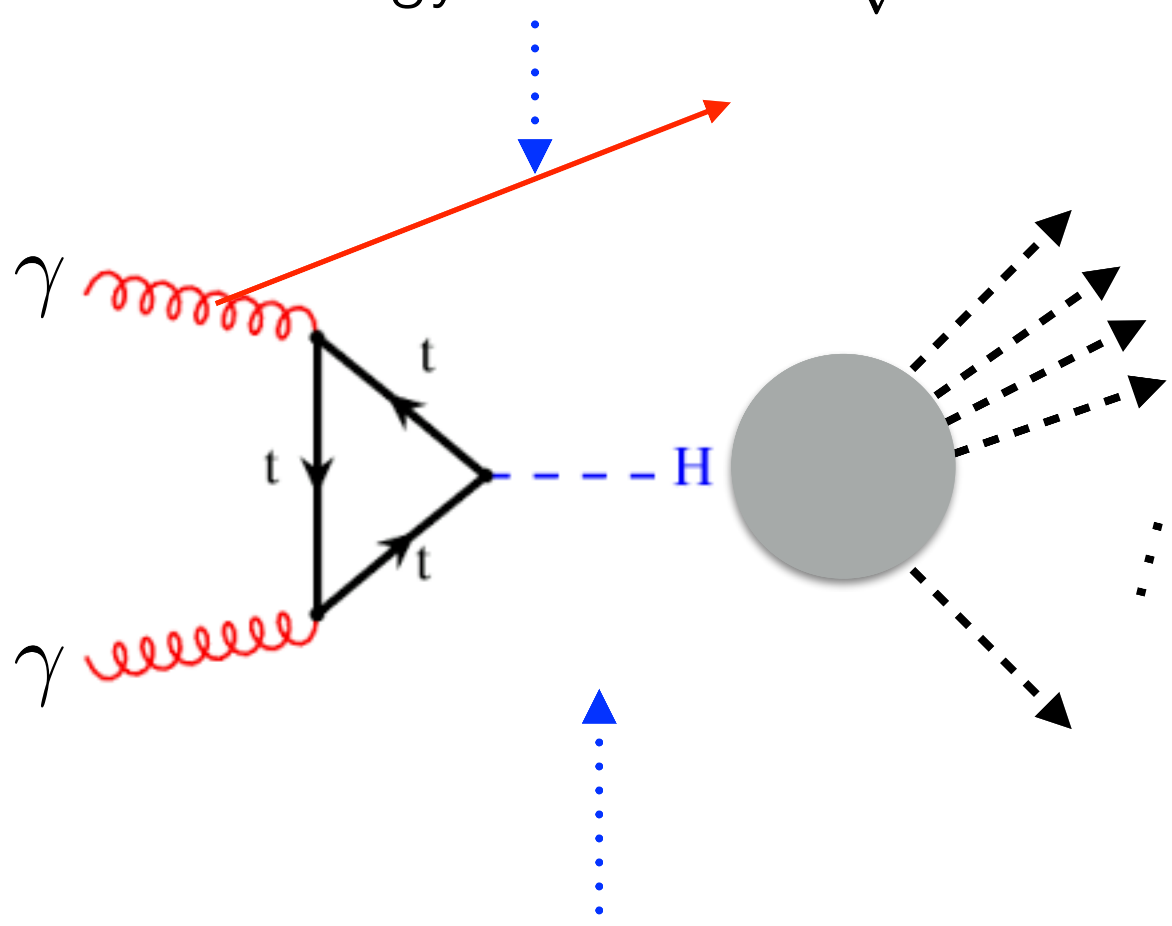

$$\frac{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{\sigma(e^+e^- \rightarrow e^+e^-)}$$

disappearing at $\sqrt{s} > E_*$

# Photon fusion at **photon-photon** colliders: ALEGRO

energy excess over $\sqrt{s_*}$ carried away by jets

$\sqrt{s'}$




n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$

$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$

Propagator with Higgspersion at $\sqrt{s_*}$

# Summary of the main idea

A conventional wisdom: in the description of nature based on a local QFT, one should always be able to probe shorter and shorter distances with higher and higher energies.

Higgsplosion is a dynamical mechanism, or a new phase of the theory, which presents an obstacle to this principle at energies above $E_*$.

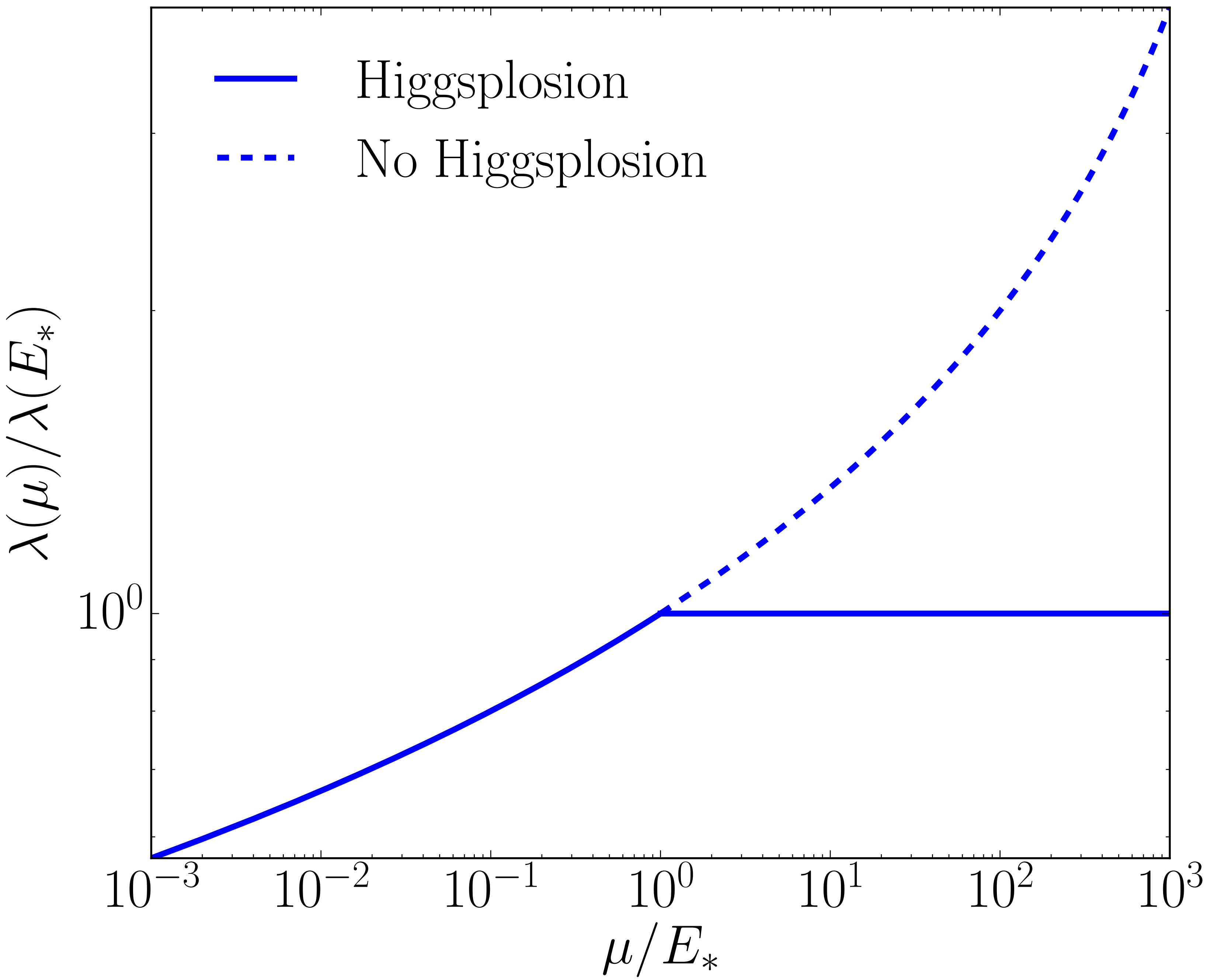
$E_*$ is the new dynamical scale of the theory, where multi-particle decay rates become unsuppressed.

Schematically, $E_* = C \frac{m}{\lambda}$, where $C$ is a model-dependent constant of $\mathcal{O}(100)$. This expression holds in the weak-coupling limit $\lambda \to 0$.

E* ~ 25 -100 TeV

# Asymptotic Safety

For all parameters of the theory (running coupling constants, masses, etc):




UV fixed point

# Higgsplosion

At energy scales above $E_*$ the dynamics of the system is changed:

1. Distance scales below $|x| \lesssim 1/E_*$ cannot be resolved in interactions;
2. UV divergences are regulated;
3. The theory becomes asymptotically safe;
4. And the Hierarchy problem of the Standard Model is therefore absent.

Consider the scaling behaviour of the propagator of a massive scalar particle

$$\Delta(x) \;:=\; \langle 0|T(\phi(x)\,\phi(0))|0\rangle \;\sim\; \begin{cases} m^2\, e^{-m|x|} & : \;\text{for } |x| \gg 1/m \\ 1/|x|^2 & : \;\text{for } 1/E_* \ll |x| \ll 1/m \\ E_*^2 & : \;\text{for } |x| \lesssim 1/E_* \end{cases} ,$$

where for $|x| \lesssim 1/E_*$ one enters the Higgsplosion regime.

This is a non-perturbative criterium. Can in principle be computed on a lattice.

# Higgsplosion

Propagators are effectively cut off at $E_*$ by the exploding width $\Gamma(p^2)$ of the propagatıng state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \lll E_*^2$.

The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the $n$ soft particle quanta of the same field $\phi$.

VVK & Michael Spannowsky 1704.03447, 1707.01531

# Summary

- The Higgsplosion / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplosion energy).
- UV-finiteness => all coupling constants slopes become flat above the Higgsplosion scale => automatic asymptotic safety
- [Below the Higgsplosion scale there is the usual logarithmic running]
- 1. Asymptotic Safety
- 2. No Landau poles for the U(1) and the Yukawa couplings
- 3. The Higgs self-coupling does not turn negative => stable EW vacuum
- No new physics degrees of freedom required — very minimal solution

# Additional slides

- The n! growth of perturbative amplitudes is not entirely surprising: the number of contributing Feynman diagrams is known to grow factorially with n. [In scalar QFT there are no partial cancellations between individual diagrams (unlike QCD).]

- Important to distinguish between the two types of large-n corrections:

(a) present case where the *leading-order* tree-level contribution to the 1*->n Amplitude grows factorially with the particle multiplicity n of the final state.

(b) *higher-order* perturbative corrections to some leading-order quantities

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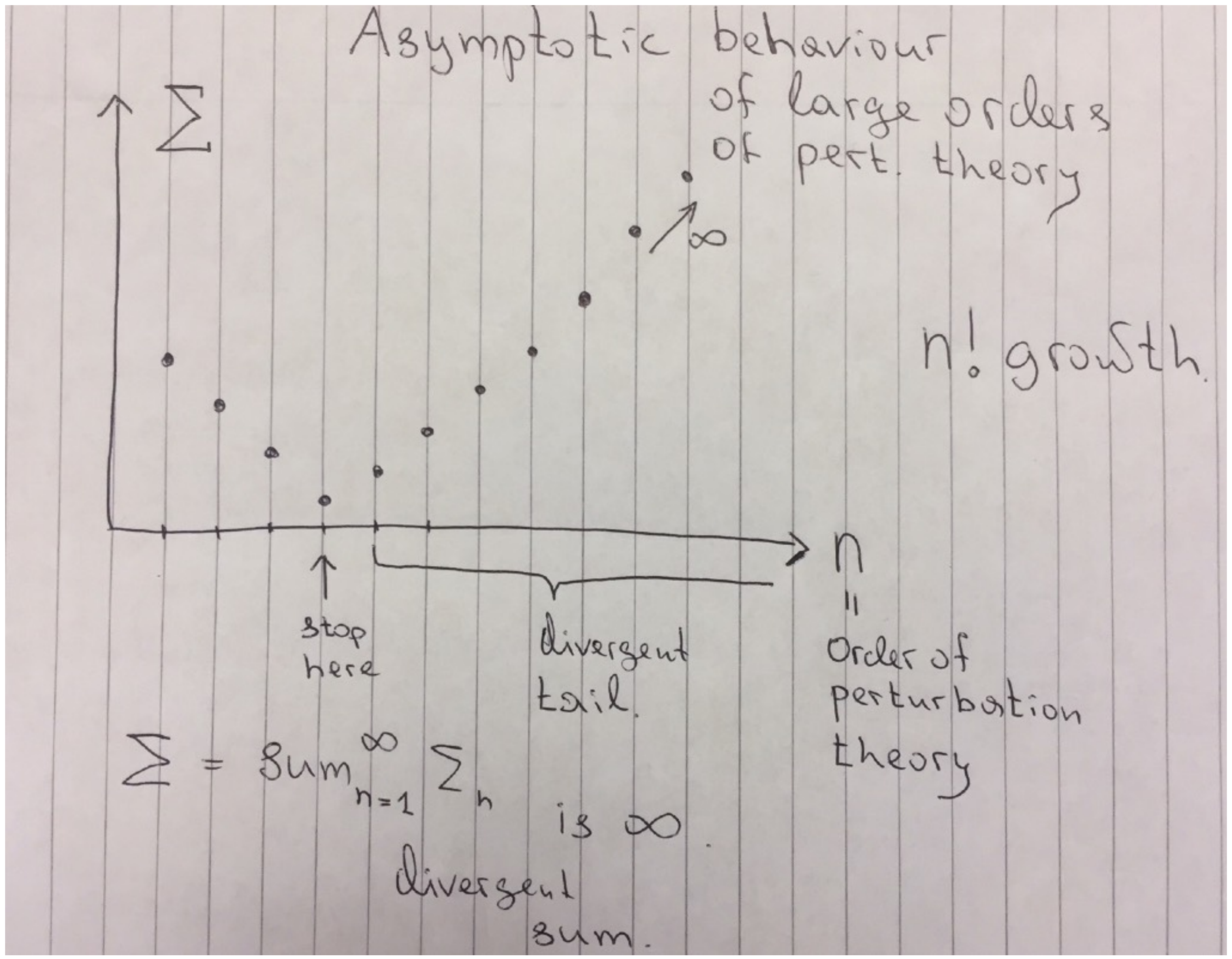
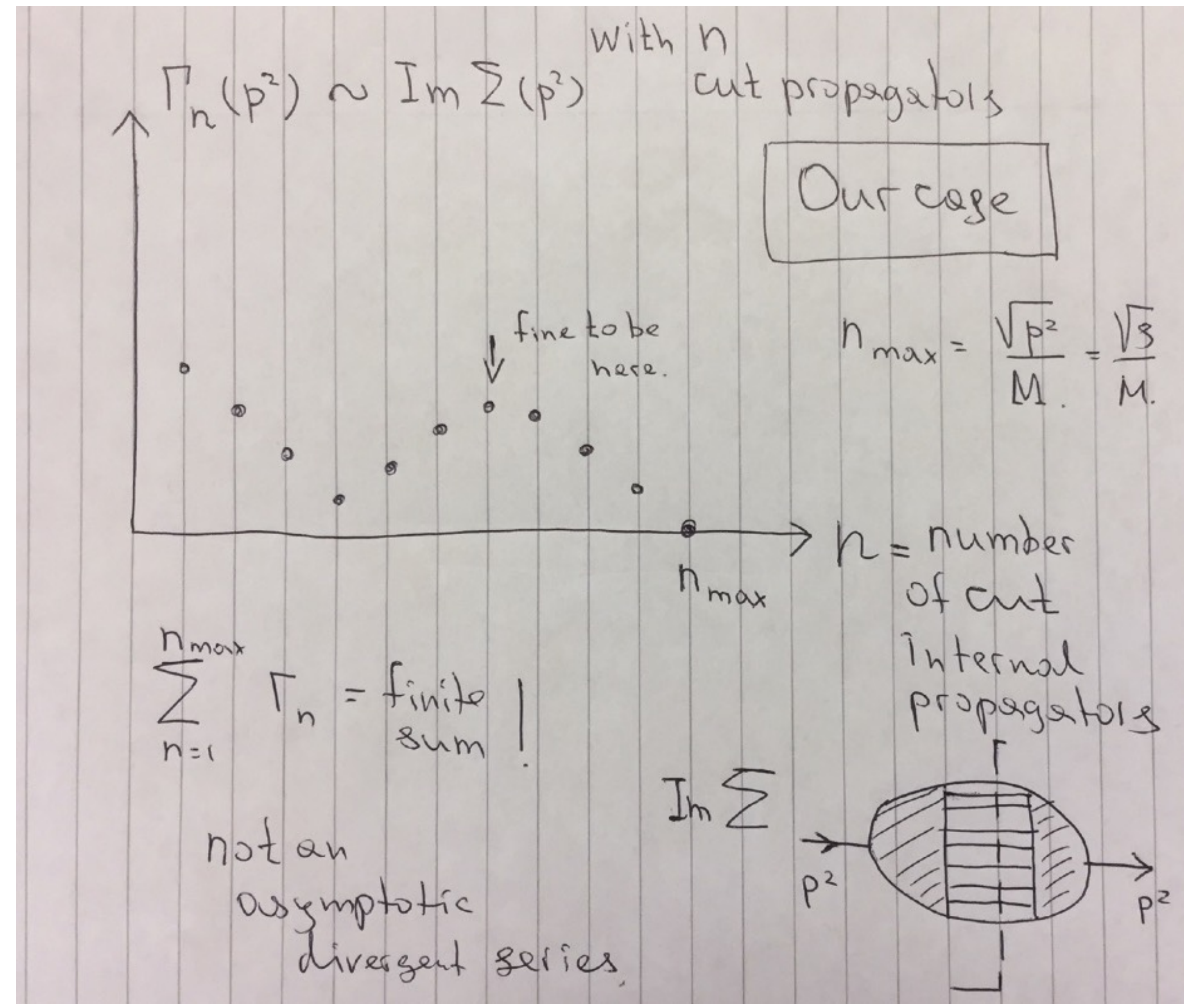
- These amplitudes were first studied in the 90s in scalar QFTs

  - But now it is realised that the characteristic energy scale for EW applications starts in the 50-100 TeV range. Future colliders: (FCC, ALEGRO e+e-, photon-photon) would provide an exciting challenge to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.

Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma_n(s)




Higgsplosion is not the same as the high orders tail of pert. theory

It is the decay width Gamma_n(s) which is the central object of interest and the driving force of Higgsplosion.