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# Higgsplosion

Valya Khoze

**IPPP** Durham

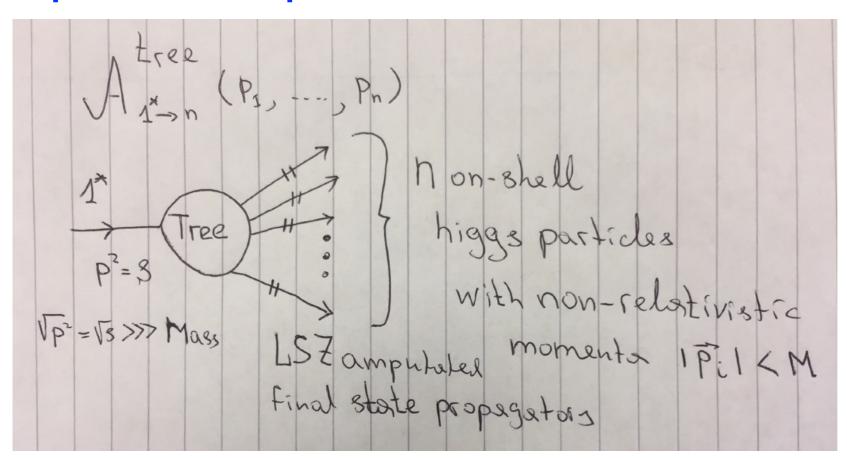
In collaboration with Michael Spannowsky

- Before the Higgs discovery, massive Yang-Mills theory violated perturbative unitarity
   — problem with high-energy growth of 2 -> 2 processes
- Discovery of the (elementary) Higgs made the SM theory self-consistent
- The Higgs brings in the Hierarchy problem: radiative corrections push the Higgs mass to the new physics (high) scale:  $m_h^2 \simeq m_0^2 + \delta m_{\rm new}^2$
- In this talk: consider n~100s of Higgs bosons produced in the final state n lambda
   >> 1. Investigate scattering processes at ~ 100 TeV energies.
- HIGGSPLOSION: n-particle rates computed in a weakly-coupled theory can become
  unsuppressed above critical values of n and E. Perturbative and non-perturbative
  semi-classical calculations. n! ~ exponential growth with n or E. (Scale n~E/m).
- A new unitarity problem caused by the elementary Higgs bosons appears to occur (?) for processes with large final state multiplicities n >> 1
- HIGGSPLOSION offers a solution to both problems: it restores the unitarity of high-multiplicity processes and dynamically cuts off the values of the loop momenta contributing to the radiative corrections to the Higgs mass.

# Organisation of the talk:

- Main idea and simple expressions for n-point amplitudes and rates
- Interpretation of tree-level results;
   quantum effects: loops and semiclassical methods.
- Higgsplosion at at e-e- and photon photon colliders
- => Summary

#### **Compute 1 -> n amplitudes @LO with non-relativistic final-state momenta:**



see classic 1992-1994 papers: Brown; Voloshin; Argyres, Kleiss, Papodopoulos Libanov, Rubakov, Son, Troitski

more recently: VVK 1411.2925

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} h)^2 - \frac{\lambda}{4} (h^2 - v^2)^2$$

prototype of the SM Higgs in the unitary gauge

Tree-level  $1^* \to n$  amplitudes in the limit  $\varepsilon \to 0$  for any n are given by

$$\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \left(1 - \frac{7}{6}n\varepsilon - \frac{1}{6}\frac{n}{n-1}\varepsilon + \mathcal{O}(\varepsilon^2)\right)$$

factorial growth

amplitude on the n-particle threshold 
$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \,=\, \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^n\vec{p_i}^2$$

In the large-*n*-non-relativistic limit the result is

kinetic energy per particle per mass

$$\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \exp\left[-\frac{7}{6}n\varepsilon\right], \quad n \to \infty, \ \varepsilon \to 0, \ n\varepsilon = \text{fixed}$$

#### Can now integrate over the n-particle phase-space

The cross-section and/or the *n*-particle partial decay  $\Gamma_n$ 

$$\Gamma_n(s) = \int d\Phi_n \frac{1}{n!} |\mathcal{A}_{h^* \to n \times h}|^2$$

The n-particle Lorentz-invariant phase space volume element

$$\int d\Phi_n = (2\pi)^4 \delta^{(4)} (P_{\text{in}} - \sum_{j=1}^n p_j) \prod_{j=1}^n \int \frac{d^3 p_j}{(2\pi)^3 2p_j^0},$$

in the large-n non-relativistic limit with  $n\varepsilon_h$  fixed becomes,

$$\Phi_n \simeq \frac{1}{\sqrt{n}} \left( \frac{M_h^2}{2} \right)^n \exp \left[ \frac{3n}{2} \left( \log \frac{\varepsilon_h}{3\pi} + 1 \right) + \frac{n\varepsilon_h}{4} + \mathcal{O}(n\varepsilon_h^2) \right]$$

We find:

$$\Gamma_n^{\text{tree}}(s) \sim \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) + \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}n\varepsilon + \mathcal{O}(n\varepsilon^2)\right]$$

Son 1995;

 $n \gg 1, \varepsilon \ll 1$ 

Libanov, Rubakov, Troitskii 1997; more recently: VVK 1411.2925

### Now include loop corrections in the exponent of Higgsplosion

The 1-loop corrected threshold amplitude for the pure n Higgs production:

$$\phi^4$$
 with SSB:  $\mathcal{A}_{1\to n}^{\text{tree+1loop}} = n! (2v)^{1-n} \left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)$ 

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\mathrm{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]$$

in the limit  $\lambda \to 0$ ,  $n \to \infty$  with  $\lambda n^2$  fixed. Here B is determined from the 1-loop calculation (as above) – Smith; Voloshin 1992):

$$B = +\lambda n \, \frac{\sqrt{3}}{4\pi}$$

# Semi-classical approach for computing Higgsplosion rate

• DT Son1995

Multi-particle decay rates  $\Gamma_n$  can also be computed using an alternative semiclassical method. This is an intrinsically non-perturbative approach, with no reference in its outset made to perturbation theory.

The path integral is computed in the steepest descent method, controlled by two large parameters,  $1/\lambda \to \infty$  and  $n \to \infty$ .

$$\lambda \to 0$$
,  $n \to \infty$ , with  $\lambda n = \text{fixed}$ ,  $\varepsilon = \text{fixed}$ .

The semi-classical computation in the regime where,

$$\lambda n = \text{fixed} \ll 1$$
,  $\varepsilon = \text{fixed} \ll 1$ ,

reproduces the tree-level perturbative results for non-relativistic final states.

Remarkably, this semi-classical calculation also reproduces the leading-order quantum corrections arising from resumming one-loop effects.

# Semi-classical approach for computing Higgsplosion rate

$$\Gamma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon)$$

The semiclassical approach is equally applicable and more relevant to the realisation of the non-perturbative Higgsplosion case where,

$$\lambda n = \text{fixed} \gg 1$$
,  $\varepsilon = \text{fixed} \ll 1$ .

This calculation was carried out for the spontaneously broken theory with the result given by,

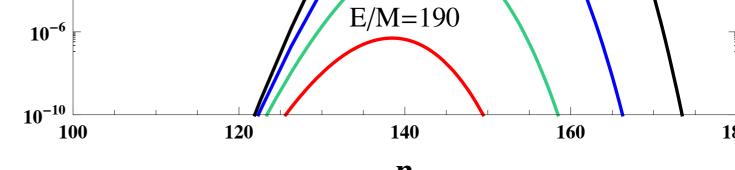
VVK 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) = \exp\left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 0.85\sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2}\log \frac{\varepsilon}{3\pi} - \frac{25}{12}\varepsilon\right)\right],$$

Higher order corrections are suppressed by  $\mathcal{O}(1/\sqrt{\lambda n})$  and powers of  $\varepsilon$ .

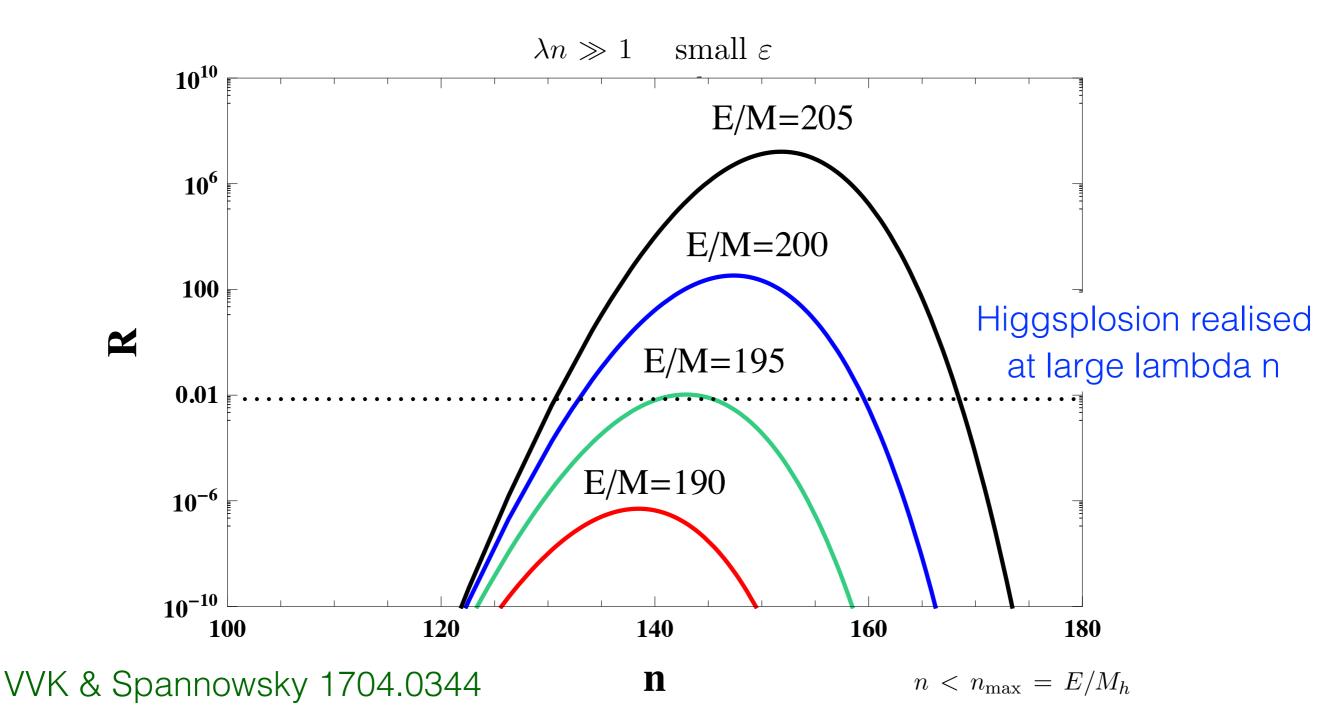
### Thus we have computed the

using the semi-classical app



VVK 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) = \exp\left[\frac{\lambda n}{\lambda} \left(\log \frac{\lambda n}{4} + 0.85\sqrt{\lambda n}\right) + \frac{1}{2} + \frac{3}{2}\log \frac{\varepsilon}{3\pi} - \frac{25}{12}\varepsilon\right)\right]$$



### Intuitive reason for Higgsplosion:

# Higgsplosion



constructive interference between n! diagrams for h\*-> nh elementary scalars in a spontaneously broken QFT

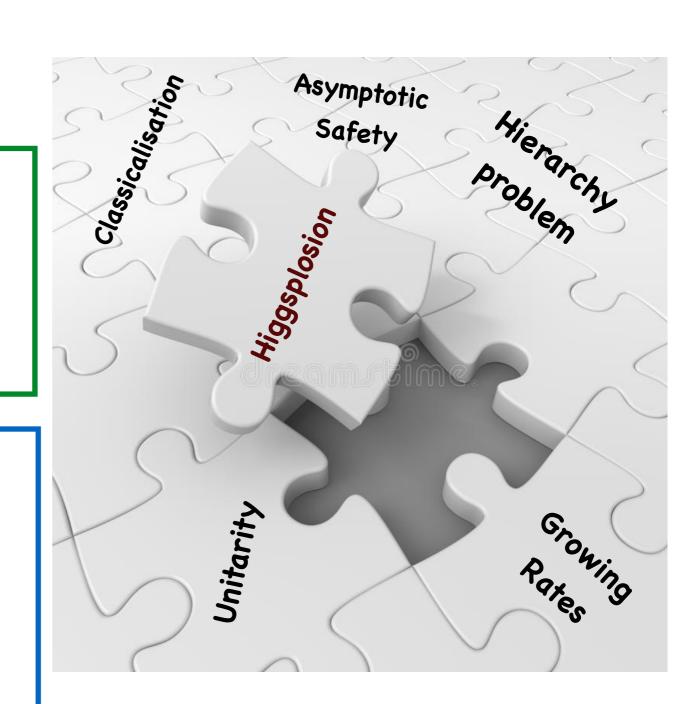
Why not observed somewhere else before?

Higgs lacks symmetry to prevent Higgsplosion:

- Gauge fields (gauge symmetry)
   e.g. simple Parke-Taylor '86 amplitudes
- Fermions (Pauli principle)
   zero at threshold and phase space scales 1/N!

Integrable systems in 1+1 dim or even 2+1 dimensional scalar QFTs: The cases in <4 dimensions are special different IR dynamics of quantum loops

Quantum Mechanics: too simple 0+1 dims



Our current understanding: Need spontaneously broken scalar QFT in at least 3+1

## HIGGSPLOSION and HIGGSPERSION

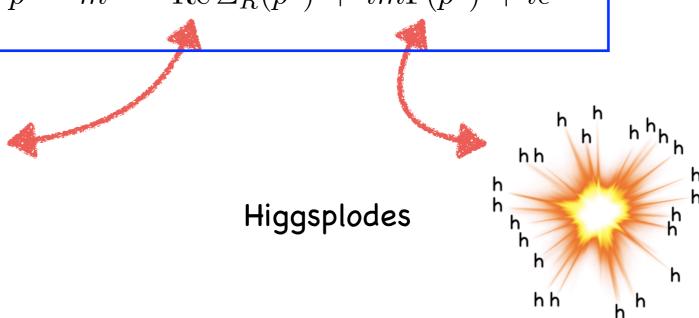
The optical theorem now relates the  $1^*$  -> nh amplitudes with the imaginary part of the self-energy (valid to all orders)

$$-\operatorname{Im}\Sigma_{R}(p^{2}) = m\Gamma(p^{2}) - \operatorname{Im}\left(\frac{p^{2}}{2}\right)$$

where 
$$\Gamma(s)=\sum_{n=2}^{\infty}\Gamma_n(s)$$
 and  $\Gamma_n(s)=\frac{1}{2m}\int\frac{d\Phi_n}{n!}\left|\mathcal{M}(1\to n)\right|^2$ 

and thus 
$$\Delta_R(p) = \frac{i}{p^2 - m^2 - \operatorname{Re}\Sigma_R(p^2) + im\Gamma(p^2) + i\epsilon}$$

No information as perturbation theory breaks down for many loops, but not possible to cancel imaginary part



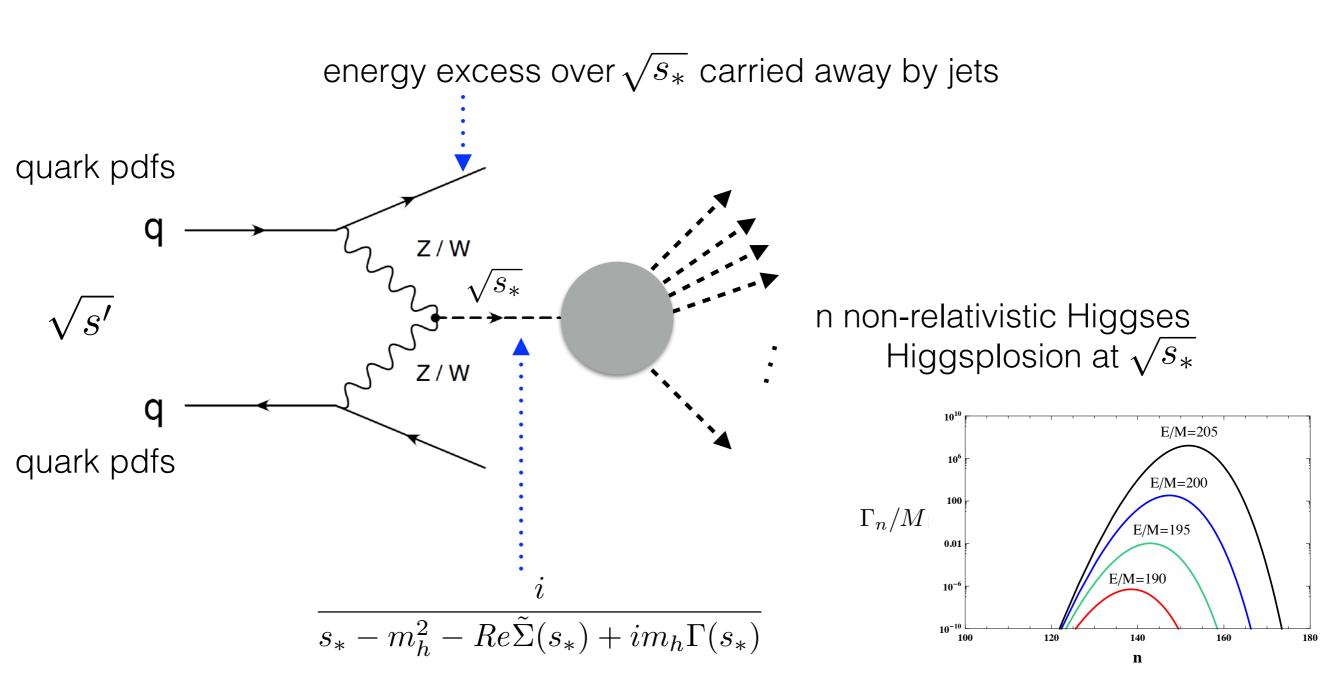
the final state is fixed. The first factor so that the total kinetic energy per particle mass  $n\varepsilon$  in the final state is fixed. The first factor ee-level amplitude (whose precisely a energy at the threshold, =on the right-hand side of Eq. (3.3) corresponds to the tree-level amplitude (or more precisely a current with one incoming off-shell leg) computed on the *n*-particle threshold,  $nM_h$ Hetic energy per particle mass  $n\varepsilon$  in the final state is fixed. If die of Eq. (3.3) corresponds to the tree-level amplitude (or more precisely also reduction of the incoming line, equipping off-shell leg) computed on the n-particle threshold,  $\mathcal{M}_{1 \to n}^{ ext{thr.}} = n! (n^2 - 1) \frac{\lambda^{\frac{n}{2}}}{M} \mathcal{M}_{h}^{-3} h * \rightarrow n \times h$ (3.7)d for any value of m is the m in m and m in m i which reaction in the level applitudes valid for any value of n [5]. The kinematic vistic limit an exponential form-factor dependence in Eq. (3.3) then produces in the non-relativistic limit an exponential form-factor of the final state  $n\varepsilon$ . But, importantly, which has an analytic dependence on the kinetic energy of the final state  $n\varepsilon$ . But, importantly, epatheelasignifications of the lincoming line, the factorial growth  $\sim \lambda^{n/2} n!$  characteristic to the multi-particle amplitude on mass threshold e interference between the diagrams in the diagram in the diag remains. Its occurrence can be traced back to the factorially growing number of Feynman diagrams at large n [14, 15, 16] and the lack of destructive interference between the diagrams in or more detail about these amphudes. the scalar theory. We refer the reader to Refs. [5, 7, 9] for more detail about these amplitudes. (3.3) over the *n*-particle phase-space at **h** The next step is to integrate the amplitudes in Eq. (3.3) over the *n*-particle phase-space at cles are non-relativistic). The relevant pression for tree-level at h large n (in the approximation where the outgoing particles are non-relativistic). The relevant alue of naisoness quantity rescribing the militial processes is (3.3) then produces in the an exponential form-factor' state  $n\varepsilon$ . But, importantly isti $\mathcal{R}_{exp}$  is  $n\varepsilon$  in the high energy hedependenceron Thie the high nergy nplitude on mass threshold and the decay rates  $\Gamma_n(s)$  and the cross-sections  $\sigma_n(s)$  are obtained from  $\mathcal{R}_n(s)$  after an aphabotain  $\sim \lambda^{n/2} \, n! \, ext{characteristic} \quad \mathsf{h} \quad ext{are obtained from } \mathcal{R}_n(s)_2$  after an  $\operatorname{ap}_{\mathbf{h}}$ owing numbers of the yearning with My and sp Following in the step of Refs.) [8, 95, no obtain (3.9) sences (alogical recent de obtain (3.9) [14, 15, 16] and the lack ce between the diagrams in h  $\Gamma_n(s) \ \nabla \mathcal{R}(\lambda; n, \varepsilon) ,_6$  $Ve^{\sigma_{\mathbf{r}}(s)}$  er the reader to  $\mathbf{k}_{\mathbf{r}}$  is. [5,  $\iota$ ,  $\mathbf{y}_{\mathbf{l}}$  for more detail about these amplitudes. growth of the large the amplitudes and lates in Eq. (3.3) over the n-particle phase  $\lim_{n\to\infty} \operatorname{particular}_n$  note that the ubiquitous factorianter  $\mathcal{R}$  above. considerational twilter of the outgoing aparticles are non-relativistic). The relevantarise our discussion so for, let us hty<sup>E</sup>desckiping the uniuitie particle mocesses is at the same time,  $\lambda \ll 1$ . It was pointed out fi phi in the literature, that in this limit the mult particle rates have a characteristic experimentia phi  $\underset{\rightarrow}{\operatorname{aracteristi}} \mathcal{R} = \underbrace{\underset{\rightarrow}{\operatorname{sonorm}}} \underbrace{\underset{\rightarrow}{\operatorname{horm}}} \underbrace{\underset{\rightarrow}{\operatorname{horm}} \underbrace{\underset{\rightarrow}{\operatorname{horm}}} \underbrace{\underset{\rightarrow}{\operatorname{$ nultiplicity limit: n weak-coupling limit above, the  $\log \lambda n$  is nultiplicity limit:

y, weak-coupling limit above, the fgc  $\lambda n$  is

or  $\lambda n$  is quantity  $\varepsilon$  is the average kinetic energy per particle  $F(\lambda n, \varepsilon)$  is a certain a priori unknown function ticle per mass in the final state of  $\mathbf{P}q$ . (3.5), and of two arguments X treedevel, the dependence and  $\sigma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon)$ .  $F^{\text{tree}}(\lambda n, \varepsilon) = f_0(\lambda n) + f(\varepsilon),$ (3.11)ticular+ prote, that the ubiquitous (factorial growth of the large-n, amplitudes translates into  $|M_n|^2 \sim n! \lambda^n \approx e^{n \log(\lambda n)}$  factor in the rate  $\mathcal{R}$  above. and the two independent functions are given by the following expressions in the Higgs model of TIU+consider the multi-particle limit  $n \gg 1$  and scale agreement with the expression Eq. (3.9), and scale same time,  $\lambda \ll 1$ . It was pointed out first in Refs. [7, 8], and then argued for extensively (3.12) $= \frac{1}{2} \operatorname{terg}(\underline{\operatorname{ufe}}_{3\pi}) + 1 \operatorname{that}_{12} \operatorname{this}_{12} \operatorname{limit}_{12} \operatorname{this}_{13} \operatorname{limit}_{13} \operatorname{this}_{13} \operatorname{limit}_{14} \operatorname{there}_{13} \operatorname{this}_{12} \operatorname{limit}_{12} \operatorname{this}_{13} \operatorname{limit}_{14} \operatorname{there}_{13} \operatorname{this}_{14} \operatorname{limit}_{14} \operatorname{there}_{13} \operatorname{there}_{13} \operatorname{there}_{14} \operatorname$ (3.13)One can further come up with various improvements in the understanding and control of brovements in the understanding and control of the property of the function of the fine particular, at tree-level the function of the particular, at tree-level the function of the function of the multi-particle rate. In particular, at tree-level the function of the fun tipn,  $is(\varepsilon)$  as started right the three higher than the multi-particle  $\varepsilon$ , i.e. near the multi-particle  $\varepsilon$  as started right than the higher than the multi-particle  $\varepsilon$ , i.e. near the multi-particle  $\varepsilon$ , i.e. near the multi-particle threshold. This point was addressed recently in Ref. [10] where the function  $f(\varepsilon)$  was computed the first threshold. This point was addressed recently in Ref. [10] where the function  $f(\varepsilon)$  was computed the first threshold. This point was addressed recently in Ref. [10] where the function  $f(\varepsilon)$  was computed the first threshold. This point was addressed recently in Ref. [10] where the function  $f(\varepsilon)$  was computed the first threshold. This point was addressed recently in Ref. [10] where the function  $f(\varepsilon)$  was computed the first threshold.

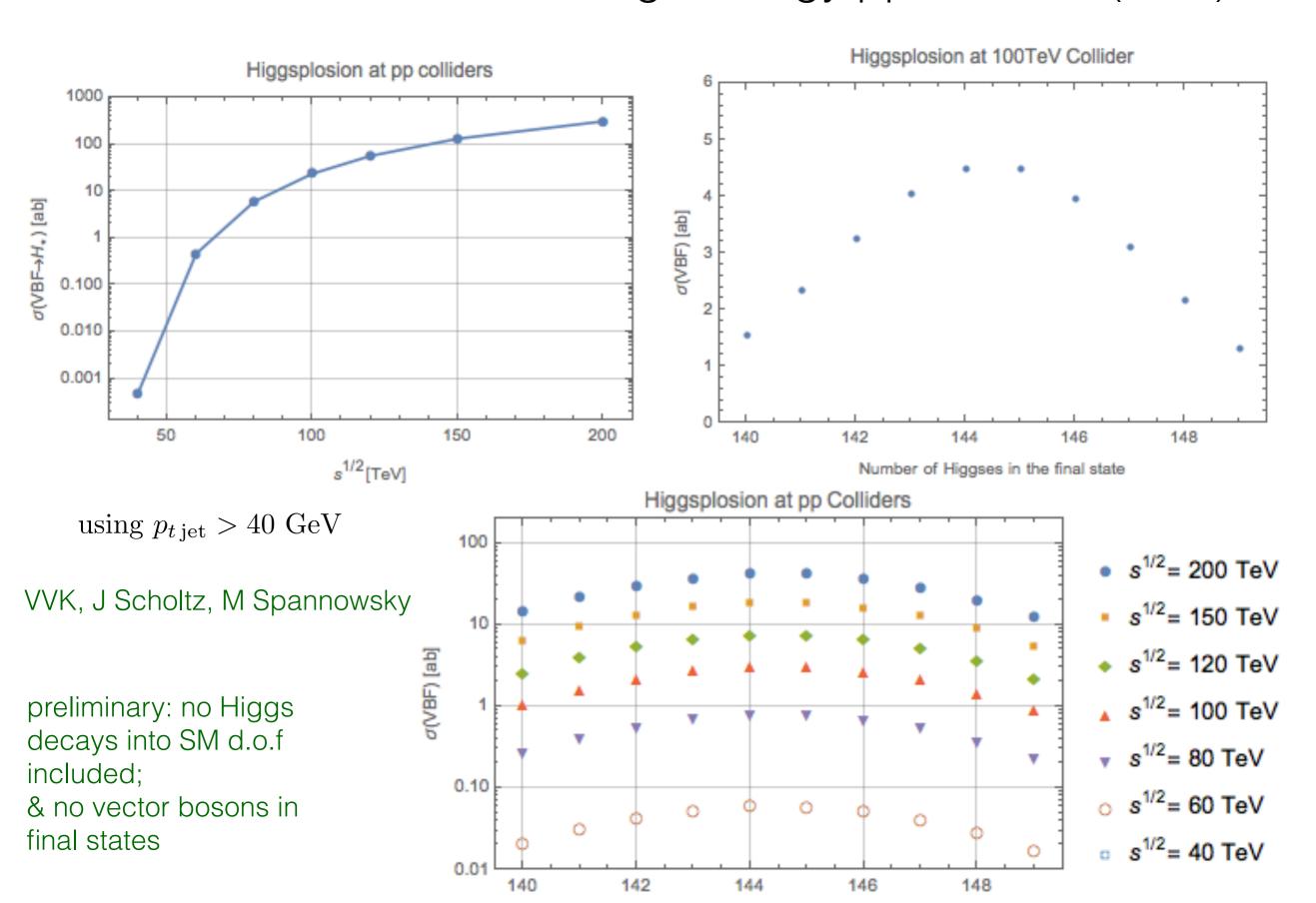
# Prospects of direct observation of Higgslposion

Vector boson fusion at high-energy pp colliders (FCC)



Propagator with Higgspersion at  $\sqrt{s_*}$ 

### Vector boson fusion at high-energy pp colliders (FCC)

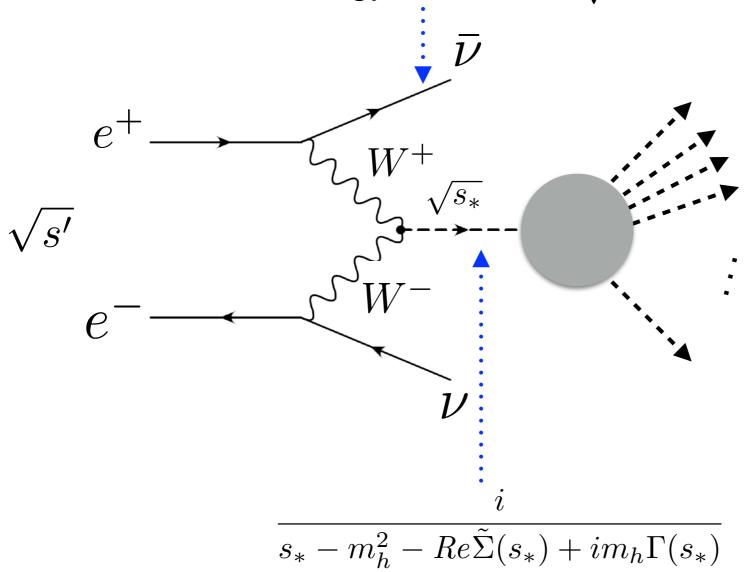


Number of Higgses in the final state

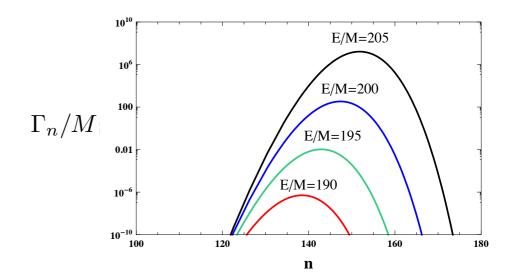
# Prospects of direct observation of Higgslposion

## Vector boson fusion at high-energy e+e- colliders: ALEGRO

energy excess over  $\sqrt{s_*}$  carried away by jets  $\vdots$ 



n non-relativistic Higgses Higgsplosion at  $\sqrt{s_*}$ 



Propagator with Higgspersion at  $\sqrt{s_*}$ 

# Vector boson fusion at high-energy e+e- colliders: ALEGRO

More generally should include multiple longitudinal vector boson production into Higgsplosion processes:

$$e^+e^-_{\rm VBF} \to h + \bar{\nu}\nu \to n \times h + \bar{\nu}\nu$$

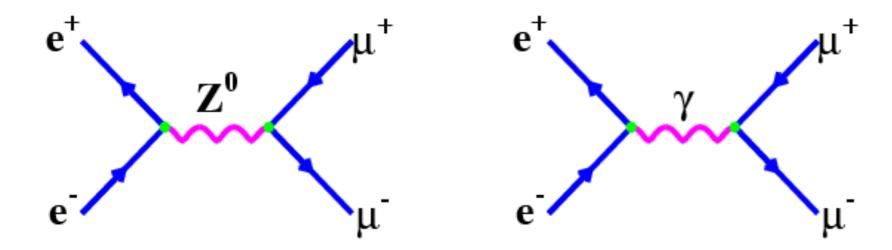
$$e^+e^-_{\rm VBF} \rightarrow Z^0 + \bar{\nu}\nu \rightarrow Z^0 + n \times h \bar{\nu}\nu$$

$$e^+e^-_{VBF} \rightarrow Z^0 + \bar{\nu}\nu \rightarrow Z^0 + n_h \times h + n' \times \bar{V}_L V_L + \bar{\nu}\nu$$

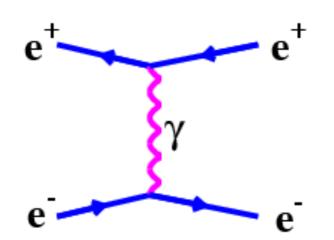
Where any of the Vector boson pairs are  $\bar{V}_L V_L = W^+ W^-$  or  $\bar{V}_L V_L = Z^0 Z^0$ . This should be the dominant process.

# One can also look for the disappearance of the **e+e-** -> **mu+ mu-** rates above E\*

Higgspersion for exclusive 2 to 2 s-chanel processes



But not for:



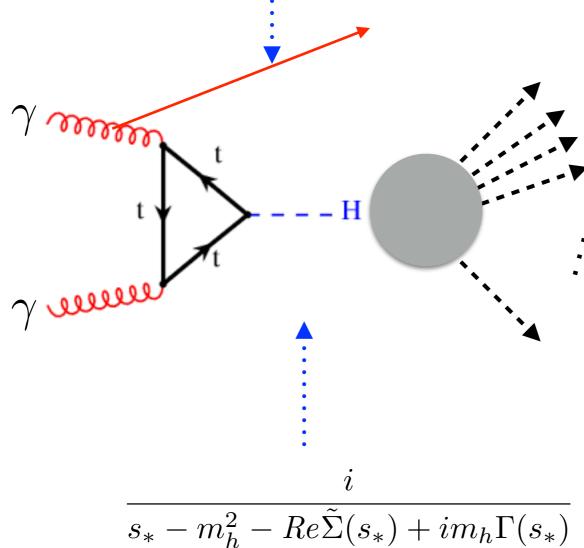
$$\frac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sigma(e^+e^- \to e^+e^-)}$$

disappearing at 
$$\sqrt{s} > E_*$$

# Photon fusion at **photon-photon** colliders: ALEGRO

energy excess over  $\sqrt{s_*}$  carried away by jets





n non-relativistic Higgses Higgsplosion at  $\sqrt{s_*}$ 

$$s_* - m_h^2 - Re\Sigma(s_*) + im_h\Gamma(s_*)$$

Propagator with Higgspersion at  $\sqrt{s_*}$ 

# Summary of the main idea

A conventional wisdom: in the description of nature based on a local QFT, one should always be able to probe shorter and shorter distances with higher and higher energies.

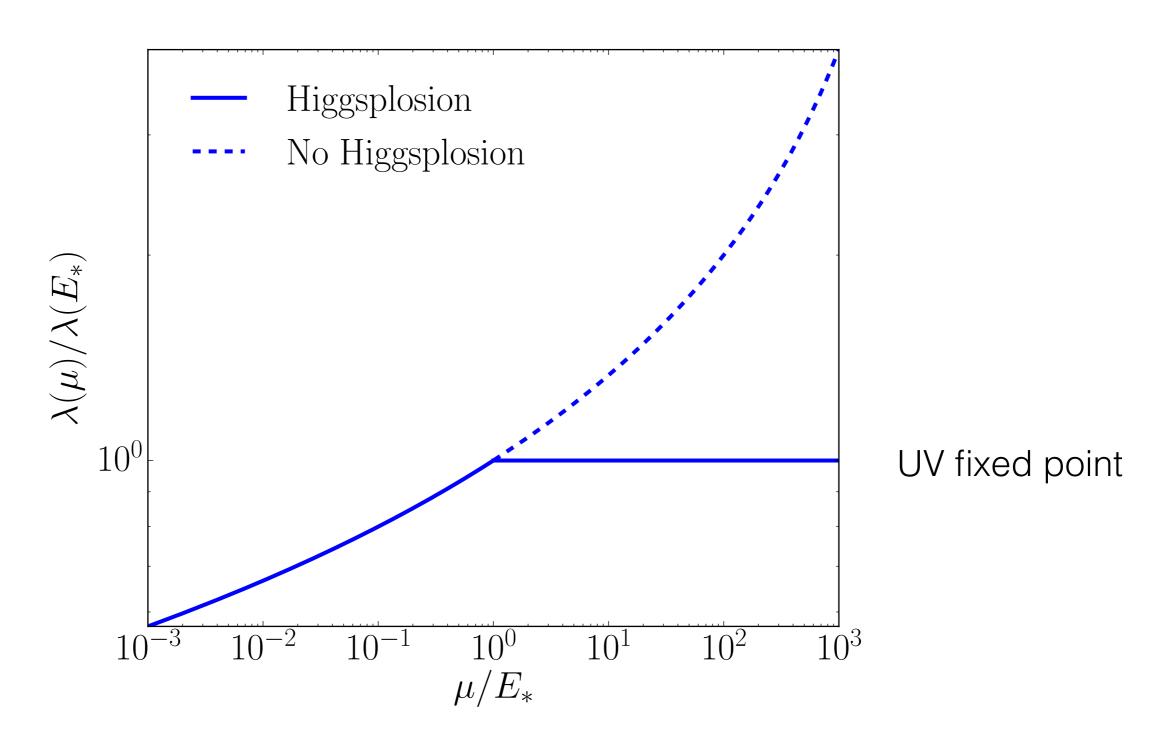
Higgsplosion is a dynamical mechanism, or a new phase of the theory, which presents an obstacle to this principle at energies above  $E_*$ .

 $E_*$  is the new dynamical scale of the theory, where multi-particle decay rates become unsuppressed.

Schematically,  $E_* = C \frac{m}{\lambda}$ , where C is a model-dependent constant of  $\mathcal{O}(100)$ . This expression holds in the weak-coupling limit  $\lambda \to 0$ .

# Asymptotic Safety

For all parameters of the theory (running coupling constants, masses, etc):



## Higgsplosion

At energy scales above  $E_*$  the dynamics of the system is changed:

- 1. Distance scales below  $|x| \lesssim 1/E_*$  cannot be resolved in interactions;
- 2. UV divergences are regulated;
- 3. The theory becomes asymptotically safe;
- 4. And the Hierarchy problem of the Standard Model is therefore absent.

Consider the scaling behaviour of the propagator of a massive scalar particle

$$\Delta(x) := \langle 0 | T(\phi(x) \phi(0)) | 0 \rangle \sim \begin{cases} m^2 e^{-m|x|} & : \text{ for } |x| \gg 1/m \\ 1/|x|^2 & : \text{ for } 1/E_* \ll |x| \ll 1/m \\ E_*^2 & : \text{ for } |x| \lesssim 1/E_* \end{cases}$$

where for  $|x| \lesssim 1/E_*$  one enters the Higgsplosion regime.

This is a non-perturbative criterium. Can in principle be computed on a lattice.

## Higgsplosion

Propagators are effectively cut off at  $E_*$  by the exploding width  $\Gamma(p^2)$  of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta  $k_i^2 \sim m^2 \ll E_*^2$ .

The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the n soft particle quanta of the same field  $\phi$ .

VVK & Michael Spannowsky 1704.03447, 1707.01531

# Summary

- The Higgsplosion / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplosion energy).
- UV-finiteness => all coupling constants slopes become flat above the Higgsplosion scale => automatic asymptotic safety
- [Below the Higgsplosion scale there is the usual logarithmic running]
- 1. Asymptotic Safety
- 2. No Landau poles for the U(1) and the Yukawa couplings
- 3. The Higgs self-coupling does not turn negative => stable EW vacuum
- No new physics degrees of freedom required very minimal solution

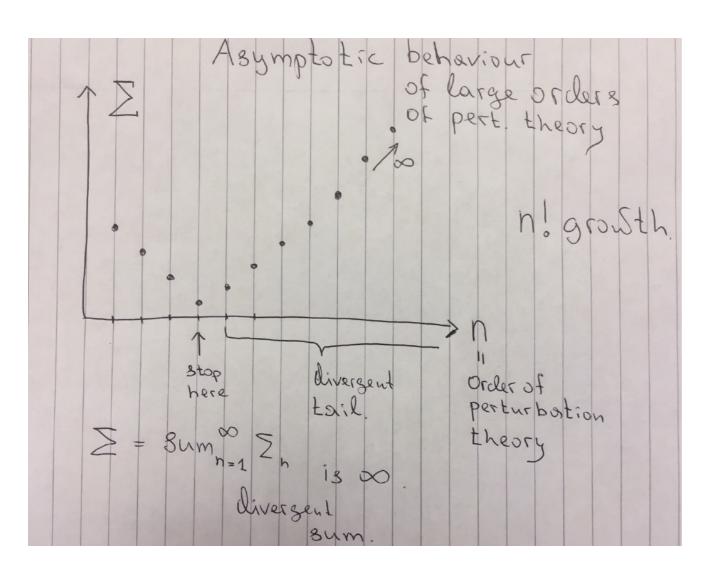
# Additional slides

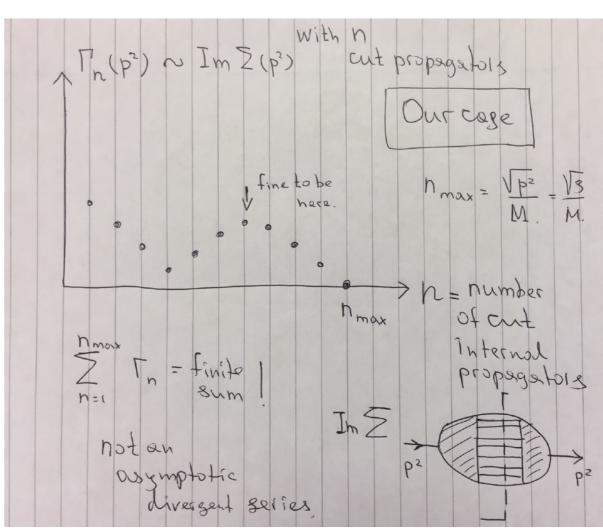
- The n! growth of perturbative amplitudes is not entirely surprising: the number of contributing Feynman diagrams is known to grow factorially with n. [In scalar QFT there are no partial cancellations between individual diagrams (unlike QCD).]
- Important to distinguish between the two types of large-n corrections:
- (a) present case where the *leading-order* tree-level contribution to the 1\*->n Amplitude grows factorially with the particle multiplicity n of the final state.
- (b) higher-order perturbative corrections to some leading-order quantities

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- These amplitudes were first studied in the 90s in scalar QFTs
  - But now it is realised that the characteristic energy scale for EW applications starts in the 50-100 TeV range. Future colliders: (FCC, ALEGRO e+e-, photon-photon) would provide an exciting challenge to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.

# Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma\_n(s)





Higgsplosion is not the same as the high orders tail of pert. theory

It is the decay width Gamma\_n(s) which is the central object of interest and the driving force of Higgsplosion.