# Consideration for a plasma stage in a PWFA linear collider

## ALEGRO workshop 2018

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**Parameter selection process for :**

E. Adli, J. P. Delahaye et al., "A Beam Driven Plasma-Wakefield Linear Collider: From Higgs Factory to Multi-TeV", Snowmass 2013, http://arxiv.org/abs/1308.1145

# Input constraints

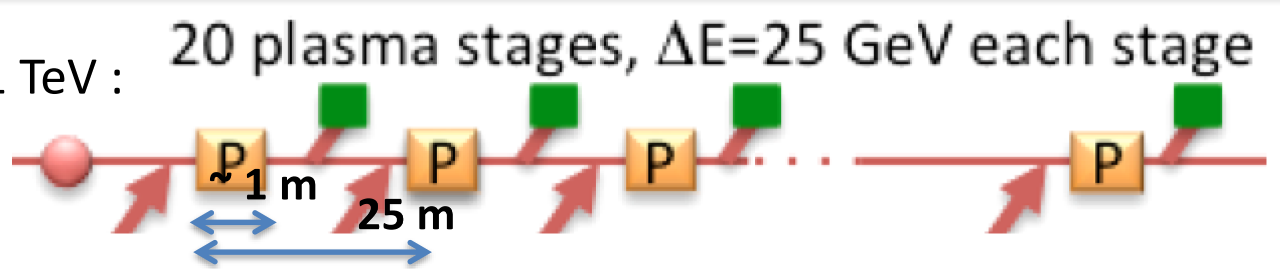
**The main beam parameters** for the 2013 PWFA-LC design are **assumed to be the ILC main beam parameters**, with some modifications (allows reuse of earlier LC studies ):

- Bunch length shortened to fit in plasma
- Charge of 1e10 particles per bunch (1/2 the ILC nominal bunch charge)
- Equal bunch spacing (“CW” collisions)

Other **input constraints** :

- **1 GeV/m** average gradient along main linac (“CLIC x 10”) with 25 GeV energy gain per plasma stage, assuming 25 m average stage length (see Carl’s talk later)
- High transfer efficiency
- Push towards low plasma density (see scalings later)
- Parameter optimization assume**s e- drive bunch and e- witness bunch in the blow out regime, and no ion motion**

For 1 TeV :




**The drive beam parameters are results** of plasma optimization process.

# Plasma density considerations

*Advantages of lower plasma density*

- plasma structure **length scale**

$$\lambda_p \sim n_0^{-1/2}$$

→ looser transverse and longitudinal tolerances; looser bunch length requirements for the drive and witness bunches

- **matched beta function** in the plasma

$$\beta_{\mathrm{mat}} \sim n_0^{-1/2}$$

→ looser optics matching, looser transverse tolerances

- **synchrotron radiation losses** in the plasma (for matched beam)

$$W' \sim n_0^{3/2}$$

→ less synchrotron radiation loss in the plasma, looser transverse tolerances (SR for transverse offset beam)

- **head erosion** in a pre-ionized plasma

$$\sim n_0^{1/4}$$

→ less head erosion of the drive bunch

- the **hosing instability** grows as

$$x/x_0 \sim e^{c(k_p s)^{1/3}(k_p z)^{2/3}}, \; k_p \sim n_0^{-1/2}$$

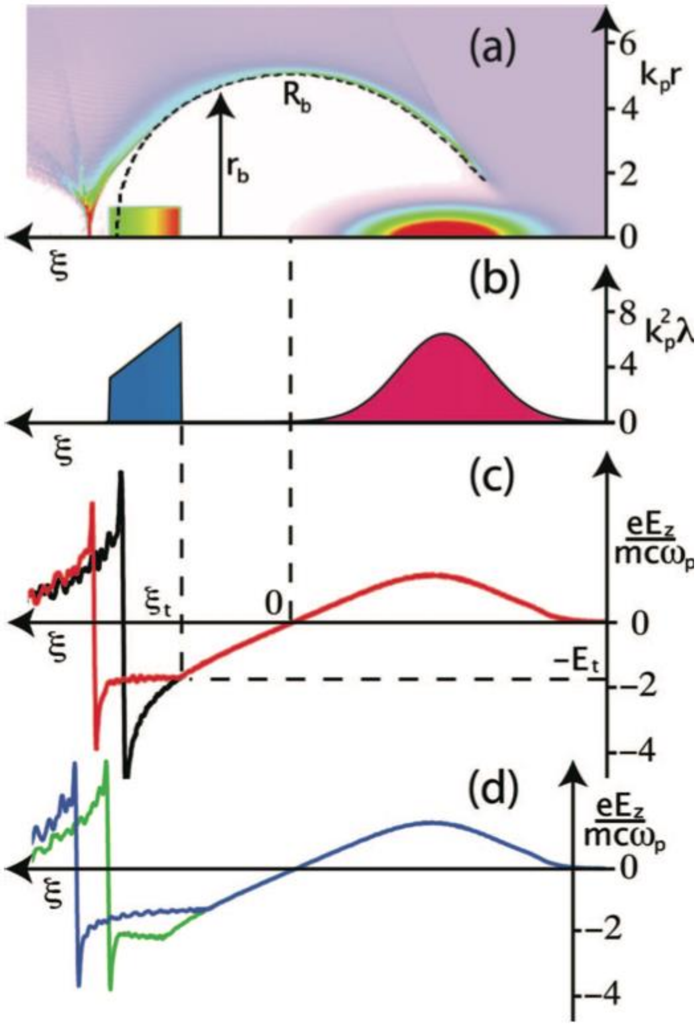
*Main disadvantage of lower plasma density :*

- **lower accelerating gradient**

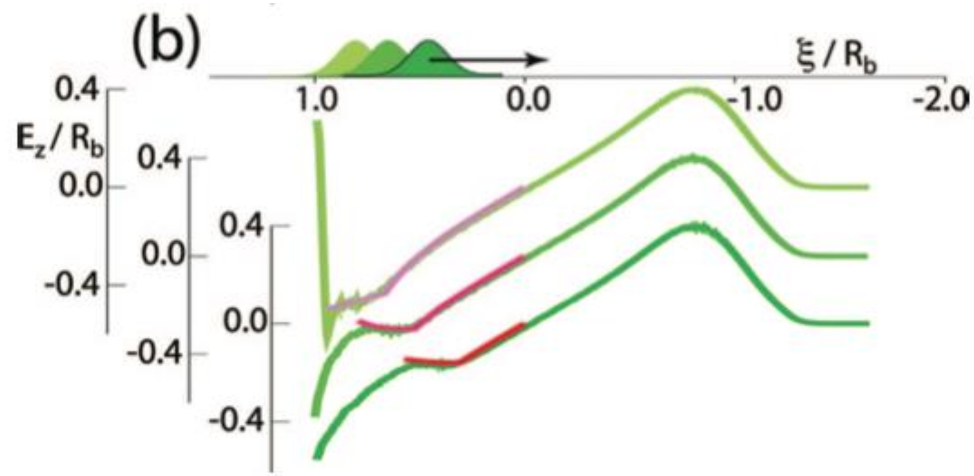
$$E_{\mathrm{wavebreak}} \sim cm_e\omega_p/e \sim n_0^{1/2}$$

# Non-linear beam loading

Tzoufras et al. : beam-loading in the blow-out regime. More **than 80% energy transfer efficiency possible** for optimally shaped trapezoidal bunch. Flattening of the longitudinal field along the witness bunch, resulting **in small energy spread**. :




Almost flat beam loading and good efficiency also possible for **Gaussian witness bunches**. For a given blow out radius, and a given bunch separation, $\Delta z$, the optimal beam loading ratio is given by the appropriate witness bunch charge, bunch length ($Q_{WB}$, $\sigma_{z,WB}$).

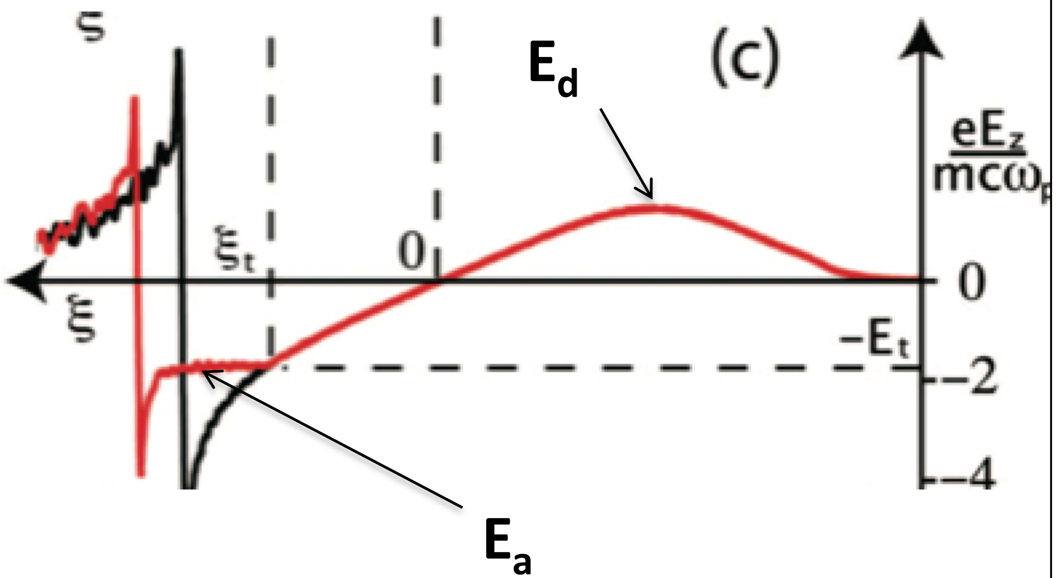



**From Tzoufras et al., Physics of Plasmas, 16, 056705 (2009)**

# Transformer ratio

**Transformer ratio :**

In the blow out regime the peak accelerating field may be several times the peak dececeleration field. The tranformer is up to a certain value a free parameter.




Peak decelerating field

$$E_d = \hat{E}_{dec}$$

Witness bunch mean energy gain

$$E_a = < E_{acc} >_{WB}$$

Tranformer ratio

$$T \equiv \frac{E_a}{E_d}$$

Drive bunch to witness bunch efficiency

$$\eta_{WB} = \frac{\Delta\mathscr{E}_{WB}}{\mathscr{E}_{0,DB}} \frac{Q_{WB}}{Q_{DB}} = T \frac{Q_{WB}}{Q_{DB}} \qquad (1)$$

Last equality valid only if most decelerated particle in drive bunch is fully depleteted.

**Consequenses of higher transformer ratio :**

+ Reduced drive beam energy $\mathscr{E}_{0,DB}$
- Increased drive beam charge $Q_{DB}$, since

$$\boxed{Q_{WB} \times E_a = const.}$$

- shorter witness bunch length; tightened tolerances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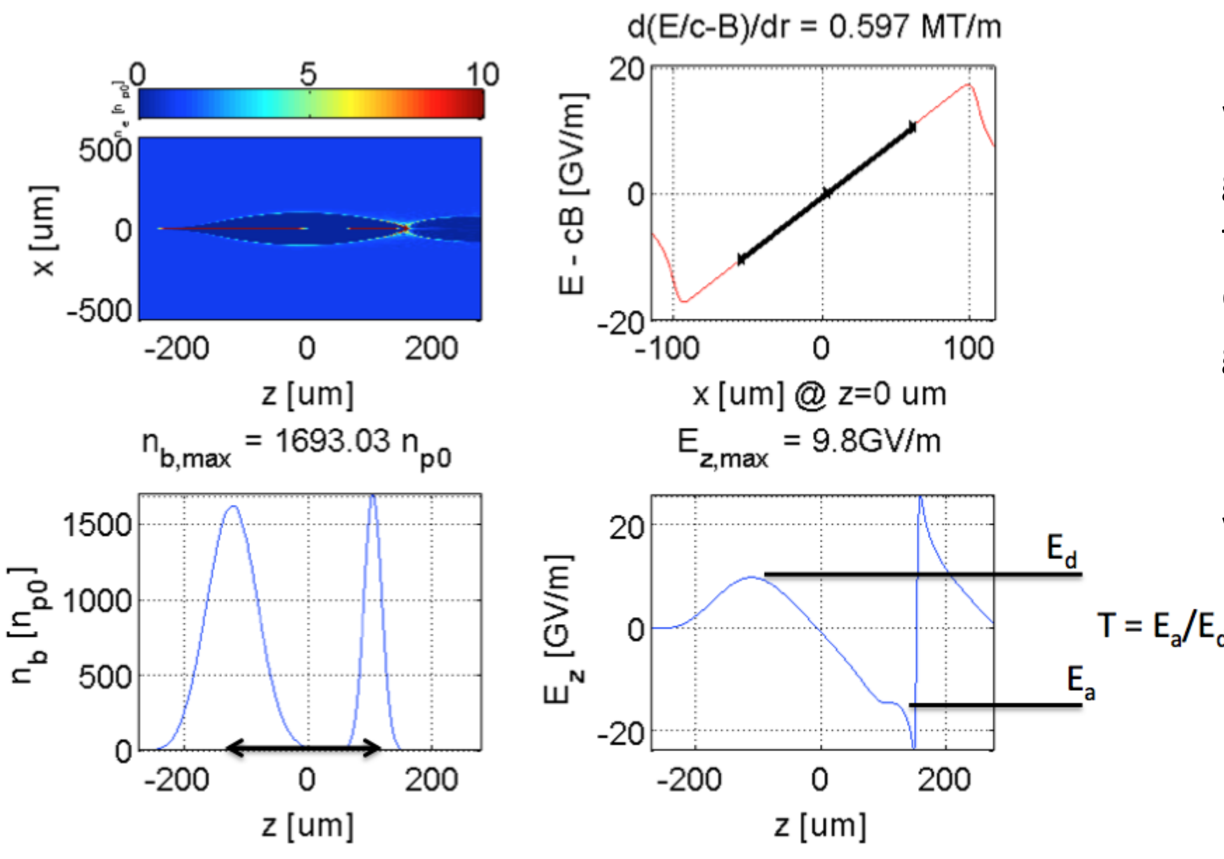
# Plasma stage optimization

**Input constraints:** main beam parameters; $Q_{WB}$=1x$10^{10}$e, $\Delta\varepsilon$=25 GeV/stage, $L_{cell}$ < few m, keep WB energy spread low, reasonable WB length

**Design choice:** plasma density $\mathbf{n_0}$, transformer ratio **T**

**Drive beams then set :** $Q_{DB}$ (charge), $\varepsilon_{0,DB}$ (energy) , $\Delta z_{DW}$ (DB-WB separation), $\sigma_{z,DB}$, $\sigma_{z,WB}$




With main beam parameters given, plasma density and transformer ratio chosen, the drive bunch parameters are given by $Q_{DB}$ x $E_{acc}$ = const., plus the requirement of equal peak current in the drive and witness bunch.

Simulations are performed using QuickPIC (UCLA)

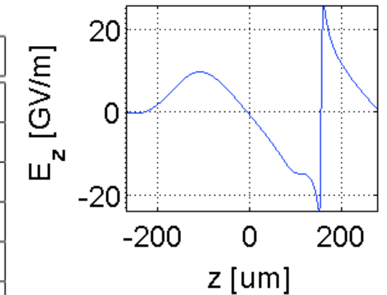
# 2013 parameters

Parameters optimized following Tzoufras recipe, for two transformer ratios T=1, T=1.5, verified using QuickPIC. No practical solution found for T >= 2.

| | T=1.5 | T=1.0 | Old value (2009) | Comments for new values |
|---|---|---|---|---|
| $n_0$ $[10^{16}/cm^3]$ | 2 | 2 | 10 | Sufficient field to keep $L_{cell} \leq 3$ m |
| $Q_{WB}$ $[10^{10}e]$ | 1.0 | 1.0 | 1.0 | Input constraint |
| $T$ | 1.5 | 1.0 | 1.0 | Reduces drive beam energy by $1/T$ |
| $\sigma_{z,DB}$ [$\mu$m] | 40 | 40 | 30 | Imposed to give $\sigma_z k_p \sim 1$ |
| $E_d$ [GV/m] | 10 | 7.6 | 25 | Function of $n_0$ and $DB$ params |
| $E_a$ [GV/m] | 15 | 7.6 | 25 | $E_d \times T$ |
| $\mathscr{E}_{0,DB}$ [GeV] | 17 | 25 | 25 | Results in full $DB$ depletion |
| $L_{cell}$ [m] | 1.7 | 3.3 | 1 | Length required for 25 GeV gain |
| $Q_{DB}$ $[10^{10}e]$ | 3.0 | 2.0 | 3.0 | Non-linear wake optimization |
| $\sigma_{z,WB}$ [$\mu$m] | 14 | 20 | 10 | Non-linear wake optimization |
| $\Delta z_{DW}$ [$\mu$m] | 225 | 187 | 110 | Non-linear wake optimization |
| $< \sigma_E/\mathscr{E} >_{WB}$ | 3% | (n/calc) | (n/avl) | Energy spread due to acc. |
| $\eta_{DW}$ | 50% | 50% | 33% | Increased efficiency |
| $\sigma_{\{x,y\},\mathrm{mat}}$ [nm] | {328,51} | {328,51} | {219,37} | E=500GeV |
| $W'_{loss,max}$ [MeV/m] | 6.1 | 6.1 | 70 | E=500GeV, $r_\beta = \sigma_{x,matched}$ |
| $\Delta r$ @ 100 MeV/m [um] | 1.3 | 1.3 | 0.26 | $\Delta r$ giving $W'_{loss} \sim 100 MeV/m$ |

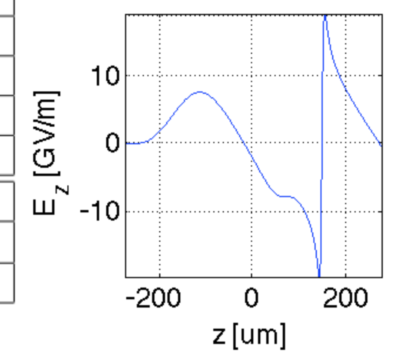
T = 1.5 :

$E_{z,max}$ = 9.8GV/m




T = 1.0 :

$E_{z,max}$ = 7.6GV/m



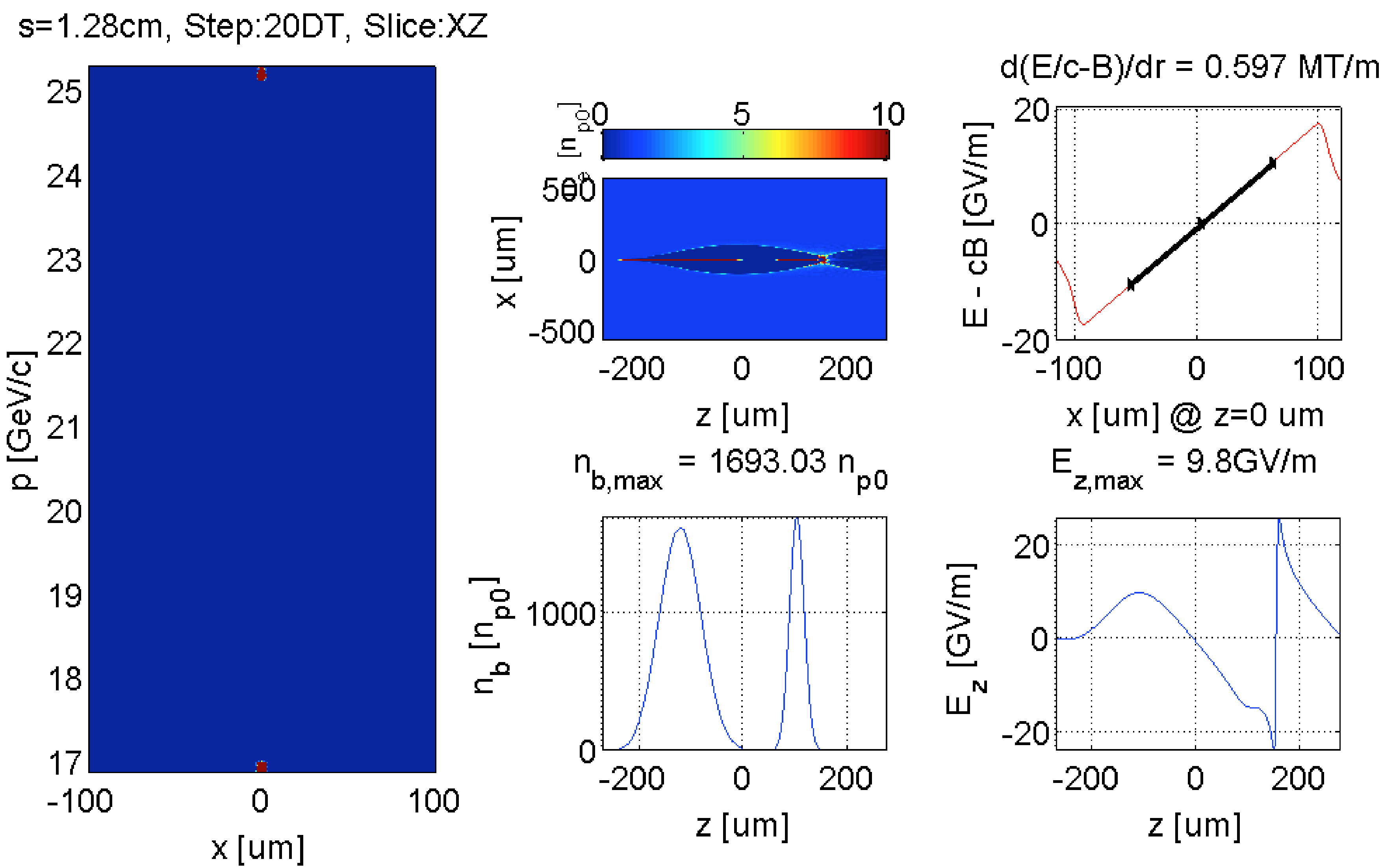

* The efficiency of the two sets is the same; the optimization is done keeping $Q_{WB} \times E_a = const.$

* The plasma density is kept constant for the two cases for comparison, resulting in 3 m long plasma cell for the $T = 1.0$ case. Ff needed, the cell length can be reduced by increasing the plasma density.

* The drive beam energy can be increased to yield smaller relative energy spread in the spent beam, at the cost of efficiency. For example, for set $T = 1.5$, increasing the drive beam energy to 20 GeV yields a minimum spent drive beam energy of 3 GeV instead of $< 1$ GeV, and an efficiency reduction from ~50% to ~40%.

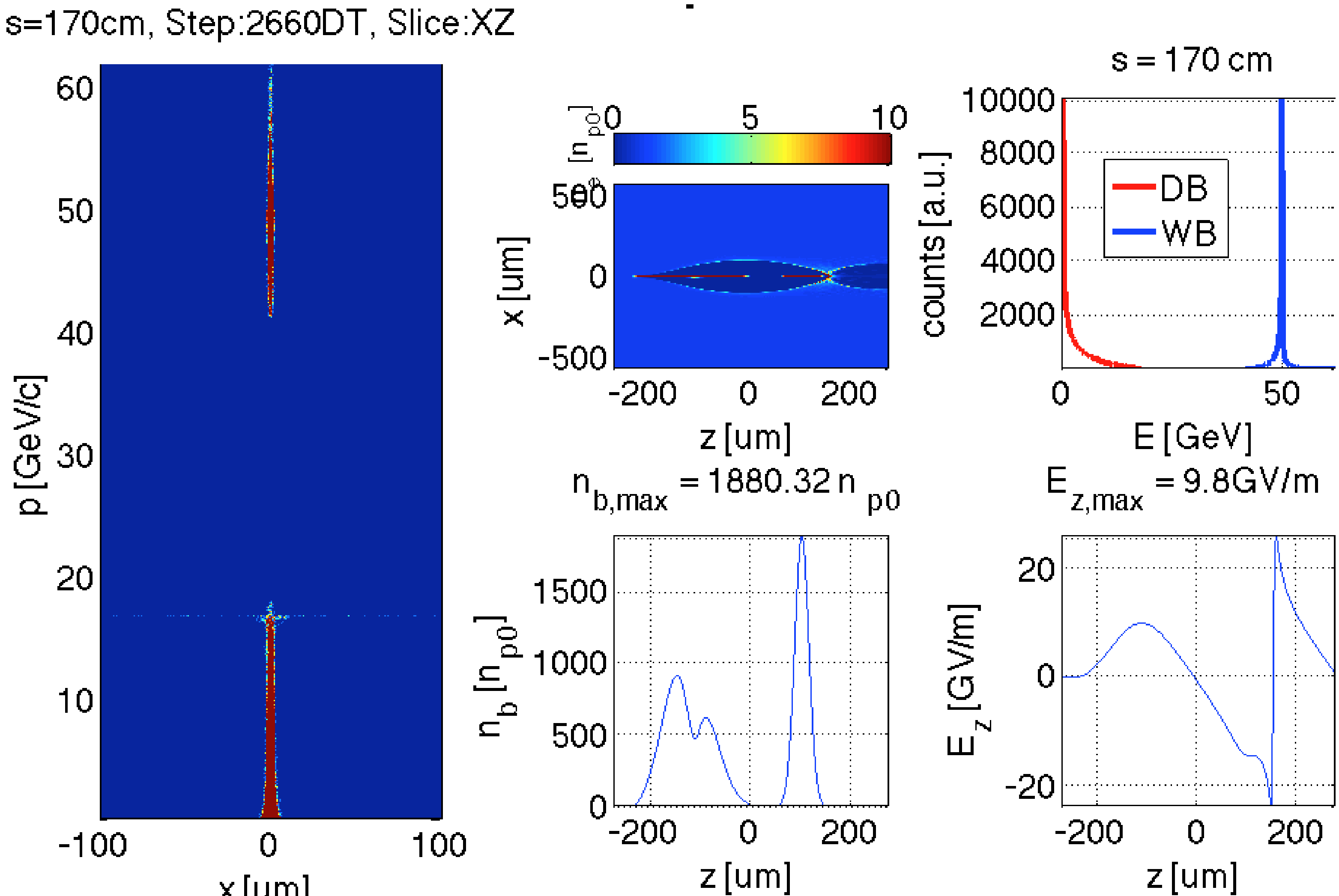
* One potential disadvantage not yet quantified is the tolerance on $\Delta z_{DW}$. For both the 2012 and the 2009 parameters, however, the tolerance is very tight, in the order of 1-10 $\mu m$ (corresponding to relative injection timing of $3 - 30$ fs).

# Full simulation – initial time step (T=1.5)




The purpose of these simulations is the study of longitudinal performance.
Input beams are perfectly aligned.

# Full simulation – final time step (T=1.5)

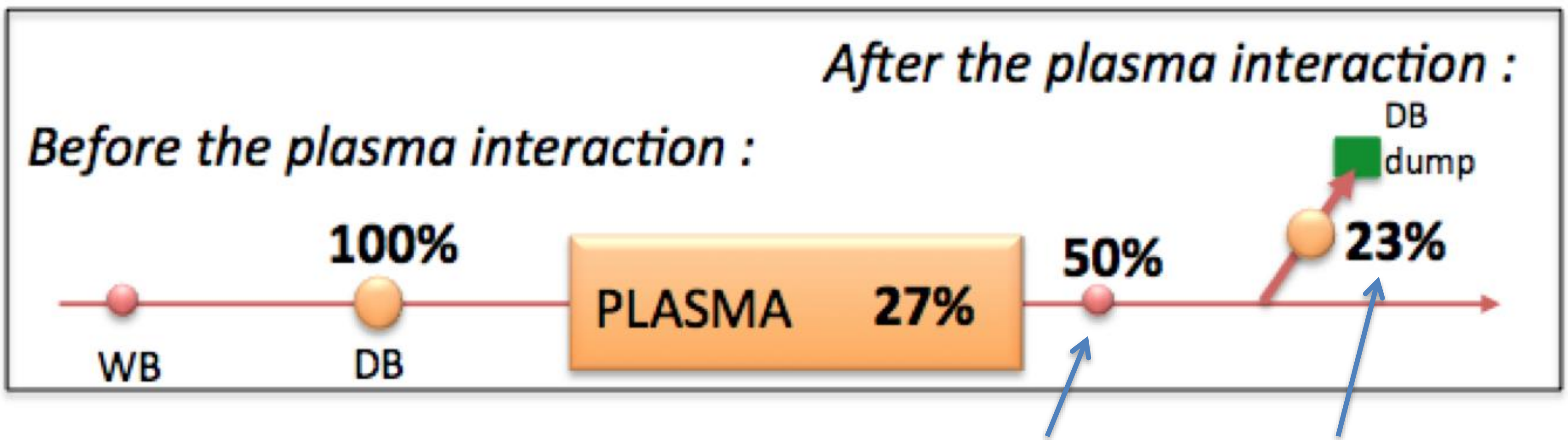



$\Delta\mathscr{E}_{WB}/\mathscr{E}_{0,WB} = 2.0$ $\eta_{WB} = 49\%$

$\sigma_{\mathscr{E}}/\mathscr{E}_{WB} = 3.1\%$ $\eta_{DB \to WB} = 62\%$

# Total energy efficiency

**Figure 2: Power flow from an e- drive bunch (DB) to Plasma and to an e- Witness Bunch (WB), as calculated by QuickPIC.**

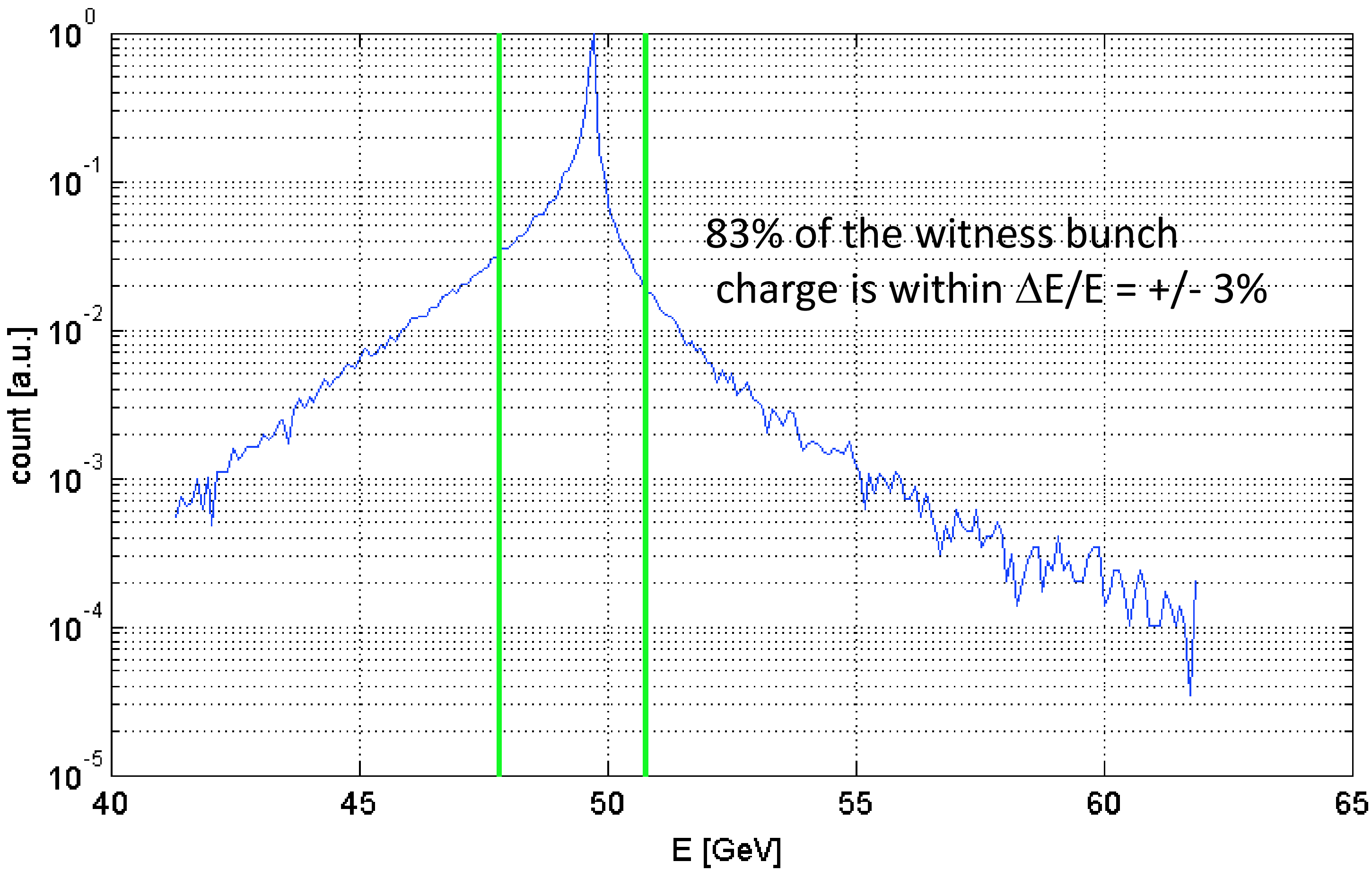



**Shaping of the WB may increase this number.**

**Shaping of the DB may reduce this number.**

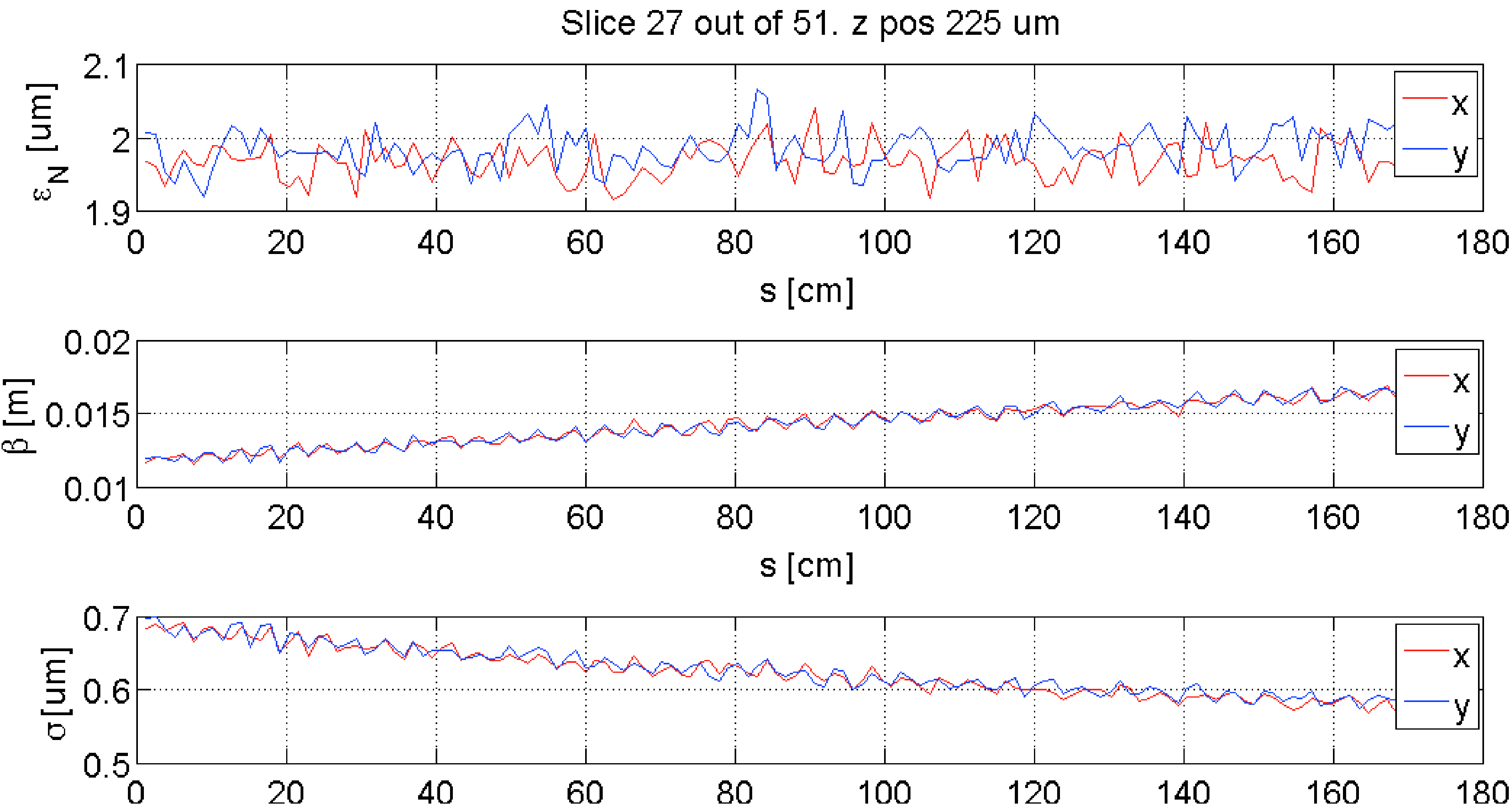
The drive beam to witness beam efficiency does not depend on the transformer ratio, for the optimization presented here.

# Final energy spectrum (T=1.5)




This is for a Gaussian bunch. The energy spread can be significantly improved by shaping the witness bunch.

# Witness bunch transverse parameters (T=1.5)




**Emittance**: preserved *(up to granularity of simulation)*

**Beta**: increases adiabatically [in the sense beta remains matched to $\beta_{mat} = \sqrt{2\gamma}/k_p$] as the energy increases. Thus, increase with $\sqrt{\gamma}$ (from 12 cm to $12cm \times \sqrt{2} = 17cm$).

**Sigma**: decreases with $1/\sqrt{\gamma}$ (from $70\mu m$ to $70\mu m \times 1/\sqrt{2} = 50\mu m$)

The small beating present may be due to small tranverse spot size compared to the simulation cell size.

# Transformer ratio

- In the arxiv-publication, we chose to use the T = 1.0 parameter set. Rationale at the time: prioritize looser tolerances (low T). Drive beam energy is an issue due to synchrotron radiation.
- High transformer ratios (> 2) requires shaped drive bunches. High transformer ratios have not been demonstrated in PWFA experiments. Plans to demonstrate with FACET-II.
- Choice: present PWFA-LC parameters with shaped drive bunches and high drive ratio?

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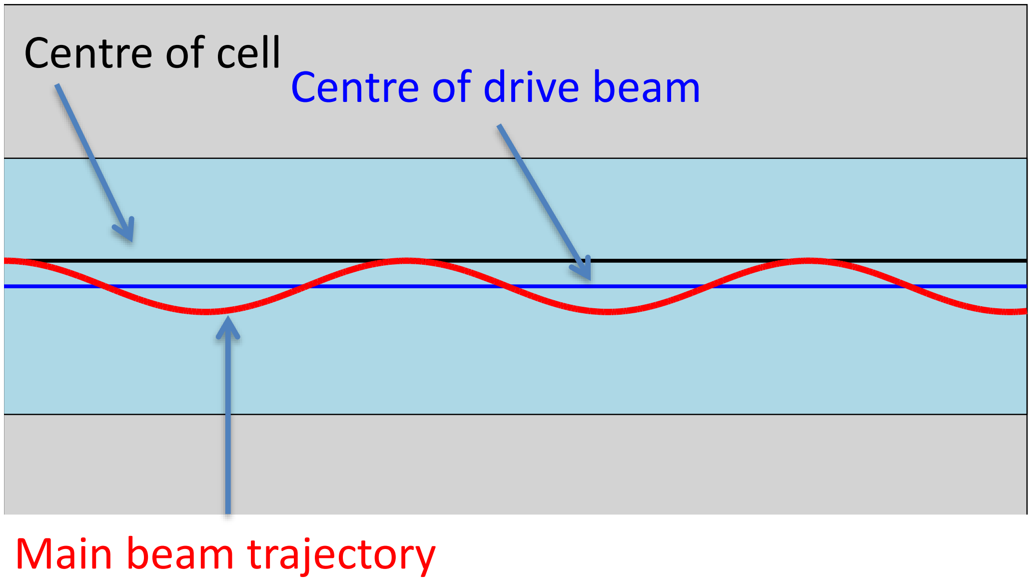
# Transverse Tolerances

- RF linear colliders: misaligned quadrupoles a major source of emittance growth.
- A plasma cell is a long, very strong quadrupole
- Laser or drive beam centre defines centre of the focusing
- This puts strong tolerances on drive-witness beam jitter

$$\sigma_y \approx 42\ \mathrm{nm} \left( \frac{\mathrm{GeV}}{\mathrm{E}} \frac{10^{16}\ \mathrm{cm}^{-3}}{\mathrm{n}_0} \right)^{\frac{1}{4}} \sqrt{\frac{\epsilon_y}{\mathrm{nm}}}$$

Based on slide from D. Schulte (EAAC 2015)




PWFA beam at 1.5TeV has $\sigma_y$=O(30 nm) for $n_0=2\times10^{16}\mathrm{cm}^{-3}$

$\Rightarrow$ Beam jitter stability O(3 nm) ?

$\Rightarrow$ Tough for laser/drive beam

$\Rightarrow$ Static misalignment is also critical

$\Rightarrow$ but depends on beam energy spread and tuning methods

Important to understand tolerances correctly

R&D programme essential on transverse alignment and stabilisation

**First order of magnitude estimate. Must be studied further in terms of emittance growth.**

# To be studied

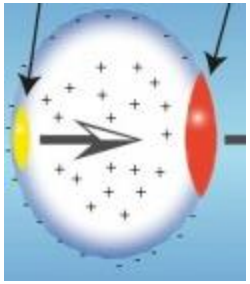
- Transverse and longitudinal tolerances for the plasma stage need to be further studied (Oslo continues this work, see Carl's presentation)
- Tolerance scaling laws should be incorporated in the plasma stage parameter optimization
- Optimization of a plasma stage with shaped bunches
  - May shape witness bunch, for lower energy spread, and higher efficiency
  - May shape drive bunch, for higher transformer ratio, and higher efficiency
- Is a baseline design on shaped bunches desired? Or, be conservative and stay with Gaussian?

# Extra

# Multiple scattering

Linear colliders require ~10 nm emittances.

- Multiple scattering may be significant, depending on beam size. Can be calculated analytically. In the quasi-linear regime, $n_b = n_0$ :



V. Lebedev and S. Nagaitsev, http://arxiv.org/abs/1304.2419

$$\varepsilon_f = \sqrt{\varepsilon_i^{\,2} + \frac{2Z(Z+1)r_e^2 N_b \Lambda_c}{\sqrt{2\pi}\sigma_s (d\gamma / ds)} (\gamma_f - \gamma_i)}$$

Acceleration to 1 TeV in $n_0$ = 1e17/cm$^3$ -> $\Delta\varepsilon$ ~ 10 um

Rules out working in the linear / quasi-linear regimes.

- However, in the blow-out regime, our matched beam sizes are very small. For small Z, emittance growth due to multiple scattering looks acceptable. Example: few nm for energy doubling from 500 GeV to 1 TeV.

Kirby, Siemann et al: http://accelconf.web.cern.ch/AccelConf/p07/PAPERS/THPMS047.PDF

- However, In the blow-out regime: matched e- beams might suffer from significant emittance growth due to ion motion. Significant when $n_b/n_0$ >> $M_i/m_e$. Topic under study, good progress. See next slide.

# Progress in Ion Motion

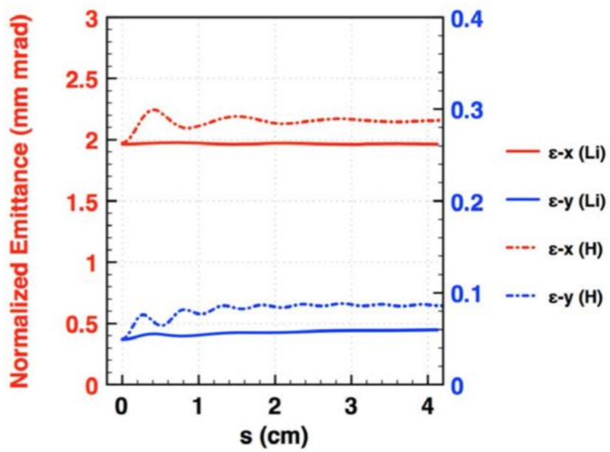
Mori at AAC 2014: https://aac2014.stanford.edu/sites/default/files/pdf-files/mori_aac_plenary_final_1.pdf

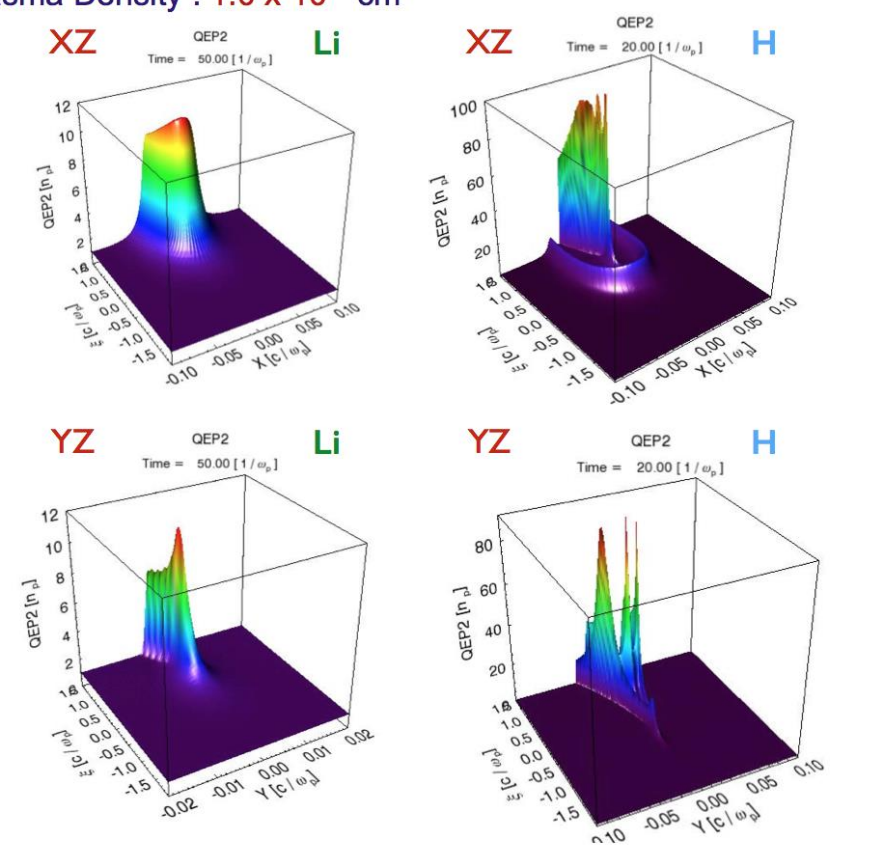
**UCLA**

## High resolution simulations show that emittance growth is not severe: Discovery

Trailing Beam: $\sigma_z$ = 10.0 μm , N = 1.0 x $10^{10}$ ,
$\sigma_x$ = 0.463 μm , $\varepsilon_{Nx}$ = **2.0 mm·mrad** , $\sigma_y$ = 0.0733 μm , $\varepsilon_{Ny}$ = **0.05 mm·mrad**
**Y = 48923.7 (25 GeV),** Plasma Density : 1.0 x $10^{17}$ $cm^{-3}$

$\sigma_x / \Delta_\perp$ = 75.9
$\sigma_y / \Delta_\perp$ = 12.0




**In Li, the emittance in x does not change, and in y direction it only increase by 20%.**

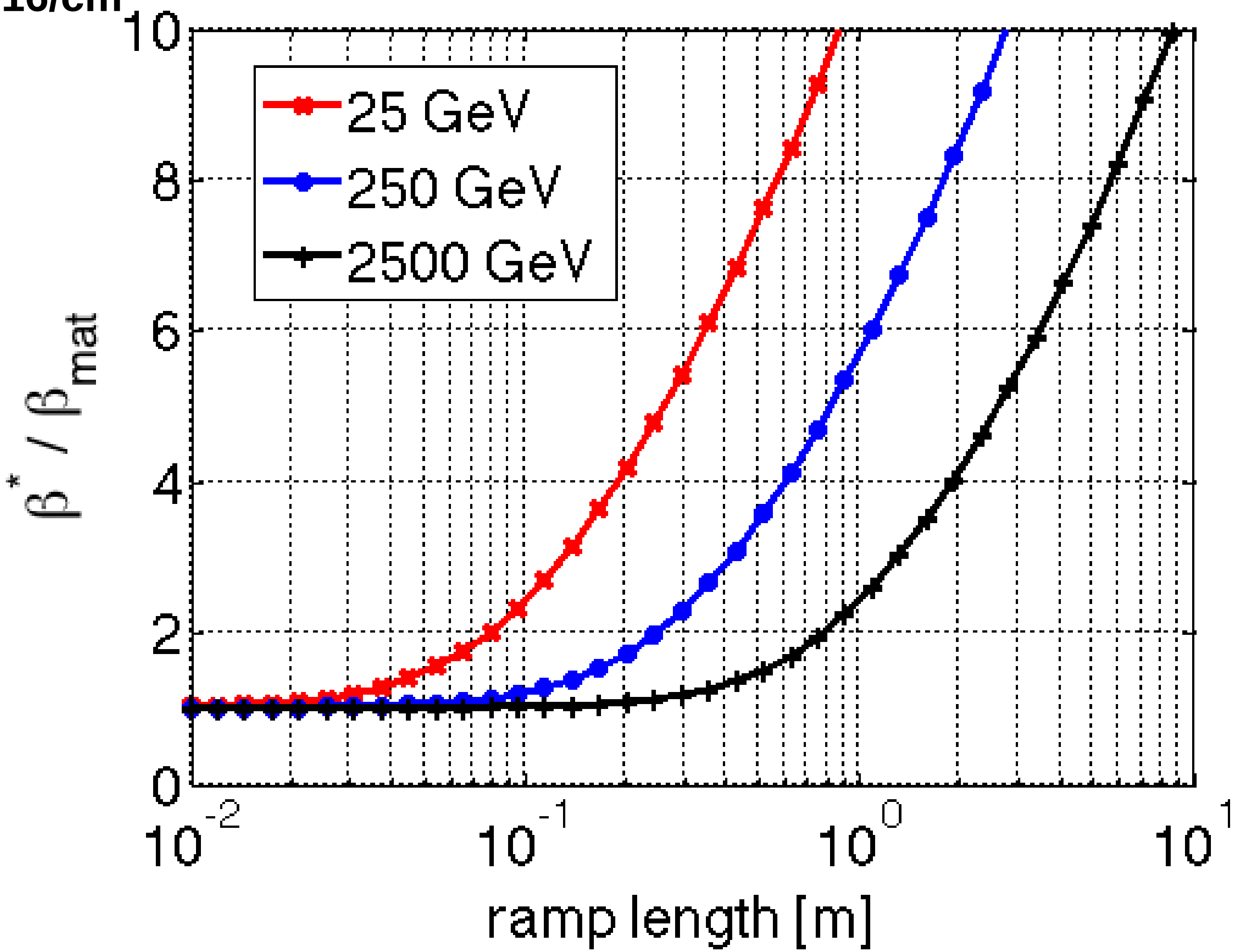
**In H, the emittance in x increase by 10%, and in y direction it increases by 70%.**

Only factor ~2 w/o any mitigation

# Beta demagnification from vacuum β* - required ramp length

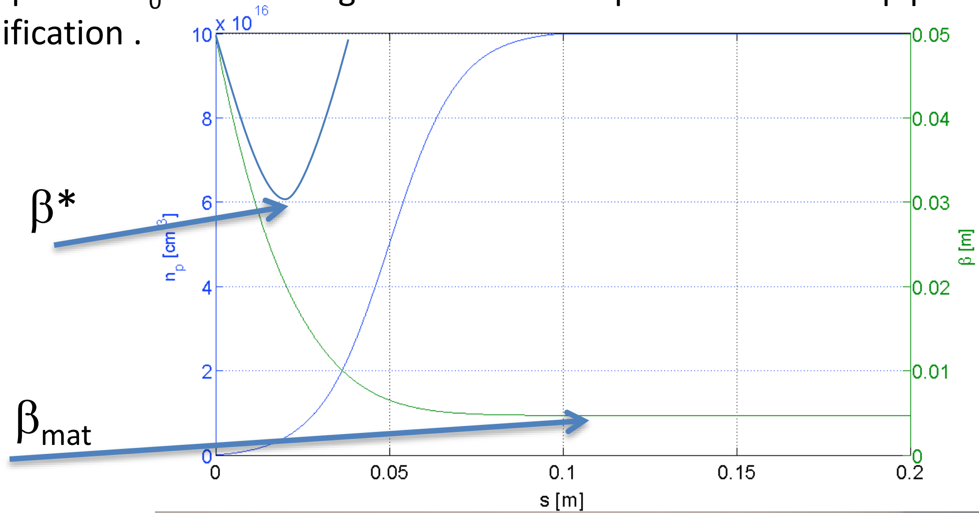
We may ask the question, for a given density (here 2e16/cm3), what is the ramp length required to reach a demagnification of a factor 10. The answer is encouraging; about 1 m for a 25 GeV beam.

**$n_0$ = 2e16/cm$^3$**



# Calculated beta demagnification from vacuum $\beta^*$ to matched value in plasma

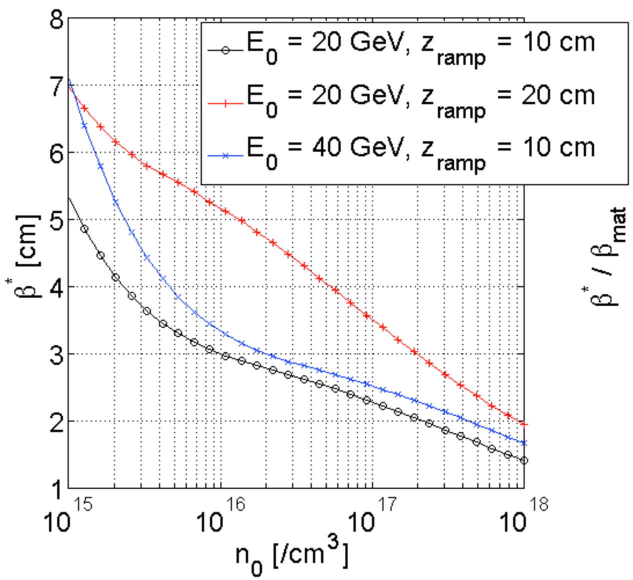
Based on a symmetric arctan-fit to the experimentally measured ramp, we assume a gradual increase in plasma density, from zero to the flat top value $n_0$. The demagnification also depends on the ramp profile; a Gaussian profile gives slightly larger demagnification .



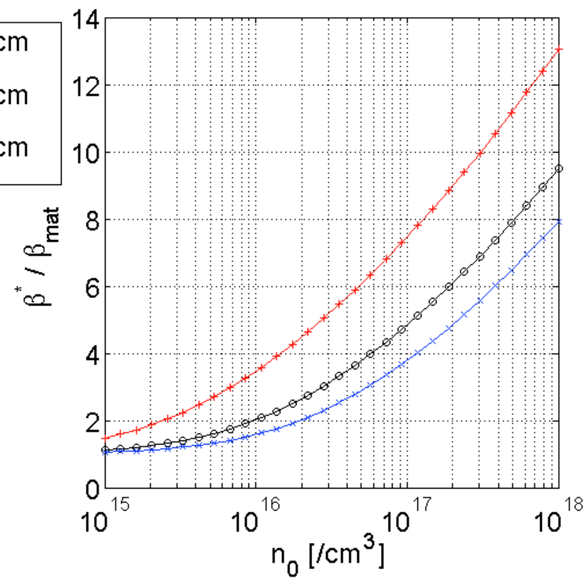

( Left: beta function in green, Plasma density in blue ).

The demagnification for three different scenarios are shown below for varying peak plasma density.

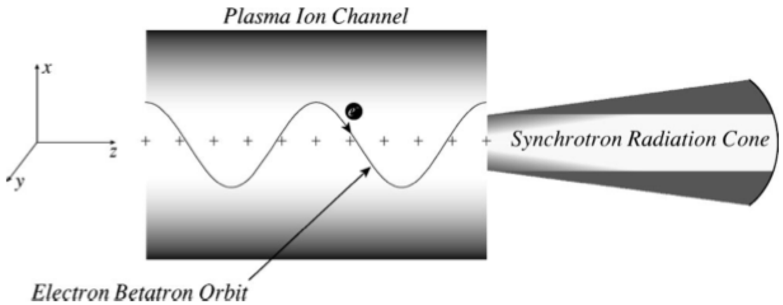
Absolute value of $\beta^*$ in vacuum :




Demagnification of $\beta^*$ due to ramp :



# Synchrotron radiation loss in plasma




Radiation loss per meter for a particle at $r = r_\beta$

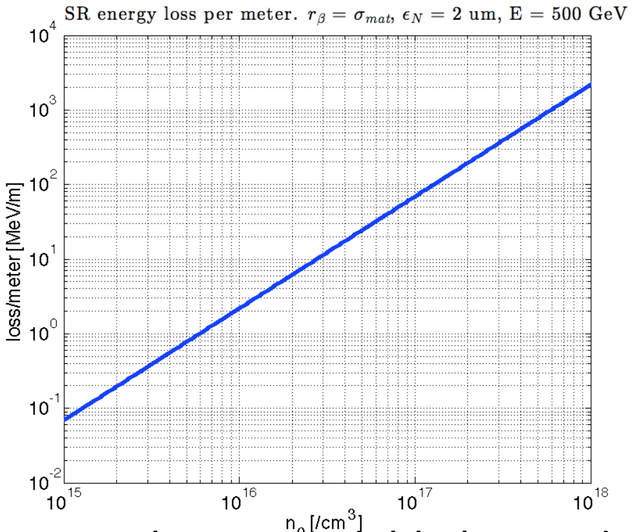
$$W' = r_e m_e c^2 \gamma^2 k_p^4 r_\beta^2 / 12$$

or in practical units

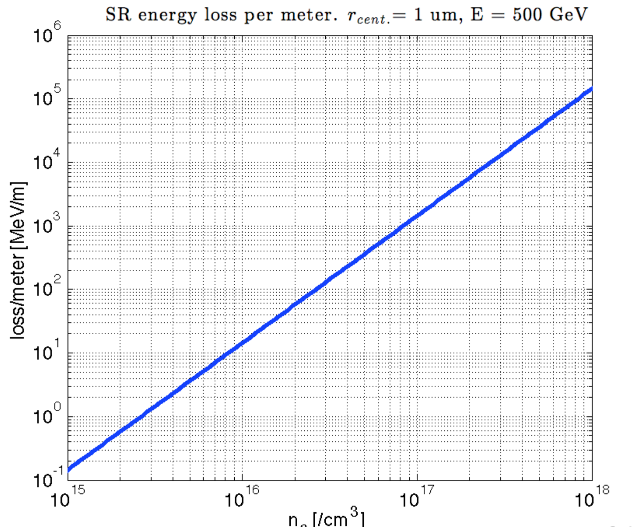
$$W' \text{ [GeV/m]} = 1505 \times \gamma^2 \text{ } [10^4] \times n_0^2 \text{ } [10^{16}/cm^3] \times r_\beta^2 \text{ [mm]}$$

Scales as $n_0^{3/2}$ for a matched beam.

Implications: unmatched parts of the beam will have significantly larger radiation loss (cf. chromatic errors in FF). Constraints centroid WB to DB offset (but $\Delta x/\sigma_x$ does not seem like a fundamental challenge).




Example: unavoidable losses due to finite beam size, here $\varepsilon_n$ = 2 um.




Example: losses due 1 um centroid offset.

# Electron-hose instability

PWFA advantage: beam creates cavity; no cavity alignment issues (one of the drivers of NC rf linear collider design).
However, BBU-type electron-hose transverse instability :

$$\left(\frac{\partial}{\partial s}\gamma\frac{\partial}{\partial s}+\gamma k_\beta^2\right)x(s,z)=\int_0^z dz'\frac{\omega_0^3}{c^2}\sin\{\omega_0(z-z')\}x(s,z')$$

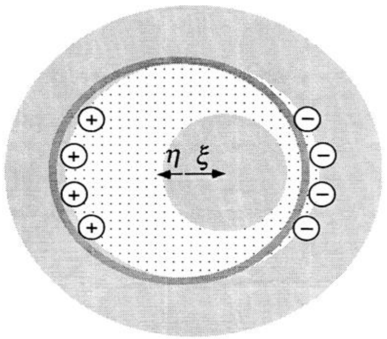


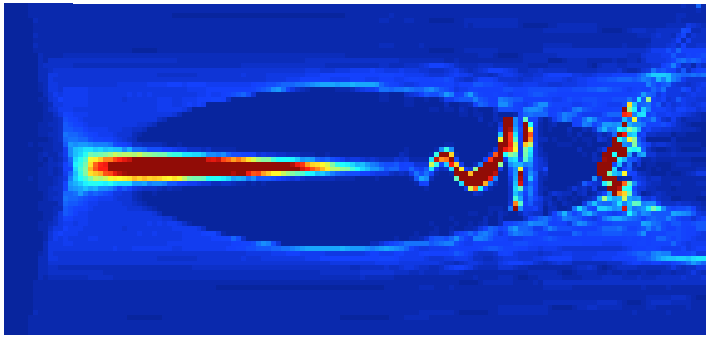
FIG. 2. A beam slice in the ion channel, displaced by an amount $\xi$ in the $x$ direction, induces a displacement $\eta$ of the channel wall, which responds as a simple harmonic oscillator with angular frequency $\omega_0$, deflecting follow-on portions of the beam.

* LHS: identical to BBU
* RHS: driving term independent of charge, cannot change $k_\beta$ -> BNS-type damping not available.

x: beam transverse displacement
s: coordinate along plasma
z: coordinate along bunch
$k_\beta$: betatron wavenumber in plasma;
$k_\beta = k_p / \sqrt{(2\gamma)}$;
$k_p = \omega_0\sqrt{2}$ is the plasma wavenumber

Asymptotic solution :

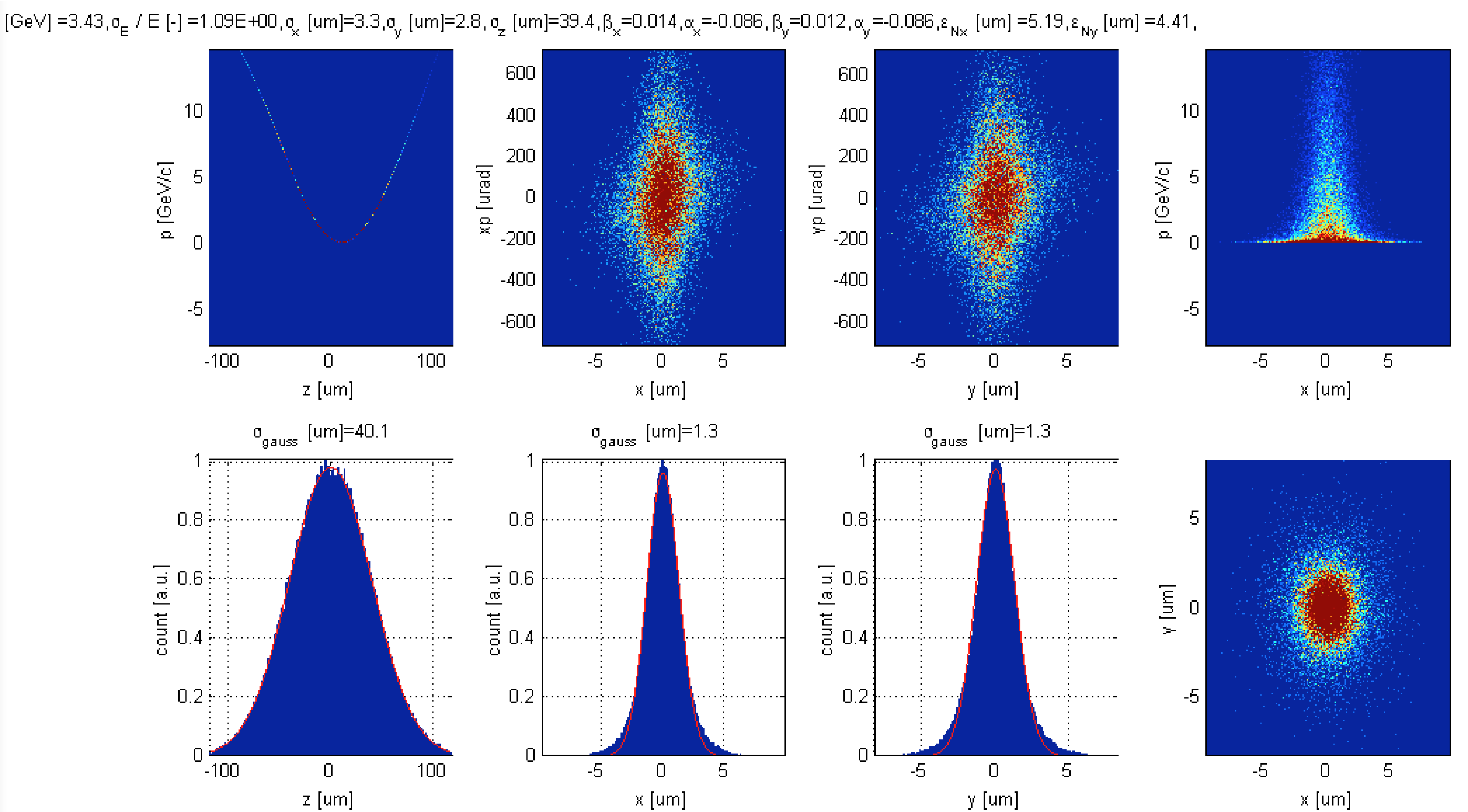
$$x/x_0 \sim e^{c(k_p s/\sqrt{\gamma})^{1/3}(k_p z)^{2/3}}$$



Mitigating factors :
- $k_p z$: reduce $\sigma_z$, reduce $n_0$
- $k_\beta s$ : increase # of stages, increase $\gamma_0$

Example : WB with offset $\Delta x/\sigma_x = 3$ (huge, for illustration), after energy doubling from 25 GeV to 50 GeV. Effects for $\Delta x/\sigma_x = 0.1$ negligible for 100 um beam. **Challenge: quantify for nm beam**.

# Full simulation T=1.5 – DB final time step



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