PWFA Interstage considerations

ALEGRO workshop 2018

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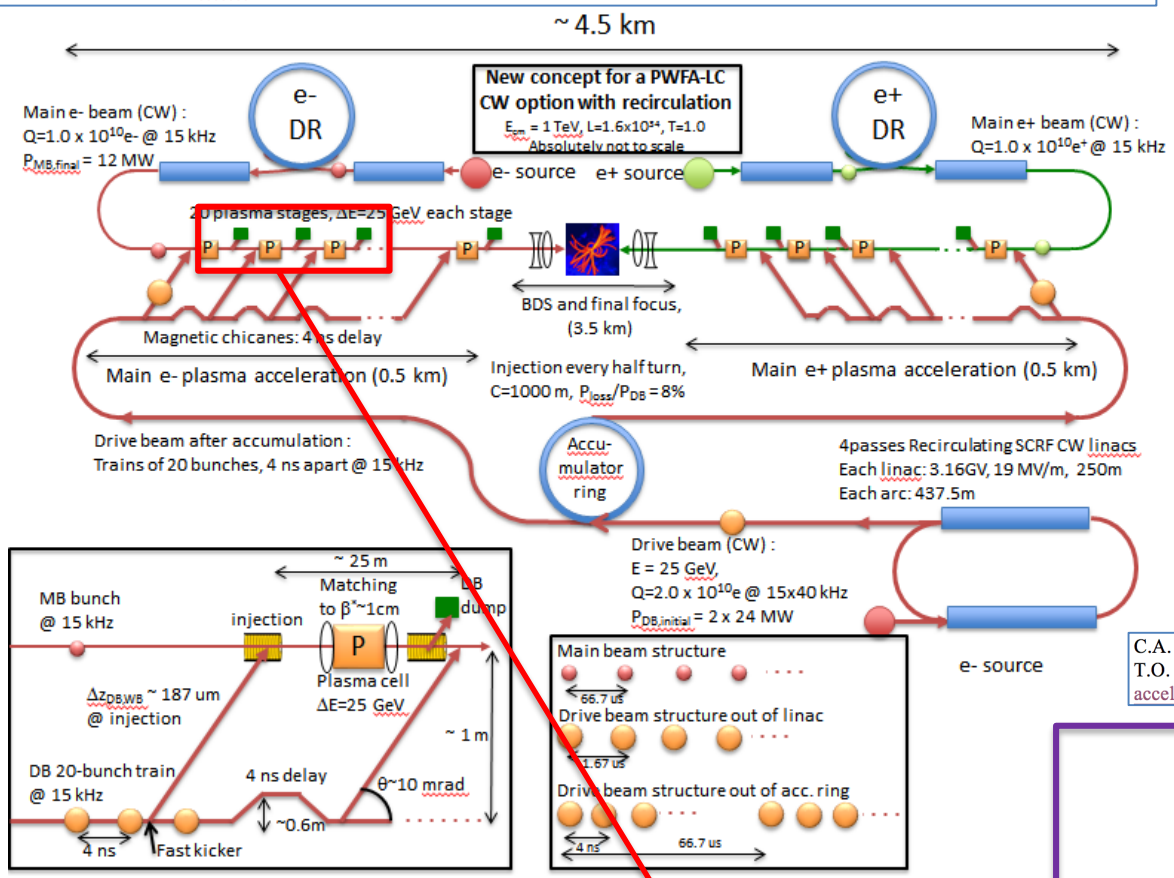
Mostly based on discussions in SLAC-Oslo PWFA-LC work 2013-2014.
Some results is published in :

C.A. Lindstrøm, E. Adli, J.M. Allen, J.P. Delahaye, M.J. Hogan, C. Joshi, P. Muggli, T.O. Raubenheimer, V. Yakimenko, "Staging optics considerations for a plasma wakefield acceleration linear collider", Nucl. Instrum. Methods Phys. Res. A **829**, 224 (2016)

C. A. Lindstrøm and E. Adli, "Design of general apochromatic drift-quadrupole beam lines", Phys. Rev. Accel. Beams **19**, 071002 (2016)

E. Adli, J. P. Delahaye et al., "A Beam Driven Plasma-Wakefield Linear Collider: From Higgs Factory to Multi-TeV", Snowmass 2013, http://arxiv.org/abs/1308.1145

In the 2013 Snowmass paper: no interstage considerations

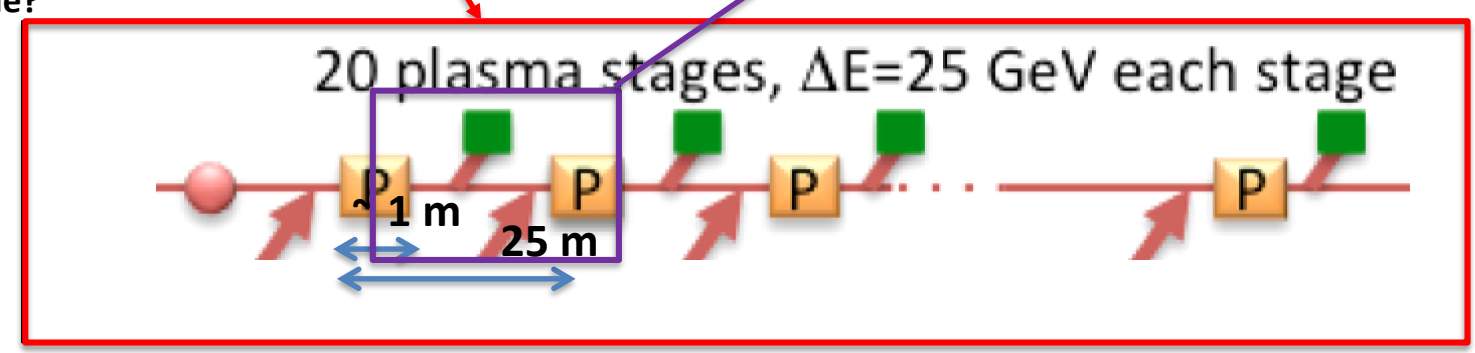


A small study was done by Oslo-SLAC in 2014-2015
Study based on Snowmass parameters.

C.A. Lindstrøm, E. Adli, J.M. Allen, J.P. Delahaye, M.J. Hogan, C. Joshi, P. Muggli, T.O. Raubenheimer, V. Yakimenko, "Staging optics considerations for a plasma wakefield acceleration linear collider", Nucl. Instrum. Methods Phys. Res. A **829**, 224 (2016)

?

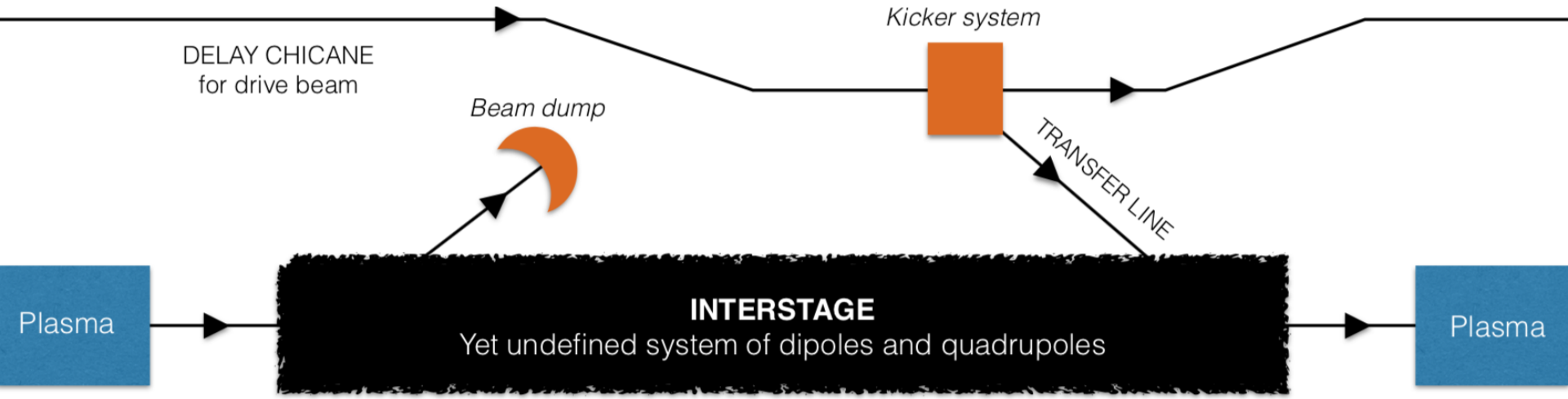
1 GV/m requires max. ~20 m interstage length.
Is this possible?



PWFA interstage:

1) inject new and extract spent drive beams
2) rematching main beam

Preserving emittance, in particular for main beam (%-level emittance growth)
Minimize length to maximize overall gradient



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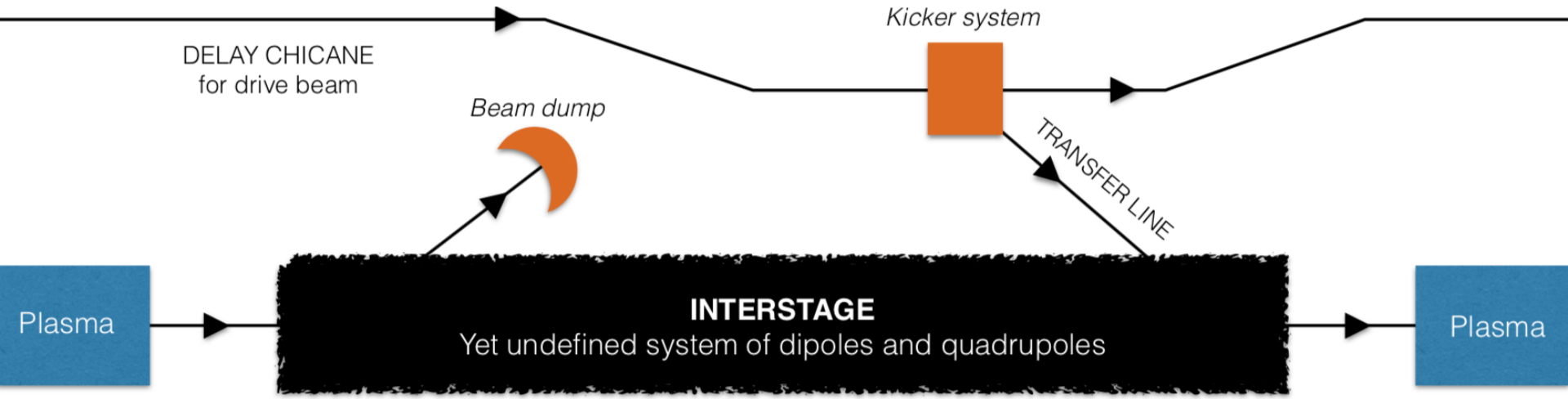

Fastest magnetic kickers: **few ns** (ILC kickers, T. Naito et al.) :

- Could be considered for DB extraction from the drive beam line
- Not an option for DB injection/extraction in the main beam line (few 100 fs drive-witness separation)
- **Assumed here:** static magnetic dipole fields, using energy to separate DB and WB,
- Other suggestions: multi-frequency TCAV, plasma-based kickers

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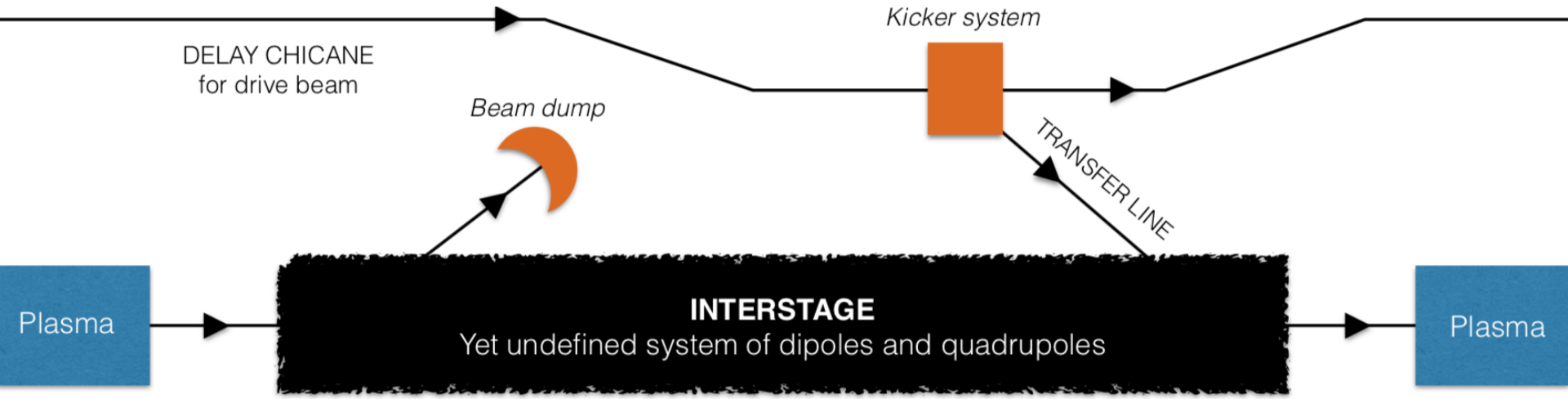

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- Could be considered for DB extraction from the drive beam line
- Not an option for DB injection/extraction in the main beam line (few 100 fs drive-witness separation)
- **Assumed here:** static magnetic dipole fields, using energy to separate DB and WB
  - 1 m space for injector/extractor dipole, 1 T field, ~6 mrad at high MB energy
- Other suggestions: multi-frequency TCAV, plasma-based kickers

PWFA interstage:

1) inject new and extract spent drive beams
2) **rematching main beam**

Preserving emittance, in particular for main beam (%-level emittance growth)
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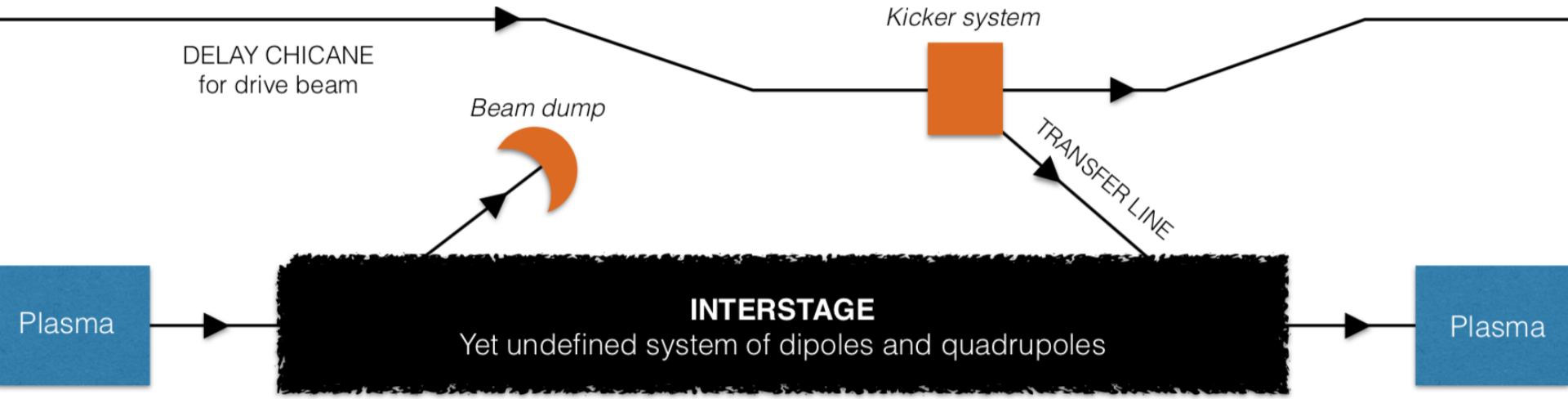


The rest of the talk

Interstage requirements

C.A. Lindstrøm, E. Adli, J.M. Allen, J.P. Delahaye, M.J. Hogan, C. Joshi, P. Muggli, T.O. Raubenheimer, V. Yakimenko, "Staging optics considerations for a plasma wakefield acceleration linear collider", Nucl. Instrum. Methods Phys. Res. A **829**, 224 (2016)

- Drive beam injection/extraction

- Collinearity

- Beta function matching

$$\beta_x(L) = \beta_y(L) = \beta_{mat}$$
$$\alpha_x(L) = \alpha_y(L) = 0$$

- Dispersion cancellation

$$D_x(L) = D'_x(L) = 0$$

- Limit bunch lengthening (R_{56})

$$R_{56}(L) \ll \frac{\sigma_z}{\delta} \approx 1 \text{ mm}$$

- Emittance preservation
  – limit chromaticity
  – limit synchrotron radiation

$$\frac{\Delta\epsilon}{\epsilon}(L) \ll 1\%$$

- Minimize length subject to all the above

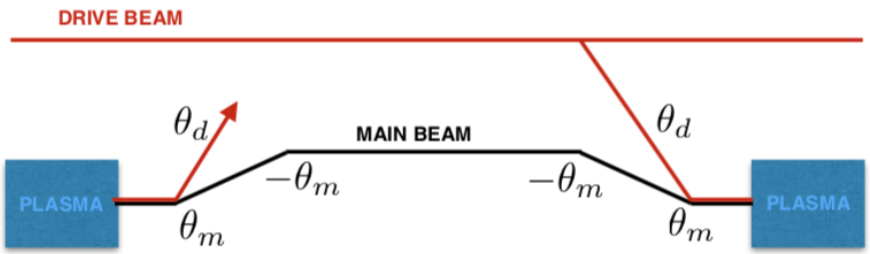
Injection/extraction of drive beams

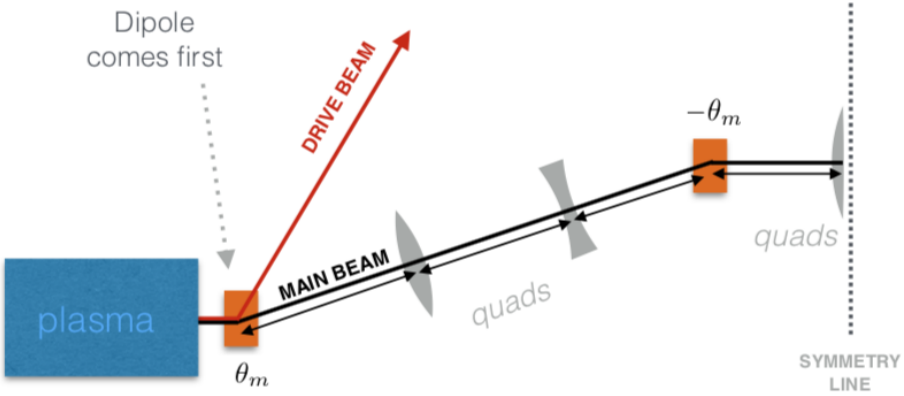
- Using dipoles:
  Filter beams by energy

- Injection and extraction are **symmetric**: greatly simplifies design.

- Injection/extraction **dipoles must be first and last magnets,** as quads would destroy the drive beam.

- Energy scaling:
  – **Dipole strength & length ~ constant** (as drive beam energy is constant)
  – **Synch. rad. power ~ E_{main}^2** (as $P_{SR} \sim E^2B^2$)





Dispersion cancellation

- Can be done by:
  - **Matching quadrupoles** (not independent of beam matching/chrom. correction)
  - **Inserting extra dipoles** (independent of beam matching/chrom. correction)

- Should be cancelled also to second order.

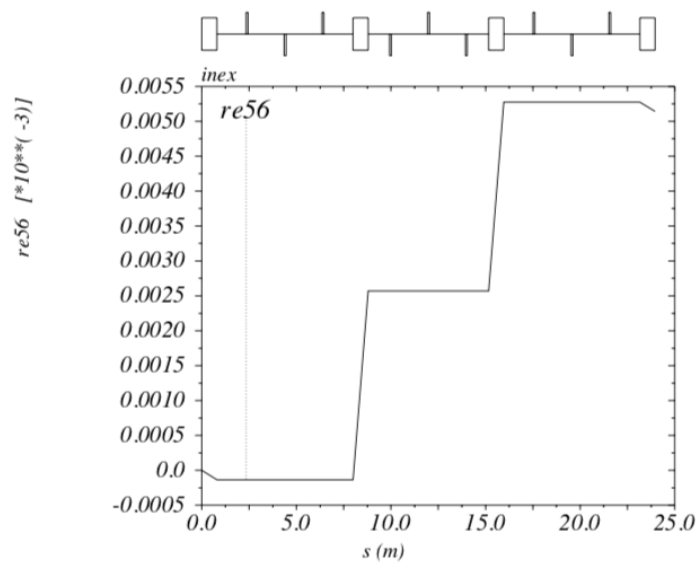
Limiting bunch lengthening (R_{56})

$$R_{56}(L) \ll \frac{\sigma_z}{\delta} \approx 1 \text{ mm}$$

- R_{56}*(energy spread) must be <u>much smaller</u> than bunch length.

- **Not a big problem** due to relatively **weak dipoles** and **short distances**.

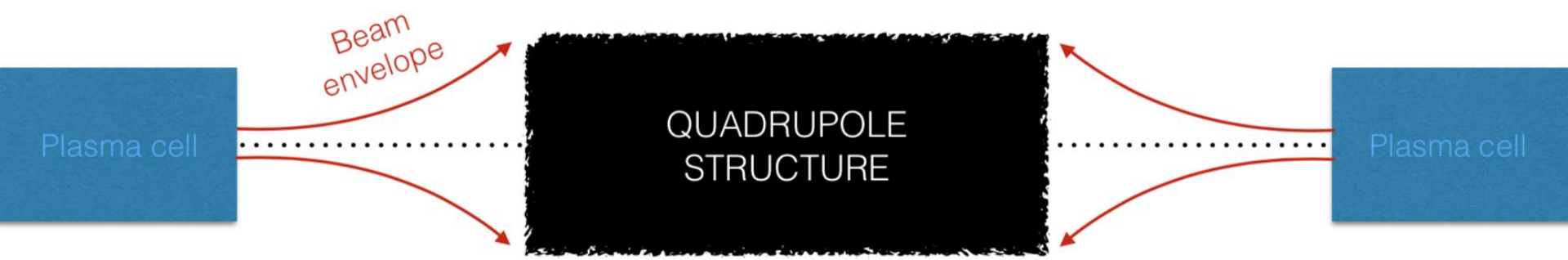
- Though if necessary, a method for matching $R_{56} = 0$:
  - **Zero dispersion in dipoles**.



Matching beam to plasma

$$\beta_x(L) = \beta_y(L) = \beta_{mat}$$
$$\alpha_x(L) = \alpha_y(L) = 0$$

- Naive matching is simple: **place quadrupoles, match strengths/separations.**
- **Strong plasma channel focusing** ⇒ small plasma betas
  ⇒ strongly diverging / large beam
  ⇒ long/strong quadrupoles ⇒ **large chromaticity** (big challenge)
- Intrinsic PWFA problem: **Strong accelerating fields** ⇒ strong focusing fields
  ⇒ large chromaticity
  ⇒ long/complex interstage ⇒ **low effective accel. gradient**
- In essence: Requires **two "final focus systems" at every stage**.



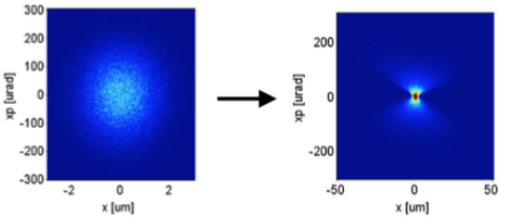
Chromaticity cancellation

- Chromaticity emerges as the **biggest challenge**.
- The PWFALC study assumes a ~1% rms. energy spread.
- Similar situation in **final focus systems** (SLC, ILC, etc).
- Conventionally **corrected using sextupoles**.

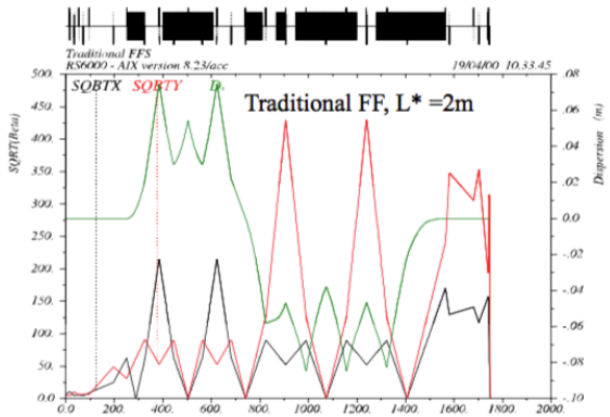
Example:

Initial phase space

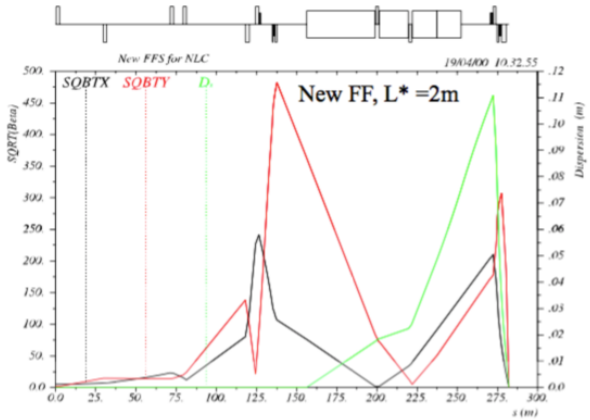
Final phase space (**1% energy spread**)



No chromaticity correction: **Emittance increases by orders of magnitude**



Traditional FF ~ 1750 m



Improved FF ~ 300 m

PWFA interstage needs two FF-like systems back-to-back in ~25m

*Figures from "Beam Delivery & beam-beam" by Andrei Seryi (SLAC)

Conventional solution: Sextupoles

- Sextupoles are **used to cancel chromaticity**.
- Effect is stronger with:
  - **Large dispersion** (requires dipoles)
  - **Large beam size** (requires strong quads or long lattices)
- **Long and complex lattices** (which might be simplified?).
- Introduces **new problems:**
  - Dipoles must ramp with energy
    - ⇒ **SR scales poorly** with energy
  - -I transforms require repeated sections
    - ⇒ unnecessarily **long lattices**
  - Thick sextupoles (imperfect -I transforms)
    - ⇒ **geometric errors** (emittance growth)
  - Sextupoles need large beam sizes
    - ⇒ **increased energy spread** from SR (Oide effect)

Sextupole B-fields:

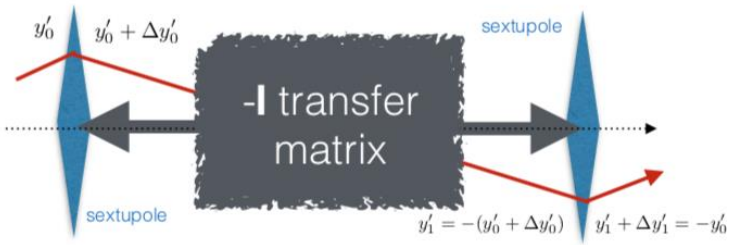
Non-linear geometric terms
NEEDS TO BE SMALL

Non-linear chromatic term
NEED TO BE CANCELLED

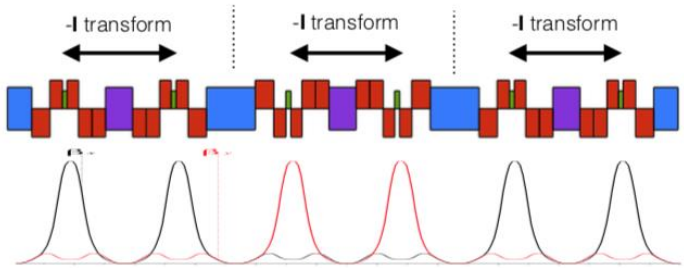
$$B_x \sim xy + \delta D_x y \qquad B_y \sim \frac{1}{2}(x^2 - y^2) + x\delta D_x + \frac{1}{2}\delta^2 D_x^2$$

Linear chromatic terms
CORRECT CHROMATICITY

Geometric term cancellation:



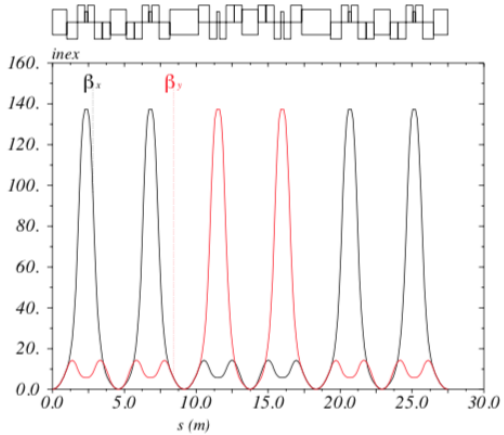
"Working" interstage using sextupoles:



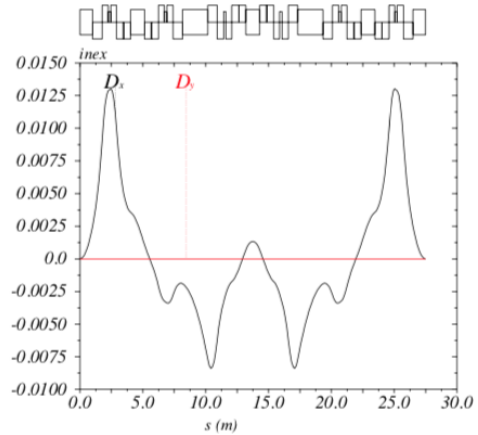
"Working" interstage using sextupoles

100 GeV, β_{mat} = 0.1m

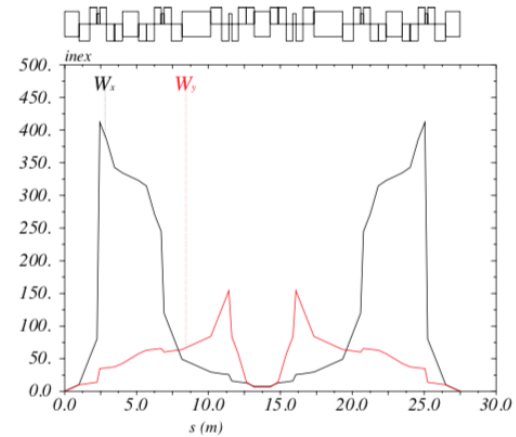
Beta functions



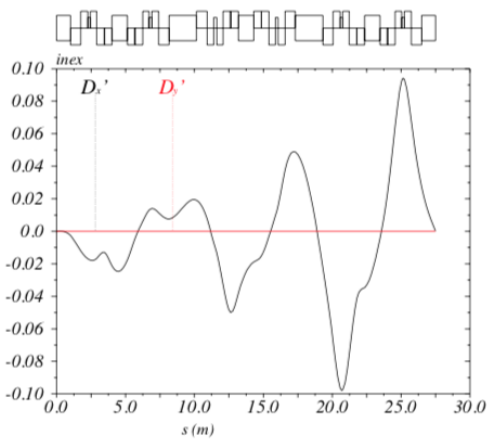
Dispersion



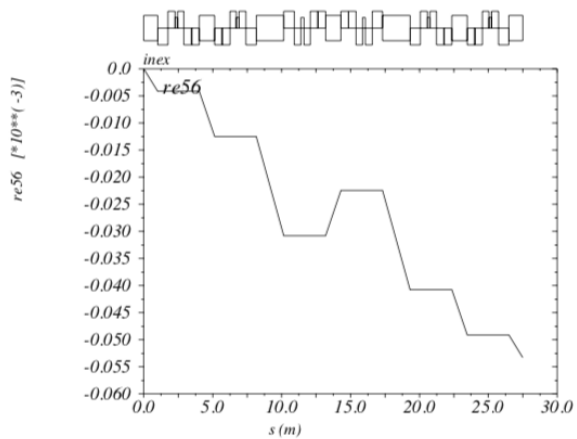
W-functions (chrom.)



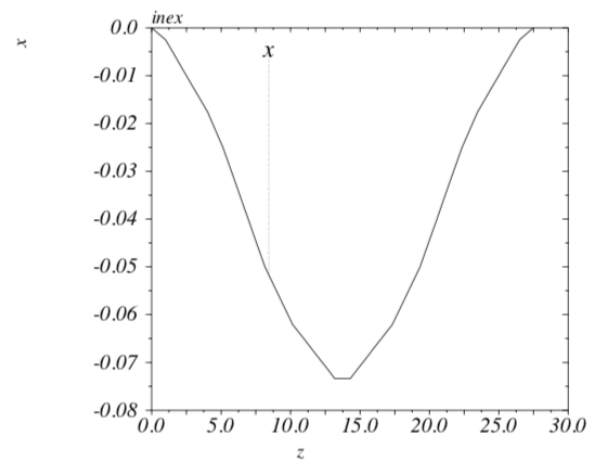
Second order dispersion



R_{56}



Footprint (x-z)



- This solution requires stronger sextupoles than currently manufacturable. Not a conceptual show-stopper.

Novel solution: Cancelling chromaticity using quadrupoles

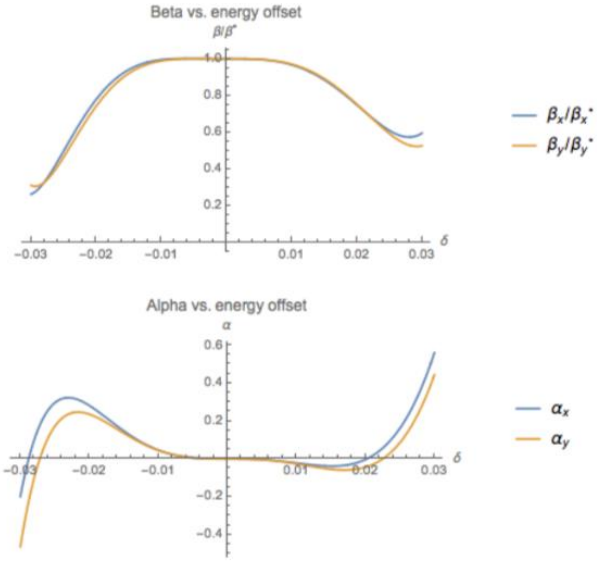
- We have developed a **new method for finding quadrupole-only lattices which cancel chromaticity** to the required order in energy offset.

- Benefits of using quadrupoles only:
  - no geometric terms ⇒ **keeps it linear**
  - no -I transforms ⇒ **much shorter**
  - no ramping dipoles ⇒ **better SR scaling**

- Similar solutions in light optics: **Superachromats** However: beam optics must simultaneously cancel chromaticity in 2 planes.

- Achieved by tailoring the energy offset (δ) expansion of β and α to be flat around δ = 0.

Simpler version used in light optics: "Superachromat" by Carl Zeiss

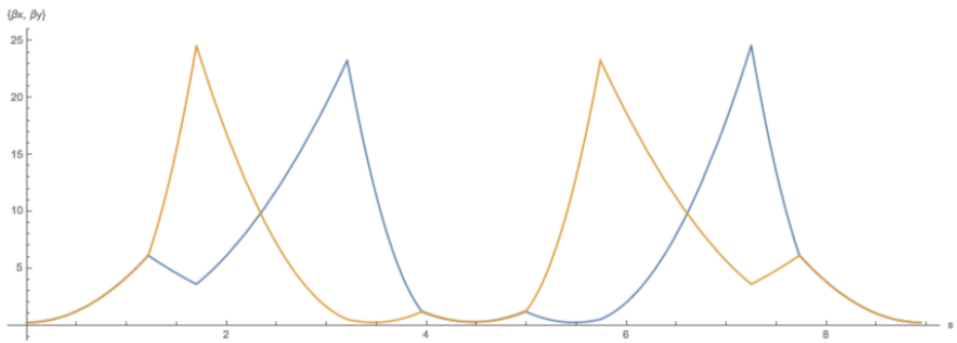


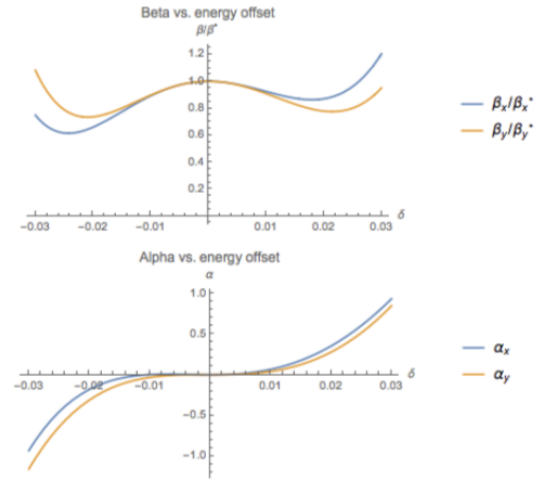
β and α vs. energy offset δ. Flat regions ⇒ no chromaticity

C. A. Lindstrøm and E. Adli, "Design of general apochromatic drift-quadrupole beam lines", Phys. Rev. Accel. Beams **19**, 071002 (2016)

Examples of chromaticity-free quadrupole lattices

- 8 quads: **cancel chromaticity to 1st order.**

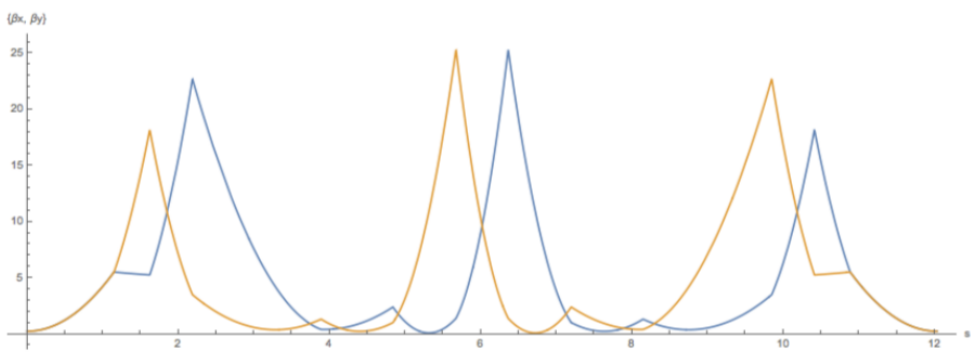


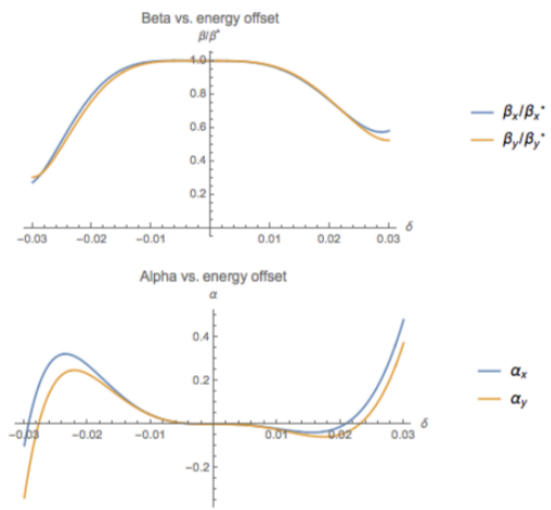


For 0.5% energy spread: $(\Delta\epsilon_x/\epsilon_x, \Delta\epsilon_y/\epsilon_y) = (0.000781, 0.000782)$

For 1.% energy spread: $(\Delta\epsilon_x/\epsilon_x, \Delta\epsilon_y/\epsilon_y) = (0.0141, 0.0142)$

- 12 quads: **cancel chromaticity to 2nd order.**





For 0.5% energy spread: $(\Delta\epsilon_x/\epsilon_x, \Delta\epsilon_y/\epsilon_y) = (0.000186, 0.000186)$

For 1.% energy spread: $(\Delta\epsilon_x/\epsilon_x, \Delta\epsilon_y/\epsilon_y) = (0.0119, 0.0119)$

C.A. Lindstrøm, E. Adli, J.M. Allen, J.P. Delahaye, M.J. Hogan, C. Joshi, P. Muggli, T.O. Raubenheimer, V. Yakimenko, "Staging optics considerations for a plasma wakefield acceleration linear collider", Nucl. Instrum. Methods Phys. Res. A **829**, 224 (2016)

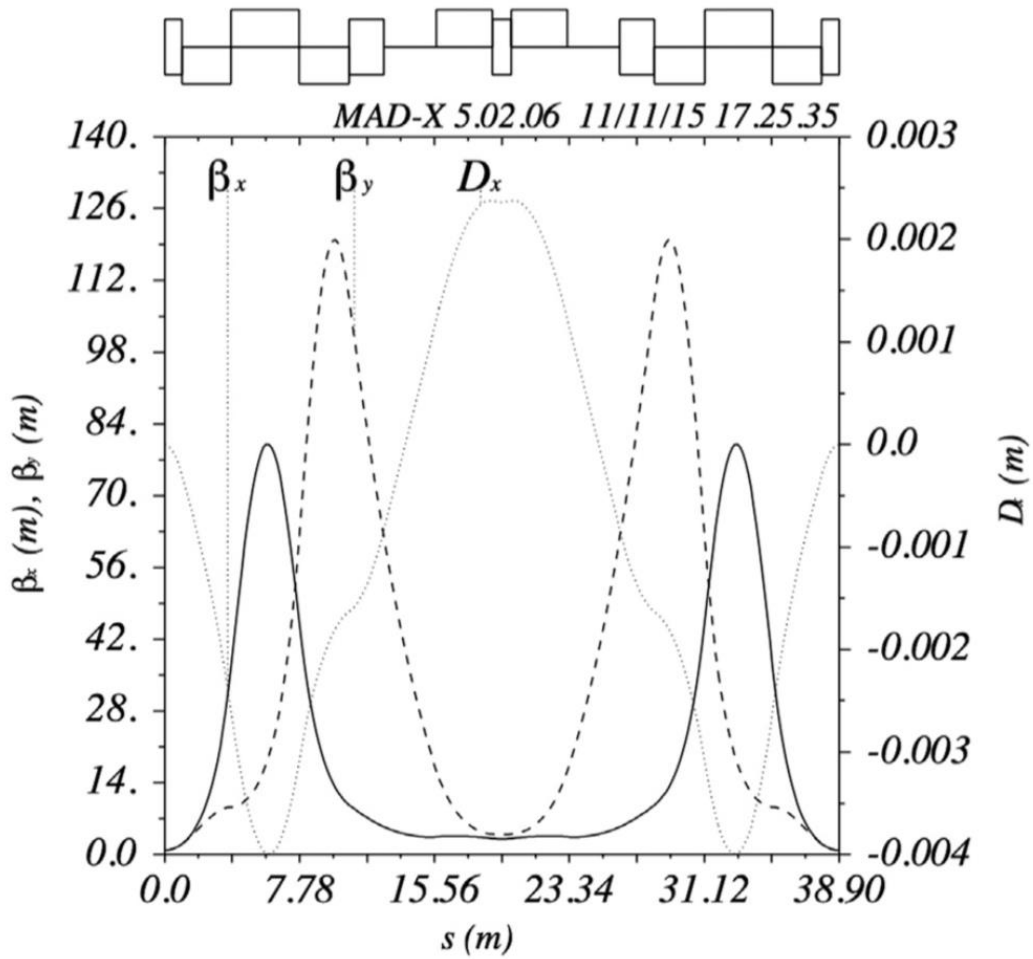


Fig. 2. Working example for 500 GeV, where 5 dipoles and 8 quadrupoles form a 39 m long C-chicane. Chromaticity is canceled by a linear lattice without sextupoles, however an uncorrected second-order dispersion leads to a 2% emittance growth.

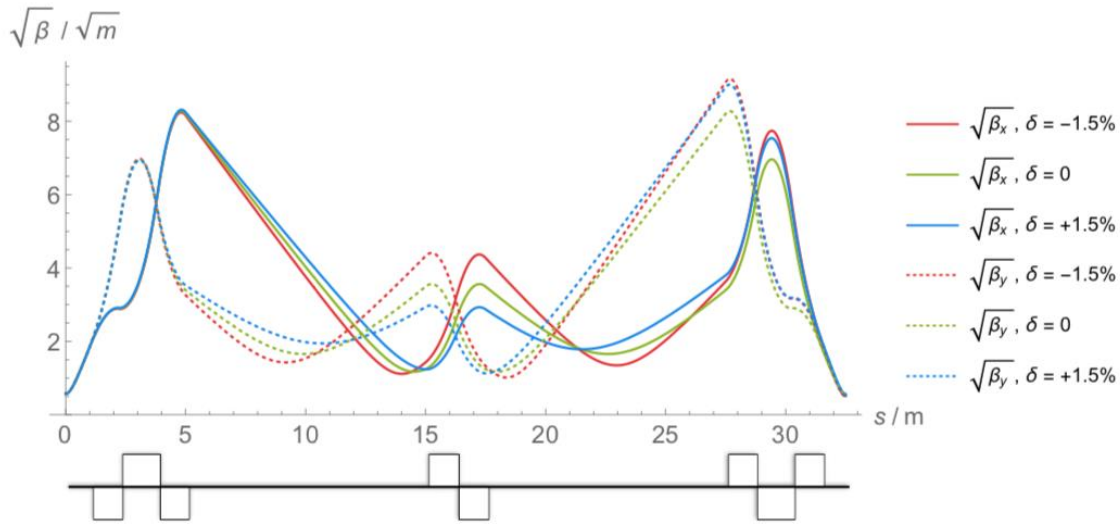
Example 1: PWFA staging (high energy collider)

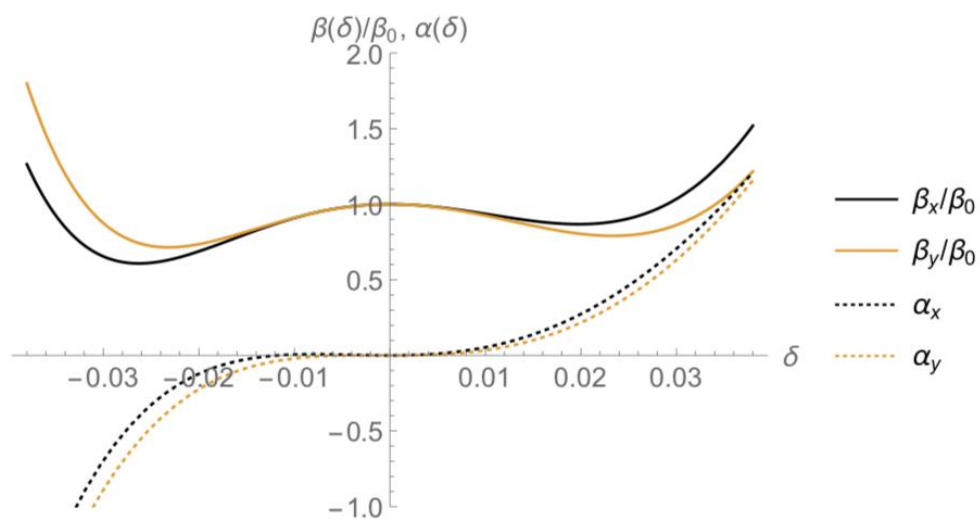
- **Suggestion #1**:
  Strong normal conducting quadrupoles (~160 T/m):

- Length 32 m, 8 quadrupoles.

- **First order apochromat** + normal matching in x/y:
  4 degrees of freedom in a anti-symmetric lattice (8 quads).

- For a 1% rms energy spread:
  0.96% emittance growth.
  (just within requirements)..

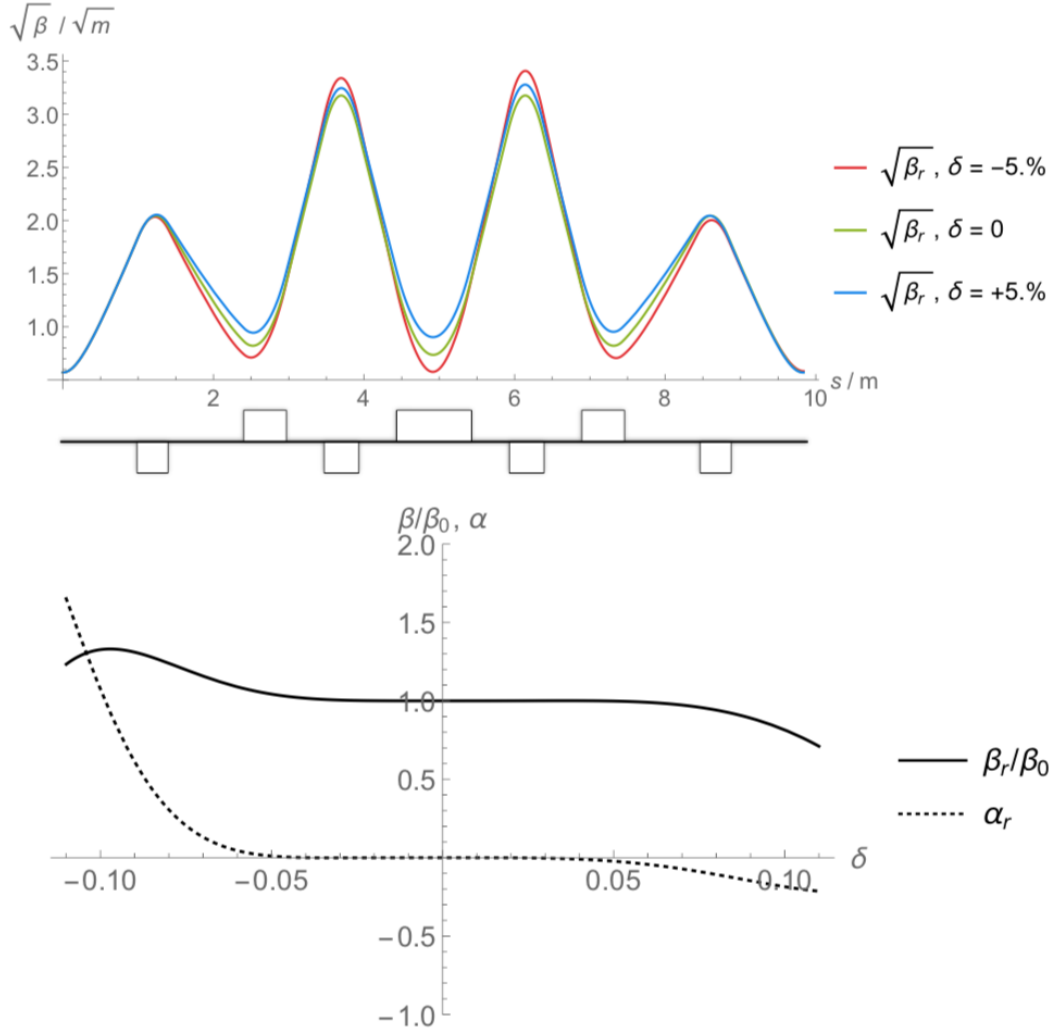




C. A. Lindstrøm and E. Adli, "Design of general apochromatic drift-quadrupole beam lines", Phys. Rev. Accel. Beams **19**, 071002 (2016)

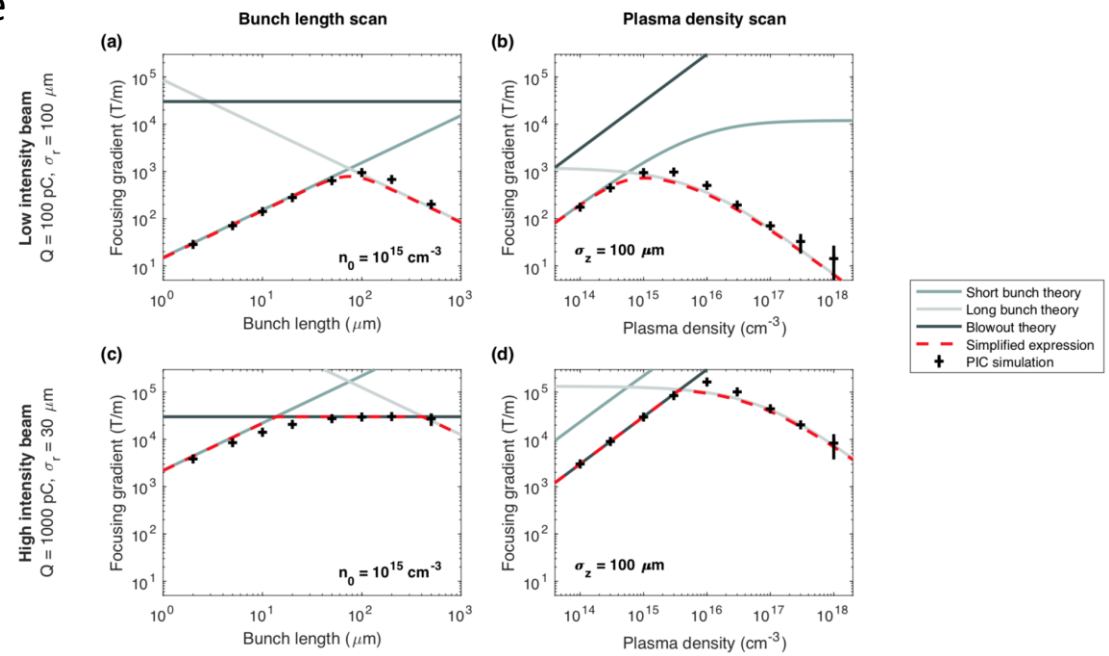
Example 1: PWFA staging (high energy collider)

- **Suggestion #2**: Plasma lenses (~3000 T/m) (very long)
- Length 10 m, 7 lenses.
- **Third order apochromat** + normal matching in only r: 4 degrees of freedom in a symmetric lattice (7 lenses).
- For a 1% rms energy spread: **0.000013% emittance growth.** ("achromatic" up to ~3% offset)

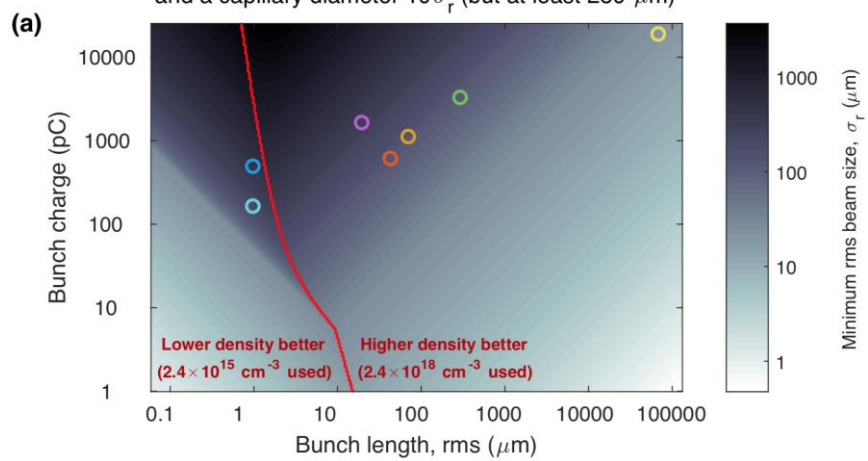


However,
due to wakes induced by very intense beams (collider beams),
active plasma lenses
may not conserve emittances to the desired level.

$$\sigma_r \gg \sqrt{\frac{2cQn_R^2}{I\left(\frac{1}{\sigma_z}+\sqrt{8\pi}k_p^2\sigma_z\right)}-\frac{2}{k_p^2}}.$$



Minimum beam size for negligible (< 3%) plasma wakefields
in an active plasma lens with 1 kA discharge current
and a capillary diameter 10σ_r (but at least 250 μm)



- ILC TDR : 111 μm, 649 T/m
- CLIC 0.5 TeV : 132 μm, 462 T/m
- CLIC 3 TeV : 125 μm, 508 T/m
- PWFA-LC : 303 μm, 87 T/m
- LPA-LC 1 : 422 μm, 45 T/m
- LPA-LC 2 : 190 μm, 221 T/m
- LHC : 20 μm, 12800 T/m

C. A. Lindstrøm and E. Adli, Submitted to Phys. Rev. Accel. Beams (2018), arXiv:1802.02750

C.A. Lindstrøm, E. Adli, J.M. Allen, J.P. Delahaye, M.J. Hogan, C. Joshi, P. Muggli, T.O. Raubenheimer, V. Yakimenko, "Staging optics considerations for a plasma wakefield acceleration linear collider", Nucl. Instrum. Methods Phys. Res. A **829**, 224 (2016)

Table 1
Energy scaling laws for the high energy regime ($E_m \gg E_d$). The same lattice is used for all energies, by scaling lengths as $\sqrt{\gamma}$, where γ is the main beam Lorentz factor.

Variable	Symbol	Energy scaling
Lattice length	L	$\sqrt{\gamma}$
Dipole, quad. length	l_d, l_q	$\sqrt{\gamma}$, $\sqrt{\gamma}$
β-functions	β	$\sqrt{\gamma}$
Spot size	σ_x	$1/\sqrt[4]{\gamma}$
Dispersion	D_x	$1/\sqrt{\gamma}$
Isochronicity	R_{56}	$1/\sqrt{\gamma}$
Chromatic amplitude	W	Const.
Emittance growth	$\frac{\Delta\epsilon}{\epsilon_0}$	Const.
Quad. field gradient	g_{max}	Const.
SR power, energy loss	P_{SR}, W_{SR}	γ, $\gamma^{1.5}$

Summary

- Both main beam matching and drive beam injection/extraction are challenging, and must be further studied
- Scaling relations and first order designs for the main beam matching has been obtained
  - In the ”high energy regime”
  - Therefore also valid for LWFA
- Plasma lensing has potential for simpler and shorter main beam matching
  - But too intense beam will lead to wakes and emittance grwoth
- Not yet studies: tolerances, CSR, radiation effects at very high energies (15 TeV beams?)
- First-order design presented here could be used as base for further tolerance, CSR studies. etc.

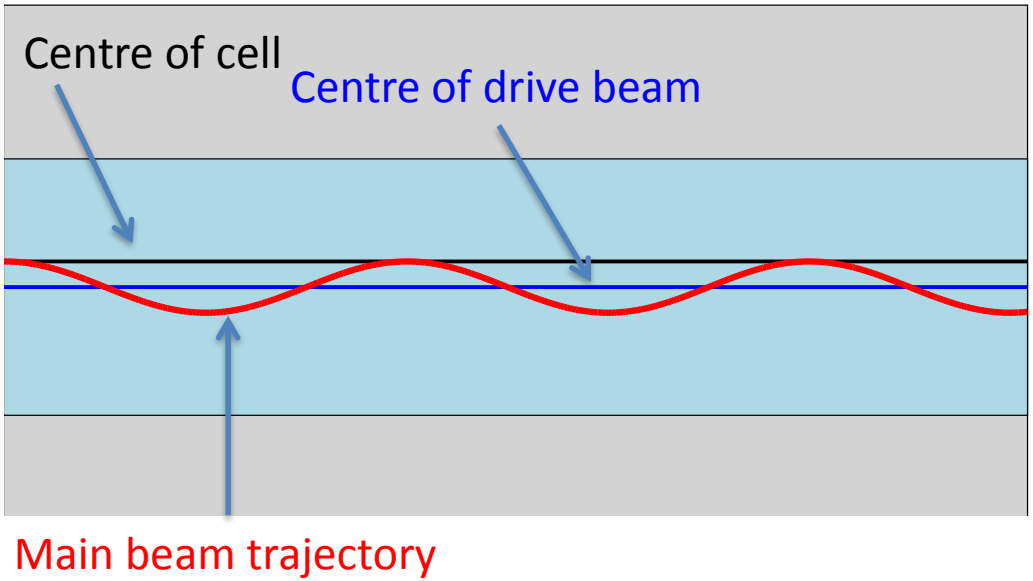
Extra

Transverse Tolerances

- RF linear colliders: misaligned quadrupoles a major source of emittance growth.
- A plasma cell is a long, very strong quadrupole
- Laser or drive beam centre defines centre of the focusing
- This puts strong tolerances on drive-witness beam jitter

$$\sigma_y \approx 42\ \mathrm{nm} \left(\frac{\mathrm{GeV}}{\mathrm{E}} \frac{10^{16}\ \mathrm{cm}^{-3}}{\mathrm{n}_0} \right)^{\frac{1}{4}} \sqrt{\frac{\epsilon_y}{\mathrm{nm}}}$$

Based on slide from D. Schulte (EAAC 2015)



PWFA beam at 1.5TeV has σ_y=O(30 nm) for $n_0=2\times10^{16}\mathrm{cm}^{-3}$

⇒ Beam jitter stability O(3 nm) ?

⇒ Tough for laser/drive beam

⇒ Static misalignment is also critical

⇒ but depends on beam energy spread and tuning methods

Important to understand tolerances correctly

R&D programme essential on transverse alignment and stabilisation

First order of magnitude estimate. Must be studied further in terms of emittance growth.

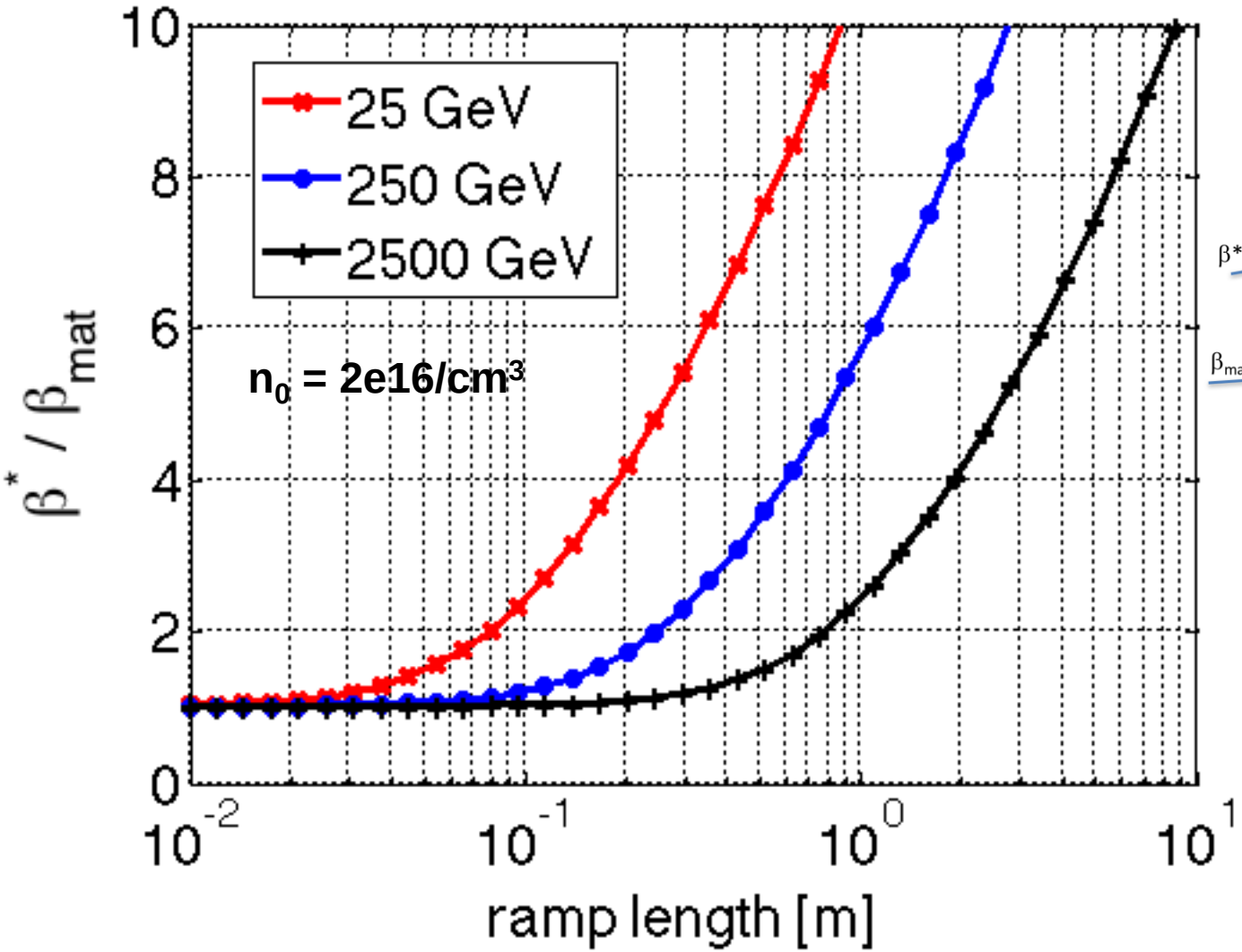
To be studied

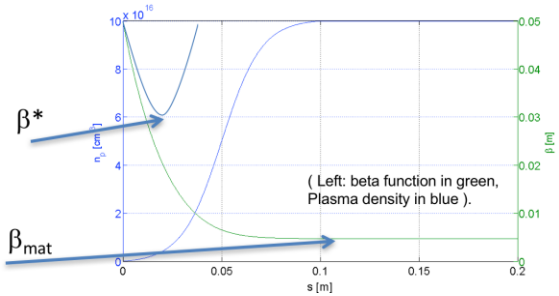
- Transverse and longitudinal tolerances for the plasma stage need to be further studied (Oslo continues this work, see Carl's presentation)
- Tolerance scaling laws should be incorporated in the plasma stage parameter optimization
- Optimization of a plasma stage with shaped bunches
  - May shape witness bunch, for lower energy spread, and higher efficiency
  - May shape drive bunch, for higher transformer ratio, and higher efficiency
- Is a baseline design on shaped bunches desired? Or, be conservative and stay with Gaussian?

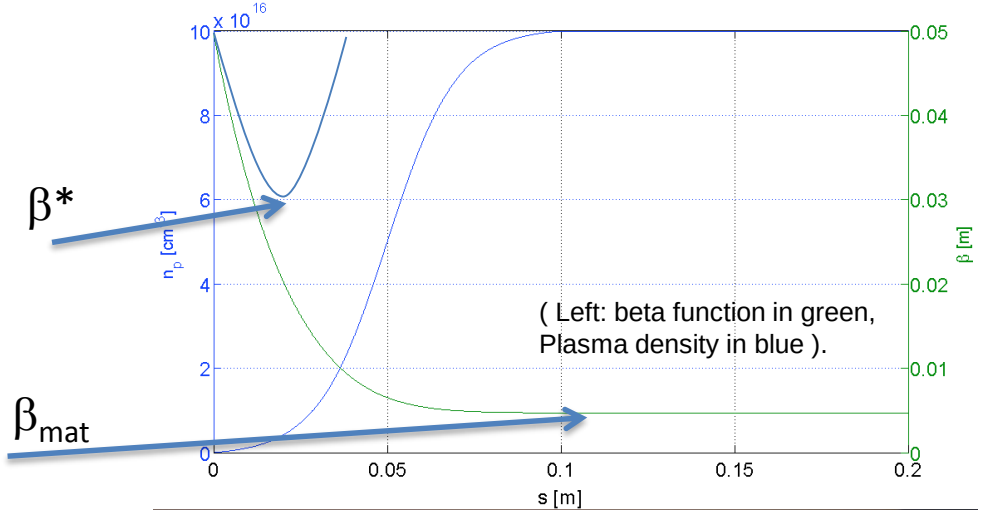
Beta demagnification from vacuum β* - required ramp length

We may ask the question, for a given density (here 2e16/cm3), what is the ramp length required to reach a demagnification of a factor 10. The answer is encouraging; about 1 m for a 25 GeV beam.

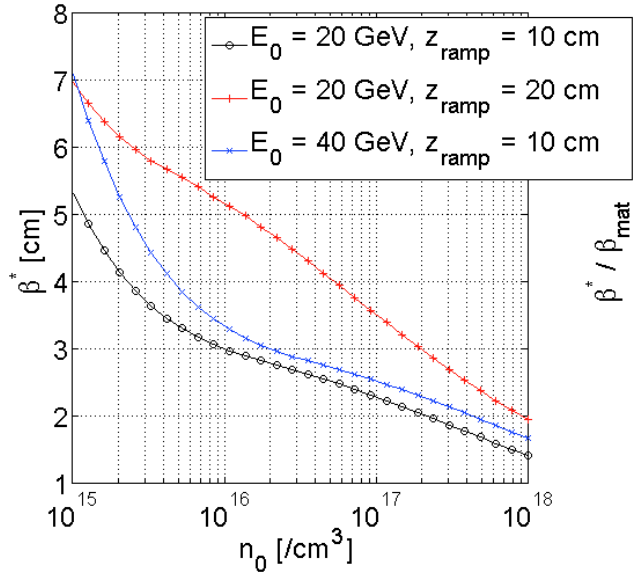
Example, Assuming atan2 plasma-profile.



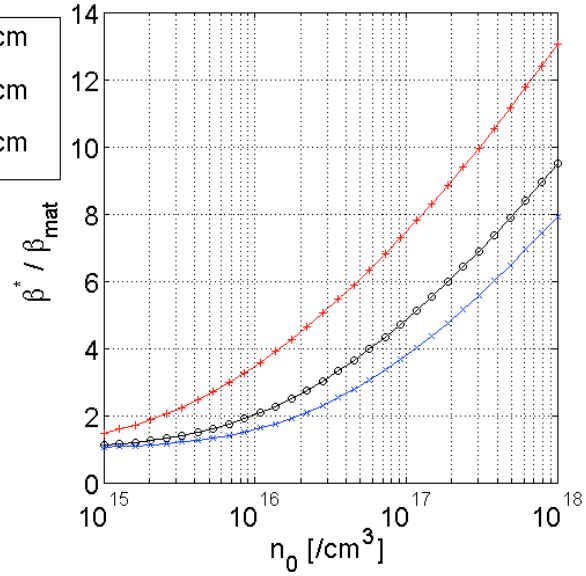




Absolute value of β* in vacuum :

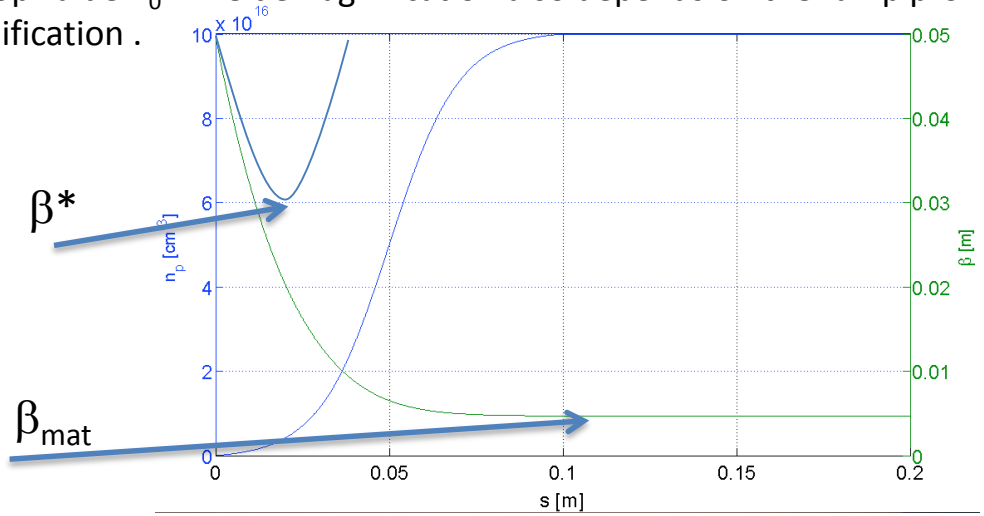


Demagnification of β* due to ramp :



Calculated beta demagnification from vacuum β* to matched value in plasma

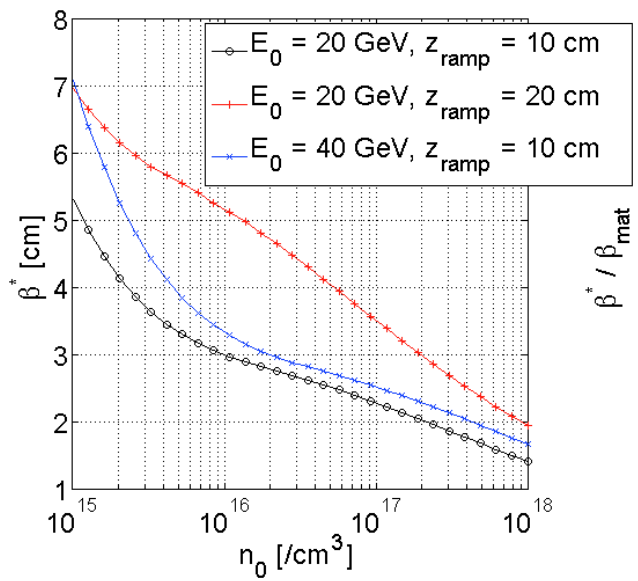
Based on a symmetric arctan-fit to the experimentally measured ramp, we assume a gradual increase in plasma density, from zero to the flat top value n_0. The demagnification also depends on the ramp profile; a Gaussian profile gives slightly larger demagnification .



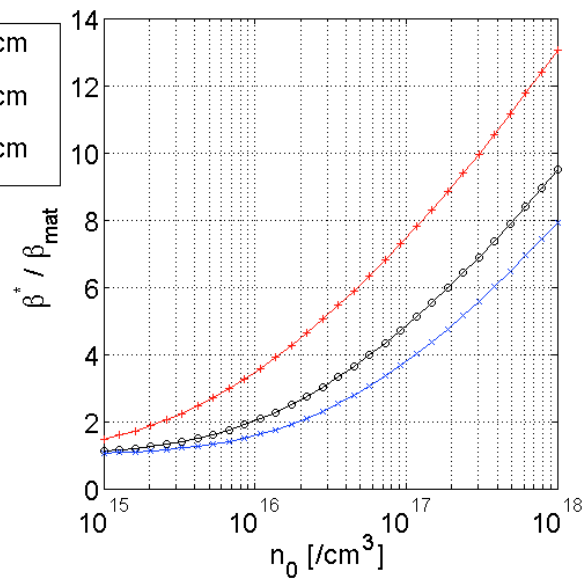
(Left: beta function in green, Plasma density in blue).

The demagnification for three different scenarios are shown below for varying peak plasma density.

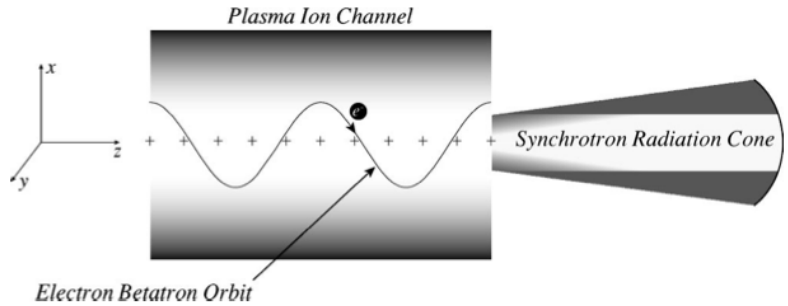
Absolute value of β* in vacuum :

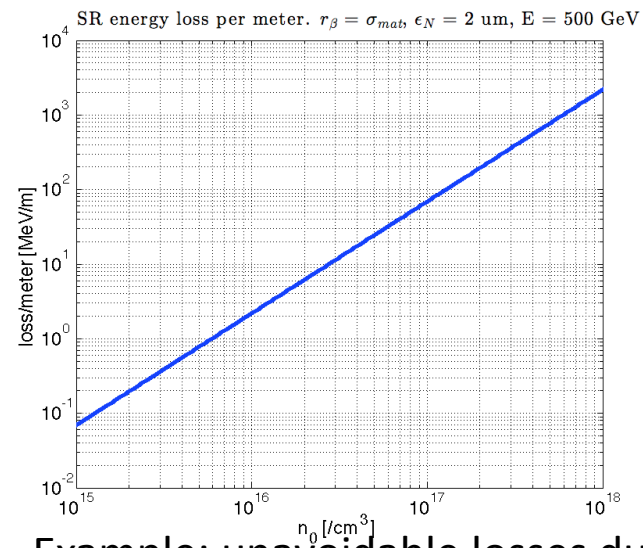


Demagnification of β* due to ramp :



Synchrotron radiation loss in plasma





Example: unavoidable losses due to finite beam size, here ε_n = 2 um.

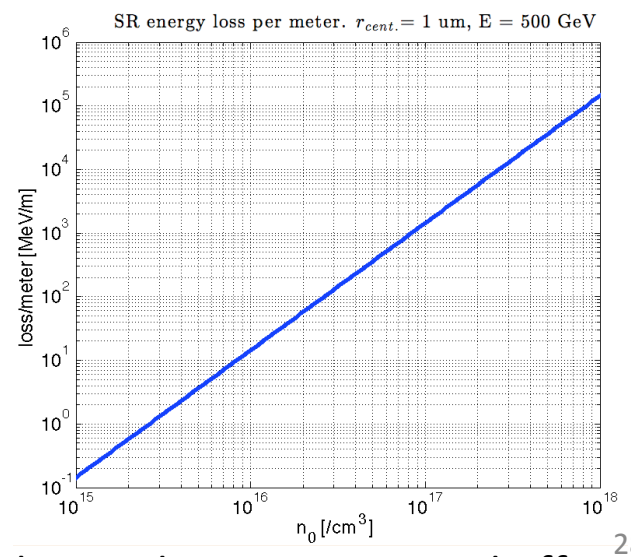
Radiation loss per meter for a particle at $r = r_\beta$

$$W' = r_e m_e c^2 \gamma^2 k_p^4 r_\beta^2 / 12$$

or in practical units

$$W' \text{ [GeV/m]} = 1505 \times \gamma^2 \ [10^4] \times n_0^2 \ [10^{16}/cm^3] \times r_\beta^2 \ \text{[mm]}$$

Scales as $n_0^{3/2}$ for a matched beam.



Implications: unmatched parts of the beam will have significantly larger radiation loss (cf. chromatic errors in FF). Constraints centroid WB to DB offset (but $\Delta x/\sigma_x$ does not seem like a fundamental challenge).

Example: losses due 1 um centroid offset.

Electron-hose instability

PWFA advantage: beam creates cavity; no cavity alignment issues (one of the drivers of NC rf linear collider design). However, BBU-type electron-hose transverse instability :

$$\left(\frac{\partial}{\partial s}\gamma\frac{\partial}{\partial s}+\gamma k_\beta^2\right)x(s,z)=\int_0^z dz'\frac{\omega_0^3}{c^2}\sin\{\omega_0(z-z')\}x(s,z')$$

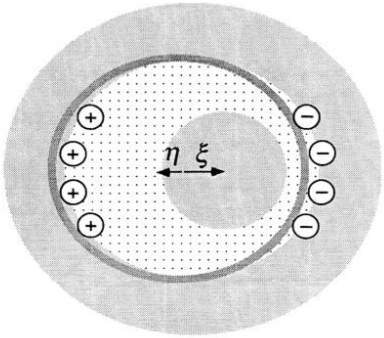


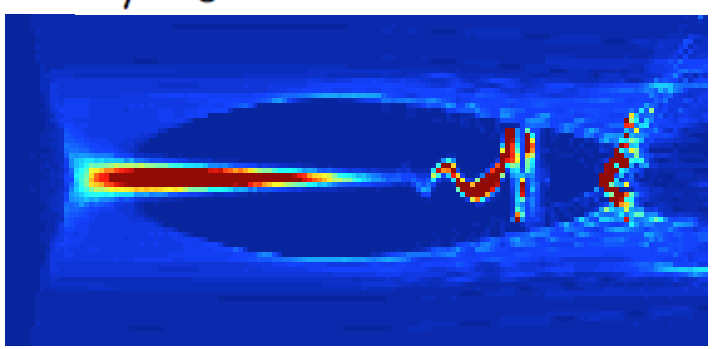
FIG. 2. A beam slice in the ion channel, displaced by an amount ξ in the x direction, induces a displacement η of the channel wall, which responds as a simple harmonic oscillator with angular frequency ω_0, deflecting follow-on portions of the beam.

* LHS: identical to BBU
* RHS: driving term independent of charge, cannot change k_β -> BNS-type damping not available.

x: beam transverse displacement
s: coordinate along plasma
z: coordinate along bunch
k_β: betatron wavenumber in plasma;
$k_\beta = k_p / \sqrt{(2\gamma)}$;
$k_p = \omega_0\sqrt{2}$ is the plasma wavenumber

Asymptotic solution :

$$x/x_0 \sim e^{c(k_p s/\sqrt{\gamma})^{1/3}(k_p z)^{2/3}}$$



Mitigating factors :

- $k_p z$: reduce σ_z, reduce n_0
- $k_\beta s$: increase # of stages, increase γ_0

Example : WB with offset $\Delta x/\sigma_x = 3$ (huge, for illustration), after energy doubling from 25 GeV to 50 GeV. Effects for $\Delta x/\sigma_x = 0.1$ negligible for 100 um beam. **Challenge: quantify for nm beam**.