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Study on the Nonlinear Vibration Characteristic of Dry-Type Air-Core Reactor

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- **Introduction**
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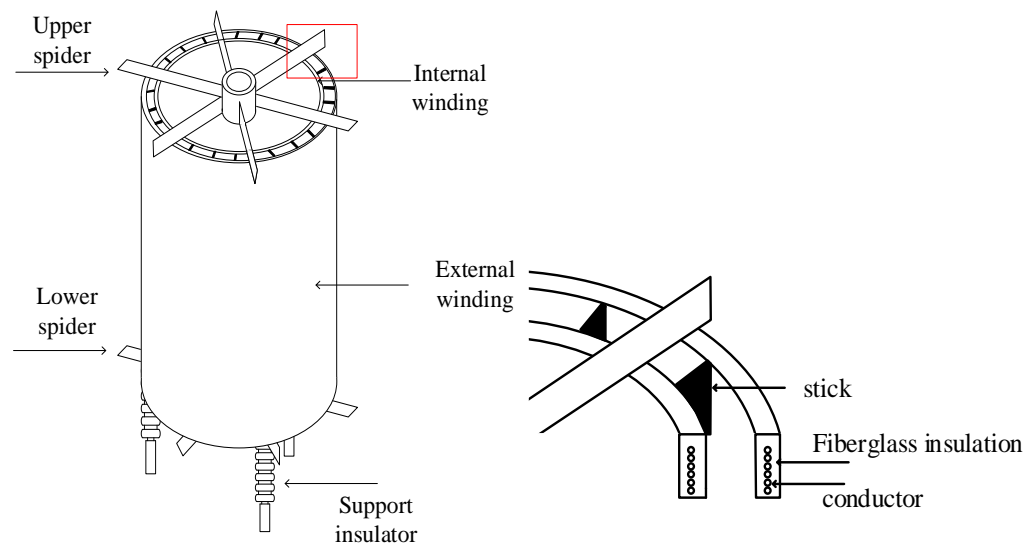
Introduction

Introduction

Dry-type air-core reactor
one of the main noise sources in HVDC converter stations.



AC filter reactor



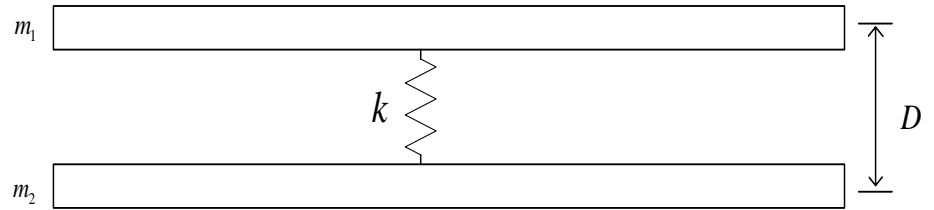
Structure of dry-type air-core reactor

Commonly accepted linear model

The vibration of dry-type air-core reactor is proportional to the electromagnetic force and the square current. $v \propto F \propto BI \propto I^2$

IEC60076-6:2007 standards suggests to decompose multiple frequency excitation into sum frequency, difference frequency and double frequency.

However, the coupling of magnetic field and vibration is not considered, which cause the nonlinear relationship between magnetic force and square current.



When the vibration happens, the magnetic field generated by the current in wires will be changed.

In this paper :

1. The problem is mathematically modeled
2. The resulting vibration spectrum is analyzed.
3. The actual vibration is measured and make the comparison.

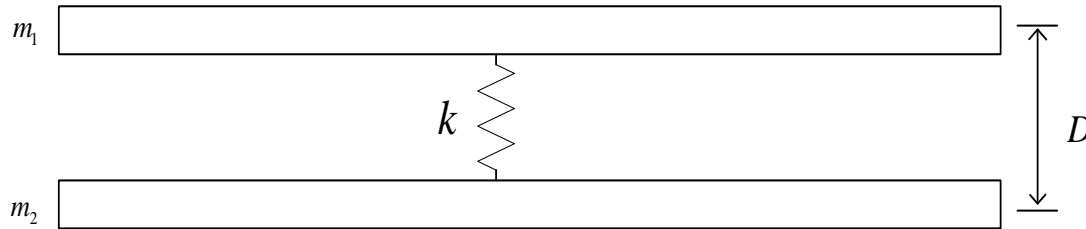


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Theoretical Analysis

Basic two-wire model



Two-wire model

Lagrangian function

$$L = E_k - E_p(r) = \frac{1}{2} m_1 r_1'^2 + \frac{1}{2} m_2 r_2'^2 - E_e(|r_1 - r_2|) - E_B$$

E_k is the kinetic energy.

E_p is the potential energy of the system.

E_e is the elastic potential energy of the system.

E_B is the magnetic potential energy.

Basic two-wire model

Vibration equation

$$\frac{d}{dt} \frac{\partial L}{\partial r'} - \frac{\partial L}{\partial r} = 0 \quad \text{Equilibrium position } r_0 \quad E_e(r_0, t) = 0 \quad \left. \frac{\partial E_e}{\partial r} \right|_{r=r_0} = 0$$

As the Taylor expansion of the elastic potential energy and the magnetic field potential energy at equilibrium position can be written as:

$$\begin{aligned} E_B &= E_B(r_0, t) + \left. \frac{\partial E_B}{\partial r} \right|_{r=r_0} (r - r_0) + \left. \frac{\partial^2 E_B}{\partial r^2} \right|_{r=r_0} (r - r_0)^2 \\ &= \frac{1}{2} \frac{\mu l}{\pi} I^2 \left(\ln\left(\frac{r_0}{r_x}\right) + \frac{1}{4} + \frac{1}{r_0} (r - r_0) - \frac{1}{2} \frac{1}{r_0^2} (r - r_0)^2 \right) \\ E_e(|r_1 - r_2|) &= \cancel{E_{e0}} + \left. \frac{\partial E_e}{\partial r} \right|_{r=r_0} (r - r_0) + \frac{1}{2} \left. \frac{\partial^2 E_e}{\partial r^2} \right|_{r=r_0} (r - r_0)^2 = \frac{1}{2} k (r - r_0)^2 \end{aligned}$$

Then we get the vibration equation of the two-wire model:

$$mr'' + k(r - r_0) + \frac{1}{2} \frac{\mu l}{\pi} I^2 \left(\frac{1}{r_0} - \frac{1}{r_0^2} (r - r_0) \right) = 0$$

Let $y = r - 2r_0$, the vibration equation can be rewritten as :

$$y'' + (\omega_0^2 - \beta I^2)y = \frac{-kr_0}{m}$$

where,

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Inherent frequency

$$\beta = \frac{\mu}{2\pi m r_0^2}$$

Small quantity

**Parameter-change
vibration equation**

Solution of vibration equation

$$y'' + (\omega_0^2 - \beta I^2)y = \frac{-kr_0}{m}$$

Method of perturbation is used here.

Taylor expansion y to the third power of β can be expressed as:

$$y(t, \beta) = y_0(t) + y_1(t)\beta + y_2(t)\beta^2 + y_3(t)\beta^3$$

$$y_0'' + \omega_0^2 y_0 = -\frac{kr_0}{m}$$

$$y_1'' + \omega_0^2 y_1 = y_0 I^2$$

$$y_2'' + \omega_0^2 y_2 = y_1 I^2$$

$$y_3'' + \omega_0^2 y_3 = y_2 I^2$$

Apply single frequency current $I = I_s \cos(\omega_s t)$

Solution of vibration equation

Expression

Frequency components

$$I^2$$

$$2\omega_s$$

$$y_0 \quad y_0'' + \omega_0^2 y_0 = -\frac{kr_0}{m}$$

$$0$$

↓ $\pm 2\omega_s$

$$y_1 \quad y_1'' + \omega_0^2 y_1 = y_0 I^2$$

$$2\omega_s$$

↓ $\pm 2\omega_s$

$$y_2 \quad y_2'' + \omega_0^2 y_2 = y_1 I^2$$

$$4\omega_s$$

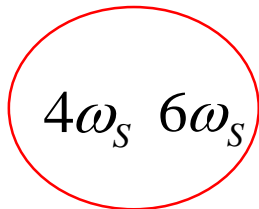
↓ $\pm 2\omega_s$

$$y_3 \quad y_3'' + \omega_0^2 y_3 = y_2 I^2$$

$$2\omega_s \quad 6\omega_s$$

Current frequency ω_s

Vibration frequency $2\omega_s$



Loading multiple frequency current,
take double frequency as an example:

Current frequency	ω_{S1}	ω_{S2}		
Vibration frequency	$2\omega_{S1}$	$2\omega_{S2}$	$\omega_{S1} \pm 3\omega_{S2}$	$2\omega_{S1} \pm 2\omega_{S2}$
		$\omega_{S1} \pm \omega_{S2}$	$3\omega_{S1} \pm \omega_{S2}$	$4\omega_{S1}$ $4\omega_{S2}$

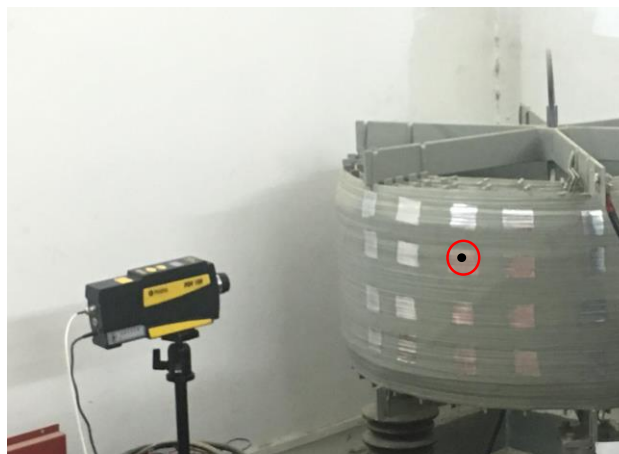
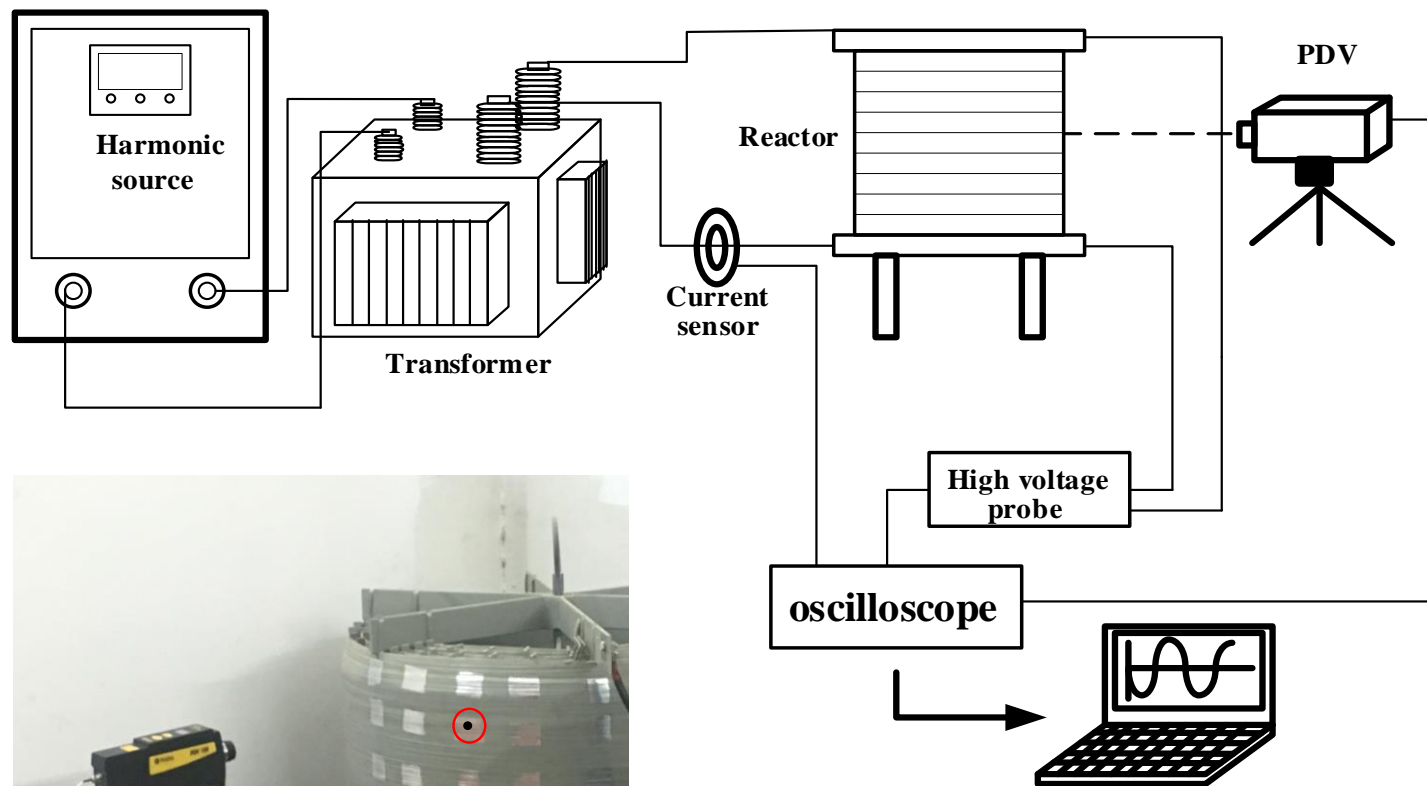


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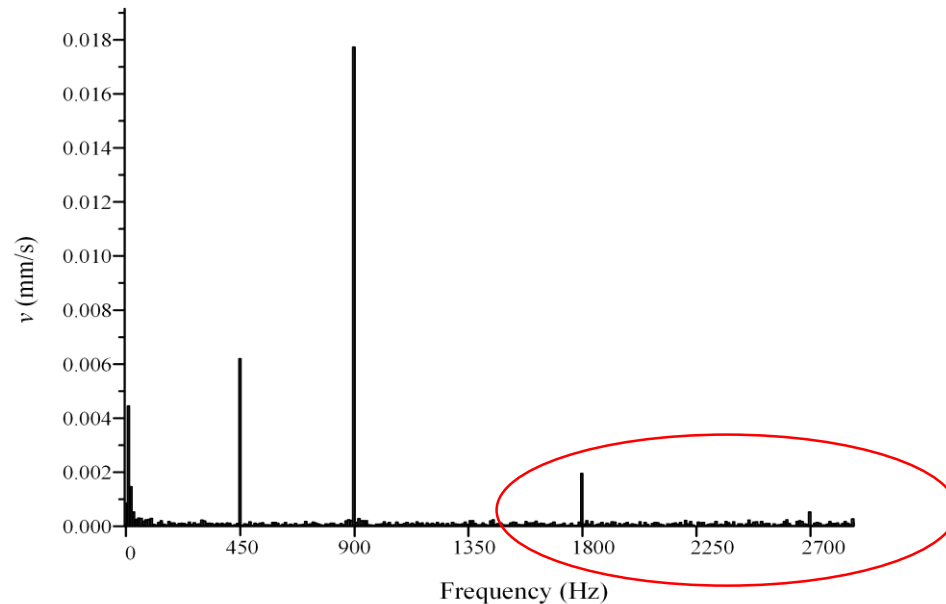
Experimental Analysis

The experimental setup



The inductance value of reactor is 1.97 mH.
The test reactor rated current is 270 A.

Single frequency current: 450Hz 80A

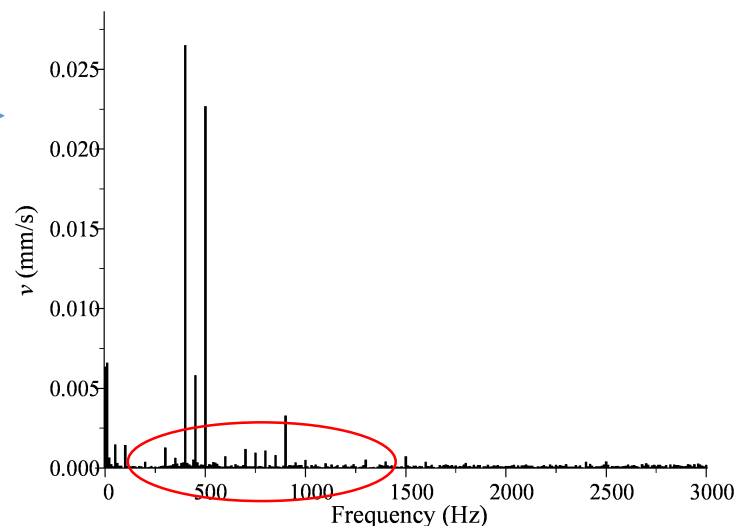


Loading single frequency current

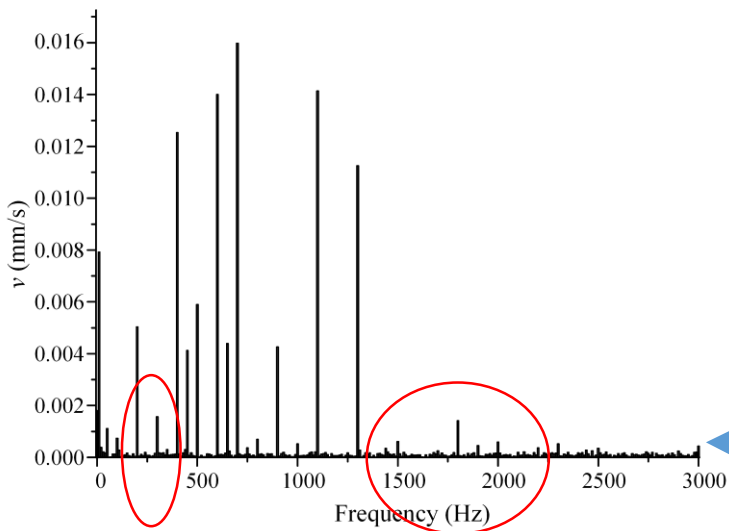
Other than double frequency component, there are still higher harmonic components in the spectrum

Vibration spectrum of reactor

Double frequency current:
50Hz 80A & 450Hz 80A



Loading double frequencies current



Loading triple frequencies current

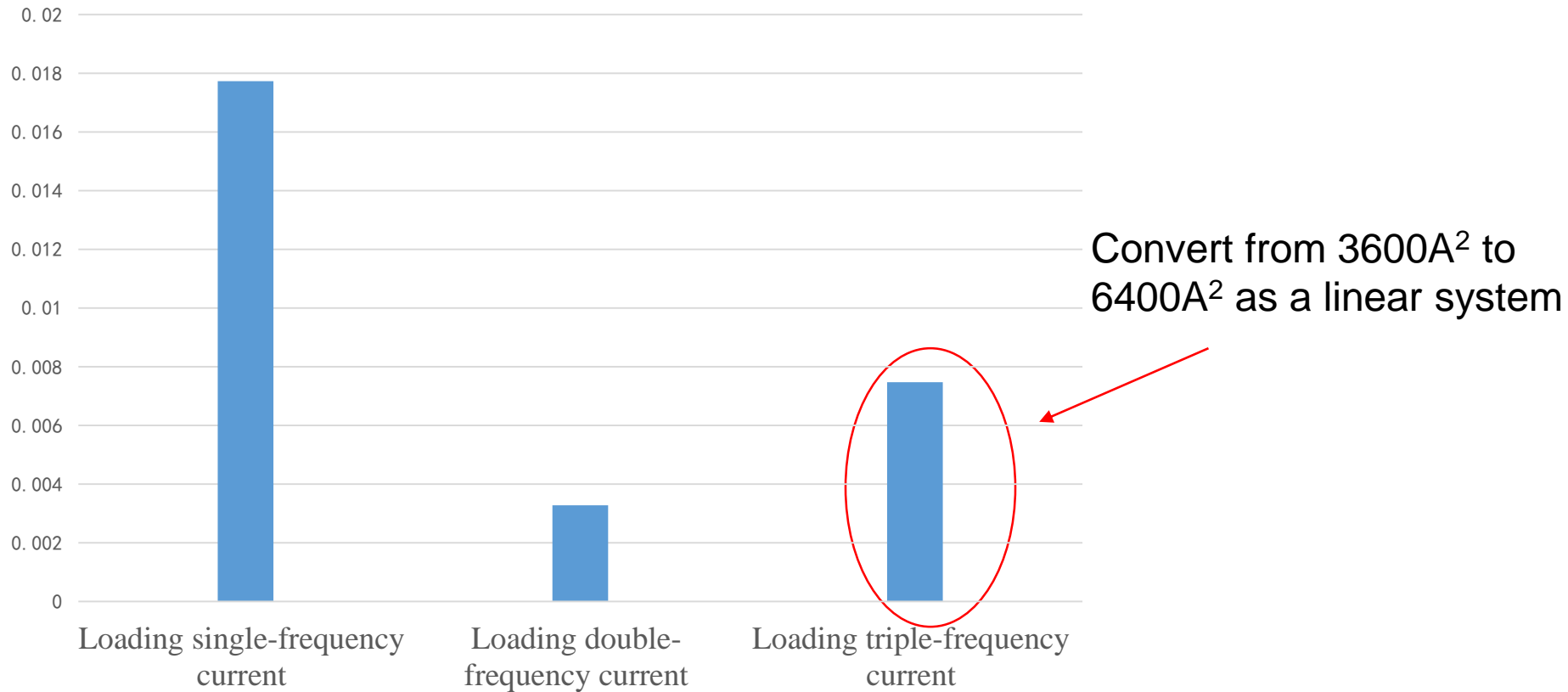
Triple frequency current:
50 Hz 60 A & 450 Hz 60 A & 650 Hz 60 A



The spectrum contains more components than the linear model and is consistent with our theoretical analysis.

Impact of nonlinearity

Applied with the same current square of 900Hz



Vibration component of 900Hz with different applying way

If the nonlinearity is not considered, huge deviation could be introduced.



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Conclusions



Conclusions

- One of the main cause of the nonlinearity of dry-type air-core reactor is the coupling of the magnetic field and the motion of the reactor.
- Other frequency exists besides the sum, difference and double components of the current frequency.
- Applied with the same current square of one frequency component, the vibration of dry-type air-core reactor verifies greatly with the change of the applying way.



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Thanks for your attention!