Perturbative QCD: a review of the latest developments

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Rencontres de Blois
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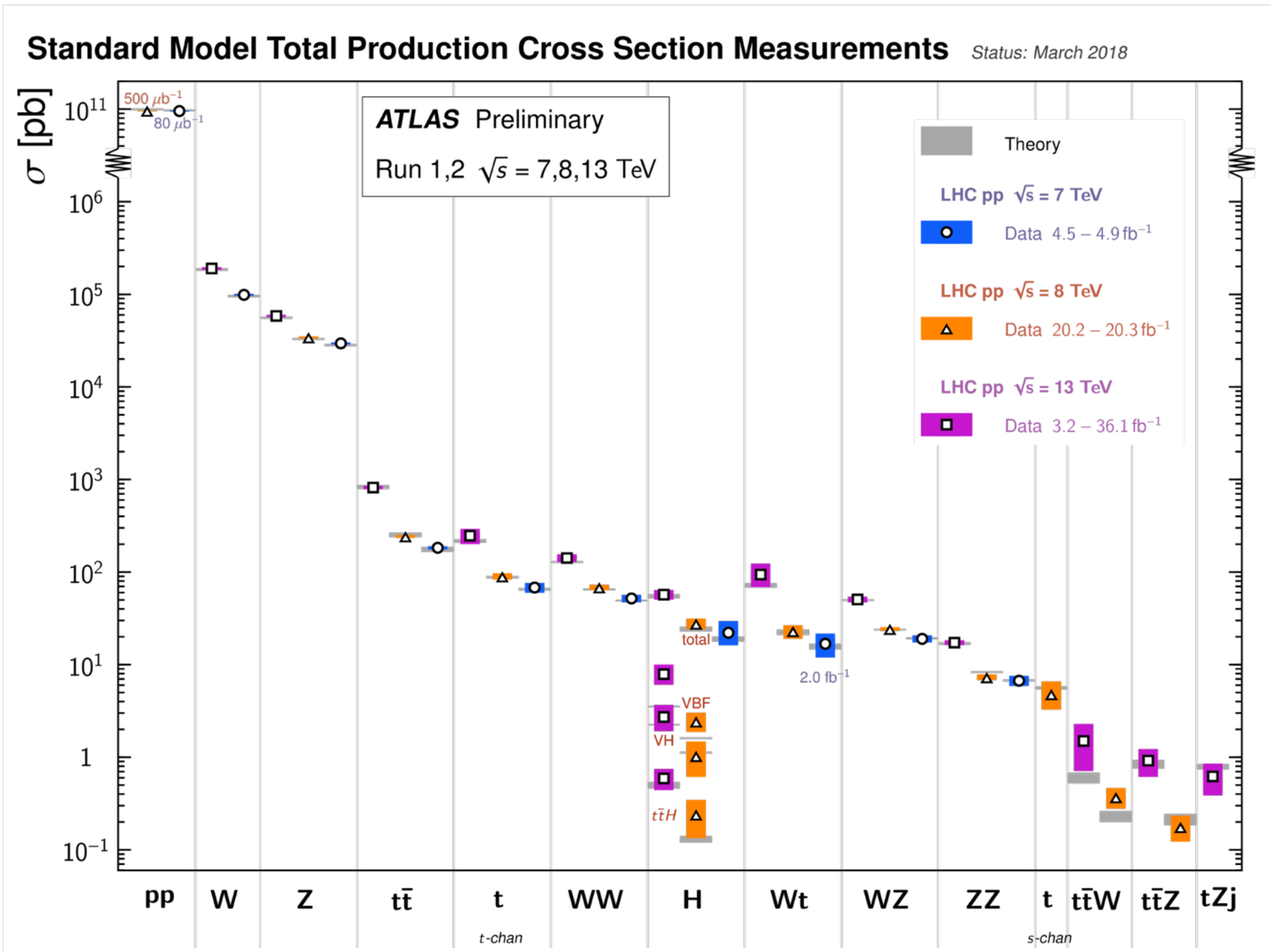
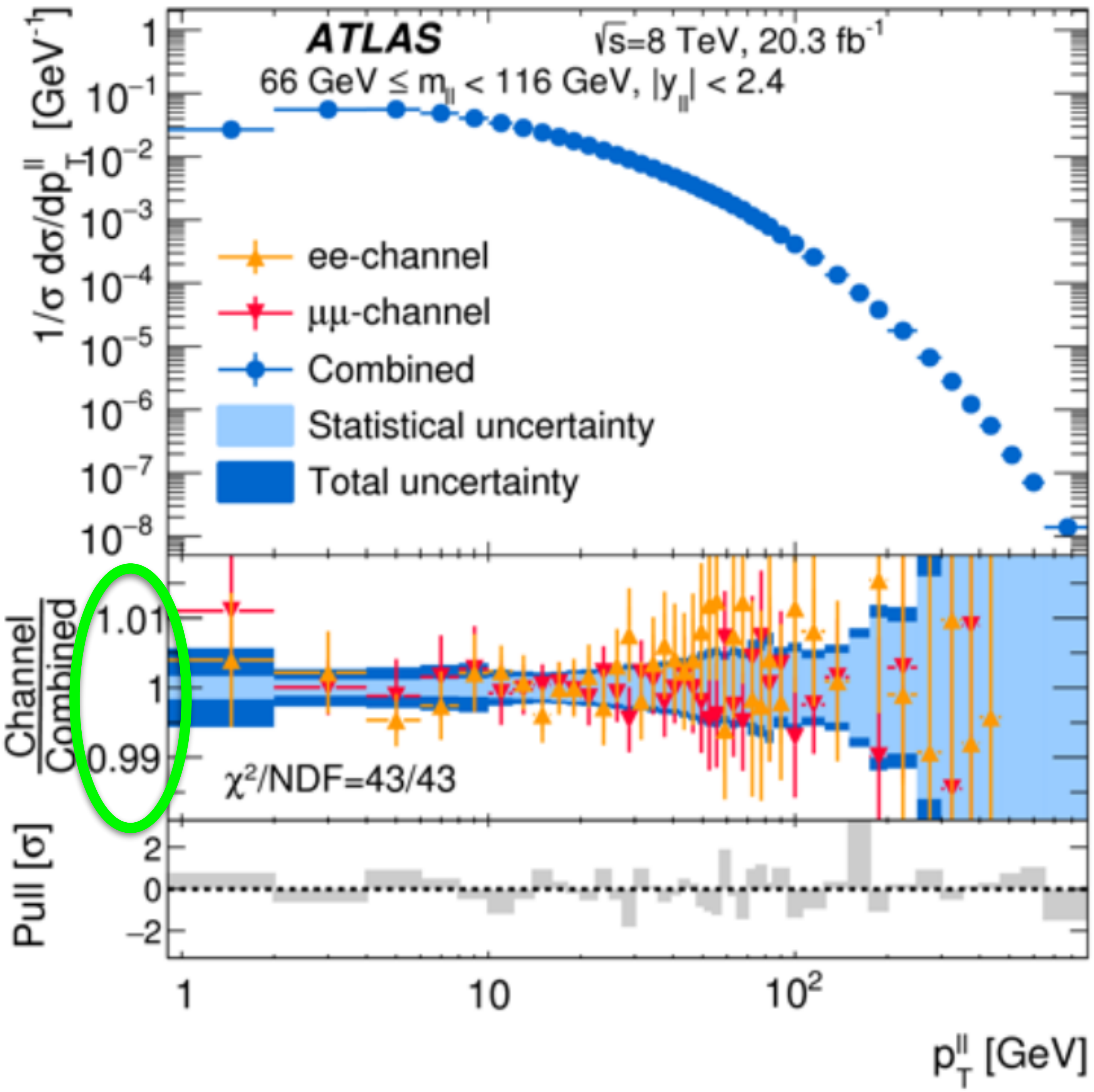
The SM versus the LHC

The Standard Model has so far done an extraordinary job in explaining data from the LHC

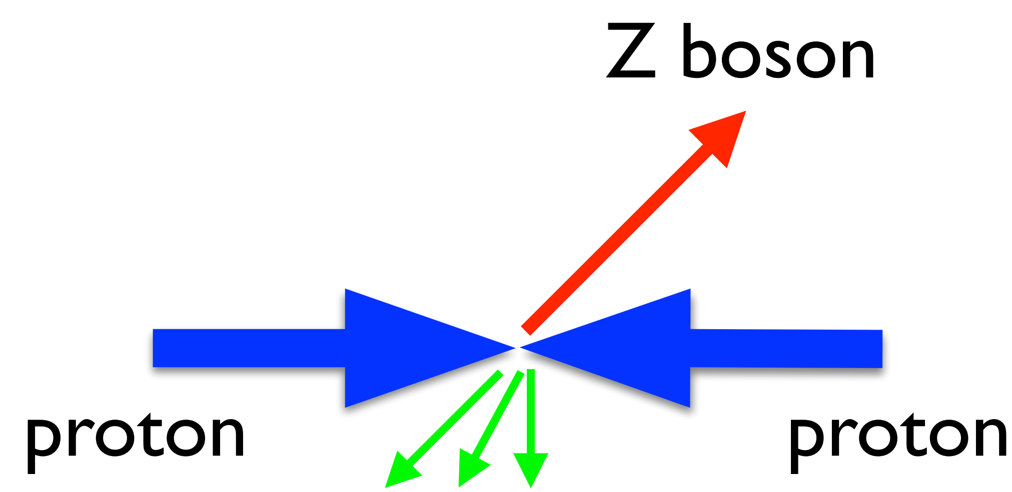


Major theme of this talk is precision theory for the Large Hadron Collider. Why is precision relevant now more than ever?

We are confronting the limitations in our ability to understand hadron collider data now!

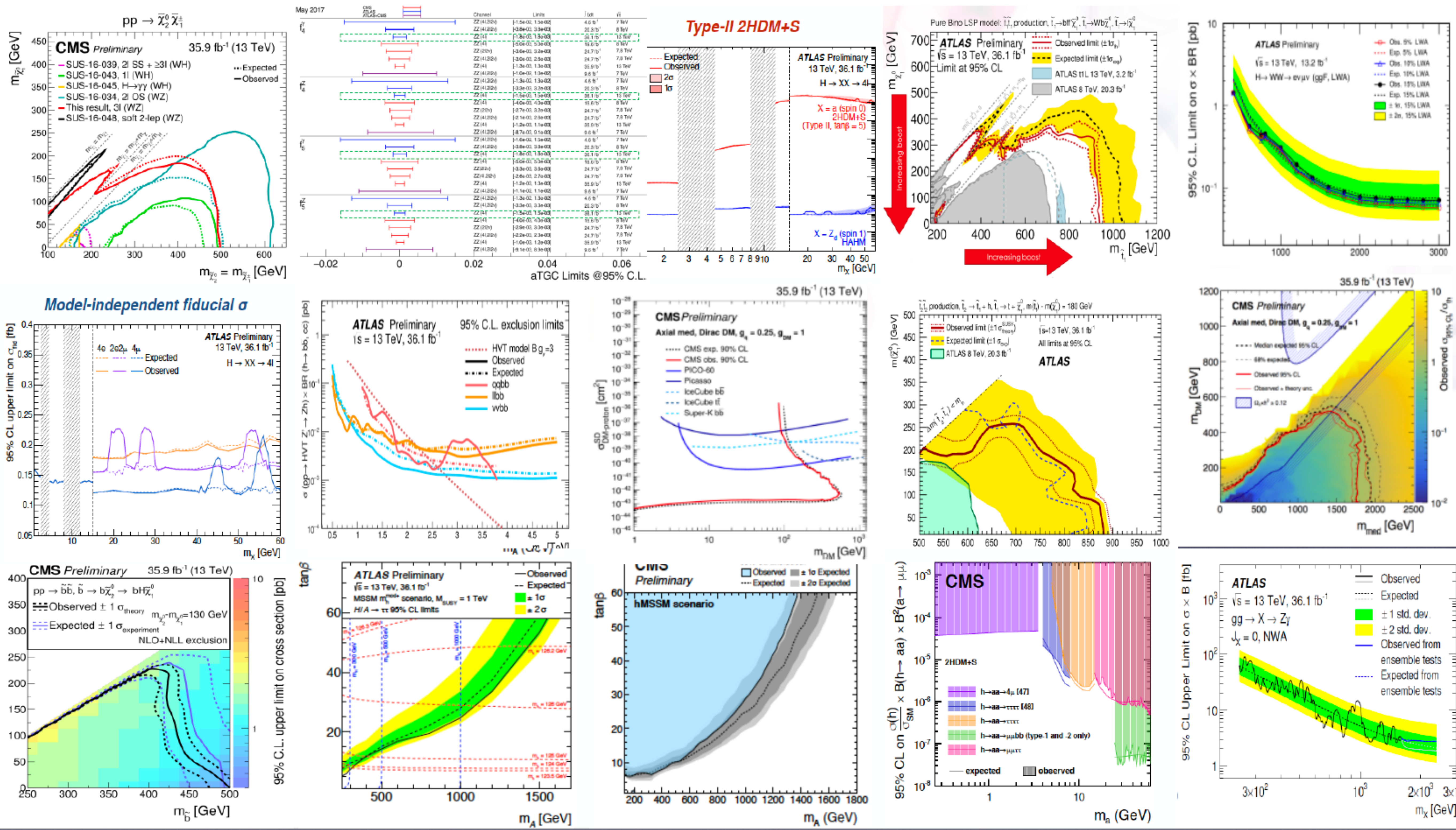


Sub-percent experimental errors over two decades of energy and 8 decades of cross section!



p_T=momentum transverse to proton direction

No hints for physics beyond the Standard Model so far; any deviations are likely small, subtle and hard to find!



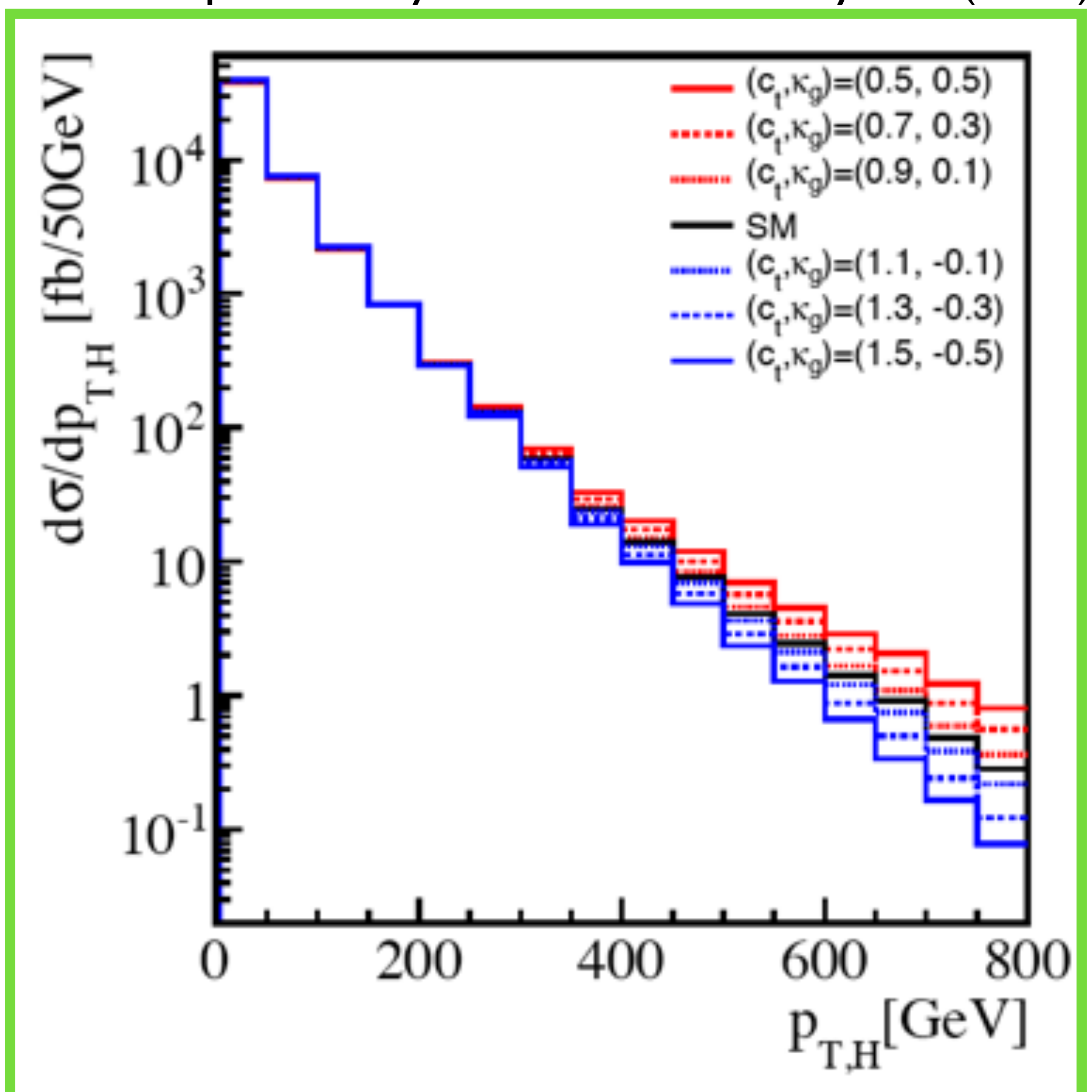
What can be hiding in a few percent % ?

- The Higgs transverse momentum is one of many examples where precision could be key in discovering and understanding new physics

$$\Delta\mathcal{L} = -c_t \frac{m_t}{v} + \kappa_g \frac{\alpha_s}{12\pi}\frac{h}{v} G^a_{\mu\nu} G^{a,\mu\nu} \quad \Rightarrow \quad \frac{\sigma(c_t, \kappa_g)}{\sigma_{\rm SM}} \approx (c_t + \kappa_g)^2$$

Schlaffer, Spannowsky, Takeuchi, Weiler, Wyman (2014)

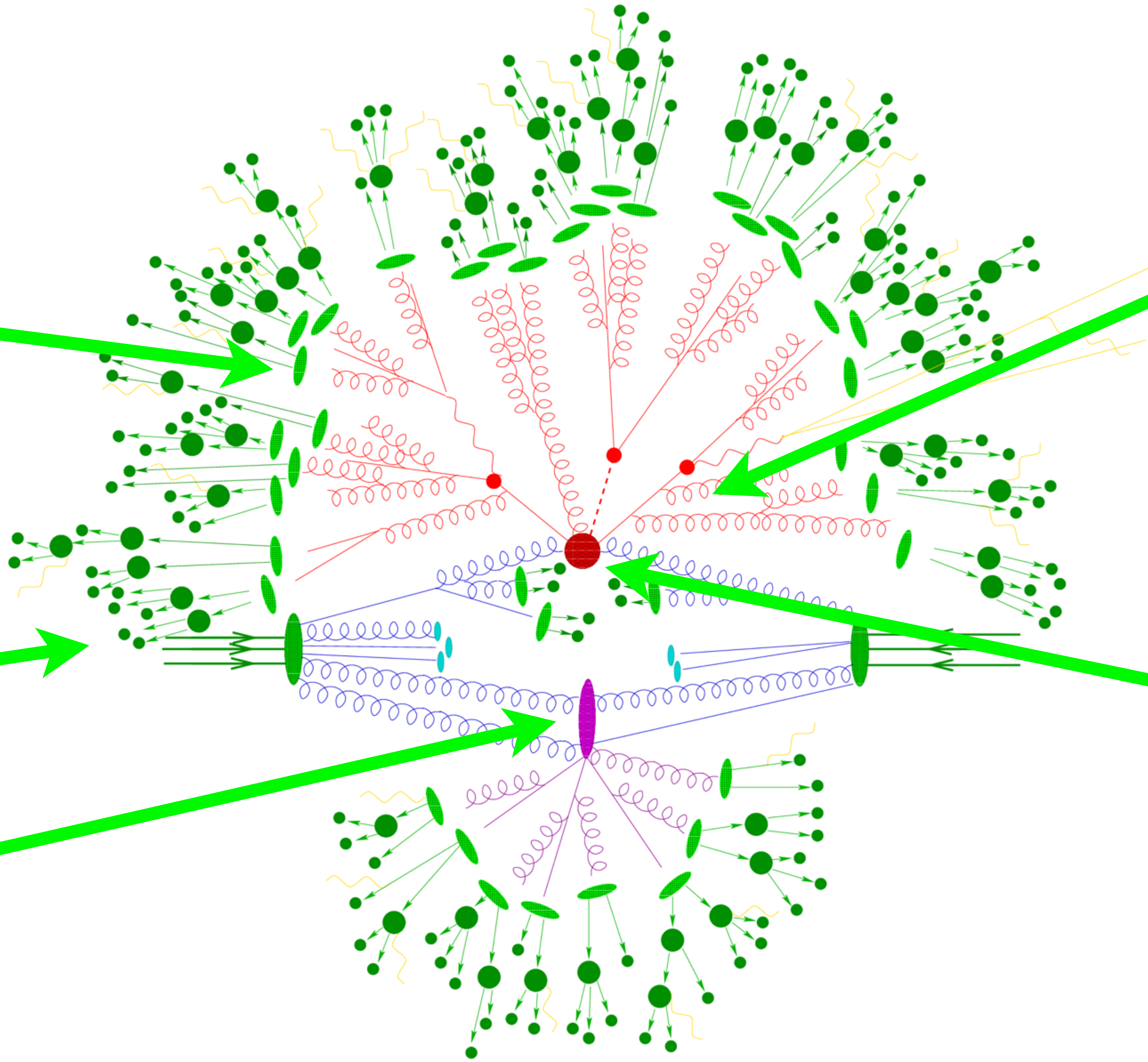
SM: $c_t=1, \kappa_g=0$



- Large changes in the high p_{TH} spectrum, while the low p_{TH} spectrum and total cross section unchanged at the few % level

- LHC has so far measured the low p_{TH} spectrum (up to ~300 GeV). It will measure the high p_{TH} spectrum as it moves to higher luminosity

A hadron collider event is rich but complicated!



Hadronization at Λ_{QCD}

Hadron decays

Multiple parton interactions

Parton-shower evolution to low energies

Hard collision (Higgs production) at short distances/ high energies

How does theory allow us to peer into the inner "hard-scattering" in this mess?

Factorization

- The key principle is factorization: separate long and short distance physics

long distance: 1/GeV

short distance: 1/TeV

$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}}$$

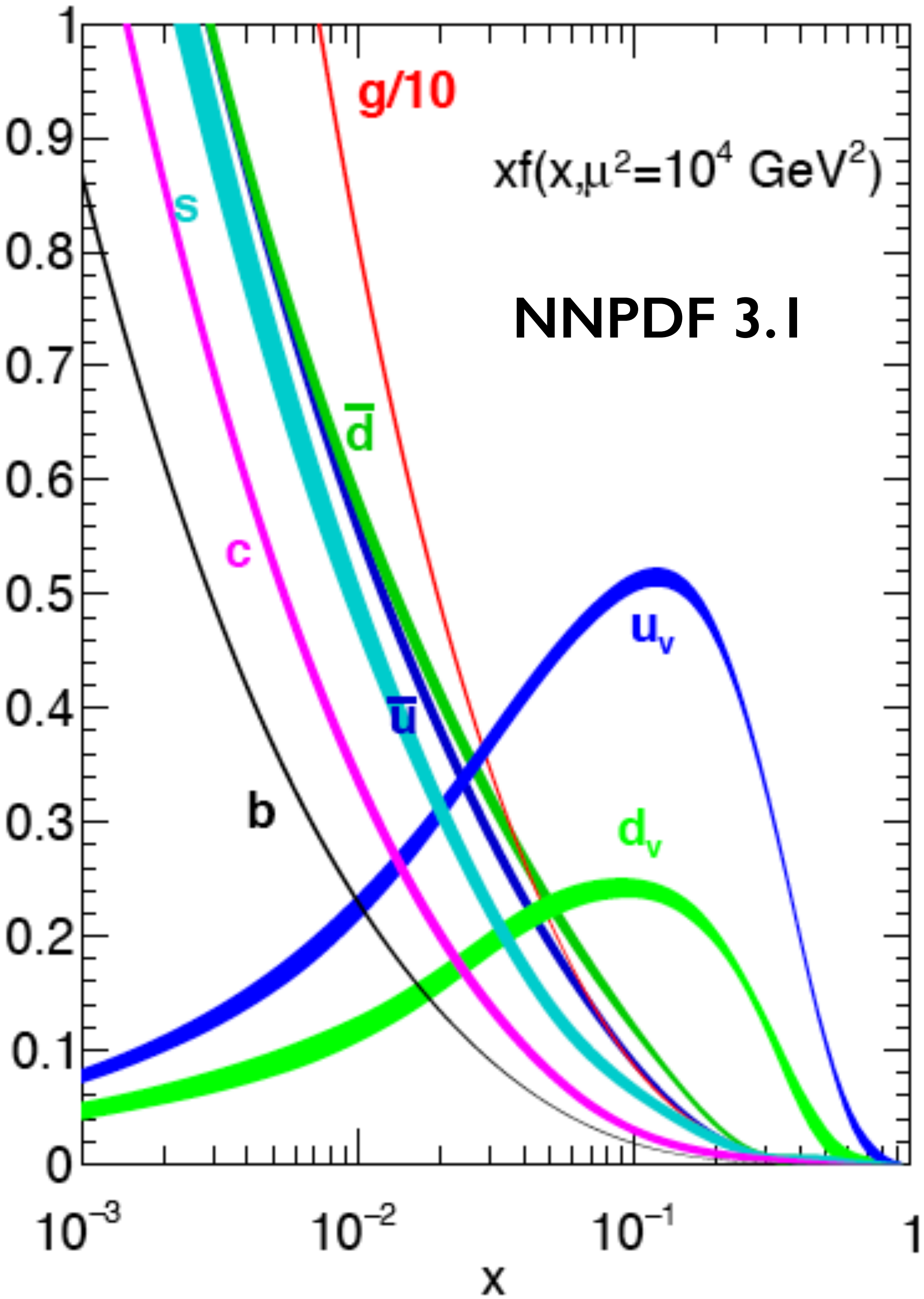
measure

extract from data

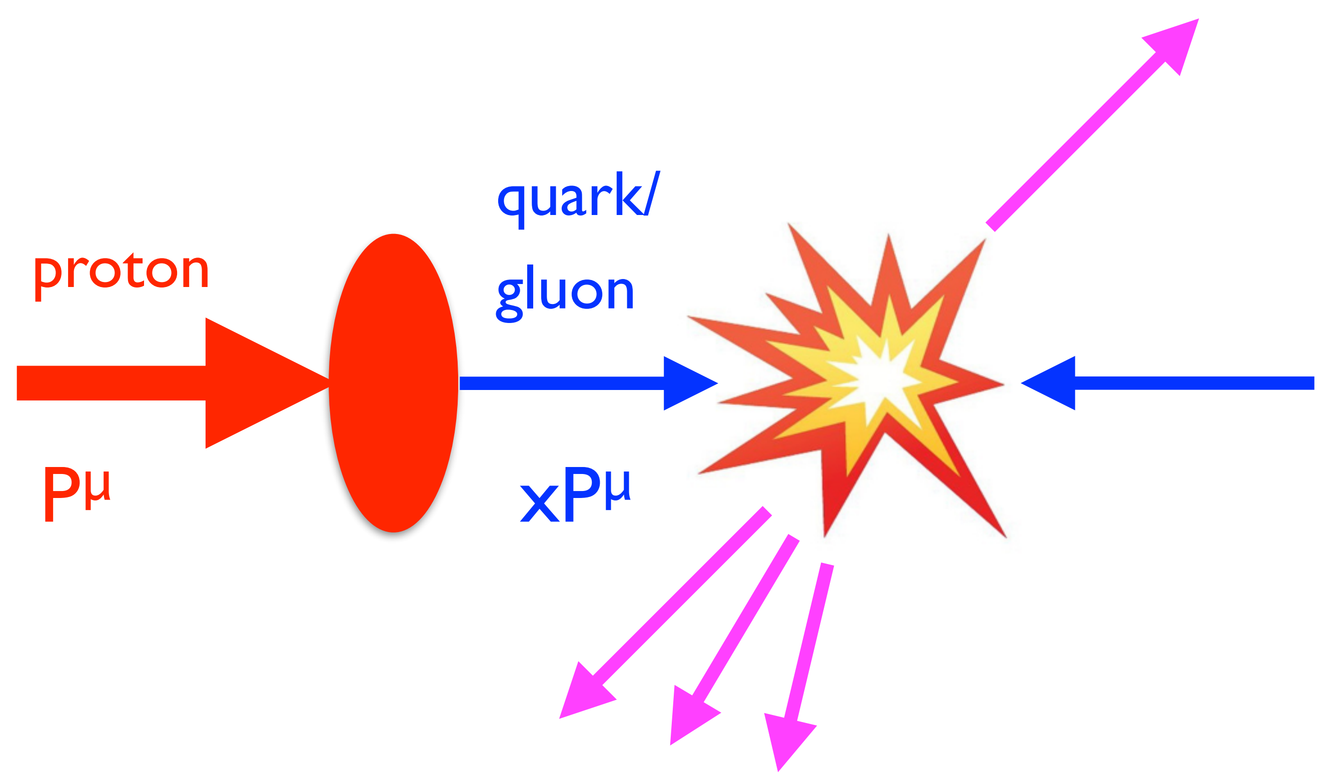
calculate

PDFs

$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}}$$

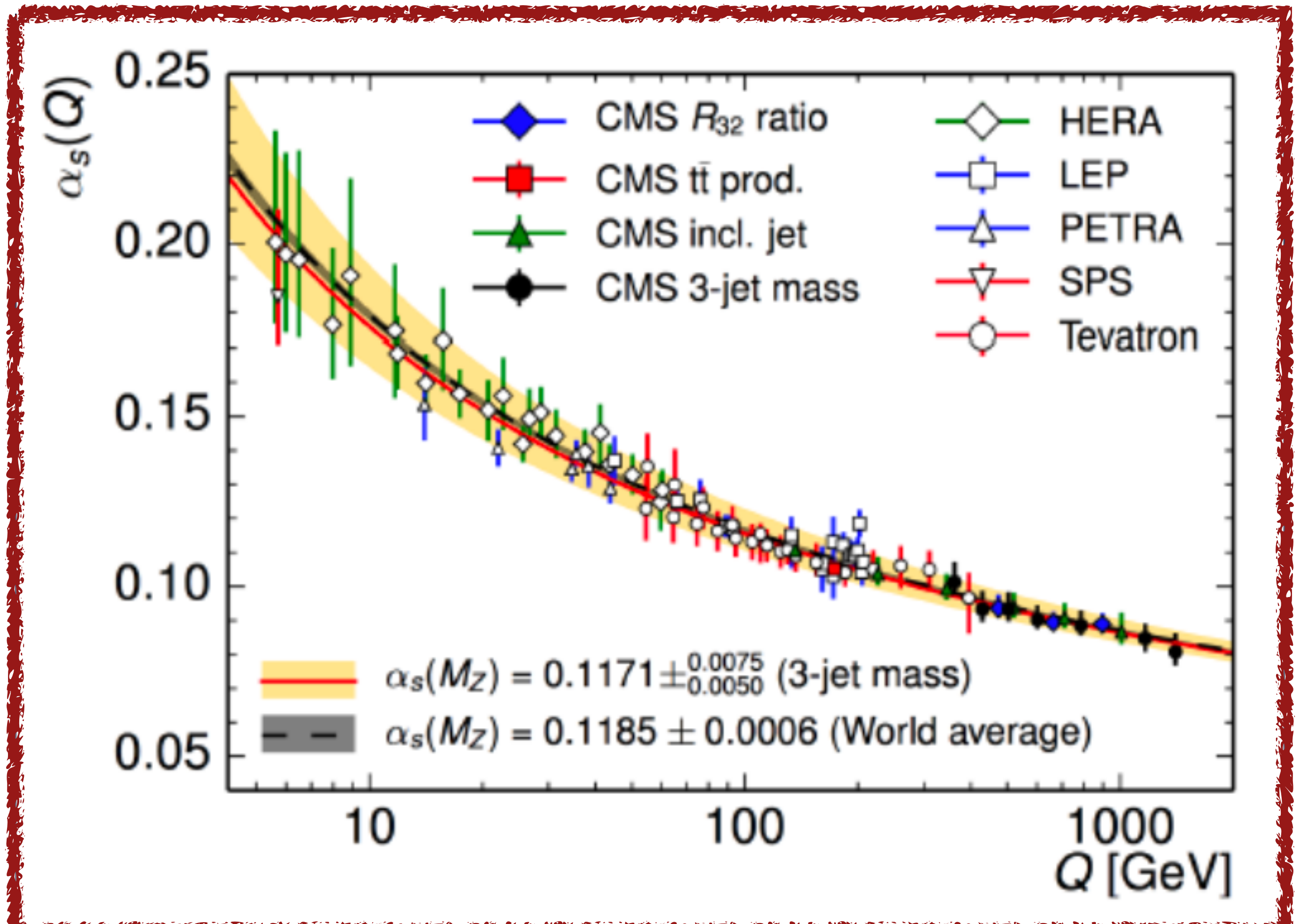


The PDFs measure the longitudinal momentum distribution of quarks and gluons inside the proton



Partonic cross section

$$\sigma_{h_1h_2\to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1;\overbrace{\mu_F^2}^{\text{factorization scale}})f_{h_1/j}(x_2;\mu_F^2)}_{PDFs}\underbrace{\sigma_{ij\to X}(x_1,x_2,\mu_F^2,\{q_k\})}_{\text{partonic cross section}}$$



Asymptotic freedom, the decrease of the QCD coupling constant with energy, allows us to compute the partonic cross section in perturbation theory

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

LO predictions | NLO corrections | NNLO corrections | NNNLO corrections

Recipe and Outline

- What must we know to make a prediction in QCD for the LHC?

$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}}$$

- Partonic cross section to high enough order in α_S
- Parton distribution functions
- The value of α_S
- For some measurements, parton showers to tie together the hard interaction scale and hadronization at Λ_{QCD}

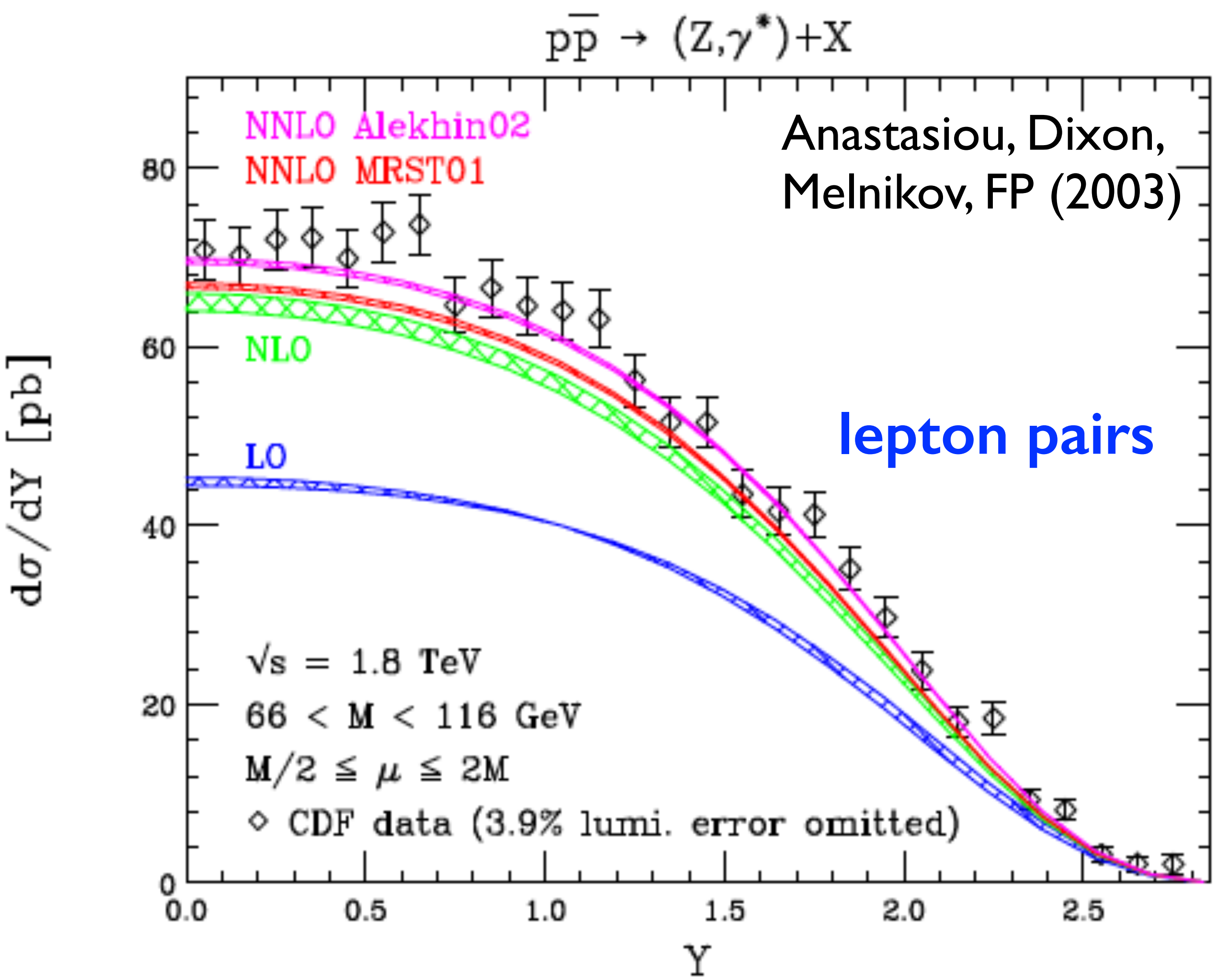
$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots \right)$$

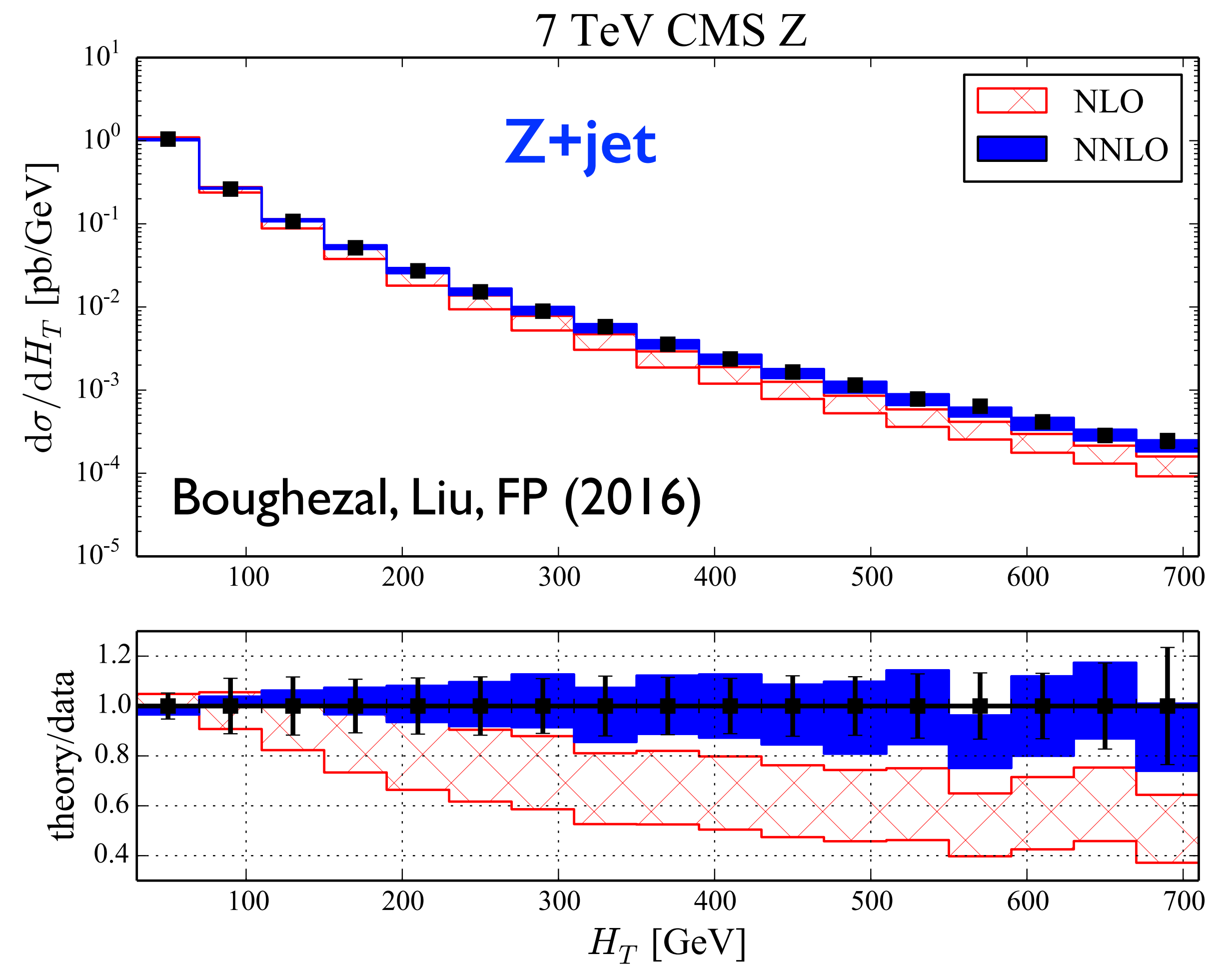
LO predictions

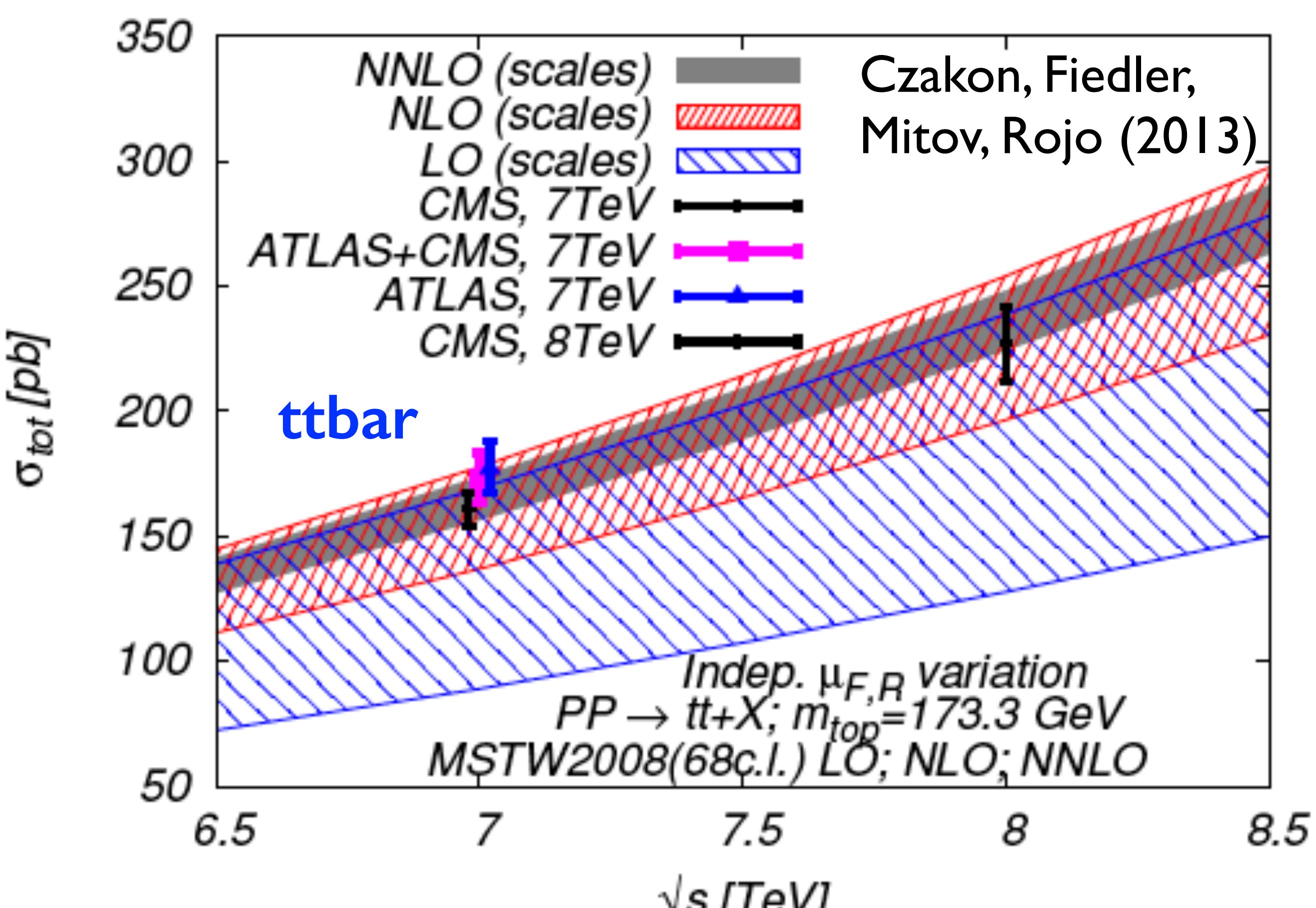
NLO corrections

NNLO corrections

NNNLO corrections







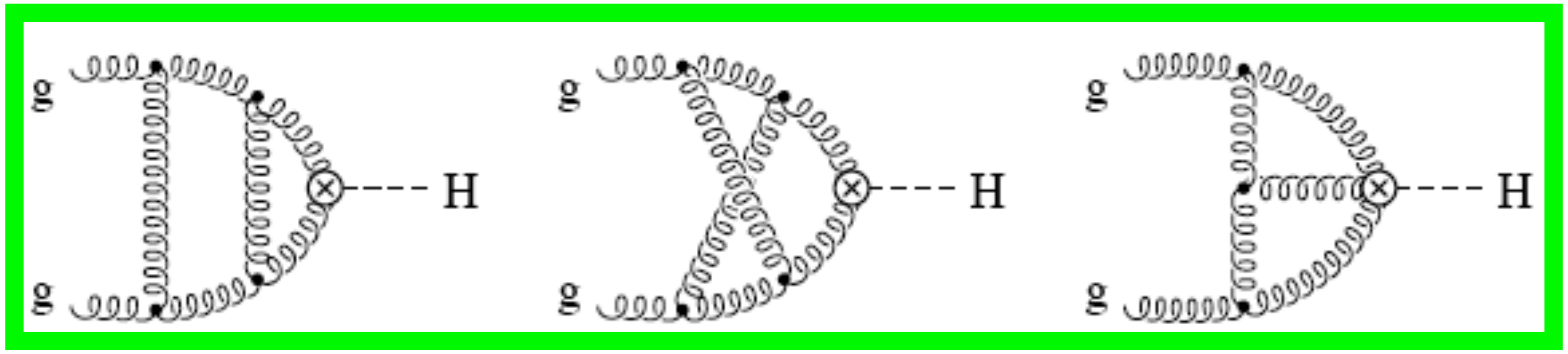
Many examples show that:

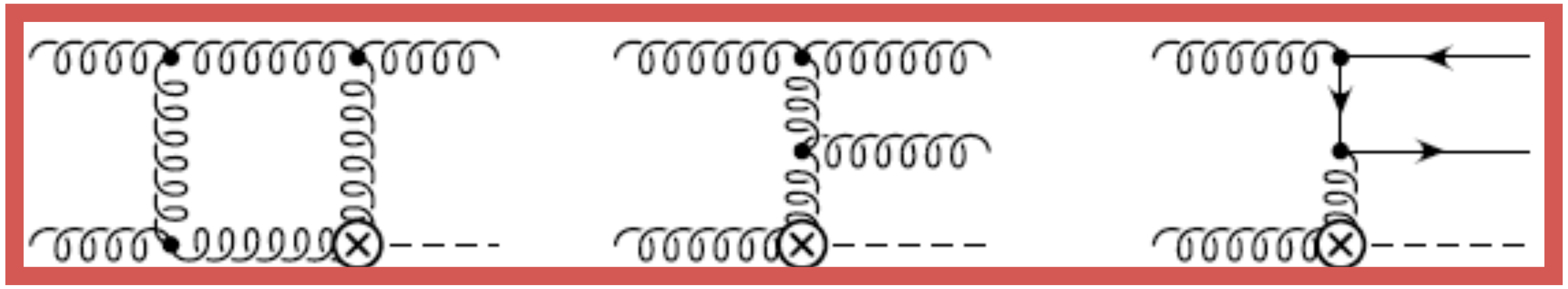
LO: rough estimate only
NLO: first quantitative estimate
NNLO: needed for precision!

Why is NNLO difficult?

- Draw and calculate all Feynman diagrams that appear at **NNLO**, or **O(α_S^2)** in perturbation theory. Higgs production as an example:

A small sample:





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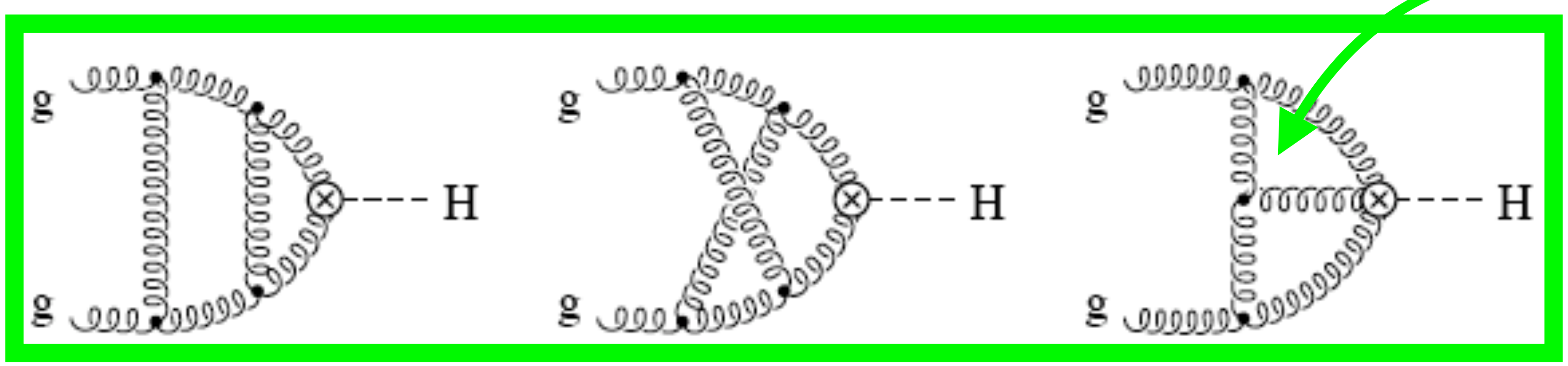

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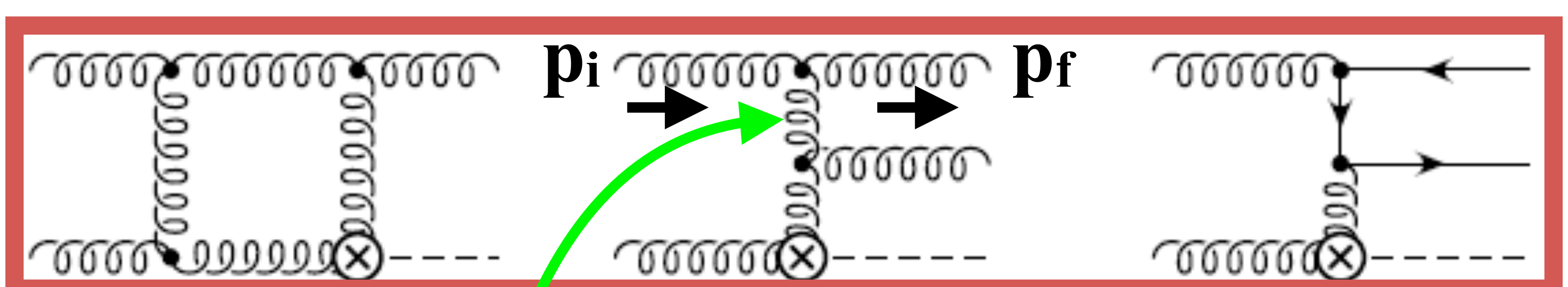

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$$\frac{1}{2p_i \cdot p_f} = \frac{1}{2E_i E_f (1 - \cos\theta_{if})}$$

Singular when $E_f \to 0$ (soft singularity) or $\theta_{if} \to 0$ (collinear singularity)

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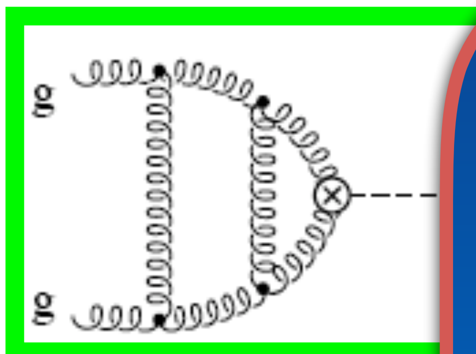
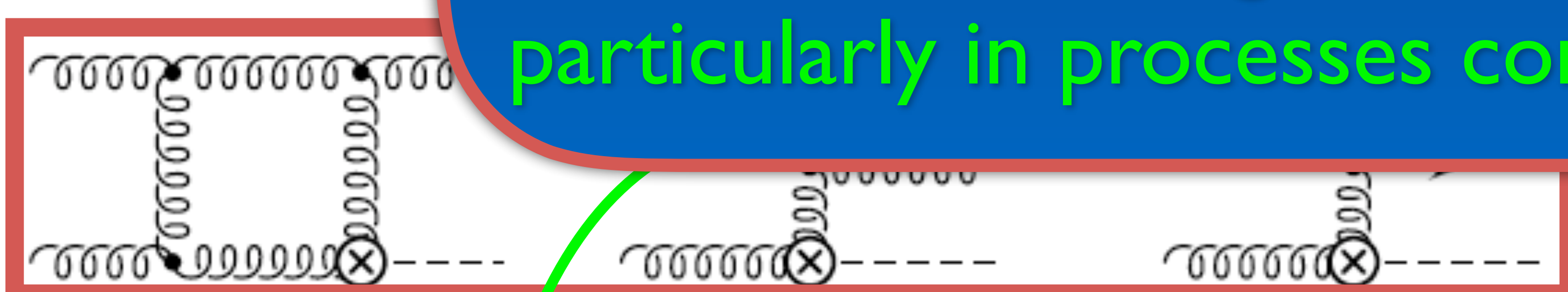


Deriving an organizing principle to extract and cancel singularities for arbitrary observables was the major obstacle in obtaining NNLO predictions, particularly in processes containing jets



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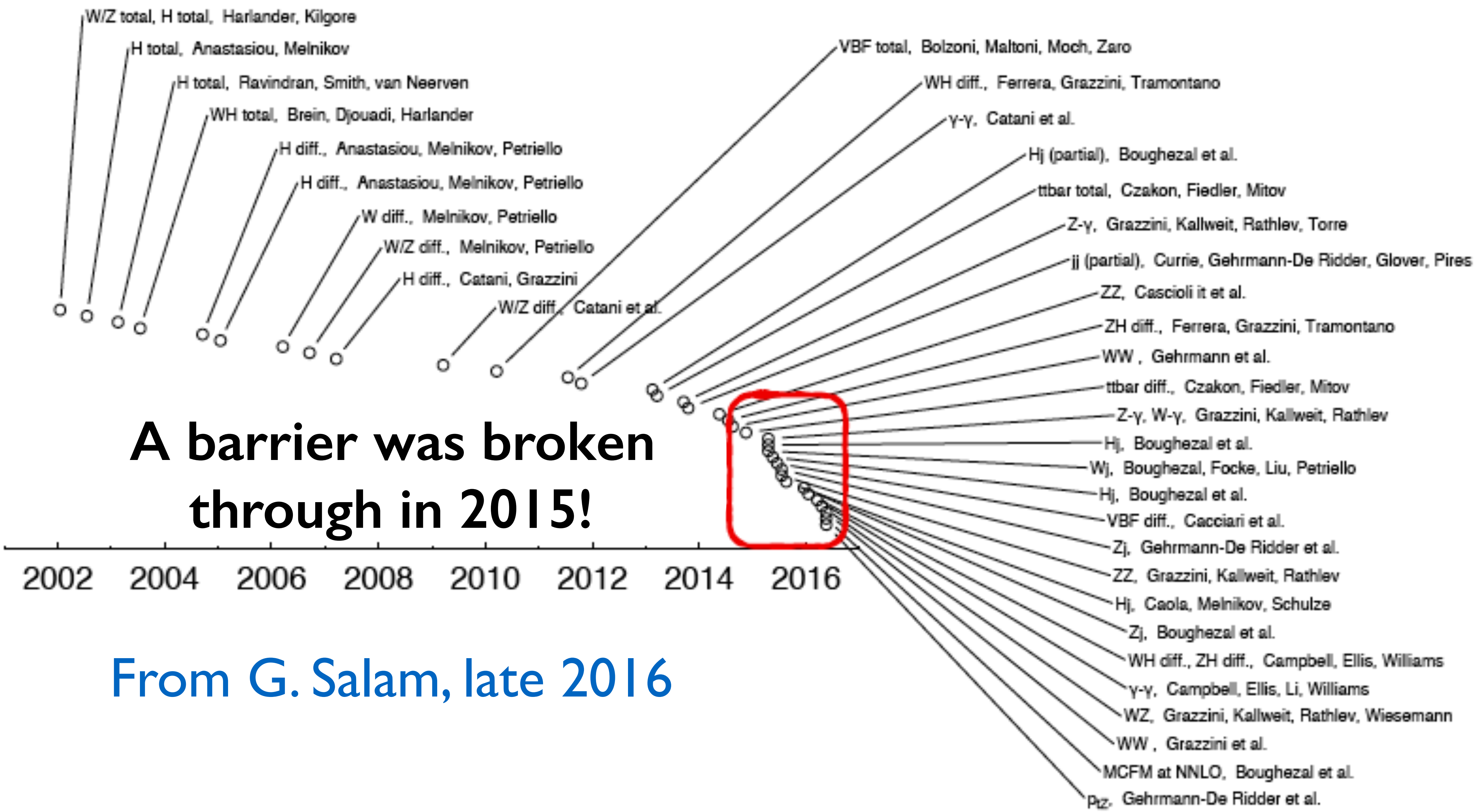
Breaking through to NNLO



From G. Salam, late 2016

This explosion of new NNLO results was made possible thanks to several new ideas!

Extraordinarily active area of research over the past several years!

- **Effective field theory methods:**
  - qT subtraction Catani, Grazzini
  - N-jettiness subtraction Boughezal, Focke, Liu, FP; Gaunt, Stahlhofen, Tackmann, Walsh

- **Subtraction methods:**
  - Sector decomposition Anastasiou, Melnikov, FP, Binoth, Heinrich
  - Antenna subtraction Kosower; Gehrmann, Gehrmann De Ridder, Glover
  - Sector Improved Residue Subtraction Czakon; Boughezal, Melnikov, FP; Czakon, Heymes; Caola, Melnikov, Rontsch
  - Colorful subtraction Del Duca, Duhr, Kardos, Somogyi, Trocsanyi
  - Projection to Born Cacciari, Dreyer, Karlberg, Salam, Zanderighi

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  - N-j ... hlhofen, T

NNLO has become the new standard for comparing theory to data for 2→2 processes, including jet production, at the LHC

- **Su**
  - S
  - Ant ... der, Glover
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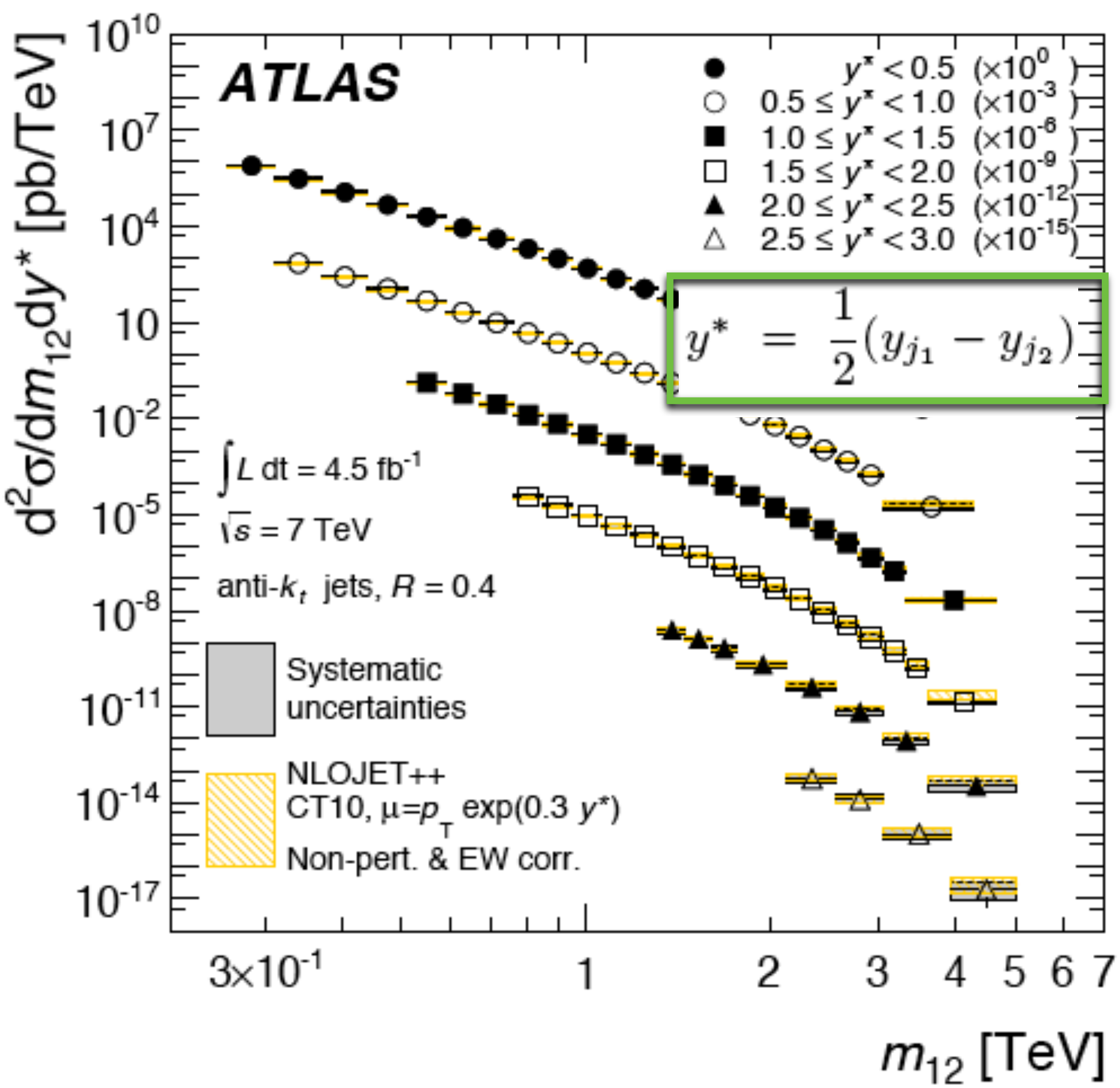
A recent example: di-jet production

- Several important LHC applications of di-jet production at the LHC, including searches for new physics in the form of new resonances or contact interactions, measurements of α_s, high-x gluon



- ✶ Enormous lever arm: measured mass range spans over an order of magnitude, cross section spans many orders of magnitude

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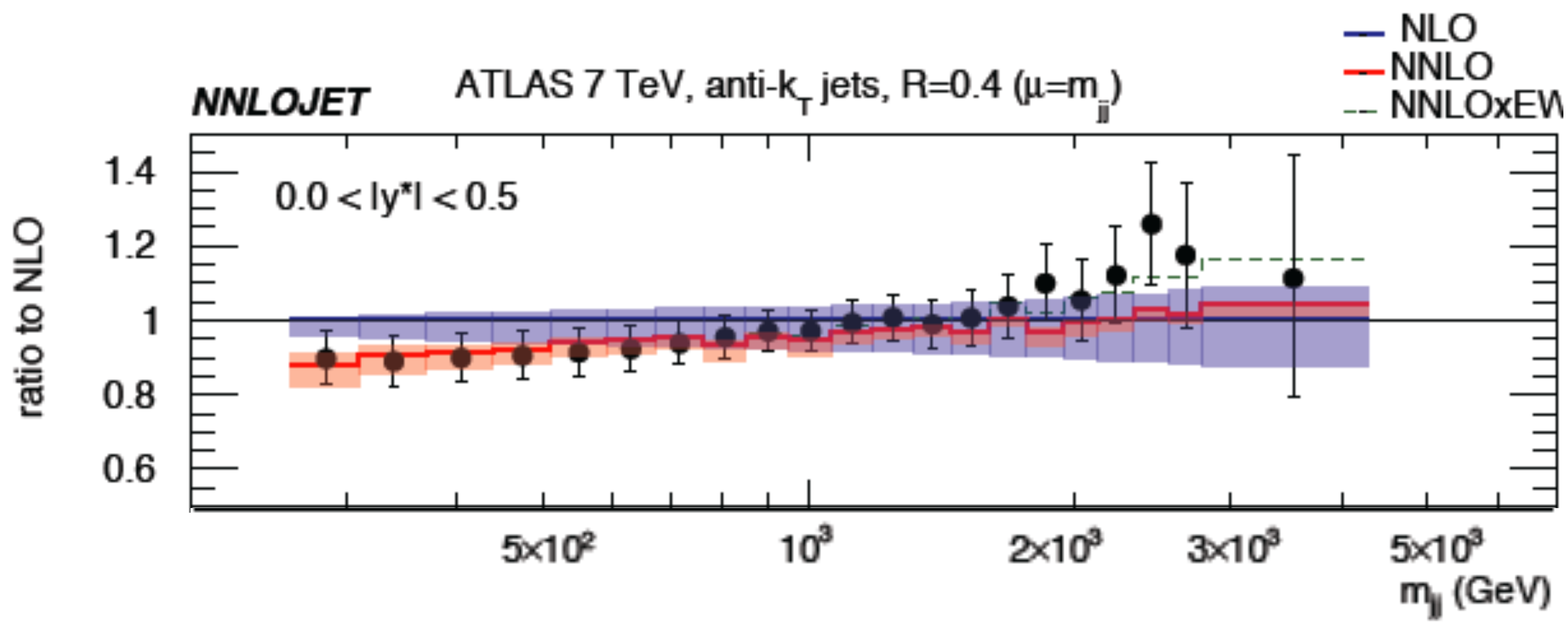

Currie, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Pires PRL 119 (2017)

- ✴ NNLO known in the leading-color approximation
- ✴ Notably improved data/theory agreement in the central y* region

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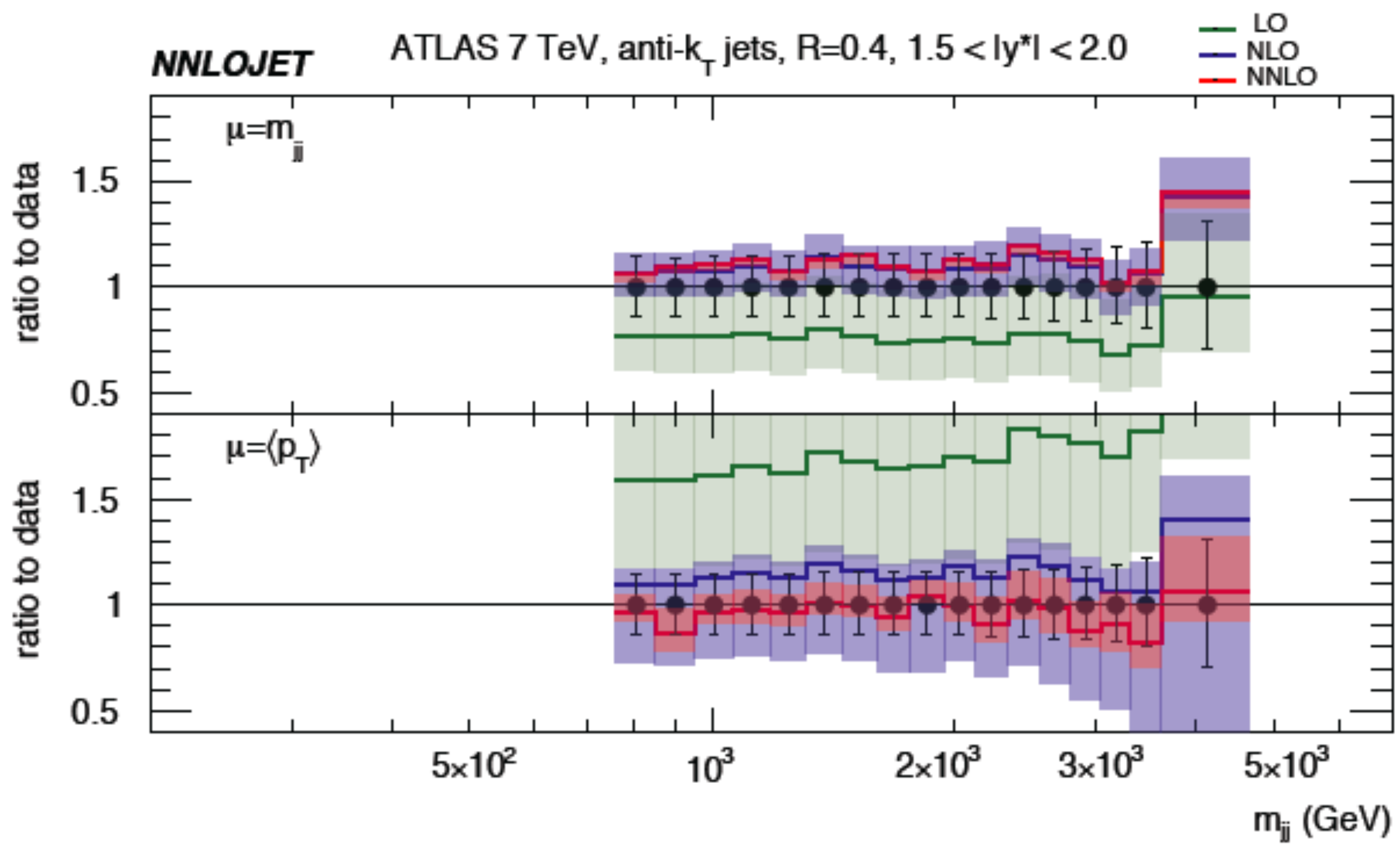
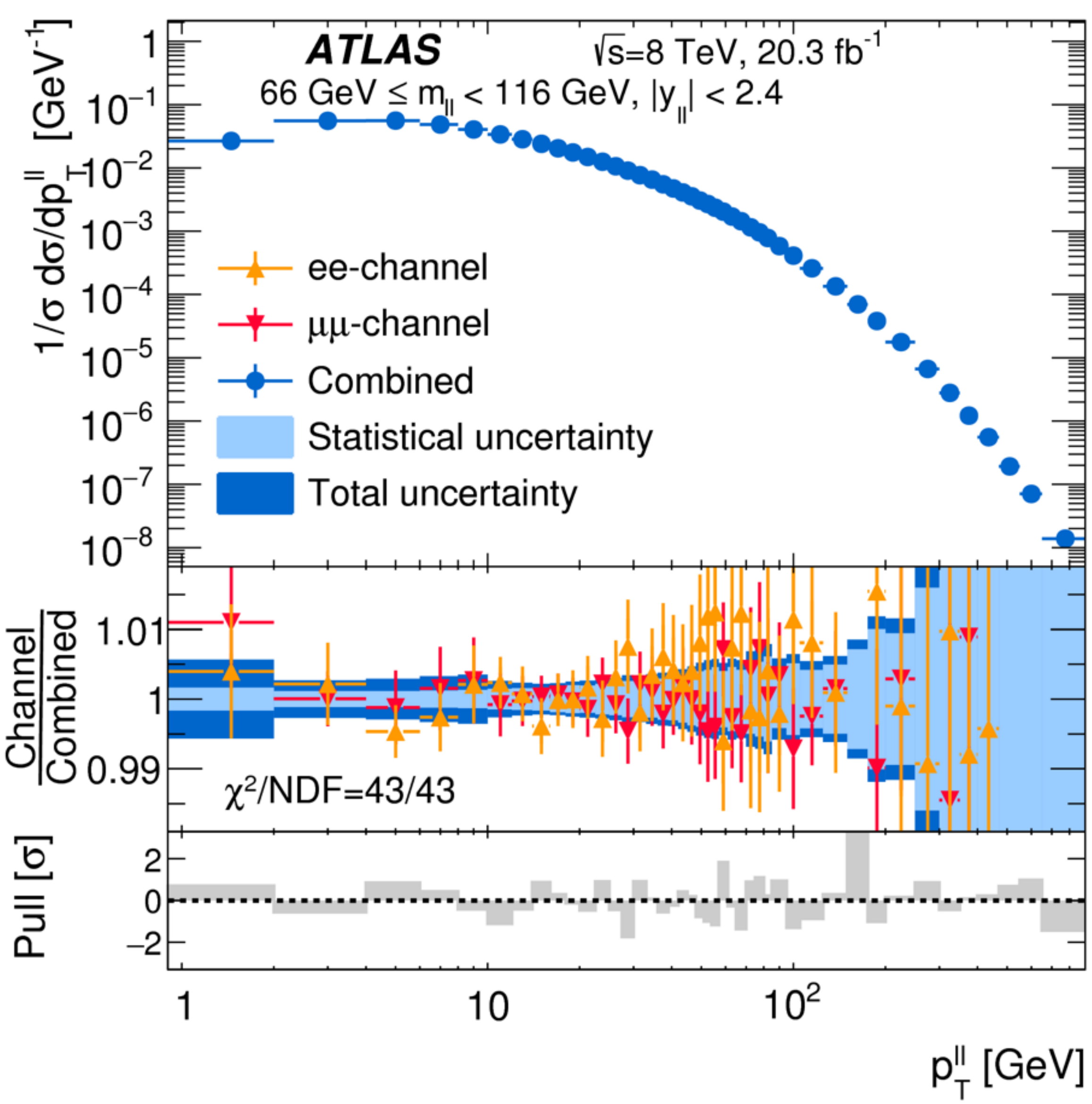


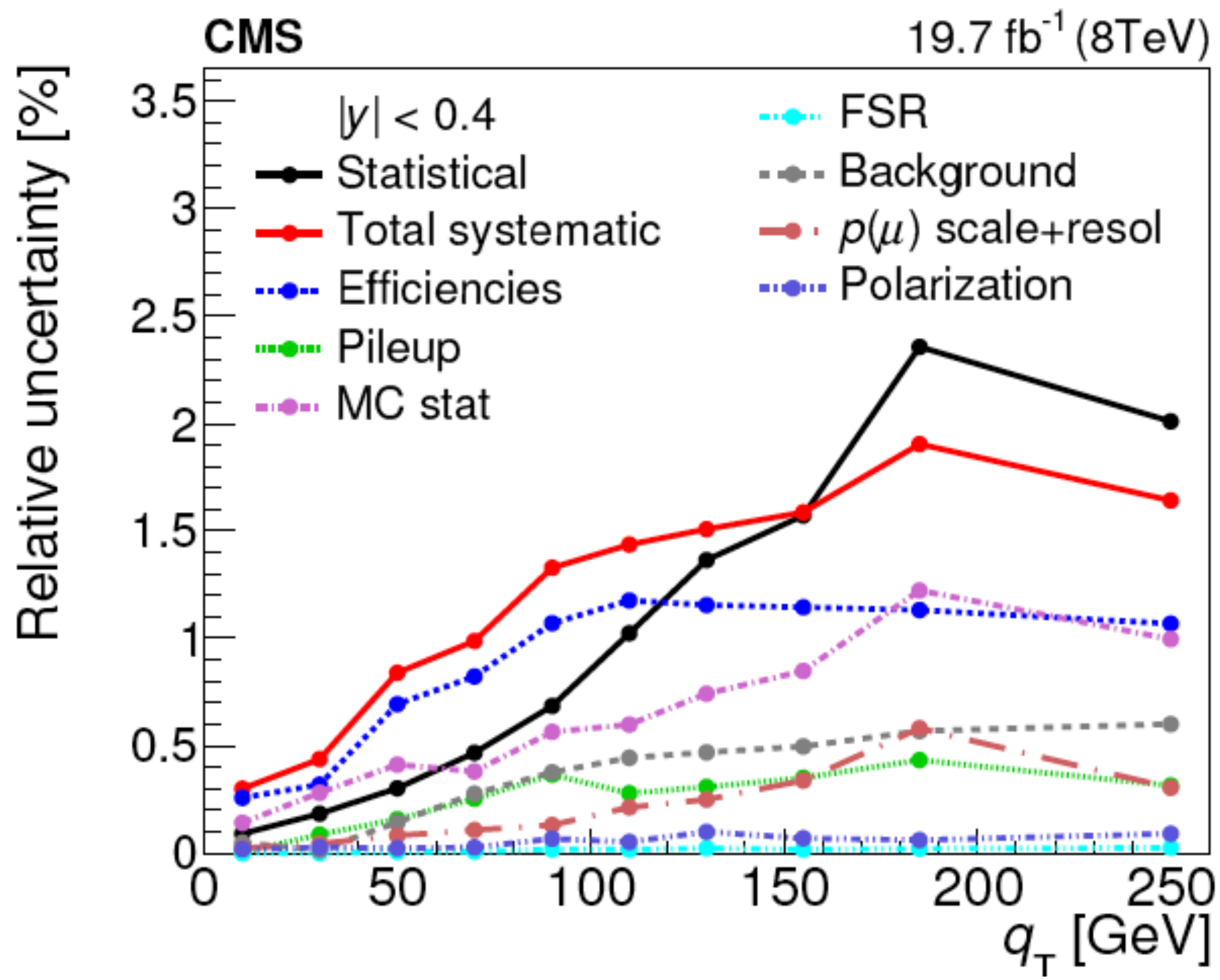
Currie, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Pires PRL 119 (2017)

Scale choices matter less when NNLO is included, even in cases when LO and NLO differ significantly!

An application: the Z-boson transverse momentum and PDF fits

- The Z-boson transverse momentum spectrum measurement has reached a remarkable precision at the LHC, with errors below 1% over a large range





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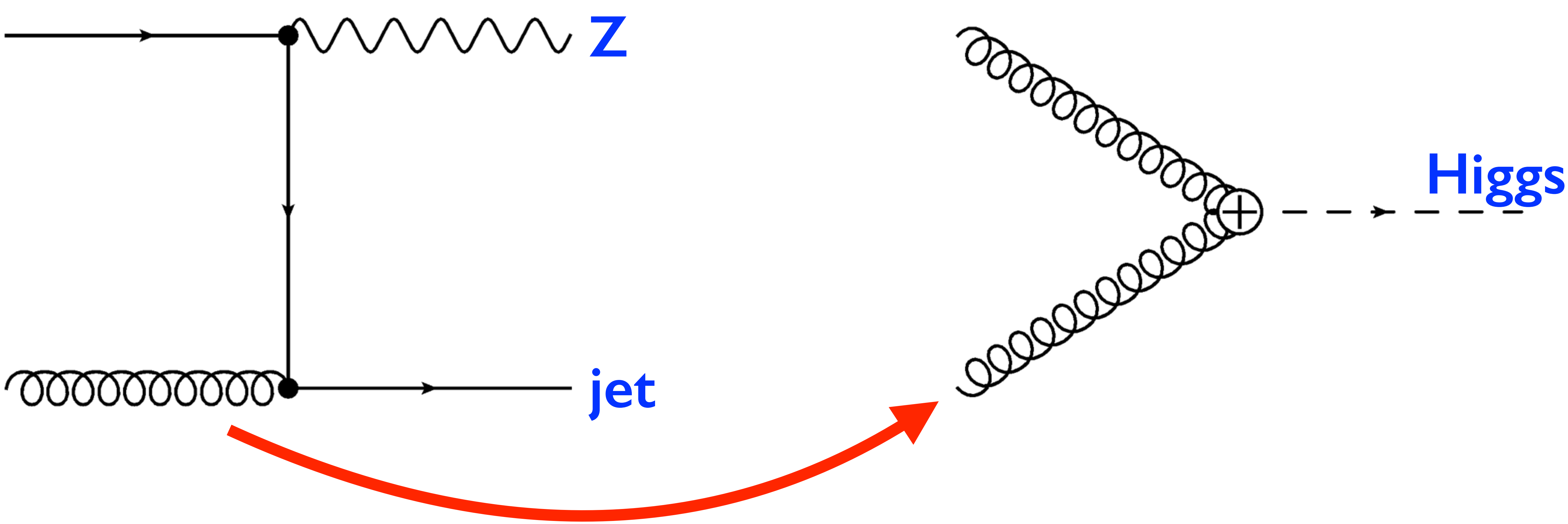
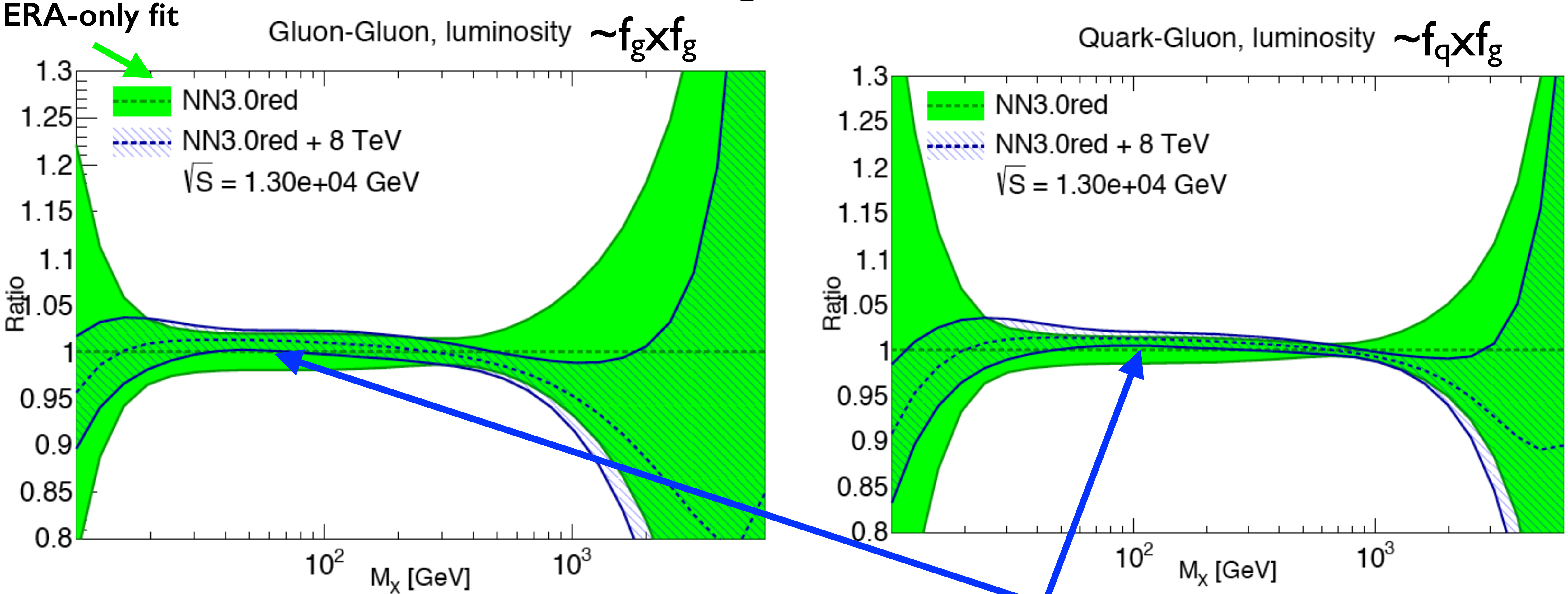


Can learn about the gluon distribution entering Higgs production from this data!

With recent theory advances we can now use this data in a NNLO global fit of PDFs!

baseline HERA-only fit



Gluon-gluon and quark-gluon luminosity errors reduced right near $M_X \sim m_H$=125 GeV!

	Before p_T^Z data	After p_T^Z data
$\sigma_{gg\to H}$ [pb]	48.22 ± 0.89 (1.8%)	48.61 ± 0.61 (1.3%)
$\sigma_{\rm VBF}$ [pb]	3.92 ± 0.06 (1.5%)	3.96 ± 0.04 (1.0%)

PDF error on Higgs cross sections reduced!

Boughezal, Guffanti, FP, Ubiali JHEP 1707 (2017)

Future directions at NNLO and beyond

- **Current topic**: 2-loop amplitudes for 2→3 processes. Currently an active subject of study, with initial results for 3-jet amplitudes appearing (Gehrmann, Henn, Lo Presti (2016); Badger, Bronnum-Hansen, Hartanto, Peraro (2017); Abreu, Febres Cordero, Ita, Page, Zeng (2017); …)

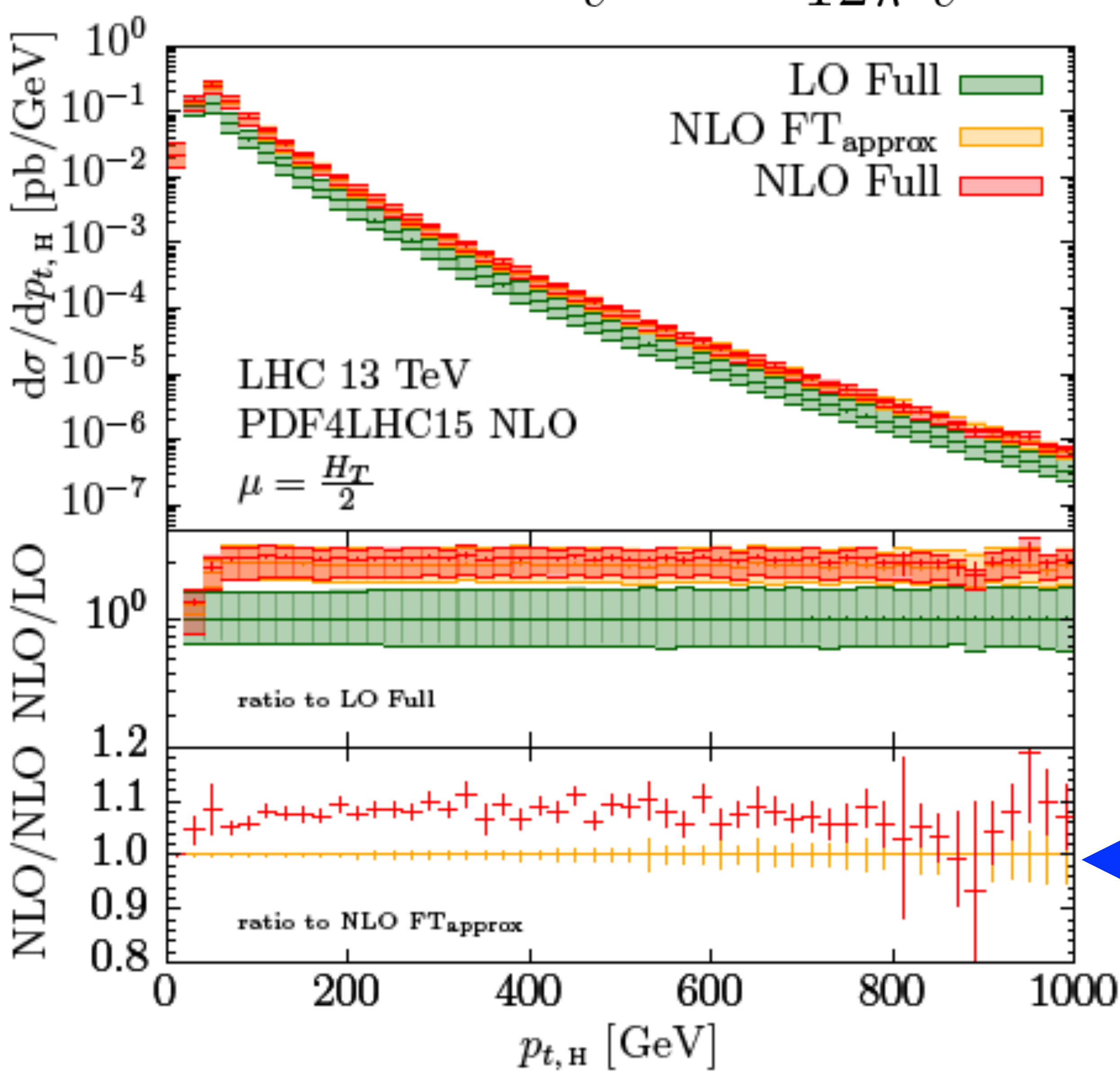
- **Current topic**: multi-scale 2-loop amplitudes with massive internal particles, relevant for Higgs, top, vector boson production. New mathematical structures beyond multiple polylogarithms appear (Remiddi, Tancredo (2016); Bonciani et al (2016); Weinzierl et al (2016-2017); Ablinger et al (2017); Broedel, Duhr, Dulat, Tancredi (2017); …)

- **New result at 3 loops:** completely analytic calculation of 3-loop inclusive gluon-fusion Higgs production in terms of elliptic integrals (Mistlberger (2018))

Recent multi-scale 2-loop result: Higgs p_T spectrum with full m_t dependence

- Critical to look for BSM effects in the Higgs sector, and to break degeneracies between couplings that appear if only the total cross section is measured:

$$\Delta\mathcal{L} = -c_t \frac{m_t}{v} + \kappa_g \frac{\alpha_s}{12\pi} \frac{h}{v} G^a_{\mu\nu} G^{a,\mu\nu} \quad \Rightarrow \quad \frac{\sigma(c_t, \kappa_g)}{\sigma_{SM}} \approx (c_t + \kappa_g)^2$$



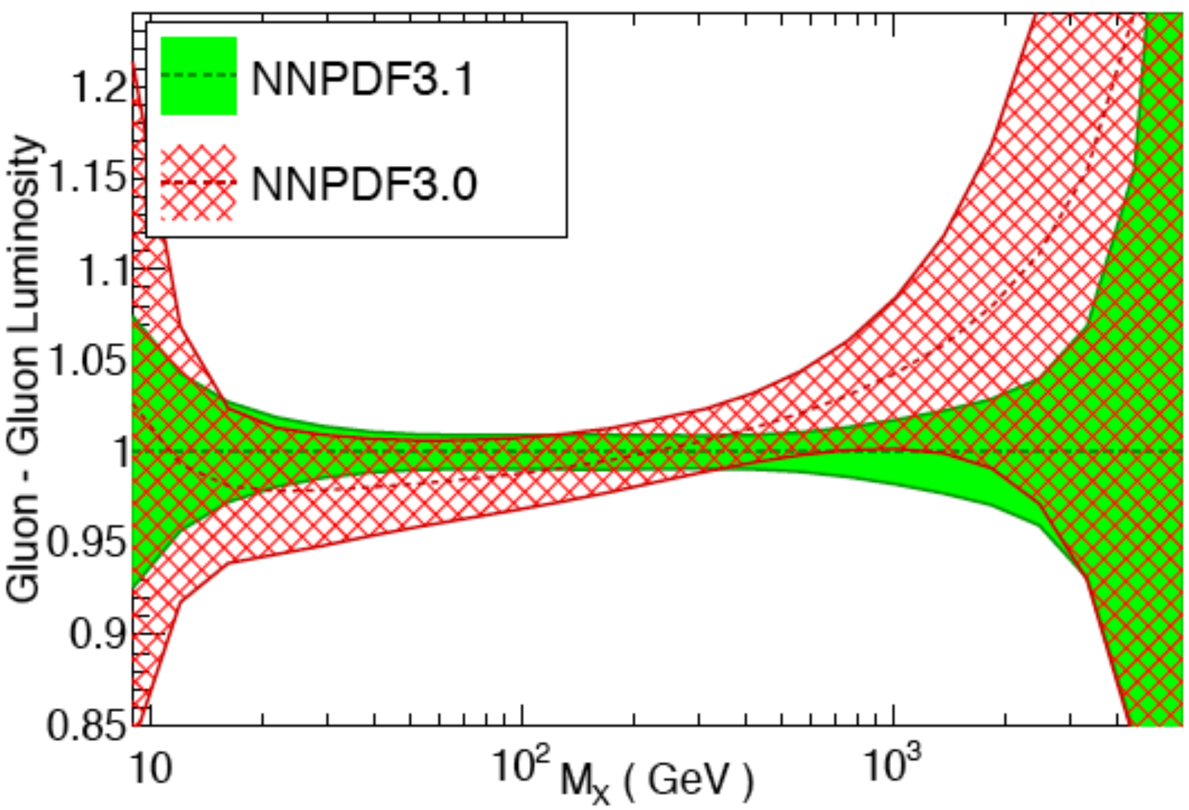
NLO for finite m_t now known, important input to future Higgs analyses! (Jones, Kerner, Luisoni (2018))

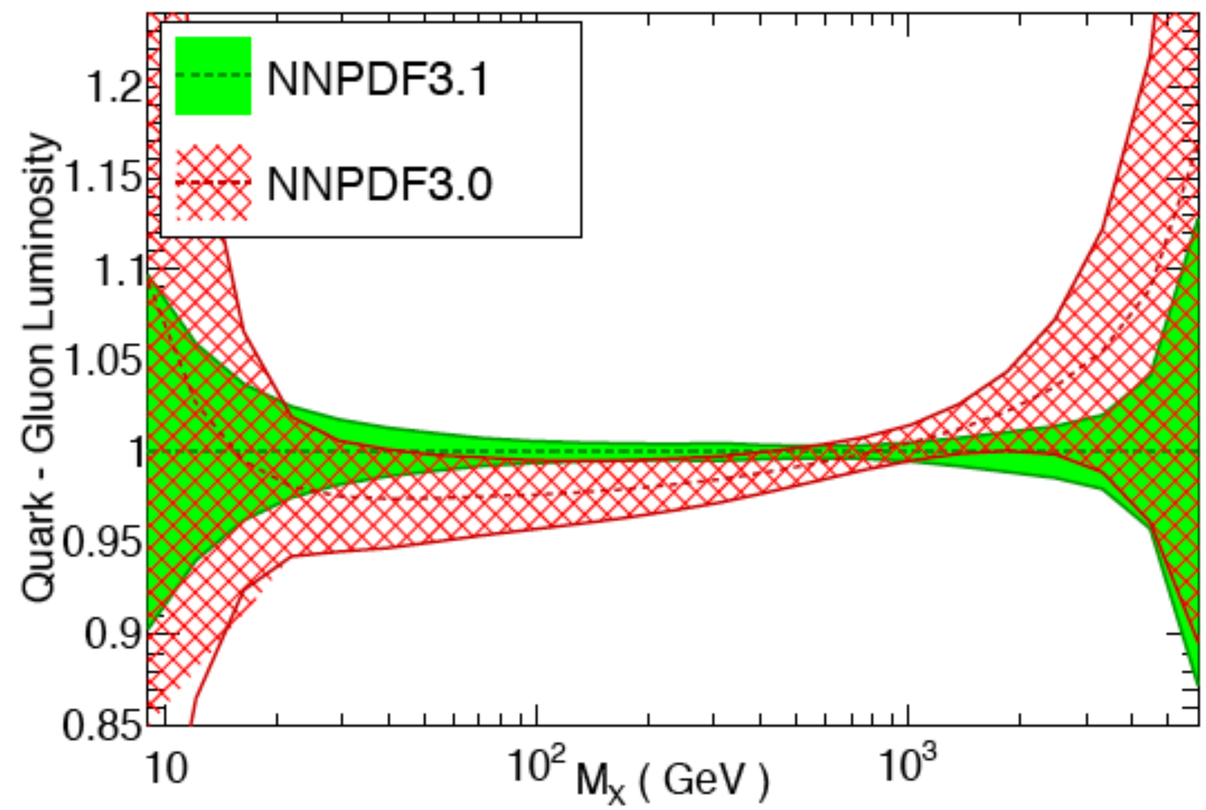
Numerical evaluation of the necessary 2-loop integrals using sector decomposition

Previous best approximation got shape correct; exact NLO larger by 6-8%

Advances in PDFs

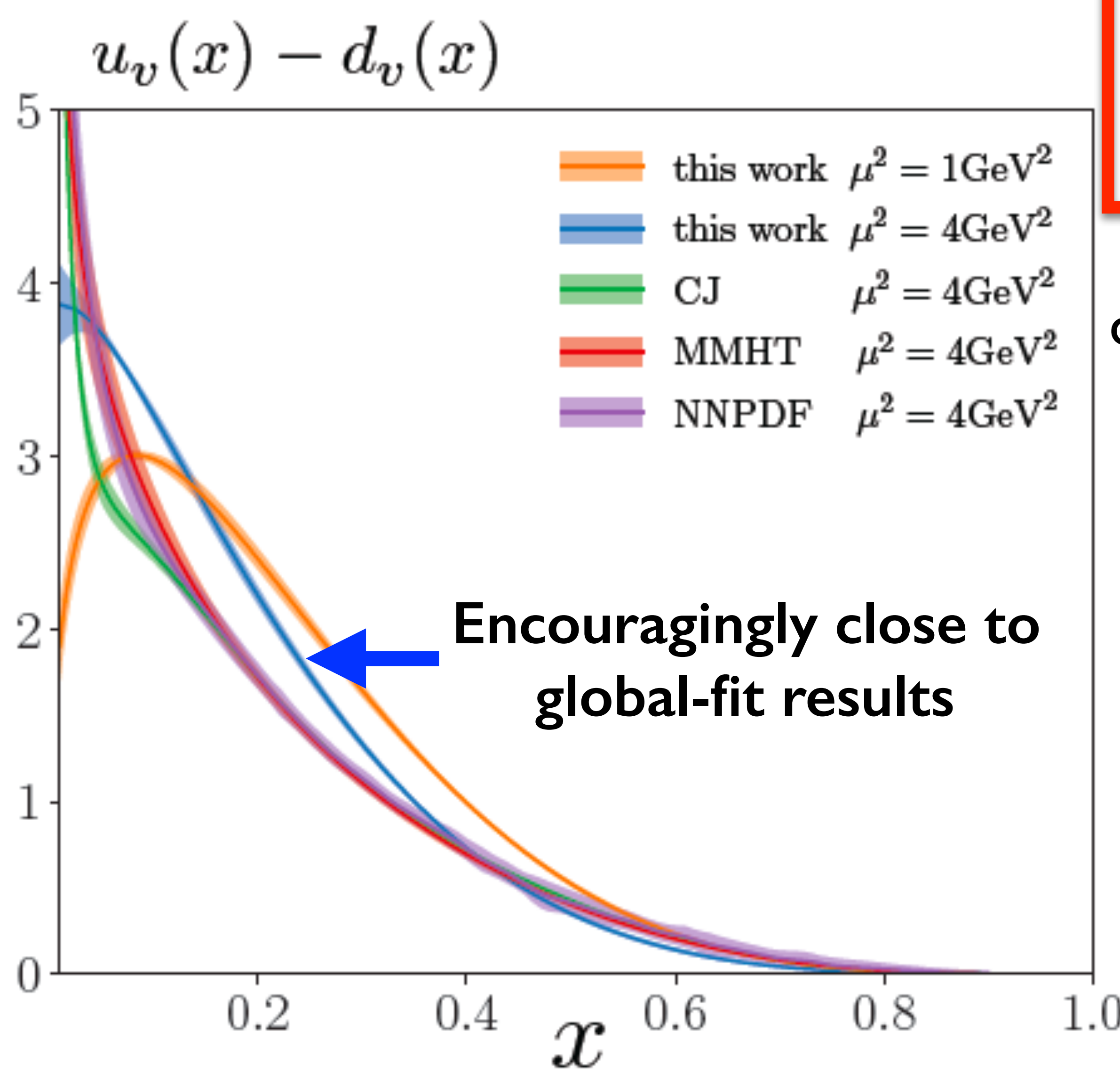
- Past year has seen updates to many global PDF determinations
  - ABMP: new W, Z, top data included; updated extraction of the strong coupling from DIS+fixed-target data: α_s=0.1145(9)
  - CTEQ: new technique to visualize impact of data sets in fits
  - MMHT: study of PDF sensitivity to jet production data
  - NNPDF: first time incorporating Z p_T and top pair data; study of the sensitivity to intrinsic charm; **significantly reduced errors!**





New direction: PDFs from the lattice

- **New idea**: x-dependent PDFs can be obtained directly from lattice QCD calculations (Ji (2013); Radyushkin (2017)); calculate related Euclidean quantities (quasi- or pseudo-PDFs) on the lattice, use effective field theory to relate them to the usual PDFs



Orginos, Radyushkin, Karpie, Zafeiropoulos (2017)

$$q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2)$$

quasi-PDF from lattice — perturbative matching coefficient — regular PDF

Proof-of-principle lattice determination of pseudo-PDFs (lattice spacing, finite volume effects not fully studied)

Significant community attention (arXiv:1711.07916);

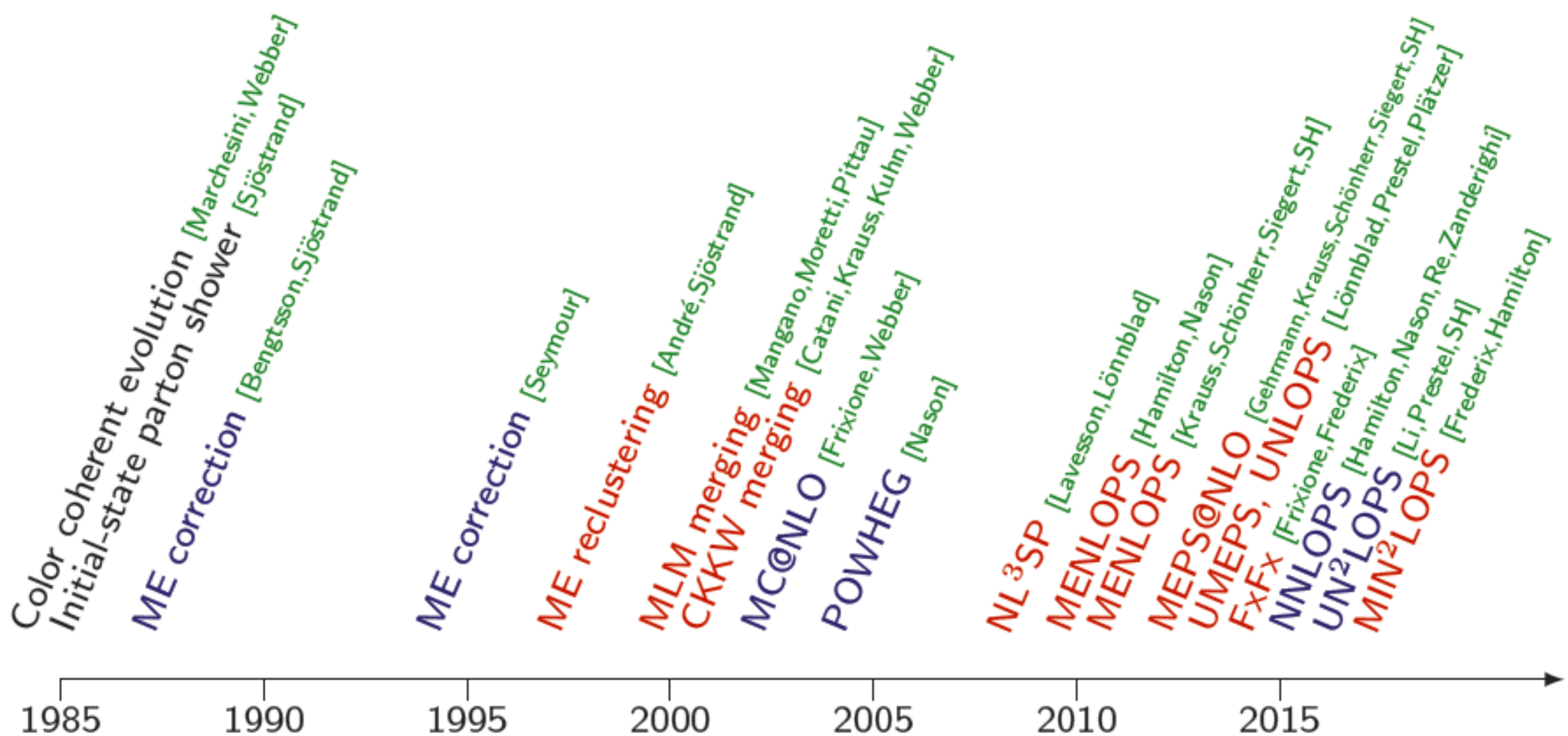
Stay tuned!

Progress on parton showers

- Parton shower Monte Carlo event generators bridge the hard interaction and the Λ_{QCD}-scale hadronization
- In measurements that probe multiple scales, parton showers resum the large logs separating these scales in a flexible way applicable to multiple observables

Merging related
Matching related

Past decade: improve description of hard interaction in parton showers through matching (more loops) or merging (more legs)



from S. Hoeche

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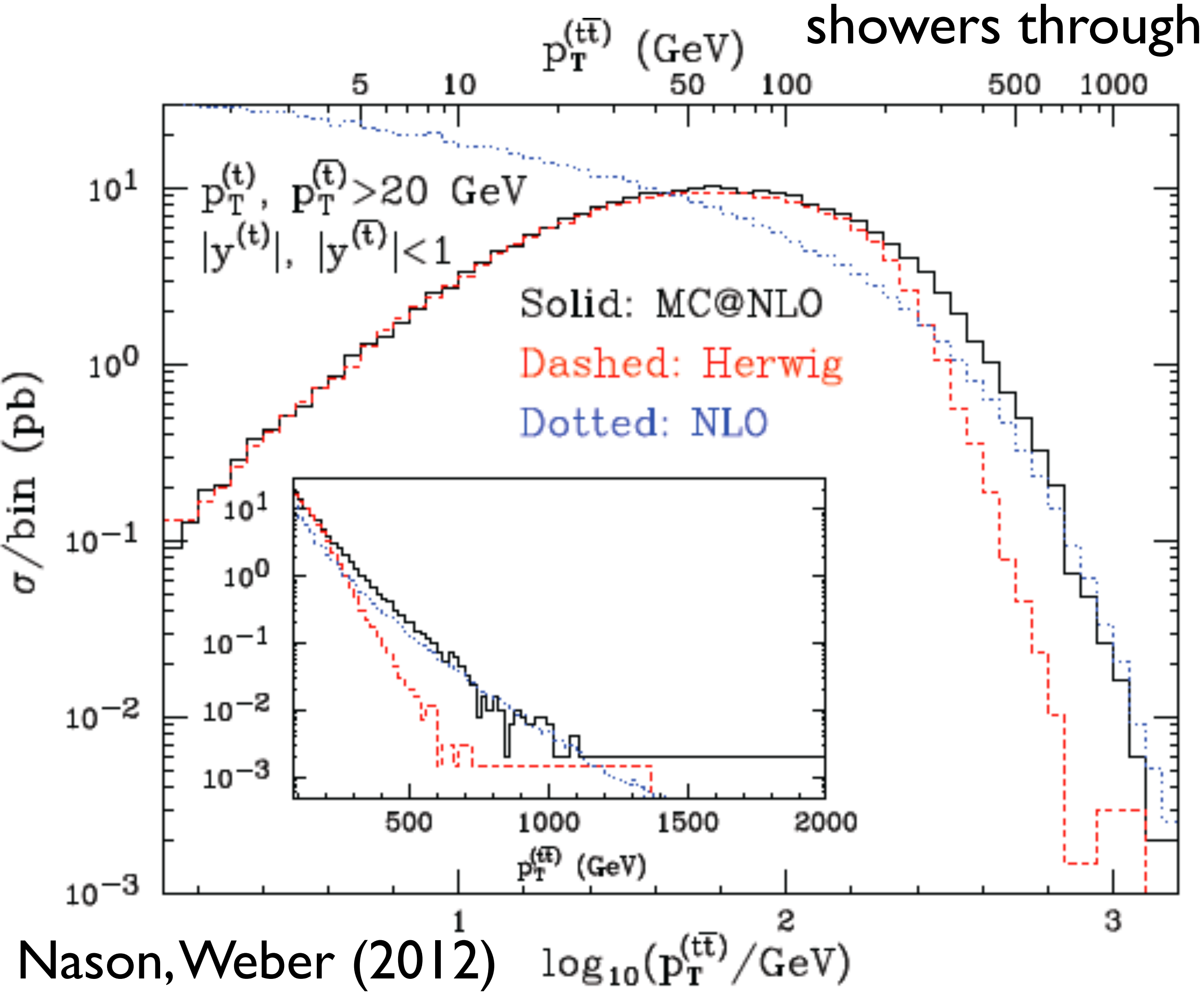


Nason, Weber (2012)

Maintains parton shower accuracy near singular regions

Matches fixed-order in hard phase-space region

Progress on parton showers

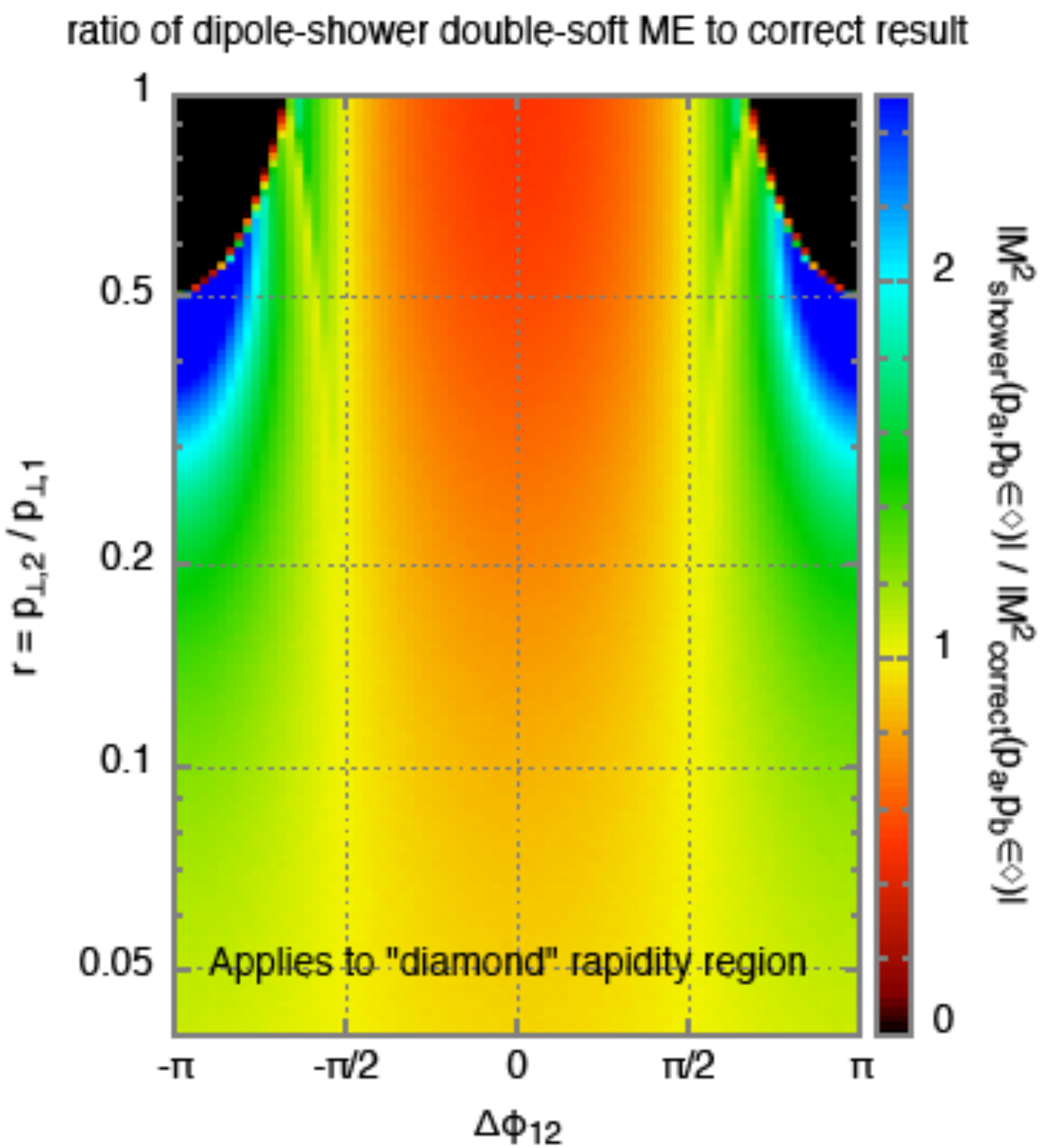
- Parton shower Monte Carlo event generators bridge the hard interaction and the Λ_{QCD}-scale hadronization
- In measurements that probe multiple scales, parton showers resum the large logs separating these scales in a flexible way applicable to multiple observables

Recent focus: revisit the accuracy of the shower itself

- Incorporate spin/color interference into showers (DEDUCTOR, Nagy, Soper (2007-2018); Martinez et al 2018; …)
- Incorporate higher-order splitting functions or $1 \to 3$ splitting into showers (Jadach et al (2016); Hoeche, Krauss, Prestel (2017); …)
- Consider different shower evolution variables (DIRE, Hoeche, Prestel (2015); VINCIA, Fischer, Prestel, Ritzmann, Skands (2016); …)

Systematic study of shower accuracy

- **New result:** a systematic framework to study shower accuracy (Dasgputa, Dreyer, Hamilton, Monni, Salam (2018)):



Two criteria:

- Do they reproduce known singular limits of multi-parton amplitudes (e.g., Catani, Grazzini (1999))?
- Do they match known analytic resummation formulae?

An example: at leading N_C, 100% mismatches in double-soft region for p_T-ordered showers (DIRE, PYTHIA)

Potential impact when measurements rely heavily on showers; stay tuned!

The strong coupling constant

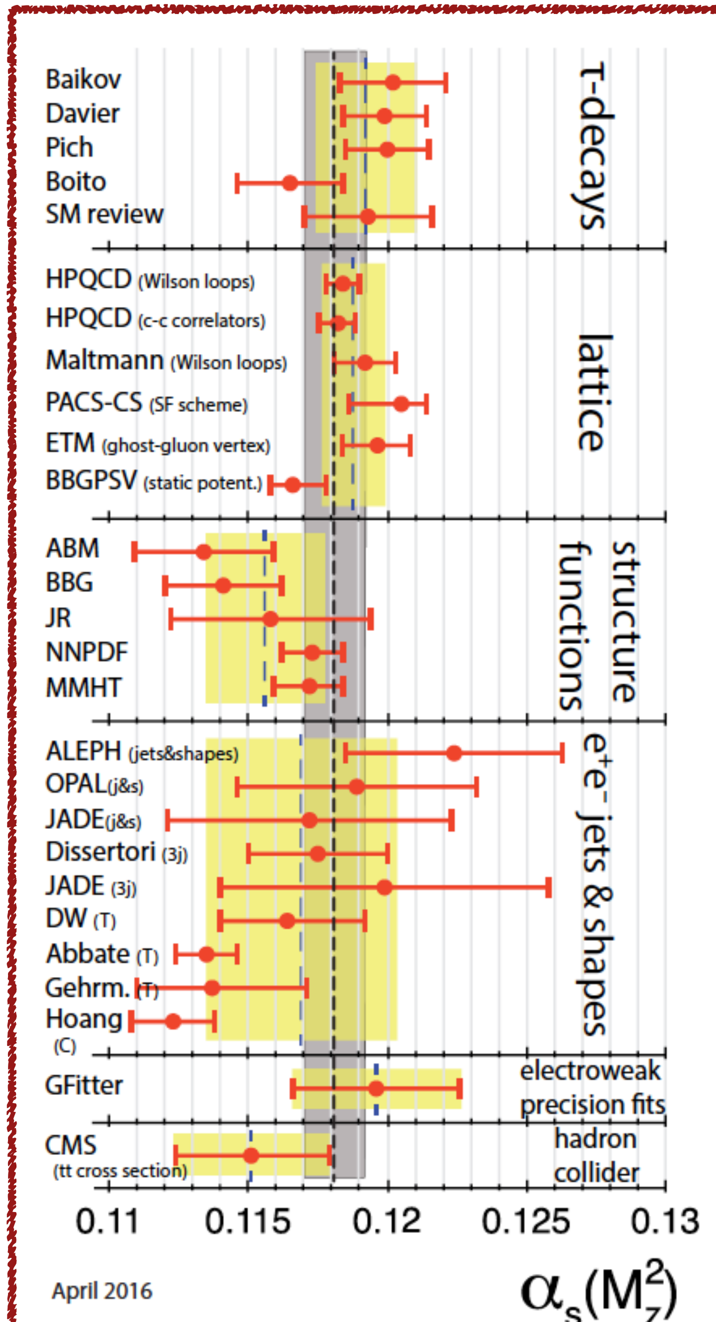
- The value of α_S at the scale M_Z cannot be predicted from QCD and must be extracted by comparing theory to data

PDG 2017 world average:

$$\alpha_S(M_Z) = 0.1181 \pm 0.0011$$

Bethke, Dissertori, Salam PDG 2017

- World average dominated by lattice determinations
- Additional contributions from tau decays, DIS, $e^+e^- \to$ jets, EW precision data, ttbar cross section



[Particle Data Group: 16]

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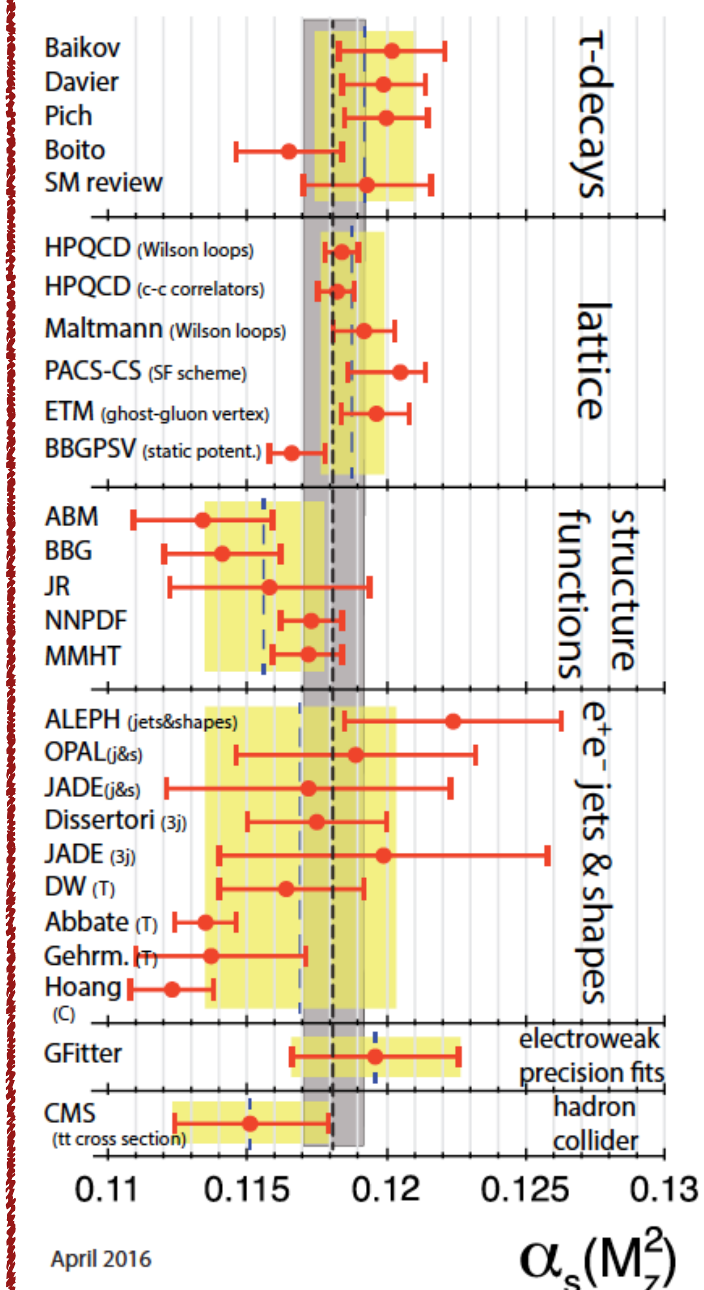

[Particle Data Group: 16]

- Error has *increased* since the PDG 2013 average of $\alpha_S(M_Z)=0.1185\pm0.0006$
- Primary reasons: low ttbar cross section, more conservative lattice averaging

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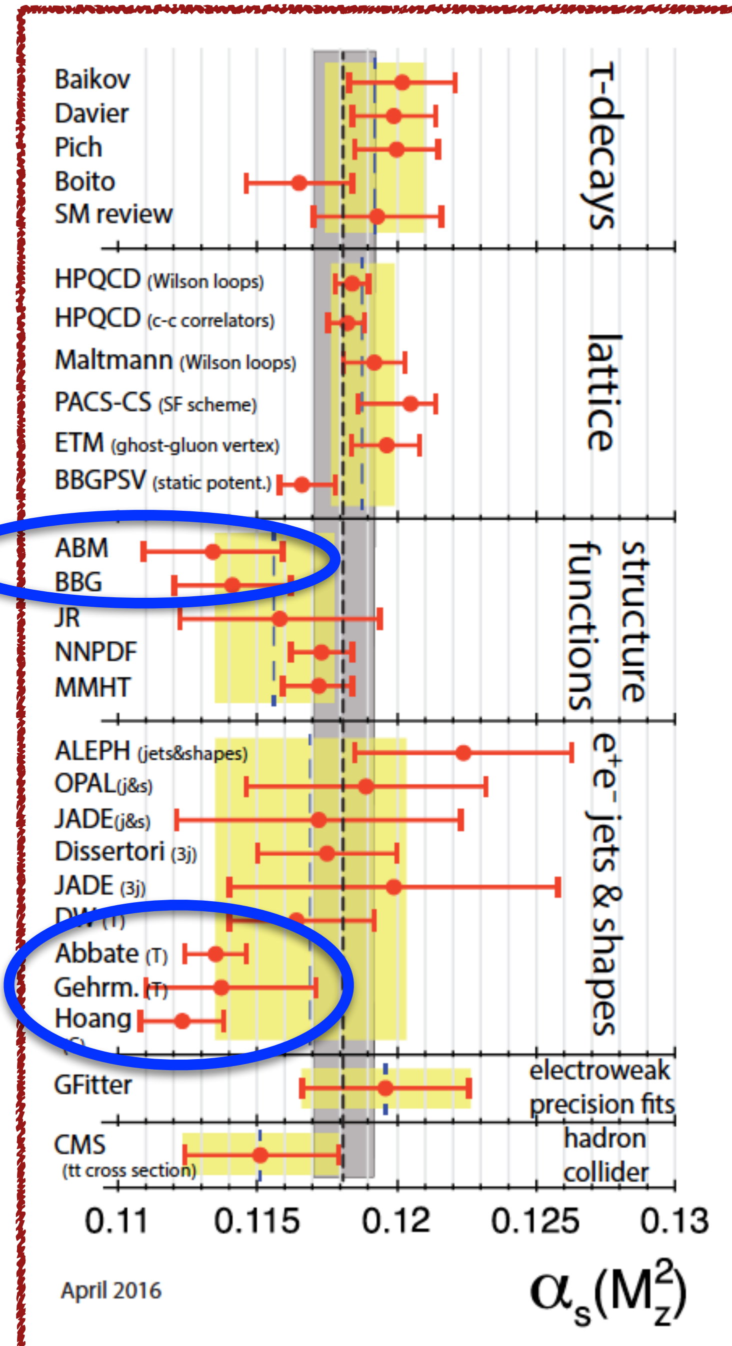
PDG 2017 world average:

$$\alpha_S(M_Z) = 0.1181 \pm 0.0011$$

Bethke, Dissertori, Salam PDG 2017

Systematically lower values from most accurate DIS and e^+e^- fits; a lingering issue!

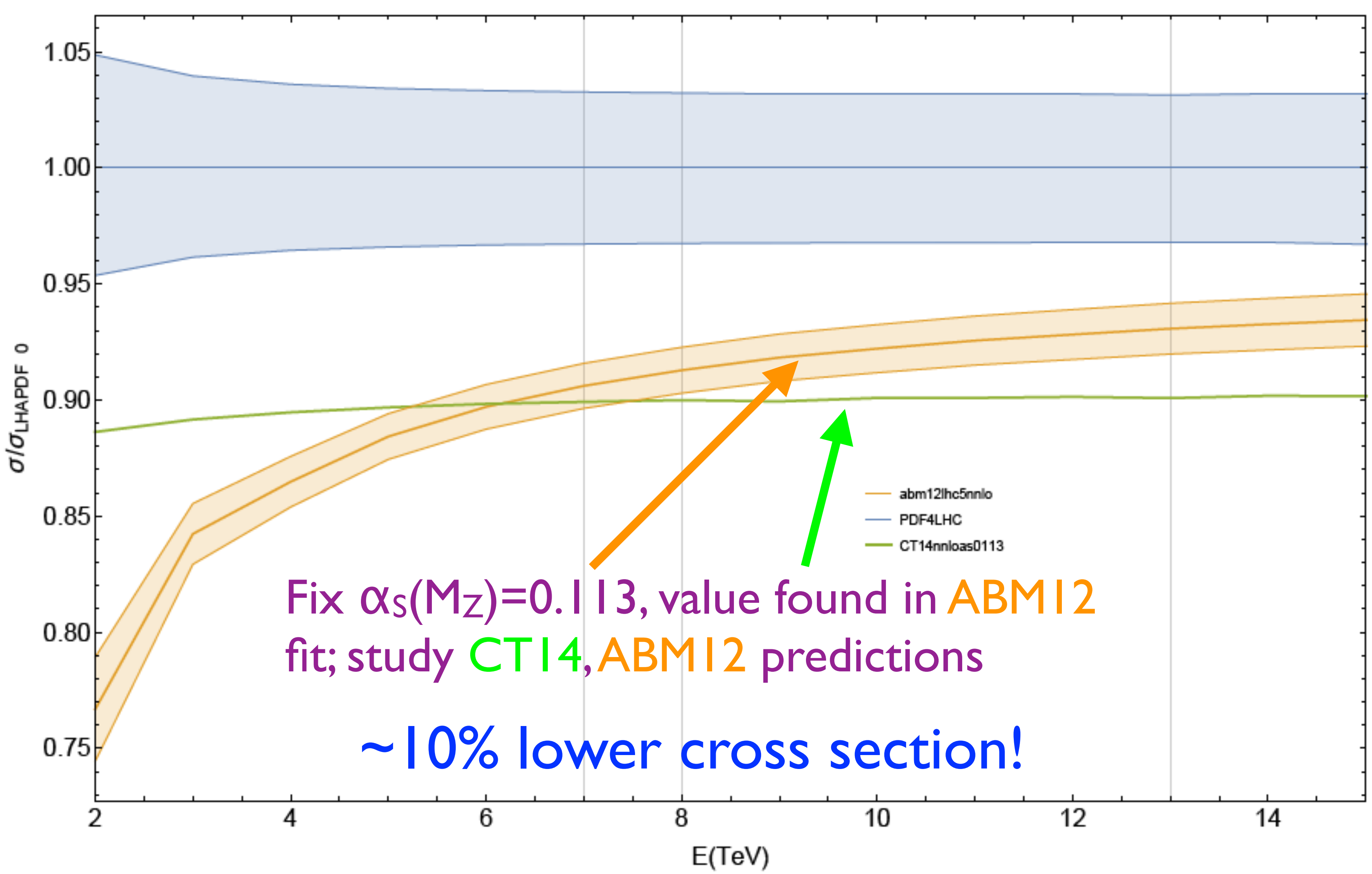


[Particle Data Group: 16]

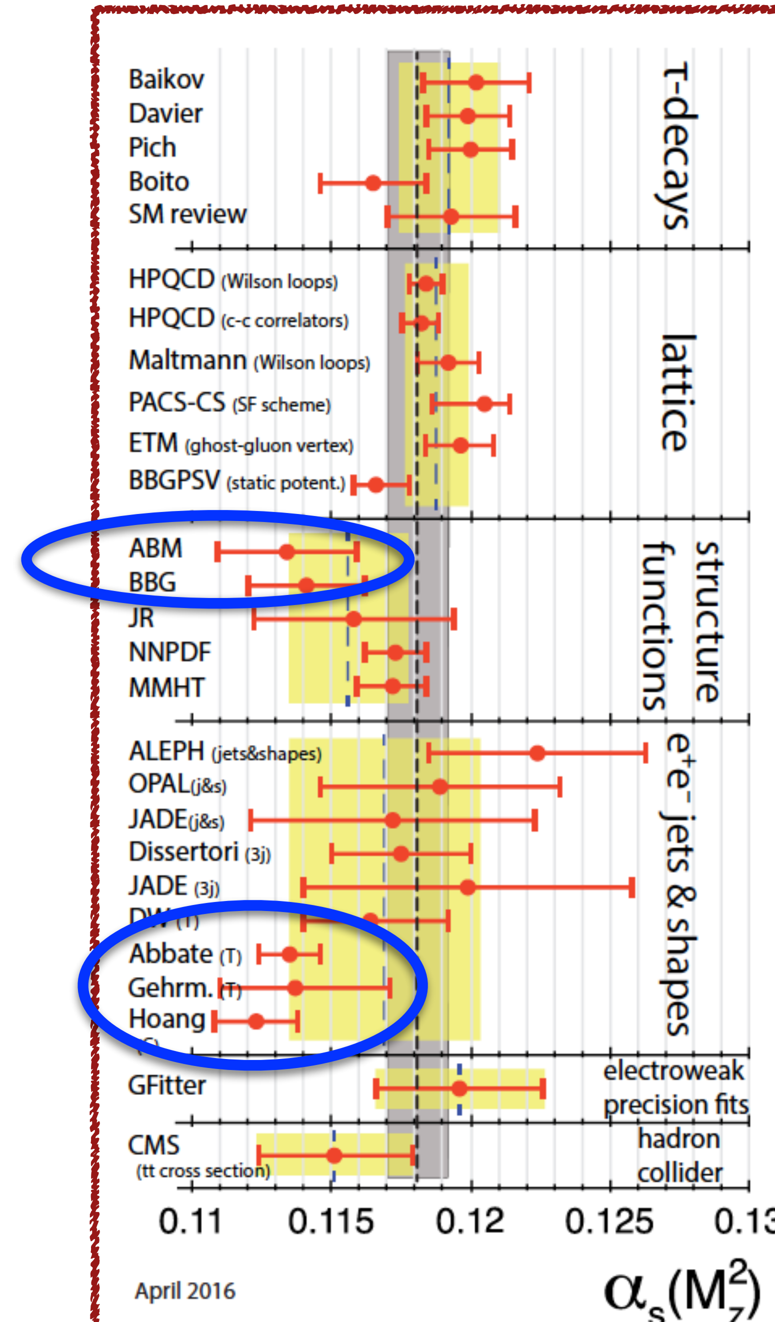
The strong coupling constant

- The value of α_S at the scale M_Z cannot be predicted from QCD and must be extracted by comparing theory to data

Could impact Higgs cross section!



Anastasiou et al (2016)



[Particle Data Group: 16]

Conclusions

- Direct searches have so far not revealed any hints of physics beyond the Standard Model. Precision searches have become critical at this point.
- Uncertainties on LHC data have reached an unprecedented level; theory predictions for various observables are quickly catching up!
- In the last couple of years several milestones were achieved:
    - $2 \to 2$ LHC processes now available at NNLO in QCD
    - LHC data were combined with NNLO predictions to achieve a more precise understanding of proton structure than ever before
    - Accuracy of parton showers being improved and quantified