Beam Energy Spread Measurement @ FCC-ee

arc cell optics	60/60	90/90	90/90	90/90
momentum compaction [10^{-5}]	1.48	0.73	0.73	0.73
horizontal emittance [nm]	0.27	0.28	0.63	1.45
vertical emittance [pm]	1.0	1.0	1.3	2.7
horizontal beta* [m]	0.15	0.2	0.3	1
vertical beta* [mm]	0.8	1	1	2
length of interaction area [mm]	0.42	0.5	0.9	1.99
tunes, half-ring (x, y, s)	(0.569, 0.61, 0.0125)	(0.577, 0.61, 0.0115)	(0.565, 0.60, 0.0180)	(0.553, 0.59, 0.0350)
longitudinal damping time [ms]	414	77	23	6.6
SR energy loss / turn [GeV]	0.036	0.34	1.72	9.21
total RF voltage [GV]	0.10	0.44	2.0	10.93
RF acceptance [%]	1.9	1.9	2.3	4.9
energy acceptance [%]	1.3	1.3	1.5	2.5
energy spread (SR / BS) [%]	0.038 / 0.132	0.066 / 0.153	0.099 / 0.151	0.15 / 0.20
bunch length (SR / BS) [mm]	3.5 / 12.1	3.3 / 7.65	3.15 / 4.9	2.5 / 3.3
Piwinski angle (SR / BS)	8.2 / 28.5	6.6 / 15.3	3.4 / 5.3	1.39 / 1.60
bunch intensity [10^{11}]	1.7	1.5	1.5	2.8
no. of bunches / beam	16640	2000	393	39
beam current [mA]	1390	147	29	5.4
luminosity [10^{34} cm^{-2}s^{-1}]	**230**	**32**	**8**	**1.5**
beam-beam parameter (x / y)	0.004 / 0.133	0.0065 / 0.118	0.016 / 0.108	0.094 / 0.150
luminosity lifetime [min]	70	50	42	44
time between injections [sec]	122	44	31	32
allowable asymmetry [%]	±5	±3	±3	±3

Initial requirements w/o energy spread

- **Targets set in the TLEP paper**
  - Precision on the Z width : 100 keV
  - Precision on the Z mass : 100 keV
  - Absolute precision on the peak cross section : 10^{-4}

- **We will run at least with three beam energies (*) around the Z pole**
  - E_{beam} = 45.6 GeV , i.e., the Z pole
    - σ_{peak} ~ 30 nb , L_{peak} ~ 100 ab^{-1}, N_{peak} ~ 3×10^{12} events
  - E_{beam} = 43.95 GeV and 47.15 GeV, for the $\alpha_{QED}(m_Z)$ measurement
    - $\sigma_{peak\pm3}$ ~ 5 – 9 nb; $L_{peak\pm3}$ ~ 25 ab^{-1}, $N_{peak\pm3}$ ~1.25 – 2.25 $\times10^{11}$ events
  - Statistics large enough to be limited by systematic uncertainties for m_Z, Γ_Z and σ_0

(*) See next slide

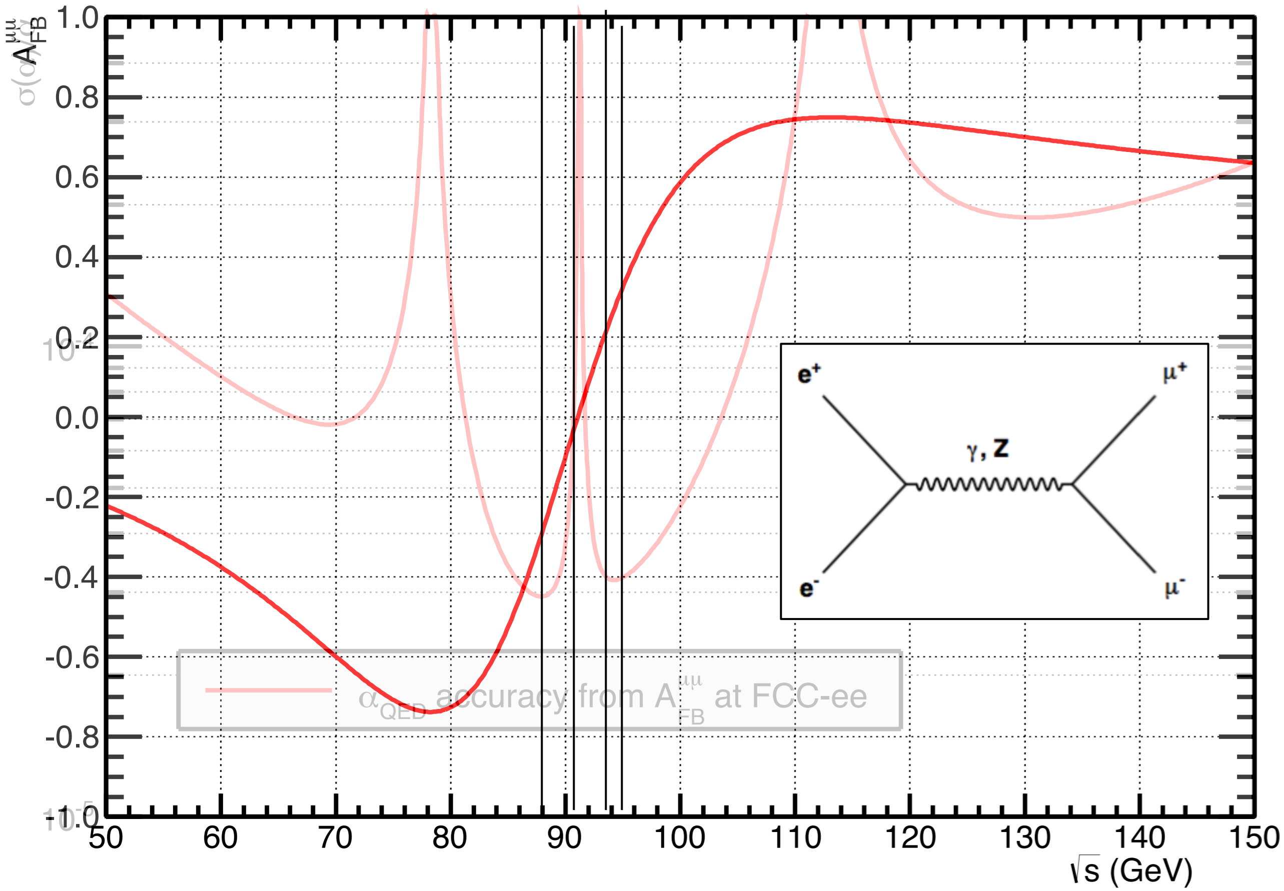
- **To reach the aforementioned targets w/o energy spread, we need**
  - A measurement of the beam energy (e^+ and e^-) with a precision of 50 keV
  - A point-to-point relative integrated luminosity measurement precision of 5×10^{-5}
  - An absolute integrated luminosity measurement precision of 10^{-4}
    - Result of a 3-parameter fit: $\sigma(m_Z)$ = 96 keV, $\sigma(\Gamma_Z)$ = 104 keV, $\sigma(\sigma_0)/\sigma_0$= 10^{-4}

Side remark : what beam energies ?

- **With the same precision on the beam energy and luminosity, no spread**
  - Result of the fit with Peak±2 GeV instead of Peak±3 GeV
    - $\sigma(m_Z)$ = 86 keV, $\sigma(\Gamma_Z)$ = 140 keV, $\sigma(\sigma_0)/\sigma_0 = 10^{-4}$
  - Result of the fit with Peak±1 GeV instead of Peak±3 GeV
    - $\sigma(m_Z)$ = 84 keV, $\sigma(\Gamma_Z)$ = 263 keV, $\sigma(\sigma_0)/\sigma_0 = 10^{-4}$
  - Target not reached for the Z width
    - Almost no difference for the mass and the peak cross section
      - ➨ Let's stick to Peak±3 GeV for the time being
- **(*) Note added (see polarization workshop)**
  - The beam energies corresponding to ~half-integer spin tunes are slighltly different
    - 25 ab^{-1} at $\sqrt{s}$ = 87.91 GeV (E_{beam} = 43.86 GeV, spin tune = 99.55) Peak - 4
    - 100 ab^{-1} at $\sqrt{s}$ = 91.19 GeV (E_{beam} = 45.6 GeV, spin tune = 103.48) Peak
    - 10 / 25 ab^{-1} at $\sqrt{s}$ = 93.85 GeV (E_{beam} = 46.93 GeV, spin tune = 107.5) Peak + 4
    - 15 / 0 ab^{-1} at $\sqrt{s}$ = 94.73 GeV (E_{beam} = 47.37 GeV, spin tune = 108.5) Peak + 5
      - ➨ Note that $\sqrt{s} < 2E_{beam}$ because of the small crossing angle

Side remark : what beam energies ?

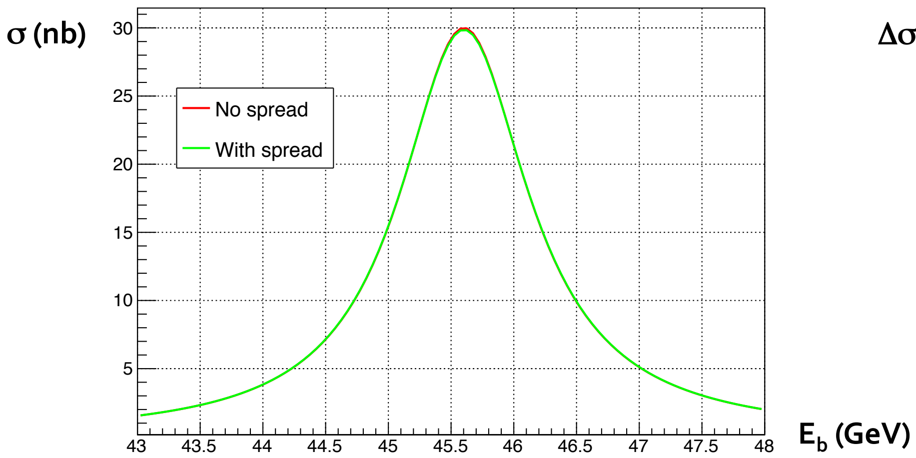
- Choice driven by $\sin^2\theta_W^{eff}$ and α_{QED} determination from $A_{FB}(\mu\mu)$

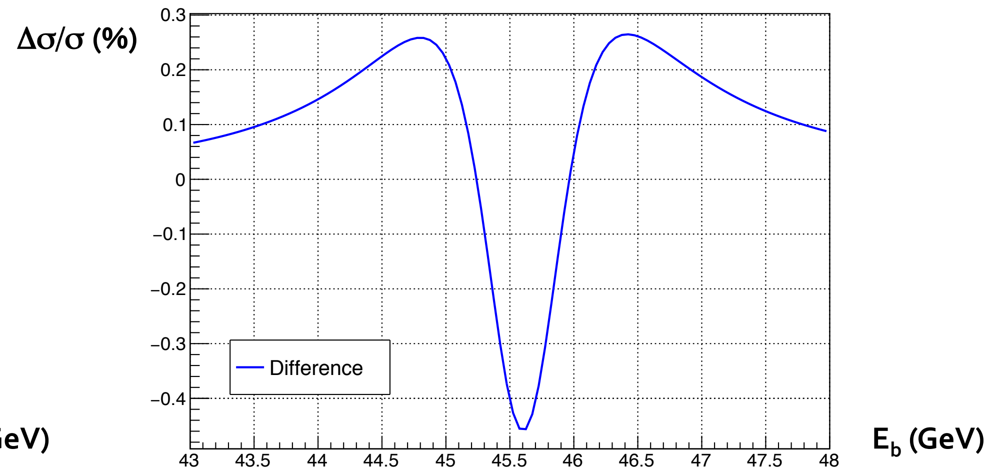


$$A_{FB}^{\mu\mu} = \frac{N_F^{\mu+} - N_B^{\mu+}}{N_F^{\mu+} + N_B^{\mu+}} \approx f(\sin^2\vartheta_W^{eff}) + \alpha_{QED}(s)\frac{s - m_Z^2}{2s} g(\sin^2\vartheta_W^{eff})$$

What happens with energy spread ?

- **Let's take the current beam energy spread with beamstrahlung**
  - E_{spread} = 0.132% E_{beam} (~60 MeV) for each beam – Spread assumed to be Gaussian
    - Cross section differs by -0.4% to +0.3% , i.e., much larger than uncertainties





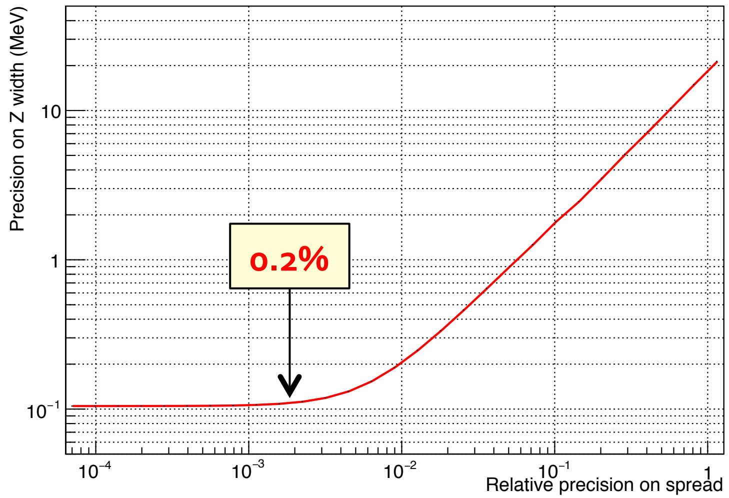
  - Dominant effect on the width measurement: reduction of the peak cross section
    - With a three parameter fit and three energies: $\Gamma_Z \rightarrow [\Gamma_Z^2 + 8E_{spread}^2]^{1/2}$
      - ➨ $\Delta\Gamma_Z > 8\Gamma_Z(E_{spread}/\Gamma_Z)^2 \times \Delta E_{spread}/E_{spread}$ (= 12 MeV × $\Delta E_{spread}/E_{spread}$ for E_{spread} = 60 MeV)

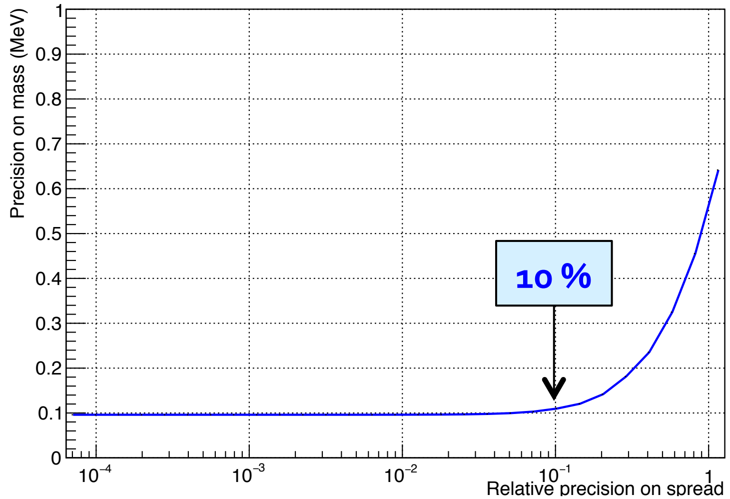
1% uncertainty of E_{spread} leads to > 120 keV uncertainty on Γ_Z !

  - Need to find a way to determine the energy spread at the peak to a few per mil
    - And fit the cross section to the convolution of a Breit Wigner with a Gaussian
      - ➨ That's a four parameter fit (m_Z, Γ_Z, σ_0, E_{spread})

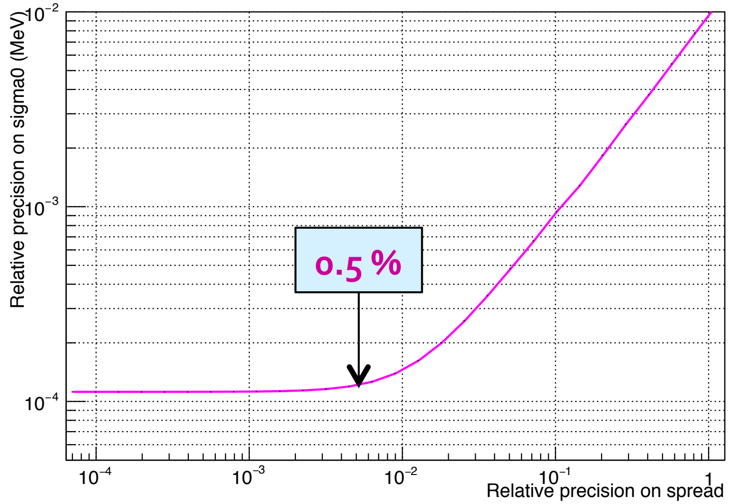
Four-parameter fit with three energies

- We need an external measurement of the beam energy spread
  - The precisions on m_Z, Γ_Z, and σ_0 depend on the precision of this measurement



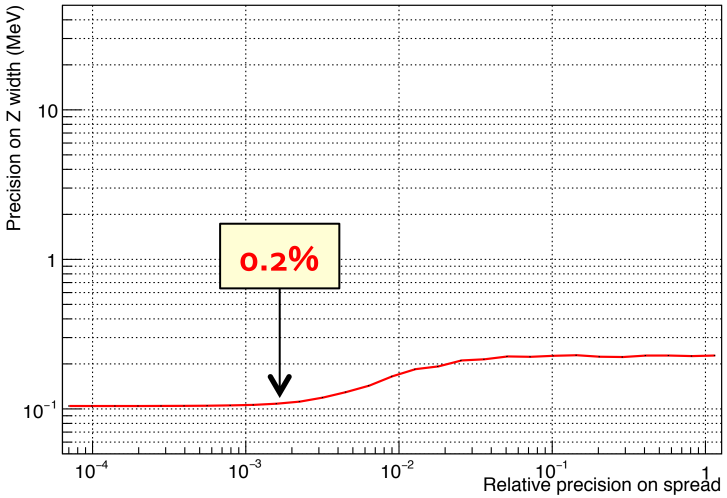


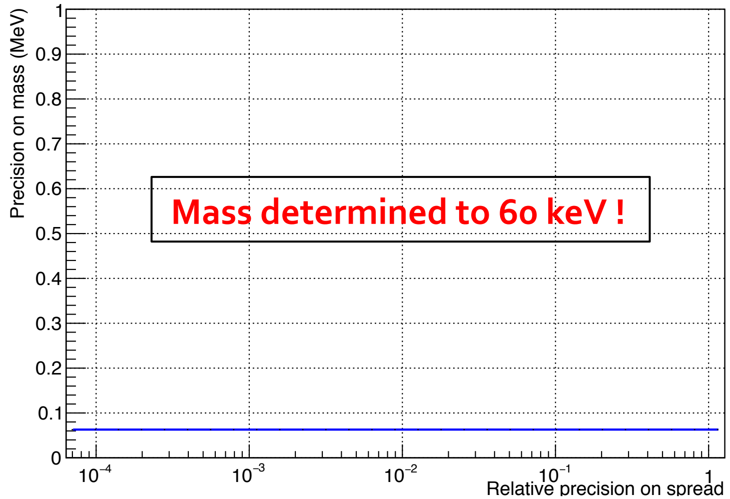
- Relative precision of 0.2% required !
  - Checked that 0.8% enough off-peak
  - Challenging beam instrumentation
- Can we help with collision data ?



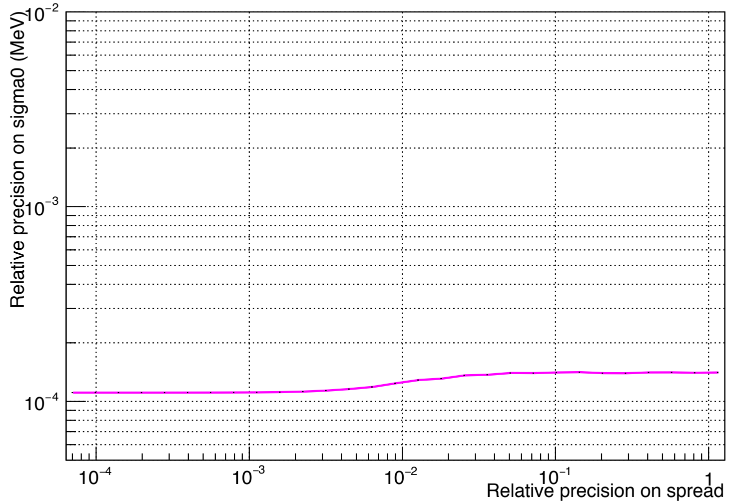
Add two energy points ?

- The optimal choice is to fit the E_{spread} by adding Peak±1
  - Energy spread determined with ~1% relative precision … not quite enough





- Still need external measurement for Γ_Z
  - ➨ Precision 200 keV otherwise
- Good to have, but…
  - Reduction in total Z statistics
  - Assumes constant & Gaussian spread
  - Assumes uniform spread for all bunches
    - ➨ Other (better) ideas required



What happens for $A_{FB}^{\mu\mu}$?

- **Convoluting A_{FB} with a Gaussian has almost no effect**
  - The sampling of $\sqrt{s}$ around $\sqrt{s_-} = 87.9$ and $\sqrt{s_+} = 93.8$ GeV, however, is not uniform
    - Has to weigh the asymmetry by the production cross section:

$$\Delta A_{\mathrm{FB}}^{\mu\mu}(s_\pm) = \frac{\int A_{\mathrm{FB}}^{\mu\mu}(s)\sigma_{\mu\mu}(s)\exp - \frac{\left(\sqrt{s}-\sqrt{s_\pm}\right)^2}{2s_\pm\delta^2} d\sqrt{s}}{\int \sigma_{\mu\mu}(s)\exp - \frac{\left(\sqrt{s}-\sqrt{s_\pm}\right)^2}{2s_\pm\delta^2} d\sqrt{s}} - A_{\mathrm{FB}}^{\mu\mu}(s_\pm)$$

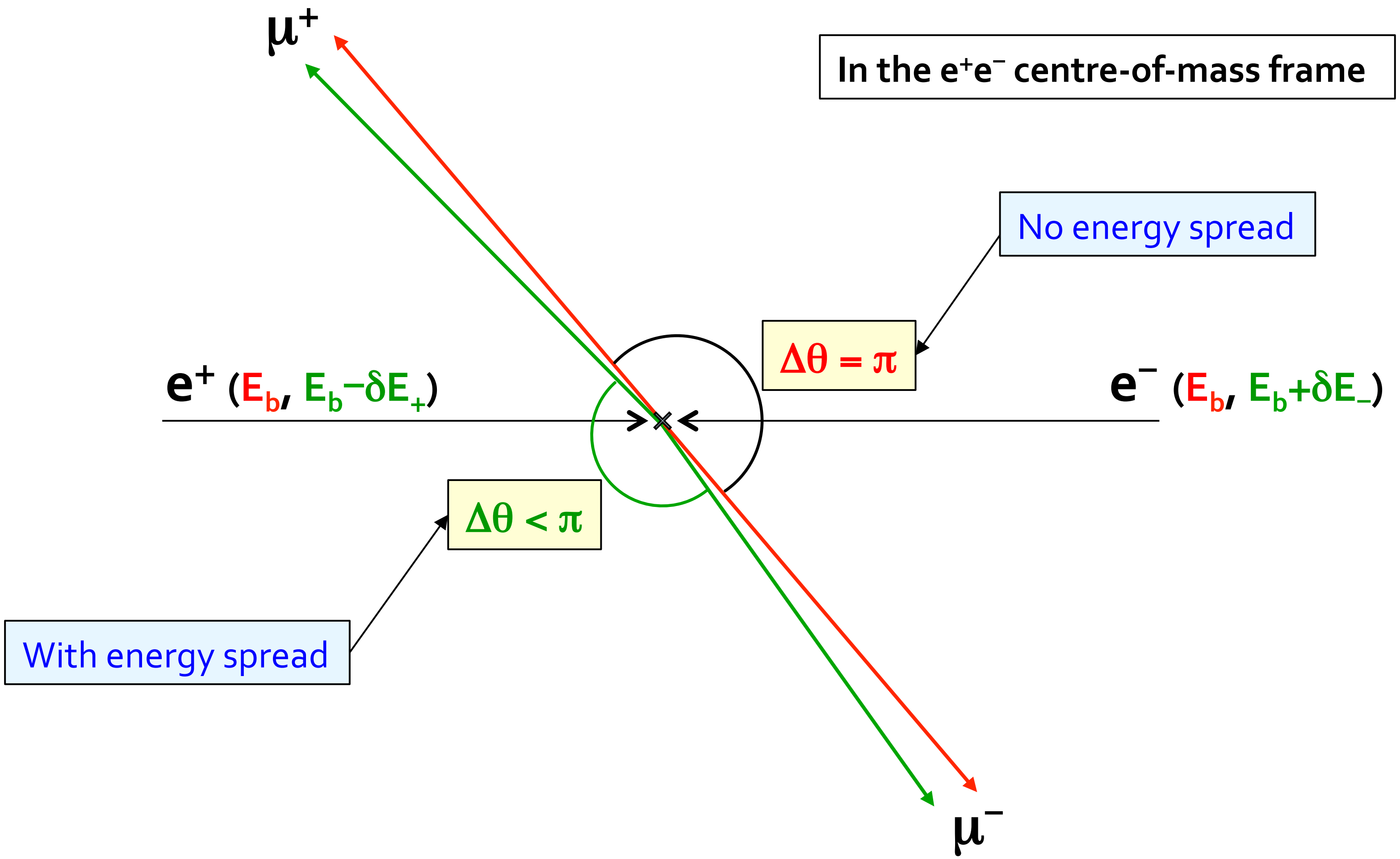
  - Effect is to increase the effective $\sqrt{s_-}$ and decrease the effective $\sqrt{s_+}$
    - And therefore decrease the absolute value of the asymmetry in both points

$$\frac{\Delta A_{\mathrm{FB}}}{A_{\mathrm{FB}}}(s_-) = -0.99 \times 10^{-3} \quad \text{and} \quad \frac{\Delta A_{\mathrm{FB}}}{A_{\mathrm{FB}}}(s_+) = -1.03 \times 10^{-3}$$

- **Need to know the energy spread to better than 1% at $\sqrt{s_\pm}$**
  - To limit the effect on α_{QED} to $< 10^{-5}$ (same as the effect from the $\sqrt{s}$ knowledge)

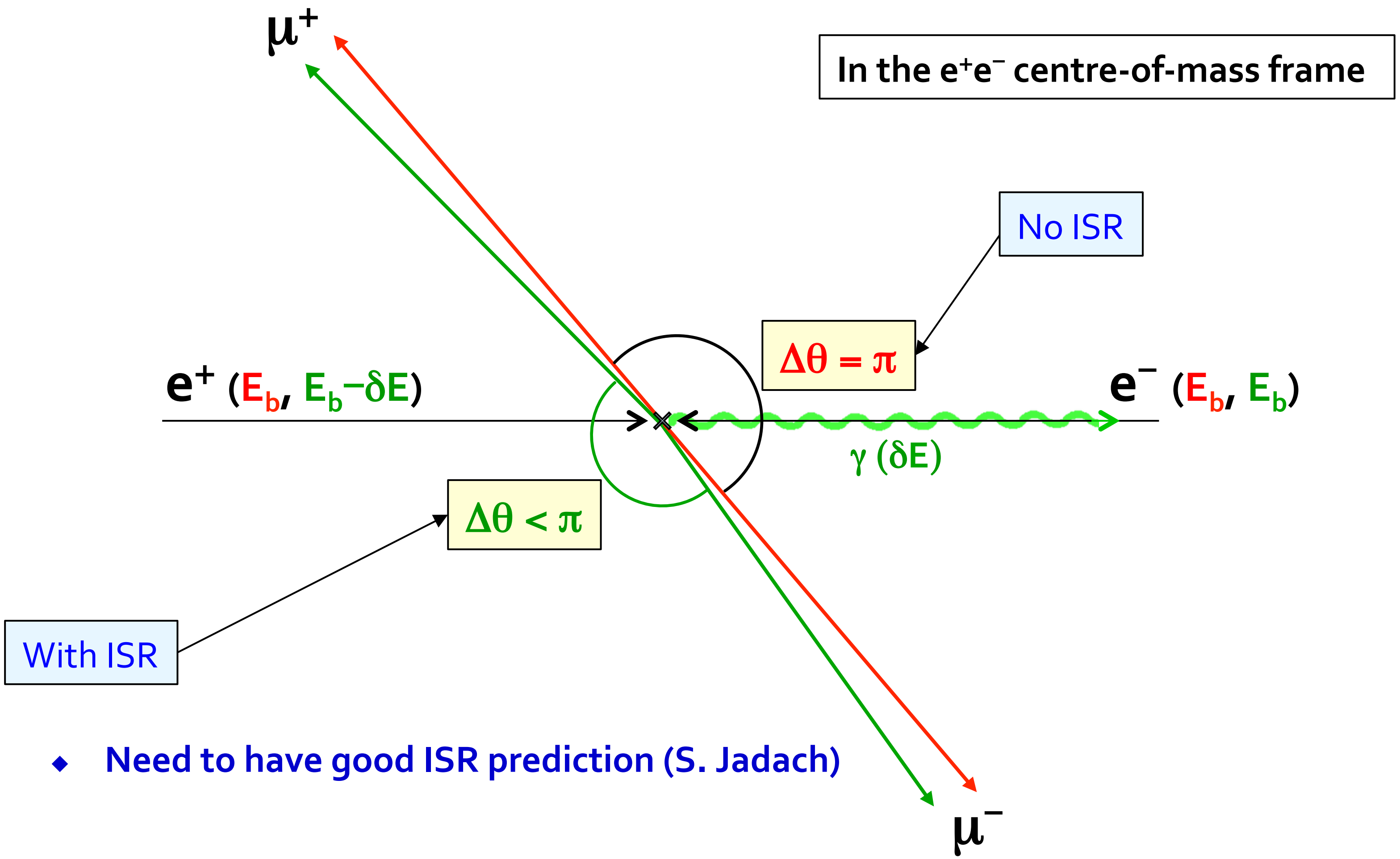
Make use of $e^+e^- \to \mu^+\mu^-$ events

- **How are the events modified with energy spread ?**



Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

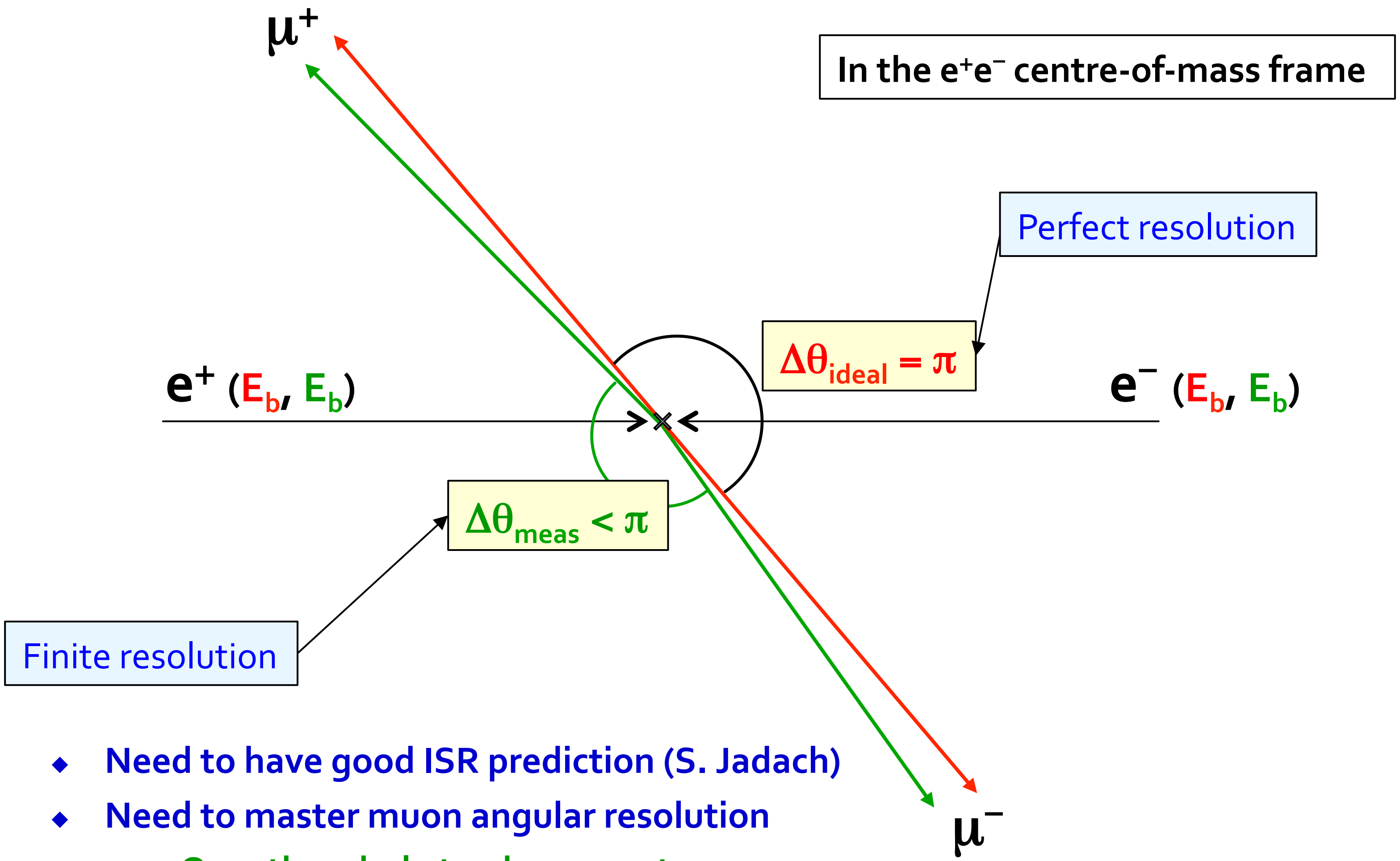
- Competes with initial state radiation (that you cannot get rid of)



- Need to have good ISR prediction (S. Jadach)

Make use of $e^+e^- \rightarrow \mu^+\mu^-$ events

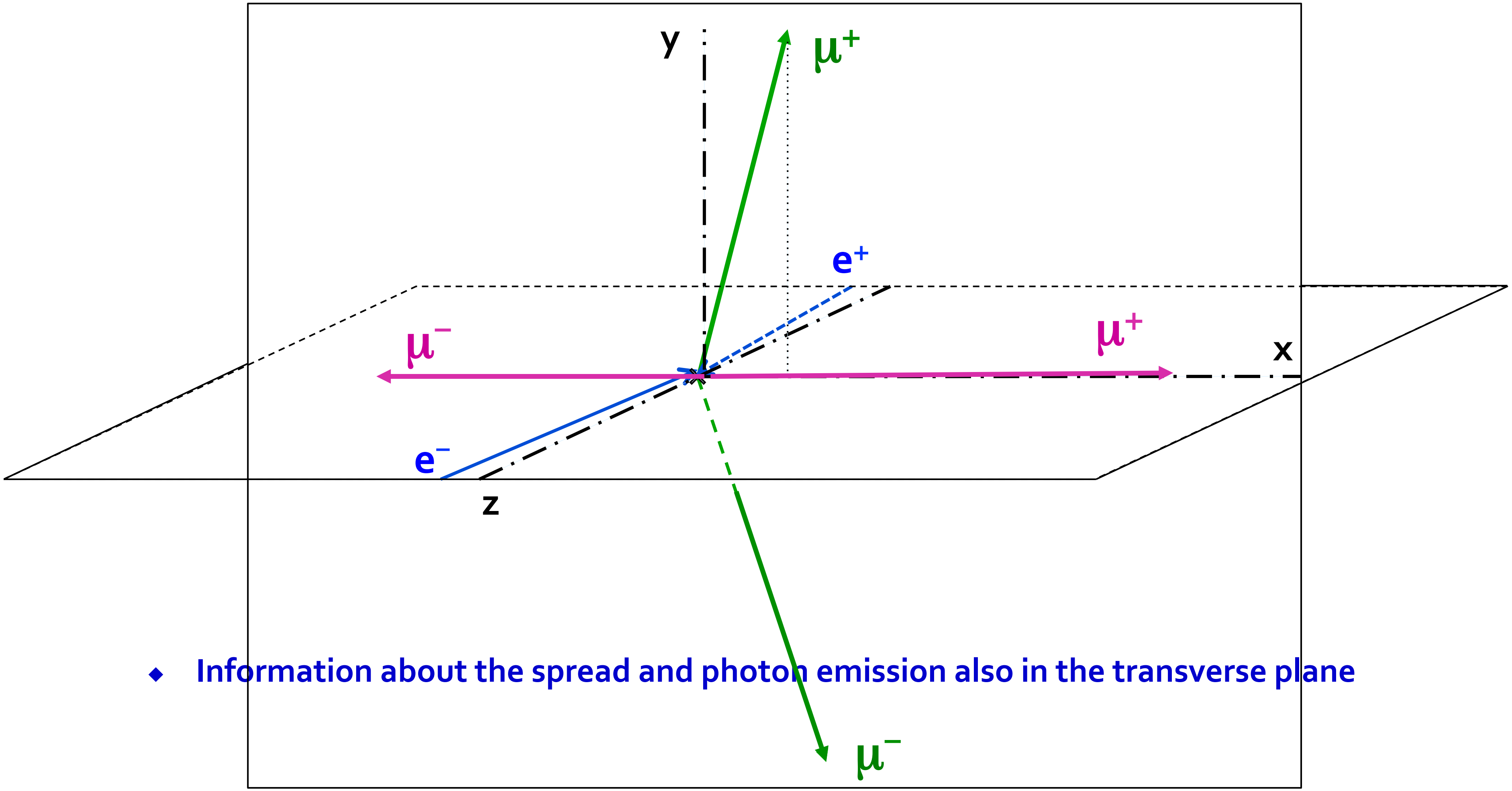
- **Also competes with muon angular resolution …**



- **Need to have good ISR prediction (S. Jadach)**
- **Need to master muon angular resolution**
  - **Over the whole tracker accpetance**

The FCC-ee frame is not the e^+e^- c.o.m frame

- An $e^+e^- \rightarrow \mu^+\mu^-$ event produced at $\theta=\pi/2$, with $\phi=0$ or $\phi=\pi/2$



- Information about the spread and photon emission also in the transverse plane

Total energy-momentum conservation

- Assuming one photon emitted along one of the two beams

$$
\begin{aligned}
E^+ \sin\theta^+ \cos\varphi^+ + E^- \sin\theta^- \cos\varphi^- + |p_z^\gamma| \tan\alpha/2 &= \sqrt{s}\tan\alpha/2, \\
E^+ \sin\theta^+ \sin\varphi^+ + E^- \sin\theta^- \sin\varphi^- &= 0, \\
E^+ \cos\theta^+ + E^- \cos\theta^- + p_z^\gamma &= 0, \\
E^+ + E^- + |p_z^\gamma| / \cos\alpha/2 &= \sqrt{s}/\cos\alpha/2,
\end{aligned}
$$

- Where $E^\pm$ are the measured energies of the $\mu^\pm$
- Where α is the beam crossing angle (nominal : 30 mrad),
- Where the z axis is the bisector of the two beam axes,
- Where the two beam axes form the (x,z) plane,
- Where $\theta^\pm$ are measured with respect to the z axis in the FCC-ee frame,
- Where $\phi^\pm$ are measured with to the x axis in the plane transverse to the z axis,
- Where $\sqrt{s}$ is the centre-of-mass energy of the collision

  - Assume initially that all axes are known perfectly / match the local (detector) axes

Total energy-momentum conservation

- P_x, P_y, E can be straightforwardly solved for α and $x_\pm = E^\pm \cos(\alpha/2)/(\sqrt{s} - |p_Z^\gamma|)$

$$\boxed{\alpha = 2 \arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+ \sin\theta^-}{\sin\varphi^- \sin\theta^- - \sin\varphi^+ \sin\theta^+}\right]}$$

$$\boxed{x_\pm = \frac{\mp \sin\theta^\mp \sin\varphi^\mp}{\sin\theta^+ \sin\varphi^+ - \sin\theta^- \sin\varphi^-}}$$

  - Independent of anything happening along the beams (ISR, E spread or asymmetry, …)

- P_z conservation then returns the reduced photon longitudinal momentum

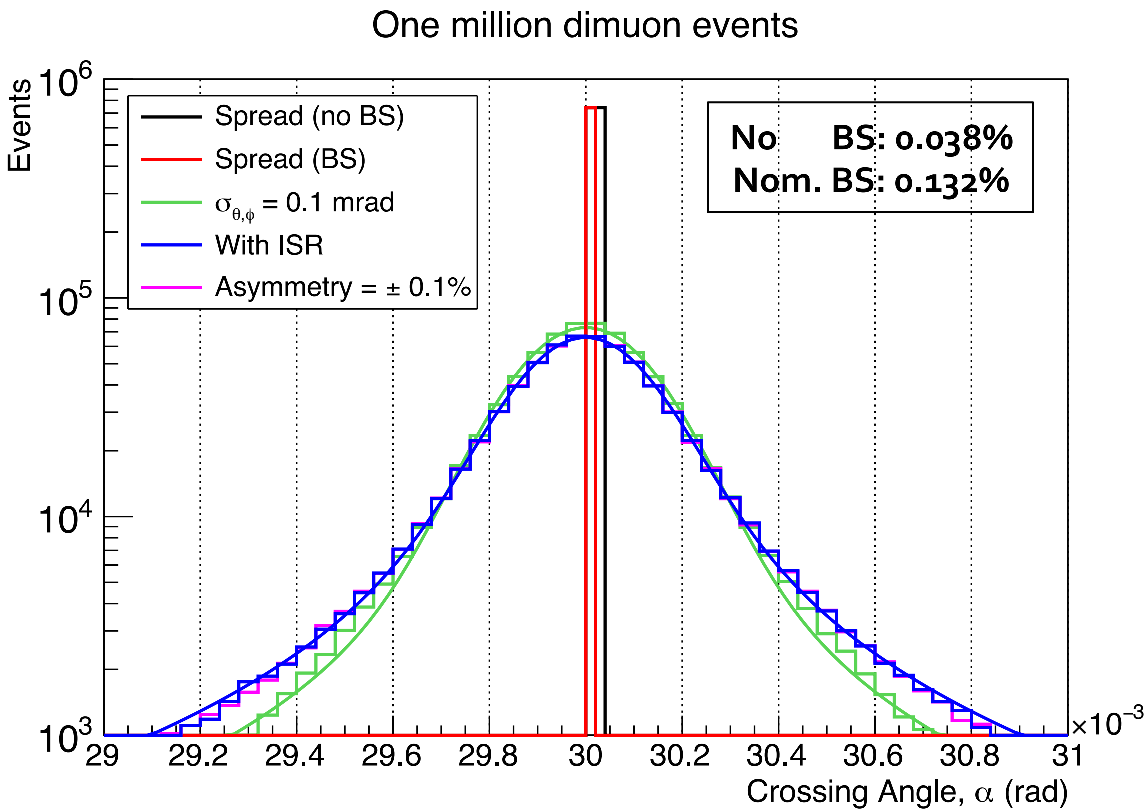
$$x_\gamma = p_z^\gamma/\sqrt{s}$$

$$\boxed{x_\gamma = -\frac{x_+ \cos\theta^+ + x_- \cos\theta^-}{\cos(\alpha/2) + |x_+ \cos\theta^+ + x_- \cos\theta^-|},}$$

  - As a function of the muon angles only (usually better determined than momenta)

Beam crossing angle determination

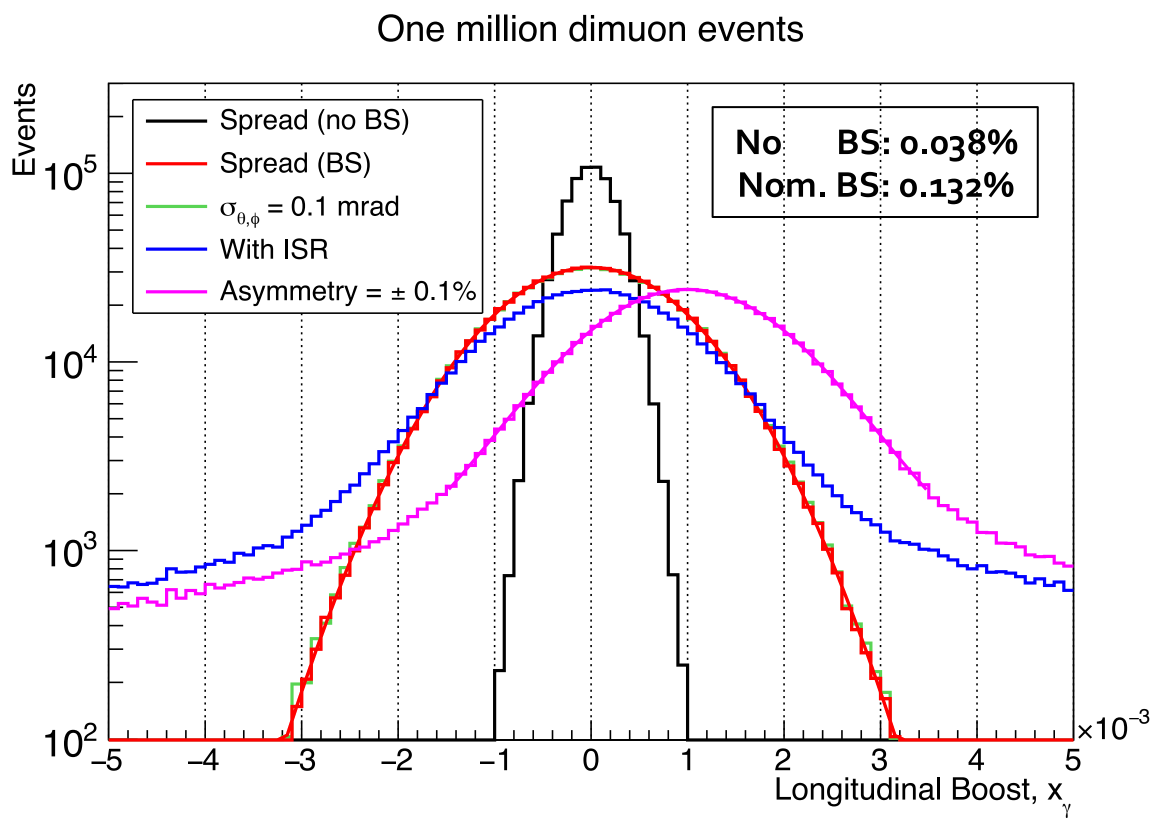
- With 10^6 dimuon events (every 5 minutes at the Z pole)



- Spread sensitive to anything happening in the transverse plane
  - ϕ resolution, p_T of emitted photons
- Mean value remarkably stable $\langle \alpha \rangle = 29.9998 \pm 0.0003$ mrad

“Photon momentum” x_γ determination

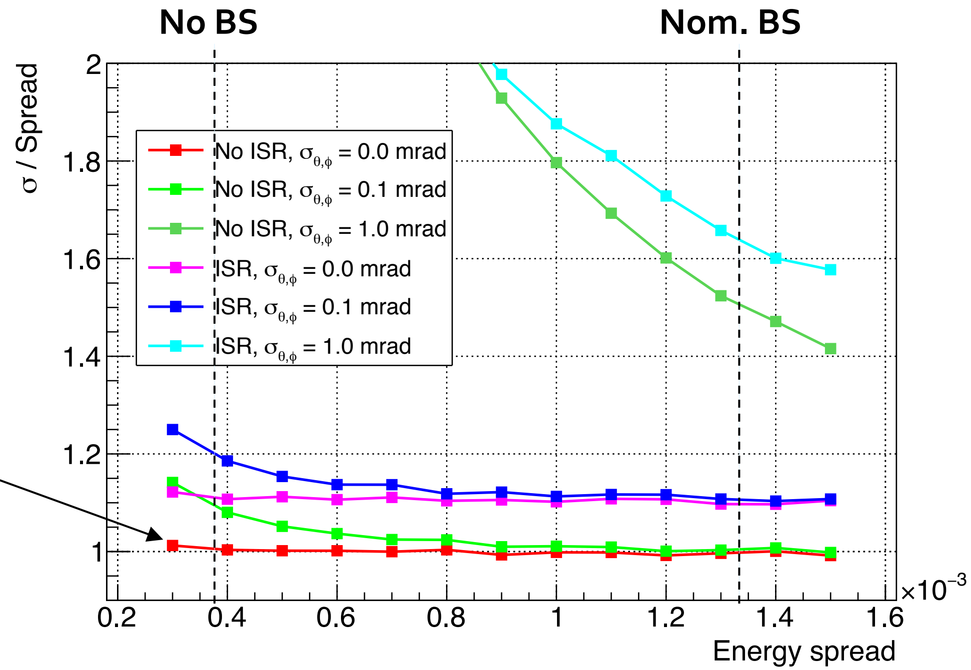
- With 10^6 dimuon events (every 5 minutes at the Z pole)



- Returns the √s spectrum (with asymmetry) for perfect angular resolution and no ISR
  - An angular resolution of 0.1 mrad seems adequately small
  - ISR slightly degrades the Gaussian core and add tails, needs to be unfolded

Effect of $\sigma_{\theta,\phi}$ and ISR

- Fit the core of the x_γ distribution with a Gaussian and compare to E_{spread}



- With $\sigma_{\theta,\phi}$ = 0 and no ISR

$$\sqrt{2}\sigma = (0.03804 \pm 0.00004)\% \quad \text{No BS}$$

$$\sqrt{2}\sigma = (0.13185 \pm 0.00011)\% \quad \text{Nom. BS}$$

  - ~0.1% relative precision

- Effect of ISR is to increase the x_γ width by 10%
  - Need to be known with a precision better than 1% for a per-mil precision on E_{spread}
- Effect of $\sigma_{\theta,\phi}$ = 0.1 mrad is to increase the x_γ width by 0.5% for nominal BS
  - Need to be known with a precision of ~10% over the whole tracker acceptance
- Effect of $\sigma_{\theta,\phi}$ = 1 mrad is devastating (similar to energy spread, difficult to control)

Control the angular resolution to 0.01 mrad ?

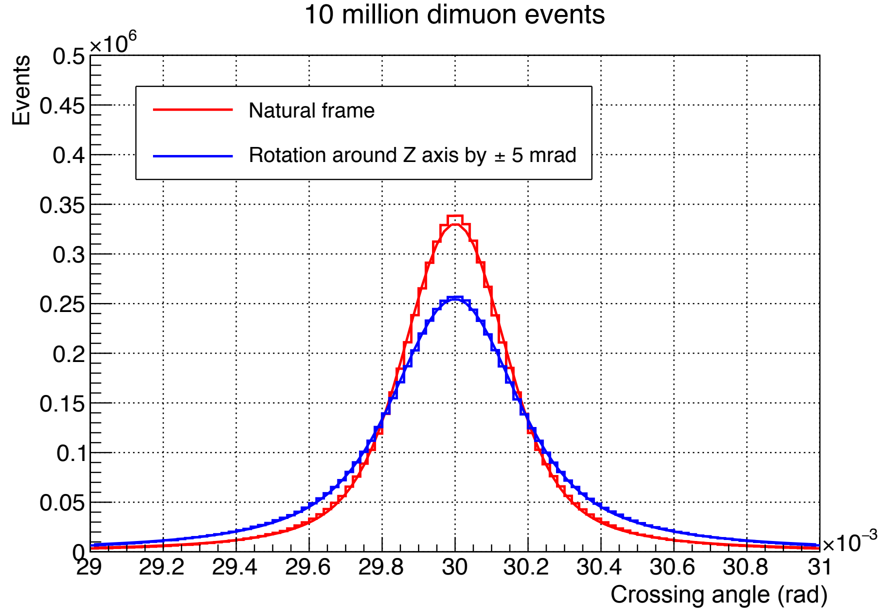
- **Q: How to measure the angular resolution to 10% or better**
  - For any value of θ and ϕ ?

- **A1: Take a muon track in dimuon events**
  - Refit it with the odd hits, on the one hand, and with the even hits, on the other
    - And compare the angles
  - Need only 100 tracks in each (θ , ϕ) bin for a 10% precision
    - 10^6 dimuon events = 5 minutes at the Z pole = bins of 3×3 (mrad)^2
  - Expected to be stable in time
    - Precision (or bin size) improves with dimuon statistics

- **A2: The crossing angle spread also helps for the ϕ resolution**

Detector alignment

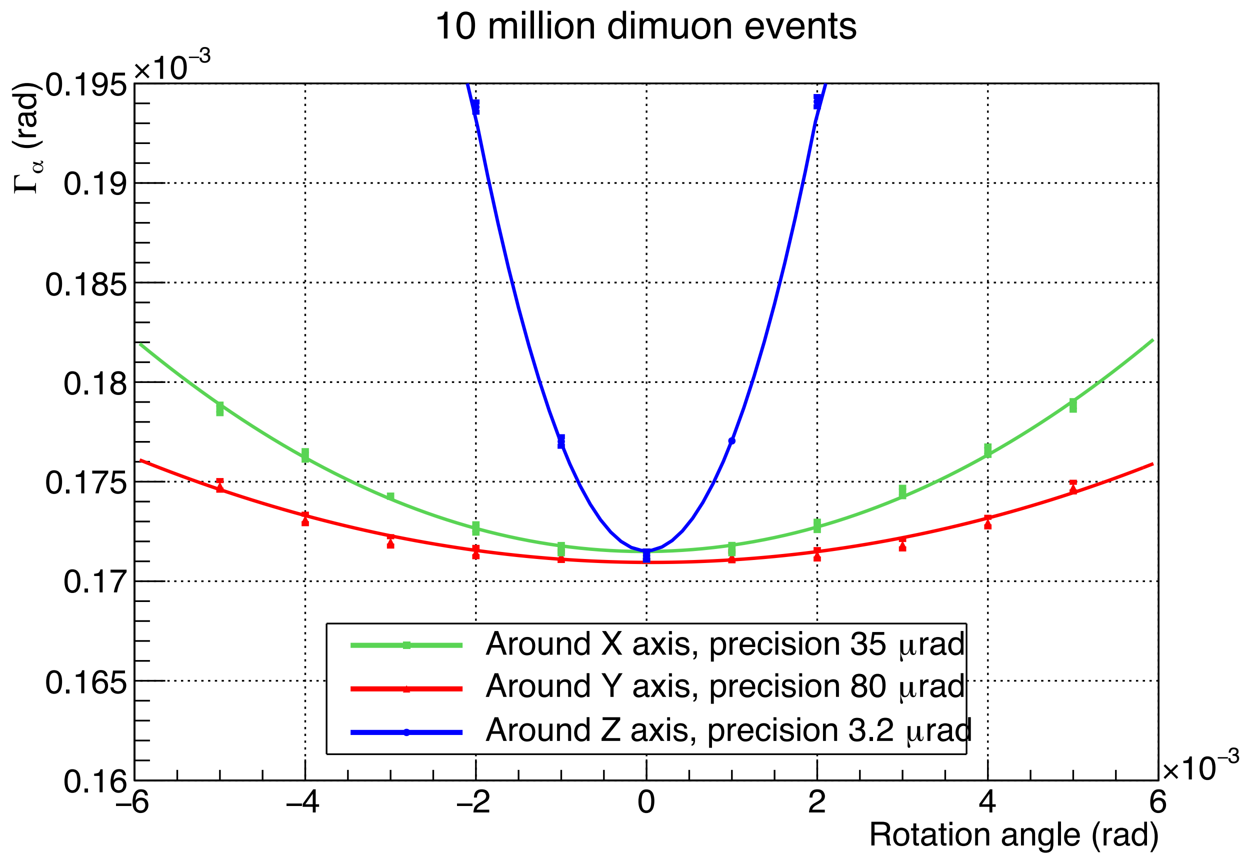
- **Absolute angle determination is (usually) not an easy task**
  - **Requires alignment of the local (detector) frame with the natural (FCC-ee) frame**
    - **Z axis = solenoid axis vs bissector of the two beam axes**
    - **(X,Z) plane = horizontal plane vs plane containing the two beam axes**
- **Spread of α increases with anything happening in the transverse plane**
  - **E.g., rotation around the Z axis changes both X and Y directions**



    - **Similarly, rotation around the X (Y) axis changes Y (X) direction**

Detector alignment, cont'd

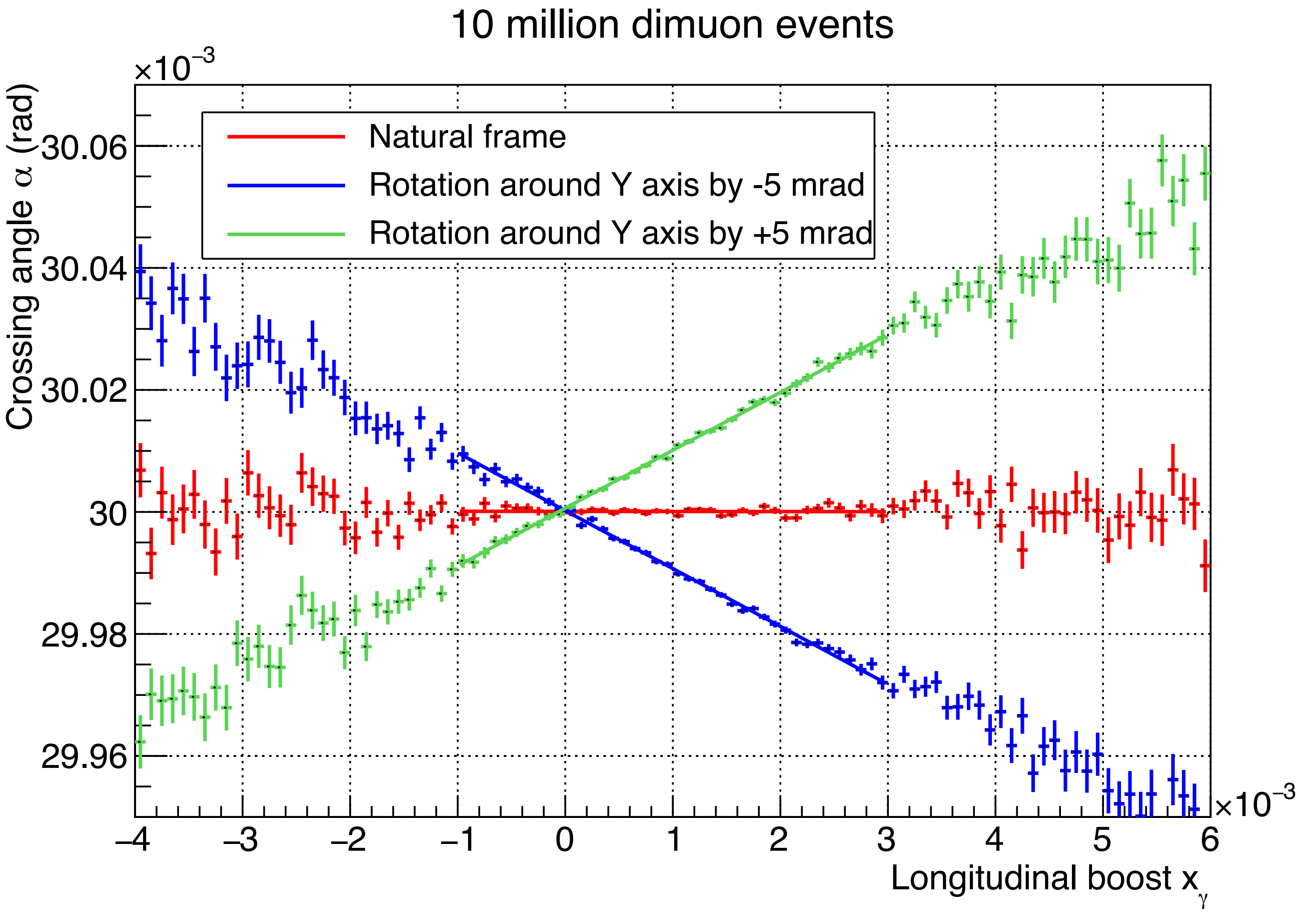
- Minimize the spread of the α distribution to find the three Euler angles



- **Note: α spread dominated by the ϕ resolution (here 0.1 mrad)**
  - **Precisions quadratically improves with the resolution in ϕ (here 0.1 mrad)**

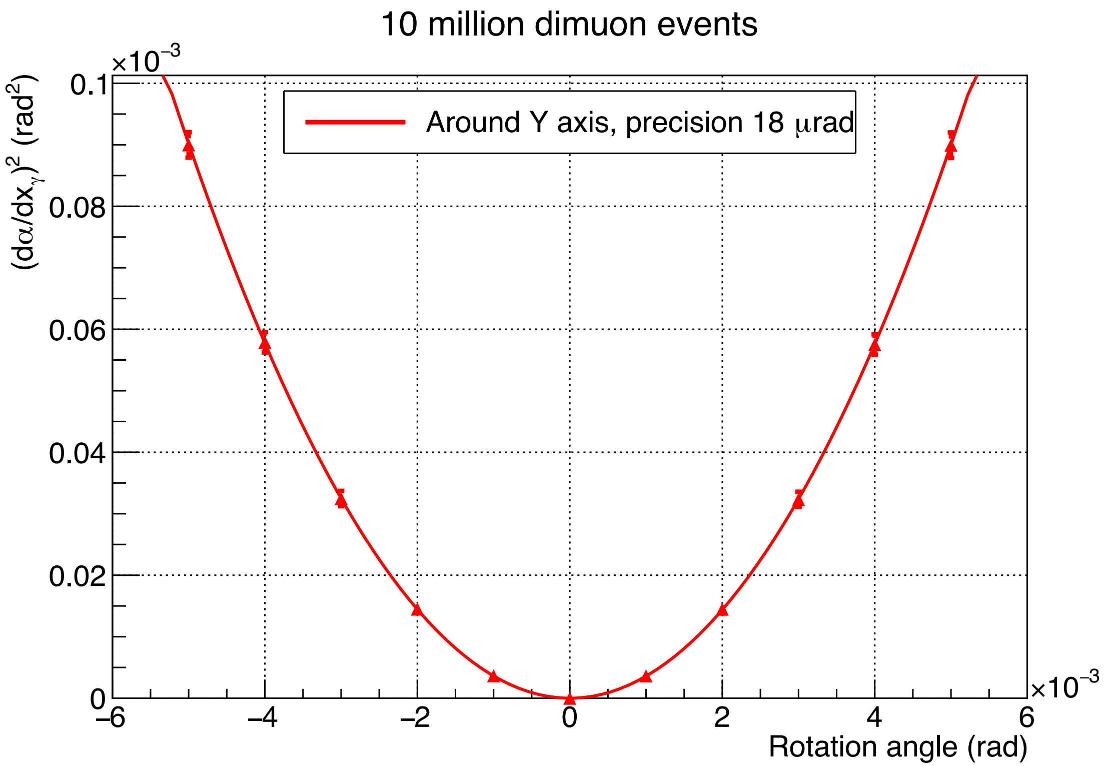
Detector alignment, cont'd

- **Improve the angle corresponding to a rotation around the Y axis**
  - **X and Z information get mixed by such a rotation**
    - **Resuting in a strong (linear) correlation between x_γ and α:**



Detector alignment, cont'd

- **Minimize the correlation between x_γ and α:**



- **Improves the precision on that crossing angle by a factor of five.**
  - **Reach a precision of 0.1 μrad on α and of 10^{-7} on x_γ**
  - **Variation of the x_γ spread already insignificant with 100 times less events**

Detector alignment, end

- A finite crossing angle is the key
  - A zero crossing angle cannot be determined with precision ($\sin\phi^+ = \sin\phi^-$, $\sin\theta^+ = \sin\theta^-$)

$$\alpha = 2 \arcsin \left[\frac{\sin(\varphi^- - \varphi^+) \sin\theta^+ \sin\theta^-}{\sin\varphi^- \sin\theta^- - \sin\varphi^+ \sin\theta^+} \right]$$

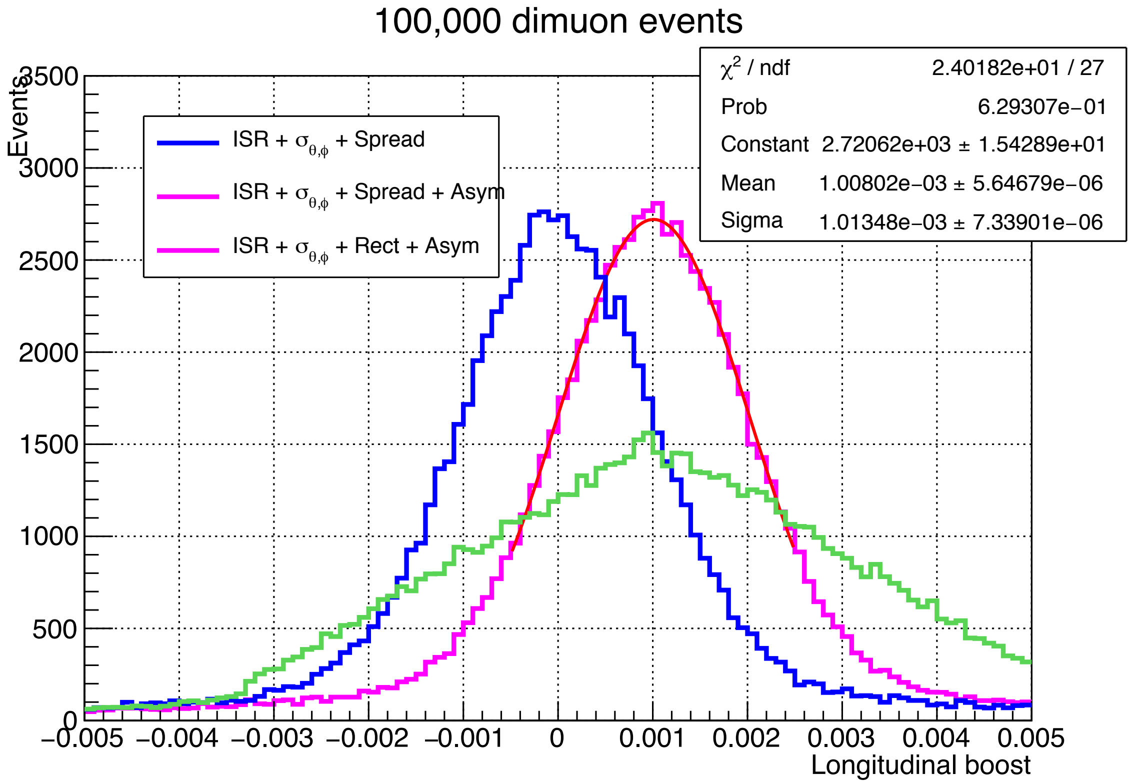
    - Spread is infinitely large even with a perfect detector alignment
    - Leads to inaccurate absolute alignment
      - ➨ With corresponding biases on the muon angles
  - With a finite crossing angle, only events with $\sin\phi^+ \sim \sin\phi^- \sim 0$ need to be rejected
    - Cut $|\sin\phi^\pm| > 0.2$ was applied in all previous plots

A permanent monitoring

- At the Z pole, $\sigma(e^+e^- \to \mu^+\mu^-) \sim 1.45$ nb
  - With $2.3 \cdot 10^{36}$ cm^{-2} s^{-1} collect 3.3 kHz of $e^+e^- \to \mu^+\mu^-$ events / detector
    - Enjoy one million events every 5 minutes
    - Monitor the beam energy spread to 0.2% precision every 4 minutes.
- At Peak ± 4, the cross section is reduced to 0.425 and 1.05 nb
  - Can afford a beam energy spread precision of 0.8% there
    - Due to fourfold smaller sensitivity of the cross section to E_{spread}
    - Need respectively 2 minutes and 45 seconds to reach this precision
- Bonuses :
  - We have to such independent monitorings (two detectors)
  - The energy spread might not be Gaussian
    - The whole spectrum is given by the x_γ distribution – see next slide
- Technical detail
  - The extraction of the Z resonance parameters will require a multi-parameter fit
    - If E_{spread} varies rapidly, one parameter per period of 1/2/3 minutes …

√s spectrum

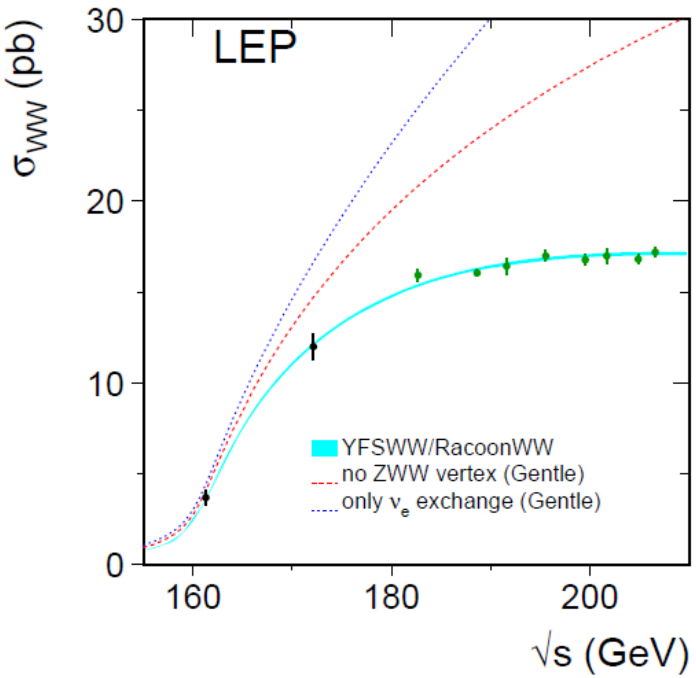
- **Shape of centre-of mass energy spread can easily be visualized**



- **A rectangular beam energy spectrum, with 0.1% asymmetry between e^+ and e^- turns, as expected, in a triangular √s spectrum**
  - **Immediately visible in the longitudinal boost distribution**

Other energies

- The W mass target precision is 500 keV
  - It is measured at threshold (as opposed to "at the peak")
    - A place where the effect of the energy spread is much smaller



    - Convolution of cross section with a Gaussian (E_{spread} = 0.153%):
      - ➨ No effect on σ_{WW} and m_W at 1st order, no effect at 2nd order, at $\sqrt{s}$ = 162.3 GeV
        (integral of an odd function at 1st order, and 2nd derivative is zero)

Other energies, cont'd

- The corresponding statistical precision on Γ_W (2.1 GeV) will be 1.5 MeV
  - With an additional run at 157.3 GeV (40% of the luminosity)
  - E_{spread} is 124 MeV, adds in quadrature to $\Gamma_W \rightarrow (\Gamma_W^2+E_{spread}^2)^{1/2}$ (reduces the slope)
    - $\Delta\Gamma_W = \Gamma_W (E_{spread}/\Gamma_W)^2 \times \Delta E_{spread}/E_{spread}$ (= 7 MeV × $\Delta E_{spread}/E_{spread}$)
      - ➨ A measurement of E_{spread} with a 7% precision is more than enough

        Contributes 0.5 MeV (quadratically) to the uncertainty on the width
      - ➨ About 650 $e^+e^- \rightarrow \mu^+\mu^-$ events suffice !

- At the WW threshold, $\sigma(e^+e^- \rightarrow \mu^+\mu^-) \sim 4$ pb
  - With $3.2{\cdot}10^{35}$ cm^{-2} s^{-1} collect 1.3 Hz of $e^+e^- \rightarrow \mu^+\mu^-$ events / detector
    - Enjoy 650 events and monitor E_{spread} to 7% precision every 8 minutes

- Note: This is only a back-of-the-envelope estimate
  - Gives the right ball park – but needs to be cross checked
    - By Paolo for the impact of energy spread on the W width precision
    - By me for the precision on E_{spread} with dimuon events at the WW threshold

Other energies, cont'd

- **The statistical precision on Γ_{top} (2 GeV) will be 25 MeV**
  - With a scan of the top threshold : 0.2 ab^{-1} around $\sqrt{s}$ = 346 GeV
  - E_{spread} is 346 MeV, adds in quadrature to Γ_{top} $\rightarrow$ $(\Gamma_{top}^2+E_{spread}^2)^{1/2}$
    - Corresponding uncertainty: $\Gamma_{top}\,(E_{spread}/\Gamma_{top})^2 \times \Delta E_{spread}/E_{spread}$ (= 60 MeV × $\Delta E_{spread}/E_{spread}$)
      - ➨ A measurement of E_{spread} with a 15% precision is more than enough

        Contributes 9 MeV (quadratically) to the uncertainty on the width
      - ➨ About 150 $e^+e^- \rightarrow \mu^+\mu^-$ events suffice !

- **At the top threshold, $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ ~ 0.8 pb**
  - With $1.8{\cdot}10^{34}$ cm^{-2} s^{-1} collect 15 mHz of $e^+e^- \rightarrow \mu^+\mu^-$ events / detector
    - Enjoy 150 events and monitor E_{spread} to 10% precision every 2 hours and 30 minutes

- **Note: This is only a back-of-the-envelope estimate**
  - Gives the right ball park – but needs to be cross checked
    - By Frank Simon for the impact of energy spread on the top width precision
    - By me for the precision on E_{spread} with dimuon events at the top threshold

- **I don't see why we would need a precise E_{spread} measurement at 240 or 365 GeV**
  - But we'll have it anyway !

Summary (extract from the CDR)

Table 1.1: Requirements on the precision of the beam energy spread measurement for nominal FCC-ee parameters. The first line indicates the precision electroweak pseudo observables relying on measurements that are significantly biassed by the beam energy spread, and the second line gives the corresponding "acceptable" uncertainty arising from the precision with which the beam energy spread is determined. The value of the acceptable uncertainty is chosen so that the FCC-ee target uncertainty on the pseudo observable determination (100 keV for the Z width, $3\,10^{-5}$ on the electromagnetic coupling constant, 1.5 MeV on the W width, and 25 MeV on the top-quark width) is increased by at most $\sim 5\%$. The centre-of-mass energies at which the measurement is performed are shown in the third line, and the precision of the energy spread measurement required to reach the acceptable uncertainty in the fourth line. The number of $e^+e^- \to \mu^+\mu-$ events needed to reach this precision is given in the fifth line. The dimuon rate determined from the luminosity (sixth line) and the $\mu^+\mu-$ production cross section (seventh line) is displayed in the eighth line. The time needed to reach the required precision on the beam energy spread is deduced in the last line.

Pseudo Observable	Γ_Z			$\alpha_{QED}(m_Z^2)$		Γ_W	Γ_{top}
Acceptable error	35 keV			10^{-5}		0.5 MeV	9 MeV
$\sqrt{s}$ (GeV)	87.9	91.2	93.8	87.9	93.8	161	350
$\sigma(\delta E)/\delta E$	0.8%	0.2%	0.8%	0.7%		7%	15%
$N_{e^+e^- \to \mu^+\mu^-}$	$5\,10^4$	$8\,10^5$	$5\,10^4$	$6.5\,10^4$		650	150
L ($10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$)	230					32	1.8
$\sigma_{\mu\mu}$ (pb)	185	1450	460	185	460	4.0	0.8
Dimuon rate (Hz)	425	3325	1050	425	1050	1.3	0.015
Time needed	2 min	4 min	< 1 min	3 min	1 min	8 min	2 h 30

Notes added

- All these numbers are valid for nominal luminosities / energy spread
  - For smaller energy spread, the precision requirement are quadratically less stringent
- Example: 0.066% instead of 0.132% at the Z pole (still BS dominated)
  - Precision required increases by a factor 4 to 0.8% instead of 0.2%
    - Number of events needed decrease by a factor of 16 to 5×10^4 instead of 8×10^5.
  - In the same time, the luminosity is reduced much less, by a factor $2^{2/3} \sim 1.58$
    - If the E spread decrease is achieved by increasing the vertical β^*
      - ➨ Keeping beam currents and vertical emittance unchanged
  - Therefore the time needed decreases to 25 seconds instead of 4 minutes
- All times displayed in the table are absolute maxima

- All these numbers were derived from the muon angles only
  - Angular resolution must be better than 0.1 mrad, + measured to better than 0.01 mrad
  - To become useful, muon momenta must be measured
    - With a precision better than the beam energy spread (~60 MeV)
    - Say 20-40 MeV precision for 45 GeV muon
      - ➨ $\sigma(1/p_T) \sim 1\text{-}2 \times 10^{-5}\ \mathrm{GeV}^{-1}$!

Pending question

- **Averaged beam energy measured with non-colliding bunches**
  - The method presented today allows the determination of
    - The crossing angle, hence $\sqrt{s}$ from the measured beam energies
    - The x_γ distribution, hence the $\sqrt{s}$ sprectrum
    - The $\langle x_\gamma \rangle$ average, hence the asymmetry between e^+ and e^- energies

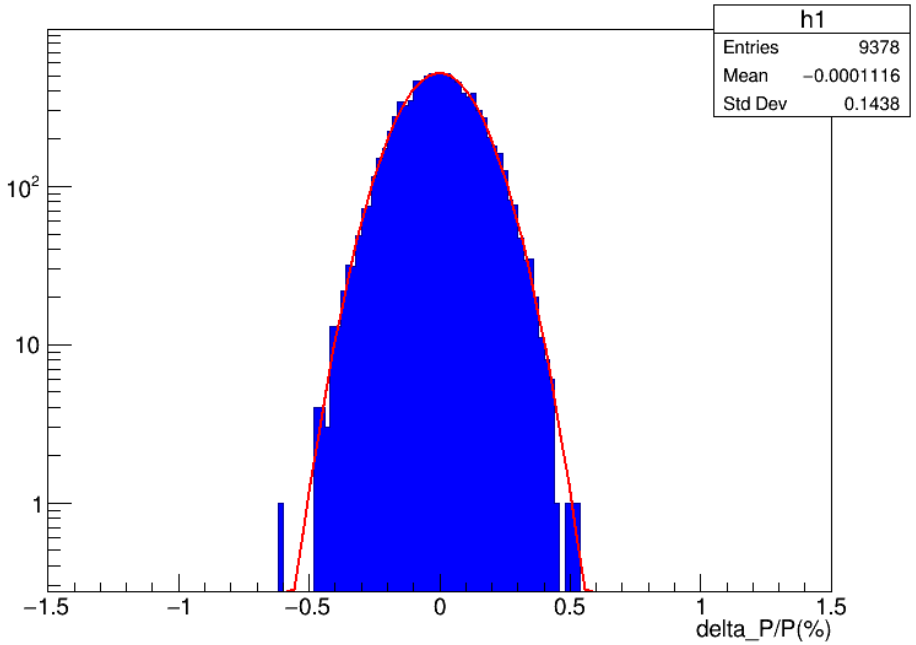
- **Energy loss during collision due to Beamstrahlung ?**
  - Being studied as we speak with multi-turn simulation
    - Possibly of the order of 0.3 MeV per beam
    - May be different for electron and positrons
    - May be different for each bunch
      - ➨ Changes the absolute beam energy with respect to the non-colliding bunches
      - ➨ Not visible with the method presented today

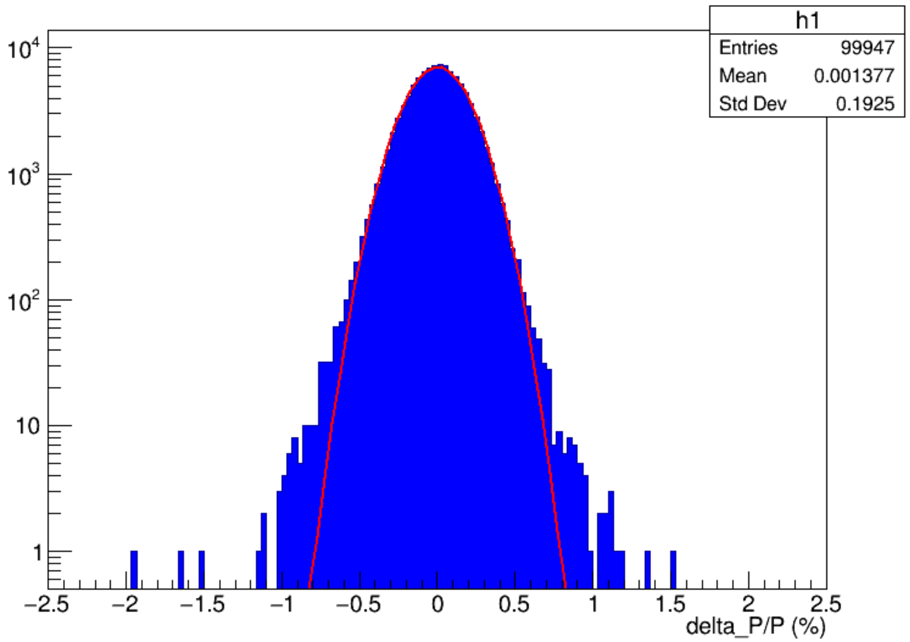
  - Need to be modeled and predicted with accuracy (5-10%)

  - Can possibly be monitored experimentally by measuring m_Z as a function of BS

Modelling the energy spread / loss

- **From Dima, for E_{beam} = 175 GeV**
  - Lattice simulation, after 1000 turns
    - Without beamstrahlung — With beamstrahlung





    - Reasonably Gaussian — Definitely not Gaussian
  - Number of events not sufficient to evaluate a possible energy loss
    - Uncertainty on the mean: 0.0014% (0.2±2.5 MeV) and 0.0019% (2.4±3.4 MeV)
  - Would need at least 1000 more statistics to get to the sub-100 keV range

Modelling the energy spread / loss, cont'd

- From Dmitry, Gaussian spread (σ_{Eo}) w/o beamstrahlung