TOWARDS FOUR-DIMENSIONAL UNSUBTRACTION AT NNLO

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Content

- Basic introduction and Loop-tree duality
- LTD/FDU approach
  - Location of IR singularities and renormalization
  - Toy-model examples
- Physical example I: $A^* \to q\bar{q}(g)$@NLO
- Physical example II: Higgs @NLO
- Extensions to two-loops (**on-going work**)
- Conclusions and perspectives

1. *Catani et al, JHEP 09 (2008) 065*
2. *Rodrigo et al, Nucl.Phys.Proc.Suppl. 183:262-267 (2008)*
3. *Buchta et al, JHEP 11 (2014) 014*

Rodrigo et al, JHEP 02 (2016) 044; JHEP 08 (2016) 160; JHEP 10 (2016) 162; arXiv:1702.07581 [hep-ph]

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Inside a "0" there could be many hidden things

Basic introduction and LTD

Theoretical motivation

- When computing **IR-safe observables,** divergences cancel through the combination of the real and virtual corrections (**KLN** theorem)
- For IR singularities, **phase-space integrals of real radiation** should originate the same structures that appear in **Feynman integrals for loop diagrams** ⇨ *Loop-tree theorems!*

Physical observable ⇨

Virtual corrections (loop integrals)

$$\int \frac{d^D q}{(2\pi)^D}$$

\+ *Real corrections (PS integrals)*

$$\int \frac{d^{D-1}\vec{q}}{(2\pi)^{D-1}\, 2q_0} = \int \frac{d^D q}{(2\pi)^D}(2\pi)\delta\left(q^2\right)\theta(q_0)$$

\+ *Renormalization counter-terms (ε poles times leading order)*

$$\frac{C_r}{\epsilon} \times d\sigma^{(0)}$$

Pole cancellation AFTER performing real-virtual integrals!!

WE WANT INTEGRAND LEVEL CANCELLATION!!!

Basic introduction and LTD

Dual representation of one-loop integrals

Loop Feynman integral

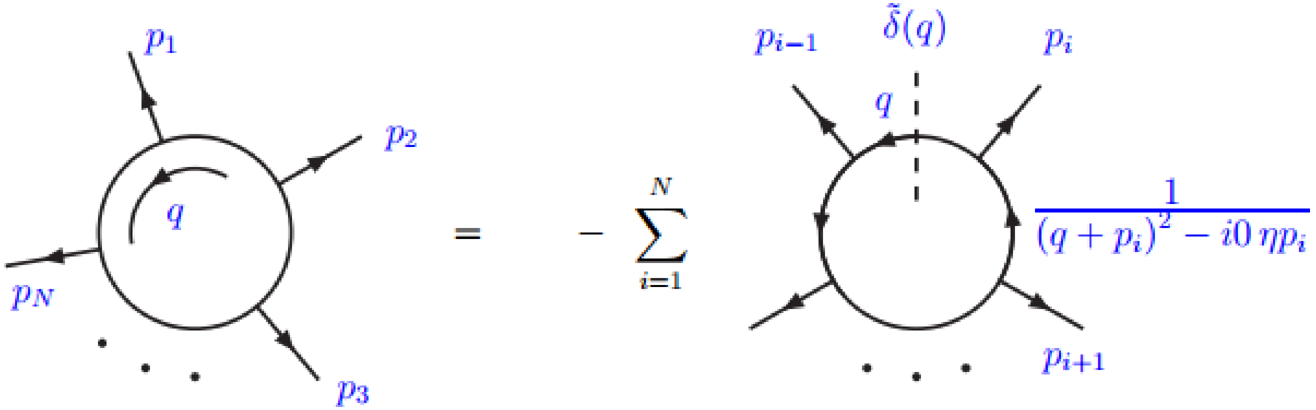
$$L^{(1)}(p_1,\ldots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\ldots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \qquad \tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$



Even at higher-orders, the number of cuts is equal the number of loops

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

Basic introduction and LTD

Derivation (one-loop)

- **Idea:** *«Sum over all possible 1-cuts»* (but with a **modified prescription…**)
  - Apply Cauchy's residue theorem to the Feynman integral:

$$L^{(N)}(p_1, p_2, \ldots, p_N) = \int_{\mathbf{q}} \int dq_0 \prod_{i=1}^{N} G(q_i) = \int_{\mathbf{q}} \int_{C_L} dq_0 \prod_{i=1}^{N} G(q_i) = -2\pi i \int_{\mathbf{q}} \sum \mathrm{Res}_{\{\mathrm{Im}\, q_0 < 0\}} \left[\prod_{i=1}^{N} G(q_i) \right]$$

  - Compute the residue in the poles with negative imaginary part:

$$\mathrm{Res}_{\{i\mathrm{-th\ pole}\}} \left[\prod_{j=1}^{N} G(q_j) \right] = \left[\mathrm{Res}_{\{i\mathrm{-th\ pole}\}}\, G(q_i) \right] \left[\prod_{\substack{j=1 \\ j\neq i}}^{N} G(q_j) \right]_{\{i\mathrm{-th\ pole}\}}$$

$$\left[\mathrm{Res}_{\{i\mathrm{-th\ pole}\}} \frac{1}{q_i^2 + i0} \right] = \int dq_0\, \delta_+(q_i^2)$$

$$\left[\prod_{j\neq i} G(q_j) \right]_{\{i\mathrm{-th\ pole}\}} = \prod_{j\neq i} \frac{1}{q_j^2 - i0\, \eta(q_j - q_i)}$$

Put on-shell the particle crossed by the cut

Introduction of «dual propagators» (η prescription, a future- or light-like vector)

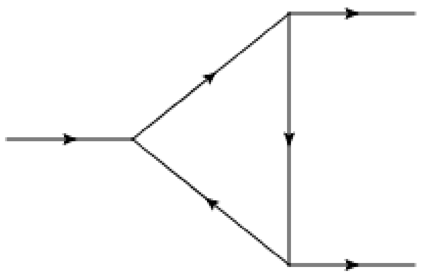
Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

LTD/FDU approach

- Two different kinds of physical singularities: **UV and IR**
  - IR divergences: *massless triangle*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2 \, s_{12}} \left(\frac{-s_{12} - \imath 0}{\mu^2} \right)^{-\epsilon}$$

IR pole

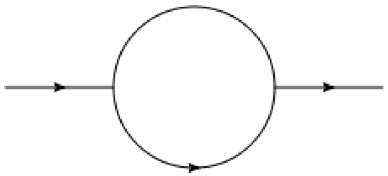


IDEA: Define a proper MOMENTUM MAPPING to generate REAL EMISSION KINEMATICS, and use REAL TERMS as fully local IR counter-terms!

  - UV divergences: *bubble with massless propagators*

$$L^{(1)}(p, -p) = \int_l \prod_{i=1}^{2} G_F(q_i) = c_\Gamma \frac{\mu^{2\epsilon}}{\epsilon\,(1-2\epsilon)} (-p^2 - \imath 0)^{-\epsilon}$$

UV pole



IDEA: Define an INTEGRAND LEVEL REPRESENTATION of standard UV counter-terms, and combine it with the DUAL REPRESENTATION of virtual terms!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach

General strategy

- To find the dual representation of Feynman integrals, we follow some steps:
  - ✓ If there are only single poles, we replace standard propagators with dual ones. Otherwise, we compute the residue and remove the energy integral:

$$\mathrm{Res}\,(f, z_0) = \frac{1}{(n-1)!}\left[\frac{\partial^{n-1}}{\partial z^{n-1}}\left((z-z_0)^n f(z)\right)\right]_{z=z_0} \;\Rightarrow\; \int d\vec{q}_i \; \mathrm{Res}\left(\prod_j G_F(q_j), q_{i,0}^{(+)}\right)$$

  - ✓ Parametrize momenta; for instance, for 1->2 processes we used

$$\begin{aligned} p_1^\mu &= \frac{\sqrt{s_{12}}}{2}(1,0,0,1) \\ p_2^\mu &= \frac{\sqrt{s_{12}}}{2}(1,0,0,-1) \\ q_i^\mu &= \xi_{i,0}\frac{\sqrt{s_{12}}}{2}\left(1, \sqrt{1-y^2}\hat{e}_T^i, y\right) \end{aligned} \;\Rightarrow\; \begin{aligned} y &\in [-1,1] \\ \xi_{i,0} &\in [0,\infty) \\ y &= 1-2v \end{aligned}$$

Scalar variables

in the massless case (analogous expressions when massive particles are present)

  - ✓ Factorize the measure in D-dimensions

$$d[\xi_{i,0}] = \frac{\mu^{2\epsilon}(4\pi)^{\epsilon-2}}{\Gamma(1-\epsilon)}\, s_{12}^{-2\epsilon}\, \xi_{i,0}^{-2\epsilon}\, d\xi_{i,0}$$

$$d[v_i] = (v_i(1-v_i))^{-\epsilon}\, dv_i$$

IMPORTANT: *We implement the method within DREG to establish a comparison with traditional results!*

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

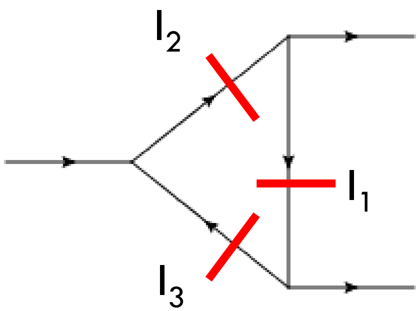
LTD/FDU approach

IR singularities

- *Reference example: Massless scalar three-point function in the time-like region*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2}\left(-\frac{s_{12}}{\mu^2} - i0\right)^{-1-\epsilon} = \sum_{i=1}^{3} I_i$$

LTD



$$I_1 = \frac{1}{s_{12}} \int d[\xi_{1,0}]\, d[v_1]\, \xi_{1,0}^{-1}\, (v_1(1-v_1))^{-1}$$

$$I_2 = \frac{1}{s_{12}} \int d[\xi_{2,0}]\, d[v_2]\, \frac{(1-v_2)^{-1}}{1-\xi_{2,0}+i0}$$

$$I_3 = \frac{1}{s_{12}} \int d[\xi_{3,0}]\, d[v_3]\, \frac{v_3^{-1}}{1+\xi_{3,0}-i0}$$

To regularize threshold singularity

- This integral is UV-finite (power counting); there are only IR-singularities, associated to soft and collinear regions
- **OBJECTIVE:** Define a *IR-regularized* loop integral by adding real corrections at integrand level (i.e. no epsilon should appear, 4D representation)

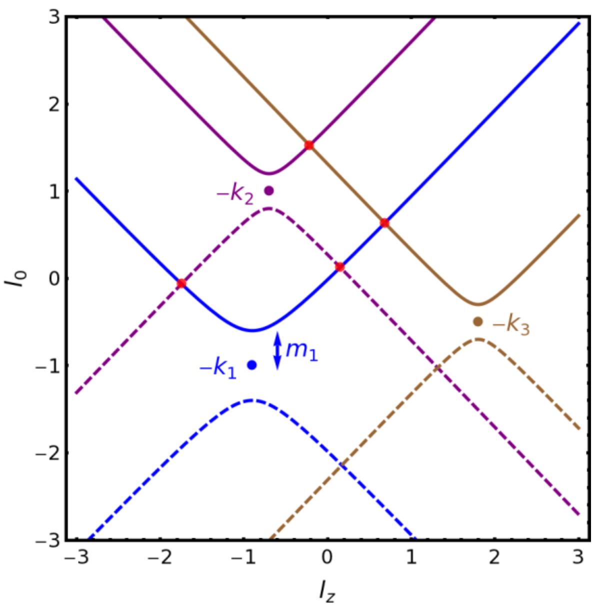
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach

Location of IR singularities in the dual-space

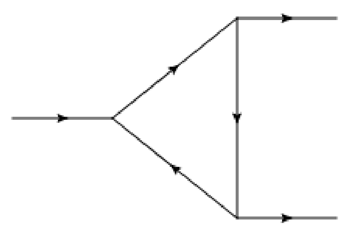
- Analize the dual integration region. It is obtained as the positive energy solution of the on-shell condition:

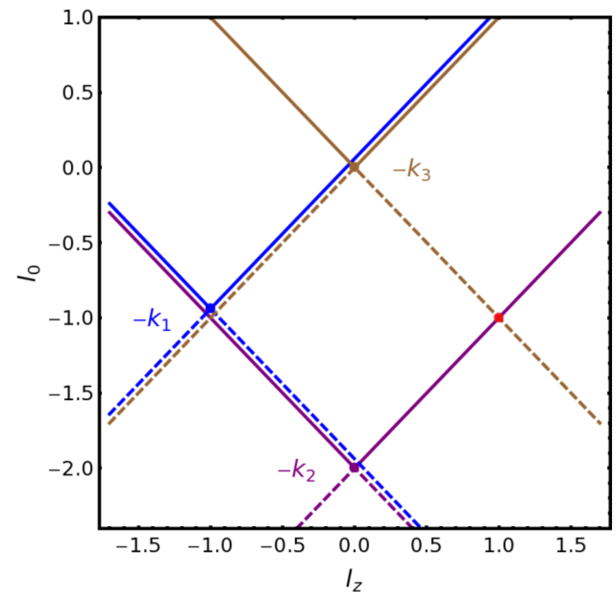
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: hyperboloids

- **Forward** (backward) on-shell hyperboloids associated with **positive** (negative) energy solutions.
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)





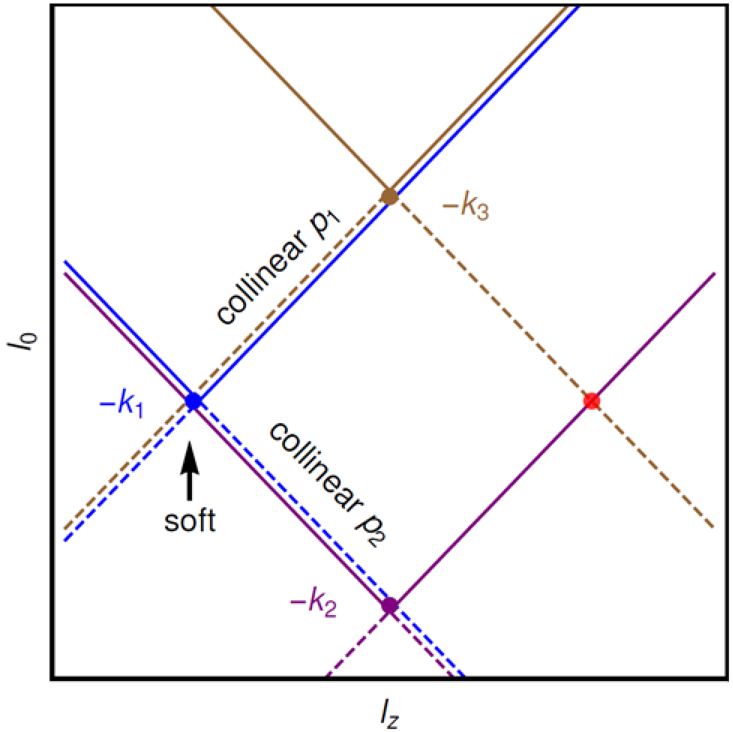
Massless case: light-cones

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach

Location of IR singularities in the dual-space

- The application of LTD converts loop-integrals into PS ones: **integration over forward light-cones**.



- Only **forward-backward** interferences originate **threshold or IR poles** (*other propagators become singular in the integration domain*)
- **Forward-forward** singularities cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a **compact region**)
- **No threshold or IR singularity at large loop momentum**

- This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable but numerically unstable)

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach

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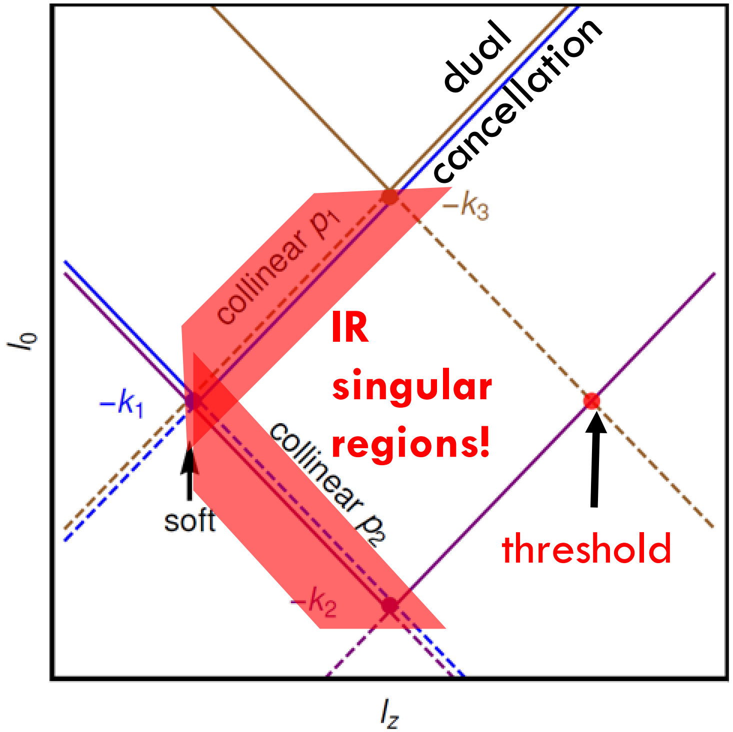

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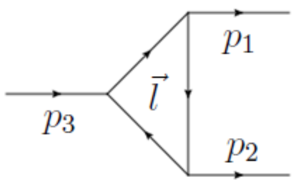
Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach: toy model

Real-virtual momentum mapping

- Suppose **one-loop** scalar scattering amplitude given by the triangle (scalar toy-model!):

Virtual

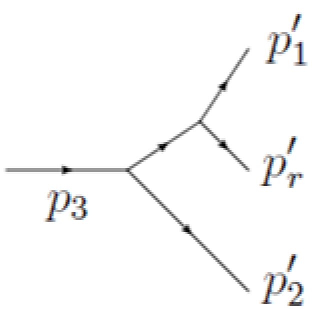


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3) \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** → **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p'_1,p'_2,p'_r;p_3)\rangle = -ig^2/s'_{ir} \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s'_{ir}\,s'_{jr}}$$

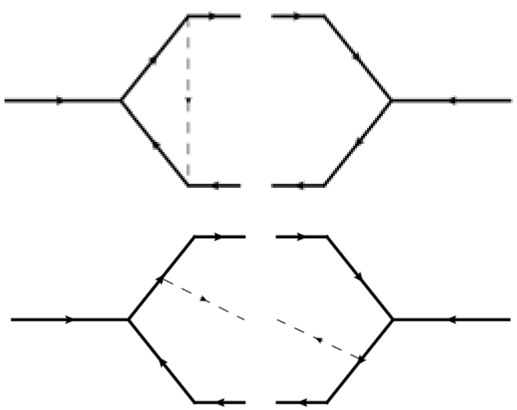
- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

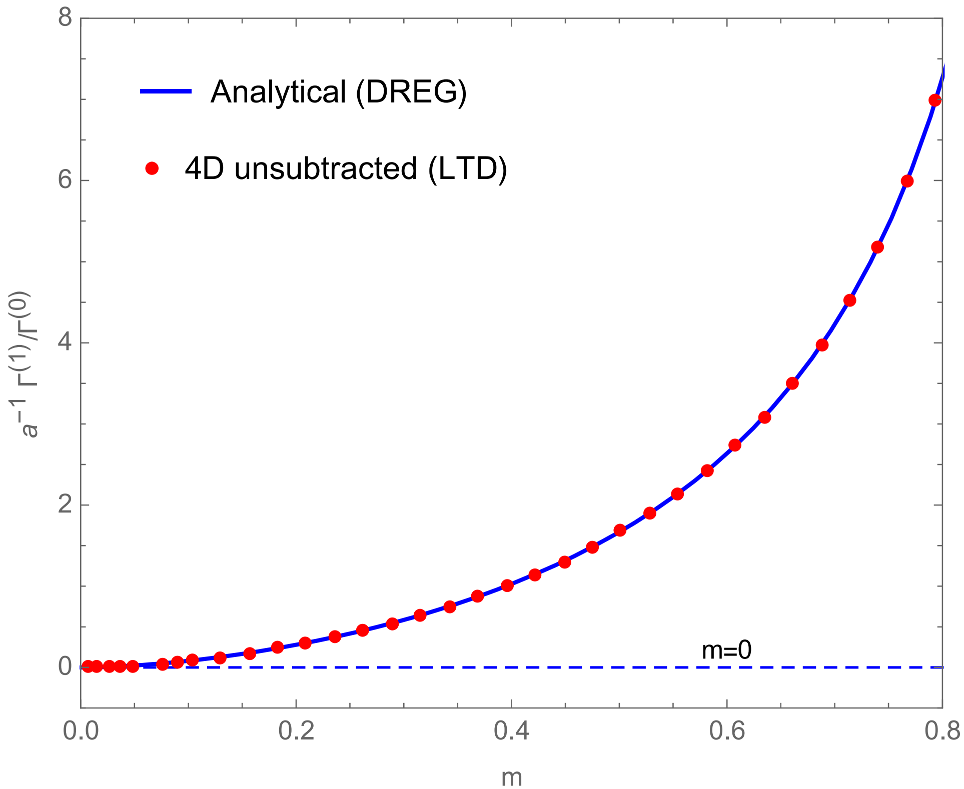
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: toy model

Example: massive scalar three-point function (DREG vs LTD)

- We combine the dual contributions with the real terms (after applying the proper mapping) to get the total decay rate in the scalar toy-model.
  - The result agrees ***perfectly*** with standard DREG.
  - **Massless limit** is **smoothly** approached due to proper treatment of **quasi-collinear** configurations in the **RV mapping**





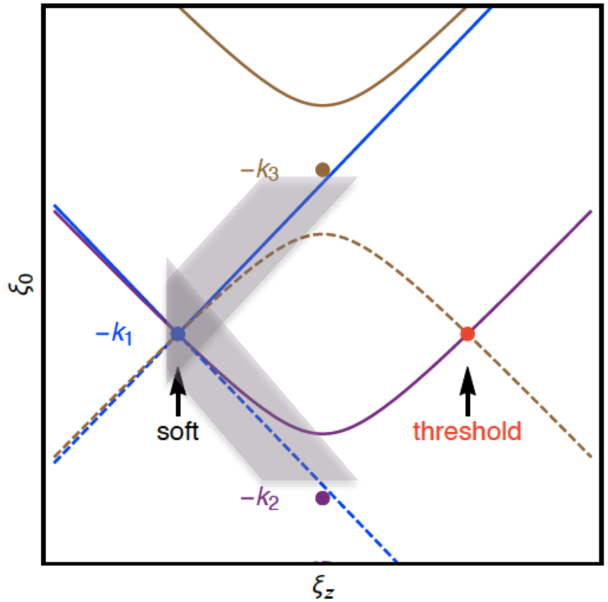
Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: toy model

Location of IR singularities: quasi-collinear limit

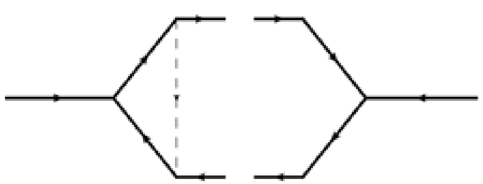
- *About the quasi-collinear configurations:* masses regulate IR singularities, but we need smooth transitions at **INTEGRAND** level to guarantee a smooth limit at **INTEGRAL** level.

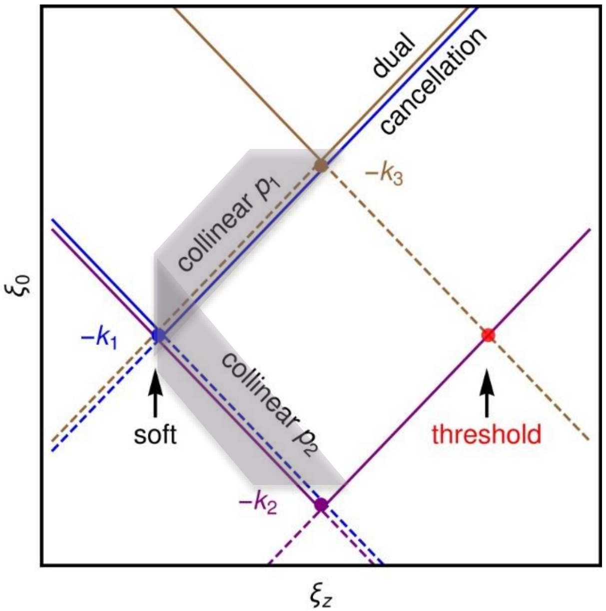
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: on-shell hyperboloids

- **Quasi-collinear** configurations lead to **Log(m²)**, which is singular in the massless limit
- We request a **smooth** behaviour in the massless limit





Massless case: light-cones

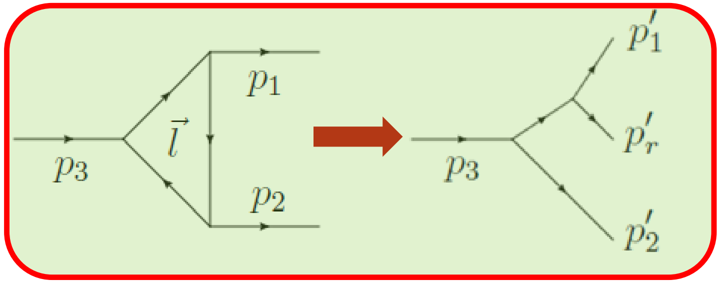
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: multileg

Real-virtual momentum mapping (GENERAL)

- **Real-virtual momentum mapping with massive particles:**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1//r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$\begin{aligned} p'^{\mu}_r &= q^{\mu}_1 \\ p'^{\mu}_1 &= (1-\alpha_1)\,\hat{p}^{\mu}_1 + (1-\gamma_1)\,\hat{p}^{\mu}_2 - q^{\mu}_1 \\ p'^{\mu}_2 &= \alpha_1\,\hat{p}^{\mu}_1 + \gamma_1\,\hat{p}^{\mu}_2 \end{aligned}$$



Impose on-shell conditions to determine mapping parameters

with $\hat{p}_i$ massless four-vectors build using p_i (simplify the expressions)

  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions!*

Repeat in each region of the partition…

Rodrigo et al, JHEP10(2016)162

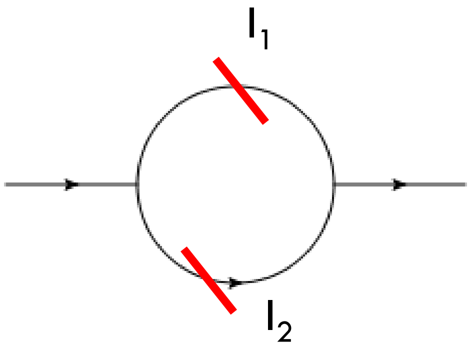
LTD/FDU approach: renormalization

UV singularities

- *Reference example: two-point function with massless propagators*

$$L^{(1)}(p,-p) = \int_\ell \prod_{i=1}^{2} G_F(q_i) = \frac{c_\Gamma}{\epsilon(1-2\epsilon)} \left(-\frac{p^2}{\mu^2} - i0 \right)^{-\epsilon} = \sum_{i=1}^{2} I_i$$

LTD



$$I_1 = -\int_\ell \frac{\tilde\delta(q_1)}{-2q_1\cdot p + p^2 + i0}$$

$$I_2 = -\int_\ell \frac{\tilde\delta(q_2)}{2q_2\cdot p + p^2 - i0}$$

To regularize threshold singularity

- In this case, the integration regions of dual integrals are two energy-displaced forward light-cones. This integral contains UV poles only!
- **OBJETIVE:** Define a *UV-regularized* loop integral by adding unintegrated UV counter-terms, and find a purely 4-dimensional representation of the loop integral

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: renormalization

Location of UV singularities and local counter-terms

- Divergences arise from the high-energy region (UV poles) and can be cancelled with a suitable renormalization counter-term. For the scalar case, we use

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}$$

Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

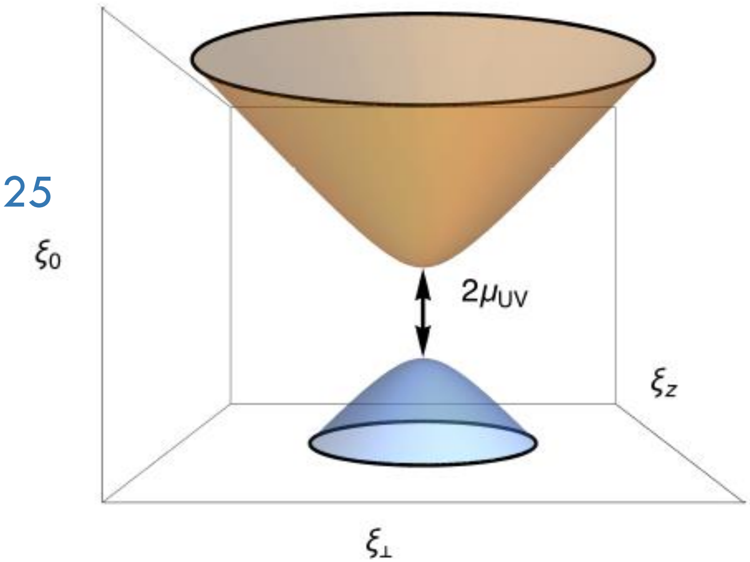
- Dual representation (**new: double poles in the loop energy**)

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde\delta(q_{\rm UV})}{2\left(q_{{\rm UV},0}^{(+)}\right)^2}$$

Bierenbaum *et al.* JHEP 03 (2013) 025

$$q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

- Loop integration for loop energies larger than $\mu_{\rm UV}$



Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- LTD must be applied to deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- **1: Expand** internal propagators around the *"UV propagator"*

$$\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2 q_{\rm UV} \cdot k_{i,\rm UV} + k_{i,\rm UV}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2 q_{\rm UV} \cdot k_{i,\rm UV})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2}\right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2}\right)$$

Becker, Reuschle, Weinzierl, JHEP12(2010)013

- **2:** Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

LTD extended to deal with multiple poles

(use residue formula to obtain the dual representation)

- **3:** Take into account **wave-function and vertex renormalization** constants (not trivial in the massive case!)

Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- Self-energy corrections with **on-shell renormalization** conditions

$$\Sigma_R(\not{p}_1 = M) = 0 \qquad \left.\frac{d\Sigma_R(\not{p}_1)}{d\not{p}_1}\right|_{\not{p}_1 = M} = 0$$

- ***Wave-function renormalization constant*** (**both IR and UV poles**):

$$\Delta Z_2(p_1) \;=\; -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2} \right) G_F(q_3) \right)$$

- ***Vertex renormalization*** (**only UV**):

$$\Gamma^{(1)}_{A,\mathrm{UV}} = g_S^2\, C_F \int_\ell (G_F(q_{\mathrm{UV}}))^3 \left[\gamma^\nu \not{q}_{\mathrm{UV}}\, \Gamma^{(0)}_A\, \not{q}_{\mathrm{UV}}\, \gamma_\nu - d_{A,\mathrm{UV}}\, \mu^2_{\mathrm{UV}}\, \Gamma^{(0)}_A \right]$$

- **Important features:**
  - Integrated results agrees with standard UV counter-terms!
  - **Smooth massless limit!**

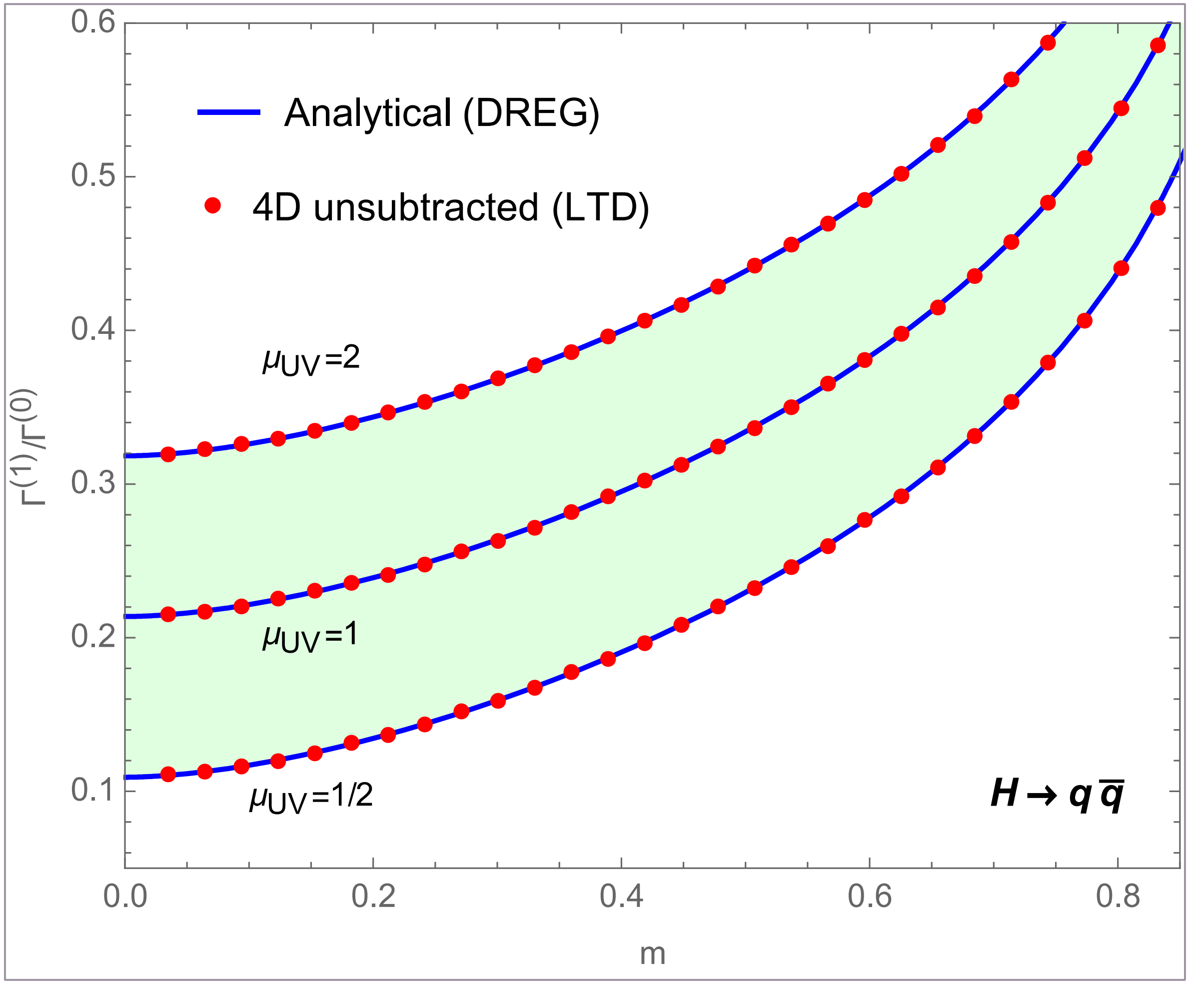
Rodrigo et al, JHEP10(2016)162

0

In the massless case, the renormalization factors are usually ignored because they are "0": but they hide a cancelation between UV and IR singularities…

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG



- *Total decay rate for Higgs into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Local version of UV counterterms succesfully reproduces the expected behaviour
  - Efficient numerical implementation

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Important remarks

- The total decay-rate can be expressed using purely **four-dimensional integrands** (which are **integrable** functions!!)
- We recover the total NLO correction**, avoiding to deal with** DREG (ONLY used for comparison with known results)
- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for **ε=0**)

    Finite integral for ε=0 $\not\Rightarrow$ Integrability with ε=0

    With FDU is true!
  - ✓ No need of tensor reduction (**avoids the presence of Gram determinants,** which could introduce numerical instabilities)
  - ✓ **Smooth transition** to the massless limit (due to the efficient treatment of **quasi-collinear** configurations)
  - ✓ **Mapped real-contribution used as a fully local IR counter-term for the dual contribution!**

Rodrigo et al, JHEP10(2016)162

Physical example: Higgs@NLO

Using LTD to regularize finite amplitudes

- Application of LTD to compute one-loop Higgs amplitudes:

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12}\, M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)}\, c_1^{(f)} + c_2^{(f)} \right) + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2}\, c_3^{(f)} \right]$$

$$\mathcal{A}_2^{(1,f)} = g_f \frac{c_3^{(f)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} - 2 \right)$$

with $q_1 = \ell + p_1,\ q_2 = \ell + p_{12}$, $q_3 = \ell,\ q_4 = \ell + p_2$ and $q_{1,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_1)^2 + M_f^2}\,,\quad q_{4,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_2)^2 + M_f^2}\,,$ $\ell_0^{(+)} = q_{2,0}^{(+)} = q_{3,0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + M_f^2}\,.$

- **Comments:**
  - Generic result valid for $gg \to H$ and $H \to \gamma\gamma$!!
  - Process dependence codified in the coefficients. **Valid for scalar, fermion and vector massive particles inside the loop!!!**

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Physical example: Higgs@NLO

Using LTD to regularize finite amplitudes

- Combine expressions (use "zero integrals" in DREG associated with Ward identities):

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \frac{s_{12} M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)} c_1^{(f)} + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} c_{23}^{(f)} \right]$$

Well defined in 4-d!!

UV divergent

$\mathcal{O}(\epsilon)$

Non-commutativity of limit and integration!!!!

- Use local renormalization (equivalent to Dyson's prescription…)

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}\Big|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\mathrm{UV}}^{(1,f)}\right)_{d=4}$$

$$\mathcal{A}_{1,\mathrm{UV}}^{(1,f)} = -g_f \int_\ell \frac{\tilde{\delta}(\ell)\, \ell_0^{(+)} s_{12}}{2(q_{\mathrm{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\mathrm{UV},0}^{(+)})^2} \frac{3\,\mu_{\mathrm{UV}}^2}{d-4}\right) c_{23}^{(f)}$$

  - Counter-term mimics UV behaviour at integrand level.
  - Term proportional to μ_{UV}^2 used to fix DREG scheme (vanishing counter-term in d-dim!!)
  - Valid also for W amplitudes in unitary-gauge (naive Dyson's prescription fails to subtract subleading terms due to enhanced UV divergences)

$$-\imath \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right) \frac{1}{q_i^2 - M_W^2 + \imath 0}$$

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

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In some amplitudes, there are contributions that vanish AFTER INTEGRATION, but they contain a non-trivial UV/IR interplay of singularities…

Physical example: Higgs@NLO

- *Infinite-mass limit used to define effective vertices.* Equivalent to explore asymptotic expansions!
- Expansions at **integrand level** are non-trivial in **Minkowski** space (i.e. within Feynman integrals) and additional factors are neccesary
- **Dual amplitudes** are expressed as **phase-space** integrals ➡ **Euclidean** space!!

$$\tilde{\delta}(q_3)\; G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{s_{12} + 2q_3\cdot p_{12} - \imath 0} \xrightarrow{M_f^2 \gg s_{12}} \tilde{\delta}(q_3)\; G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{2q_3\cdot p_{12}} \sum_{n=0}^{\infty} \left(\frac{-s_{12}}{2q_3\cdot p_{12}}\right)^n$$

Expansion of the dual propagator (q_3 on-shell)

- *Example*: Higgs amplitudes with heavy-particles within the loop

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}(s_{12} < 4M_f^2)\Big|_{d=4} = \frac{M_f^2}{\langle v \rangle} \int_\ell \tilde{\delta}(\ell) \left[\frac{3\,\mu_{\mathrm{UV}}^2\, \ell_0^{(+)}}{(q_{\mathrm{UV},0}^{(+)})^5}\, \hat{c}_{23}^{(f)} + \frac{M_f^2}{(\ell_0^{(+)})^4} \left(\sum_{n=0}^{\infty} Q_n(z) \left(\frac{s_{12}}{(2\ell_0^{(+)})^2}\right)^n\right) c_1^{(f)}\right]$$

$$z = (2\boldsymbol{\ell}\cdot \mathbf{p}_1)/(\ell_0^{(+)}\sqrt{s_{12}}) \;\text{ and }\; Q_n(z) = \frac{1}{1-z^2}\left(P_{2n}(z) - 1\right)$$

Reproduces all the known-results!!

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Extension to two loops

Introducing the notation

- Dual amplitudes can be defined at **higher-orders** (even with multiple poles)

  Bierenbaum, Catani, Draggiotis, Rodrigo; JHEP 10 (2010) 073

- *Standard example:* two-loop N-point scalar amplitude

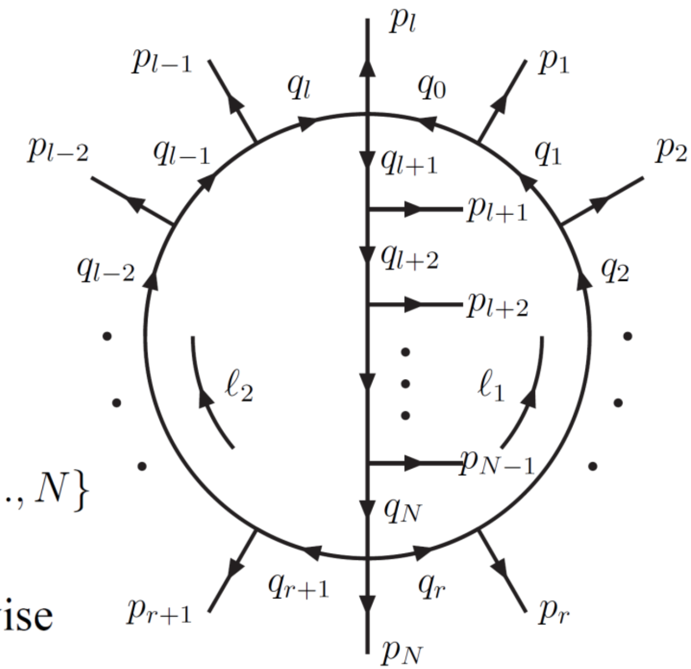
$$L^{(2)}(p_1, p_2, \dots, p_N) = \int_{\ell_1} \int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3)$$

$$\int_{\ell_i} \bullet = -\imath \int \frac{d^d \ell_i}{(2\pi)^d} \bullet \quad , \quad G_F(\alpha_k) = \prod_{i \in \alpha_k} G_F(q_i)$$

- Three possible sets of momenta, according to their dependence on l_1, l_2 or l_1+l_2 (*integration variables*)

$$\alpha_1 \equiv \{0, 1, \dots, r\} \, , \, \alpha_2 \equiv \{r+1, r+2, \dots, l\} \, , \, \alpha_3 \equiv \{l+1, l+2, \dots, N\}$$

$$q_i = \begin{cases} \ell_1 + p_{1,i} & , i \in \alpha_1 \\ \ell_2 + p_{i,l-1} & , i \in \alpha_2 \\ \ell_1 + \ell_2 + p_{i,l-1} & , i \in \alpha_3 \end{cases} \quad \text{with} \quad \begin{array}{ll} \ell_1 & \text{anti–clockwise} \\ \ell_2 & \text{clockwise} \end{array}$$



Generic two-loop diagram

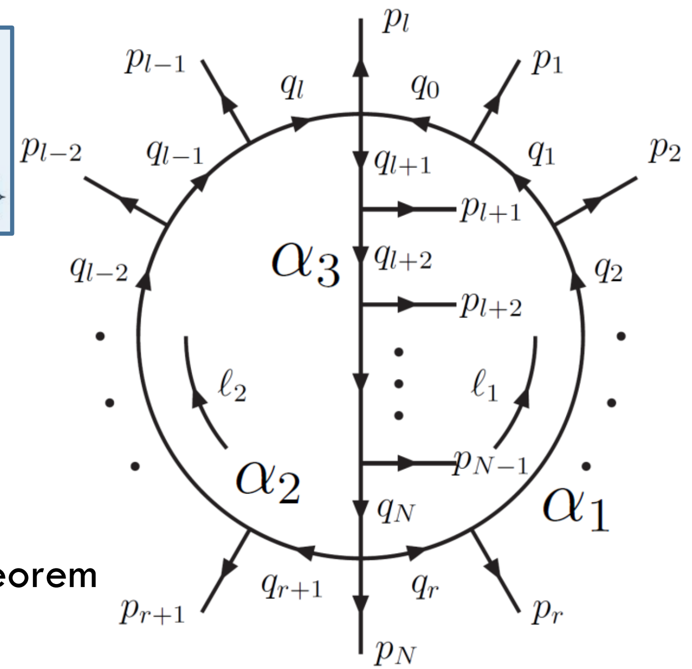
Driencourt-Mangin, Rodrigo, G.S., Torres Bobadilla, in progress

Extension to two loops

Cuts and LTD formula

- **"The number of cuts equals the number of loops"**
- *Derivation:* "Iterate" the one-loop formula and use propagator properties
- *Standard example:* two-loop N-point scalar amplitude

$$L^{(2)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1} \int_{\ell_2} \{G_D(\alpha_2)\, G_D(\alpha_1 \cup \alpha_3) + G_D(-\alpha_2 \cup \alpha_1)\, G_D(\alpha_3) - G_F(\alpha_1)\, G_D(\alpha_2)\, G_D(\alpha_3)\}$$



where we used $G_D(\alpha_k) = \sum_{i \in \alpha_k} \tilde{\delta}(q_i) \prod_{\substack{j \in \alpha_k \\ j \neq i}} G_D(q_i; q_j)$

- **Remarks and subtleties:**
  - Modified prescription depends on loop momenta.
  - Not a "trivial" iteration: connection with Cauchy's theorem and multivariable residues.
  - *Thesis:* "Virtual-real amplitudes mapped with one-loop formulae" (*partial cancellations*), but a new mapping required for double-real emission.

Driencourt-Mangin, Rodrigo, G.S., Torres Bobadilla, in progress

Conclusions and perspectives

- ✓ Loop-tree duality allows to treat **virtual and real** contributions **in the same way** (implementation simplified)
- ✓ Physical interpretation of **IR/UV singularities** in loop integrals (light-cone diagrams) and **proper disentanglement**
  - ✓ **Combined virtual-real terms are integrable in 4D!!**
  - ✓ **Asymptotic expansions made easy!!!**
  - ✓ **Partial 2-loop results (Higgs/heavy quarks)**

- **Perspectives:**
  - Automation of multileg processes at NLO (ongoing) and beyond (…)
  - *Carefull comparison with other schemes*

"Workstop-Thinkstart meeting"
UZH, Zurich, Sep. 2016
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Thanks!!!!
mytaxi