Electroweak precision measurements: theory overview

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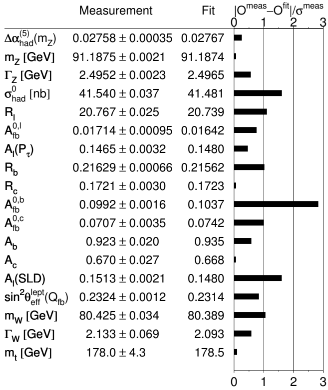
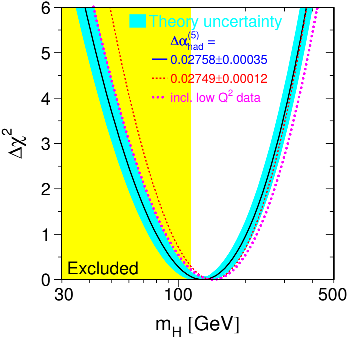
LHC EWWG meeting
13-14 December 2017, CERN

LEP/SLC legacy at the $\mathbf{Z}$ pole





LEP EWWG, SLD WG, ALEPH, DELPHI, L3, OPAL, Phys. Rept. 427 (2006) 257

- given precisely measured parameters, e.g. α, G_μ, M_Z, m_f, $(\Delta\alpha_h)$, $\alpha_s(M_Z)$ and, after LHC run I, m_H, all other quantities can be computed with high precision through perturbative calculations
- in particular, two **SM parameters**: M_W and $\sin^2\vartheta^l_{eff}$

M_W calculated in the Standard Model

$$
\begin{aligned}
M_W^2 &= \frac{M_Z^2}{2}\left\{1+\left[1-\frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}\,(1+\Delta r)\right]^{1/2}\right\} \\
M_W^2 &= 80.357 \pm 0.009 \pm 0.003\mathrm{GeV}
\end{aligned}
$$

- one loop $\mathcal{O}(\alpha)$ calculation

A. Sirlin, PRD22 (1980) 971

- two loop $\mathcal{O}(\alpha\alpha_s)$

A. Djouadi, C. Verzegnassi, PLB195 (1987) 265

- three loop $\mathcal{O}(\alpha\alpha_s^2)$

L. Avdeev et al., PLB336 (1994) 560;

K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394

- $\mathcal{O}(\alpha^2)$ for large top / Higgs mass

R. Barbieri et al., PLB288 (1992) 95; NPB409 (1993) 105

G. Degrassi, P. Gambino, A. Vicini, PLB383 (1996) 219

- exact $\mathcal{O}(\alpha^2)$

A. Freitas et al., PLB495 (2000) 338; NPB632 (2002) 189
M. Awramik, M. Czakon, PLB568 (2003) 48; PRL89 (2002) 241801

A. Onishchenko, O. Veretin, PLB551 (2003) 111; M. Awramik et al., PRD68 (2003) 053004

- $\sin^2\vartheta^l_{eff}$ defined at the Z peak from $\bar{\ell}\gamma^\mu(g_v - g_a\gamma_5)\ell Z_\mu$

$$\sin^2\vartheta^l_{eff} = \frac{1}{4|Q_l|}\left[1 - \mathrm{Re}\left(\frac{g_v}{g_a}\right)\right]$$

- it is calculated within the Standard Model: 0.23147 ± 0.00017
- with the input parameters: α, G_μ, M_Z, $m_{\rm top}$, $m_{\rm Higgs}$ $\alpha_s(M_Z)$
  - at one loop $\mathcal{O}(\alpha)$

    A. Sirlin, PRD22, (1980) 971, W.J. Marciano, A. Sirlin, PRD22 (1980) 2695

    G. Degrassi, A. Sirlin, NPB352 (1991) 352, P. Gambino and A. Sirlin, PRD49 (1994) 1160
  - at higher orders:
    - $\mathcal{O}(\alpha\alpha_s)$

      A. Djouadi, C. Verzegnassi, PLB195 (1987) 265
      B. Kiehl, NPB353 (1991) 567; B. Kniehl, A. Sirlin, NPB371 (1992) 141, PRD47 (1993) 883

      A. Djouadi, P. Gambino, PRD49 (1994) 3499
    - $\mathcal{O}(\alpha\alpha_s^2)$

      L. Avdeev et al., PLB336 (1994) 560;

      K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394; NPB482 (1996) 213
    - $\mathcal{O}(\alpha\alpha_s^3)$

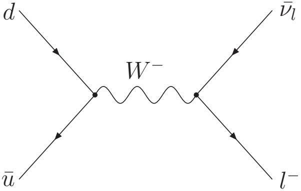
      Y. Schröder, M. Steinhauser, PLB622 (2005) 124;

      K.G. Chetyrkin et al., hep=ph/0605201; R. Boughezal, M. Czakon, hep-ph/0606232
    - $\mathcal{O}(\alpha^2)$ for large Higgs / top mass

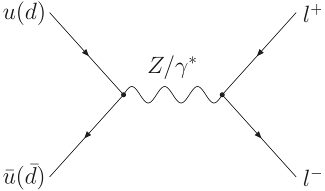
      G. Degrassi, P. Gambino, A. Sirlin, PLB394 (1997) 188
    - exact $\mathcal{O}(\alpha^2)$

      M. Awramik, M. Czakon, A. Freitas, JHEP0611 (2006) 048

- at hadronic colliders we have the opportunity to perform direct measurements of both M_W and $\sin^2\vartheta^l_{eff}$ through **Drell-Yan** processes
- CC

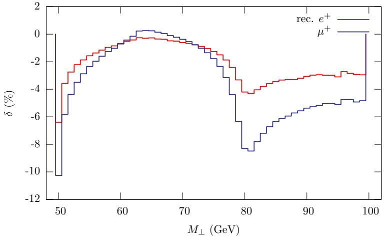
  NC

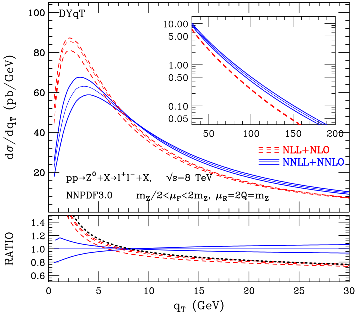
- large cross sections
- experimentally clean signals
- **the experimental precision challenges theoretical predictions**

M_W measurement: main relevant observables

- M_T^W mainly sensitive to QED FSR
- $p_\perp^W$, $p_\perp^Z$ (and their interplay), mainly sensitive to ISR QCD



C.M. Carloni Calame et al., JHEP 12 (2006) 016



S. Catani et al., JHEP 1512 (2015) 047,

- issues relevant to M_W discussed during past meetings

Higher-order corrections (for M_W fit)

$$
\begin{aligned}
d\sigma &= d\sigma_0 \\
&+ d\sigma_{\alpha_s} + d\sigma_{\alpha} \\
&+ d\sigma_{\alpha_s^2} + d\sigma_{\alpha\alpha_s} + d\sigma_{\alpha^2} + \ldots
\end{aligned}
$$

- multi-photon emission from the final state $\rightarrow \delta M_W \simeq 10$ MeV for $\mu\nu_\mu$ final state

Carloni Calame et al., PRD 69 (2004) 037301, JHEP 0710 (2007)

- mixed QCD-EWK corrections

Dittmaier, Huss, Schwinn, NPB 885 (2014) 318, NPB 904 (2016) 216

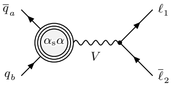
- NNLO EWK effects

C.M. Carloni Calame et al., Phys.Rev. D96 (2017) 093005

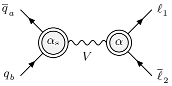
  - EWK input scheme
  - lepton pair emission

fixed order $\mathcal{O}(\alpha_s\alpha)$ in pole approximation

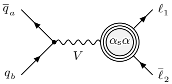
- two main classes of contributions:
  - factorizable
  - non-factorizable



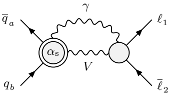
(a) Factorizable initial–initial corrections



(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections



(d) Non-factorizable corrections

S. Dittmaier, A. Huss and C. Schwinn, arXiv:1601.02027

a) not known but expected to be very small

($\mathcal{O}(\alpha)$ corrections in PA $\Longrightarrow$ $M_\perp$ and $M(l^+l^-)$ insensitive to QED ISR
in addition $M_\perp$ and $M(l^+l^-)$ mildly affected by NLO QCD corrections)

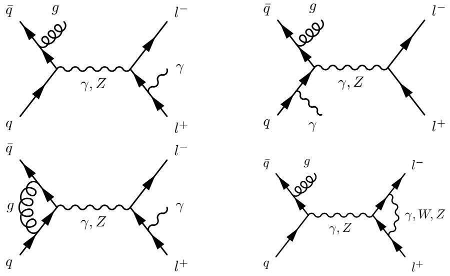
b) this gives the bulk of the contribution

c) no real contributions $\Longrightarrow$ no impact on shape of $M_\perp$ and $M(l^+l^-)$

d) numerical impact below 0.1%

$\mathcal{O}(\alpha_s\alpha)$ corrections through Monte Carlo

- The POWHEG-BOX includes NLO QCD & EW corrections interfaced to QCD/QED shower, i.e. NLOPS EW $\oplus$ QCD accuracy
  1. POWHEG_W_ew_BMNNP, CC DY
     Barzè et al, JHEP 1204 (2012) 037
  2. POWHEG_W_ew_BW, CC DY
     Bernaciak and Wackeroth, PRD 85 (2012) 093003
  3. POWHEG_Z_ew_BMNNPV, NC DY
     Barzè et al, EPJC 73 (2013) 6, 2474
  4. independent implementation
     Mück and Oymanns, JHEP 1705 (2017) 090
- correctly taken into account the NLO contribution with one additional radiation in the soft/collinear limit



- comparison POWHEG-BOX-V2 vs NNLO in pole approx

C.M. Carloni Calame et al., Phys.Rev. D96 (2017) 093005

$$d\sigma_{\mathrm{POWHEG}} = d\sigma_0 \left[1 + \delta_{\alpha_s} + \delta_\alpha + \sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right],$$

$$\Delta M_W{}^{\alpha_s\alpha}(\mu^+\nu_\mu) = -16.0 \pm 3.0 \text{ MeV} \qquad \text{vs} \qquad \delta_{\mathrm{NNLO}} = -14 \text{ MeV}$$

Dittmaier, Huss, Schwinn, NPB 885 (2014) 318, NPB 904 (2016) 216

- summary of residual effects present in $(\mathrm{QCD}\oplus\mathrm{EW})_{\mathrm{NLOPS}}$ but missing in $\mathrm{QCD}_{\mathrm{NLOPS}} \otimes$ QEDPS

		ΔM_W (MeV)	
	QED FSR model	M_T	p_T^ℓ
Tevatron	PYTHIA	+5 ± 2	+17 ± 5
	PHOTOS	-2 ± 1	-8 ± 5
LHC	PYTHIA	+6.2 ± 0.8	+29 ± 4
	PHOTOS	-0.6 ± 0.8	-2 ± 4

- differences in shifts induced by PYTHIA QEDPS and PHOTOS disappear when used on top of QCD⊕EW NLO

NNLO effect: lepton pair corrections

C.M. Carloni Calame et al., Phys.Rev. D96 (2017) 093005

- emission of a photon converting to a lepton pair
$\sim \mathcal{O}(\alpha^2 L^2) \sim$ two-photon contribution

- contribution implemented in HORACE v3.1 (now also available in POWHEG_ew, with PYTHIA8 QEDPS) through QED running of $\alpha(Q^2)$ included in the Sudakov form factor

alternative implementation: N. Davidson, T. Przedzinski and Z. Was, CPC 199 (2016) 86, arXiv:1011.0937

- $\Delta M_W(\mu^+\nu) \sim 5 \pm 1$ MeV (from $M_\perp$) and $\sim 3 \pm 2$ MeV (from $p_\perp^\ell$)

NNLO uncertainty: input parameter scheme

- pert. EW calculations require a coherent set of input param. in the gauge sector, e.g.
  - $\alpha(0)$, M_W and M_Z
  - G_μ, M_W and M_Z to be preferred in the CC DY
  - we can define

$$\begin{aligned} \alpha_\mu^{tree} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2\vartheta \\ \alpha_\mu^{1l} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2\vartheta \,(1-\Delta r) \end{aligned}$$

  - three possible different expression for the cross section, starting to differ at $\mathcal{O}(\alpha^2)$

$$\begin{aligned} \alpha_0 \;:& \qquad \sigma = \alpha_0^2\sigma_0 + \alpha_0^3(\sigma_{SV}+\sigma_H)\,, \\ G_\mu\ I \;:& \qquad \sigma = (\alpha_\mu^{tree})^2\sigma_0 + (\alpha_\mu^{tree})^2\alpha_0(\sigma_{SV}+\sigma_H) - 2\Delta r(\alpha_\mu^{tree})^2\sigma_0\,, \\ G_\mu\ II \;:& \qquad \sigma = (\alpha_\mu^{1l})^2\sigma_0 + (\alpha_\mu^{1l})^2\alpha_0(\sigma_{SV}+\sigma_H) \end{aligned}$$

- differences present at NLO, after matching with higher orders, become much smaller

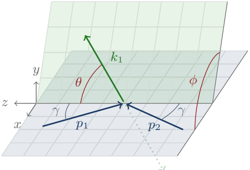
$$\Delta M_W \sim 2\ \mathrm{MeV} \pm 1 - 2\ \mathrm{MeV}$$

$pp \to \ell\bar{\ell}$ as a calibration tool

- the matrix element for the production and decay of a spin one vector boson can be parameterized by an expansion on second order polinomials with nine coefficients (corresponding to nine polarization terms)

$$
\begin{aligned}
\frac{d\sigma}{dq_T^2\, dy\ \, d\cos\vartheta\ \, d\phi} \quad &= \quad \frac{3}{16\pi}\,\frac{d\sigma^{\text{unpol.}}}{dq_T^2\ \, dy\ \, d\cos\vartheta\ \, d\phi}\Big\{1+\cos^2\vartheta \\
&+ \quad \frac{1}{2}A_0(1-3\cos^2\vartheta) + A_1\sin 2\vartheta\cos\phi \\
&+ \quad +\frac{1}{2}A_2\sin^2\vartheta\cos 2\phi + A_3\sin\vartheta\cos\phi \\
&+ \quad A_4\cos\vartheta + A_5\sin^2\vartheta\sin 2\phi \\
&+ \quad A_6\sin 2\vartheta\sin\phi + A_7\sin\vartheta\sin\phi\Big\}
\end{aligned}
$$

- ϑ and ϕ refer to a $\ell\bar{\ell}$ rest frame
- orientation given by the Collin-Soper frame



angular coefficients

- **can be measured**

at low energies: NA10, E615, NuSea

at Tevatron: arXiv 1103.5699; at LHC for Z: arXiv:1606.00689, arXiv:1505.03512

at LHC for W: arXiv:1203.2165, arXiv:1104.3829

- **can be predicted theoretically**

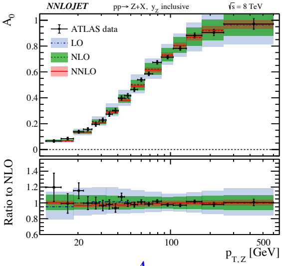
$$\langle f(\vartheta,\phi)\rangle = \frac{\int_1^1 d\cos\vartheta \;\; \int_0^{2\pi} d\phi \;\, d\sigma(\vartheta,\phi) f(\vartheta,\phi)}{\int_1^1 d\cos\vartheta \;\; \int_0^{2\pi} d\phi \;\, d\sigma(\vartheta,\phi)}$$

$$A_0 = 4 - 10\langle\cos^2\vartheta\rangle \quad A_1 = 5\langle\sin^2\vartheta\cos\phi\rangle \quad A_2 = 10\langle\sin^2\vartheta\cos 2\phi\rangle$$
$$A_3 = 4\langle\sin\vartheta\cos\phi\rangle \quad A_4 = 4\langle\cos\vartheta\rangle \quad A_5 = 5\langle\sin^2\vartheta\sin 2\phi\rangle$$
$$A_6 = 5\langle\sin 2\vartheta\sin\phi\rangle \quad A_7 = 4\langle\sin\vartheta\sin\phi\rangle$$

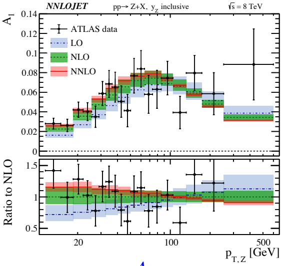
- QCD NNLO predictions with NNLOJET

$\Longrightarrow$ next slide

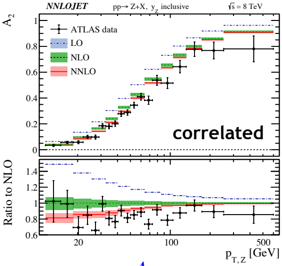
A. Gehrmann-De Ridder et al., arXiv:1708.00008



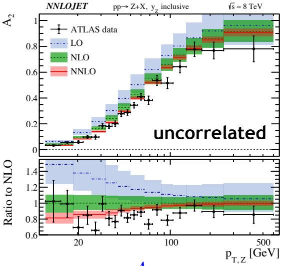
A_0



A_1



A_2



A_2

- correlated and uncorrelated refer to μ_F and μ_R variations on numerator and denominator
- at NNLO the two options give very similar results

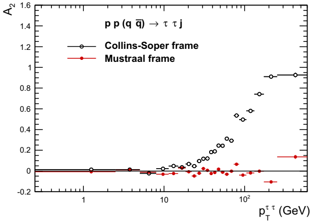
Mustraal frame vs. CS frame

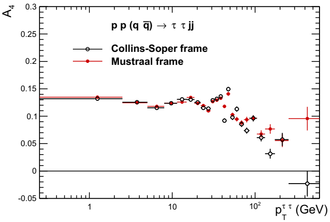
- an alternative lepton pair rest-frame introduced for e^+e^- collisions

  Berends, Kleiss, Jadach, CPC 29 (1983) 185

  with the aim of minimizing the effect of ISR radiation
- recently the frame definition has been extended to h-h collisions

  Eur.Phys.J. C76 (2016) 473
- checked on $\tau^+\tau^- j$ and $\tau^+\tau^- jj$: w.r.t CS frame, A_i remain very small over a large range of transverse momentum, except for A_4, where they are equivalent $\Longrightarrow$ it could be useful to disentangle EW effects from pure QCD ones



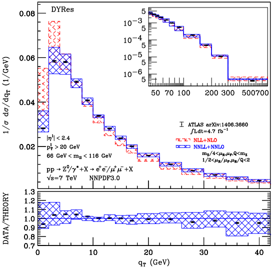


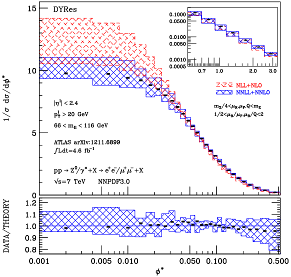
lepton pair $p_\perp$: two regimes

already covered by Marek Schönherr yesterday

- large $p_\perp$ ($\gtrsim 20$ GeV), where pert. th. is reliable
  state of the art is NNLO QCD
- small $p_\perp$ ($\lesssim 20$ GeV): ∼90% of the cross section!
  - resummation of $\log\left(\frac{M_V}{q_\perp}\right)$ is needed
  - also sensitivity to the non-perturbative model of the MC Evt Gen
  - **experimental precision higher than the theoretical one**
  - precision in the calculations reached NNLL+NNLO and N3LL+NNLO

S. Catani et al., T. Becher et al.

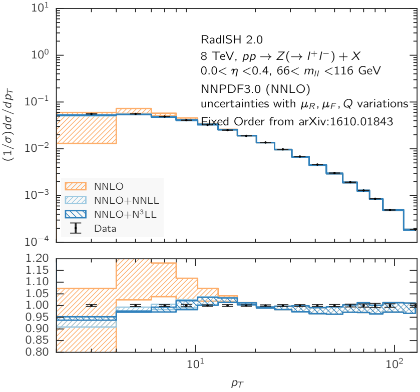


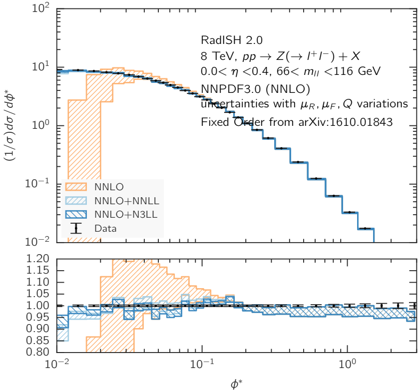


from talk by G. Ferrera at Orsay meeting

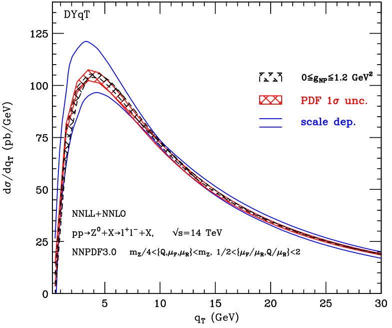
Work in progress towards N3LL+NNLO in direct space

W. Bizon, P. Monni, E. Re, L. Rottoli and P. Torrielli, arXiv:1705.09127, arXiv:1604.02191, work in progress





PDF uncertainties and NP effects



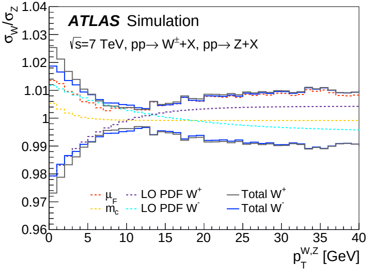
NNLL+NNLO result for Z q_T spectrum at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\, GeV^2$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \rightarrow \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T.
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3GeV$ (i.e. below the peak).

Extrapolating from Z to W



Focus on low $p_T^W \lesssim 30\,\mathrm{GeV}$ relevant for m_W

- $\simeq 2\%$ uncertainties in p_T^W translate into $\simeq 10\,\mathrm{MeV}$ uncertainty in m_W

$\Rightarrow$ Use precise Z measurement to get best possible prediction for W

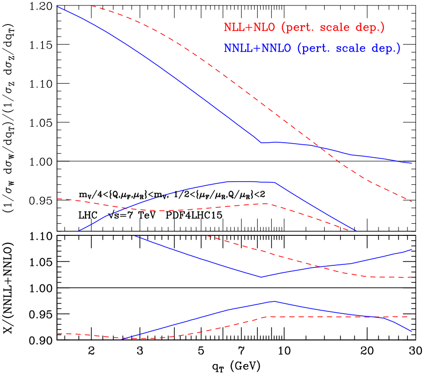
- One way to think about it

$$\frac{d\sigma(W)}{dp_T} = \left[\frac{d\sigma(W)/dp_T}{d\sigma(Z)/dp_T}\right]_{\text{theory}} \times \left[\frac{d\sigma(Z)}{dp_T}\right]_{\text{measured}}$$

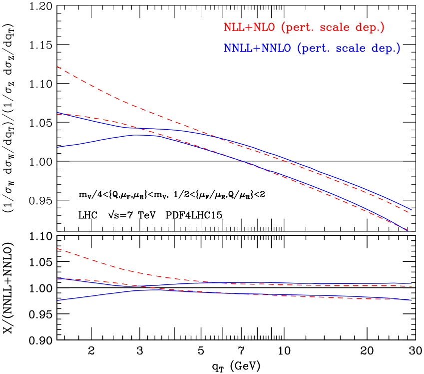
  - There is no direct resummation for ratio, it is always a derived quantity
  - Relies on ratio being more precise than individual processes, which relies on theory uncertainties being strongly correlated between processes

- More general: Use common theory framework and fit to Z data
  - Not restricted to a specific combination (like ratio)
  - Tuning Pythia on Z data is one example of this
  - Requires explicit information on correlations between processes

W/Z ratio q_T spectrum: perturbative scale uncertainty

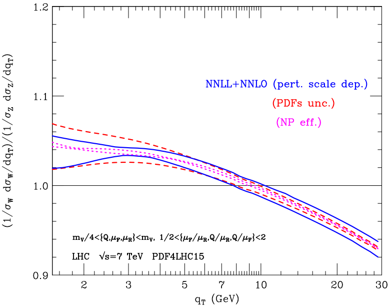


DYqT resummed predictions for the ratio of W/Z normalized q_T spectra. Uncorrelated perturbative scale variation band.



DYqT resummed predictions for the ratio of W/Z normalized q_T spectra. Correlated perturbative scale variation band.

W/Z ratio: the q_T spectrum



Ratio of NNLL+NNLO results for W/Z q_T spectra at the LHC. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- Ratio of W/Z observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- *Correlated* ($\mu^W/M_W = \mu^Z/M_Z$) scale variations by factor 2 (avoiding ratios larger than 2) gives reasonable estimate of pert. uncertainty (nice overlap of scale variation bands for $q_T > 3\ GeV$).
- PDF uncertainty dominates at very small (q_T $q_T \lesssim 5\ GeV$).
- Non trivial interplay of perturbative and NP effects.

work in progress on $b\bar{b}$ contribution estimation (present in Z but not in W)

E. Bagnaschi, F. Maltoni, A. Vicini and M. Zaro

- gauge boson production in association with heavy quarks has a non-trivial phenomenology for both exclusive and inclusive signatures

- a detailed discussion of the QCD effects and uncertainties is crucial:
  → matching NLO-QCD with QCD-PS has a sizeable impact on the distributions
  → matching and Parton Shower uncertainties are often under control but not negligible

- a combination of 5FS and 4FS results has been attempted to improve the description of the bottom quark contributions to the ptZ distribution with respect to the plain 5FS approach, with a shape distortion at the O(1%) level

- the information transfer from NC-DY to CC-DY has been estimated assuming that two perfect Parton Shower tunes are possible
  a realistic estimate is more cumbersome and will yield also an indicator of quality of the data description (e.g. a χ^2 value)
  → the present study offers only a qualitative statement about the MW sensitivity to b-quark effects
  (the shifts do not exceed the 3-5 MeV level in size and in general are smaller)

direct measurement of $\sin^2 \vartheta_{eff}^{(l)}$

- from the general parameterization

$$
\begin{aligned}
\frac{d\sigma}{dq_T^2\, dy\, d\cos\vartheta\, d\phi} &= \frac{3}{16\pi} \frac{d\sigma^{\text{unpol.}}}{dq_T^2\, dy\, d\cos\vartheta\, d\phi} \Big\{ 1 + \cos^2\vartheta \\
&+ \frac{1}{2} A_0 (1 - 3\cos^2\vartheta) + A_1 \sin 2\vartheta \cos\phi \\
&+ + \frac{1}{2} A_2 \sin^2\vartheta \cos 2\phi + A_3 \sin\vartheta \cos\phi \\
&+ A_4 \cos\vartheta + A_5 \sin^2\vartheta \sin 2\phi \\
&+ A_6 \sin 2\vartheta \sin\phi + A_7 \sin\vartheta \sin\phi \Big\}
\end{aligned}
$$

- integrating over the azimuthal angle ϕ

$$
\begin{aligned}
\frac{d\sigma}{dq_T^2\, dy\, d\cos\vartheta\, d\phi} &= \frac{3}{8} \frac{d\sigma^{\text{unpol.}}}{dq_T^2\, dy\, d\cos\vartheta\, d\phi} \Big\{ 1 + \cos^2\vartheta + \frac{1}{2} A_0 (1 - 3\cos^2\vartheta) + A_4 \cos\vartheta \Big\} \\
&\Downarrow \\
A_{FB}(M,y) &= \frac{\sigma^+(M,y) - \sigma^-(M,y)}{\sigma^+(M,y) + \sigma^-(M,y)} = \frac{3}{8} A_4(M,y)
\end{aligned}
$$

- $A_4 \sim$ couplings of the vector and axial-vector currents $\Longrightarrow \sin^2\theta$
- A_{FB} is the natural observable for direct $\sin^2\theta$ measurement
- which $\sin^2\theta$?
  - at EW LO $\sin^2\theta = \sin^2\theta_{OS} = 1 - \frac{M_W^2}{M_Z^2} = \sin^2\theta_{eff}$

$\sin^2\vartheta_{eff}$ beyond EW LO

- becomes flavour dependent
  - $\sin^2\vartheta^u_{eff} = K_u \sin^2\vartheta_{OS}$
  - $\sin^2\vartheta^d_{eff} = K_d \sin^2\vartheta_{OS}$
  - $\sin^2\vartheta^l_{eff} = K_l \sin^2\vartheta_{OS}$
- the EW NLO expression of A_{FB} contains all $\sin^2\vartheta^i_{eff}$

  $$\Downarrow$$

  we can not scan directly $\sin^2\vartheta^l_{eff}$ with a template fit
- moreover, we can not scan $\sin^2\vartheta^l_{eff}$ leaving fixed the other parameters of the calculation (G_μ, M_W, M_Z e.g. in the G_μ scheme), in the same way as $\sin^2\vartheta_{OS}$ is not fixed when we do a M_W mass fit
- with a EW NLO Monte Carlo working with the G_μ scheme, like e.g. POWHEG_ew, we can fit M_W from A_{FB} and then calculate $\sin^2\vartheta^l_{eff}$

proposal (for POWHEG_ew)

- precise fitting formulae have been derived both for M_W and $\sin^2 \vartheta^l_{eff}$ as functions of the input parameters:
  - G_μ, $\alpha(0)$, M_Z
  - M_H, m_t, $\Delta\alpha(M_Z)$, $\alpha_s(M_Z)$

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- the coefficients have been tuned on available two-loops complete calculations for M_W and $\sin^2 \vartheta^l_{eff}$

$$
\begin{aligned}
M_W &= M_W^0 - c_1 \left(\log \frac{M_H}{100\,\mathrm{GeV}}\right) - c_2 \left(\log \frac{M_H}{100\,\mathrm{GeV}}\right)^2 \\
&+ c_3 \left(\log \frac{M_H}{100\,\mathrm{GeV}}\right)^4 - c_4 \left(\frac{\Delta\alpha}{0.05924} - 1\right) \\
&+ c_5 \left[\left(\frac{m_t}{174.3\,\mathrm{GeV}}\right)^2 - 1\right] - c_6 \left[\left(\frac{m_t}{174.3\,\mathrm{GeV}}\right)^2 - 1\right]^2 \\
&- c_7 \left(\log \frac{M_H}{100\,\mathrm{GeV}}\right) \left[\left(\frac{m_t}{174.3\,\mathrm{GeV}}\right)^2 - 1\right] \\
&- c_8 \left(\frac{\alpha_s(M_Z)}{0.119} - 1\right) + c_9 \left(\frac{M_Z}{91.1875\,\mathrm{GeV}} - 1\right)
\end{aligned}
$$

$$
\begin{aligned}
M_W^0 &= 80.3768\ \mathrm{GeV} \qquad c_1 = 0.05619\ \mathrm{GeV} \qquad c_2 = 0.009305\ \mathrm{GeV} \\
c_3 &= 0.0005365\ \mathrm{GeV} \qquad c_4 = 1.078\ \mathrm{GeV} \qquad c_5 = 0.5236\ \mathrm{GeV} \\
c_6 &= 0.0727\ \mathrm{GeV} \qquad c_7 = 0.00544\ \mathrm{GeV} \qquad c_8 = 0.0765\ \mathrm{GeV} \quad c_9 = 115.0\ \mathrm{GeV}
\end{aligned}
$$

- the above eq. reproduces (the calculated) M_W within 0.3 MeV when 65 GeV $\leq M_H \leq 1$ TeV and the other parameters move within their 2σ errors: $\Delta_{1\sigma} m_t = 5.1$ GeV; $\Delta_{1\sigma}\alpha = 36 \cdot 10^{-5}$

$$
\begin{aligned}
\sin^2\vartheta^l_{eff} \quad = \quad & \sin^2\vartheta^l_0 + d_1\left(\log\frac{M_H}{100\,\mathrm{GeV}}\right) + d_2\left(\log\frac{M_H}{100\,\mathrm{GeV}}\right)^2 \\
+ \quad & d_3\left(\log\frac{M_H}{100\,\mathrm{GeV}}\right)^4 + d_4\left[\left(\frac{M_H}{100\,\mathrm{GeV}}\right)^2 - 1\right] + d_5\left(\frac{\Delta\alpha}{0.05907} - 1\right) \\
+ \quad & d_6\left[\left(\frac{m_t}{178.0\,\mathrm{GeV}}\right)^2 - 1\right] + d_7\left[\left(\frac{m_t}{178.0\,\mathrm{GeV}}\right)^2 - 1\right]^2 \\
+ \quad & d_8\left[\left(\frac{m_t}{178.0\,\mathrm{GeV}}\right)^2 - 1\right]\left[\left(\frac{M_H}{100\,\mathrm{GeV}}\right)^2 - 1\right] \\
+ \quad & d_9\left(\frac{\alpha_s(M_Z)}{0.117} - 1\right) + d_{10}\left(\frac{M_Z}{91.1876\,\mathrm{GeV}} - 1\right)
\end{aligned}
$$

$$
\begin{aligned}
\sin^2\vartheta^l_0 &= 0.2312527, & d_1 &= 4.729\cdot 10^{-4}, & d_2 &= 2.07\cdot 10^{-5} \\
d_3 &= 3.85\cdot 10^{-6}, & d_4 &= -1.85\cdot 10^{-6}, & d_5 &= 2.07\cdot 10^{-2} \\
d_6 &= -2.851\cdot 10^{-3}, & d_7 &= 1.82\cdot 10^{-4}, & d_8 &= -9.74\cdot 10^{-6} \\
d_9 &= 3.98\cdot 10^{-4}, & d_{10} &= -6.55\cdot 10^{-1}
\end{aligned}
$$

the above eq. reproduces (the calculated) $\sin^2\vartheta^l_{eff}$ within $4.5\cdot 10^{-6}$ when 10 GeV $\leq M_H \leq 1$ TeV and the other parameters move within their 2σ errors: $\Delta_{1\sigma} m_t = 5.1$ GeV; $\Delta_{1\sigma}\alpha = 36\cdot 10^{-5}$

- we can solve the second equation for one parameter, e.g. $\Delta\alpha$, and introduce its expression in the first giving

$$M_W = f(\sin^2\vartheta^l_{eff})$$

- given an input value for $\sin^2\vartheta^l_{eff}$, we can convert it to a M_W value and use as input for the POWHEG_ew matrix element calculations in the G_μ scheme (or any other code working with M_W as input parameter) where everywhere $M_W = M_W(\sin^2\vartheta^l_{eff})$

$\sin^2\theta^l_{eff}$	M_W (GeV)
0.2305	80.415
0.2310	80.389
0.2315	80.363
0.2320	80.337

Let's start the discussion session