Second Workshop on
Beyond 3rd Generation
Standard Model

Vector-like Quarks:
What are they?
What can they do for you?

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(See Miguel Nebot's talk)

What they are:

Consider the Standard Model (SM) and add New Quarks with charges either $Q = 2/3$ or $Q = -1/3$ which are $SU(2)_L$ isosinglets.

$$\left.\begin{matrix} U_L\,,\ U_R \\ D_L\,,\ D_R \end{matrix}\right\} \text{ Singlets under } SU(2)_L\,!$$

Vector-Like as above defined, violate the Principle of Natural Flavor Conservation (Glashow and Weinberg) in Z-mediated neutral currents.

No Problem!! Vector-Like quarks provide a Mechanism, for Natural Suppression of FCNC entirely analogous to the Seesaw mechanism for ν masses!

What vector-like quarks can do for you:

- Introduce "naturally small" violations of 3x3 unitarity of V^{CKM}
- Imply the appearance of tree-level Z-mediated FCNC, with couplings "naturally suppressed"
- Provide a simple solution to the "Strong CP problem"
- Imply "New Physics" which may be "visible" at the next round of experiments at

  Tevatron
  LHCb, Atlas, CMS
  Super-B factory
  etc...

- Arise in a variety of extensions of the SM

The case of a $Q = -1/3$ vector-like quark:

Charged currents interactions:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \; \overline{u}_L \gamma^\mu V d_L W_\mu^{+} + \text{h.c.}$$

$u = (u, c, t)$; $d = (d, s, b, D)$

$$g/\sqrt{2} \; (\overline{u} \; \overline{c} \; \overline{t}) \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uD} \\ V_{cd} & V_{cs} & V_{cb} & V_{cD} \\ V_{td} & V_{ts} & V_{tb} & V_{tD} \end{bmatrix} \gamma^\mu \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix}$$

V is a 3×4 matrix, consisting of the first three lines of a 4×4 unitary matrix. Without loss of generality, one can go to the weak-basis where m_u is diagonal, real. Then:

$$\begin{bmatrix} d^0 \\ s^0 \\ b^0 \\ D^0 \end{bmatrix} \begin{bmatrix} & & V & \\ \hdashline U_{41} & U_{42} & U_{43} & U_{44} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix}$$

Example :

Non-orthogonality of first two columns of V:

$$V_{ud} V_{us}^{*} + V_{cd} V_{cs}^{*} + V_{td} V_{ts}^{*} = - u_{41} u_{42}^{*}$$
$$= Z_{ds}$$

$$u_{41} \sim m/M$$
$$u_{42} \sim m/M$$

m → standard quark mass

M → vector-like quark mass

$$Z_{ds} \approx m^2/M^2$$

Important point : "Deviations of unitarity" and FCNC are related and they are both naturally suppressed by a mechanism entirely analogous to the seesaw mechanism.

No fine-tuning is required!!

The case of a $Q = 2/3$ quark

$$\mathcal{L}_W = -\, g/\sqrt{2}\; \overline{u}_L \gamma^\mu V d_L W_\mu^+ + \text{h.c.}$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W}\left[\overline{u}_L \gamma^\mu V V^\dagger u_L - \overline{d}_L \gamma^\mu d_L - 2\sin^2\theta_W J^\mu_{em}\right] Z_\mu$$

where $\quad u \equiv (u, c, t, T)\,,\; d \equiv (d, s, b)$

V is a 4×3 submatrix of the 4×4 unitary matrix which enters the diagonalization of the up-type quark mass matrix:

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{bmatrix}$$

$V V^\dagger \neq 1 \longrightarrow$ FCNC in the up sector.

Consider a quark mixing matrix of arbitrary size :

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \cdots \\ V_{td} & V_{ts} & V_{tb} & \cdots \\ \vdots & & & \ddots \\ & & & \end{bmatrix}$$

In the 3×3 sector of the CKM matrix there are, obviously, 9 phases. Of these, 5 phases can be removed by rephasing of quark fields.

N° independent physical phases : $9-5=\boxed{4}$

It is convenient to express these phases in terms of 4 rephasing invariant phases.

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^{*} V_{cd}\right)$$

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^{*} V_{td}^{*}\right)$$

$$\beta_s \leftarrow \chi \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^{*} V_{tb}^{*}\right)$$

$$\beta_K \leftarrow \chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^{*} V_{cs}^{*}\right)$$

A convenient phase convention:

G.C.B., L. Lavoura, J. Silva

$$\arg V^{CKM} = \begin{bmatrix} 0 & \chi' & -\gamma & \cdots \\ \pi & 0 & 0 & \cdots \\ -\beta & \pi+\chi & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \vdots & \vdots & & \end{bmatrix}$$

If 3×3 unitarity is assumed

One can reconstruct the full CKM matrix from the knowledge of the 4 rephasing invariant phases β, γ, χ, χ'.

R. Aleksan, B. Kayser, D. London

One can also reconstruct the full CKM matrix from 4 independent moduli, with a two-fold ambiguity, reflecting the fact that one can evaluate, from the moduli, the strength of CP violation, but not the sign.

One has 13 rephasing invariant quantities in the 3×3 sector of a CKM matrix of arbitrary size:

9 moduli + 4 rephasing inv. phases.

The SM with 3 generations predicts **exact relations** among these quantities:

Examples:

(db)
$$\boxed{\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma}\,\frac{|V_{tb}|}{|V_{ud}|}}$$

$\chi \equiv \beta_s$

(ct)
$$\boxed{\sin\chi = \frac{|V_{td}|}{|V_{ts}|}\,\frac{|V_{cd}|}{|V_{cs}|}\,\sin\beta}$$

(uc)
$$\boxed{\sin\chi' = \frac{|V_{ub}||V_{cb}|}{|V_{us}||V_{cs}|}\,\sin\gamma}$$

(db)
$$\boxed{|V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|}\,\frac{\sin\beta}{\sin(\gamma+\beta)}}$$

Important point:

Prior to the measurement of γ there was no solid evidence for the fact that V^{CKM} is complex, independently of the presence of New Physics.

One can calculate:

$$|\mathrm{Im}\, Q| = F\,(4 \text{ independent moduli})$$

where Q is any invariant quartet of V^{CKM}

Can take as independent "moduli":

$$|V_{us}|,\ |V_{cb}|,\ |V_{ub}|,\ |V_{td}|$$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

General formula for Im Q

$$|\mathrm{Im}\, Q|^2 = |V_{\alpha i}|^2 |V_{\beta j}|^2 |V_{\alpha j}|^2 |V_{\beta i}|^2 - \left(\mathrm{Re}\, Q_{\alpha i \beta j}\right)^2$$

$$\mathrm{Re}\, Q_{\alpha i\beta j} = \frac{1}{2}\left\{ 1 - |V_{\alpha i}|^2 - |V_{\beta j}|^2 - |V_{\alpha j}|^2 - |V_{\beta i}|^2 + |V_{\alpha i}|^2 |V_{\beta j}|^2 + |V_{\alpha j}|^2 |V_{\beta i}|^2 \right\}$$

$$Q_{\alpha i\beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}$$

If one uses the present "data" on

$|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$

one obtains indeed

$$\mathrm{Im}\, Q \neq 0$$

However, this conclusion is not valid because New Physics may contribute to $B_d - \overline{B}_d$ mixing, thus affecting the extraction of $|V_{td}|$ from the experimental value of $B_d - \overline{B}_d$ mixing

The measurement of

$$\gamma \equiv \arg\left(-V_{ud} V_{ub}^{*} V_{cd}^{*} V_{cb}\right)$$

provides clear evidence for a complex CKM matrix, even if one allows for New Physics contributions to

B_d-$\bar{B}_d$, B_s-$\bar{B}_s$ mixings

EPS prize to Kobayashi and Maskawa!!

- There is room for New Physics, say at 20% level in B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixings

- Contributions to γ could be large !!

Excellent probe of New Physics

In most of tests of the Flavour sector of the SM 3×3 unitarity of V^{CKM} plays a crucial rôle.

Can one have a self-consistent extension of the SM, where there are naturally small violations of 3×3 unitarity? Yes!

Consider an extension of the SM, with the addition of a $Q = 2/3$ isosinglet quark.

$$\left.\begin{matrix} T_L \\ T_R \end{matrix}\right\} \to \text{both isosinglets under } SU(2)_L$$

In the mass eigenstate basis, the charged and neutral currents can be written:

$$\mathcal{L}_W = -\, g/\sqrt{2}\; \bar{u}_L \gamma^\mu V d_L W^+_\mu + h.c.$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W}\left[\bar{u}_L \gamma^\mu (VV^\dagger) u_L - \bar{d}_L \gamma^\mu d_L - \right.$$
$$\left. - 2\sin^2\theta_W J^\mu_{em}\right] Z_\mu$$

Let $\mathcal{U}$ be the 4×4 unitary matrix $\mathcal{U}$ which enters in the diagonalization of the up-type mass matrix (in the weak-basis where down quark mass matrix is diagonal, real)

$$\mathcal{U} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \mathcal{U}_{14} \\ V_{cd} & V_{cs} & V_{cb} & \mathcal{U}_{24} \\ V_{td} & V_{ts} & V_{tb} & \mathcal{U}_{34} \\ V_{Td} & V_{Ts} & V_{Tb} & \mathcal{U}_{44} \end{bmatrix} \qquad VV^\dagger \neq 1$$

$V \rightarrow 4\times3$ matrix entering in charged currents

There are naturally small violations of 3X3 unitarity leading to small FCNC couplings in the up quark sector.

$$\bar{u}_L \gamma^\mu (V V^\dagger) u_L =$$

$$\left(\bar{u}\ \bar{c}\ \bar{t}\ \bar{T}\right)_L \begin{bmatrix} 1-|u_{14}|^2 & -u_{14}u_{24}^* & -u_{14}u_{34}^* & -u_{14}u_{44}^* \\ & 1-|u_{24}|^2 & -u_{24}u_{34}^* & -u_{24}u_{44}^* \\ & & 1-|u_{34}|^2 & -u_{34}u_{44}^* \\ & & & 1-|u_{44}|^2 \end{bmatrix} \cdot \gamma_\mu \begin{bmatrix} u \\ c \\ t \\ T \end{bmatrix}_L$$

In this extension of the SM, one may have:

$$\sin\chi = O(\lambda)$$

From orthogonality of the second and third columns of V', one obtains

$$\sin\chi = \frac{|V_{ub}||V_{us}|}{|V_{cb}||V_{cs}|}\sin(\gamma-\chi+\chi') + \underbrace{\frac{|V_{Tb}||V_{Ts}|}{|V_{cb}||V_{cs}|}\sin(\sigma-\chi)}_{NP}$$

In order to obtain χ of order λ one needs:

$$V_{Tb} = O(\lambda)$$
$$V_{Ts} = O(\lambda^2)$$
$$\sigma \equiv \arg\left(V_{Ts}V_{cb}V_{Tb}^*V_{cs}^*\right) = O(1)$$

From orthogonality of second and third rows:

$$\sin\chi = \frac{|V_{cd}||V_{td}|}{|V_{cs}||V_{ts}|}\sin\beta + \frac{|U_{24}||U_{34}|}{|V_{cs}||V_{ts}|}\sin\delta$$

$$\delta = \arg\left(V_{tb}^* U_{24}^* V_{cb} U_{34}\right)$$

χ of order λ requires

$$|U_{24}||U_{34}| = O(\lambda^3)$$
$$\sin\delta = O(1)$$

Clear-cut prediction:

FCNC couplings of the type:

$$\bar{c}_L \gamma^\mu t_L Z^\mu$$

are proportional to

$$|U_{24} U_{34}|$$

On the other hand, in order to have

$$x = O(\lambda)$$

one requires, in the context of the model

$$|U_{24} U_{34}| \approx \lambda^3$$

$$\Downarrow$$

this leads to rare top decays $t \to c Z$ at rates such that they can be observed at LHC

- J. A. Aguilar-Saavedra, J. F. Botella, M. Nebot, G.C.B Nucl. Phys. B706, 204 (2005)
- F. J. Botella, M. Nebot, GCB arXiv 0805.3995 (2008)

See M. Neubert

Recent exciting result !!

Observation of a large phase in B_s mixing amplitude at CDF and DØ collaborations, recently analysed by the UTfit collaboration

Model independent parametrization of New Physics contributions to B_s mixing:

$$\frac{\langle B_s | H_{eff}^{full} | \bar{B}_s \rangle}{\langle B_s | H_{eff}^{SM} | \bar{B}_s \rangle} = \frac{A_s^{SM} e^{-i(2\beta_s)} + A_s^{NP} e^{2i(\phi_s^{NP} - \beta_s)}}{A_s^{SM} e^{-2i\beta_s}}$$

$$\equiv C_{B_s} e^{2i\phi_s}$$

$$\phi_s(\text{rad.}) = (.60 \pm .27)\,\text{rad}$$

In the SM, $\beta_s = 0.018 \pm 0.001$

Some details of the model

$$m_T < 500 \text{ GeV}$$

$$B(t \to cZ) \approx 10^{-4}$$

There are $\Delta C = 2$ FCNC contributing to $D^0 - \bar{D}^0$ mixing

One may account for the observed size of $D^0 - \bar{D}^0$ mixing without having to invoke long distance contributions

The model accommodates all present experimental data on:

ε_K $\varepsilon'/\varepsilon_K$, ΔM_{B_d} ΔM_{B_s} $A_{J/\psi K_s}$,

$K^+ \to \pi^+ \nu \bar{\nu}$ $B \to X_s e^+ e^-$ etc

$B \to X_s \mu^+ \mu^-$